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I hereby recommend that the thesis prepared under my supervision by Sidney Sam Kuniansky *entitled* A Study of the Flow of Liquids Through V-Notch Weirs

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SYMBOLS

A	Area of notch wet by liquid	Feet^2
a	Constant in V-notch equations	Feet
a'	Constant in V-notch equations (2.5a)	Feet
B	Weir flow coefficient for semicircular weir equations with variable exponent, n	Ft.^{3-n} per sec.
C	General weir flow coefficient	Dimensionless
D	Weir flow coefficient for V-notch equation with variable exponent, n	Ft.^{3-n} per sec.
d	Diameter of semicircular notch	Feet
E	Weir flow coefficient for V-notch equations	$\text{Ft.}^{3-2.5}$ per sec.
F	Weir flow coefficient for V-notch equations with variable exponent, n	$\text{Ft.}^{2.5-n}$ per sec.
f	Head of liquid above notch crest	Centimeters
g	Acceleration due to gravity	32.17ft. per sec.^2
H	Head of liquid above notch crest	Feet
h	Head due to velocity of approach	Feet
K	Value of constant for semicircular notch for each $\frac{d}{H}$	Dimensionless
k	Proportionality constant	Dimensionless
L	Crest length of rectangular notches	Feet
l	Length	Feet
m	Mass	Pounds
n	Exponent of head, variable for different notches	Dimensionless
P	Height of notch crest above channel bottom	Feet

p	Number of observations made with a given notch	Dimensionless
Q	Volume rate of flow	Ft.^{3} per sec.
q	Volume rate of flow	Gal. per min.
r	Constant exponent	Dimensionless
S	Slope of side of V-notch expressed as a decimal	Dimensionless
s	Number of end contractions for rectangular notch	Dimensionless
t	Time	Seconds
U	Viscosity	Lb. per ft.sec.
u	Constant in V-notch equations	Ft.$^{1-r}$
v_0	Velocity of approach	Ft. per sec.
W	Mass rate of flow	Lb. per sec.
ρ	Density	Lb. per ft.^{3}
θ	Angle of V-notch weir	Dimensionless

ABSTRACT

In carrying out the experimental work, two major changes were made in the procedure considered as standard by most weir flow investigators. Instead of weighing the overflow from the weir, the writer measured the rate of flow with the use of two water meters, each covering a definite range, and the characteristics of the two meters were carefully determined. The second innovation was the use of a cathetometer for measuring the head by sighting the water level in a gage glass connected appropriately to the tank. As a rule, most investigators use a hook gage and a stilling box for measuring the head.

Equations of the form $Q = DH^n$ were used by most investigators to represent their data. However, the writer found that an equation of the form $Q = E(H+a)^{2.5}$ gave much better results. The value of <u>a</u> was so small that no quantitative considerations were made involving that term.

The results of rather limited experiments with solutions of different surface tensions led to the conclusion that the surface tension of the liquid has very little effect upon weir flow.

The experimental data were extremely accurate in comparison with data taken by other investigators. The calculated results are compared with the observed values of the rate by the use of the expression $dq/q^{.6}$ which is a better index for

comparing the closeness of the values of the calculated and observed rates than percentage variation in view of certain limitations in the experimental data.

INTRODUCTION AND HISTORY

A weir is a notch or opening in an obstruction placed in a channel through which the liquid carried by the channel is constrained to flow. The shape of the opening is used to serve as a means of classifying different weirs, such as triangular or V-notch weir, rectangular weir, etc. Usually the notch is cut in a plate which is stationed in a vertical position, although this is not necessary.

Very little of the history of the weir has been compiled, the only complete history available dealing with dams or spillways used for diverting large quantities of water. Discussions relating to dams and spillways are not included within the scope of this treatise.

The word weir was derived from the Anglo-Saxon wer and the German wehr, and was originally used to designate a dam or some other similar artificial obstruction built across a water channel for such reasons as creation of a storage basin, making the channel deeper and hence more navigable, or providing water for power and irrigation purposes.[2] As used today, the term weir is reserved for those obstructions which are used for measuring the volume rate of flow of some liquid in contrast to those which merely serve to provide an exit for liquid in some confined region. Not until about the eighteenth century was an attempt made to measure the flow of liquids through notches by estimating the head or depth above the bottom of a notch cut out in a rectangular shape in the bottom of a dam. It was from this time on that the meanings of the two words, weir and dam, took on individual significance.

The basis of almost all formulae used in subsequent weir treatises comes from an argument in 1717 by Poleni who regarded the flow of water over notches (rectangular notches in his original work) as the sum of the elemental flows through a series of horizontal slits, one above the other.[3] Experimental attempts to determine the exact relationship between the head and the rate of flow were not made until the nineteenth century - the best known of the earlier investigations being made in France by Boileau, Poncelot, Eytelwein, and Bazin.[4] Professor James Thomson,[5] carrying out experiments on a rectangular notch using exceedingly rough equipment, arrived at expressions for the relation between head and rate of discharge which were remarkably accurate. Thomson also carried out experiments on the 90° V-notch. It was these latter experiments that prompted Barr[6] to carry out numerous experiments on the 90° V-notch. These were extremely complete and accurate; nevertheless his treatment of the data obtained was such that he failed to realize the full worth of his experiments. This same statement applies to almost all work done on the V-notch, and, to an extent not quite as great, to weir experiments in general. Very little research work was done in the United States on weir flow until James B. Francis[7] began his experiments on rectangular weirs at Lowell, Massachusetts, in 1848. The work done by him is still considered to be among the best done to date. Since the work by Francis, much research has been done on weirs, with most of the work dealing with rectangular notches. The result of most of this work has been countless formulae for which not too great accuracy is claimed. The conclusion drawn by almost all of the experimenters whose work has come to the attention of the author is that the weir, as an instrument for measuring the rate of flow, cannot be depended upon to give greater than one percent accuracy.

An exception to the abandoning of the weir as an accurate means of measuring the rate of flow of liquids is found in the work of Yarnell,[8] who built a V-notch meter for the Richmond Station of the Philadelphia Electric Company for which meter an accuracy of one-half of one percent was guaranteed, whether using a 90° weir plate or smaller ones. This weir meter, with a maximum capacity of one million pounds of water per hour, was to be used as a standard piece of testing equipment for calibrating orifices and other types of meters in the station.

This guarantee of one-half percent accuracy is for duplication of results and not for obtaining a general formula to hold with that degree of accuracy. This restriction does not in any way detract from the value of the experiments; however, it does give some indication of the strict empiricism met everywhere in weir research. Where there are theoretical approaches to the subject, the results are rather unsatisfactory. In 1903 Hermanek[3] applied some of the principles of hydrodynamics to weir flow without arriving at any results which relieved the situation. Several other attempts to apply the principles of hydrodynamics have been made, particularly by Searle, Mise, and Richardson. However, in spite of this work, experimenters have found empirical formulae more accurate than theoretical expressions.

GENERAL DISCUSSION

Some of the terminology commonly used in weir discussions is arrived at logically; others have illogical evolutions. The channel of approach is the confining volume for the flow of the liquid immediately upstream from the weir plate. The weir plate is the obstruction placed in the channel and the notch is the opening in this weir plate through which the liquid flows. The mean lineal velocity of the liquid in the approach channel is termed the velocity of approach, the equivalent head due to the velocity of approach is termed the velocity head (and this quantity, when appreciable, should be measured at the same distance upstream as the observed head). The total head is the sum of the observed head and the velocity head. Paradoxically the bottom of the notch is called the crest. The overflowing stream is called the nappe, a French word meaning sheet. A term rarely met is the velocity of retreat, this being the mean lineal velocity of the liquid just as it leaves the weir.

There are two broad general classifications of weirs - the weir with free overfall, which has the water level below the weir at a height less than that of the notch crest, and the submerged weir, which has the water level on the downstream side of the weir plate above the notch crest. Submerged weirs are not included in this treatise. Weirs with free overfall are subdivided in several different ways. The water in flowing over the notch comes into contact with the crest and sides of the notch. Classified with reference to the form of the crest, weirs may be sharp-crested or broad-crested. The sharp-crested weir has a sharp upstream edge so that the liquid in passing through comes in contact with only a line; in contrast, the broad-crested weir has either a rounded upstream edge or a crest so broad that the flowing liquid contacts a surface. Sharp-crested weirs are used almost exclusively for measuring the rate of flow of liquids whereas broad-crested weirs are usually part of a dam or some similar structure where the measurement of flow is a secondary function.

Sharp-crested rectangular weirs are divided into two general classes, suppressed weirs and sharp-edged weirs. Both of these types have crest contraction which is the contraction of the underside of the nappe caused by the action of the vertical components of velocity just upstream from the weir. In the sharp-edged weir, the sides of the notch have sharp upstream edges so that the nappe is contracted in width and the weir is said to have end contractions. If the weir has a length equal to the width of the channel immediately upstream from the weir, the nappe suffers no contraction in width and the weir is said to have end contractions suppressed. Much work has been done on both suppressed and sharp-edged weirs, most weir research being done on these two. It is obvious that the velocity of approach factor is greater in the case of the former.

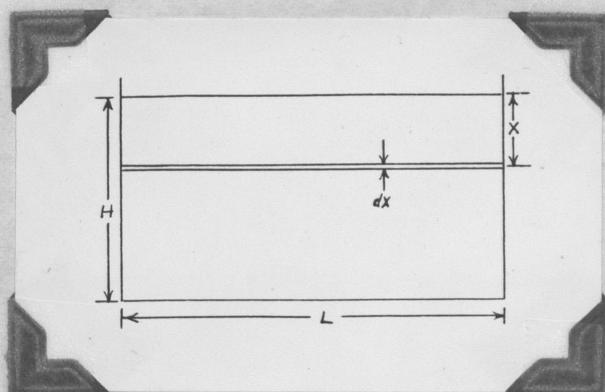
The usual triangular or V-notch weir is sharp-edged and located in the channel in such a way that the nappe is completely contracted on the two sides and at the bottom. The description of the nappe as given by Dodge and Thompson[9] is expressive of the overflowing stream: "The nappe is triangular at the plane of the weir, changing to a crescent-like cross section in a short distance, and finally, if allowed to fall far enough, to a nearly circular jet."

In this treatise, the interest is in only sharp-edged weirs in general, and V-notch weirs in particular. All of the following remarks must be interpreted with this fact in mind. There is a downstream curvature to the surface of the liquid in the vicinity of the weir, which is called surface contraction, the curved surface sometimes being referred to as the down-drop curve. Surface curvature is not perceptible at more than twice the head upstream from the weir.[10] The vertical contraction of the nappe includes both surface and crest contractions. Incomplete contraction of the nappe occurs when the weir crest is so near the bottom or the sides of the weir are so near the walls of the channel that the velocity components parallel to the face of the weir are affected. Results obtained by Barr[6] show that the prevention of inward flow of water at the sides of the notch, whether caused by narrowness of the approach channel or by the roughness of the up-stream surface of the notch, produces an increase in quantity of flow over the notch. All the factors listed above as tending to increase the rate of flow enlarge the vena contracta of the nappe.

THEORETICAL FORMULAE FOR DIFFERENT NOTCH SHAPES

The derivation of theoretical formulae for sharp-edged notches probably preceded even the earliest researches on weirs, and, despite many new attempts, these original formulae have not been improved upon by later considerations of theoretically derived weir flow equations. This does not imply that there are no improvements to be made. Indeed, erroneous assumptions in the theoretical equations to be derived will be pointed out.

Rectangular Notches



Consider the flow across any element of area Ldx in the plane of the weir at a depth x below the water surface, the depth being measured far enough upstream so that the surface is horizontal. The volume of liquid passing through the element of area per unit time is equal to the product of the area and the velocity of flow through the area.

$$dQ = v\,dA$$

but $v = \sqrt{2gx}$, neglecting the velocity of approach.

$$Q = \int_0^H \sqrt{2gx}\, dA$$

This formula is general for notches of all shapes and it is from this step that specific derivations will be started.

For the rectangular notch dA = Ldx, therefore

$$Q = \int_0^H \sqrt{2gx} \cdot L dx = \frac{2}{3} L\sqrt{2g}\, H^{1.5}$$

Because of various factors, most of which involve the specific characteristics of each individual set-up, but among which must also be included errors due to invalid assumptions already made, a factor C must be added usually as a single multiplier. This constant seems to vary slightly with the head which would lead one to believe the true weir flow equation must be of a different form than the one above.

$$Q = \frac{2}{3} L\sqrt{2g}\, CH^{1.5}$$

The velocity of approach factor may be included without any difficulty; however, it would seem desirable to eliminate this factor in weir research so as to remove a complicating feature. Applying Bernoulli's Equation to the region between the vertical plane of the weir plate and a vertical plane upstream where the surface is practically horizontal,

$$\frac{v_0^2}{2g} + x = \frac{v^2}{2g}$$

$$v = \sqrt{2g}\left(\frac{v_0^2}{2g} + x\right)^{.5}$$

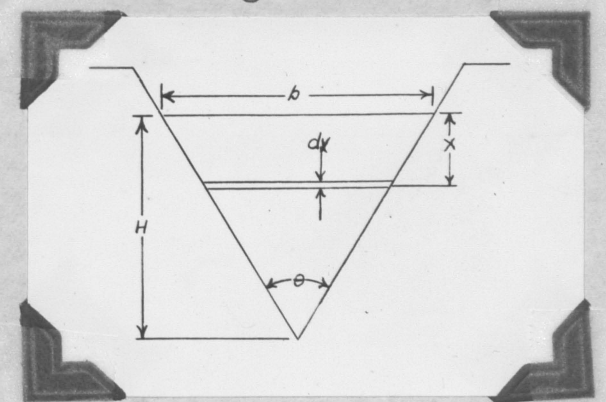
Replacing $\frac{v_0^2}{2g}$ by h, where h is regarded as the head due to the velocity of approach,

$$Q = \int v dA = \int_0^H \sqrt{2g}\,(h + x)^{.5} dx$$

$$Q = \frac{2}{3}\sqrt{2g}\left[(H + h)^{1.5} - h^{1.5}\right]$$

The theoretical formula above assumes v_0 to be uniform in all parts of the channel section and therefore have the value, $v_0 = \frac{Q}{A}$, where A is the area of the cross-section. Corrections for this and other assumptions will be pointed out in a discussion of weir equations as proposed by experimenters to be given in the next section.

Triangular Notches



$$Q = \int \sqrt{2gx}\, dA$$

$$Q = \int_0^H \sqrt{2gx}\ \frac{b}{H}\,(H-x)dx$$

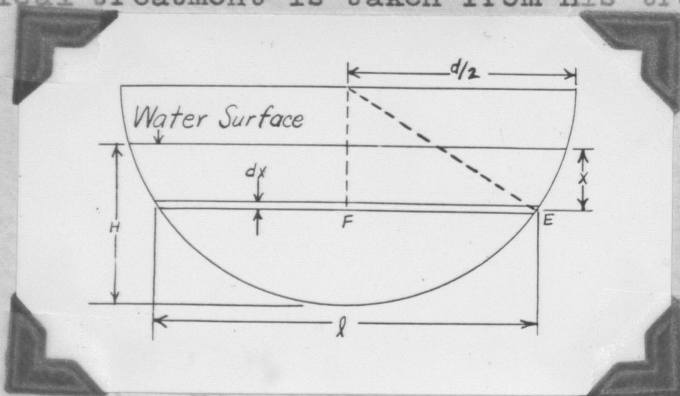
$$Q = \left(\frac{2}{3}\sqrt{2g}\ H^{2.5} - \frac{2}{5}\sqrt{2g}\ H^{2.5}\right)\frac{b}{H}$$

but $b = 2\left(\tan \frac{\theta}{2}\right). H$, and

$$Q = \frac{8}{15}\left(\tan\frac{\theta}{2}\right)\sqrt{2g}\ H^{2.5}$$

Semicircular Notches

One of the few instances where this type of notch was used in experimental work was by Greve[2], and the following theoretical treatment is taken from his treatise.



$$Q = \int_0^H \sqrt{2gx}\ l\,dx$$

$$\text{but} \left(\frac{l}{2}\right)^2 + \left(x + \frac{d}{2} - H\right)^2 = \left(\frac{d}{2}\right)^2$$

$$l = 2\sqrt{2Hx + Hd - H^2 - xd - x^2}$$

$$Q = \int_0^H 2\sqrt{2Hx + Hd - H^2 - xd - x^2}\ \sqrt{2gx}\,dx$$

$$Q = 2\sqrt{2g}\int_0^H \sqrt{x(-H^2 + Hx + Hx - x^2 + Hd - xd)}\ dx$$

$$Q = 2\sqrt{2g}\int_0^H \sqrt{x[-H(H - x) + xH - x) + d\ (H - x)]}\,dx$$

$$Q = 2\sqrt{2g}\int_0^H \sqrt{x(x + d - H)(H - x)}\ dx$$

Let $x = H(1-t)$, $d = fH$, then $dx = -Hdt$

$$Q = 2\sqrt{2g}\int_1^0 \sqrt{H(1 - t)[H(1-t) + fH-H][H - H(1 - t)]}\ (-Hdt)$$

$$Q = 2\sqrt{2g}\ H^{2.5}\int_0^1 \sqrt{(1-t)(f-t)t}\ \ dt$$

$$Q = 2\sqrt{2g}\ KH^{2.5}$$

Values of K must be computed for various ratios of $\frac{d}{H}$ by the use of some numerical integrating method as Simpson's Rule. Some values of K obtained by Greve from Simpson's Rule are:

d/H	2.0	2.5	3.0	3.5	4.0	4.5	5.0
K	.4773	.5518	.6175	.6768	.7313	.7817	.8288
d/H	5.5	6.0	6.5	7.0	7.5	8.0	8.5
K	.8746	.9168	.9537	.9967	1.034	1.070	1.106
d/H	9.0	9.5	10.0				
K	1.141	1.177	1.205				

There are other forms of notches which have been investigated, such as the parabolic notch[2] and the trapezoidal notch. However, because of the difficulty of machining these notches accurately, they are not used extensively and therefore warrent no discussion here. The semicircular notch is given in this discussion, in spite of the fact that it also has been used in research work to only a very limited extent; however, it is included here because of its potentialities. Of all available forms, the semicircular notch can probably be most accurately machined and its true crest most accurately determined. For this reason, in light of work done by the author, this notch might eventually be of great importance in additional research work.

