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THE VISCOSITY OF SODIUM

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by

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ABSTRACT

A viscometer for liquids of low viscosity was built to be used at temperatures up to ca. 1000°C. with the liquid maintained under a vacuum or inert gas atmosphere. The viscometer follows the method first suggested by Helmholz (8) and later successfully developed by Chiong and Andrade (4). In this method, the liquid is enclosed in a sphere; and the sphere is set in rotatory oscillation about a vertical axis. From measurements of the damping of the oscillations and the period and moment of inertia of the rotating pendulum, absolute values of the viscosity are calculated using the equations derived by Chiong and Andrade(4).

The completed viscometer, fabricated from type 303 stainless steel, was used to measure the viscosity of sodium over a temperature range of from 328°C. to 877°C. The following results were obtained:

Table I

Values of the viscosity of sodium as a function of temperature

Temperature (°C)	Viscosity (cps.)	Temperature (°C)	Viscosity (cps.)
327	0.332	626	0.192
328	0.334	628	0.209
331	0.322	677	0.199
383	0.277	677	0.180
385	0.281	724	0.186
437	0.263	728	0.184
439	0.254	777	0.166
459	0.241	777	0.180
462	0.240	825	0.150
488	0.244	826	0.154
493	0.244	876	0.164
537	0.235	877	0.165
587	0.225		

These data are plotted in Figure XII, page 57.

The above data were found to fit equally well two different equations for viscosity, one of which was proposed by Andrade (1) and the other by Batschinski (2).

Andrade Equations: Temperature Range

(1) $nv^{1/3} = (.001046 \pm .000032)e^{(814.8 \pm 27.7)/vT}$ 328-877

(2) $nv^{1/3} = (.001073 \pm .000034)e^{(793.5 \pm 8.3)/vT}$ 328-493

Batschinski Equation:

(3) $n = .0006478/v - 0.9371$

In these equations v = specific volume (ml./g.)

T = absolute temperature (°K.)

n = viscosity (poises)

Using Equation (1), the deviation between the observed and calculated values of the viscosity varied between 0.31 and 9.06 per cent, with an average deviation of 3.75 per cent. Equation (2), obtained from the data at the lower end of the temperature range, yields deviations between 0 and 6.1 per cent, with an average deviation of 2.53 per cent.

The Batschinski Equation yields calculated values of the viscosity which vary from the observed values by from 0 to 8.09 per cent, with an average deviation of 4.03 per cent.

INTRODUCTION

The purpose of this investigation was to construct a viscometer to measure the viscosity of molten metals and salts accurately at temperatures up to 1000°C. Since sodium was selected as the first material for study, many of the more common types of viscometers were considered unsuitable.

Most measurements of viscosities at high temperatures have been made by capillary flow methods. The most convenient capillary viscometers are those in which the molten material is forced to flow through a capillary tube into a wider reservoir tube containing electrical contacts at a fixed distance apart (3), (13). By means of these contacts, the time required for a known volume of liquid to flow through the capillary is automatically recorded. One difficulty with this type of instrument is that considerable error may be introduced by liquid from a previous determination adhering to the walls of the reservoir tube.

Other capillary viscometers have their own peculiar disadvantages. Common to all of them, however, is the ease with which the capillary becomes clogged, especially when corrosive, readily oxidizable materials are used. Sodium, the material under investigation, is not only easily oxidized, but also has a small viscosity (less than one centipoise). For materials of low viscosity, the product of the capillary radius and the rate of flow through the capillary must be

made small in order to avoid turbulence. This can be accomplished either by using a long, fine capillary or by using a small driving pressure. These expedients are undesirable inasmuch as a long, fine capillary is susceptible to clogging; and a small driving pressure is hard to reproduce and measure accurately.

Another important objection to capillary flow viscometers is that they are reliable only for relative viscosities and must be calibrated with a liquid of known viscosity at different temperatures. In order to obtain absolute values of viscosity by capillary flow methods, the Hagenbach correction and corrections for the ends of the capillary must be applied; and these corrections are uncertain. Furthermore, the mean effective liquid head must be determined; and this is not easily done.

Because of the difficulties mentioned above and because of the fact that no transparent material will contain sodium at high temperatures, capillary flow methods were rejected.

Perhaps the second most popular type of high temperature viscometer which has been used in the past is the concentric cylinder apparatus of Margules (9). This method has been successfully used to measure the viscosities of slags, glasses, and other materials having moderately large viscosities. However, the ordinary concentric cylinder viscometer is not sufficiently sensitive for measuring viscosities less than one centipoise. A concentric cylinder viscometer can be made

sufficiently sensitive only by reducing the annular space between the two concentric cylinders to the point where it is difficult to align the inner, rotating cylinder.

Much consideration was given to a method used by Dantuma (6) which involves measuring the logarithmic decrement of the amplitude of oscillations of a sphere suspended in the liquid whose viscosity is being measured. Dantuma applied this method with apparent success to fused sales such as KNO_3 and NaCl; and some investigators have reported using the method successfully with metals (11). However, Fawsitt (7), using the method on metals, reported that large errors are possible due to the presence of impurities on the surface skin. Such impurities tend to make the surface very viscous; and since the surface is in contact with the member suspending the sphere, high values for the viscosity result. Fawsitt stated that even invisible traces of impurities on the surface can easily introduce errors of ten per cent, and that the presence of visible impurities makes the experiment hopeless. Since viewing the surface would present some difficulties in an apparatus made of opaque materials, this method was set aside. The most promising procedure found in the literature for use with easily oxidized metals appeared to be one developed by Chiong and Andrade (5) when they measured the viscosity of sodium from its melting point up to 350°C.

In this method, the liquid is enclosed in a sphere suspended in a vacuum; and the viscosity is determined by

observing the damping of the oscillations about the vertical axis. The equations relating the viscosity and the experimental quantities are:

$$n = \frac{a^2 R^2 \pi d}{4 q^2 T} \left\{ 1 - (1-\mu)^{1/2} \right\}^2$$

$$\mu = \frac{3 q I \delta}{2\pi^2 a^2 R^5 d} \left(T^2 / T_o^2 + 1 \right)$$

$$q = 2 - \frac{gR - 1}{(gR-1)^2 + h^2 R^2}$$

$$a = 1 - \delta / 4\pi$$

$$g = \sqrt{\frac{\pi d a}{T n}}$$

$$h = \sqrt{\frac{\pi d}{T n} \left(1 + \delta / 4\pi \right)}.$$

n = viscosity (poises)
R = radius of sphere (cm.)
I = moment of inertia of rotating pendulum (gm.-cm.^{2})
d = density of liquid (gm/cm^3)
T = period of oscillation of pendulum with the sphere filled with liquid (seconds)
T_o = period of oscillation of pendulum with the sphere evacuated (seconds)
δ = $\log_e$ decrement of the damping of oscillations

The above equations were obtained by Chiong and Andrade from a solution of the general equations of motion of a viscous liquid for a sphere of liquid rotating about the vertical axis with no slipping between the surface of the sphere and the container wall. For a detailed derivation, reference may be made to Helmholz (8) and Chiong and Andrade (4). To obtain these equations it was assumed that the external damping due to gases surrounding the pendulum and to imperfect elasticity of the wire suspending the sphere is negligible in in comparison to the damping due to the liquid enclosed in the sphere. It is further assumed that the quantity $\exp(-\pi d/Tn)$ is negligibly small in comparison to $\exp(\pi d/Tn)$. The latter assumption is important in that it affects the design of the pendulum and limits the instrument to liquids of low viscosity.

It was decided to use Chiong and Andrade's method; and a radical modification of their instrument was designed and constructed.

DESCRIPTION OF APPARATUS

The viscometer consists essentially of a hollow spherical vessel suspended in a gas-tight container by means of a long bifilar suspension wire. The vessel contains the test liquid and is caused to undergo rotatory oscillation. The oscillations of the sphere are observed by mounting a mirror on the suspension and reflecting a beam of light from the mirror onto a glass scale.

The outward appearance of the viscometer is shown in Figure I. Figure I shows a brass plate (15) mounted on a supporting frame (21) by means of push-pull levelling screws (17). To the lower side of this plate is attached a 38-inch length of 2 1/2-inch stainless steel pipe (type 304) (19) which passes through the three-inch tube furnace (23). This length of pipe serves as a "crucible" in which the hollow sphere is suspended. The lower end of the crucible is closed off with a plug holding a twelve-inch thermocouple well which extends up the center of the pipe. Gas-tight connections of the plug to the crucible and of the crucible to the plate are accomplished by "O"-ring gaskets.

The bifilar suspension wire is housed in the tube (12). This housing is a 56-inch length of 1 3/4-inch brass tubing which is sealed to the plate (15) with an "O"-ring. Only the lower portion of the tube may be seen in the photograph.

The window (13) in the lower portion of the suspension wire housing is seen in alignment with the galvanometer lamp

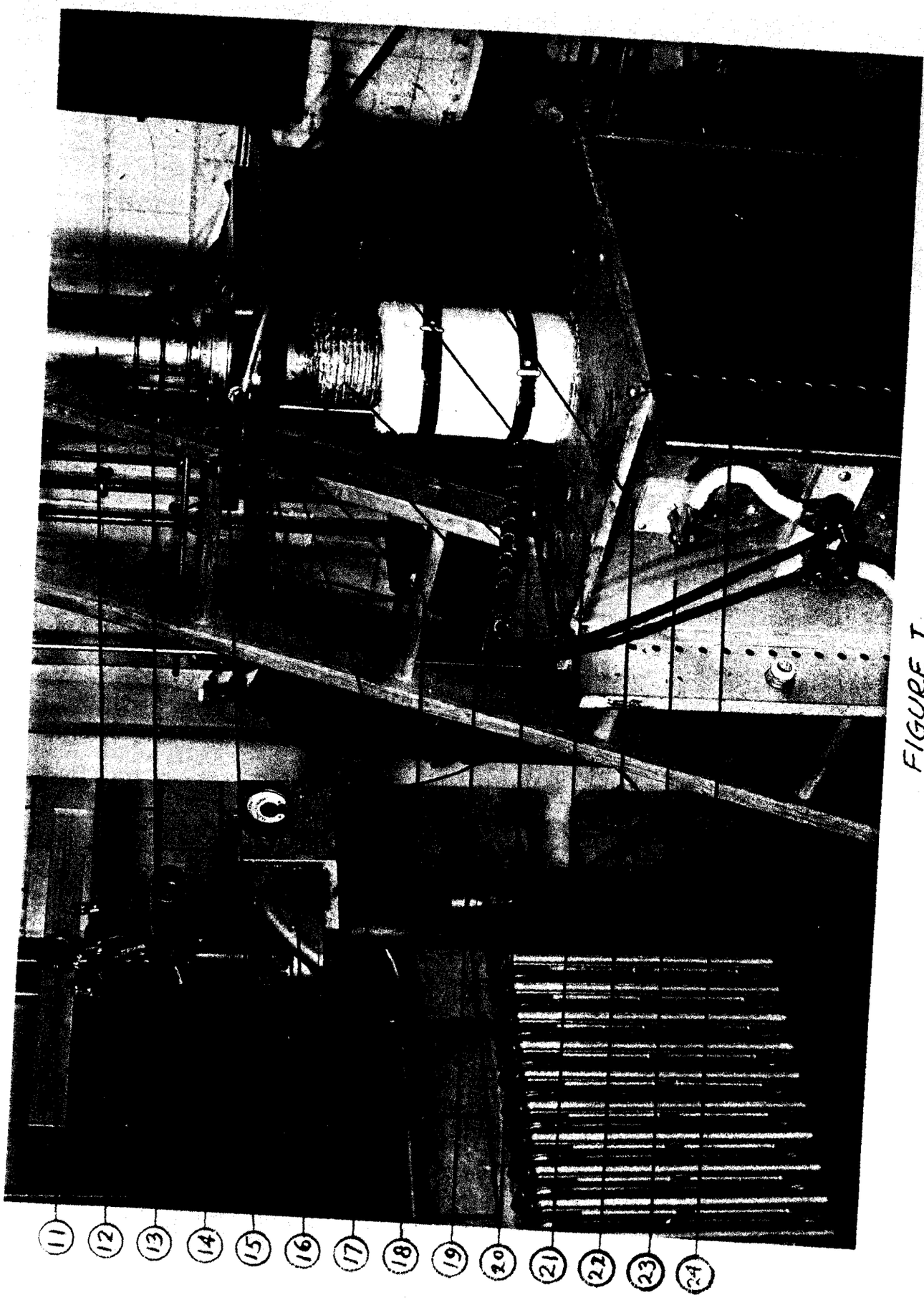


FIGURE I

and scale device (14) mounted on a stand approximately one meter away from the window. By means of this window and lamp and scale, a beam of light is reflected from the mirror mounted on the suspension. Also shown is a photocell (11) mounted on the scale and an electrical timer (16) by means of which the period of oscillation is measured.

