# Specific Energy, Lorentz Factor & WIMP Annihilation

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**Abstract.** The time dilation formulas of both the Special Relativity and the General Relativity could be understood as children of a common expression that uses a factor dependent on the specific energy. Should such factor be used to define the relativistic mass, the equation that arises is an approximation of the mass and energy relation. An entangled mathematical definition of mass that is finally compared to the equations that define Dark Matter annihilation into charged states via loop-level processes.

**Keywords.** Specific energy — relativistic time — relativistic mass — dark matter

## 1. Introduction

In the present paper, an alternative mathematical perspective of the Lorentz Factor is proposed.

The way time and mass is understood has been accurately predicting most of the research it has been carried out over the last century. But there are still many uncertainties in the universe for which it lacks sufficient understanding of these two variables, such like Dark Matter. Thus, the mathematical formulation obtained from this alternative development is benchmarked using the current experimental knowledge of weakly-interacting massive particles, WIMPs (considered to be potential candidates to explain the nature of Dark Matter). More specifically, WIMP annihilation into charged states producing photons via loop-level processes ($\chi\chi \rightarrow \gamma X$).

## 2. Methodology

The Lorentz Factor $\gamma$ (Einstein 1905; Einstein 1915; Cenko *et al.* 2015) depends on both the velocity ($v$) and the speed of light ($c$),

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

But instead of using the velocity ($v$), the specific kinetic energy of the particle will be considered. Understanding the specific energy $\varepsilon$ or $\varepsilon_{A_{r_B}}$ of a particle $A$ as that which arises taking as reference a point $B$. Which can be calculated dividing the kinetic Newtonian energy of the particle $A$ by its mass, $m_A$.

$$\varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{E^{\,Newt}{}_{kin}|_{Ref\ B}}{m_A} \tag{2}$$

Thus,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{\frac{1}{2}\, m_A \left(v_{A_{r_B}}\right)^2}{m_A} = \frac{\frac{1}{2}\, m_A v^2}{m_A} = \frac{v^2}{2} \tag{3}$$

Rewriting the Lorentz Factor as a function of this variable $\varepsilon$, the expression (1) will be as follows,

$$\gamma = \frac{1}{\sqrt{1-\frac{\varepsilon}{c^2/2}}} \tag{4}$$

This transformation does not introduce any new insight, the formulas (1) and (4) are indeed the same.

It is also well-known that the relativistic time (Einstein 1916; Francis *et al.* 2013) depends on the Lorentz Factor as defined by (5),

$$T' = T\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

Should the suggested transformation be used, the previous expression becomes,

$$T' = T\,\frac{1}{\sqrt{1-\frac{\varepsilon}{c^2/2}}} \tag{6}$$

Where the specific energy of (6) is entirely specific kinetic energy. Hence, it is not referring to any energy source other than the one due to the velocity of the particle.

Let it be studied the formula that defines time dilation due to the gravitational effect,

$$T' = T\,\frac{1}{\sqrt{1-\frac{2GM}{R\,c^2}}} \tag{7}$$

The previous expression could also be rewritten as (6) where the particle considered is only submitted to the effect of gravity, and thus its specific energy is only gravitational,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{m_A\, g\, R}{m_A} = g\, R = \frac{GM}{R^2}\, R = \frac{GM}{R} \qquad (8)$$

It should be pointed out that it is curious that these equations have both this common ancestor. Thus, the development that follows is now focused in the possible effect of the specific energy, no matter what type it is, in the Lorentz Factor.

To understand better the previous intention, a bigger frame should be considered. For such purpose, in (9) is expressed the equation that relates the relativistic mass or total mass ($m_{Tot}$) with the rest mass ($m_0$) and the Lorentz Factor (Roche 2005),

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (9)$$

In the same way, equation (9) can be rewritten using the factor defined by (4) to relate $m_{Tot}$ and $m_0$ as follows,

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \qquad (10)$$

The specific energy of (10) is again entirely kinetic. But the author poses the following question: What if it was not only kinetic energy? In such case, (10) would be an equation that relates the rest mass, the relativistic mass, and the total energy of a certain particle. But the famous equation (11) already relates those variables (Rainville, Thompson, Myers *et al.* 2005).

$$E = m_{extra}\, c^2 \qquad (11)$$

Where the extra mass ($m_{extra}$) is the difference between the total mass ($m_{Tot}$) and the rest mass ($m_0$), that is,

$$m_{Tot} = m_0 + m_{extra} \qquad (12)$$

Expressing (11) in a different form,

$$E = (\, m_{Tot} - m_0 \,)\, c^2 \qquad (13)$$

To make (13) comparable to (10) both equations should have the variable of specific energy $\varepsilon$. For such purpose, (13) can be modified as follows,

$$\varepsilon = \frac{E}{m_{Tot}} = \frac{m_{Tot} - m_0}{m_{Tot}}\, c^2 \qquad (14)$$

$$\varepsilon = \left(1 - \frac{m_0}{m_{Tot}}\right) c^2 \qquad (15)$$

Equations (15) and (11) are the same, but it would be truly remarkable if this last equation (15) is exactly equal to equation (10), which is proposed in this research. If that was the case, it could lead to a deep understanding of the Lorentz Factor. The possible influence of the specific energy in time and mass.

