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THE CHARACTERISTICS OF THE PROCEDURES OF GOOD AND POOR PROBLEM SOLVERS IN SIXTH GRADE ARITHMETIC

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THE DEFINITION OF THE PROBLEM

Introductory Statement

Problem solving in arithmetic, dealing with a wide range of situations, is a vital part of the arithmetical growth of elementary school pupils. In solving arithmetic problems, pupils must summon from previous background experiences those experiences which apply and must utilize them in such manner as to produce a correct solution. A pupil's arithmetical success depends to a great degree upon the way he goes about doing this. Children, however, do not have equivalent experiential backgrounds for problem solving. Some homes offer repeated arithmetical opportunities and responsibilities, while others provide little chance for such development. Such environmental differences, together with variations in intellectual ability, teaching methods, and other school experiences, have resulted in a considerable range in the abilities of pupils to solve problems in arithmetic.

Teachers of arithmetic have long been aware of these differences, and they have sought in many ways to isolate and study their origins and effects. They have used standardized tests to survey pupils' problem-solving efficiency, diagnostic tests to determine specific capabilities and deficiencies, reading tests, vocabulary tests, and tests measuring computational skills. While all of these measures have in one way or another been productive of better understanding of the arithmetical processes employed by pupils, teachers have been puzzled for many years about exactly what a pupil does when he solves a problem. In other words the question is: What are the problem solving procedures which precede, accompany and follow the solution of an arithmetic problem?

The Purpose of the Study

The purpose of this study is to investigate the characteristics of the procedures of good and poor problem solvers while they are engaged in solving verbal problems in arithmetic. Specifically, this study attempts to isolate and describe these characteristics of procedure of good and poor sixth grade problem solvers which show evidence of thoughtful and meaningful understanding, as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error. These characteristics include:

1. Insight as a factor in developing thoughtful and meaningful understanding.

2. Thinking processes of pupils as they selected a method, followed it through, and evaluated the results.

3. Number relationships and computational skills employed by pupils in associating number ideas with the problem situation.

The Significance of the Study

This study outlines for teachers of arithmetic hitherto unreported information about the characteristic procedures of good and poor problem solvers, and it also suggests a technique that may be employed by teachers in furthering their own study of individual pupil performances in arithmetic problem solving. Group techniques of studying pupils' problem solving procedures have not been effective in isolating individual behavior for determining exactly what pupils do when they solve arithmetic problems. Only through individual diagnostic interview techniques can the actual understanding, thought processes, and number relationships which function during problem solving be observed. Once isolated, the individual behavior can be studied in relation to the manner in which the problem solving procedures affect a pupil's success in problem solving.

The contribution of this study is, therefore, twofold. It introduces additional evidence of the behavior of pupils while solving arithmetic problems, and it suggests a method of securing additional information about this subject.

Definition of Terms

In this study certain terms have been used in such manner that definition seems advisable. "Good and poor problem solvers" are two groups of pupils selected from the upper and from the lower twenty-seven percent of the sixth grade pupils from six elementary schools in the city of Cincinnati, Ohio. These groups were selected by means of a standardized test in arithmetic problem solving. A good problem solver is one who made a score in the upper twenty-seven percent of all the pupils tested. Similarly, a poor problem solver is one who made a score in the lower twenty-seven percent of all the pupils tested. "Problems in arithmetic" is a term which refers to the eight selected verbal problems prepared for use in the investigation. The term "sixth grade arithmetic," as used in this study, refers to the general subject matter content of arithmetic as it is taught at the sixth grade level in the public schools.

Scope of the Study

Any study must of necessity be limited in scope. Obviously, the investigation could not include arithmetic problem solving at all grade levels. This study, therefore, is confined only to problem solving in sixth grade arithmetic. Verbal arithmetic problems at the sixth grade level include a wide range of computational abilities and reasoning processes. The conclusions of this study are derived from the performance of good and poor problem solvers in the solution of eight representative verbal problems. These problems of one and two-step complexity involve only whole numbers and utilize the four basic skills in computation: namely, addition, subtraction, multiplication, and division.

The aspects of problem-solving behavior are numerous and diverse. Two characteristics, however, are readily observed. One is that of thoughtful and meaningful understanding; the other is mechanical and almost purposeless attack. This study will describe those activities of good and poor problem solvers in arithmetic which show evidence of thoughtful and meaningful understanding as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error.

Sources of Data

Data for this study have been obtained from tests, personal observations, tape recordings, and written responses made by pupils on individual work sheets. Specifically, these sources are:

1. <u>The Iowa Every Pupil Test of Basic Skills, Test D: Basic</u>

Arithmetic Test, Form C, a standardized test, furnished the initial data for selecting good and poor problem solvers.