The atmosphere in the viscometer is controlled through the gas connection (18) to the brass plate (15). A length of 3/8-inch copper tubing (22) extends from this connection to an oil diffusion pump (24) just visible at the lower left in the photograph. A vacuum of less than one micron, as measured by a thermocouple gauge, was obtained by means of a Megavac backing pump and this diffusion pump. All connections in the vacuum lines were silver soldered with the exception of the connection to the brass plate and the thermocouple gauge connection. The latter two connections are screwed 1/8-inch pipe connections caulked with glyptal.

The upper regions of the viscometer are kept cool by circulating water through the copper coils (20) soldered to the crucible. Not visible is another cooling coil placed around the plug at the bottom end of the crucible to avoid overheating the "O"-ring gasket there.

Bifilar Suspension

The bifilar suspension is shown in Figure II. The suspension head (27) is in the shape of a truncated cone which fits into a ground, tapered seat at the top of the suspension

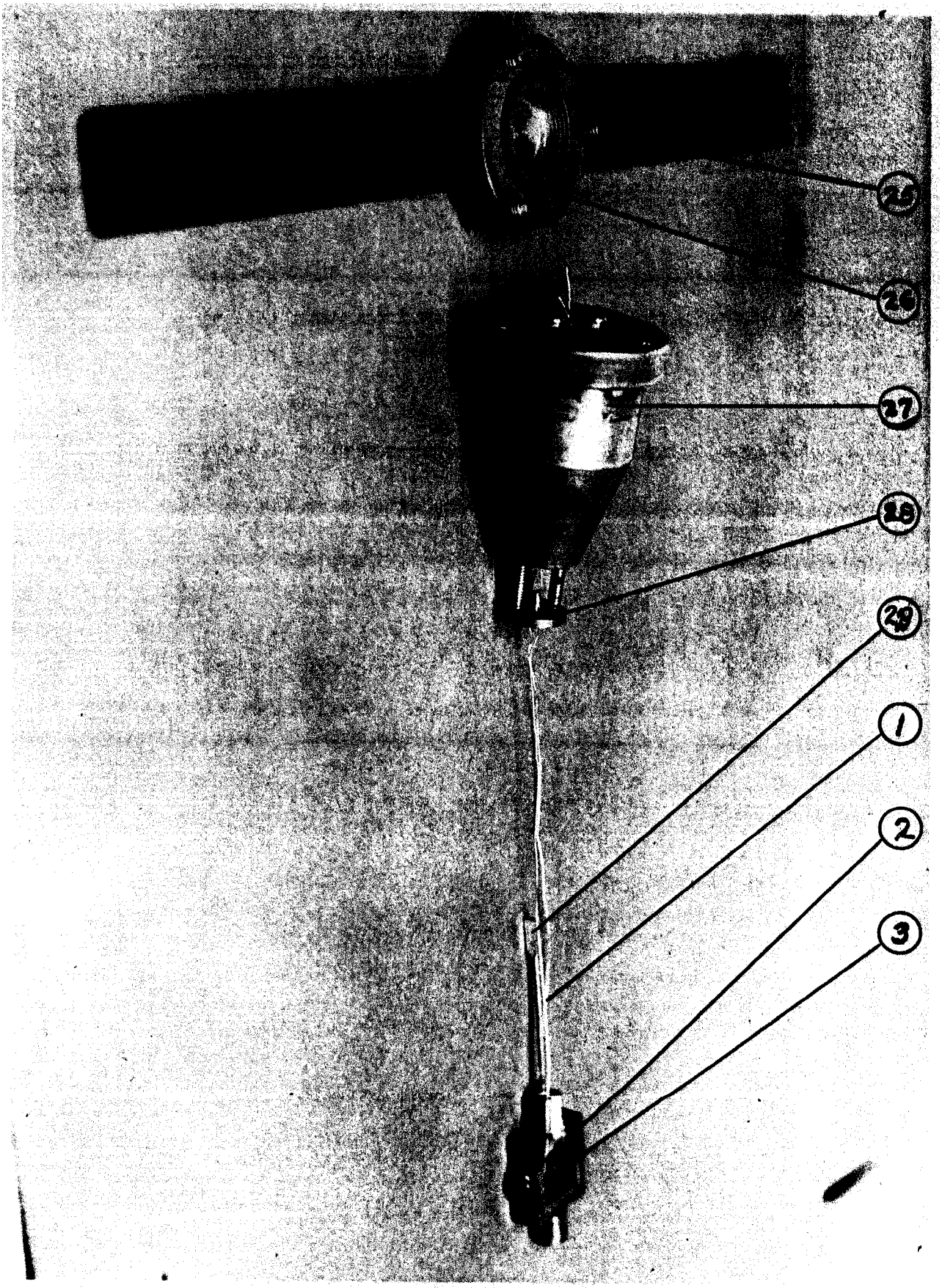


FIGURE II

housing previously described. A coaxial conical hole is turned in the small end of the cone. The ends of the suspension wire (1) are fixed at opposite ends of a diameter of this hole by means of the conical plug (28) fitting tightly into it. This method of suspension ensures that the two ends of the suspension wire are equidistant from the axis of the suspension head and that, when the instrument is levelled, the axis is truly vertical.

Two holes, 3/16-inch in diameter, are drilled through the length of the truncated cone; two threaded holes are made in the conical plug for holding the wires to the suspension head. Two 3 1/2-inch screws are inserted through the holes in the truncated cone and screwed into the plug so as to hold the plug firmly against the truncated cone. Another hole is drilled axially through the suspension head thus enabling the two wires to be drawn through the suspension head. This permits convenient adjustment of the length of the suspension wire. When the suspension is adjusted to the proper length, the two screws holding the conical plug are tightened to hold the wires firmly in place.

In order to prevent leakage through the holes drilled through the suspension head, the cap (26) is screwed to the top of the head. A gas-tight seal is obtained with the "O"-ring shown fitted in the cap.

The pendulum is set in rotatory oscillation by turning the cone in its tapered seat at the top of the suspension housing. The handle (25) is screwed to the cap to facilitate the turning of the cone. Apiezon grease is used between

the cone and its ground seat both to obtain a tight seal and to ease the turning of the cone.

The suspension wire (1) fits around a circumferential groove on the cylindrical bar (3) which is bolted to the brass spider (2). A piece of 1/16-inch nickel rod is soldered to the top of the spider, and the mirror (29) is glued to this rod. Triangular slots are cut in the sides of the spider so that the suspension wire just touches against both sides of the spider. With the suspension hanging vertically in place, the wire is sealed to both sides of the spider with glyptal. The actual suspension wire is a 10 mil stainless steel wire and is approximately 115 inches long. The lower part of the suspension, which includes the hollow sphere, is attached to the bottom of the spider.

Pendulum Assembly

Figure III is a drawing of the pendulum assembly. The spider (2), with its grooved wheel around which the suspension wire fits, is shown at the top of the drawing. The stem machined on the hollow sphere (10) is shown at the bottom of the drawing. All of this assembly is housed in the crucible pipe.

Cylindrical rings of equal mass and known moments of inertia fit tightly around the neck of the yoke (9). By means of these, the moment of inertia of the entire pendulum may be measured according to a procedure to be described.

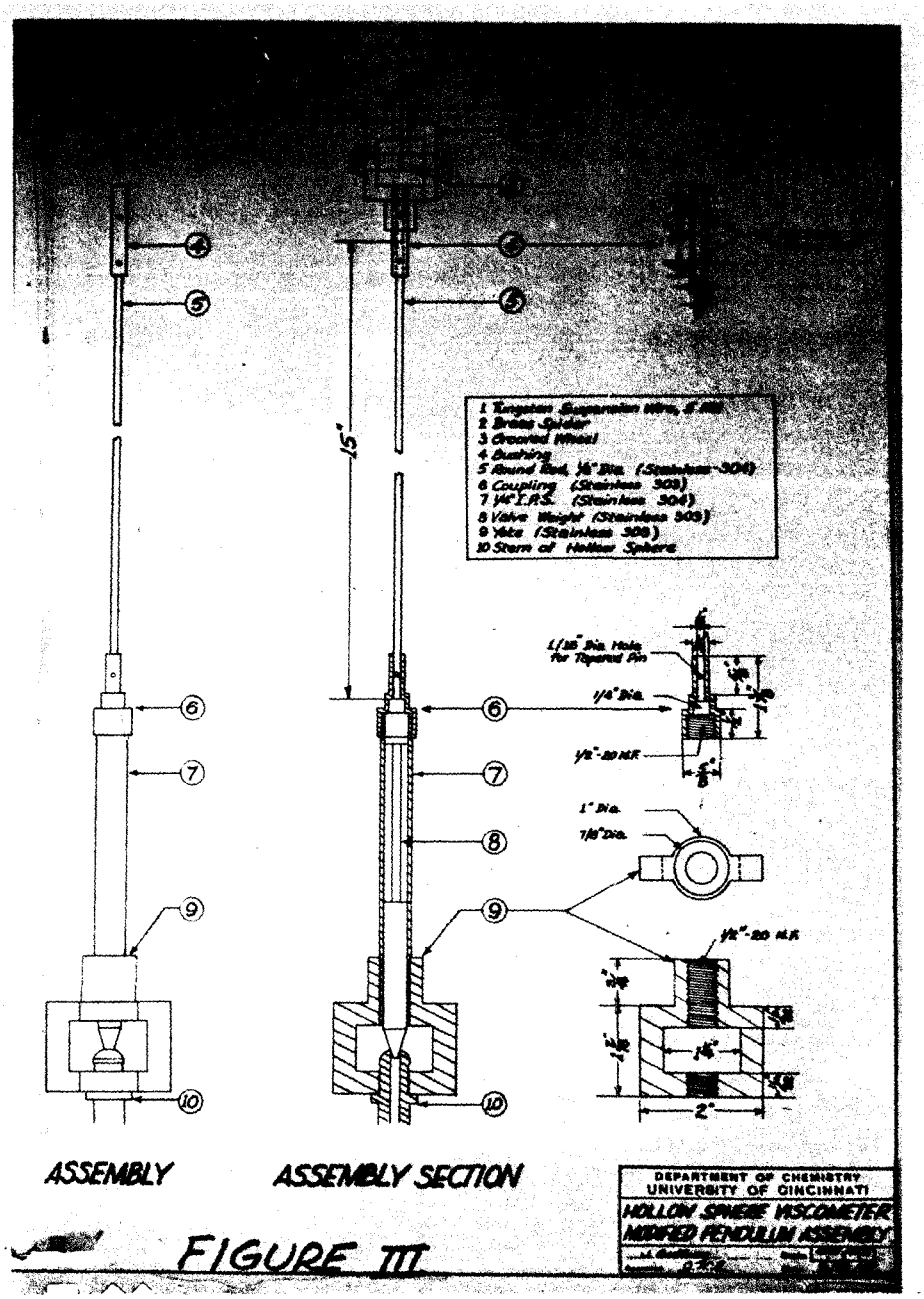


FIGURE III

Of particular interest in this drawing is the device for sealing off the stem of the hollow sphere. The valve weight (8) is made with a conical tip to fit into the hole in the stem of the hollow sphere. The valve weight is guided by its close fit into the length of 1/4-inch stainless pipe (7) and is held down by a tungsten spring (not shown) placed in the coupling (6). As the temperature is increased, the liquid in the sphere expands and forces the valve weight up. Liquid overflows to relieve the pressure; and the valve weight descends again to close off the stem. This device permits the measurement of the viscosity over a wide temperature range without disassembly and refilling of the sphere. At the same time, it prevents vaporization and loss of the liquid in the sphere.

The construction of the hollow sphere is shown in Figure IV. The assembled sphere is one made of stainless steel (type 303) which has been used to measure the viscosity of sodium. Shown in front of it is a disassembled nickel sphere of similar design. The upper hemisphere (31) is provided with threads for a nut (30) which is slotted to fit a lip projecting from the lower hemisphere. To obtain a tight seal, the surfaces at which the two hemispheres meet are tapered and ground. The end of the stem of the upper hemisphere is machined to serve as the inner member of a gas-tight ball and socket joint for filling the sphere.

