Mathematical Influence of the Specific Energy in the Lorentz Factor

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The time dilation formulas of both the specific relativity and the general relativity could be understood as children of a common expression that uses a factor dependent on the specific energy. Surprisingly when such factor is used to define the relativistic mass, the equation that arises is an extraordinary approximation of the mass and energy relation. An entangled mathematical perspective of both time and mass that opens up again the question of what their definition really is, and which are the factors that modify such variables.

Keywords: specific energy, relativistic time, relativistic mass

1. Introduction

The way we understand time and mass has been accurately predicting most of the research it has been carried out over the last century, but there still many uncertainties in the universe that scape from our understanding of these two variables. It has been said for several years that we need an alternative perspective of the current theories to be able to tackle and reconcile both observations and predictions of some of the most challenging known complexities, like the existence of dark matter.

2. The Time Equation

The Lorentz Factor γ, depends on both the velocity (v) and the speed of light (c) (Ref. [1]),

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

But instead of using the velocity (v), the specific kinetic energy of the particle will be considered. Understanding the specific energy ε or $\varepsilon_{A_{r_B}}$ of a particle A as that which arises taking as reference a point B. Which can be calculated dividing the kinetic Newtonian energy of the particle A by its mass, m_A.

$$\varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{E^{\ Newt}{}_{kin}|_{Ref\ B}}{m_A} \tag{2}$$

Thus,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{\frac{1}{2}\, m_A \left(v_{A_{r_B}}\right)^2}{m_A} = \frac{\frac{1}{2}\, m_A v^2}{m_A} = \frac{v^2}{2} \tag{3}$$

Rewriting the Lorentz Factor as a function of this variable ε, the expression (1) will be as follows,

$$\gamma = \frac{1}{\sqrt{1-\frac{\varepsilon}{c^2/2}}} \tag{4}$$

This transformation does not introduce any new insight, the formulas (1) and (4) are indeed the same.

We also know that the relativistic time depends on this Lorentz factor as defined by (5),

$$T' = T\ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

And if the suggested transformation is used, the previous expression is,

$$T' = T\ \frac{1}{\sqrt{1-\frac{\varepsilon}{c^2/2}}} \tag{6}$$

All the specific energy of the previous expression is specific kinetic energy, entirely. It is not referring to any energy source other than the one due to the velocity of the particle.

But let us study for a moment the formula that defines time dilation due to the gravitational effect, according to the General Relativity Theory (Ref. [2] and [3]),

$$T' = T\ \frac{1}{\sqrt{1-\frac{2GM}{R\,c^2}}} \tag{7}$$

Surprisingly, the previous expression could also be rewritten as (6) where the particle considered is only submitted to the effect of gravity, and thus its specific energy its entirely gravitational,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{m_A\, g\, R}{m_A} = g\, R = \frac{GM}{R^2}\, R = \frac{GM}{R} \tag{8}$$

It is curious that these equations have both this common ancestor. It could be a coincidence, but as every coincidence in physics, it must be studied. Said that, the following question arises, could it be possible that the energy, no matter what type it is, is the variable that modifies the Lorentz Factor?

To understand better this question, a bigger frame should be considered. So, let us take the equation that relates the relativistic mass or total mass (m_{Tot}) with the rest mass (m_0) and the Lorentz Factor,

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{9}$$

In the same way, equation (9) can be rewritten using the factor defined by (4) to relate m_{Tot} and m_0 as follows,

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{10}$$

The specific energy of (10) is entirely kinetic, but what if it was not only kinetic energy?