2. Evidence of meaningful understanding, mechanical manipulation, guesswork, or trial and error was provided by the eight problems with were formulated by the investigator.

3. Tape recordings of the vocalizations of the pupils while they were engaged in reading the problems, solving them, and evaluating their answers supplied the basis for interpreting the oral reactions of the problem solvers.

4. Pencil and paper solutions on the individual work sheets used by the pupils provided additional evidence of thought processes and mechanical activity.

5. Notes taken during the interview were useful in the recall of emotional behavior, physical reaction, and the extra computational aids used by the pupils.

Methods of Research

The basic method of research used in this study was the modified case-study method. Individual pupils were studied through a systematic observational technique designed to localize and record each pupil's reaction during his attempts at problem solving. Total patterns of behavior were surveyed and the observed behavior, written, spoken, physical and emotional, of the good and poor problem solvers was evaluated and classified. Anecdotal excerpts from individual case studies were used in support of the evidence derived from the survey of total behavior patterns.

Organization of the Dissertation

This dissertation presents a report of the observed characteristics of procedure of good and poor problem solvers in working eight selected problems in sixth grade arithmetic. In Chapter I the problem is defined, with specific reference to the purpose and significance of the study, definition of terms, scope of the study, sources of data and methods of research. In Chapter II the contributions made by related studies are enumerated. Chapter III outlines the hypothesis upon which the study is based, the considerations relating to this hypothesis, the preliminary investigations in establishing the procedures, and the steps followed in conducting the study. Chapters IV, V, and VI, respectively, describe the results of the investigation as they apply to each of the three general types of procedure being investigated. The summary of the study, conclusions, recommendations for further research and the implications for teachers appear in Chapter VII.

CHAPTER II

A SURVEY OF THE RELATED LITERATURE

The Purpose of the Chapter

The nature of problem solving has received the attention of a number of writers in the field of arithmetic. The observations and conclusions of these writers have emphasized not only the importance of problem solving as an arithmetical accomplishment, but also the significance of specific skills and procedures in attaining competence in arithmetic problem solving. The literature in these areas is voluminous, but not all of it was pertinent to the purpose of this investigation. Only those aspects of the literature are reported that the writer believes to be related to the expressed objective of the study, or to the techniques employed in developing it. The objective, as set forth in Chapter I, is to isolate and describe those characteristics of the procedures of good and poor problem solvers in arithmetic which show evidence of thoughtful and meaningful understanding, as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error. The techniques of investigation are those of interview and observation of individual pupils engaged in problem solving.

The literature has made three important contributions to the development of this study. These contributions are classified into three categories: the importance of meaning and understanding in problem solving in arithmetic, reports of research studies associated with the purpose of this investigation, and techniques suggested by authorities for studying the arithmetic problem solving behavior of pupils.

The Importance of Meaning and Understanding in Problem Solving in Arithmetic

Writers have long been aware of the significance of meaning and understanding in arithmetic problem solving. As early as 1922, Stone (24:170-89) recognized that understanding is essential before a pupil can solve a problem efficiently:

> The problem is not concrete to the child unless he is able to form a clear mental picture of the situation described. It may be that he has not read the problem correctly, or that he may have read it carefully and yet, through lack of experience in the situation described, he may not be able to form a picture of the situation.

Greene and Buswell (14:275) shed further light on the question of meaning and understanding in arithmetic when they suggested that teachers try to find out the mental processes used by pupils in working problems:

> One can scarcely overemphasize the importance of discovering the mental processes which lie back of pupil's answers in arithmetic. Intelligent teaching can proceed only from an analysis of pupils' methods of work. Consequently, when serious difficulties are encountered by the pupils, the only final answer to the trouble is a detailed analysis of how the difficulties were produced, followed by an attempt to improve the pupils' work by some changes in the methods involved.
>
> Furthermore, one must not overlook the fact that pupils in many cases secure correct answers by methods which are very crude and cumbersome and which should be replaced by more efficient methods of thinking. For example, the writer has observed pupils, who in multiplying nine times nine, counted nine nines on their fingers...

Brueckner (6:329) referred to meanings as the "social significance" of arithmetic, and expressed concern over the tendency of teachers to overemphasize the mechanical aspects rather than concentrate on meaningful experiences:

> The most commonly recognized goal of arithmetic teaching, that of mastery of the facts and skills of arithmetic, has been accorded such importance that the social significance of arithmetic has often been neglected. The value of facts and skills of arithmetic cannot be denied, but the possession of these without the ability to use them in quantitative situations which life offers is evidence that the school is not meeting the needs of those whom it seeks to educate.