Let equations (10) and (15) be compared. To do so, equation (10) will be rewritten as,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \qquad (16)$$

Now (15) will be substituted in (16) and if both are the same, the resulting combined equation should lead to an identity,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{2\,\frac{m_0}{m_{Tot}} - 1}} \qquad (17)$$

Calling $y$ the quotient of $\frac{m_0}{m_{Tot}}$,

$$\frac{1}{y} = \frac{1}{\sqrt{2\,y - 1}} \qquad (18)$$

Simplifications end in two functions, one on the left of equality and one on the right,

$$f_1 = \frac{1}{y} \qquad (19)$$

$$f_2 = \frac{1}{\sqrt{2\,y - 1}} \qquad (20)$$

The functions defined by $f_1$ and $f_2$ are not the same. Thus, equations (10) and (15) are not the identical. To see how big the difference is, in Fig. 1 both are plotted together having $y$ as the independent variable.

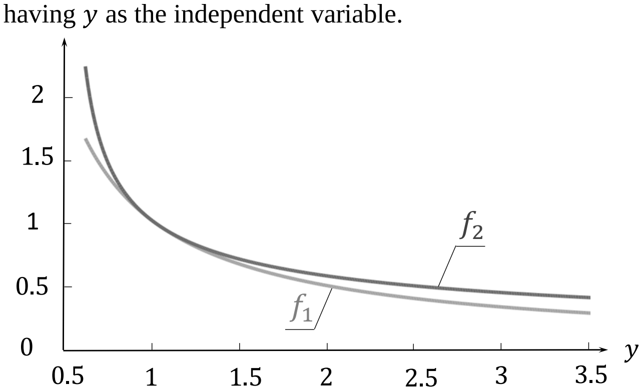


**Figure 1.** Tangency of functions $f_1$ y $f_2$

The resulting plot is impressive, since both functions are actually tangent at the point where $y$ is equal 1 (Fig 1.). It is thus observed that for those values close to 1, the error incurred is very small, while when being far from 1, the error can be very high.

In other words, when $m_{Tot}$ and $m_0$ are similar (usual in small particles, where $\varepsilon$ is often small), both formulas are applicable, (10) and (15). In fact, for the low-mass experiments carried out on Earth (small values of $m_{extra}$), if (15) was correct, no significant difference between (10) and (15) would be appreciated in the measurements.

Both formulas are very similar, and that is remarkable given the fact that their origins are completely different.

Each one understands the mass in its own way, and this is indeed the most interesting point to think about. Equation (11) says that mass is a linear property, that can be calculated adding up their parts. Simply with a sum. Indeed, (11) and (12) together they form a system of two equations. Formula (11) without (12) is meaningless. But equation (10) is suggesting that mass depends on its energy, and that the sum of the parts $(m_0 + m_{extra})$ is not the same as the whole $(m_{Tot})$.

To see more in detail the relation between these two versions of the theory, (10) and (15) will be compared using $E$ instead of $\varepsilon$. To do so, (16) will be transformed as follows,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{E}{m_{Tot}\ c^2/2}}} \tag{21}$$

$$1 - \frac{E}{m_{Tot}\ c^2/2} = \left(\frac{m_0}{m_{Tot}}\right)^2 \tag{22}$$

$$E = m_{Tot}\ \frac{c^2}{2}\left(1 - \left(\frac{m_0}{m_{Tot}}\right)^2\right) \tag{23}$$

$$E = \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) \tag{24}$$

This equation (24) is exactly the same as (10), but allows the calculation of the bonding energy, being $m_0$ and $m_{Tot}$ known.

Let it be studied the difference between (13) and (24) in a three-dimensional space being $m_0$ and $m_{Tot}$ the independent variables.