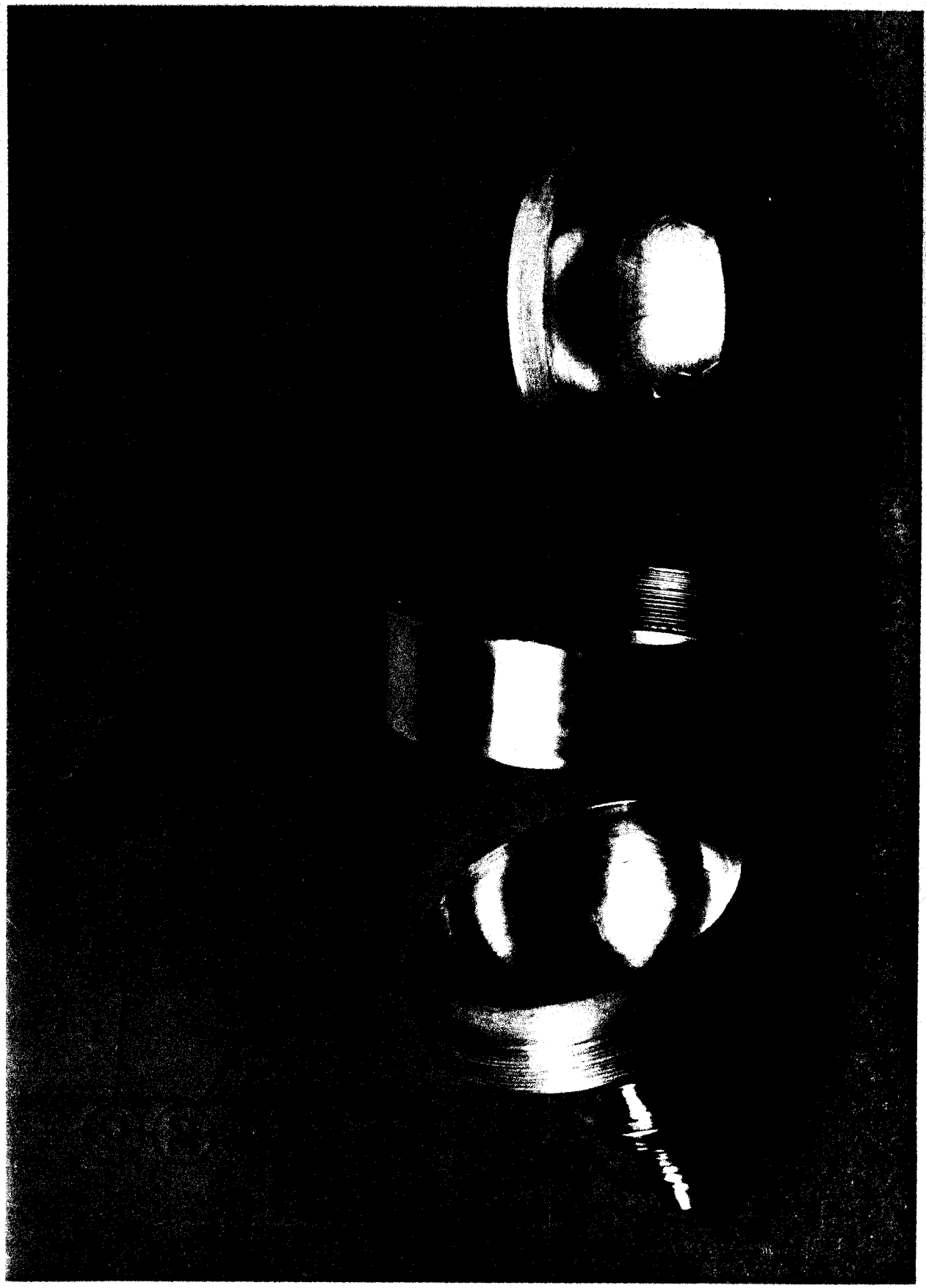


FIGURE IV

Apparatus for timing the period of oscillations electrically

Automatic timing of the period of oscillations of the pendulum is accomplished by means of the electrical circuit schematically shown in Figure V. Light is reflected from the mirror mounted on the pendulum onto a photoelectric cell mounted at the center of the scale of a galvanometer type of lamp and scale device. As the pendulum turns, the spot of light moves back and forth on the scale. When it strikes the photoelectric cell, the timer is started. At any desired number of periods later, the coincidence of the spot of light with the photoelectric cell may be caused to stop the timer.

For starting the timer, switch S is set at position 2. Light striking the photoelectric cell (P) causes contacts S_1 and S_2 to close. Since contact S_3 is normally closed, the timer will now start. At the same time, contact S_4 closes, thus shorting out S_2 and S and keeping the timer running.

The timer is stopped only when switch S is put in position 1. With S in this position, the closing of contact S_2 activates relay R_3 to open the normally closed contact S_3. The opening of S_3 stops the timer and opens contact S_4 to keep the timer stopped.

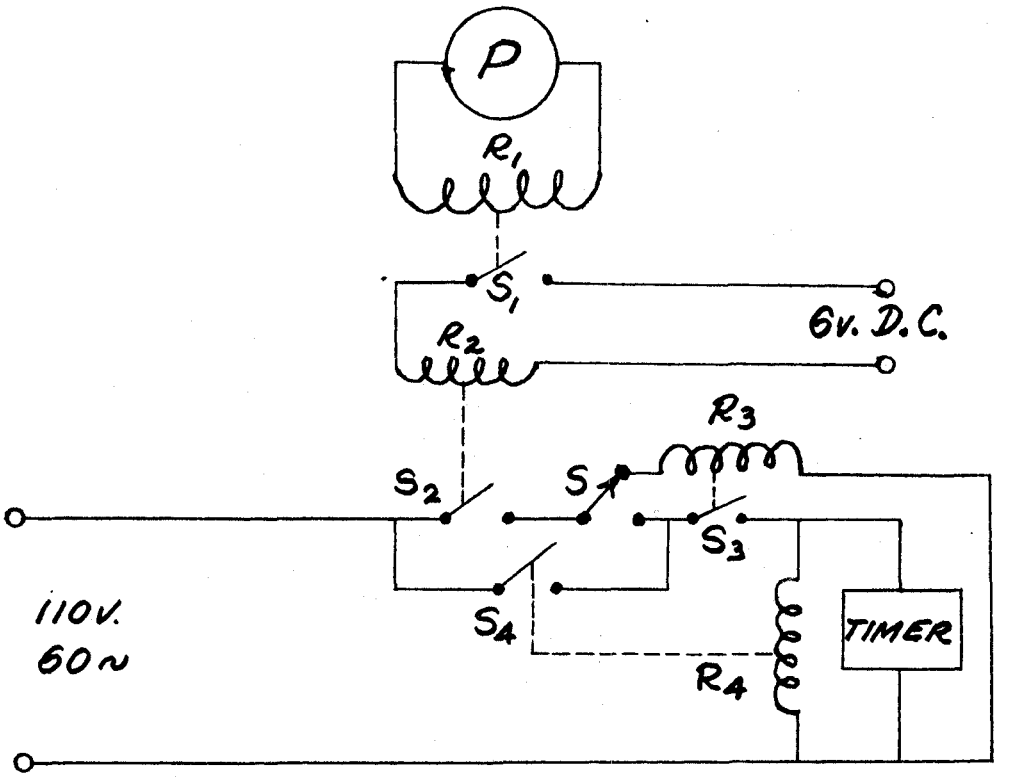
The electrical timer is a direct reading, indicating counter, graduated in 1/10ths of a second, and driven by a synchronous motor. The photoelectric cell is a relay type cell made by the Weston Company. Relay R_1 is a sensitive

FIGURE V

Electrical Circuit for

Measuring the Period of Oscillations

of the Pendulum



R_1, R_2, R_3 and R_4 are relay coils; and S_1, S_2, S_3 and S_4 are their respective contacts.

R_1 is a sensitive micro-relay.
R_2 is an auxiliary relay with a 6 volt D.C. coil voltage.
R_3 and R_4 are 110 volt 60 cycle relays with single pole double throw contacts (S_3 and S_4).
S is a single pole double throw switch.
S_3 is normally closed.
S_1, S_2 and S_4 are normally open.

galvanometer type, which is adjustable for any condition of light or dark operation. It is also made by the Weston Company. R_2 is a direct current relay which serves as an auxiliary power relay.

To obtain a brighter spot of light on the scale, a 50-watt, 120-volt projector type bulb was substituted for the 5-volt, 22-candlepower light bulb ordinarily used in lamp and scale devices.

EXPERIMENTAL PROCEDURES

Purification of Sodium

Two methods of purifying sodium are customarily used. The first is vacuum distillation using a still made of Pyrex, nickel, or certain of the stainless steels. Triple distillation is recognized almost as a standard procedure for purifying sodium; and if the distillate is taken off in fractions, it is possible to remove other alkali metals as well as any contaminating oxides or salts present.

The second is filtration of reagent grade sodium through a sintered disk at low temperatures, not exceeding 110°C. Provided the temperature is kept low, the oxides and contaminating salts are satisfactorily removed; and the only impurities are any heavy metals or other alkali metals which were present in the original sodium. Since a good grade of sodium will be contaminated by a minimum of other metals, filtration of a reagent grade of sodium may be expected to be a satisfactory method of purification. Indeed, it has been found that filtration is as satisfactory a procedure as distillation (10); for this reason and because of the simplicity of the procedure, filtration was selected as the method of purification.

Reagent grades of Merck and Mallinckrodt sodium were used. The purity of this sodium conforms to the specifications of the American Chemical Society which allow for the

following maximum impurities:

Chloride	.0015%	Sulfate	.002%
Nitrogen	.003%	Heavy Metals	.001%
Phosphate	.0005%	Iron	.001%

Filtrations were made using both Pyrex and stainless steel sintered disks. The filtrations made in Pyrex were advantageous in that it was possible to observe the shiny appearance of the filtered sodium. However, either sodium or its oxide attacked the Pyrex disk causing pieces of it to flake off; and eventual disintegration of the disk resulted after a few filtrations. No difficulties were encountered in filtrations through sintered disks of stainless steel. A great many filtrations were carried out with no apparent attack on the stainless steel.

The most successful filtrations were made using a stainless steel Buchner type funnel with a sintered stainless steel disk of coarse porosity. The funnel used had a capacity of 100 ml. and an inside diameter of 6 cm. A stainless steel socket joint (size 12/5) was soldered to the delivery tip of the funnel; and a brass flange was soldered to the top of the funnel. A tight connection was made between the filter and sphere through the socket joined to the filter and the ball machined on the stem of the sphere. The filter was connected to vacuum and helium lines by means of 1/4-inch copper tubing connected to a brass plate which served as a cap for the funnel. A rubber gasket was used to obtain

a tight seal at the flanged connection.

The sodium filtration was performed in an electric oven. After the filter was sealed to the hollow sphere, pieces of freshly cut sodium in an amount in excess of the capacity of the hollow sphere were placed on the sintered disk. The funnel cap was set in place and connected to the vacuum line; and the assembly was placed in the cool oven. In an hour's time, the pressure in the sphere and funnel assembly was reduced to about 10 microns. At this pressure the oven was turned on and brought up to 110°C., where it was maintained within five degrees for a period of at least ninety minutes. This extended heating period was to allow sufficient time for all the sodium to melt and form an even liquid layer over the sintered disk. Filtration was then accomplished by allowing helium to leak through the copper lines down onto the layer of sodium on the disk. Very little helium pressure was required to force the sodium through the sintered bed into the evacuated sphere.

After the filtration, the oven was turned off; and the sphere and filter were allowed to cool to room temperature. When the sphere was disconnected from the filter, shiny sodium metal was seen to extend from the stem of the sphere solidly up into the delivery tip of the funnel. The sodium was cut flush with the opening in the stem of the sphere so that only the small surface (1/8-inch in diameter) was in contact with the air before the sphere was suspended in the viscometer.

The lowest temperature at which the viscosity was determined was 328°C. Since the sphere was full at room temperature, any oxides or carbonates formed at the exposed tip were removed as the sodium expanded and overflowed from the sphere on heating up to the temperature at which determinations were made.

Determination of the Damping of the Swings

The logarithmic decrement of the damping of oscillations is defined by:

$$(1)\quad \text{Log}_e\text{decrement} = \delta = (\text{Log}_e A_i - \text{Log}_e A_j)/(j - i)$$

or

$$(2)\quad \text{Log}_e A_i = \text{Log}_e A_j + (j - i)\delta$$

where A_i = amplitude of the ith swing

A_j = amplitude of the jth swing

From (2) it is apparent that the logarithmic decrement is the slope of the straight line obtained by plotting the logarithms of the amplitudes of the swings versus the swing numbers. In this investigation, the amplitudes of several successive swings were noted; and the method of least squares was used to find the slope.

The viscometer was assembled with the sphere filled with sodium; and the system was evacuated to a pressure of less than one micron. The electric furnace was then turned on; and the temperature was brought up to the lowest temperature at which viscosity determinations were to be made. After the temperature had been held constant for at least two hours, the sphere was set in rotatory oscillation by

turning the suspension head at the top of the apparatus. (See description of bifilar suspension, page 10.) The swings of the sphere were observed by following a spot of light reflected from the mirror mounted on the pendulum onto a glass scale of a galvanometer type of lamp and scale apparatus.