Brownell (2:72) recommended a review of methods of instruction, and pointed out that teachers were failing to bring about an understanding in their teaching:

> Of late we have begun to suspect that there is something wrong in our instruction. The pupils who have been subjected to it do not seem to show the expected proficiency. Some item seems to have been missing in our teaching, and that item, we are coming to believe, is understanding.

Brownell and Moser (3:156) suggested that the term be competently described, in order that the teachers who work with arithmetic be informed as to precisely what is meant by meaningful arithmetic:

> The term "meaningful arithmetic" is seriously in need of competent analysis. Research may yet discover and arrange in order of importance the arithmetical meanings which must be taught. Only when a program of "meaningful arithmetic" is defined in this way can it be discovered just how "meaningful" it is. When (or if) this analysis and this list of essential meanings are undertaken, at the top of the list of essential meanings will be found those which govern our procedures in computation.

Sueltz and others (31:141) stressed meaning and understanding in problem solving and indicated that pupils must understand if arithmetic is to be significant:

> In order to obtain correct and final judgments and answers in a mathematical situation, the pupil must (a) know what computational processes to use and (b) use these processes with facility and accuracy. In both of these stages, it is the presence of the factors of meaning and understanding that raises the performance of the pupil above that of a computing machine.

The same writers advocated that means must be found for measuring meaning and understanding in arithmetic. They pointed out research trends toward studies which attempt to measure arithmetical meanings: (31:141)

> The measurement of meanings and understandings is beginning to creep into arithmetic. Of twenty-seven studies examined, eight showed that the author was deliberately trying to measure beyond the traditional scope of problem solving and computation. New procedures in teaching and evaluation will have to be developed as the schools broaden their vision of the function and scope of mathematics.

The preceding discussion has emphasized the thinking of representative writers in the field of arithmetic concerning meaning and understanding. Careful interpretation of the quotations, however, reveals that although these writers recognized the importance of arithmetical understanding, none actually attempted to define the term or to say actually what is meant by it. The literature seems rather to point out the effects of meaning and understanding, than to isolate and identify specific kinds of behavior. The vagueness and indefiniteness, however, with which the subject has been treated is intriguing to one who is interested in research in arithmetic and suggests an area of investigation which promises opportunity to establish clearer interpretation of the terms.

Research Studies Associated with the Purpose of the Investigation

After making an exhaustive search through the literature, the writer discovered only a limited number of research studies which seemed to be related to the nature and purpose of the present investigation.

White (35:451-5) placed arithmetic problems in vocabulary settings ranging from some which were quite familiar to the pupils to settings which were entirely outside their experiences. As long as the problems were easy, the settings made little difference in the ability of the pupils to work them. As the problems became more difficult, unfamiliar vocabulary settings became more of a handicap in the pupils' efforts to solve them. Brownell and Stretch (4:83) reported a similar experiment, from which they concluded:

> The data secured in this investigation offers no ground for the reasonable belief that problems are made unduly difficult for children by being given unfamiliar settings, except under certain circumstances which have been named. (Numerical relationships of an intermediate degree of difficulty, number of times a given operation has been met, and limitations as to time.)

These writers (Brownell and Stretch) further concluded that if a teacher used only language with which the child was familiar in presenting arithmetic problems, there would be no opportunity for the child to grow in vocabulary usage and in the applications to which he could put his new-found learning.

Johnson (17:97-110) experimented with seventh grade pupils in the use of vocabulary teaching materials as a technique for improving success in arithmetic. He found that teaching vocabulary terms improves arithmetic vocabulary as well as solution of problems involving that vocabulary. The use of vocabulary exercises does not, however, bring about a general improvement in arithmetical learnings, nor is there any evidence of transfer of training between words taught and other words not taught. He reported further that teachers using vocabulary teaching aids can bring about greater growth than if they are not used, that experienced teachers are not necessary for the use of the materials, and that regular and systematic use of the materials brings best results.

Glennon (12:64-74) reported research with an instrument for measuring understanding and meaning in arithmetic. He constructed a multiple-choice test of eighty items covering five areas of meaning and understanding basic to the computational processes taught in grades one to six. The test was validated in part on the combined judgments of experts in the subject matter of arithmetical problems and in part by the observation of its ability to distinguish between pupils who understand the meanings and those who did not. The test was administered to 1139 subjects at seven levels: Grades 7, 8, 9, 10, 11 and 12, teachers college freshmen, college seniors and in-service teachers. Very small differences were noted in scores at the various levels of administration. Glennon concluded that the data showed very little growth in arithmetical meaning and understanding at the levels of learning indicated, and that the experiment revealed the meager degree with which teachers are bringing about arithmetical meanings at these levels.