We already know the equation that relates the rest mass, the relativistic mass, and the total energy of a certain particle, and that is the famous equation of A. Einstein,

$$E = m_{extra}\, c^2 \tag{11}$$

Where the extra mass (m_{extra}) is the difference between the total mass (m_{Tot}) and the rest mass (m_0), that is,

$$m_{Tot} = m_0 + m_{extra} \tag{12}$$

Expressing (11) in a different form,

$$E = (\, m_{Tot} - m_0 \,)\, c^2 \tag{13}$$

To make (13) comparable to (26) we need to have in both equations the variable of specific energy ε, so, we can modify (13) as follows,

$$\varepsilon = \frac{E}{m_{Tot}} = \frac{m_{Tot} - m_0}{m_{Tot}}\, c^2 \tag{14}$$

$$\varepsilon = \left(1 - \frac{m_0}{m_{Tot}}\right) c^2 \tag{15}$$

Equations (15) and (11) are the same, but it would be truly remarkable if this last equation (15) is exactly the equal to the equation (10), which is proposed in this research. If that was be the case, then we would be facing a deep understanding of the Lorentz Factor that up to know has not been suggested. The possible influence of the specific energy in time and mass.

So, let us compare both equations (10) and (15). To do so, we first rewrite (10) as,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{16}$$

Now (15) will be substituted in (16) and if both are the same, the resulting combined equation should lead to an identity,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{2\, \frac{m_0}{m_{Tot}} - 1}} \tag{17}$$

Calling y the quotient of $\frac{m_0}{m_{Tot}}$,

$$\frac{1}{y} = \frac{1}{\sqrt{2\, y - 1}} \tag{18}$$

Simplifications end in two functions, one on the left of equality and one on the right,

$$f_1 = \frac{1}{y} \tag{19}$$

$$f_2 = \frac{1}{\sqrt{2\, y - 1}} \tag{20}$$

Then, it is quite obvious that f_1 and f_2 are not the same functions. Thus, the equations (10) and (15) are not the same. To see how big the difference is, let us plot them together having y as the independent variable, as it can be seen in (Fig 1.)

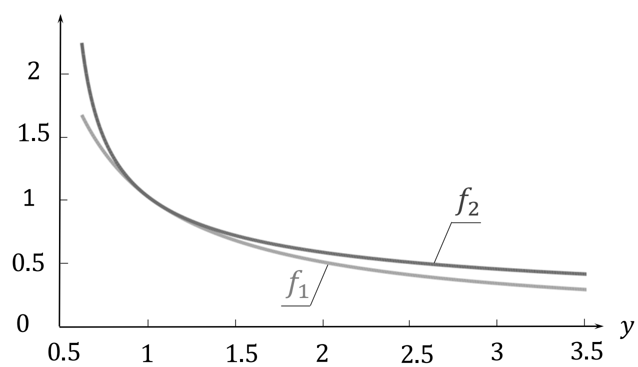

Fig. 1. Tangency of functions f_1 y f_2

The resulting plot is impressive, since both functions are actually tangent at the point where y is equal 1 (Fig 1.). It is thus observed that for those values close to 1, the error incurred is very small, while when being far from 1, the error can be very high.

In other words, when m_{Tot} and m_0 are similar (usual in small particles, where ε is often small), both formulas are applicable, (10) and (15). In fact, for the experiments we can carry out here on Earth we will not have any significant difference in the measurements. Because they would all involve extremely small values of m_{extra}.

Both formulas are very similar, and that is remarkable given the fact that their origins are completely different.

Each one understands mass in its own way, and this is indeed the most interesting point to think about. Equation (11) says that mass is a linear property, that can be calculated adding up their parts. Simply with a sum. Indeed, (11) and (12) together they form a system of two equations. Formula (11) without (12) is meaningless. But equation (10) is suggesting that mass depends on its energy, and that the sum of the parts ($m_0 + m_{extra}$) is not the same as the whole (m_{Tot}).