PROMINENT WEIR RESEARCHES AND THEIR RESULTS

Rectangular Notches

A great number of experiments have been carried out on weirs and a large number of equations have been proposed for the relation between head and rate of flow. By far the greater amount of research was done on rectangular weirs, and the equations developed for flow over this notch have, apparently, been given more thorough consideration than equations for other notch forms. All formulae, unless otherwise stated, apply only to water and involve the units of feet, pounds, and seconds, Q being volumetric rate of flow.

The work accepted as standard for rectangular weirs was done by James B. Francis[7] at Lowell, Massachusetts, between 1848 and 1852. The length of the crest, L, varied from 3.7 to 17 feet, being about 10 feet for most of his experiments. The head, H, varied from 8 inches to 1.6 feet. Experiments were carried out on sharp-crested weirs, with and without end contraction. After the first series of experiments, Francis assumed the formula $Q = FLH^n$, with n having a mean value of 1.47. However, as a result of later experiments Francis adopted 1.5 as the value of n. As a result of over eighty experiments, he proposed the two following equations:

$$Q = 0.622 \times \frac{2}{3}\left(L - \frac{sH}{10}\right)\sqrt{2g}\left[(H+h)^{1.5} - h^{1.5}\right]$$

and

$$Q = 0.622 \times \frac{2}{3}\left(L - \frac{sH}{10}\right)\sqrt{2g}\, H^{1.5}$$

The above equations are applicable to weirs with and without end contractions, the factor s being the number of end contractions. For a suppressed weir, s is zero, while for a sharp-edged weir, s is two. The factor $\frac{H}{10}$ was evaluated empirically to fit the data obtained. The first equation provides for the effect of velocity of approach which supplies the velocity head, h, should this factor be appreciable. Both of these equations are seen to be of the same form as the equations previously derived theoretically to cover the cases of rectangular weirs with and without appreciable velocity of approach. In order that the factors $0.622 \times \left(L - \frac{2H}{10}\right)$ properly take care of the flow of water over sharp-edged weirs, the width of the approach channel and the depth beneath the crest should both be greater than 3 H. The velocity head, h, is calculated from $h = \frac{v_0^2}{2g}$, where v_0 is the mean linear velocity in the upstream channel. Thus Q is first determined from the measured H, and this value of Q is used to determine v_0, knowing the area of the channel. A second approximation of v_0 carried out in the same way is sufficiently accurate. The coefficient in the equation is an average value which differed from the extreme values from more than eighty experiments by about three percent.

In 1877-1879, Fteley and Stearns[11] carried out a number of experiments on both sharp-edged and suppressed rectangular weirs. As a result of over fifty experiments on the former type, these experimenters did not find that the $\frac{H}{10}$ factor in Francis' equations was satisfactory. Because of their inability to obtain a satisfactory expression, their conclusion was that weirs with end contractions should not be used, and they recommended the use of suppressed weirs. As a result of numerous experiments on suppressed weirs as well as consideration of Francis' work, the following equations were proposed:

$$Q = 3.31\ LH^{1.5} + 0.007L$$

$$Q = 3.31\ L(H + 1.5h)^{1.5} + 0.007L$$

the 1.5 factor used in correcting for the velocity of approach, h, is an average value, the value changing slightly with the head. Fteley and Stearns list values for various heads ranging from 0.2 to 2.0 feet. These formulae have not been found quite as satisfactory as the ones proposed by Francis.

Bazin[4] carried out a series of excellent experiments in 1886 on suppressed rectangular weirs. As a result of his own experiments and those of his predecessors, he arrived at the following formulae:

$$Q = \left(0.405 + \frac{0.00984}{H}\right) L\sqrt{2g}\ H^{1.5}$$

$$Q = \left(0.405 + \frac{0.00984}{H}\right) L\sqrt{2g}\ (H + 1.69h)^{1.5}$$

The second equation, correcting for the velocity of approach, was further modified by Bazin to bring in the height of the crest above the bottom of the channel, this measurement being given the symbol P below.

$$v_0 = \frac{Q}{A} = \frac{Q}{(P + H)L}$$

$$h = \frac{v_0^2}{2g} = \frac{Q^2}{2g(P + H)^2L^2}$$

Substituting this method of obtaining the velocity of approach in the Bazin equation correcting for this factor, the following working equation is obtained after simplification:

$$Q = \left[0.405 + \frac{0.00984}{H}\right]\left[1 + 0.55\left(\frac{H}{P + H}\right)^2\right] L\sqrt{2g}\, H^{1.5}$$

The form used by Bazin in expressing the weir coefficient as a function of H is of particular interest to the author inasmuch as the same form was found to be the best method of expressing the weir coefficient of V-notch weirs.

Hamilton Smith[12] in 1886, as a result of careful study of work by himself, Francis, Fteley and Stearns, and others, proposed the following formulae for rectangular weirs:

$$Q = C \times \frac{2}{3} L\sqrt{2g}\, H^{1.5}$$

$$Q = C \times \frac{2}{3} L\sqrt{2g}\, (H + 1.4h)^{1.5}$$

The values of C for the two formulae are given by Smith for a wide range of conditions of change in H or change in weir characteristics. These equations, with their accompanying tables for selecting C, have proven to be very valuable.

All of the formulae presented so far have given $\frac{3}{2}$ as the exponent of H, this being in accordance with views of the author concerning use of the theoretical value of the exponent in weir flow equations for all types of notches. However, many experimenters have proposed formulae with the value of the exponent slightly lower than the theoretical one in attempts to obtain formulae which fit their data more closely. Very little of this has been practiced in connection with rectangular weirs, this, however, being a common practice in the study of V-notches.

Alfred A. Barnes in 1916 made use of an exponent less than 1.5 in weir formulae proposed by him. After a very thorough examination of all work done before his time, Barnes proposed the following formulae:

Sharp-edged weirs-

$$Q = 3.324\left(H + \frac{v_0^2}{70}\right)^{1.49} L^{1.11}\left[L + 2\left(H + \frac{v_0^2}{70}\right)\right]^{-0.11}$$

Suppressed weirs-

$$Q = 3.324\left(H + \frac{v_0^2}{50}\right)^{1.49} L^{0.98}\left[L + 2\left(H + \frac{v_0^2}{50}\right)\right]^{0.02}$$

In the above formula, v_0, the velocity of approach, is obtained by first calculating Q neglecting this factor, then obtaining v_0 from $\frac{Q}{A}$. A second approximation of v_0 is sufficiently accurate for use in the above formulae. The formula for sharp-edged weirs agreed with the observed discharges within one percent in 80 out of 82 experiments, and in only ten instances was the error greater than one-half of one percent. This formula is remarkably accurate for heads as low as one inch - providing of course that the nappe springs clear of the downstream side of the plate. This accuracy is about as great as has been obtained by any formula yet proposed for rectangular weirs. The formula for suppressed weirs when compared with 92 experiments gives a maximum error of 1.7 percent and in 73 cases the formula agreed within one percent. Both of these formulae differ materially from the theoretical equations. The closer agreement of the equation for sharp-edged weirs is in direct disagreement with the statement by Fteley and Stearns that the sharp-edged weir does not give as good results as the suppressed weir.

The formulae listed previously are regarded as the principal equations for flow over rectangular weirs, although many more have been proposed, some of which will be given later. It is incorrect to state that a certain formula is better than another for general use because the various formulae were derived for different combinations of crest length, crest height, head, velocity of approach, etc. An individual formula should be derived for each separate weir set-up, the form, however, being similar to one of those given. Gibson[14] gives the following table showing results relative to those obtained by the Francis formula as applied to a ten feet long suppressed weir.

Length	Crest Height	Head	Approach Velocity	Francis	Fteley-Stearns	Bazin	Smith	Barnes
10ft	4ft	1.0ft	.68ft/sec	1.000	1.001	1.010	1.002	1.005
10ft	2ft	1.0ft	1.16ft/sec	1.000	1.015	.985	1.018	1.010
10ft	4ft	4.0ft	2.15ft/sec	1.000	1.050	1.050	1.115	1.004

It is most apparent that at low velocities of approach, the formulae, with the exception of the one proposed by Bazin, all agree very closely. In this country, present day practice is to use the Fteley and Stearns formulae for heads up to 0.6 feet and the Francis formulae for higher heads. The Bazin formula is popular in Europe, and is probably the most useful of all for short weirs and those having considerable velocity of approach. Barnes' formulae are based upon a consideration of widely different data, and hence are particularly useful for weirs not conforming to the general proportions or conditions of any of the other investigators.

Among other formulae worthy of mention is one by Gourley and Crimp for sharp-edged weirs. These investigators propose the same exponent for H as proposed by Francis early in his investigations. The Gourley-Crimp[15] formula is

$$Q = 3.10L^{1.02}H^{1.47}$$

King[10] has proposed the following formula for suppressed weirs which has the same advantage enjoyed by the Bazin Formula of giving the velocity of approach correction without any preliminary calculations.

$$Q = 3.34LH^{1.47}\left[1 + .56\left(\frac{LH}{A}\right)^2\right],$$

where LH is the cross section of the water over the notch and A is the area of the channel upstream from the notch where the measurement of H is made. Many more formulae could be given, but enough have been listed to show the great uncertainty as to a correct and best form for an equation to give the rate of discharge of water over rectangular weirs.

Semicircular Notches

Very few instances of the use of semicircular weirs have come to the attention of the author. One of the few experiments on this type of notch was carried out by Greve[2] at Purdue University. The general formula used by Greve in expressing his data was

$$Q = BH^{n}$$

where B and n are constants, different for notches of different diameter. The value of B becomes larger with notches of larger diameters, but there seems to be no orderly change in n with increase in diameter of the notch, the average value being 1.87 and giving rise to the following general equation:

$$Q = BH^{1.87}$$

upon taking into account the relation of B to the diameter, d, of the notch Greve arrives at the following equation:

$$W = 179(d)^{0.637}H^{1.87}$$

W being in pounds per second.

The maximum difference between W as calculated from the above equation and as observed experimentally rangesfrom +0.79 to -1.79 percent.

Cone[16] carried out experiments with the circular weir, his data being summarized by O'Brien[17]. The range in head was from .2d to d, d being the diameter of the notch. The following equations were proposed;

$$Q = BH^{n}$$

$$B = 2.612(d)^{0.8177}$$

$$n = 1.793(d)^{0.0769}$$

Comparison of the values of B and n as deduced from experiment and as calculated from the above formulae can be made from the following table:

Diameter	B		n	
	Experiment	Formula	Experiment	Formula
0.4495	1.461	1.481	1.696	1.700
1.0025	2.652	2.612	1.792	1.793
1.5011	3.729	3.649	1.870	1.850
1.9995	4.482	4.603	1.875	1.890

The reason for including the semicircular notch in this treatise will become evident later when the limitations of the experiments performed by the author are described.

Triangular Notches

The numerous formulae expressing the relation between head and rate of flow for V-notch weirs are essentially of the same form, there being much less variety of forms than in the case of rectangular notches. Here the variable exponent operating on H comes into extensive use. Professor James Thomson[5] proposed the following formulae:

$Q = 2.536H^{2.5}$, for a right-angle notch

$Q = 5.29H^{2.5}$, for a notch with $\tan\frac{\theta}{2} = 2$.

The constants given above are average values, in both notches the values of the constant decreasing very slightly with diminishing head. The decrease was about one percent for the right-angle notch for values of the head ranging from two to seven inches, the decrease being about two percent over the same range in head for the second notch.

The facts above would seem to indicate that the constant should be represented as a function of the head, with the head function exerting a greater influence the smaller the notch. This is accomplished by most investigators by using the form $Q = DH^n$, where n is just slightly less than 2.5 and decreases with notch size. The author will later propose the form $Q = E(H + a)^{2.5}$, which is the same as $Q \approx E(1 + \frac{2.5a}{H})H^{2.5}$, since a is so small that the first term of the expansion is sufficient. Whenever possible, all equations in the future will be discussed with reference to the above form.

Barr[6], carrying out extensive research on the right-angle notch with heads varying from two to ten inches, arrived at practically the same results as Thomson. It might be well to mention here that by having the channel of approach wide enough and deep enough, the factor of approach velocity is completely negligible. For this reason the velocity head correction will be noticeably absent from V-notch weir flow equations. Barr, in his experiments, found that the velocity of approach ceased to be a factor in the use of right-angle notches when the channel depth was about four times the head, and when the channel width was about eight times the head. Barr also discovered that roughening the upstream surface of the weir plate with coarse emery increased the discharge by about 2.4 percent with a three inch head and by 1.7 percent with a four inch head.

Barnes[13], from a consideration of the data obtained by Thomson and Barr, proposes the following formulae:

$$Q = 2.48H^{2.48}, \text{ for } 90^\circ \text{ notch}$$

$$Q = 1.244H^{2.48}, \text{ for } 53^\circ 8' \text{ notch}$$

These formulae fit all of Barr's experimental data with an error of less than one-half of one percent. Since the constants as given by Thomson are a mean value, the empirical formulae of Barnes are to be preferred. Whenever the velocity of approach exceeds 0.5 feet per second, Barnes recommends adding a correction, $0.01\ v_0^2$, to H, where v_0 is the mean velocity of approach measured at the same distance upstream as the head.

Strickland[18], in commenting on Barr's data on right-angle notches, calculated values of the coefficient in the equation $Q = EH^{2.5}$, using the expression $E = m + \frac{n}{\sqrt{H}}$ and obtained the same values of the coefficient for the various heads as did Barr from experimental results. The expression arrived at by Strickland is:

$$Q = \left(2.4170 + \frac{.067}{\sqrt{H}}\right) H^{2.5}$$

where H is in feet and Q is in cubic feet per second. Strickland's values of the coefficients differ from Barr's observed values by not more than one part in 3,038 with heads ranging from two to ten inches. This equation is quite similar in form to that found most satisfactory by the author. The surprising fact is that, with the remarkable results obtained by Strickland available, few investigators have taken the trouble to investigate equations of similar form further.

Thornton[19], also as a result of an analysis of Barr's data, has proposed a formula which is of exactly the same form as that found by the author to be the best. Thornton represents Barr's data by the following equation for right-angle notches:

$$Q = \left(2.4760 + \frac{0.016}{H}\right) H^{2.5}$$

where Q is in cubic feet per second and H is in feet. Mawson[20], using the same data and the same meanings for Q and H as did Thornton proposed the following equation:

$$Q = \left(2.41380 + \frac{0.068206}{H^{0.4996}}\right) H^{2.5}$$

Cone[16], carried out ninety-eight separate tests on five notches with heads ranging from 0.2 foot to 1.35 feet. He arrived at the following equations:

$$Q = 4.400H^{2.487} \text{ for } 120^\circ \text{ notch}$$

$$Q = 2.487H^{2.481} \text{ for } 90^\circ \text{ notch}$$

$$Q = 1.446H^{2.471} \text{ for } 60^\circ \text{ notch}$$

$$Q = 0.6848H^{2.448} \text{ for } 30^\circ \text{ notch}$$

$$Q = 0.6405H^{2.445} \text{ for } 28^\circ 4' \text{ notch}$$

The nappe fell free in all instances except in the case of the 120° notch. Cone claims that for this reason the 120° notch is impractical. Using data from all the notches except the largest, Cone arrived at the following general equations:

$$Q = (0.025 + 2.462S)H^n$$

$$n = 2.5 - \frac{0.0195}{S^{0.75}}$$

where S is the slope of the sides expressed as a decimal.

Some research by Yarnall[8] has already been mentioned in the Introduction. Yarnall, using notches of 90°, 53°8', 27°, and 13°8' got very good duplication of results. He was able to guarantee calculation of the discharge within one-half of one percent by the use of plots of H vs the weir coefficient in the equation $Q = EH^{2.5}$ Yarnall made no attempts to develop a formula to represent his results; however O'Brien, in a discussion following the article by Yarnall, has expressed the data in equations of the following form:

$$Q = DH^n$$

The following table compares the values of D and n as deduced from Yarnall's data as compared with those calculated with the use of Cone's general equation which was given previously.

Angle	D		n	
	Cone	Yarnall	Cone	Yarnall
90°	2.49	2.48	2.48	2.49
53°8'	1.256	1.242	2.47	2.48
27°	0.5912	0.613	2.443	2.475
13°8'	0.283	0.3312	2.40	2.42

Cone's equation was derived from data from notches ranging from 28° to 90°, hence it is not surprising that the constants for smaller notches as calculated from his formula do not agree too closely with values of the constants when calculated from experimental data.

Gourley and Crump[15] carried out investigations on four V-notch weirs having side slopes (vertical to horizontal) of 5 to 1, 3 to 1, 2 to 1, and 1 to 1. The range in head was from 0.15 foot to 1 foot, and the velocity of approach was negligible. These experimenters proposed the following formula:

$$Q = 2.48 \left(\tan\frac{\theta}{2}\right) H^{2.47}$$

Greve[2] carried out a large number of experiments on notches varying from 10° to 120°. However, due to excessive clinging of the smaller notches, data is listed for notches of central angles of over 20°. The data were expressed in an equation of the following form:

$$W = \rho D H^{n},$$

with W in pounds per second. A summary of the results is given in the following table:

Angle	25°3'	36°53'	40°00'	44°24'	45°23'	53°55'	59°7'
ρD	34.86	52.24	56.99	63.87	65.44	79.53	88.65
n	2.460	2.457	2.468	2.471	2.464	2.476	2.478
Angle	69°38'	81°52'	94°39'	98°45'	102°20'	110°00'	118°11'
ρD	108.6	135.3	169.1	181.7	193.6	222.4	260.0
n	2.460	2.469	2.474	2.479	2.466	2.468	2.469

A run on the 45°23' notch was repeated after removing the weir to demonstrate that the results could be duplicated. The values of ρD and n for this second run were exactly the same as the ones listed above. A second run was also made on the 53°55' weir, the type of baffles being changed to determine the effect of alterations in the approach channel. The values of ρD and n obtained in the second test were exactly the same as those obtained in the first one, hence, as would be expected, the type of baffling employed does not influence the results, provided that the velocity of approach is sufficiently small. A general equation as proposed by Greve is as follows:

$$W = 156\left(\tan\frac{\theta}{2}\right)^{.996} H^{2.47},$$

where W is in pounds per second. The difference between the actual rate of discharge and that computed by the above formula varied from +0.78 percent to -0.97 percent. This is exceedingly good considering the fact that a general formula was used for all notches, and not the specific formula calculated for each notch.