Johnson (18: 110-15) attempted to identify some intellectual factors most closely related to the ability to solve verbal problems in arithmetic at the eighth grade level. He found the greatest correlation ratio (from .45 to .51) between problem solving and general vocabulary. The second-highest correlation ratio (.37 to .47) was found between problem solving and arithmetical reasoning. When, however, a problem scale without numbers was used in place of a regular problem scale, the order of relationships was reversed; i.e., correlations with reasoning were higher than correlations with vocabulary.

Both testing and oral interviews were used by Burch (8:145) in evaluating pupil responses to problem tests that required four analytical steps in working the problems. The four analytical steps were: What is given? What is to be found? What is the estimated answer? and How is the problem to be solved? He found that pupils tended to score higher in problem solving in the test in which he did not require responses to the analytical steps, and that even when pupils had been taught to use the steps, they did not do so unless under compulsion.

Sueltz and Benedick (30:24-33) constructed a test to measure four areas of mathematical ability: understandings, judgments, computation, and problem solving. The results from testing two thousand sixth graders in three eastern states revealed glaring weaknesses in all of the four areas tested. Approximately the same results were obtained when the tests were repeated in higher grades. The authors concluded that more than the ability to compute is needed to use arithmetic functionally.

Flanders (11:385) used extensive records of classroom procedures obtained by means of recording devices. He attempted to show that there are relationships between the language of thinking and the language of communication. Among other things, the study shows a significant correlation between the kinds of statements made about percents and the pupils' individual learning status in that area. This study seems to indicate that an ability to express ideas verbally is an index of the child's ability to use the idea in a logical pattern.

Walker (34:36-41) studied the behavior of slow-learning children in arithmetic problem solving. He reported that slow-learning children are inferior in insights, in ability to form relationships, in ability to generalize, in discrimination, and in comprehension and ability to use the higher mental processes. They are also inferior in their ability to define terms and use them correctly. They do their best work with terms that are most closely related to their daily life. Slow-learning children were found to be less able to select and use the proper arithmetical processes and to differentiate extraneous from relevant material in problem solving. Their concepts of time were more limited than those of normal pupils, and what they understood was closely related to their own experience. He found that memorization was difficult, retention inadequate, and attention span so short as to complicate teaching. Analysis of their errors showed that they made the same errors as normal pupils, but to a greater extent. Their work was characterized by carelessness, technical incompetence, and developmental errors, such as counting on the fingers. Their reading was, of course, retarded, and this in turn reflected their arithmetical achievement.

After partialling out mental age, Eagle (9:75-9) discovered that at grade levels from four to nine the knowledge of the vocabulary of mathematics and the ability to interpret graphs and formulas were most closely related to success in arithmetic problem solving. Fay (10:541-7) found that when mental and chronological ages were controlled, those who were higher in reading skills were not any better in arithmetic problem solving than those inferior in reading.

Gowan (13:281-3) asked seven thousand trainees at a Naval Training Station to solve certain arithmetic problems and then tested them to determine the degree of accuracy they employed in judging the correctness of their answers. They were found to be low in ability to recognize when an answer is reasonable, to determine degrees of accuracy, and to accept the absolute necessity for having the correct answer.

Hansen (15:111-18) classified the factors that constitute the ability to solve verbal problems in arithmetic into three general groups, arithmetical, mental and reading. He selected the good and the poor problem solvers among sixth grade pupils in ten midwestern communities and compared their abilities to use each of the factors of ability in solving problems in arithmetic. He found the superior achievers to be significantly better in all except four of the factors, namely, speed of reading to predict outcomes, comprehension in reading to predict outcomes, speed in reading to note details, and comprehension in reading to note details.

Using ninety excellent and ninety poor achievers, Koenker (20: 578-86) experimented with sixth grade pupils in the use of two-figure long division. He found superior achievers to be significantly better than poor achievers in the mechanics of long division, i.e., one figure division, estimating the quotient figure, placing the first quotient figure, multiplication as in division, subtraction as in division, comparison of products and partial dividends, finding errors in division, vocabulary in division, and fundamental operations. In none of these operations was an inferior achiever superior to a high achiever.