When the swings had decreased in amplitude until the spot of light traveled only about 30 cm. on the glass scale, the furnace was turned off; and the amplitudes of approximately twenty consecutive swings were recorded. The total time required for the twenty swings was automatically recorded by means of the photocell-electrical timer apparatus described on page 17; and the period of oscillation was found by dividing the total elapsed time by the number of cycles.

A distance of travel of thirty centimeters on the glass scale corresponds to a maximum displacement of about 8 1/2 degrees on both sides of the rest position of the pendulum. So long as the maximum displacement is held this small, the motion of the pendulum is both simple harmonic and sufficiently slow to permit the amplitudes to be read easily to within $\pm$ 0.04 cm.

Since the glass scale on which the swings were observed was flat, it was necessary to convert the observed amplitudes to the arc lengths to which they corresponded. This was done from a table constructed from the equation:

$$s = 2r \arctan(T/r)$$

where s = arc length

r = distance from mirror to glass scale(114.8cm)

T = one-half the observed amplitude

This is a small correction for swings of low amplitude.

The logarithmic decrement of the damping found by the above procedure includes both the damping due to the liquid sodium and that due to residual gases surrounding the pendulum and to imperfect elasticity of the suspension wire. The sum of the latter two damping effects is known as the external damping; and the logarithmic decrement found with the sphere filled must be corrected by subtracting from it the logarithmic decrement of the external damping. The external damping was found by observing the swings of the pendulum with the sphere empty and pieces of sodium metal in the crucible to obtain the same vapor atmosphere as in the above procedure.

All determinations were made with the furnace turned off because it was found that the electrical field greatly influenced the swings of the pendulum. Highly irregular swings, probably caused by the on-off action of the relay controlling the input to the furnace, resulted when the furnace controller was used.

Moment of Inertia

The moment of inertia of the pendulum was calculated from measurements of the periods of oscillation of the pendulum with two different inertia pieces of equal mass and different, known moments of inertia added to the pendulum. The period of oscillation of the pendulum is proportional to the square root of the quotient of its moment of inertia and the restoring force. Since the total mass of the pendulum is the same, regardless of which inertia piece is used, the restoring force is also constant. We may then write the equation:

$$T_1/T_2 = \sqrt{(I + I_1)/(I + I_2)}$$

where T_1 = period of oscillation when inertia piece #1 is used

T_2 = period of oscillation when inertia piece #2 is used

I_1 = moment of inertia of piece #1

I_2 = moment of inertia of piece #2

I = moment of inertia of the rest of the pendulum

The two inertia pieces used were carefully machined in the form of cylindrical rings of different dimensions. One inertia piece weighed 92.9368 grams and had inside and outside diameters of 0.877 and 2.0905 inches, respectively at 28°C. The other piece weighed 93.4492 grams and had inside and outside diameters of 0.878 and 1.242 inches, respectively at 28°C. The moment of inertia of a cylindrical ring is

given by the equation:

$$I = 1/2\ M\ (r_1^2 + r_2^2)$$

where M = mass of inertia piece

r_1 = inside radius of inertia piece

r_2 = outside radius of inertia piece

For the two inertia pieces described above, the moments of inertia at any temperature are given by the equations:

$$I_1 = 385.15 \left\{1 + \beta(t - 28)\right\}^2$$

$$I_2 = 174.25 \left\{1 + \beta(t - 28)\right\}^2$$

where β = coefficient of thermal expansion

t = temperature, °C.

With the sphere empty, the period of oscillation was measured over the temperature range with first one and then the other inertia piece added to the pendulum. From this the moment of inertia was calculated at various temperatures. The contribution to the total moment made by the spherical ball of sodium was calculated by means of the equation:

$$I = 2/5\ (4/3\pi r^3\ d)\ r^2$$

where r = radius of sphere

d = density of sodium

Substituting in this equation the value of the radius of the sphere at 28°C., and expressing this radius as a function of temperature, the equation for the moment of inertia of the sodium sphere is obtained in the form:

$$I = 89.738 \left[1 + \beta(t - 28)\right]^5 d$$

Since moments of inertia are additive, the calculated moment of the sodium ball is added to the measured moment of the rest of the pendulum. The contribution of the moment of inertia of the sodium to the total moment is small.

Temperature Measurement and Control

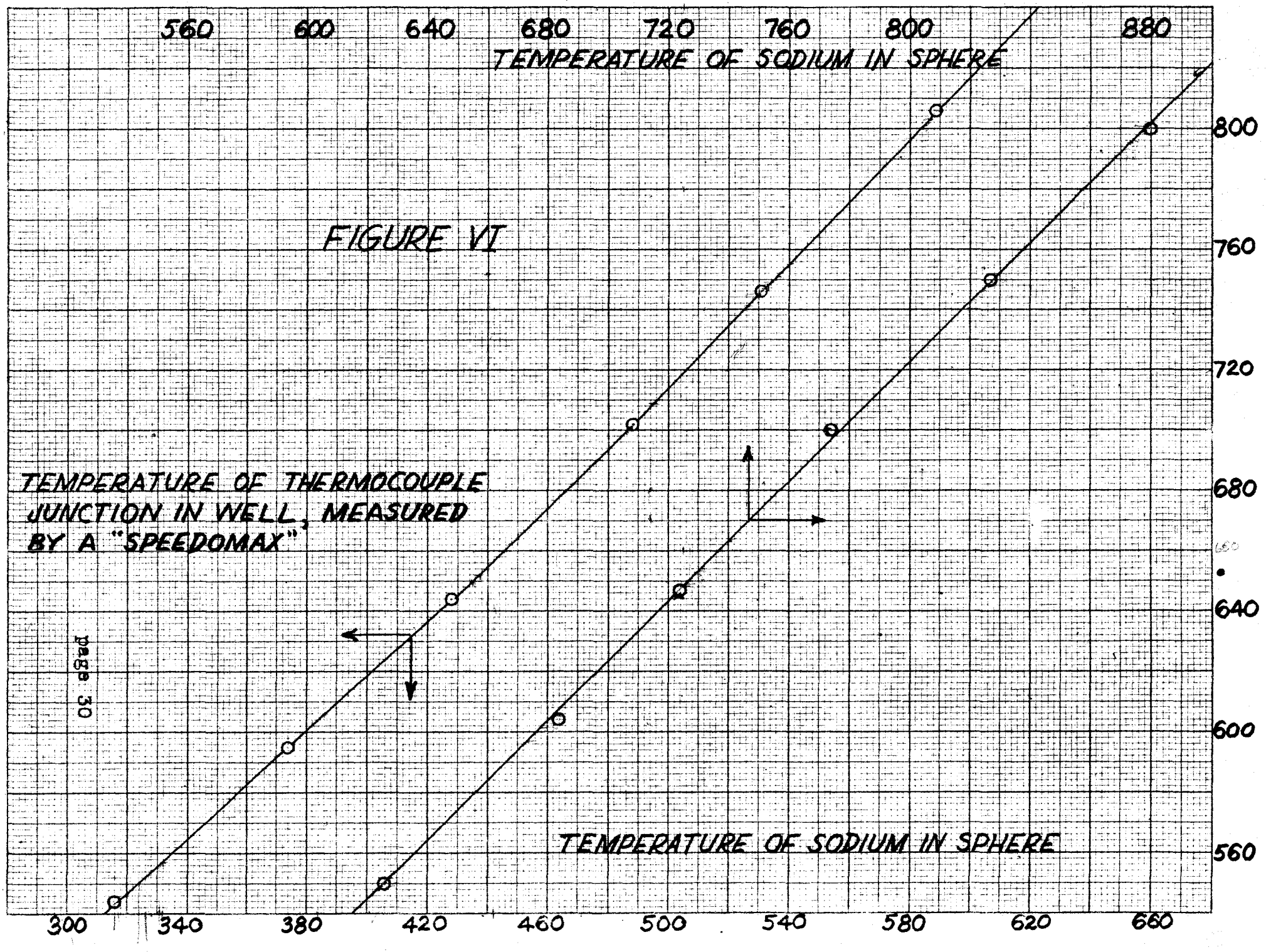
Since the suspended sphere must be completely free to move, it was not feasible to measure the temperature of the sodium during the determination of its viscosity. Instead, a chromel-alumel thermocouple was inserted into a well in the bottom of the crucible, directly under the sphere and about 1/4-inch below it. The temperature was measured and controlled by means of this thermocouple and a Leeds & Northrup Speedomax recorder-proportional controller.

After all the data necessary for the viscosity calculations had been accumulated, a study was made to determine the actual temperature of the sodium at each of the points at which measurements had been made. This was done by filling the sphere with plumber's solder and inserting a chromel-alumel couple into the sphere. The sphere was suspended as it had been in the viscosity determinations; and the temperatures measured with the lower thermocouple and Speedomax instrument were compared with the actual temperatures inside the sphere. The actual temperatures inside

the sphere were determined by measuring the output of the thermocouple inserted in the sphere with a potentiometer capable of measuring to the nearest one thousandth of a millivolt. The temperatures of the two thermocouples are compared graphically in Figure VI.

In this study the two temperatures were compared at fifteen to thirty minute intervals until both temperatures had become constant and remained so. Usually, after the Speedomax had indicated a constant temperature for an hour, the inner couple also gave a constant output. This is shown by the data listed in TABLE II. Since all viscosity data were taken only after a minimum time of two hours at constant temperature, it may safely be assumed that a steady temperature had been attained.

All the viscosity data were taken with the furnace turned off. Since the average run required from four to five minutes, there was the danger that the temperature of the sodium had dropped in the course of the run. When the data were taken, the temperature recorded by the Speedomax was noted at the beginning and end of each run. The difference between these two temperatures was small at lower temperatures but became appreciable at the higher temperatures. The temperature drop which occurred inside the sphere with the furnace turned off was measured and found to be slightly less than that indicated by the lower thermocouple. The temperatures inside the sphere and at the lower thermocouple hot junction are listed as functions of the time after the furnace was turned off in TABLE III.



Measurement of the Radius of the Sphere

The radius of the sphere was measured by weighing the sphere filled with mercury and empty. From the weight of the mercury required to fill the sphere, the radius was calculated, assuming the sphere to be perfectly spherical. Measurements of the radius were made before the sphere was used in the viscometer and after completion of all the viscosity determinations. The sphere was found to have a radius of 2.2180 cm. before it was used in the viscometer. After all the viscosity data were taken, the radius was again measured and found to be 2.2165 cm. In the viscosity calculations, the radius was arbitrarily taken to be 2.2170 cm. at room temperature and corrected for thermal expansion.

The slight change in the radius of the sphere after repeated use indicates that there was no appreciable attack by the sodium on the sphere. Thus contamination of the sodium by solution or corrosion of the stainless steel sphere could not be very great. The sphere was opened after it had been used; and its inner surface was found to be discolored but not pitted or roughened in any way.

EXPERIMENTAL DATA

TABLE II

Comparison of the temperature of the liquid in the sphere with the temperature of the controlling thermocouple in the bottom of the viscometer. These data correspond to the graph in Figure VI.

t	T_l	E_s	T_c	T_s	t	T_l	E_s	T_c	T_s
15	244	11.857	26.3	316	0	550	24.974	27.5	627
	243	11.857	26.3	316	1/2	550	25.064	27.6	630
3/4	301	14.479	24.6	378	0	605	27.609	24.6	687
1	301	14.499	24.4	378	1/3	606	27.548	24.7	686
5/4	299	14.511	24.1	378	1/2	606	27.595	24.6	687
3	296	14.420	23.2	375	5/6	608	27.595	24.9	687
	295	14.430	23.2	374	5/4	605	27.486	24.8	684
3/4	342	16.635	23.2	426	3/2	604	27.504	25.1	685
5/4	344	16.723	23.3	428	0	647	29.159	25.3	724
3/2	342	16.739	23.2	430	1/8	646	29.159	25.6	725
0	403	19.405	23.3	493	1/4	647	29.127	25.8	724
1/4	401	19.229	23.7	489	1/2	647	29.125	26.1	724
1	402	19.253	24.1	490	15	700	31.140	27.8	774
5/4	401	19.207	24.3	489		700	31.133	26.2	773
3/2	402	19.207	24.6	490	0	749	33.243	24.4	822
15	446	20.967	26.2	532	1/3	750	33.243	25.1	823
	446	20.997	26.2	533	3/2	750	33.243	25.7	824
0	505	23.526	24.6	591	0	801	35.514	26.4	880
1/2	504	23.473	24.5	589	1	798	35.361	26.8	877
1	506	23.550	24.6	591	5/4	800	35.321	27.1	876
3/2	506	23.544	24.6	591	3/2	800	35.364	27.3	878

Legend:

t is the time in hours at which the bottom thermocouple was at the control temperature.