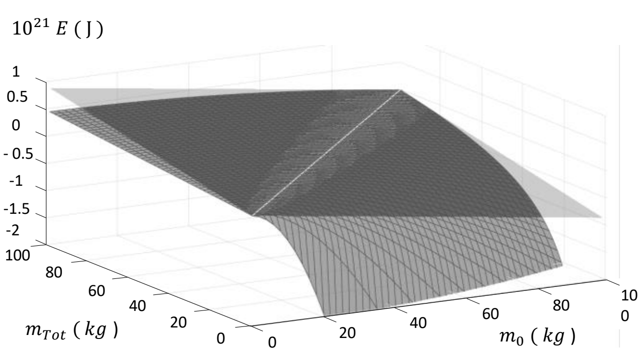


**Figure 2.** Differences between equations (13) and (24)

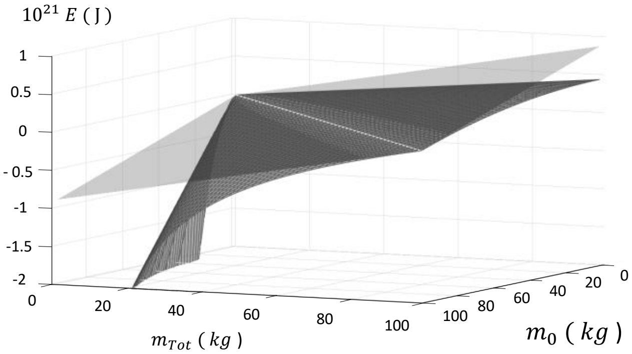


**Figure 3.** Differences between equations (13) and (24)

These representations are of vital importance. The plane represents equation (13) while the curved surface refers to equation (24).

Both equations provide the same value of the energy when $m_0$ and $m_{Tot}$ are equal, and therefore $E$ is zero. In the immediate vicinity of this line, (24) is a good approximation of (13). But when leaving the adjoining margin, the errors become much more noticeable, even reaching infinitely different values.

Interestingly, the plane defined by (13) is tangent to the curved surface (24). In fact, it is tangent along the entire line $E = 0$. An uncommon feature, between both three-dimensional functions.

Both surfaces predict a negative energy in case $m_0 > m_{Tot}$, positive when $m_0 < m_{Tot}$ and null in case the masses are equal, as explained.

The following development shows how the plane is tangent, across the straight line, to the curved surface.

The equation of the plane tangent to a given point $P$ of a function $F$ is given by,

$$F_x|_P(x - x_P) + F_y|_P(y - y_P) + F_z|_P(z - z_P) = 0 \tag{25}$$

Being $F$,

$$F = E - \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) = 0 \tag{26}$$

Renaming with $x, y, z$,

$$F = z - \frac{c^2}{2}\left(y - \frac{x^2}{y}\right) = 0 \tag{27}$$

Being the point $P$ any point belonging to the line $m_0 = m_{Tot}$, thus $P(m, m, 0)$. Calculating the partial derivatives,

$$F_x = -\frac{c^2}{2}\left(-\frac{2\,x}{y}\right) \tag{28}$$

$$F_y = -\frac{c^2}{2}\left(1 + \frac{x^2}{y^2}\right) \tag{29}$$

$$F_x = 1 \tag{30}$$

Substituting $P$,

$$F_x|_P = -\frac{c^2}{2}\left(-\frac{2\,m}{m}\right) = c^2 \quad (31)$$

$$F_y\big|_P = -\frac{c^2}{2}\left(1 + \frac{m^2}{m^2}\right) = -c^2 \quad (32)$$

$$F_z|_P = 1 \quad (33)$$

Finally,

$$c^2\,(\,m_0 - m) - c^2\,(m_{Tot} - m) + (E - 0) = 0 \quad (34)$$

$$E = c^2\,(m_{Tot} - m) - c^2\,(\,m_0 - m) \quad (35)$$

$$E = (m_{Tot} - m_0)\,c^2 = m_{extra}\,c^2 \quad (36)$$

Checking in this way that the plane is indeed tangent to the curved surface.

But still there is some hidden relation between equations (13) and (24) that has not been covered. In order to see it, the variable $m_{Tot}$ will be extracted from equation (24), obtaining,

$$m_{Tot}{}^2 - \frac{2\,E}{c^2} m_{Tot} - m_0{}^2 = 0 \quad (37)$$

$$m_{Tot} = \frac{\frac{2\,E}{c^2} \pm \sqrt{\frac{4\,E^2}{c^4} + 4m_0{}^2}}{2} \quad (38)$$

Considering only the positive value of the mass,

$$m_{Tot} = \frac{E}{c^2} + \sqrt{\frac{E^2}{c^4} + m_0{}^2} \quad (39)$$

Assuming that $\frac{E^2}{c^4}$ is negligible in comparison to $m_0{}^2$, then,

$$m_{Tot} = \frac{E}{c^2} + m_0 \quad (40)$$

Which in the end leads to,

$$E = (m_{Tot} - m_0)\,c^2 \quad (41)$$

In such a way that the equation (13) could be understood as an approximation of (24), if $\frac{E^2}{c^4}$ is much smaller than $m_0{}^2$.