To see more in detail the relation between these two versions of the theory, we will do the process the other way around. Instead of comparing (10) and (15) with ε let us compare them with E. To do so, we need to transform (16) as follows,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{E}{m_{Tot}\ c^2/2}}} \tag{21}$$

$$1 - \frac{E}{m_{Tot}\ c^2/2} = \left(\frac{m_0}{m_{Tot}}\right)^2 \tag{22}$$

$$E = m_{Tot}\ \frac{c^2}{2}\left(1 - \left(\frac{m_0}{m_{Tot}}\right)^2\right) \tag{23}$$

$$E = \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) \tag{24}$$

This equation (24) is exactly the same as (10), but allows the calculation of the bonding energy, being m_0 and m_{Tot} known.

Let it be studied the difference between (13) and (24) in a three-dimensional space being m_0 y m_{Tot} the independent variables.

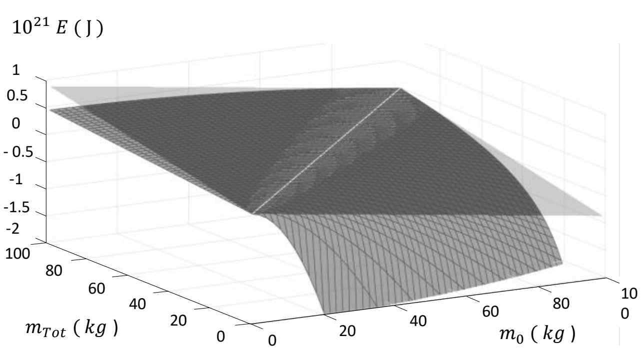

Fig. 2. Differences between equations (13) and (24)

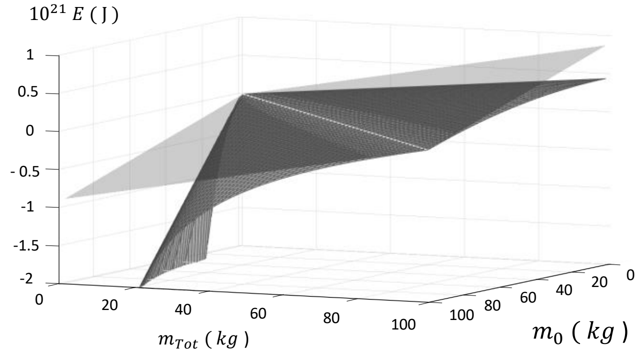

Fig. 3. Differences between equations (13) and (24)

These representations are of vital importance. The plane represents equation (13) while the curved surface refers to equation (24).

Both equations provide the same value of the energy when m_0 and m_{Tot} are equal, and therefore E is zero. In the immediate vicinity of this line, (24) is a good approximation of (13). But when leaving the adjoining margin, the errors become much more noticeable, even reaching infinitely different values.

Interestingly, the plane defined by (13) is tangent to the curved surface (24). In fact, it is tangent along the entire line $E = 0$. An uncommon beautiful relationship, between both equations.

In fact, both surfaces predict a negative energy in case $m_0 > m_{Tot}$, positive when $m_0 < m_{Tot}$ and null in case the masses are equal, as explained.

The following development shows how the plane is tangent, across the straight line, to the curved surface.

The equation of the plane tangent to a given point P of a function F is given by,

$$F_x|_P(x - x_P) + F_y\big|_P(y - y_P) + F_z|_P(z - z_P) = 0 \tag{25}$$

Being F,

$$F = E - \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) = 0 \tag{26}$$

Renaming with x, y, z,

$$F = z - \frac{c^2}{2}\left(y - \frac{x^2}{y}\right) = 0 \tag{27}$$

Being the point P any point belonging to the line $m_0 = m_{Tot}$, thus $P(m, m, 0)$. Calculating the partial derivatives,

$$F_x = -\frac{c^2}{2}\left(-\frac{2\,x}{y}\right) \tag{28}$$

$$F_y = -\frac{c^2}{2}\left(1 + \frac{x^2}{y^2}\right) \tag{29}$$

$$F_x = 1 \tag{30}$$

Substituting P it is obtained,

$$F_x|_P = -\frac{c^2}{2}\left(-\frac{2\,m}{m}\right) = c^2 \quad (31)$$

$$F_y\Big|_P = -\frac{c^2}{2}\left(1 + \frac{m^2}{m^2}\right) = -c^2 \quad (32)$$