T_l is the control temperature in °C. It is the temperature of the lower couple as measured by a Speedomax recorder.

E_s is the output voltage of the thermocouple in the sphere, expressed in millivolts.

T_c is the temperature of the cold junction of the sphere thermocouple in °C.

T_s is the temperature of the liquid in the sphere.

TABLE III

Drop of temperature with time after the furnace is turned off.

Time (minutes	0	2	3	3 1/2	4	4 1/2	5
T_s	316	316	316	316	316	315	315
T_l	243						241
T_s	374	374	374	374	374	373	372
T_l	295						291
T_s	430	430	430	429	428	---	426
T_l	342						336
T_s	490	490	490	489	487	---	485
T_l	402						395
T_s	533	533	532	531	530	---	527
T_l	446						437
T_s	591	591	590	---	589	---	586
T_l	506						497
T_s	630	629	626	624	622	620	
T_l	550					538	
T_s	685	685	---	680	675	670	
T_l	604					590	
T_s	724	723	720	716	---	706	
T_l	647					629	
T_s	773	767	762	758	754		
T_l	700				680		
T_s	824	820	810	805	802		
T_l	750				739		
T_s	878	873	863	856			
T_l	800			789			

Legend: T_s is the temperature of the liquid in the sphere.
T_l is the temperature at the lower thermocouple hot junction.

TABLE IV

Period of oscillation of the pendulum and the logarithmic decrement (base 10) of the damping of oscillations as functions of the temperature. The sphere is filled with sodium; inertia piece #1 is added to the pendulum; and the pressure in the viscometer is less than one micron. These data correspond to the graph in Figure VII.

T_1	T_s	T	δ	T_1	T_s	T	δ
254-250	328	17.635	.00855	507-498	587	17.69	.00789
253-252	327	17.602	.00854	506-497	587	17.67	.00791
257-253	331	17.600	.00844	557-544	626	17.67	.007628
305-300	385	17.64	.008219	558-545	628	17.687	.00741
306-302	386	17.60	.008199	608-592	677	17.700	.00745
303-300	383	17.59	.008185	608-592	677	17.68	.00774
352-348	437	17.633	.00800	658-639	727	17.737	.007645
354-350	439	17.625	.007896	660-633	724	17.70	.007962
377-370	462	17.647	.007735	662-641	728	17.687	.007619
374-370	459	17.647	.007751	709-687	777	17.708	.007536
409-400	493	17.637	.007862	709-687	777	17.746	.007332
404-399	488	17.630	.007869	758-736	825	17.762	.007162
408-400	492	17.641		810-788	877	17.783	.007271
455-446	537	17.66	.00794	810-785	876	17.788	.007283
455-444	537	17.659	.00802				

Legend:

T_1 is the temperature range over which each set of data were taken, as indicated by the lower thermocouple.

T_s is the effective temperature in the sphere.

T is the period of oscillation in seconds

δ is the $\log_{10}$ decrement of the damping of oscillations.

FIGURE VII

PLOT OF PERIOD OF OSCILLATION OF THE PENDULUM VS. TEMPERATURE SPHERE FILLED WITH SODIUM - INERTIA WGT #1 ADDED

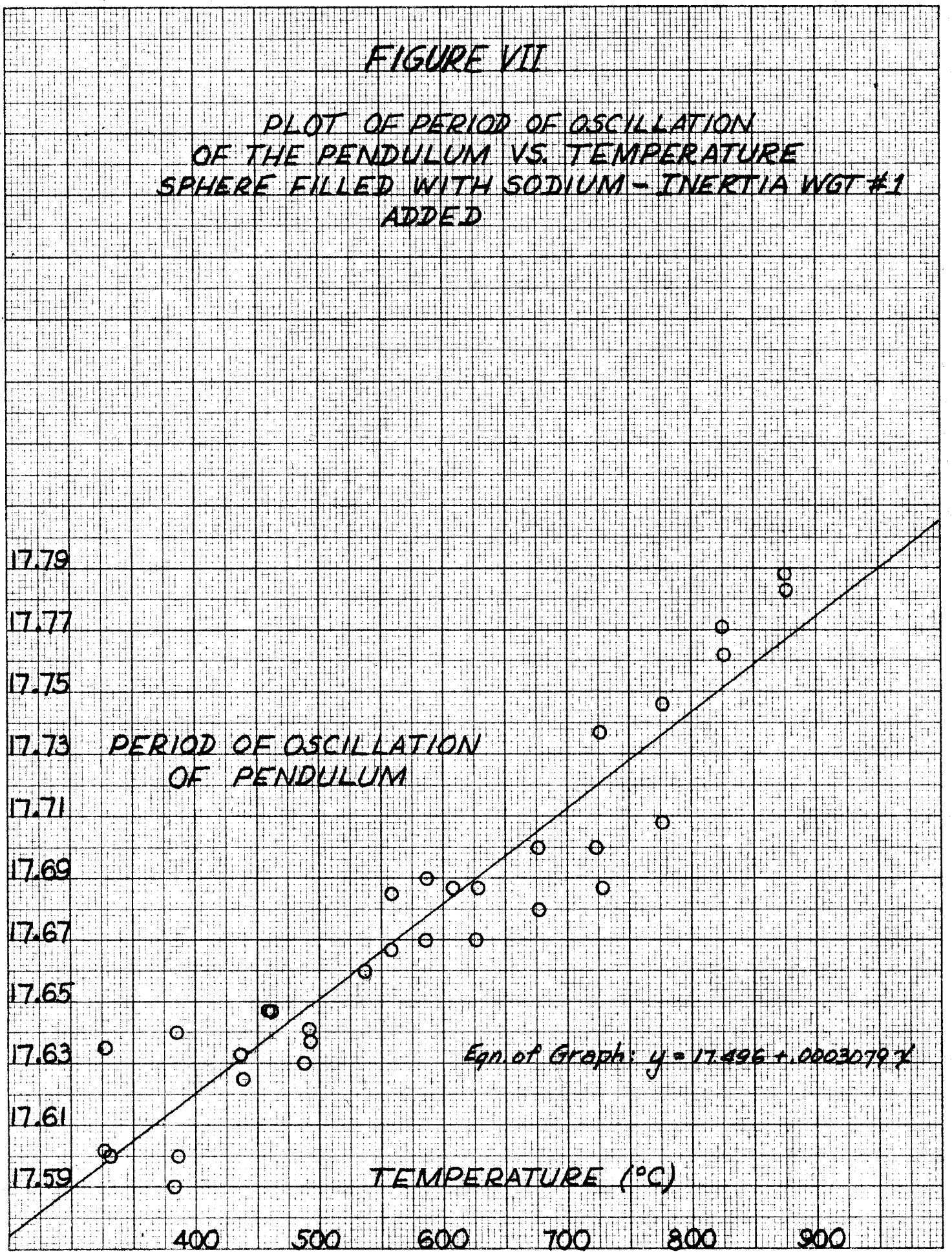


TABLE V

Period of oscillation of the pendulum with the sphere empty as a function of the temperature of the sphere. Inertia piece #1 added to the pendulum. These data correspond to the graph in Figure VIII.

T_1	T_s	T	T_c	$T_c - T$
256-252	330	17.44	17.432	.008
255-252	329	17.433	17.432	.001
306-301	386	17.481	17.455	.026
304-300	384	17.43	17.454	.024
303-300	383	17.47	17.454	.016
356-351	442	17.465	17.479	.014
356-352	442	17.472	17.479	.007
352-344	435	17.471	17.476	.005
405-397	489	17.493	17.498	.005
405-400	490	17.494	17.498	.004
454-444	536	17.519	17.518	.001
452-440	533	17.512	17.517	.005
507-499	587	17.525	17.539	.014
507-498	587	17.531	17.539	.008
556-536	624	17.571	17.555	.016
559-546	630	17.592	17.557	.035
602-589	673	17.588	17.575	.013
610-598	681	17.556	17.579	.023
660-645	731	17.591	17.600	.009
660-643	730	17.608	17.600	.008
710-683	776	17.593	17.618	.025
710-692	779	17.625	17.620	.005
760-742	828	17.645	17.640	.005
812-785	877	17.669	17.661	.008

Legend:

T_1 is the temperature range over which each period was measured, as indicated by lower thermocouple. (°C)

T_s is the effective temperature in the sphere (°C)

T is the observed period of oscillation in seconds

T_c is the period of oscillation calculated from the equation:

$$T_c = 17.294 + .00041794 T_s$$

FIGURE VIII

PLOT OF PERIOD OF OSCILLATION OF THE PENDULUM VS. TEMPERATURE SPHERE EMPTY - INERTIA WGT. #1 ADDED

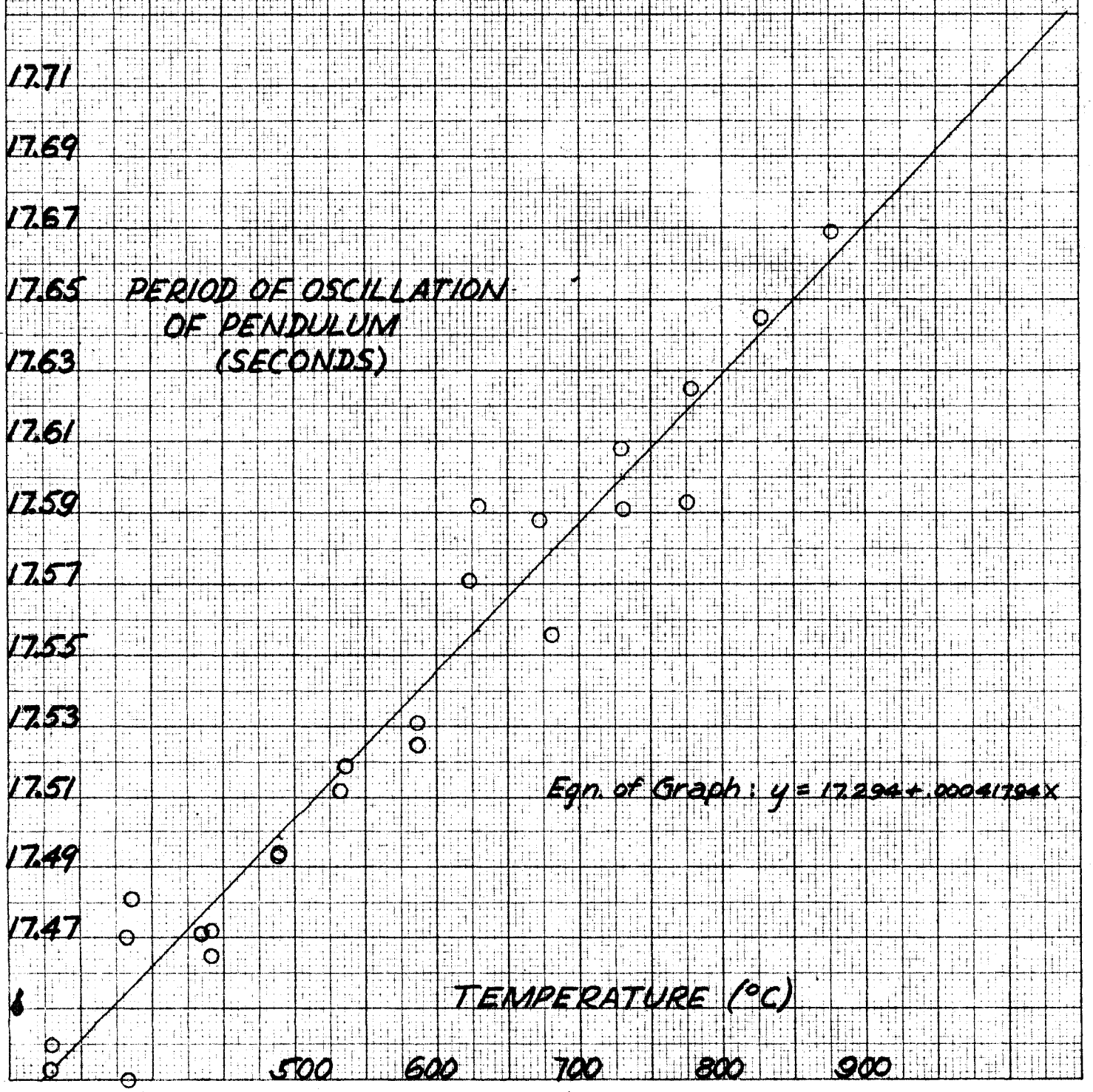


TABLE VI

Period of oscillation of the pendulum with the sphere empty as a function of the temperature of the sphere. Inertia piece #2 added to the pendulum. These data correspond to the graph in Figure IX.