On the other hand, it should be remembered that $E = m\,c^2$ is actually a specific case in which the particle considered has no velocity according to the reference. If it has velocity, the expression becomes,

$$E^2 = (m\,c^2)^2 + (pc)^2 \quad (42)$$

Being $p$ the linear momentum of the study particle. However, this generalization is not necessary with the proposed equation (24), since it already considers all the specific energy according to the desired reference.

If the previous holds, if this proposed theory is indeed correct, it would imply that the energy cannot be converted into matter nor vice versa, but rather the energy affects the weight of matter.

## 3. WIMP annihilation via loop-level processes

Many are the challenges that, yet have not been solved with the current understanding of mass. Dark Matter is indeed one of those challenges (Bertone 2010; Metzler, Evrard, Navarro 1996; Merritt 2006; Navarro, Frenk, White 1997; de Blok *et al.* 2001; Wang *et al.* 2016). There are indisputable differences between predictions and observations related to certain gravitational effects in the universe, and in order to use current theories of gravitation, it would be necessary to have more mass than what it is observed. Indeed, that is the origin of Dark Matter, it should exist to reconcile both observations and theories. But its nature is so complex that its existence has never been proved.

The so-called weakly-interacting massive particles, or WIMPs are studied for being a potential candidate to explain the nature of Dark Matter (Sanders 1990; Borriello, Salucci 2001; Zaharijas, Hooper 2006; Gnedin *et al.* 2004). WIMP annihilation into charged states produces photons via loop-level processes. When two WIMP particles $\chi\chi$ annihilate each other at close to zero relative velocity into $\gamma X$, an energy of $E_\gamma$ is released (Abdo *et al.* 2010; Goodman Ibe, *et al.* 2010; Coogan, Profumo, Shepherd 2015), as it is given by (43).

$$E_\gamma = m_\chi \left(1 - \frac{m_X{}^2}{4\,m_\chi{}^2}\right) \quad (43)$$

Where $m_\chi$ refers to the mass of a single WIMP particle $\chi$, measured in energy units. And $m_X$ refers to the remaining mass after the annihilation, measured also in energy units. Thus, the total mass of Dark Matter before and after the annihilation can be expressed as $m_{Tot}$ and $m_0$ respectively ((44) and (45)), where $m_{Tot}$ is twice the $m_\chi$ (since there are two WIMP particles). Applying the transformation to get $m_{Tot}$ and $m_0$ in mass units instead of energy units, the resulting masses are,

$$m_{Tot} = \frac{2\,m_\chi}{c^2} \quad (44)$$

$$m_0 = \frac{m_X}{c^2} \quad (45)$$

Substituting $m_\chi$ and $m_X$ in (43),

$$E = \frac{c^2}{2}\,m_{Tot}\left(1 - \frac{m_0{}^2}{m_{Tot}{}^2}\right) \quad (46)$$

Which is exactly the same equation as the one predicted in this theory (24).

Equation (24) has been obtained from a mathematical development that understands mass as a dependent variable of the specific energy. But equation (43) is a well-known relation that has been developed from the observed data of Dark Matter in nearby halos. The fact that they are both the same it is not only an impressive result, but also a clarification of what the real nature of Dark Matter could be.

## 4. Conclusions

This development studies from a mathematical perspective the possible influence of the specific energy in the Lorentz Factor and its implication in the definition of time and mass. The major differences between the Theory of Relativity and the one proposed are shown in Table I. Which ultimately summarizes that both formulas in the right column are the same equation but ordered in two different ways, while the two on the left are different formulas. Going back to Fig. 2 and Fig. 3, it can be seen that there is a very important difference between both solutions. That although in the small world of the experiments carried out on Earth both theories predict similar results, in the vastness of outer space they lead to completely different predictions.

**Table I.** Differences between theories

| Theory of Relativity | Theory proposed |
|---|---|
| $m_{Tot} = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ | $m_{Tot} = \dfrac{m_0}{\sqrt{1-\frac{\varepsilon}{c^2/2}}}$ |
| $E = (m_{Total} - m_0)\, c^2$ | $E = \dfrac{c^2}{2}\left(m_{Tot} - \dfrac{{m_0}^2}{m_{Tot}}\right)$ |

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