Greve also compiled the following table of values of ρD and n for right-angle notches calculated from the published data of several investigators and fitting the same equation form as given above in connection with Greve's own experiments.

	Barr	Cone	Gaskell	Greve	King	Thomson	Yarnall
ρD	153.6	155.1	154.8	156.0	157.1	157.2	154.6
n	2.475	2.481	2.488	2.470	2.470	2.491	2.491

There is evidently considerable divergence of results which is blamed on variations in the physical characteristics of the various set-ups. The main difficulty seems to lie in the machining of the notch itself, in particular the machining of the vertex. For the last named reason, the readings of the head might quite conceivably all be incorrect by a constant amount. With this in mind, the application of a formula of the type

$$Q = E(H + a)^{2.5}$$

or $$Q = D(H + a)^{n}$$

might give a better comparison of the various data. The first of the above forms would serve as a better means of comparing data, with the correction factor "a" taking care of any constant error in measuring H, as well as the accepted fact that the weir coefficient varies with the head. Elaboration of these ideas will be forthcoming in subsequent discussions of the author's results under conclusions.

RESTRICTIONS IN THE USE OF V-NOTCH WEIRS

Judging from the researches and formulae given in the preceding section, it would seem unsafe to assume that precise rates of discharge for a particular set-up would insue by application of a formula developed from someone else's data. Since V-notch weirs are used for measurements of rates of flow relatively lower than those ordinarily measured by rectangular weirs, in almost all cases of use of the former type a calibration can be carried out for each particular set-up. The author is certain, however, that with the incorporation of certain restrictions to eliminate the velocity of approach, with the use of sharp-edged V-notch weirs with truly accurate apices and smooth upstream faces, with a method of measuring the true head with reference to the crest of the notch, and with equations of correct forms available, it should be possible to get accuracy of one-half of one percent with the use of derived general equations based on data obtained from set-ups abiding by the restrictions given above.

Since the machining of a notch as specified above is an extremely difficult job, the separate calibration and development of equations for each set-up would seem desirable. Once such a test has been carried out, there should be no difficulty in reproducing results within at least one-half of one percent. In designing a set-up, certain facts should be observed so as to make the velocity of approach negligible and so as not to interfere with the normal contraction of the nappe.

Many experiments have been made to determine the channel size necessary to make the approach velocity negligible. Barnes[13] specifies that the crest should be at least 6 inches above the level on the downstream side; that the floor of the channel of approach should be at least 12 inches for heads of less than 9 inches and 1.5 feet for higher heads, that the width of the channel should be 4 feet for heads less than 9 inches and 6 feet for higher heads, and that the upstream edge should be sharp and free of projections. Diederichs and Andrae[21] claim that the width has no effect if it is greater than 8H, and that the depth of the approach channel should be greater than 12 inches. Staus[22] found the same dimensions to be satisfactory and he also claims that the head should be greater than 2 inches when dealing with notches less than 20° in order to prevent clinging. Marks[23] claims that in order to obtain an essentially horizontal surface, the head should be measured at a distance upstream from the weir of at least 4H.

Weirs enjoy their greatest use as parts of hydraulic edifices such as spillways, dams, etc. For these purposes the rectangular weir is almost always used because of its greater capacity. However the triangular notch is the one used to the greatest extent by industry. One of its great advantages is its greater accuracy at small rates of flow and its wide capacity without appreciable sacrifice in accuracy. With weirs it is possible to measure liquids whether they are hot or cold, corrosive, or contain suspended material; however the weir must be calibrated for any change in physical properties of the liquid being metered. One rather widespread use of the V-notch is by power plants to meter the station water rate.

A major objection to the weir is that it measures instantaneous rates of flows and not the total quantity of flow. For this reason it is of no use unless the liquid being metered is at a constant head. Also not enough is known of the effect of viscosity, density, etc. to permit the use of weirs with any degree of assurance to measure various liquids without calibrating the weir with the liquid to be metered.

STUDIES OF THE EFFECT OF VISCOSITY

Very little has been attempted in the direction of establishing the quantitative effect of viscosity upon the flow of liquids through weirs, and the results of the researches that have been made are almost worthless because of the limited variations in viscosity ranges considered. A number of equations have been proposed to include the effect of viscosity, but the importance of these equations will remain in doubt until more complete data are available.

An equation derived by dimensional analysis has been proposed by Professor Tour of the University of Cincinnati, the derivation of which will be given here. His attack is based upon evaluation of the dimensionless constant, or weir coefficient, used as an added factor in the theoretical equation to force it to conform with experimental results.

$$Q = C \frac{8}{15} \sqrt{2g} \left(\tan \frac{\theta}{2}\right) H^{2.5}$$

This dimensional treatment of the constant in the above equation seems to be the most desirable approach, particularly since the physical properties of the liquid in no way entered into the derivation of the theoretical equation. It might be pointed out here that this approach is also applicable should the correct weir equation be found to have one of the following forms:

$$Q = C \sqrt{2g} \left(\tan \frac{\theta}{2}\right) (H + a)^{2.5}$$

$$Q = C\sqrt{2g}\left(\tan\frac{\theta}{2}\right)\left(H + \frac{u}{H^r}\right)^{2.5}$$

where u and r are constants.

The variables which might most conceivably affect the coefficient C are the viscosity of the liquid, gravity, the density of the liquid, and the head of liquid. The symbols for these variables and their dimensions are

viscosity - U - $ml^{-1}t^{-1}$
gravity - g - lt^{-2}
density - ρ - ml^{-3}
H - head - l

assuming that a product relationship exists:

$$C = k(U^a g^b \rho^c H^d)$$

$$C = k\left[ml^{-1}t^{-1}\right]^a \left[lt^{-2}\right]^b \left[ml^{-3}\right]^c \left[l\right]^d$$

mass: $0 = a + c$

length: $0 = -a + b - 3c + d$

time: $0 = -a - 2b$

$a = -2b$

$c = 2b$

$d = 3b$

or

$$C = kU^{-2b}g^b\rho^{2b}H^{3b} = k\left(\frac{U^2}{g\rho^2H^3}\right)^{-b}$$

If the contention of most experimenters is accepted, i.e., that the flow of liqids through V-notch weirs is best and most accurately represented by equations of the form

$$Q = F \cdot \frac{8}{15}\sqrt{2g}\left(\tan\frac{\theta}{2}\right)H^n, \quad n < 2.5,$$

then the values of the exponents in the expression for C just derived could be evaluated from the values of n obtained from the experiments. This could be done by rewriting the above equation in the theoretical form:

$$Q = \left(\frac{F}{H^{2.5-n}}\right)\frac{8}{15}\sqrt{2g}\left(\tan\frac{\theta}{2}\right)H^{2.5}$$

Then the dimensionless weir coefficient C can be expressed as:

$$C = \frac{F}{H^{2.5-n}} = k\left(\frac{U^2}{g\rho^2H^3}\right)^{-b}$$

From the value of the exponent of H,

$$2.5 - n = -3b$$

$$b = \frac{n-2.5}{3}$$

The value of n can be determined experimentally. The expression for the weir coefficient becomes

$$C = k\left(\frac{U^2}{g\rho^2H^3}\right)^{\frac{2.5-n}{3}}$$

However, it is the writer's contention that, if the derived expression for the weir coefficient is at all applicable, the value of the exponent b will have to be determined experimentally and without the consideration of n as given above.

Gibson[14] suggests that the viscosity effect can be included by the following consideration:

$$Q \propto H^{2+m} U^{\frac{1-2m}{3m}}$$

If the value of m is assumed to be 0.47, an increase in viscosity of 100%, corresponding to the change in temperature of water from 90°F to 40°F, could increase the rate of flow by 2%. The fact that increasing the viscosity increases the rate of flow is surprising, but nevertheless is an experimentally proven fact. This might be explained by the fact that increasing the viscosity also increases the size of the vena contracta of the nappe.

Equations somewhat similar to the one developed by Tour were developed by Mawson[24]. He considered the volumetric discharge to be a function of the density of the liquid, the viscosity of the liquid, the head, and the acceleration of gravity. Mawson arrives at the following equations by the application of dimensional analysis:

$$\frac{Q}{H^2 \sqrt{gH}} = X$$

for non-viscous liquids, X being a constant, and

$$\frac{Q}{H_2 \sqrt{gH}} \propto \left(\frac{U}{H\rho \sqrt{gH}}\right)^d$$

where d is a constant. The last equation is of exactly the same form as that derived by Tour. Mawson's continuation from this point seems to be based on rather obviously incorrect premises. According to Mawson if the liquid is non-viscous, the second equation becomes the same as the first. The second equation is then written as

$$\frac{Q}{H^2\sqrt{gH}} = X + Y\,f\left(\frac{U}{\rho H\sqrt{gH}}\right)$$

This applies to a particular V-notch, since the angle is not included. The argument behind the last expression, combining the expressions for both viscous and non-viscous liquids, seems to be faulty. A non-viscous liquid does not exist, and it would therefore seem that an expression for such a liquid could not have any meaning.

Using Barr's data[6], Mawson examines his equation to see if it satisfies an exponential law when

$$\frac{Q}{H^{2.5}} = X + Y\left(\frac{U}{\rho H^{1.5}}\right)^n$$

Mawson calculates the values of E corresponding to those of Barr in the equation

$$Q = EH^{2.5}$$

and arrives at the following expression for a right-angle notch:

$$E = 2.4141 + \frac{0.06878}{H^{0.4966}}$$

The values of E as calculated from the above expression agree within 1 part in 3000 with those arrived at experimentally by Barr. Taking into account the average temperature of the water during the experiments, Mawson arrives at the following expression for a right-angle notch:

$$\frac{Q}{H^{2\cdot5}} = 2.4141 + 2.7382\left(\frac{U}{\rho H^{1\cdot5}}\right)^{0\cdot33107}$$

It is claimed that the above expression can be applied to liquids other than water. This is probably incorrect, at least based on its development. The application of Barr's data is simply a repitition of the work of Strickland[18] mentioned earlier, in which he calculates Barr's values for the coefficients using

$$E = a + \frac{b}{\sqrt{H}}$$

Where a and b are constants. Mawson's expression is correct in that an increase in viscosity increases the rate of flow; when H = 1 foot, an increase in viscosity of 100% increased Q by 5.5%.

Equations have been derived by Smith[25] and by Eaton[26], both of which seem so invalid and devoid of interest that no mention will be made of them here. Eshbach[27] gives an equation somewhat similar to the two already given. The following is the equation, given along with the statement that it can be established by dimensional analysis:

$$Q = g^{\cdot5}H^{2\cdot5}f\left(\theta, \frac{H^{1\cdot5}g^{\cdot5}\rho}{U}\right)$$

Yarnall[8] has obtained data which showed qualitatively that increasing the viscosity increases the rate of discharge. This same result was obtained by a number of experimenters, none of whom had enough or sufficiently good data with which to work. Merriman[28] claims to have knowledge of weir experiments, where, for discharges of about 200 cuft/sec, it appeared that the effect of changing the temperature from 76°F to 33°F was to decrease the rate of discharge by about 0.75%. However he admits the inaccuracy of the data, in view of which fact the conclusions drawn should not be given too much consideration. Switzer[29] ran tests on a 54 degree and a 90 degree notch with water; the temperature of which ranged from 39°F to 165°F. The conclusion drawn from his data was that the change in temperature over the range tested had no appreciable effect upon the rate of discharge. Cozzens[30] gives a graph of temperature of water plotted against a factor by which the volume calculated from the equation

$$Q = 2.53H^{2.5}$$

must be multiplied to take care of change in temperature. According to his graph, increasing the temperature,hence lowering the viscosity, also increases the rate of discharge. This is contrary to most views.

From observations of the present status of the study of the effect of viscosity, it is apparent that until accurate data are available for the flow of liquidsof various viscosities, not much can be done to settle this problem. Experiments on glycerine-water solutions are now being carried out at the University of Cincinnati, using the same equipment as was used by the writer. With the completion of these experiments, maybe a quantitative effect of viscosity will be established.

EQUIPMENT

Weir Plate Assembly

The weir plates used in this investigation were a set of sharp-edged V-notches with varied angles. The angles available were 5°, 10°, 15°, 20°1.1', 29°54.3', 44°58.0', 60°2.9', 90°12.4',120°; however, because of excessive clinging of the nappe throughout too much of the available range, experimental work done with the 5°, 10°, 15°, and 120° notches is omitted from this treatise.

The weir plate assembly consisted of three separate pieces, the weir plate, the supporting plate which extended completely across the channel, and the bolting strip used to bolt the weir plate to the large supporting plate. All these plates were 1/8 inch thick hardened bronze.

The notches were cut in plates of the shape of isosceles right triangles, the hypotenuse forming the upper (horizontal) edge of the plate, the plate thus resting on the 90° angle, inclined 45° to the horizontal. The top of each plate was 24 inches across, and the vertical distance from the top to the apex of the 90° angle was 12 inches. The notches were cut into the hypotenuse so that the vertical distance from the top to the apex of each notch was 8 inches, except in the case of the 120° notch where the distance was 6 inches. Brazed and tapped into the plate around the sides (except for the horizontal side) were 1/4 inch brass studs, 1 inch in length, which were machined flush with the upstream side, presenting a smooth surface to the water in the channel. The edges of the notch itself were beveled at an angle of 45° and to such a depth that the actual notch edge was 1/64 of an inch. The studs on each of the weir plates were in exactly the same positions so as to make the plates interchangeable.

The large supporting plate was a 1/8 inch bronze plate, 40 inches high and 42 inches wide. On the top side, in the center, a right triangle 12 inches in depth was cut out so that the weir plate fitted snugly upon the side of the triangle. Along the sides of the triangular part were placed studs of the same type as on the weir plate, also machined flush on the upstream side.

A bolting strip was supplied to fasten the weir plate on to the supporting plate. This strip of 1/8 inch bronze was in the shape of a right triangle with a similar and smaller right triangle cut out of the metal so as to make the resulting strip 2 inches wide, measured perpendicular to the sides. The strip was constructed so as to overlap the junction of the weir plate and the supporting plate, and in the strip were holes which mated with the studs on the two plates. A rubber gasket of the same size as the strip was supplied between the strip and the two plates. The strip was then bolted to the supporting plate by the use of wing nuts, thus providing a slot into which the weir plates fit. When the weir plate is inserted and bolted to the strip, the completed weir plate assembly presents a smooth upstream face. A clear picture of this assembly can be obtained from the sketch given with the appendices.

Weir Box And Accessories

The inside of the weir box was 94-3/4 inches long, 40 inches deep, and 42 inches wide. The box was made of 1/2 inch transite, a material made of compressed asbestos and cement. The bottom of the tank rested upon 1/2 inch iron rods which extended through the sides and were bolted onto 4 inch iron channels which formed the base for the box. On each of the four sides and extending along the entire length of each side were three 4 inch iron channels which were bolted to the box. The channels along the sides of the tank were connected by 1/2 inch iron rods which keep the sides of the tank in place. These rods extended around and through the tank, but in such a way that no rods extended through the tank in the approach channel proper. This can be seen in one of the photographs given in the appendix. This transite box was lined with 20 ounce copper sheet to keep the transite tank from leaking.

The supporting plate fitted snugly into the tank and was bolted to the tank by 1-1/2 inch brass angles, a rubber gasket being placed between the two surfaces so as to prevent leaking from the approach chamber into the discharge compartment. The bolts were countersunk into the upstream face of the large supporting plate, and the angles connecting the plate to the box were on the downstream side; thus the smooth upstream surface of the weir assembly was maintained. In the bottom of both the discharge compartment and the approach channel was a 1 inch drain, the two outlets being connected to the same 1 inch copper service pipe, with a cock between the two drains. This 1 inch line was connected to the pump and was used for draining purposes.

The inlet to the tank, a 3 inch copper pipe, was located in the back end, the end farthest from the weir plate, and was 12 inches above the bottom. Inside the tank was a three-sided distribution box, 4 by 4 inches, made of 1/4 inch transite, the open end being against the back of the tank and the box extending across the entire width of the tank. There was an opening of 1/8 inch between the back of the weir box and the distribution box to allow the water to flow in the weir box after having its direction of flow changed through 180°. At 14-1/2 inches from the back was a 1/4 inch galvanized iron screen of 1 inch mesh, which extended completely across the tank and was bolted to 1-1/2 inch brass angles which, in turn, were bolted to the tank. Into the small back compartment were packed 1 inch ceramic Raschig rings which served to break up eddy currents and give a perfectly smooth surface.

Between the screen and the weir plate was a space, 55 inches in length, which served as an approach channel. This space was free of all obstructions so as to allow a smooth flow of the liquid to the face of the weir. This compartment was of sufficient length, depth, and width to allow full contraction of the vena contracta. Off this approach channel at a distance of 33 inches upstream from the weir plate and 9 inches above the bottom of the tank, was the connection for the gage glass located in front of the tank and connected to the opening in the side of the tank by 1/4 inch copper tubing. The gage connection to the tank was flush with the side of the tank so as to measure only the static pressure head in the tank, and not any velocity head which might be present. The gage glass itself was a 3/4 inch I.D. glass tube, open at the top, which was mounted on a white background.

The discharge compartment was 25 inches long, 42 inches wide and 40 inches deep. The outlet pipe, a 2 inch copper pipe, was located in this compartment at the front end at about 10 inches from the bottom extending to within a few inches of the weir plate, and then turning down so that the water actually entered the outlet pipe at a distance of two inches from the floor.

To brace the weir plate and prevent it from giving because of the pressure on the upstream side, two 3 inch cadmium-plated iron channels, bolted to the weir plate with the bolts being countersunk on the upstream side, were braced against the front of the tank by 1/2 inch cadmium-plated iron rods which were tied to the transite by having nuts on the inside and outside of the wall. In this discharge compartment was also a baffle made of 1/4 inch transite which had three sides (the opening being towards the weir plate) and a bottom which was 12 inches from the bottom of the tank. The side of the baffle nearest the outlet fitted tightly against the weir plate, forcing the liquid to go out the other side, where an opening was provided for this purpose, and around the back of the baffle to the outlet from the tank. This circuitous path supplied ample time for the dissipation of any bubbles which might be formed by the occlusion of air when the discharge stream flowed into the discharge compartment.