T_1	T_s	T	T_c	$T_c - T$
250-249	324	16.467	16.465	.002
251-248	325	16.457	16.466	.008
254-251	328	16.473	16.467	.006
251-246	325	16.472	16.466	.006
300-295	380	16.483	16.488	.005
304-300	384	16.500	16.490	.010
350-344	435	16.520	16.511	.009
353-352	438	16.518	16.512	.006
359-355	444	16.500	16.514	.014
402-401	486	16.530	16.532	.002
404-397	490	16.519	16.533	.014
404-398	490	16.531	16.533	.002
406-399	492	16.515	16.534	.019
455-442	537	16.557	16.553	.004
454-447	537	16.561	16.553	.008
505-498	587	16.567	16.573	.006
505-497	587	16.569	16.573	.004
558-546	632	16.600	16.592	.008
559-542	632	16.600	16.592	.008
610-592	678	16.600	16.611	.011
610-597	681	16.627	16.612	.015
659-636	724	16.618	16.629	.011
658-640	726	16.633	16.630	.003
712-684	777	16.646	16.651	.005
715-694	782	16.660	16.653	.007

Legend:

T_1 is the temperature range over which each period was measured, as indicated by the lower thermocouple.

T_s is the effective temperature in the sphere. (°C)

T is the observed period of oscillation in seconds.

T_c is the period of oscillation calculated from the equation:

$$T_c = 16.332 + .0004108T_s$$

FIGURE IX

PLOT OF PERIOD OF OSCILLATION OF THE PENDULUM VS. TEMPERATURE SPHERE EMPTY - INERTIA WGT. #2 ADDED

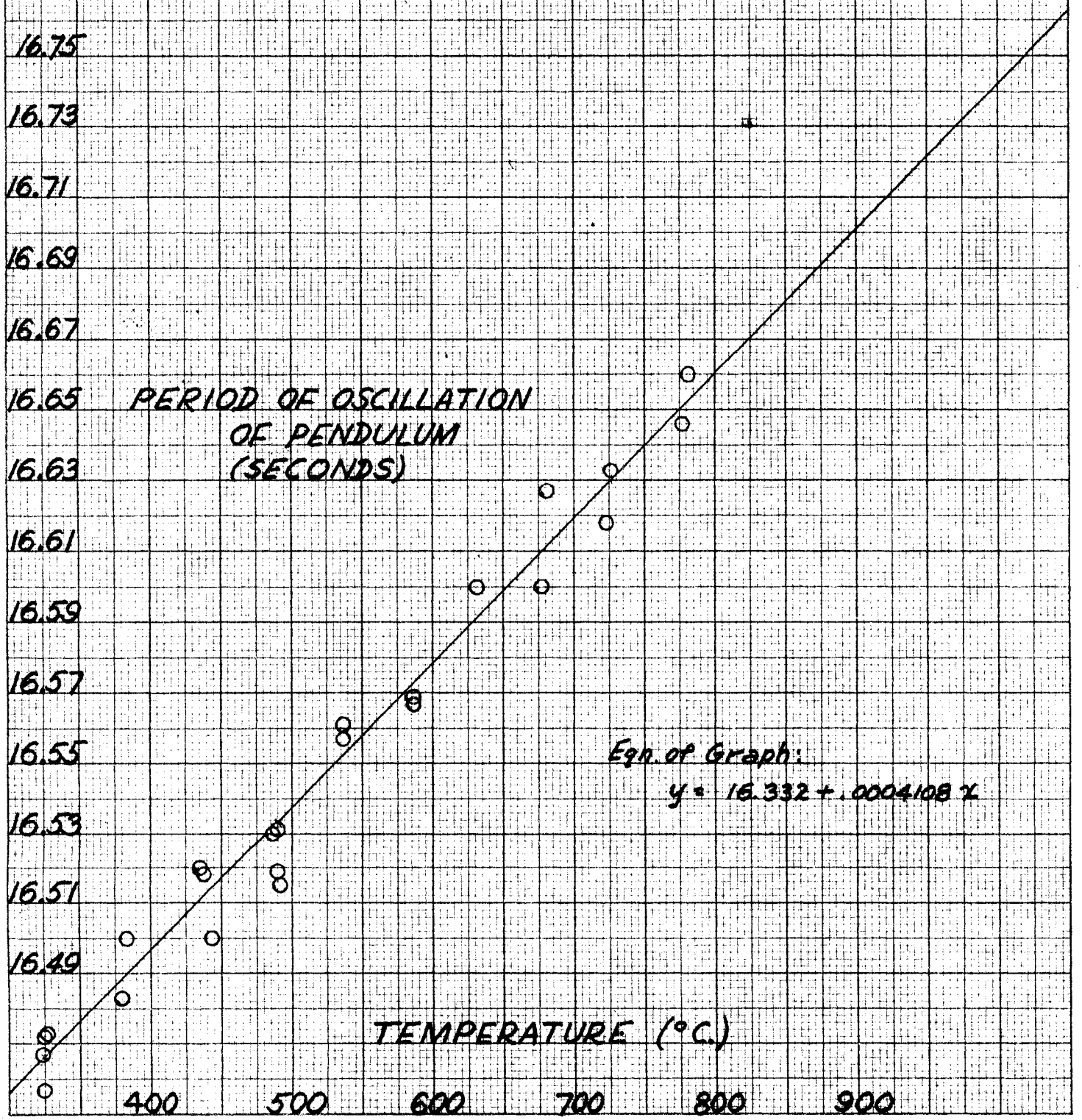


TABLE VII

Moment of inertia of the pendulum as a function of the temperature of the liquid in the sphere. These data correspond to the graph in Figure X.

T_s	I_1	I_{Na}	I	I_t	I_c	$I_c - I_t$
328	389.4	80.4	1592.4	2062.2	2061.1	1.1
331	389.4	80.4	1592.7	2062.5	2061.4	1.1
385	390.2	70.5	1596.4	2066.1	2066.5	0.6
383	390.2	79.5	1596.1	2065.8	2066.3	0.5
437	391.0	78.7	1601.8	2071.5	2071.5	0.0
462	391.3	78.3	1604.3	2073.9	2073.9	0.0
459	391.3	78.3	1604.0	2073.6	2073.6	0.0
488	391.7	77.9	1605.1	2074.7	2076.3	1.6
492	391.8	77.8	1605.6	2075.2	2076.8	1.6
537	392.4	77.2	1612.3	2081.9	2081.0	0.9
558	392.8	76.8	1613.1	2082.7	2083.0	0.3
587	393.3	76.4	1616.2	2085.9	2085.8	0.1
608	393.6	76.1	1618.3	2088.0	2087.8	0.2
626	393.8	75.7	1618.4	2087.9	2089.5	1.6
677	394.9	75.1	1625.1	2095.1	2094.4	0.7
727	395.7	74.2	1630.6	2100.5	2099.1	1.4
724	395.7	74.3	1628.4	2098.4	2098.8	0.4
728	395.7	74.2	1630.7	2100.6	2099.2	1.4
777	396.4	73.4	1634.1	2103.9	2103.9	0.0
826	397.2	72.6	1636.3	2106.1	2108.6	2.5
876	398.0	71.8	1644.9	2114.7	2113.4	1.3

Legend:

T_s is the temperature in the sphere

I_1 is the moment of inertia of inertia piece #1

I_{Na} is the moment of inertia of the sodium sphere

I is the combined moment of inertia of the rest of the pendulum

I_t is the total moment of inertia of the pendulum

I_c is the calculated total moment of inertia of the pendulum, given by the equation:

$$I_c = 2029.8 + .09536T_s$$

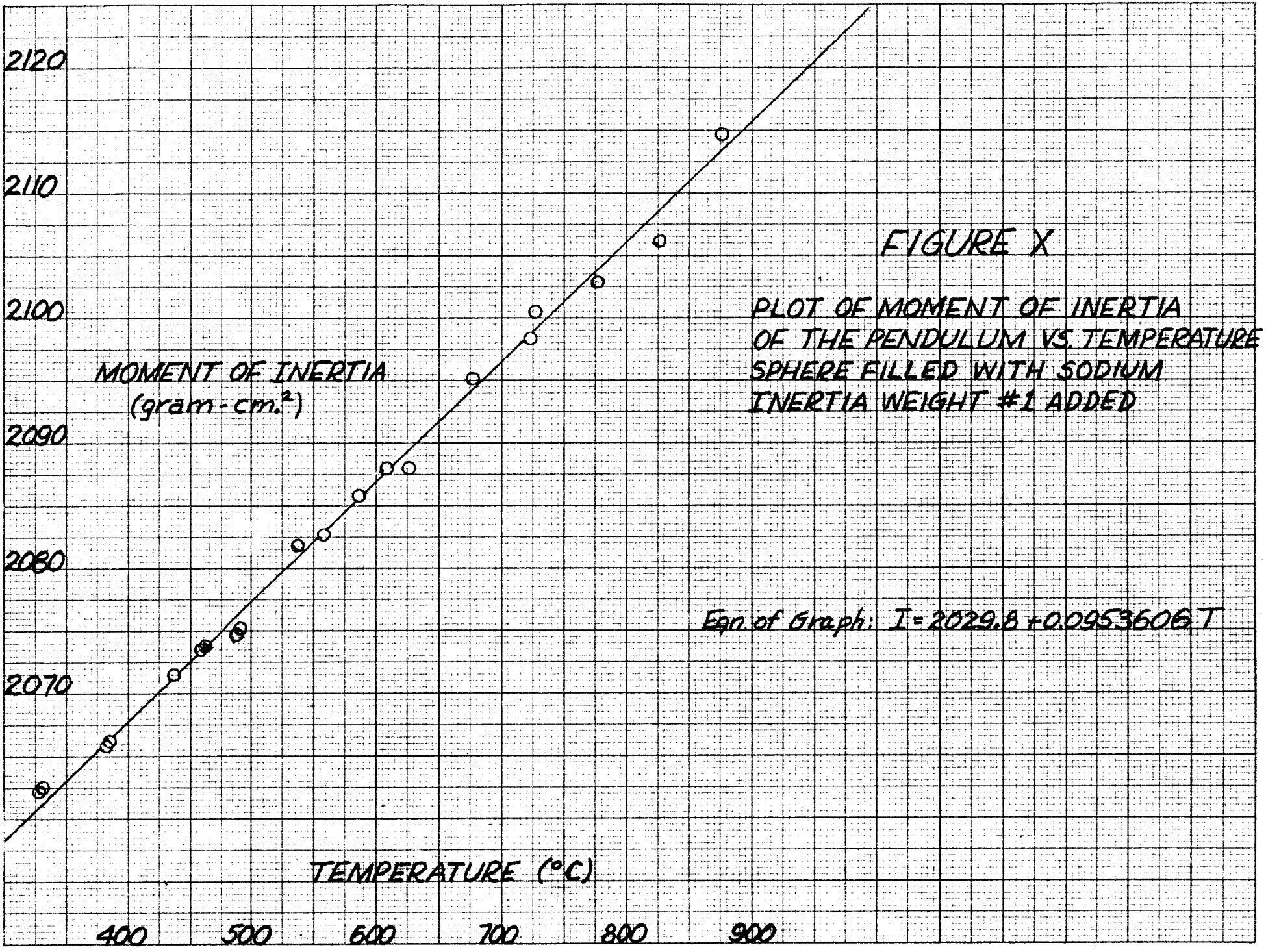


FIGURE X

PLOT OF MOMENT OF INERTIA OF THE PENDULUM VS. TEMPERATURE SPHERE FILLED WITH SODIUM INERTIA WEIGHT #1 ADDED

Eqn. of Graph: $I = 2029.8 + 0.0953606\,T$

TABLE VIII

Logarithmic decrement (base 10) of the external damping of oscillations of the pendulum as a function of the temperature of the sphere. These data are shown graphically in Figure XI.

T_1	T_s	δ_x
250-248	324	.00096
255-252	329	.00106
302-297	382	.00126
350-348	435	.00124
350-347	435	.00132
402-390	484	.00140
401-395	485	.00141
500-492	582	.00166
503-495	585	.00178
554-542	626	.00164
554-541	626	.00172
609-590	679	.00203
609-590	679	.00180
658-643	730	.00206
712-686	779	.00226
711-687	779	.00186
759-738	828	.00210
802-780	872	.00230
805-782	875	.00191

Legend:

T_1 is the temperature range over which the external damping was measured, as indicated by the lower thermocouple.

T_s is the effective temperature in the sphere

δ_x is the $\log_{10}$ decrement of the external damping of oscillations.