Treacy (33) in reporting the results of his study of good and poor achievers in problem solving, presented evidence to show ability in problem solving is definitely associated with vocabulary. He measured the problem solving abilities of 244 seventh grade children, and through the use of a selection test chose eighty high and eighty low achievers. Good achievers were significantly superior to low achievers on several factors--four of which were reading skills definitely associated in one way or another with vocabulary.

Some work has been done in comparing the behavior of good and poor achievers in areas other than arithmetic. Bond (1:746) found that there was a greater incidence of hearing impairment among lower achievers in reading. He concluded that this may indicate a possible causal relation, or it may reveal merely another difficulty for which the non-achiever must compensate. Russell (24) matched sixty-nine pairs of good and poor problem solvers to study their relative behavior in spelling. He found that the greatest single difference between good and poor spellers was visual perception. When he attempted to analyze what he meant by visual perception, he concluded that so many aspects of it were so closely related to intelligence, that perhaps the real difference between good and poor spellers is one of intelligence. He found his most reliable differences in academic achievement items. He believed that many of these items did not cause spelling difficulty, but were rather associated with it.

The foregoing studies are limited, both in number and in scope, but they indicate the complexity and the depth of the problem by revealing the many aspects these studies have assumed. Their chief contribution to this study has been to give direction to the investigation and to serve as guides in interpreting the data.

Techniques Suggested for Studying Pupils

Individual differences in the ways in which pupils go about solving arithmetic problems is a question that has been discussed at length by writers in the field of arithmetic. Extensive diagnostic testing programs have been outlined, and many techniques have been offered as aids in studying pupils individually. Brueckner (5:33), for example, has summarized these techniques by placing them in six groups:

1. Use of standardized diagnostic tests of various kinds.

2. Analysis of available records.

3. Observation of pupils at work.

4. Analysis of oral and written responses.

5. Interviews and questionnaires to get information that is otherwise not available.

6. Laboratory procedures.

The diagnostic interview has come to be a rather common technique for studying pupils. Teachers have learned that an intensive study of the pupil while he is working problems can be productive of information about the pupil and his methods of work. Brueckner and Grossnickle (7:456) cited eleven specific abilities that could be observed during an interview designed to study problem-solving behavior:

1. The ability of pupils to read problems orally, and the extent of vocabulary difficulty.

2. Knowledge of denominate numbers, their relation and the ability to apply them.

3. Knowledge of formulas used.

4. Ability to name the procedures used.

5. Ability to estimate answers.

6. Ability to remember facts and solve problems stated orally by the examiner.

7. Ability to locate information in the index, appendix, dictionary or supplemental book.

8. Informational background and experiential background of

the uses of numbers.

9. Methods of attack on selected test problems.

10. Ability to formulate original positions based on the original test data.

11. Ability to use drawings and diagrams in visualizing the problems.

Monroe (22:314) pointed out that diagnosis in arithmetic is concerned with two major problems:

1. Determination of the extent to which desirable educational objectives are achieved.

2. Identification of factors that might be interfering with the optimum growth of the individual.

He further stated that:

> Any procedure that yields dependable information concerning the status of a pupil with respect to some phase of his development may be useful, but usually a diagnostic procedure is one that provides relatively detailed measures.

<u>The use of the interview technique</u>.-- The interview may be specifically useful as a technique for studying about pupils' procedures in solving arithmetic problems. Spitzer, (27:193-4) in particular, stressed the usefulness of this technique to teachers in determining the arithmetic habits of their pupils:

> One of the best indications of the mastery of a subject is the ability to make significant comments or ask significant questions about it. As a pupil works in an arithmetic class, the teacher has many opportunities to observe the work and thereby obtain a knowledge of his grasp of various phases of arithmetic.
>
> Another indication of achievement in a field is interest in that field, for few human beings maintain an interest for any length of time in a field where they possess little knowledge. Still another indication of achievement is the degree of confidence displayed when work is undertaken or assigned.

Hildreth also advocated (16:843) the use of oral methods in determining the procedures used by pupils in solving problems:

> Although the oral method is necessarily an individual one, it has advantages over any other method for diagnostic purposes... Applied to arithmetic, it enables the examiner to judge the speed of response to separate number items, It reveals hesitation, confusion, repetition, omissions, irregularities in methods of attack, indicates eye movements up or down the addition columns, and the child's facility in the number combinations, with or without a pencil. In the solution of written problems, the method reveals difficulty, slowness and irregularity, as well as the ability to read and pronounce the names of numbers and number terms.