Places inside the tank where there might be a possibility of leakage were covered with Sarva, a black coating very similar to an asphalt base paint.

Pump, Motor, Meters, and Piping

The pump used was a Worthington centrifugal pump with a 2 inch inlet and outlet. It was driven by a General Electric synchronous motor, 3 phase, 60 cycle, 220 volts, 18 amps, 1800 r.p.m., 3 horsepower motor, with a thermostatically controlled switch to avoid overheating. The pump and motor assembly had a capacity of 125 gallons of water per minute when operating against the pressure drop of the meters, line, and full weir tank.

Because of the wide range of the notches, two meters were necessary. For rates up to 20 gallons per meter, a 3/4 inch Empire (Rotary Piston) Meter was used and for higher rates a 2 inch Empire Rotary Piston Meter was used, both these meters being products of the National Meter Company.

Two inch copper pipe was run from the tank outlet to the pump, thence to the 2 inch meter, then around to the side of the tank where it was connected by a rubber connection to a 3 inch copper pipe which led to the tank inlet, thus providing for recirculation of the liquid. Between both the tank outlet and the pump, and the meters and the tank inlet were provided short sections of rubber piping so as to lessen the chance of motor, pump, and meter vibrations being transmitted to the weir tank. Around the large meter was a 1 inch by-pass which connected the small meter into the system. All valves used on the system were brass gate valves. Coming up from a trench which runs lengthwise under the tank was a 1 inch copper line which connected the drains from the approach channel and the discharge compartment to the pump inlet.

Auxiliary Equipment and Instruments

For the purposes of calibrating the meters a 750 gallon copper tank was available. This was connected to the meter outlet by a 2 inch copper line and a 2 inch copper line was also provided as a drain from the tank. The tank was supplied with a 1/2 inch gage glass behind and directly onto which was affixed a strip of coördinate paper to enable the change in height of the liquid in the tank to be measured. A Beckmannthermometer lens was clamped on the gage glass to aid in reading the height of liquid in the tank.

For purposes of reading the head on the weir, as indicated by the 3/4 inch gage glass connected to the tank, a French-make cathetometer was used. This cathetometer rested upon three screws which served to align the cathetometer so that the triangular-shaped rod on which the telescope moves was perfectly vertical. Affixed to the telescope barrel was an extremely delicate spirit level which enabled the telescope to be maintained in a horizontal position. A scale with millimeter divisions was made directly onto the triangular brass rod. The telescope base, which was attached directly to the barrel and moved with the telescope, contained a vernier scale which had twenty markings, enabling readings to be made with the limiting accuracy of 0.005 cm.

The temperature of the water in the approach channel was read with an ordinary 120°C mercury thermometer. A Swiss-make stop watch with .1 of a second markings was used to get rate readings from the meter. The surface tension of the dreft solution, which was used in a special run, was measured with a commercial stalagmometer for which an accuracy of one-half of one percent was claimed.

EXPERIMENTAL PROCEDURE

Location of Notch Crests and Measurements of Notch Angles

The location of the notch crests, a most important measurement, was carried out by two independent methods. First the notch crest was determined by mounting the weir plate in position in the supporting plate and measuring the vertical distance between the crest of the notch and its top right hand corner (looking downstream) by means of the cathetometer which was set inside the empty tank. The reason for using the top right hand corner as a reference point was that the weir plate extended just a little higher than the front of the tank, and the edge could be sighted with the cathetometer when the instrument was in position to read the head as indicated by the gage glass.

The depths of the notches were checked by the use of another method, which also gave the angle of the notch. The notch was taken out of the weir plate assembly and placed in positions such that the following readings could be taken with the use of the cathetometer: the horizontal distance across the top of the notch, the horizontal distances across the notch at three points lower down, and the distances along the edge of the notch from the top to each of the three places where horizontal distances were measured. From these readings, three separate calculations of the angle of each notch were made. Knowing the angle and the horizontal distance across the top of the notch, by the use of trigonometric relationships, the depth of the notch was calculated. Correction was also made for the fact that the weir plate, when in place, was not lined up so that the two top edges of the notch were perfectly horizontal. The notches were only slightly out of line, but this could be detected with the cathetometer and corrections for this were made, also using the cathetometer. This correction factor was applied in such a way as to give the vertical distance from the crest of the notch to right hand edge of the notch (looking downstream). The results of these measurements checked extremely closely with those obtained by direct measurements.

Calibration of Meters

The meter calibrations were extremely important in that the error in the meter readings appears in the final results to the same degree. In order to calibrate the meters it was necessary to first calibrate a 750 gallon copper tank which was used to measure the flow through the meters. The copper tank was first filled with water to the top and the reading on the gage recorded, the reading being made with the aid of a BeckmannThermometer lens which enabled the observer to read the head to 0.01 cm. Since the readings on the gage were all handled in the end as differences, care was taken to read the same portion of the miniscus every time. After the gage reading was made of the full tank, water was allowed to flow from the tank into a 30 gallon can, resting on scales which could be read to 0.1 pound. When the can was nearly full, the valve from the tank was closed, care being taken that no water loss occurred in the hose. The weight of the water was noted, along with its temperature, and the reading on the tank gage was taken. Thus it was possible to tell what volume of water the tank held per divisionfor the range covered by the water removed. This procedure was continued until the tank was emptied. Another run was made exactly similar to the one described above and with the water at the same temperature. It was found that the tank was not exactly uniform, but the two runs checked one another to within about .02 percent. In using the tank to calibrate the meter, an average of the two tank calibrations was used, and the tank calibration was computed over the exact same range as was represented by each individual meter calibration.

The procedures used in calibrating both the large and the small meter were the same. First the line was opened from the weir box through the meter to be calibrated and into the tank. A small amount of water was run through this system and the valve closed. Readings were then taken of both the meter and the gage glass on the tank. The valve was then quickly opened to a definite opening so as to give the rate of flow for which this particular meter calibration was intended. The stop watch was then started at a definite reading on the meter and then stopped just before the run was to be terminated. These readings served togive the rate of the flow through the meter. The valve was then quickly closed and the meter and tank gage again read. From the initial and final readings of the glass gage on the tank, the exact amount of water having passed through the meter can be computed. In doing this the average gallons per division for the tank was obtained from the tank calibration by considering the exact same range in the tank as was represented during the run. The amount recorded by the meter was obtained from the initial and final meter readings. The rate as obtained from the stop watch readings was based on the meter gallons and not the correct gallons as indicated by the tank, but the uncorrected rate was the desired one. The correction was not represented as a percentage error but as the ratio $\frac{\text{meter gallons}}{\text{tank gallons}}$. The reason for this was that the uncorrected rate could be corrected by dividing by this correction divisor, this dividing operation being desired because of the relative ease of division in comparison with multiplication with the use of an electric calculator. The correction curve then plotted was uncorrected rate versus the correction divisor mentioned above. The temperature of the water used to calibrate the meter was exactly the same as that of the water used to calibrate the tank.

The absolute error in the small meter was about 2 percent, while that of the large meter varied from -0.7 to +1.0 percent.

In the runs made, however, checks were obtained to within .04 percent for meter calibrations made before and after a series of weir experiments, hence the use of an average calibration curve should involve an error of not more than .02 percent. The calibrations changed continuously, so the meters were calibrated both before and after each series of runs. As a result a great many meter calibrations were made, but this was rewarded by the accuracy of the meter calibrations as applied to a particular weir run.

Determination of the True Head

The reading taken by sighting the cathetometer on the gage glass connected to the weir tank was not of the true bottom of the miniscus but of a point a little higher where a very sharp line was formed in the liquid in the gage glass. This was corrected to the true height of the liquid in the tank by the use of a suspended needle which was lowered until the point of the needle just touched the water when the weir tank was full and the water still. The cathetometer was sighted on the top of the needle and then on the sharp line in the miniscus of the water in the gage glass. The length of the needle was determined with the used of the cathetometer, and this subtracted from the cathetometer reading of the top of the needle when the point was just touching the water surface gives the true height of the liquid with reference to the cathetometer. The gage glass reading was compared with this true height, and the correction for the gage reading determined. This correction was a constant for all runs with water. The value of this correction is of such great importance that a great number of determinations of its value was made in a number of different ways. These different determinations all resulted in the same correction, which result was extremely satisfactory.

The true head was then obtained by sighting the cathetometer on the top right hand corner of the weir notch (looking downstream) which served as a reference point. The vertical distance from this edge to the crest had been determined, its determination having been described in an earlier part of this section, and subtracting from this distance the distance from the top right hand corner of the notch to the corrected gage reading gives the corrected head.

This method of measuring the true head is quite different from the hook gage used by almost all other weir experimenters. However, in spite of its apparent complications, this method is really much simpler than the hook gage. Once the notch depth and the correction to be applied have been established, it is only necessary to read the reference point at the start of each run, until this notch is replaced, and the gage readings during the run. The availability of the cathetometer to measure vertical distances, whether or not the points are in the same vertical plane, is a feature which contributed greatly to the accurate results obtained.

Calibration of Weirs With Water

One unique feature of this research was that the water passing over the weir was recirculated, and not collected and weighed. The recirculation method was made possible by the use of meters, corrected by calibrations, to measure the rate of flow. The use of a constant head source of water was made unnecessary because of the use of a centrifugal pump geared to a synchronous motor. These features made the weir set-up relatively simple and still added no measurable errors.

Before the insertion of a weir plate into the weir plate assembly, the plate was soaped and rinsed well to clean the surface. The plate was then inserted into position and clamped to the bolting strip by wing nuts. The cathetometer was then sighted on the reference point and this reading checked a number of times. Then the calibration of the weir was started by setting the valve so as to get the highest head possible, this being determined in the cases of the larger notch by the capacity of the pump and in the cases of the smaller notches by the height of the tank. After about a minute when the delivery through the meter reached a constant rate, a run was begun by starting the stopwatch at a recorded meter reading. After the completion of head readings corresponding to the rate of flow, the stopwatch was stopped at a second recorded meter reading. Thus, knowing the time for this definite amount of water to pass through the meter, the rate of flow over the weir plate could be obtained after meter corrections had been applied to the uncorrected rate.

About five minutes after the run was started, the time varying with the notch and the head, the first reading of the gage glass was made, using the cathetometer. Several minutes later another reading was taken, and if this did not agree exactly with the first reading, another reading would be taken several minutes later on the premise that the first reading was taken before equilibrium conditions were established. To the credit of the motor-pump assembly, the head, after equilibrium conditions were reached, did not vary enough to be detected by the cathetometer which measures 0.005 cm.

During the runs at each head, the temperature of the water in the approach channel was taken. The temperature was maintained at a constant value (just around room temperature) by the addition of cooler water from the line to the system.

After the run at the highest head, runs were made at lower heads until the head became so low that the nappe clung to the weir plate instead of springing clear. When this occurred, the calibration of the particular notch was considered concluded.

Calibration of 20° Notch With Dreft Solution

Here the calibration was carried out in exactly the same manner as described in the calibrations with water, with a few additional operations. Enough dreft was added to lower the surface tension of the water by almost 50 percent.

Readings of the surface tension were taken with the aid of a stalagmometer at least twice for each change in head. Because of the accumulation of foam in the discharge compartment, it was necessary to ladle this out almost continuously.

The calibration of the 20° notch with dreft and an accompanying check calibration with water were performed at a much earlier date than the experiments with water which are included in this treatise. At the time of the performance of the dreft experiment, the leveling device of the cathetometer was not perfectly aligned with the barrel of the telescope, and, as a result, the runs made were comparable with one another but not with the subsequent runs made with water after the cathetometer level and telescope were adjusted. Since the experiment disclosed that surface tension had no measurable effect upon the flow of liquids over weirs, the experiment on dreft and its accompanying check with water were deemed satisfactory in spite of a constant error in the head.

PROCEDURE IN CALCULATING RESULTS

Discussion of Equations

The first attempt to correlate results was by the use of the formula accepted by most researchers,

$$Q = DH^n \quad \text{.....................................(1)}$$

Using least squares, the values of D and n were calculated from the log form of the above equation, that is from

$$\log Q = \log D + n \log H$$

The resulting formulae gave values of Q agreeing only fairly well with the observed values, and the values of n varied around 2.5 without any systematic change. For these reasons attempts were made to get other forms to express the data.

Although the head as measured was the one assumed by all workers in this field, that is the vertical distance from the notch crest to the surface of the liquid measured far enough upstream so that it was horizontal, there remained the possibility that this was not the true head. With this in mind the following equations were used:

$$Q = E(H+a)^{2.5} \quad \text{.....................................(2)}$$

$$Q = D(H+a)^n \quad \text{.....................................(3)}$$

Using least squares, the values of D, a, and n were calculated. The resulting equations both gave values of Q agreeing very closely with the observed values. For this reason the results given here are expressed with the use of these two equations.

That equations (2) and (3) are equivalent to the following equations,

$$Q = E(1+ \frac{a^1}{H})H^{2.5} \quad \ldots\ldots (4)$$

$$Q = D(1+ \frac{a^1}{H})H^n \quad \ldots\ldots (5)$$

can be readily shown. The value of a is extremely small compared with H so equation (1) can be simplified as follows:

$$Q = E(H+a)^{2.5} \ldots$$

$$Q = E(1+\frac{a}{H})^{2.5}H^{2.5}$$

$$\ln Q = \ln E+2.5\ln H+2.5(\ln(\frac{1}{1} + \frac{a}{H})$$

Since a is so small, the second term of the expansion of $\ln(1+ \frac{a}{H})$ can be neglected, giving the following formula:

$$\ln Q - \ln E - 2.5\ln H - 2.5 \frac{a}{H} = 0 \quad \ldots\ldots (6)$$

Equation (6) was the form used in the calculations of E and a by least squares for their values in equation (2). Starting with equation (4), it is obvious that the equation for application of least squares would be

$$\ln Q - \ln E - 2.5\ln H - \frac{a^1}{H} = 0 \quad \ldots\ldots (7)$$

thus the only difference between equations (4) and (2) being that a^1 is equal to 2.5a. The same similarity exists between equations (3) and (5).

An equation of the form

$$Q = E(H + \frac{u}{H^r})^{2.5} \quad \ldots\ldots (8)$$

might conceivably give better results than equation (2). Due to the limitations of the data, no attempt was made to apply the formula.

Expression of Results

It is customary to compare experimental results with a derived equation by calculating the percentage variation in the observed and calculated values of the rate of discharge. This procedure was followed in representing the results of the writer's experiments. Due to the nature of the errors in obtaining values of Q and H, expression of results as percent error of calculated rates of discharge seemed an unfair method of comparing the accuracy of the weir at high and low heads. The inherent error in Q as obtained in the experiment was a constant percentage error, whereas the inherent error in the measured head was a constant, this fact causing a greater percentage error in the values of the head at lower rates of discharge. In view of this, the following derivation was suggested by Professor Tour to get a better way of expressing the results.

$$Q = E(H+a)^{2.5}$$

$$\frac{dQ}{dH} = 2.5E(H+a)^{1.5}$$

$$(H+a)^{1.5} = \left(\frac{Q}{E}\right)^{\frac{1.5}{2.5}}$$

$$\frac{dQ}{dH} = 2.5\left(\frac{Q}{E}\right)^{.6}$$

$$\frac{dQ}{Q^{.6}} = 2.5\frac{dH}{E^{.6}} = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Equation (9) shows that for a constant error in H, i.e., dH being a constant, the expression $dQ/Q^{.6}$ gives a constant regardless of the range of the head. For this reason, this expression was used to express the comparison between the calculated and experimental values of the rate of discharge.

Derivations of Least Square Equations

In order to get the best expressions for the two formulae desired, least square equations were derived. For the derivation of the least square equations representing equation (2), its modified form as represent by equation (6) was used.

$$\text{residue} = \ln Q - \ln E - 2.5 \ln H - 2.5\frac{a}{H} = 0$$

$$\frac{\partial\, \text{residue}^2}{\partial \ln E} = \cancel{2}(\ln Q - \ln E - 2.5 \ln H - 2.5\frac{a}{H})(\cancel{-1}) = 0$$

$$\frac{\partial\, \text{residue}^2}{\partial a} = \cancel{2}(\ln Q - \ln E - 2.5 \ln H - 2.5\frac{a}{H})\left(-\frac{\cancel{2.5}}{H}\right) = 0$$

The final equations used, in terms of logarithms to the base 10, were:

$$\begin{aligned} &\Sigma(\log Q) - p \log E - 2.5\,\Sigma(\log H) - \frac{2.5a}{2.302585}\Sigma\left(\frac{1}{H}\right) = 0 \\ &\Sigma\left(\frac{\log Q}{H}\right) - \log E\,\Sigma\left(\frac{1}{H}\right) - 2.5\,\Sigma\left(\frac{\log H}{H}\right) - \frac{2.5a}{2.302585}\Sigma\left(\frac{1}{H^2}\right) = 0 \end{aligned} \qquad (10)$$

where p is the number of observations being summed up.

The derivation of the least square equations for calculating E, n, and a of equation (3) is as follows:

$$\text{residue} = \ln Q - \ln D - n\ln H - \frac{an}{H} = 0$$

$$\frac{\partial\,\text{residue}^2}{\partial \ln D} = 2\left(\ln Q - \ln D - n\ln H - \frac{an}{H}\right)(-1) = 0$$

$$\frac{\partial(\text{residue})^2}{\partial a} = 2\left(\ln Q - \ln D - n\ln H - \frac{an}{H}\right)\left(-\frac{n}{H}\right) = 0 \ldots\ldots\ldots\ldots(11)$$

$$\frac{\partial\,\text{residue}^2}{\partial n} = 2\left(\ln Q - \ln D - n\ln H - \frac{an}{H}\right)\left(-\ln H - \frac{a}{H}\right) = 0 \ldots(12)$$

In equation (12) the second factor, $\frac{an}{H}$, multiplied by the part in parenthesis is equal to zero as shown in equation (11).