FIGURE XI

PLOT OF LOG_{10} DECREMENT OF THE EXTERNAL DAMPING VS. TEMPERATURE

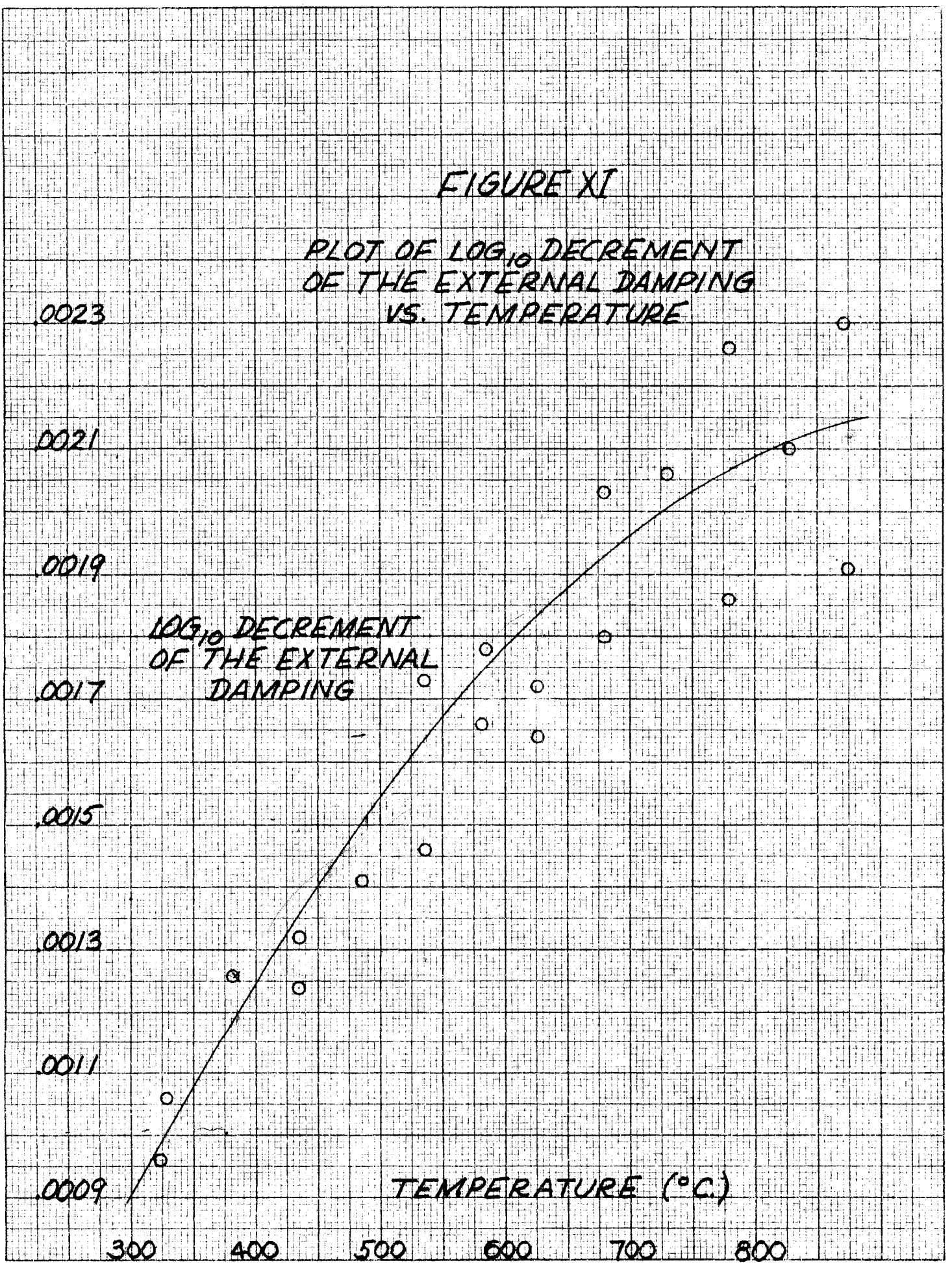


TABLE IX

Experimental Data Used to Calculate the Viscosity of Sodium

t	T	T_o	δ_t	δ_x	R	d	I	n_o	n_N
328	17.635	17.432	.01969	.00233	2.2292	.8720	2061.1	0.334	0.310
331	17.600	17.433	.01944	.00233	2.2293	.8711	2061.4	0.322	
327	17.602	17.431	.01967	.00233	2.2292	.8720	2061.0	0.332	
385	17.640	17.455	.01893	.00290	2.2315	.8578	2066.5	0.281	0.277
383	17.590	17.454	.01885	.00290	2.2314	.8582	2066.3	0.277	
437	17.633	17.477	.01843	.00295	2.2355	.8449	2071.5	0.263	0.251
439	17.625	17.477	.01819	.00295	2.2336	.8444	2071.7	0.254	
459	17.647	17.486	.01785	.00299	2.2345	.8394	2073.6	0.241	0.245
462	17.647	17.487	.01781	.00299	2.2347	.8387	2073.9	0.240	
488	17.630	17.498	.01812	.00322	2.2357	.8323	2076.3	0.244	0.235
493	17.637	17.500	.01810	.00322	2.2359	.8310	2076.8	0.244	
537	17.660	17.518	.01829	.00371	2.2381	.8202	2081.0	0.235	0.220
587	17.690	17.539	.01817	.00396	2.2402	.8078	2085.8	0.225	0.206
628	17.670	17.556	.01757	.00387	2.2418	.7977	2089.7	0.209	0.196
626	17.687	17.556	.01707	.00387	2.2418	.7982	2089.5	0.192	0.197
677	17.680	17.577	.01775	.00442	2.2450	.7856	2094.4	0.199	0.187
677	17.700	17.577	.01716	.00442	2.2450	.7856	2094.4	0.180	
724	17.737	17.597	.01760	.00474	2.2470	.7739	2098.8	0.186	0.175
728	17.687	17.598	.01754	.00474	2.2471	.7730	2099.2	0.184	
777	17.746	17.619	.01688	.00474	2.2493	.7608	2103.9	0.166	
777	17.708	17.619	.01735	.00474	2.2493	.7608	2103.9	0.180	
825	17.771	17.639	.01636	.00484	2.2514	.7489	2108.6	0.150	
826	17.762	17.639	.01650	.00484	2.2514	.7489	2108.6	0.154	
877	17.788	17.661	.01678	.00484	2.2535	.7361	2113.4	0.165	
876	17.783	17.660	.01675	.00484	2.2535	.7364	2113.3	0.164	

Legend for TABLE IX:

t = temperature (°C.)
T = period of oscillation of the pendulum with the sphere filled with sodium (seconds)
T_o = period of oscillation of the pendulum with the sphere empty. (seconds)
δ_t = total logarithmic decrement of the damping of oscillations (base e)
δ_x = logarithmic decrement of the external damping (base e)
R = radius of the sphere (cm.)
I = moment of inertia of the pendulum (gm.-cm.^{2})
n_o = observed viscosity (centipoises)
n_N = values of the viscosity of sodium obtained by the Naval Research Laboratory. (NRL Report No. C-3287, page 14)
d = density of sodium (gm./ml.) Values for the density were obtained from NRL Report No. C-3287, page 37.

The radius of the sphere was corrected for expansion due to increased temperature. Coefficients of thermal expansion of Type 303 stainless steel were obtained from Nickel Alloy Steels, 2nd edition, Section 7, page 17, The International Nickel Co., Inc., New York, N. Y. (1947).

Sample Calculation of Viscosity

Data (from TABLE IX):

Temperature ..... 328°C. = t
Moment of Inertia .....2061.1 gm-cm^2 = I
Period (sphere filled)... 17.635 seconds = T
Period (sphere empty).... 17.432 seconds = T_o
Total Damping ($\log_e$)..... 0.01969 = δ_t $\quad \delta_t - \delta_x = \delta$
External Damping ($\log_e$).. 0.00233 = δ_x
Radius of Sphere..... 2.2292 cm. = R
Density of sodium ..... 0.872 gm/ml = d
Trial value of viscosity of sodium ....0.324 cps. = n

Equations:

$$n = \frac{a^2R^2\pi d}{4q^2 T}\left\{1 - (1-\mu)^{\frac{1}{2}}\right\}^2$$

$$\mu = \frac{3qI\,\delta}{2\pi^2 a^2 R^5 d}\,(T^2/T_o^2 + 1)$$

$$q = 2 - \frac{(gR-1)}{(gR-1)^2 + h^2R^2}$$

$$g = \sqrt{\frac{\pi d}{Tn}\,(1 - \delta/4\pi}$$

$$a = 1 - \delta/4\pi$$

$$h = \sqrt{\frac{\pi d}{Tn}\,(1 + \delta/4\pi)}$$

$\delta/4\pi$	0.001381
$1 - \delta/4\pi$	0.998619
$\pi ad/Tn$	47.879
$\pi d/Tn\,(1 + \delta/4\pi)$	48.012
g	6.9193
gR - 1	14.4245
$(gR-1)^2$	208.07
h^2R^2	238.59
$(gR-1)/[(gR-1)^2 + h^2R^2]$	0.0323
q	1.9677

$1+T^2/T_o^2$ 2.0234

$1.5 I_q \delta (1 + T^2/T_o^2)$ 213.74

$\pi^2 a^2 R^5 d$ 473.72

μ 0.45119

$(1 - \mu)$ 0.54881

$(1 - \mu)^{\frac{1}{2}}$ 0.74082

$\left\{1 - (1 - \mu)^{\frac{1}{2}}\right\}^2$ 0.067174

$a^2 R^2 \pi d \left\{1 - (1 - \mu)^{\frac{1}{2}}\right\}^2$ 0.9119

$4q^2 T$ 273.12

n 0.334

DISCUSSION OF DATA

The accuracy of the measured viscosities of sodium listed in TABLE IX depends on the accuracy with which the temperatures, periods of oscillation, and logarithmic decrements of the damping were measured. The temperatures and the periods of oscillation were measured with sufficient accuracy to make the errors from these sources negligible in comparison with the error caused by inaccuracies in the values of the logarithmic decrements of the damping.

Discussion of Temperature Measurement

Using the experimental procedure outlined on page 28, the temperature of the sodium at the beginning of each run could be measured within four degrees. This error is allowed on the basis of a check made on the thermocouple inserted in the sphere, using the melting point of $K_2Cr_2O_7$ as a standard temperature. The measured melting point of the $K_2Cr_2O_7$ was 400°C., which is 2.5 degrees greater than the standard figure of 397.5°C. for the melting point of $K_2Cr_2O_7$.

Because the furnace had to be turned off during the runs, the temperature dropped; and all data were actually taken over a temperature range. The magnitudes of the temperature ranges are listed in TABLE III; and this table was used to estimate the effective temperatures of the sodium at which data were taken.

An observation of TABLE III reveals that at temperatures up to 533°C. inside the sphere, the temperature does not

start to drop until three to four minutes after the furnace was turned off. Since this time represents about two-thirds of the time required for the runs to be made, and since even after three to four minutes the temperature drop is small, the beginning temperature was taken as the effective temperature of the run.

At temperatures above 533°C., the overall temperature drop during the runs was greater. The effective temperatures in this range were taken as the weighted averages of the temperature-time data given in TABLE III. For most of the runs made above 533°C., the weighted average of the temperature during the run is very nearly equal to the beginning temperature minus forty per cent of the temperature drop. It is felt that temperatures estimated in this way should not be in error by more than a few degrees.

Because of the necessity of the temperature estimation described above, an accuracy of only ±6°C. is claimed for the temperatures at which the viscosities are reported. Since the viscosity of sodium varies from 0.334 centipoises at 325°C. to 0.164 centipoises at 876°C., an error of six degrees in the temperature will cause an error of less than one per cent in the value of the viscosity.

Discussion of the Measurement of the Periods of Oscillation

Three series of measurements of the period of oscillations were made. The first of these series was made with

the sphere filled and inertia piece #1 added to the pendulum. The results of this series are listed in TABLE IV. Series 2 and 3 were made with the sphere empty and with inertia pieces #1 and #2, respectively, added to the pendulum. The results of Series 2 and 3 are shown in TABLES V and VI, respectively.

For each series of runs, the period of oscillation was found to be a linear function of the temperature of the sphere, as shown by the graphs in Figures VII, VIII, and IX which correspond to Series 1, 2 and 3, respectively. The equations of the graphs, found by the method of least squares, are:

$$\text{Series 1: } T = 17.496 + .0003079T_s$$

$$\text{Series 2: } T_1 = 17.294 + .0004179T_s$$

$$\text{Series 3: } T_2 = 16.332 + .0004108T_s$$

wherein T is the period of oscillation and T_s is the temperature in the sphere.