Torgerson (32:125) identified certain limitations to the interview technique, and suggested that care be exercised in the use of it:

> The interview method, like the observational method, has all the limitations inherent in a subjective approach. Flexibility in method is essential for minimum individualization, and for this reason the interview should not be standardized. Skill in the use of a method will minimize the subjective factors which destroy its value.

It is apparent from this discussion that diagnosis of pupils' problem solving experiences is helpful in determining the processes used by the pupils in solving the problems. Among other techniques of diagnosis, that of the diagnostic interview was suggested and the potentialities of the individual oral interviews were indicated. These suggestions were helpful in selecting areas to be observed and in isolating specific objectives for definite study.

Summary

The writings of recognized authorities and the research studies reported have made three important contributions to this investigation:

1. The significance of the term "meaning and understanding" was established through its interpretation by recognized writers in the field. Although there was little evidence that any of these attempted

to define the term as such, there was a notable tendency to recognize the importance of meaningful arithmetic.

2. Research studies associated with the purpose of this investigation, although limited in number and scope, indicated the complex and diverse areas involved. General procedures and specific objectives reported in those investigations were especially helpful in organizing and developing this study.

3. The diagnostic interview was shown to be of significant value in studying individual differences in pupil behavior during problem solving. It was generally agreed that diagnostic interviews on an individual basis, with well-established objectives for the interview in mind, may give valuable clues regarding procedures employed by pupils.

CHAPTER III

PROCEDURES USED IN THE STUDY

The Purpose of the Chapter

The purpose of this chapter is to outline the procedures followed in organizing and conducting the investigation and in evaluating the data that resulted from it. The chapter presents the hypothesis upon which the study is based, and describes the preliminary studies used in structuring the plan of research. It indicates the steps taken in selecting the schools in which the study was conducted, in choosing the pupils for the sample, in preparing the materials used in the study, in administering the research procedures. Plans for the organization and interpretation of the data are also included in the chapter.

Preliminary Considerations

The basic hypothesis of the study.-- The basic hypothesis of the investigation was that problem solving procedures could be isolated and studied, if observed systematically while the pupils are engaged in working arithmetic problems. The design of the study and the procedures employed throughout the investigation have been influenced by considerations related to this hypothesis. Some of these are based on known research in the field of arithmetic; others have been derived from personal experience and from suggestions of persons skilled in educational research. They are the guideposts which furnished direction to the study in the initial stages. Stated concisely, these guideposts are:

1. The sample must be adequate. It must be sufficient in number to include a typical representation of sixth grade pupils in the public schools.

2. Standardized investigational procedures must be followed to insure uniformity. The same questions must be asked and the same problems given to all pupils.

3. Arithmetic problems must be typical of those worked by sixth grade pupils in their daily class work. The problem-solving period must approximate the normal classroom situation as closely as possible.

4. Initial selection must be based on the results of the administration of a standardized test in arithmetic problem solving. Inasmuch as patterns of behavior become more sharply defined when the extremes are studied, good and poor achievers must be selected from the upper and the lower ends of the distribution on the selection test.

5. Observed behavior and oral responses must be accurately recorded. Recorded observations, observed behavior, and written solutions must be given methodical study within a reasonable time after the observational period to insure reconstruction of all aspects of the interview with each individual pupil.

6. Analysis of the behavior of the pupils must be made by an investigator trained in observation of pupils in a problem-solving situation.

Preliminary Investigations

Reasons for the preliminary studies.-- With the foregoing considerations in mind, preliminary studies were planned in selected elementary schools to determine the particular methods of investigational procedure which would be used when the design of the study was in final form. Seven elementary schools were selected for this purpose, three schools in

Cincinnati, Ohio, three in Greenville, Ohio, and one in Hamilton, Ohio. Preliminary work was begun in these schools in March, 1952.

Preliminary investigation at the Kirby Road School.-- The first exploratory study was made at the Kirby Road School in Cincinnati, Ohio. Forty-two fifth grade pupils were given the problem solving test of the Sueltz Functional Evaluation in Mathematics, Elementary Level (Appendix A, p.197). Eleven pupils who made the highest scores and eleven who made the lowest scores were selected for special study. Four groups of five pupils each were then given the National Achievement Test in Arithmetic, Grades 3-6, with instructions to work the problems in the test (Appendix A, p.198). Thirty minutes was allowed each pupil. The small unit of five pupils was planned to allow the observer time to watch each pupil as he worked the problems. When the pupils had finished their work, each was asked to explain how he worked the problems. Several obvious difficulties were apparent in this procedure. With five pupils engaged in problem solving, it was impossible to spend enough time with any one pupil to isolate specific details of his problem-solving behavior. The presence of other pupils in the room influenced some pupils and distracted them from their work. Controlling unoccupied members of the group while others were explaining their problems was difficult. Only general observations could be made, and these were too obvious to be of any real value. The allotted time was inadequate. This led to a varied experience for each pupil, since each reached a different level of achievement during the time allowed.