The final equations, in terms of logarithms to the base 10, are:

$$\Sigma(\log Q) - p\log D - n\Sigma(\log H) - \frac{an}{2.302585}\Sigma\left(\frac{1}{H}\right) = 0$$

$$\Sigma\left(\frac{\log Q}{H}\right) - \log D\,\Sigma\left(\frac{1}{H}\right) - n\Sigma\left(\frac{\log H}{H}\right) - \frac{an}{2.302585}\Sigma\left(\frac{1}{H^2}\right) = 0 \ldots(13)$$

$$\Sigma(\log Q \log H) - \log D\,\Sigma(\log H) - n\Sigma(\log H)^2 - \frac{an}{2.302585}\Sigma\left(\frac{\log H}{H}\right) = 0$$

Units Used in Expressing Results

The units used in expressing the results of the writer's experiments were:

f = centimeters

q = gallons per minute.

These were the units in which the measurements of the head and discharge rate were measured, and for convenience these same units were used in all calculations arising from the data.

EXPERIMENTAL RESULTS

An attempt was made to express the experimental results by equations of the form of equation (1). This was not successful in that the resulting expressions did not fit the data at all closely. Expressing the data by equations similar to (2) and (3) gave the following expressions:

20°1.1' notch:

$$q = .039609(f + .1020)^{2.5} \qquad (14)$$

$$q = .041104(f + .0502)^{2.4896} \qquad (15)$$

29°54.3' notch:

$$q = .058360(f + .0433)^{2.5} \qquad (16)$$

$$q = .062673(f - 0.0453)^{2.4794} \qquad (17)$$

44°58.0' notch:

$$q = .090072(f + .0189)^{2.5} \qquad (18)$$

$$q = .085398(f + .0796)^{2.5160} \qquad (19)$$

60°2.9' notch:

$$q = .123713(f - .0133)^{2.5} \qquad (20)$$

$$q = .124380(f - .0103)^{2.5008} \qquad (21)$$

90°12.4' notch:

$$q = .213561(f + .0229)^{2.5} \qquad (22)$$

$$q = .236765(f - .0956)^{2.4682} \qquad (23)$$

The results of the 29°54.3', 44°58', 60°2.9' notches were recalculated, several runs included in the above equations being omitted because of their values of $dq/q^{.6}$ being rather high. In accordance with the opinion set forth in the section "Expression of Results", $dq/q^{.6}$ was used as a criteria for comparing data in preference to the use of percent variation. The equations obtained disregarding the runs furthest off by the above method of comparison were:

28°54.3' notch:

$$q = .058372(f + .0429)^{2.5} \quad (16a)$$

$$q = .062403(f - .0384)^{2.4806} \quad (17a)$$

44°58' notch:

$$q = .090199(f + .0120)^{2.5} \quad (18a)$$

$$q = .087483(f + .0477)^{2.50916} \quad (19a)$$

60°2.9' notch:

$$q = .124734(f - .0129)^{2.5} \quad (20a)$$

$$q = .128053(f - .0424)^{2.4920} \quad (21a)$$

The experiment on the 20°1.1' notch with dreft solution and an accompanying check with water gave the following results:

With dreft:

$$q = .039450(f - .2490)^{2.5} \quad (24)$$

$$q = .040090(f - .2700)^{2.4953} \quad (25)$$

With water:

$$q = .039528(f - .2468)^{2.5} \quad (26)$$

$$q = .039574(f - .2483)^{2.4997} \quad (27)$$

For the equations of the form of equation (2), excluding the run on the 20°1.1' notch because of the inconsistant data on the measurements of the angle of the notch, the following general equation gives the value of the coefficient:

$$E = .21314(\tan\frac{\theta}{2})^{.97938} \qquad (28)$$

Results obtained by the use of equation (28) are given in the following table:

notch	29°54.3'	44°58.0'	60°2.9'	90°12.4'
E obs.	.058360	.090072	.124713	.213561
E calc by (28)	.058492	.089830	.12457	.21388

DISCUSSION OF RESULTS

It is evident from comparison of the equations obtained of the forms of (2) and (3) that, as far as accuracy in expressing the results in the experiment, there is not much to choose between the two. In equations of the form of (3) there is no direct relationship between the exponent and the angle, nor is the exponent a constant. This is in contradiction to results of many experimenters who have chosen to express their results in a form similar to (1). Greve[2] gives a constant exponent of 2.48 for all notches; this, however, is not generally acknowledged by most experimenters, and does not agree with results obtained by the writer.[1] There is no reason why this exponent should be constant at any value unless it be 2.5, in which case the coefficient must be expressed as a function of the head so as to fit experimental data. The great majority of writers express their results in the form of equation (1) and the exponents obtained vary with the angle, the value decreasing with decreasing notch angles. If there were a constant error in measuring the absolute head, one would expect the exponent to vary regularly with the angle, since the effect of the constant error in head becomes increasingly significant the greater the angle.

The use of equation (2) gives very good comparison of results, the constant a serving to take care of constant errors in reading the head. This is shown by comparing the results of the 20°1.1' notch with water and the run with dreft solution and its accompanying check with water. These experiments on the 20°1.1' notch were made after the method of measuring the head had been revised and corrected, the head readings in the case of the run with dreft solution and the check with water all being incorrect by a constant amount. As the results illustrate, this error is taken care of very nicely by expressing the results in equations similar to (2). Some discrepancy in the values of the coefficient can be attributed to the fact that the constant a is much greater in the case of the dreft solution runs because of the inherent constant error in measuring the head. As seen in the derivation of the least square equations from (6), the greater the value of a, the greater the error involved in leaving out all but the first term of the expansion involving this constant.

When the exponent is allowed to be variable as in equation (3), one would expect better agreement of the formulae with experimental data than would be obtained with equations with the exponent fixed at 2.5. This, of course, being true because of allowing an additional factor to vary so as to more closely conform to experimental data which admittedly is far from perfect and hence might require an exponent in the equations expressing the results to take care of some experimental errors. The slight benefit given by using a variable exponent is illustrated in the following table:

	Exponent = 2.5		Variable exponent	
Notch Angle	Av. percent error in q	Equation	Av. percent error in q	Equation
20°1.1'	0.0991	(14)	0.0897	(15)
	0.1419	(24)	0.1381	(25)
	0.1335	(26)	0.1271	(27)
29°54.3'	0.1967	(16)	0.1824	(17)
	0.1300	(16a)	0.1162	(17a)
44°58.0'	0.1666	(18)	0.1569	(19)
	0.0939	(18a)	0.0826	(19a)
60°2.9'	0.1567	(20)	0.1565	(21)
	0.1040	(20a)	0.1033	(21a)
90°12.4'	0.0820	(22)	0.0770	(23)

The above table clearly indicates that using the variable exponent, which is undoubtedly incorrect, gives results not appreciably better than using 2.5 as the value of the exponent. This and all other discussions, unless otherwise stated, refer only to the comparison of equations (2) and (3).

The above table also shows the accuracy obtained in the experiments performed by the writer. Actually the average percent variations tabulated are not a true evaluation of the average accuracy - this being even better than the figures would indicate. Reference to the data will confirm the fact that the greatest percent errors occur at the lowest heads- this is as pointed out in the section, "Expression of Results". The true evaluation of the data in these experiments is represented by the values of $dq/q^{.6}$. Further inspection of the data indicates that there is no pronounced change in $dq/q^{.6}$ over the entire range covered in each series of runs. The author does not represent this method of evaluating data as one whichshould always be used in expressing experiments with V-notch weirs. However, in view of the error in measuring the head being a constant absolute error and not a constant percentage error, the expression $dq/q^{.6}$ is the correct way to evaluate the accuracy of these experiments in the light of the limitations in measurements for the experiments carried out by the writer.

The expression for the coefficient E as represented by equation (28) contains the same fault of which the author has already complained in connection with the expression of V-notch weir data - the use of the variable exponent to conform to experimental results and to obscure, to some extent, experimental errors. The exponent of the tangent of half the notch angle should undoubtedly enter to the first power if the notches are very accurately machined and the measurements of the angles correct. The discrepancy in the results of the author is probably due to a small degree to inaccurate measurements of the angles, an error of several minutes causing an appreciable change in the exponent. However, the most likely cause of the variation from one in the value of the exponent is probably the notch being set in the weir plate assembly in such a way that the bisector of the notch angle was 22' from the vertical. This fact should cause no other difficulty since the liquid section flowing through the notch is still triangular in cross-section.

The equations (16a), (17a),.....(21a) were calculated with the omission of several runs in each series because of relatively high values of $dq/q^{.6}$. A purpose served by the equations is to further emphasize the inadequacy of the use of a variable exponent. For example, as shown in equations (21) and (21a), the leaving out of serveral points changes the coefficient from 0.124380 to 0.128053, the exponent from 2.5008 to 2.4920, and the constant "a" from - .0103 to -.0424. However in the corresponding equations, (20) and (20a), the coefficient changes from .124713 to .124734, the exponent is constant at 2.5, and the constant "a" changes only from -.0133 to .0129. Even comparison of experimental data from the same set is shown to be difficult with the use of the variable exponent equation.

The constant a in equations (2) and (3) takes care of a multitude of sins, foremost of which is the inability to get accurate readings of the head with reference to the true bottom of the notch. It was because of this reason that the author chose to represent results by (2) and (3) in preference to equations (4) and (5), although the similarity of these two sets is pointed out in the section on "Discussion of Equations". No general relationship was obtainable from the values of a for different notches. The fact that the true head was not the one recorded due to inability to get the true bottom of the notch might have spoiled any generalities which might have been gleaned from values of a. However, the only instances where there was any doubt as to the correct bottom of the notch arose in the case of the 20°1.1 and the 90°12.4' notches. The values of a for the other notches, in the case of equations of form similar to (2), decrease with increasing notch angle and the value becomes negative for the 60°2.9' notch. Since the magnitude of a is so small, quantitative assurance of the values given can not be made. However, qualitatively there seems to be a definite trend from positive to negative values which would tend to indicate that attempts to evaluate a as a function of the notch angle, head, physical properties of the liquid, etc., must result in a fairly complicated expression. The presence of both positive and negative values would lead one to believe that there are two opposing factors involved in the expression for a. However, in view of the smallness of this value, it is extremely difficult to conceive of experiments accurate enough to enable a to be evaluated in more general terms.

The experiments on solutions of surface tension lower than water were too limited to settle the effect of surface tension; however, it does appear that the effect of surface tension is rather small, leaving the same difficulty here as in the case of getting generalizations for <u>a</u>. There seems to be very little reason for expecting surface tension to have any appreciable effect on the rate of flow. The only places where surface tension could come into play are at the points of contact between the liquid and the notch edge. Due to the fact that the notch edges are sharp and the nappe springs clear, any tendency for the interfacialtension between the liquid and the notch edge to alter the contraction would seem to have been overcome by the velocity of the liquid approaching the notch. Thus, the conclusion is that, for sharp-edge weirs where there is no clinging, a change in the surface tension should have no measurable effect on the rate of flow.

In conclusion, it must be admitted that weir flow as approached theoretically still leaves much to be desired. Any further developments, it seems, must come from experiments of extremely great accuracy. However, this in no way interferes with the use of weirs as a metering device for in that use its calibration is empirical. The effect of viscosity has not been established and no data on experiments with different liquids have been published, although such experiments have been performed. Evidently the results were not consistent. Experiments on glycerine-water solutions are now under way at the University of Cincinnati, using the same equipment as was used by the writer. It is hoped that the results of these experiments will shed a little light on the role viscosity plays in weir flow.

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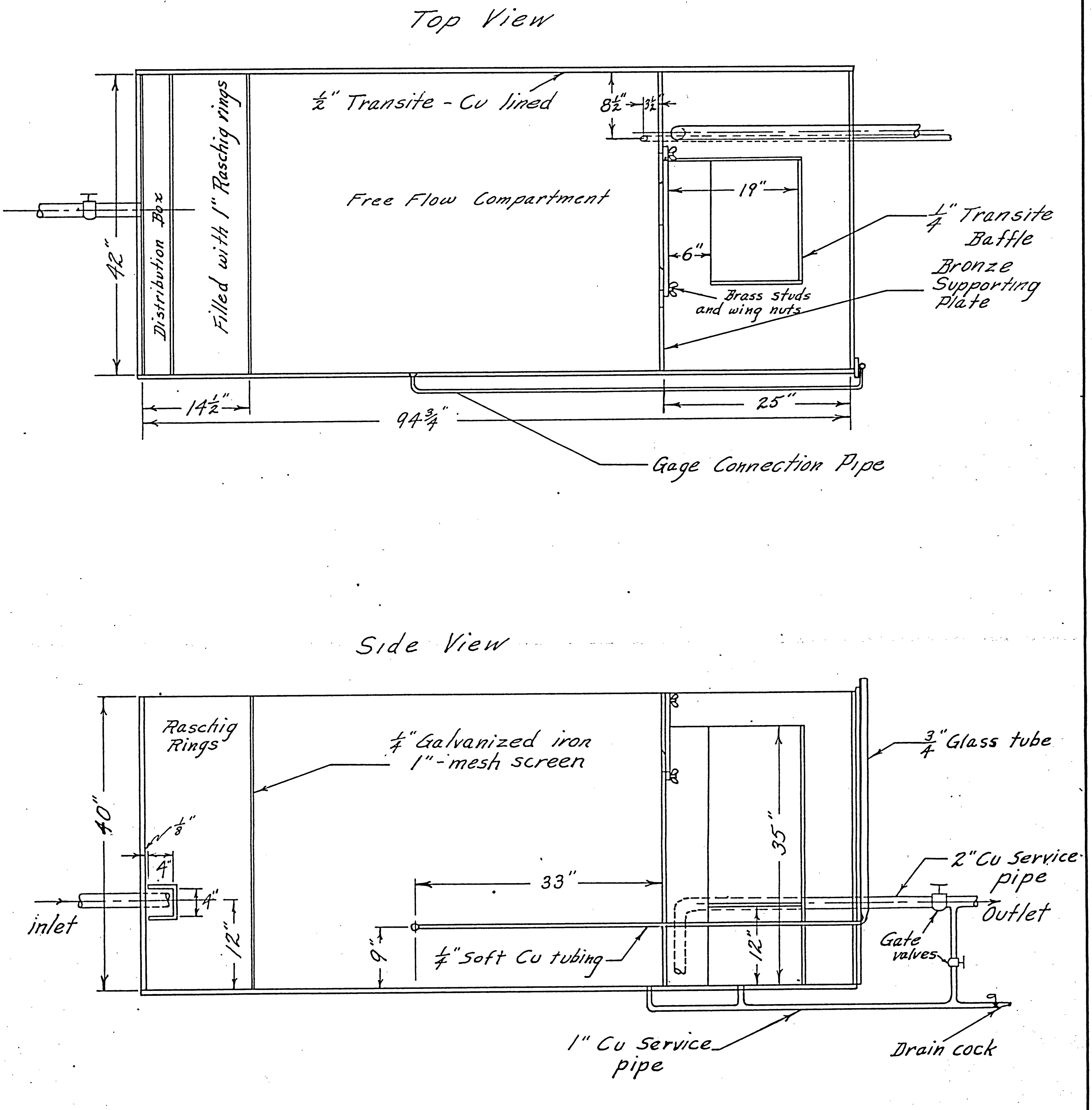
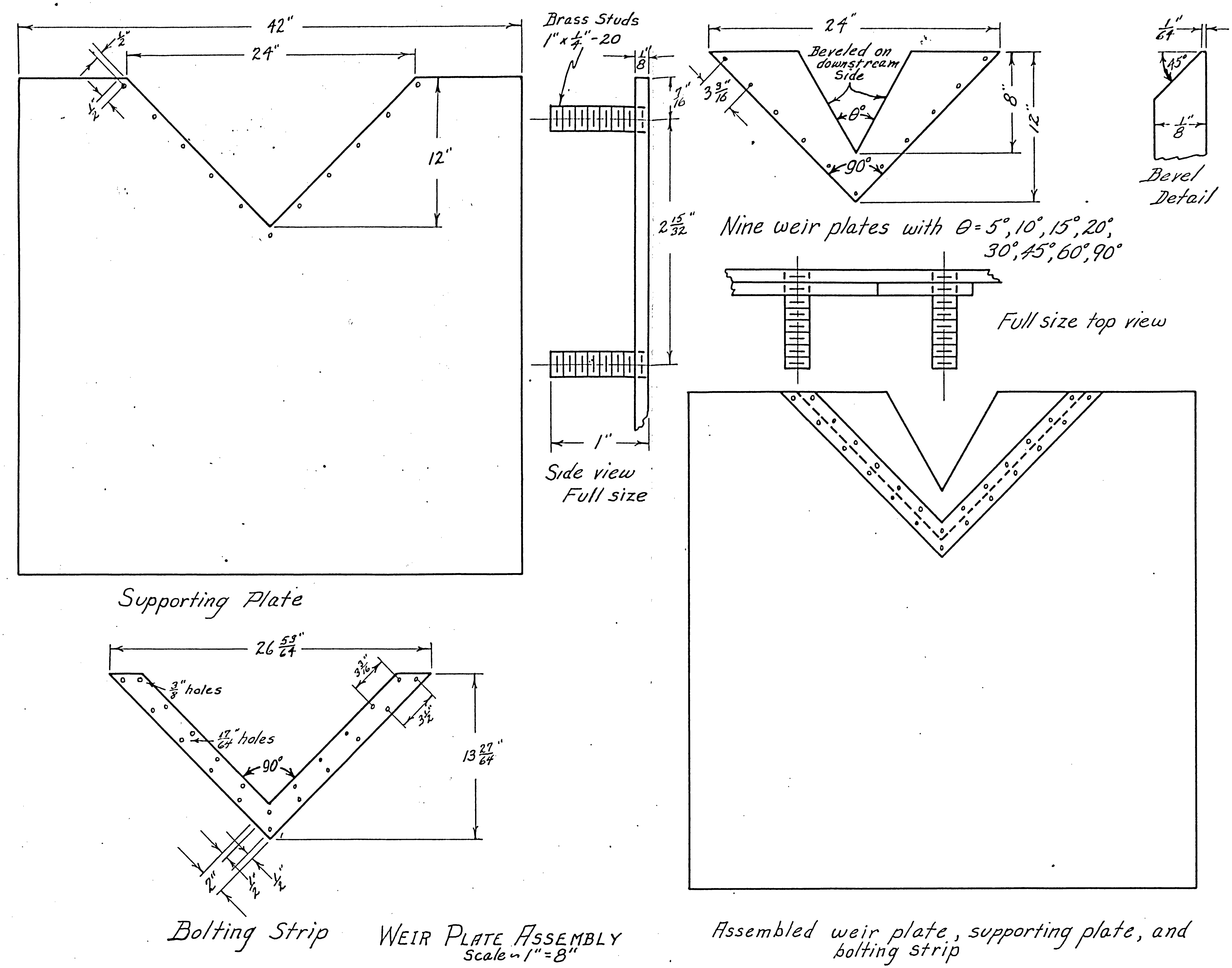
Appendices