The period of oscillation was found by measuring the time interval required for approximately twenty complete periods. Since the total time interval was measured with an accuracy of ± 0.2 seconds, the error in the measurement of the periods of oscillation should be about $\pm$ 0.01 seconds. The average errors found for Series 1, 2 and 3 were ± 0.013, $\pm$ 0.012, and ± 0.008 seconds, respectively. Since the periods were all in the neighborhood of seventeen seconds, an error of ± 0.01 seconds is an error of approximately ± 0.06 per cent.

Any errors made in the measurement of the periods in Series 2 and 3 are reflected in the calculation of the moment of inertia (See page 26.). The moment of inertia was calculated at various temperatures using values of T_1 and T_2 obtained from the equations for the periods of oscillation in Series 2 and 3, respectively. These calculated values are listed in TABLE VIII under the column headed "I". The total moment of inertia, "I_t" in TABLE VIII, is found by adding the moments of inertia piece #1 and of the sodium sphere to the values of "I".

I_t is a linear function of the temperature as shown by the graph in Figure X. The equation of this graph, found by the method of least squares, is:

$$I_t = 2029.8 + 0.09536T_s$$

The average error in the total moment is ± 0.8, Since the total moment of inertia is of the order of 2000 gm-cm^2, the average error is about ± 0.04 per cent.

Discussion of the Damping Determinations

The total damping of the oscillations of the pendulum at various temperatures is shown in TABLE IV. The logarithmic decrements (base 10) of the damping listed here were measured with the sphere filled with sodium. They include both the external damping and that due to the viscosity of the sodium rotating inside the sphere.

From TABLE IV it is apparent that the logarithmic

decrement decreased regularly as the temperature was increased from 327°C. to 459°C. There was an increase in the damping at 493°C.; and as the temperature was increased above 493°C., the damping decreased, but in an irregular manner.

The irregularities in the total damping are attributed to the external damping for which measured values are listed as a function of temperature in TABLE VIII. At lower temperatures, the external damping was small in comparison to the total damping and did not greatly affect its magnitude. Probably because of this, the precision with which the damping was measured at temperatures up to 459°C. varied from a few hundredths of a per cent up to about one per cent. At higher temperatures, the external damping became larger because of the increased viscosity of the gases surrounding the pendulum; whereas the total damping decreased. Consequently, the size of the external damping affected appreciably the measured value of the total damping. Furthermore, the fluctuations in the measured external damping became larger at the higher temperatures, as indicated in TABLE VIII by the greater discrepancy between external dampings measured at approximately the same temperatures. This was reflected in the decreased precision of measurement of the total damping at temperatures above 493°C. The precision in the higher temperature range varied from 0.1 to 3 per cent.

Any error in the measured damping is reflected by an error approximately 2.3 times as large in the calculated viscosity. Therefore, the viscosity values listed in TABLE IX at temperatures up to 493°C. may be in error by approximately two per cent. The viscosity values listed above 493°C. may be incorrect by as much as seven per cent because of the lack of precision in the measurement of the damping.

It should also be mentioned here that the magnitude of the external damping at higher temperatures casts some doubt on the reliability of the viscosities reported at the upper end of the temperature range. In deriving the equations for calculating the viscosity it was assumed that the external damping could be neglected in comparison with the damping due to the liquid enclosed in the sphere. Since the external damping increased with temperature, the viscosity equations became more and more approximate as the temperature increased.

Rectification of Viscosity Data

The measured viscosities of sodium were fitted to equations of the type proposed by Andrade (1) and by Batschinski (2). Using the method of least squares on the viscosities measured over the entire temperature range, the following equations were obtained:

Andrade Equation

$$(1)\quad nv^{1/3} = (.001046 \pm .000032)e^{(814.8 \pm 27.7)/vT}$$

Batschinski Equation

(2) $n = .0006478/(v - .9371)$

where n = viscosity of sodium, poises

v = specific volume of sodium, ml/gram

T = absolute temperature, ^{o}K.

The viscosities calculated from these equations are listed in TABLE X and plotted in Figure XII as a function of the temperature.

The per cent deviations between the calculated and observed viscosities are also listed in TABLE X. Using Equation (1), the deviation between the observed and calculated values of the viscosity varied between 0.31 and 9.06 per cent with an average deviation of 3.75 per cent. Equation (2), the Batschinski Equation, yields calculated values of the viscosity which deviate from the observed values by from 0 to 8.09 per cent with an average deviation of 4.03 per cent.

Equation (1) fits the data at the upper and lower ends of the temperature range better than Equation (2). On the other hand, Equation (2) gives the best fit at the middle of the temperature range. By observation, Equation (1) seems to be the more reliable equation.

Since the viscosities measured at lower temperatures were believed to be more reliable than those at the higher temperatures, the constants in the Andrade Equation were also evaluated by using the viscosity values between 325 and 493°C. The following equation was obtained:

$$(3)\quad nv^{1/3} = (.001073 \pm .000034)e^{(793.5 \pm 8.3)/vT}$$

TABLE X

Comparison of the experimentally determined values of the viscosity of sodium with those calculated by the Andrade and Batschinski equations. These data are shown graphically in Figure XII.

T	n_o	n_A	n_B	r_A	r_B
328	.334	.326	.309	-2.4	-8.1
327	.332	.326	.309	-1.8	-7.4
331	.322	.323	.307	0.3	-4.9
383	.277	.289	.284	4.2	2.5
385	.281	.287	.283	2.1	0.7
439	.254	.260	.262	2.3	3.0
437	.263	.261	.263	-0.8	0.0
459	.241	.251	.255	4.0	5.5
462	.240	.250	.254	4.0	5.5
488	.244	.240	.246	-1.7	0.4
493	.244	.238	.244	-2.5	0.0
537	.235	.223	.230	-5.4	-2.2
587	.225	.209	.215	-7.7	-4.6
628	.209	.199	.204	-5.0	-2.4
626	.192	.200	.205	4.0	6.3
677	.199	.189	.193	-5.3	-3.1
677	.180	.189	.193	4.8	6.7
724	.186	.181	.182	-2.8	-1.6
728	.184	.180	.182	-2.2	-1.4
777	.166	.172	.172	3.5	3.5
777	.180	.172	.172	-4.6	-4.6
825	.150	.166	.162	9.1	7.4
826	.154	.166	.162	6.6	4.9
877	.165	.159	.154	-3.8	-7.1
876	.164	.159	.154	-3.1	-6.5

Legend:

T = temperature (°C.)
n_o = observed viscosity of sodium
n_A = viscosity of sodium calculated from the Andrade equation fitted to the observed data
n_B = viscosity of sodium calculated from the Batschinski equation fitted to the observed data
r_A = per cent deviation between n_A and n_o
r_B = per cent deviation between n_B and n_o

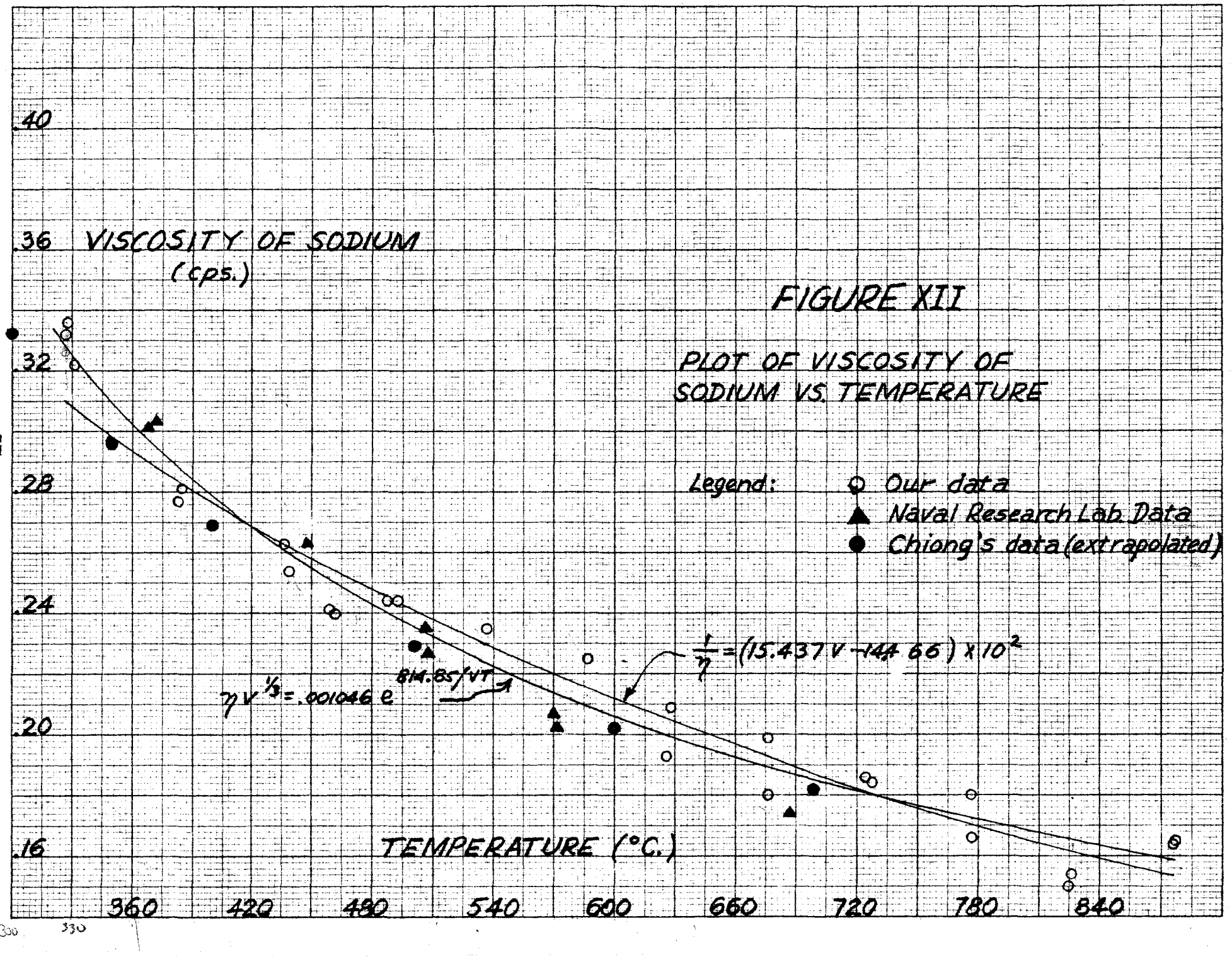


FIGURE XII

PLOT OF VISCOSITY OF SODIUM VS. TEMPERATURE

Viscosities calculated using Equation (3) deviate from the observed viscosities in the temperature range from 328-493°C. by from 0 to 6.1 per cent with an average deviation of 2.53 per cent.

SUMMARY

The viscosity of sodium was measured over a temperature range of from 327 to 877°C. using a viscometer based on the method of Chiong and Andrade (3). The measured viscosities were found to fit an equation of the type proposed by Andrade (1):

$$nv^{1/3} = (.001046 \pm .000032)e^{(814.8 \pm 27.7)/vT}$$

where n = viscosity of sodium, poises

v = specific volume of sodium, ml/g

T = absolute temperature, °K.

The percentage deviation between the observed viscosity and the viscosity calculated from the above equation varied between 0.31 and 9.06 with an average deviation of 3.75 per cent. This average deviation agrees with the average error expected in the measured viscosities because of the error in the measurement of the logarithmic decrements of the damping.

The damping was measured with greater precision at temperatures less than 493°C. because of the small external damping. Since the precision of measurement of the damping was always less than one per cent in this temperature range, the viscosities reported below 493°C. are believed to be in error by no more than 2 1/2 per cent.

At temperatures greater than 493°C., the external damping was sufficiently large that precise measurement of the damping due to the sodium became impossible. In this temperature range, lack of precision in the measurement of the damping

was responsible for a maximum deviation of 7 per cent in the measured viscosity. The average error at temperatures above 493°C. is about ±5.0 per cent.

The accuracy of the viscometer depends largely on the accuracy with which the damping due to the enclosed liquid can be measured. This, in turn, is largely determined by the amount of external damping. Therefore, reduction of the external damping appears to be necessary to increase the sensitivity of the instrument. Reduction of the external damping may be accomplished by reducing the pressure of the gases surrounding the pendulum and by decreasing the external surface of the pendulum. Since the viscosity determinations were carried out with a pressure of less than one micron in the viscometer, any further reduction in the pressure will be both too expensive and too troublesome to be worth the small decrease in external damping it will effect. On the other hand, some reduction in the external damping may be readily accomplished by decreasing the size of some of the parts of the pendulum.

The viscometer may be used without modification to measure viscosities of one centipoise or higher with an accuracy of one per cent or better. A viscosity of this magnitude is sufficient to render the external damping negligible at all temperatures in comparison to the damping due to the viscosity of the enclosed liquid.

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