As the result of this investigation, it was obvious that: A smaller number of pupils must be studied at one time to insure a more systematic observation of the pupils. Definite procedures for studying the behavior of individual pupils and of reporting that behavior were necessary. A list of problems that would permit completion and discussion within the attention span of sixth graders was needed. The general idea of the investigation, however, showed promise.

Investigation at the Central Fairmount School.-- A second preliminary study was made at the Central Fairmount School, also in Cincinnati. The sixth grade teacher was asked to choose six of the best problem solvers and six of the poorest problem solvers in his arithmetic classes. The investigator then prepared a list of ten problems, believed to be typical of those worked by sixth grade pupils (Appendix B, p. 201). The number of problems selected was purely arbitrary, but the purpose was to keep the number small so that all of the solutions and explanations of those solutions could be completed within the attention span of a sixth grade pupil. Two pupils were interviewed at one time. While one pupil solved a problem, the other pupil explained the one he had just completed. Then the other pupil was interviewed while his partner worked all ten problems. This procedure proved more effective than working with groups of five and an unspecified number of problems, but several weaknesses still remained. Two pupils could not be interviewed at one time if the interviewer were to give each his undivided attention. Close attention was necessary to secure details of the problem-solving procedures employed by the pupils. Talking tended to be distracting to both pupils and interviewer, and audible discussions of the problems not only disturbed the other pupil, but also furnished him with clues to the solutions. There had been no definite plan of observation, nor any systematic checklist of behavior to be observed.

Certain conclusions were reached as the result of the Central Fairmount investigation: Because of the disturbing factors in interviewing two pupils at one time, it would be necessary to conduct all interviews on an individual basis. Adequate objective data, based upon systematic observations, would require full and complete written notes. Further experimentation with plans for recording the procedures of pupils would be needed.

Investigation at the Whittier School.-- Procedures followed at the Central Fairmount School had failed to reveal any definite information about problem solving procedures by the pupils interviewed. Another preliminary study was arranged at the Whittier School. In this investigation eight sixth grade pupils, selected by their arithmetic teacher in a random manner were interviewed, one at a time. The problems were similar to the ones used at the Central Fairmount School (Appendix B, p. 202). These problems were presented to the pupils singly, on individual cards, for solution. On separate cards, the investigator attempted to write all the vocalizations of the pupil and note any significant behavior or remarks. The results of this study brought the following conclusions: It was impossible to record all of the utterances of the pupil as he said them, unless the pupil were asked to repeat much that he said. Hints or suggestions, intended to start or to correct pupils' procedures, influenced the actual thinking done by the pupil and made it impossible to separate the pupils' thinking from that of the investigator. Solving and discussing ten problems consumed too much time and the interviews became tiresome to both pupils and interviewer.

The results of the Whittier investigation prompted the following decisions: A mechanical recorder would be useful in securing an exact record of the utterances of the pupil and of the interviewer during the problem solving period. Non-directive interviewing would eliminate participation of the interviewer in the problem-solving experiences. Not more than eight problems could be solved and discussed within a reasonable attention span for sixth grade pupils. Greater contrast, and as the result, greater clarity in differences in behavior could be observed when high and low achievers were studied than when random selections were made.

Investigation at the East Elementary School, Greenville, Ohio.-- Early investigations had revealed that some of the problems used were not appropriate for the particular purpose of the study. Continuous selection of problems had been made as each of the preliminary studies was organized. Problems that had proved too difficult as well as those which were too easy were replaced by others believed to be within a reasonable range of difficulty. When the list approached the point at which it was believed that the problems were within the comprehension of sixth grade pupils, these problems were submitted to fifty-one sixth grade pupils in the East Elementary School in Greenville, Ohio (Appendix B, p. 203). Not one of the twelve problems on the list was missed by all of the pupils, and none was worked correctly by all of the pupils.

Further evaluation of the problems.-- Forty sixth grade pupils in the South School in Greenville, Ohio, and ninety sixth grade pupils in the Filmore School in Hamilton, Ohio, were given the same list of problems. Results from both schools gave additional evidence that enough pupils had worked the problems correctly to justify conclusions that they were within the comprehension of sixth grade pupils. After tabulating the frequency of errors in these problems, the two that were missed most often and the two that were missed least often were eliminated. The remaining eight problems were those which were used in the experiment. These problems may be seen in Appendix B, p. 203.