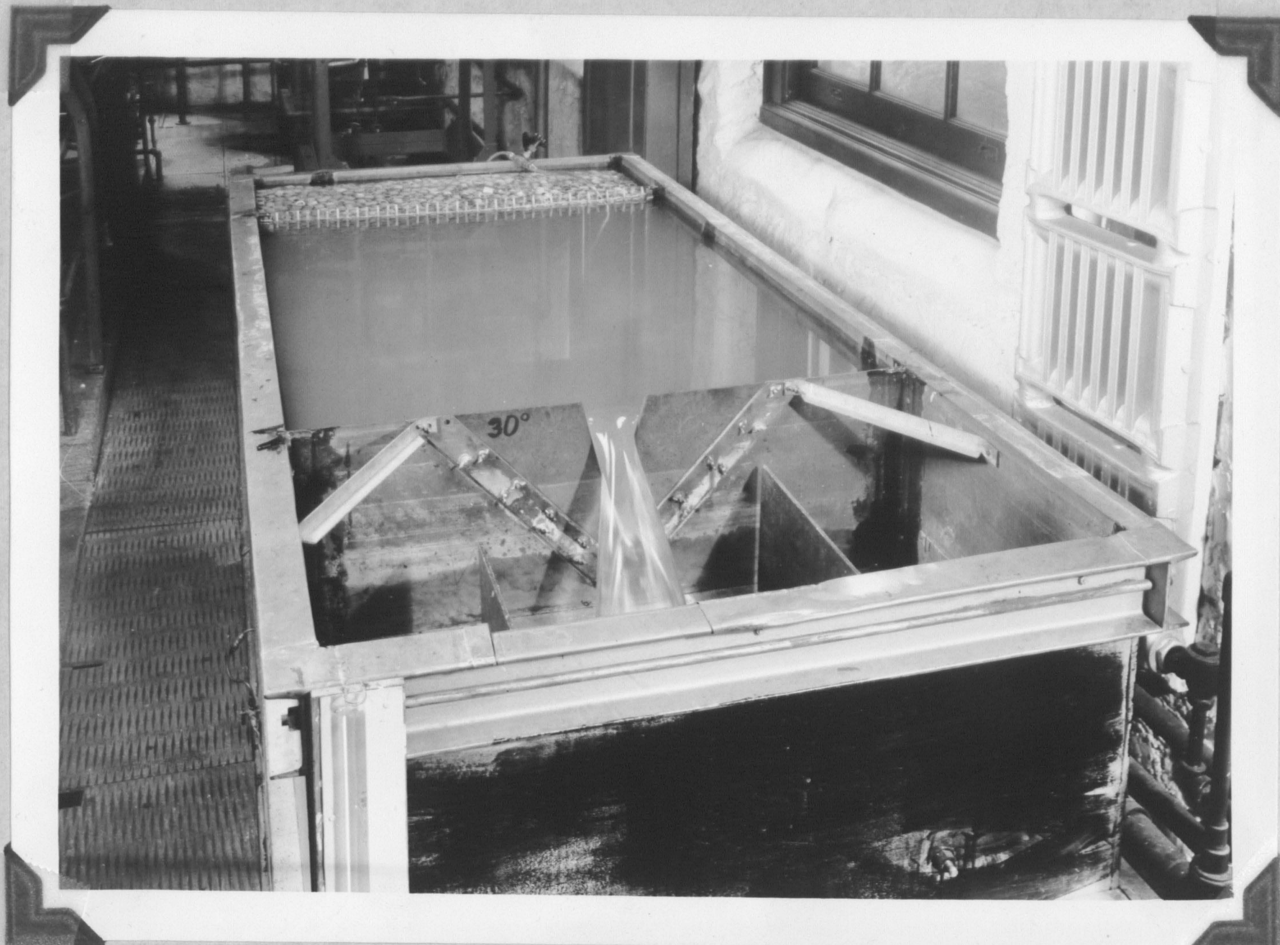


WEIR BOX SKETCH
Scale ~ 1" = 15"



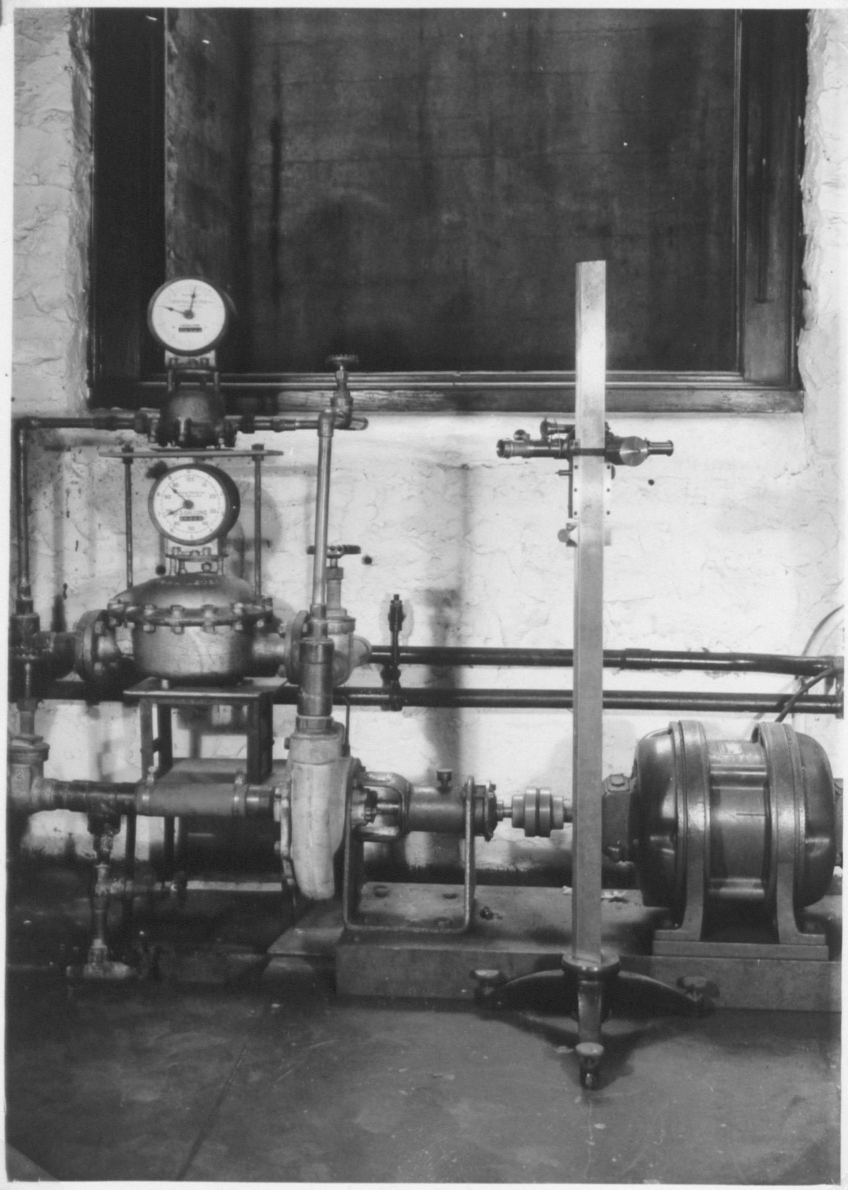
WEIR PLATE ASSEMBLY
Scale ~ 1" = 8"

A view of the general construction of the tank. The stillness of the water surface is evident from the reflection in the water.

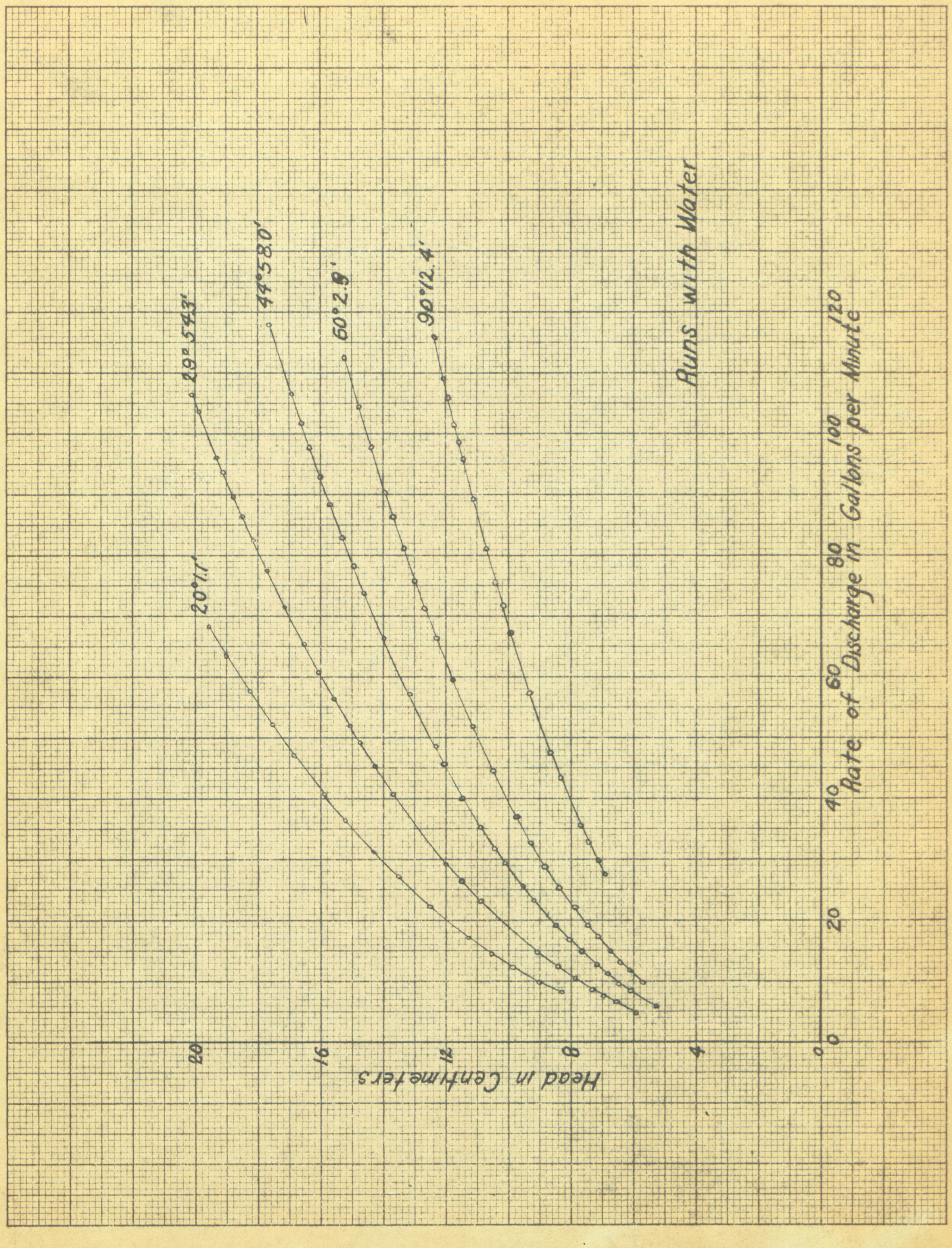


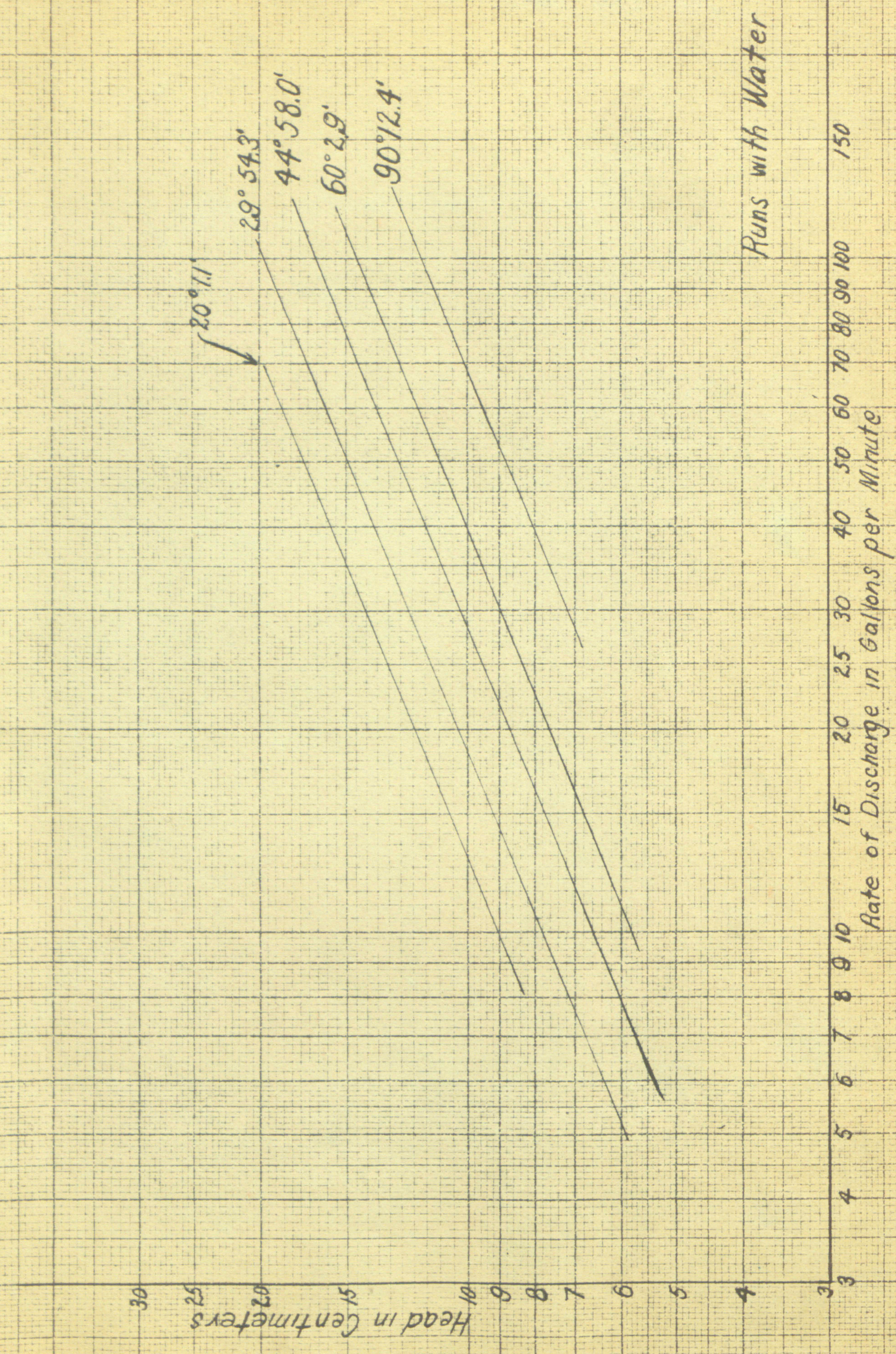
A view of the method of bracing the tank, the weir plate arrangement, and the pattern of the nappe.

Details of the nappe and also of the weir plate assembly.

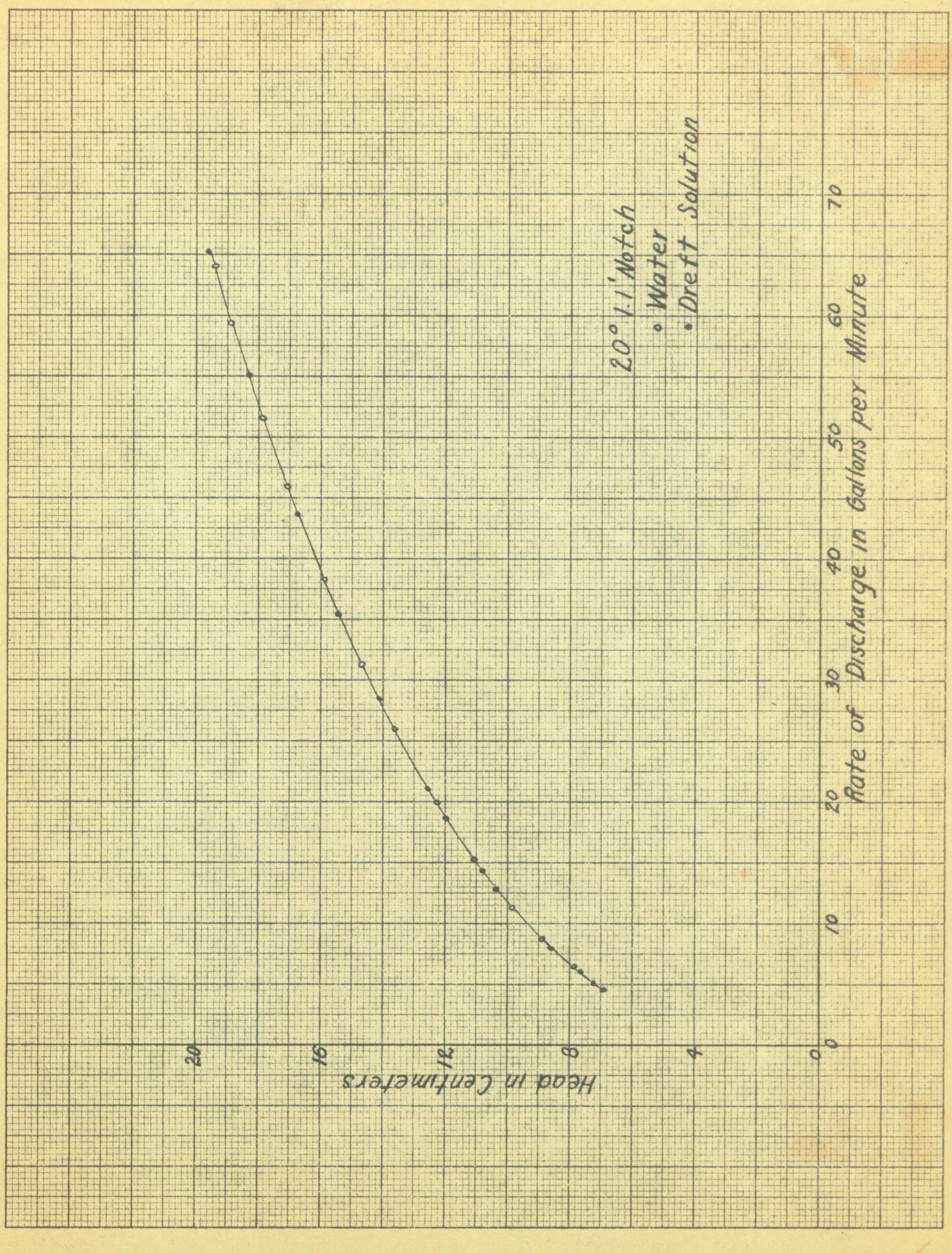


Pump, motor, meters, and cathetometer.