$$F_z|_P = 1 \quad (33)$$

Finally,

$$c^2\,(\,m_0 - m)\, - c^2\,(m_{Tot} - m)\, + \,(E - 0) = 0 \quad (34)$$

$$E = c^2\,(m_{Tot} - m) - c^2\,(\,m_0 - m) \quad (35)$$

$$E = (m_{Tot} -\, m_0)\,c^2 = m_{extra}\,c^2 \quad (36)$$

Checking in this way that the plane is indeed tangent to the curved surface.

But still there is some hidden relation between equations (13) and (24) that has not been covered. In order to see it, the variable m_{Tot} will be extracted from equation (24), obtaining,

$${m_{Tot}}^2 - \frac{2\,E}{c^2}m_{Tot} - {m_0}^2 = 0 \quad (37)$$

$$m_{Tot} = \frac{\frac{2\,E}{c^2} \pm \sqrt{\frac{4\,E^2}{c^4} + 4{m_0}^2}}{2} \quad (38)$$

Considering only the positive value of the mass,

$$m_{Tot} = \frac{E}{c^2} + \sqrt{\frac{E^2}{c^4} + {m_0}^2} \quad (39)$$

Assuming that $\frac{E^2}{c^4}$ is negligible in comparison to ${m_0}^2$, then,

$$m_{Tot} = \frac{E}{c^2} +\, m_0 \quad (40)$$

Which in the end leads to,

$$E = (m_{Tot} -\, m_0)\,c^2 \quad (41)$$

In such a way that the equation (13) could be understood as an approximation of (24), if $\frac{E^2}{c^4}$ is much smaller than ${m_0}^2$.

On the other hand, it should be remembered that $E = m\,c^2$ is actually a specific case in which the particle considered has no velocity according to the reference. If it has velocity, the expression becomes,

$$E^2 =\, (m\,c^2)^{\,2} +\, (pc)^2 \quad (42)$$

Being p the linear momentum of the study particle. However, this generalization is not necessary with the proposed equation (24), since it already considers all the specific energy according to the desired reference.

If the previous holds, if this proposed theory is indeed correct, it would imply that the energy cannot be converted into matter nor vice versa, but rather the energy affects the weight of matter. And in fact, there are many challenges that, yet we have not been able to solve with the current understanding of mass. Dark Matter for example can be an interesting concept that helps clarifying the idea of this section. We know there are indisputable differences between predictions and observations related to certain gravitational effects in the universe, and in order to use current theories of gravitation, it would be necessary to have more mass than what it is observed. Indeed, that is the origin of Dark Matter, it should exist to reconcile both observations and theories. But its nature is so complex that we have never been able to prove its existence. Matter that does not interact, not even electromagnetically, that cannot be seen and still creates a gravitational field. But what if the mass is greater than what we thought it was? What if is heavier the greater its energy is?

The answers to those questions deserve much more than just the last lines of this short paper. Nevertheless, the author has considered to include these final comments to open the path, using the theory proposed in this research, to future investigations that the readers might see of interest.

2. Conclusions

The major differences between the Relativity Theory and the one proposed are shown in the table below.

Table I. Differences between theories

Relativity Theory	**Theory proposed**
$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$	$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}}$
$E = (\,m_{Total} - m_0\,)\,c^2$	$E = \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right)$

Which ultimately summarizes that both formulas in the right column are the same equation but ordered in two different ways, while the two on the left are different formulas. Going back to Fig. 2 and Fig. 3, it can be seen that there is a very important difference between both solutions. That although in the small world of experiments on Earth both theories predict similar results, in the vastness of outer space they lead to completely different predictions.

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