Runs with Water
Rate of Discharge in Gallons per Minute
0
20
40
60
80
100
120
Head in Centimeters
0
4
8
12
16
20
20°1.1'
29°54.3'
44°58.0'
60°2.9'
90°12.4'



Runs with Water
20°1.1'
29°54.3'
44°58.0'
60°2.9'
90°12.4'
Rate of Discharge in Gallons per Minute
3 4 5 6 7 8 9 10 15 20 25 30 40 50 60 70 80 90 100 150
Head in Centimeters
3 4 5 6 7 8 9 10 15 20 25 30


20° 1.1' Notch
° Water
• Dreft Solution
Rate of Discharge in Gallons per Minute
0
10
20
30
40
50
60
70
Head in Centimeters
0
4
8
12
16
20

DATA

Weir Tests With Water

20°1.1' notch - Feb. 25, 1941, Water temp. = 20.8-21.1°C:

	head cm.	corrected rate gal/min	calc.rate, eq(14)	calc rate eq(15)	From calc results of eq(14): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(14): $\frac{Qcalc-Qobs}{Qobs} \times 100$	From calc. results of eq(15): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc. results of eq(15): $\frac{Qcalc-Qobs}{Qobs} \times 100$
1)	19.568	68.07	67.968	67.957	-.0081	-.150	-.0090	-.166
2)	19.038	63.47	63.482	63.482	+.0010	+.019	-.0009	-.017
3)	18.298	57.47	57.523	57.510	+.0044	+.092	+.0034	+.070
4)	17.568	51.99	51.988	51.982	-.0002	-.004	-.0008	-.015
5)	16.865	46.93	46.969	46.970	+.0039	+.083	+.0040	+.085
6)	15.923	40.65	40.718	40.725	+.0074	+.167	+.0081	+.185
7)	15.215	36.37	36.369	36.378	-.0001	-.003	+.0009	+.022
8)	14.330	31.30	31.341	31.353	+.0052	+.131	+.0067	+.169
9)	13.515	27.17	27.102	27.114	-.0094	-.250	-.0077	-.206
10)	12.460	22.17	22.154	22.164	-.0025	-.072	-.0009	-.027
11)	11.213	17.067	17.058	17.065	-.0016	-.053	-.0004	-.012
12)	10.510	14.538	14.531	14.536	-.0014	-.048	-.0004	-.014
13)	9.808	12.270	12.246	12.248	-.0053	-.196	-.0049	-.179
14)	9.005	9.916	9.9137	9.9127	-.0005	-.020	-.0008	-.030
15)	8.293	8.072	8.0881	8.0844	+.0046	+.198	+.0034	+.149
				Average	.00371	.0991	.00349	.0897

29°54.3' notch — Feb. 26, 1941, water temp. = 19.7-20.1°C.

	head cm.	corrected rate gal/min	calc. rate, eq(16)	calc rate eq(17)	From calc results of eq (16) $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$ X100	From calc. results of eq(17) $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$ X100
1)	20.088	106.007	106.1189	105.9498	+.00682	+.1055	-.00348	-.0540
2)	19.811	103.508	103.8015	103.6453	+.01814	+.2836	+.00849	+.1364
3)	19.311	96.095	96.1739	96.0562	+.00510	+.0821	-.00251	-.0404
4)	19.111	93.610	93.7082	93.6032	+.00645	+.1049	-.00045	-.0073
5)	18.781	89.751	89.7245	89.6372	-.00178	-.0295	-.00766	-.1268
6)	18.493	86.338	863315	86.2598	-.00045	-.0075	-.00539	-.0906
7)	18.148	82.507	82.3706	82.3157	-.00966	-.1653	-.01355	-.2319
8)	17.693	77.351	77.3161	77.2806	-.00227	-.0451	-.00517	-.0911
9)	17.156	71.501	71.5963	71.5811	+.00735	+.1333	+.00618	+.1120
10)	16.523	65.224	65.1895	65.1940	-.00281	-.0529	-.00245	-.0460
11)	15.588	56.402	56.3769	56.4034	-.00223	-.0445	+.00013	+.0025
12)	16.043	60.643	60.5692	60.5863	-.00629	-.1217	-.00483	-.0935
13)	15.078	51.920	51.8903	51.9255	-.00278	-.0572	+.00051	+.0106
14)	14.733	49.051	48.9810	49.0209	-.00677	-.1427	-.00281	-.0614
15)	14.258	45.070	45.1390	45.1838	+.00702	+.1531	+.01158	+.2525
16)	13.701	40.714	40.8714	40.9188	+.017030	+.3866	+.02227	+.5057
17)	12.006	29.377	29.4115	29.4598	+.00454	+.1174	+.01689	+.2818

29°54.3' notch - (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(16)	calc rate eq(17)	From calc results of eq(16) $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc results of eq(16) $\frac{Qcalc-Qobs}{Qobs}$X100	From calc. results of eq(17) $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc. results of eq(17) $\frac{Qcalc-Qobs}{Qobs}$X100
18)	11.503	26.434	26.4375	26.4830	+.00049	+.0132	+.00687	+.1854
19)	10.893	23.116	23.0828	23.1237	-.00504	-.1436	+.00117	+.0333
20)	9.073	14.7194	14.64402	14.66568	-.01500	-.5122	-.01070	-.3648
21)	8.406	12.1682	12.11057	12.12421	-.01286	-.4542	-.00982	-.3616
22)	7.851	10.2600	10.21873	10.22571	-.01022	-.4025	-.00848	-.3343
23)	7.375	8.7821	8.78730	8.74869	.00945	-.3963	-.00907	-.3803
24)	7.011	7.6633	7.71346	7.71076	+.01479	+.6551	+.01400	+.6198
25)	6.538	6.4838	6.48476	6.47703	+.00031	+.0148	-.00222	-.1049
26)	5.893	4.9863	5.01075	4.99706	+.00931	+.4893	+.00408	+.2146
				Average	.007114	.1967	.00711	.1824

29°54.3 notch (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(16a)	calc rate eq(17a)	From calc results of eq(16a): (Qcalc-Qobs)/(Qobs)	From calc results of eq(16a): (Qcalc-Qobs)/Qobs X100	From calc results of eq(17a): (Qcalc-Qobs)/$(Qobs)^{.5}$	From calc results of eq(17a): (Qcalc-Qobs)/Qobs X100
1)	20.088	106.007	106.1355	105.9806	+.00783	+.1212	-.00161	-.0249
2)	19.311	96.095	96.1884	96.0841	+.00604	+.0972	-.00071	-.0113
3)	19.111	93.610	93.7222	93.6291	+.00737	+.1198	+.00125	+.0204
4)	18.781	89.751	89.7377	89.6622	-.00090	-.0148	-.00598	-.0989
5)	18.493	86.338	86.3452	86.2830	+.00050	+.0083[illegible]	-.00479	-.0637
6)	17.693	77.351	77.3279	77.3007	-.00170	-.0299	-.00370	-.0650
7)	17.156	71.501	71.6069	71.5989	+.00817	+.1481	+.00755	+.1369
8)	16.523	65.224	65.1989	65.2095	-.00205	-.0385	-.00118	-.0222
9)	15.588	56.406	56.3848	56.4163	-.00153	-.0305	+.00127	+.0254
10)	16.043	60.643	60.5782	60.6004	-.00552	-.1069	-.00363	-.0702
11)	15.078	51.920	51.8975	51.9371	-.00210	-.0433	+.00160	+.0329
12)	14.733	49.050	48.9877	49.0317	-.00612	-.1290	-.00187	-.0393
13)	12.006	29.377	29.4151	29.4665	+.00501	+.1297	+.01177	+.3047
14)	11.503	26.434	26.4407	26.4892	+.00094	+.0253	+.00774	+.2088
15)	7.851	10.2600	10.2195	10.2303	-.01002	-.3947	-.00735	-.2895
16)	7.375	8.7821	8.7479	8.7532	-.00929	-.3894	-.00785	-.3291
17)	6.538	6.4838	6.4851	6.4814	+.00042	+.0200	-.00078	-.0370
18)	5.893	4.9863	5.0109	5.0012	+.00938	+.4934	+.00591	+.3109
				Average	.00472	.1300	.00425	.1162

44°58.0' notch - Feb. 26, 1941, water temp. = 21.8-22.1°C

	head cm.	corrected rate gal/min	calc.rate, eq(18)	calc rate eq(19)	From calc results of eq(18)		From calc results of eq(19)	
					$\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs} X100$	$\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs} X100$
1)	17.648	118.065	118.1648	118.3277	+.00570	+.0845	+.01500	+.2225
2)	16.625	101.644	101.7952	101.8913	+.00945	+.1488	+.01545	+.2433
3)	16.930	106.449	106.5222	106.6377	+.00445	+.0688	+.01147	+.1117
4)	16.355	97.827	97.7178	97.7981	-.00705	-.1126	-.00185	-.0295
5)	16.020	92.845	92.7953	92.8956	-.00328	-.0535	+.00096	+.0157
6)	15.708	88.394	88.3477	88.3985	-.00315	-.0524	+.00031	+.0051
7)	15.308	82.852	82.8368	82.8239	-.00107	-.0183	-.00135	-.0231
8)	14.938	78.094	77.9276	77.9485	-.01203	-.2131	-.01065	-.1863
9)	14.608	73.882	73.7001	73.7105	-.01369	-.2482	-.01298	-.2321
10)	13.963	66.154	65.8420	65.8351	-.02522	-.4716	-.02579	-.4822
11)	12.073	45.621	45.7955	45.7621	+.01763	+.3825	+.01426	+.3093
12)	12.350	48.509	48.4636	48.4321	-.00442	-.0936	-.00749	-.1585
13)	11.433	40.043	39.9744	39.9383	-.00750	-.1713	-.01144	-.2615
14)	10.880	35.333	35.3219	35.2859	-.00131	-.0314	-.00555	-.1333
15)	10.420	31.636	31.7121	31.6772	+.00957	+.2402	+.00519	+.1302

44°58.0' notch (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(18)	calc rate eq(19)	From calc results of eq(18): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(18): $\frac{Qcalc-Qobs}{Qobs}$ X100	From Calc results of eq(19): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From Calc results of eq(19): $\frac{Qcalc-Qobs}{Qobs}$ X100
16)	10.075	29.132	29.1566	29.1232	+.00325	+.0844	-.00116	-.0302
17)	9.560	25.561	25.5786	25.5487	+.00252	+.0689	-.00176	-.0481
18)	9.190	23.100	23.1796	23.1527	+.01210	+.3446	+.00801	+.2281
19)	8.485	19.0184	18.99473	18.97424	-.00405	-.1246	-.00755	-.2324
20)	8.088	16.8477	16.85481	16.83844	+.00130	+.0421	-.00171	-.0552
21)	7.678	14.7767	14.80397	14.79197	+.00543	+.1848	+.00304	+.1035
22)	7.190	12.5270	12.56780	12.56120	+.00895	+.3257	+.00756	+.2730
23)	6.845	11.0983	11.11771	11.11484	+.00458	+.1748	+.00389	+.1487
24)	6.430	9.5145	9.51263	9.51523	-.00049	-.0200	-.00008	-.0031
25)	6.068	8.2315	8.23337	8.23859	+.00054	+.0231	+.00200	+.0863
26)	5.225	5.7031	5.67183	5.68452	-.011010	-.5488	-.00654	-.3261
				Average	.006913	.1666	.007040	.1569

44°58.0' notch (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(18a)	calc rate eq(19a)	From calc results of eq(18a): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(18a): $\frac{Qcalc-Qobs}{Qobs}$X100	From calc results of eq(19a): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(19a): $\frac{Qcalc-Qobs}{Qobs}$X100
1)	16.355	97.827	97.7510	97.8003	-.00485	-.0777	-.00171	-.0273
2)	16.020	92.845	92.8406	92.8654	-.00029	-.0047	+.00135	+.0220
3)	15.708	88.394	88.3754	88.4063	-.00126	-.0210	+.00084	+.0139
4)	15.308	82.852	82.8604	82.8824	+.00059	+.0101	+.00215	+.0369
5)	12.350	48.509	48.4643	48.4496	-.00435	-.0921	-.00578	-.1222
6)	10.420	31.636	31.7044	31.6878	+.00861	+.2162	+.00652	+.1637
7)	10.075	29.132	29.1478	29.1321	+.00209	+.0826	+.00001	+.0003
8)	9.560	25.561	25.5686	25.5548	+.00109	+.0297	-.00089	-.0243
9)	8.485	19.0184	18.9829	18.9744	-.00606	-.1867	-.00752	-.2316
10)	8.088	16.8477	16.8427	16.8364	-.00092	-.0297	-.00208	-.0671
11)	7.678	14.7767	14.7916	14.7877	+.00296	+.1008	+.00219	+.0744
12)	7.190	12.5270	12.5555	12.5540	+.00625	+.2275	+.00592	+.2155
13)	6.845	11.0983	11.1054	11.1065	+.00168	+.0640	+.00193	+.0749
14)	6.430	9.5145	9.5006	9.5040	-.00360	-.1461	-.00272	-.1104
15)	6.068	8.2315	8.2217	8.2270	-.00277	-.1191	-.00127	-.0547
				Average	.00316	.0939	.00285	.0826

60°2.9' notch - Feb. 27, 1941, water temp = 22.8-23.1°C

	head cm.	corrected rate gal/min	calc.rate, eq(20)	calc rate eq(21)	From calc results of eq(20): $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc results of eq(20): $\frac{Qcalc-Qobs}{Qobs}$X100	From calc results of eq(21): $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc results of eq(21): $\frac{Qcalc-Qobs}{Qobs}$X100
1)	15.231	112.342	112.6637	112.6696	+.01893	+.2863	+.01928	+.2916
2)	14.781	104.419	104.5187	104.5230	+.00613	+.0955	+.00639	+.0996
3)	14.393	97.895	97.7881	97.7914	-.00683	-.1092	-.00662	-.1058
4)	13.941	90.253	90.2839	90.2861	+.00207	+.0342	+.00222	+.0367
5)	13.681	86.207	86.1292	86.1309	-.00537	-.0909	-.00525	-.0883
6)	13.358	81.065	81.1304	81.1316	+.00468	+.0807	+.00477	+.0822
7)	12.988	75.668	75.6232	75.6237	-.00334	-.0592	-.00330	-.0585
8)	12.662	71.049	70.9747	70.9761	-.00576	-.1046	-.00565	-.1026
9)	12.303	66.219	66.0337	66.0332	-.01497	-.2798	-.01501	-.2806
10)	11.793	59.600	59.3946	59.3938	-.01767	-.3445	-.01775	-.3460
11)	9.766	36.967	37.0445	37.0431	+.00888	+.2096	+.00872	+.2059
12)	9.308	32.770	32.8473	32.8460	+.00953	+.2359	+.00937	+.2319
13)	8.858	28.920	29.0148	29.0136	+.01259	+.3278	+.01286	+.3237
14)	8.401	25.380	25.4109	25.4100	+.00443	+.1217	+.00431	+.1182
15)	7.933	22.003	22.0132	22.0125	+.00160	+.0464	+.00149	+.0432

60°2.9' notch (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(20)	calc rate eq(21)	From calc results of eq(20): $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc results of eq(20): $\frac{Qcalc-Qobs}{Qobs} X100$	From calc results of eq(21): $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	From calc results of eq(21): $\frac{Qcalc-Qobs}{Qobs} X100$
16)	7.473	18.9938	18.95463	18.95427	-.00670	-.2064	-.00675	-.2070
17)	7.193	17.2684	17.22573	17.22550	-.00773	-.2473	-.00776	-.2484
18)	6.778	14.8505	14.84347	14.84338	-.00139	-.0471	-.00141	-.0478
19)	6.431	13.0302	13.01243	13.01276	-.00381	-.1366	-.00373	-.1335
20)	6.151	11.6141	11.63928	11.64096	+.00579	+.2170	+.00618	+.2316
21)	5.731	9.7500	9.74912	9.74978	-.00023	-.0090	-.00005	-.0021
				Average	.007068	.1567	.007089	.1565

60°2.9' notch (cont)

	head cm.	corrected rate gal/min	calc.rate, eq(20a)	calc rate eq(21a)	From calc results of eq(20a) $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$ X100	From calc results of eq(21a) $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$ X100
1)	14.781	104.419	104.5433	104.5117	+.00764	+.1190	+.00571	+.0888
2)	14.393	97.895	97.8114	97.7896	-.00534	-.0854	-.00674	-.1077
3)	13.941	90.253	90.3055	90.2935	+.00352	+.0582	+.002272	+.0449
4)	13.681	86.207	86.1500	86.1428	-.00393	-.0661	-.00443	-.0745
5)	13.358	81.065	81.1502	81.1483	+.00610	+.1051	+.00596	+.1028
6)	12.988	75.668	75.6418	75.6453	-.00195	-.0346	-.00169	-.0300
7)	12.663	71.049	70.9937	71.0010	-.00428	-.0778	-.00372	-.0676
8)	8.401	25.380	25.4182	25.4291	+.00549	+.1505	+.00705	+.1935
9)	7.933	22.003	22.0197	22.0279	+.00261	+.0759	+.00390	+.1132
10)	7.473	18.9938	18.9603	18.9657	-.00573	-.1764	-.00480	-.1479
11)	7.193	17.2684	17.2310	17.2345	-.00677	-.2166	-.00614	-.1963
12)	6.778	14.8505	14.8481	14.8488	-.00048	-.0161	-.00034	-.0114
13)	6.431	13.0302	13.0167	13.0152	-.00289	-.1036	-.00321	-.1151
14)	6.151	11.6141	11.6431	11.6398	+.00666	+.2497	+.00590	+.2213
15)	5.731	9.7500	9.7525	9.7466	+.00063	+.0256	-.00087	-.0349
				average	.00427	.1040	.00421	.1033

90°12.4' notch - Feb. 27, 1941, water temp. = 23.8-24.3°C:

	head cm.	corrected rate gal/min	calc. rate, eq(22)	calc rate eq(23)	From calc results of eq(22) $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$X100	From calc results of eq(23) $\frac{Qcalc-Qobs}{(Qobs)^{.6}}$	$\frac{Qcalc-Qobs}{Qobs}$X100
1)	12.389	115.962	115.9092	115.9337	-.00305	-.0455	-.00163	-.0244
2)	12.097	109.048	109.2117	109.1610	+.00980	+.1501	+.00677	+.1036
3)	11.957	106.004	106.0853	106.0447	+.00495	+.0767	+.00248	+.0384
4)	11.734	101.215	101.2172	101.1917	+.00014	+.0022	-.00146	-.0230
5)	11.599	98.404	98.3365	98.3190	-.00430	-.0686	-.00542	-.0864
6)	11.477	95.793	95.7761	95.7655	-.00107	-.0176	-.00178	-.0287
7)	10.449	75.713	75.7853	75.8114	+.00539	+.0955	+.00734	+.1300
8)	10.204	71.423	71.4302	71.4602	+.00056	+.0101	+.00287	+.0521
9)	9.949	67.174	67.0605	67.0931	-.00909	-.1690	-.00648	-.1204
10)	8.664	47.491	47.4990	47.5216	+.00079	+.0168	+.00302	+.0644
11)	8.632	43.553	43.4778	43.4940	-.00781	-.1727	-.00612	-.1355
12)	7.712	35.565	35.5353	35.5339	-.00348	-.0835	-.00365	-.0874
13)	7.439	32.456	32.4822	32.4726	+.00325	+.0807	+.00205	+.0511
14)	7.182	29.777	29.7572	29.7393	-.00258	-.0665	-.00492	-.1266
15)	6.972	27.588	27.6361	27.6111	+.00657	+.1744	+.00316	+.0837
				Average	.00419	.0820	.00394	.0770

20°1.1' notch with dreft solution - Nov. 13, 1941, water temp. = 25.0-25.5

	head cm.	corrected rate gal/min	calc.rate, eq(24)	calc rate e_q(25)	From calc results of eq(24): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(24): $\frac{Qcalc-Qobs}{Qobs}$ X100	From calc results of eq(25): $\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	From calc results of eq(25): $\frac{Qcalc-Qobs}{Qobs}$ X100
1)	12.398	20.364	20.2954	20.2972	-.01514	-.3369	-.01475	-.3280
2)	10.381	12.877	12.8910	12.8919	+.00344	+.1087	+.00366	+.1157
3)	7.226	5.0725	5.07244	5.06981	-.00003	-.0020	-.00082	-.0530
4)	15.453	35.455	3 5.5584	35.5551	+.02000	+.2916	+.01936	+.2820
5)	14.608	30.851	30.8218	30.8209	-.00584	-.0946	-.00602	-.0976
6)	12.558	20.980	20.9703	20.9720	-.00215	-.0462	-.00178	-.0381
7)	12.003	18.697	18.6858	18.6876	-.00252	-.0599	-.00212	-.0503
8)	10.806	14.293	14.2856	14.2870	-.00177	-.0518	-.00144	-.0420
9)	8.673	8.1529	8.1254	8.1345	-.00752	-.3373	-.00503	-.2257
10)	7.731	6.0497	6.0407	6.0402	-.00263	-.1488	-.00278	-.1570
11)	19.605	65.079	65.0258	64.9945	-.00892	-.0817	-.01175	-.1298
				Average	.006360	.1419	.006319	.1381

Surface tension- using stalagmometer;

Dreft solution: 80.5-82.7 drops

Water at 24.0°C: 47.7 drops

20°1.1' notch with water - check on run with dreft solution
Nov. 6, 1941, water temp. = 26.0-27.5°C

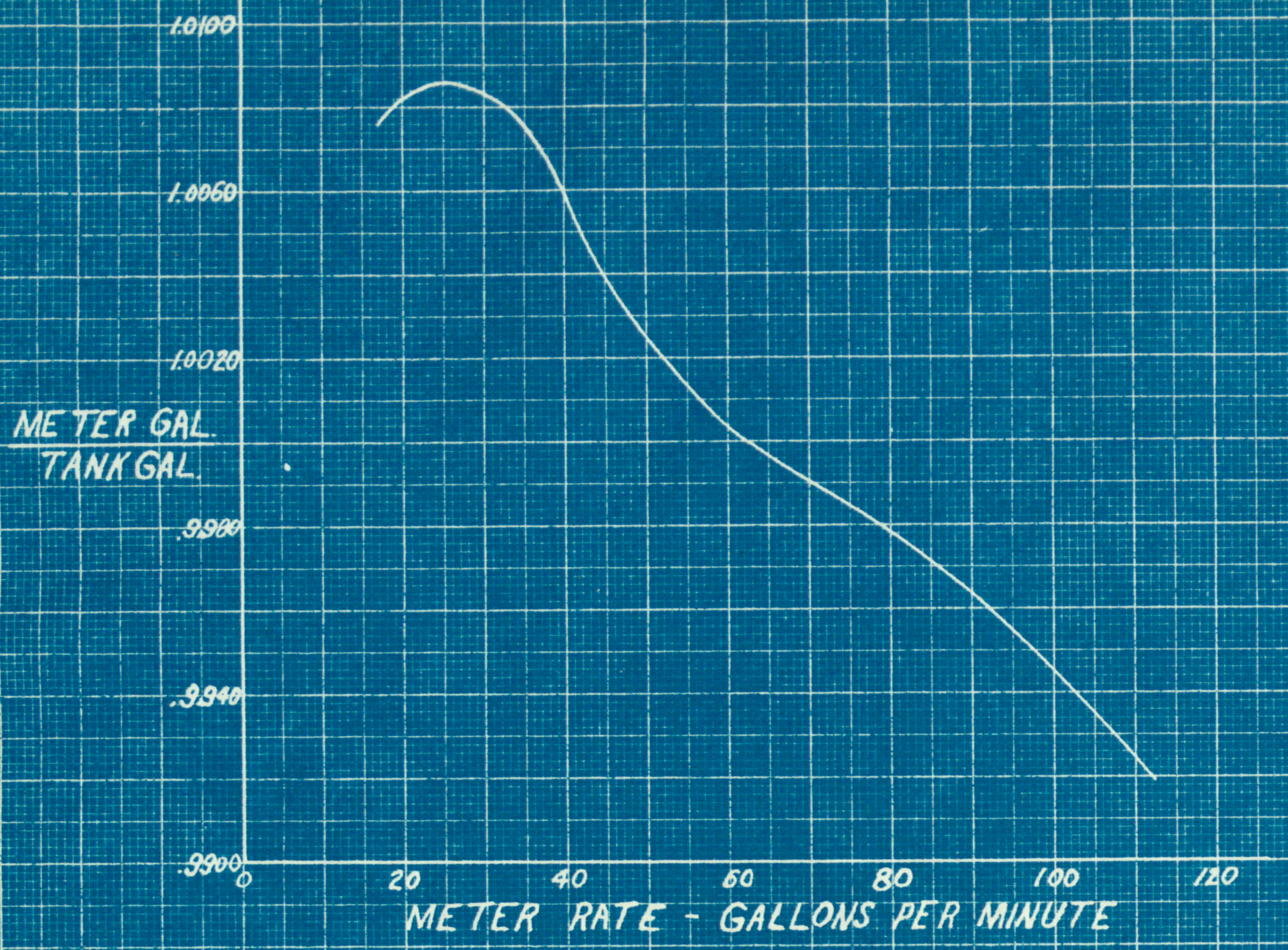
	head cm.	corrected rate gal/min	calc.rate, eq(26)	calc rate eq(27)	From calc results of eq(26)		From calc results of eq(27)	
					$\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$ X100	$\frac{Qcalc-Qobs}{(Qobs)^{.8}}$	$\frac{Qcalc-Qobs}{Qobs}$
1)	18.913	59.438	59.5350	59.5022	+.01662	+.1632	+.01100	+.1080
2)	17.863	51.516	51.4852	51.4845	-.00546	-.0598	-.00559	-.0611
3)	17.048	45.860	45.7355	45.7351	-.02271	-.2715	-.02278	-.2724
4)	15.908	38.361	38.3770	38.3675	+.00305	+.0417	+.00123	+.0169
5)	14.628	31.039	31.0020	31.0022	-.00740	-.1192	-.00736	-.1186
6)	13.648	25.940	25.9858	25.9876	+.00955	+.1766	+.00992	+.1836
7)	12.373	20.253	20.2402	20.2405	-.00283	-.0632	-.00276	-.0617
8)	11.056	15.174	15.1840	15.1843	+.00237	+.0659	+.00243	+.0679
9)	9.868	11.357	11.3489	11.3496	-.00205	-.0713	-.00187	-.0652
10)	8.883	8.6930	8.6638	8.6639	-.00787	-.3359	-.00785	-.3348
11)	7.848	6.3090	6.2966	6.2966	-.00360	-.1964	-.00360	-.1961
12)	6.936	4.5761	4.5744	4.5743	-.00053	-.0371	-.00056	-.0393
				Average	.007003	.1335	.006413	.1271

Weir Plate Data

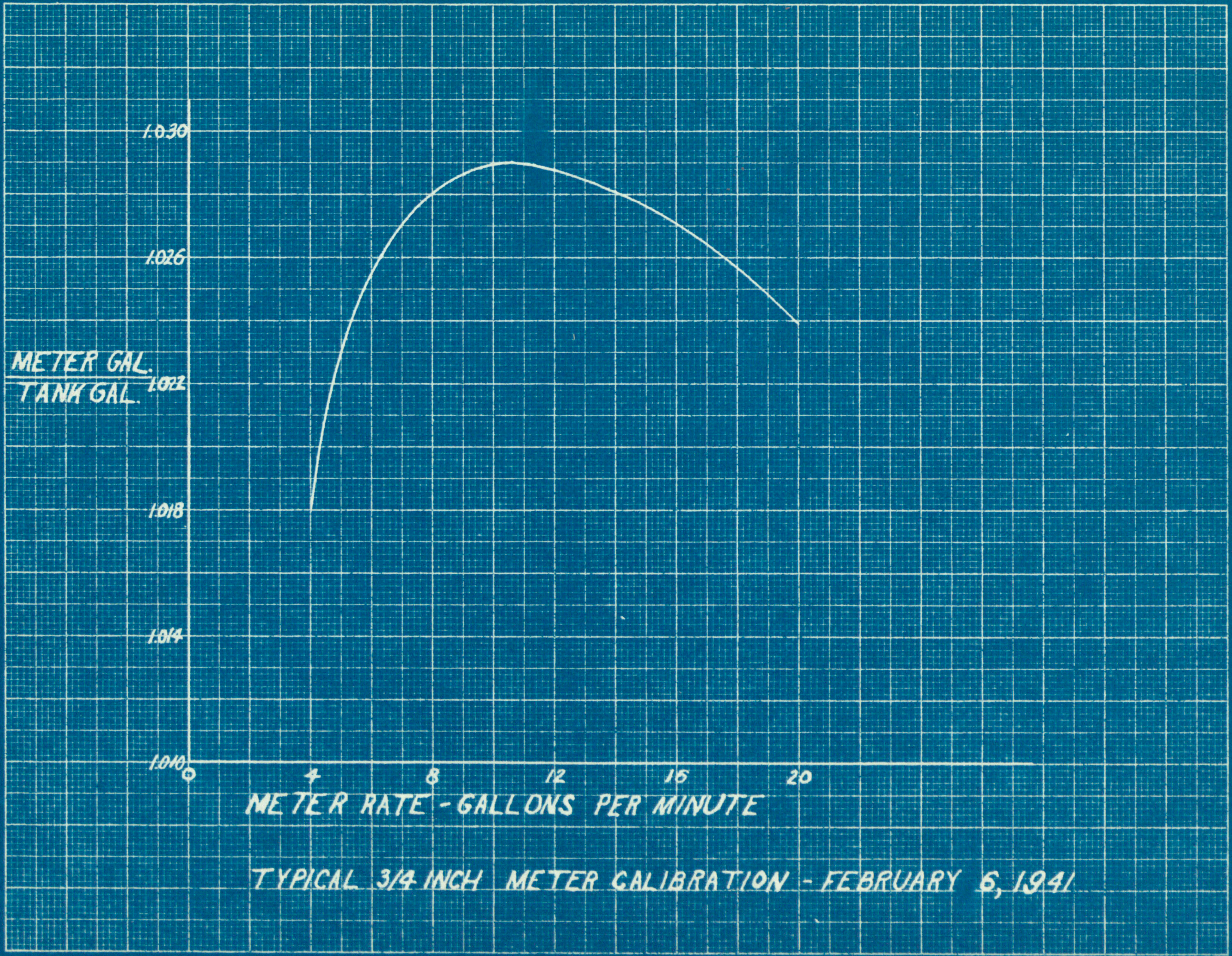
Exact notch angle	Notch depth, cm	Vertical distance from right-hand edge of notch (looking downstream) to the bottom of the notch, cm.
20°1.1'	20.388	20.375
29°54.3'	20.342	20.303
44°58.0'	20.310	20.250
60°12.4'	20.271	20.200
90°12.4'	20.339	20.209

When the notch is set in place, the top of the notch is 22' from horizontal, hence the necessity for the correction on the true notch depth to get the vertical distance from the reference point (top right-hand edge of notch) to the bottom of the notch.

Correction for gage reading for getting head is -0.060cm. This correction was required because the true bottom of the miniscus in the gage glass was not the reading taken.



TYPICAL 2 INCH METER CALIBRATION - FEBRUARY 6, 1941


METER GAL. / TANK GAL.
1.030
1.026
1.022
1.018
1.014
1.010
0
4
8
12
16
20
METER RATE - GALLONS PER MINUTE
TYPICAL 3/4 INCH METER CALIBRATION - FEBRUARY 6, 1941

Tabulation Of V-notch Weir Flow Equations With Water

Equations' Author	Angle	
The Writer:	20°1.1':	$Q = 0.45261(H+.003346)^{2.5}$
		$Q = 0.45329(H+.001647)^{2.4896}$
	29°54.3':	$Q = 0.66688(H+.001421)^{2.5}$
		$Q = 0.66748(H-.001486)^{2.4794}$
	44.58.0':	$Q = 1.02925(H+.000620)^{2.5}$
		$Q = 1.03068(H+.002611)^{2.5160}$
	60°2.9':	$Q = 1.42509(H-.000436)^{2.5}$
		$Q = 1.42517(H-.000338)^{2.5008}$
	90°12.4':	$Q = 2.44035(H+.000751)^{2.5}$
		$Q = 2.42692(H-.003136)^{2.4682}$
	20°1.1':	$Q = 0.45079(H-.008097)^{2.5}$ (Dreft Solution)
		$Q = 0.45081(H-.008858)^{2.4953}$ (Dreft Solution)
		$Q = 0.45168(H-.008097)^{2.5}$
		$Q = 0.45175(H-.008146)^{2.4997}$
Thomson:	90°:	$Q = 2.536H^{2.5}$
	126°52':	$Q = 5.29H^{2.5}$
Barr:	90°:	$Q = 2.48H^{2.48}$
	53°8':	$Q = 1.244H^{2.48}$
Strickland:	90°:	$Q = \left(2.4170+\frac{.067}{H^{.5}}\right)H^{2.5}$
Thornton:	90°:	$Q = \left(2.4760+\frac{.016}{H}\right)H^{2.5}$
Mawson:	90°:	$Q = \left(2.41380+\frac{0.068206}{H^{0.4985}}\right)H^{2.5}$
Cone:	120°:	$Q = 4.400H^{2.487}$
	90°:	$Q = 2.487H^{2.481}$
	60°:	$Q = 1.446H^{2.471}$

Cone:	30°:	$Q = 0.6848H^{2.448}$
	28°4':	$Q = 0.6405H^{2.445}$
Gourley and Crimp:	θ:	$Q = 2.48(\tan \frac{\theta}{2})H^{2.47}$
Greve:	118°11':	$W = 260.0H^{2.469}$
	110°00':	$W = 222.4H^{2.468}$
	120°20':	$W = 193.6H^{2.466}$
	98°45':	$W = 181.7H^{2.479}$
	94°39':	$W = 169.1H^{2.474}$
	81°52':	$W = 135.3H^{2.469}$
	69°38':	$W = 108.6H^{2.460}$
	59°7':	$W = 88.65H^{2.478}$
	53°55':	$W = 79.53H^{2.476}$
	45°23':	$W = 65.44H^{2.464}$
	44°24':	$W = 63.87H^{2.471}$
	40°00':	$W = 56.99H^{2.468}$
	36°53':	$W = 52.24H^{2.457}$
	25°3':	$W = 34.86H^{2.460}$
	θ:	$W = 156(\tan \frac{\theta}{2})^{.996}H^{2.47}$

Calibration of Copper Storage Tank - March 4, 1941

Tank range, divisions	Weight of water, lbs.	lbs./ division	Water temp., °C
141.89-135.47	257.7	4.014	8.4
135.47-129.62	235.0	4.017	8.4
129.62-123.70	236.65	3.998	
123.70-117.74	240.6	4.037	8.4
117.74-111.81	238.25	4.018	
111.81-105.86	238.55	4.009	8.4
105.86-99.88	240.2	4.017	
99.88-93.94	239.3	4.029	8.4
93.94-87.93	240.7	4.005	
87.93-81.95	238.9	3.995	8.5
81.95-76.00	237.7	3.995	"
76.00-70.01	240.25	4.011	8.5
70.01-64.03	239.25	4.001	
64.03-58.08	238.55	4.009	8.6
58.08-52.08	239.45	3.991	
52.08-46.07	240.15	3.996	8.7
46.07-40.09	239.8	4.010	
40.09-34.05	241.6	4.000	8.8
34.05-28.06	239.65	4.001	
28.06-22.08	240.0	4.013	9.0