Testing the procedures for the final study.-- Before deciding upon final plans for the investigation, the writer made another test of the procedures. The arithmetic teacher of the North Elementary School in Greenville, Ohio, chose two good problem solvers and one poor problem solver. These pupils were interviewed individually, and they were asked to work each of the eight problems, vocalizing their thinking processes. A tape recorder was employed to record their utterances which began with the oral reading of the problem and ended with a questioning period following the solution of each problem. Three facts were revealed as the result of the North School experience. The tape recorder was effectual in securing an exact record of the vocalizations of both the pupil and the interviewer during the observation period. It was apparent that procedures could be further standardized by the preparation of a uniform set of questions to be asked all pupils after the solution of the problems.

Selection of the Schools

Selecting representative schools.-- The city of Cincinnati was chosen for the study, and the investigator was granted permission to conduct the investigation in the Cincinnati Public Schools. Two decisions had been made arbitrarily: (1) Approximately one hundred pupils would be needed for the investigation; (2) These pupils would be selected from not fewer than six elementary schools. The Supervisor of Elementary Arithmetic and the Director of the Appraisal Services for the Cincinnati Public Schools assisted in selecting the schools. The selection was based on the following: The median percentile ranking as measured by the Kuhlman-Anderson Group Intelligence Test administered during the previous semester (21), of the sixth grade pupils in the six proposed schools was approximately the same as that of the sixth grade pupils in the school system as a whole. Geographically, the schools were well-distributed throughout the city. The socio-economic levels of the pupils in the several schools were representative of the principal socio-economic groups within the city. School populations in the proposed schools contained representatives of the various social, racial and cultural groups in the city.

There were three general locations: "Downtown" schools in or near the business district of the city. "Residential" schools well within the city limits in chiefly residential neighborhoods. "Suburban" schools at or near the city limits where a separate community spirit seemed to prevail. Both large and small schools were represented. Table 1 shows the number of each school, the number of sixth grade pupils, the median percentile rankings of sixth grade pupils on the Kuhlman-Anderson Group Intelligence Test, and the kind of community served by the school. The intelligence tests were given routinely during the 1951-52 school year by the Appraisal Services of the Cincinnati Public Schools. The median for the six selected schools is slightly lower than for all the sixth grade pupils in the city school system--53.9 as compared with 54.6 for the city as a whole.

TABLE 1

THE SCHOOL NUMBER, NUMBER OF SIXTH GRADE PUPILS, MEDIANS OF SIXTH GRADE PERCENTILE RANKINGS IN INTELLIGENCE, AND CLASSIFICATION OF EACH SCHOOL BY LOCATION

School Number	Number of Sixth Grade Pupils Enrolled	Median Percentile Ranking of Sixth Grade Pupils in Intelligence	Classification of the Location of the School
1.	168	35.7	Downtown
2.	80	46.8	Downtown
3.	133	62.0	Suburban
4.	73	57.8	Residential
5.	51	60.2	Residential
6.	91	74.5	Suburban
	596 Total Pupils	53.9 Median for all Schools	

Selection of the Pupils

Method used in selecting the pupils.-- The specific purpose of this study, as stated in Chapter I, is to investigate the characteristics of the procedures of good and poor problem solvers which show evidence of thoughtful and meaningful understanding as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error. In compliance with the expressed purpose of the study, it was necessary to adopt some plan for separating the sixth grade pupils into groups of good and poor problem solvers. Kelley (18:17-24) has developed a statistical procedure for dividing a group of pupils in such manner that high achievers and low achievers can be differentiated from the average achievers. This plan provides that the upper twenty-seven percent of a group be considered high achievers, and the lower twenty-seven percent of a group be considered low achievers. The plan proposed by Kelley was therefore adopted for use in this investigation.

A standardized test of arithmetic problem solving, The Iowa Every Pupil Test of Basic Skills, Test D: Basic Arithmetic Skills, Form O, Part III, was given to all sixth grade pupils in the six selected schools (Appendix A, p. 199). This test had not been previously used at the sixth grade level in the Cincinnati Schools, it was inexpensive, easily administered, and could be scored objectively. Part III of the test measured problem solving, the area in which the investigation was being conducted. This part was composed of thirty verbal problems in arithmetic with a difficulty range from grade five to nine.

Distribution of scores on the selection test.-- Out of a possible score of thirty on the selection test, the highest made by any pupil was twenty-four, and the lowest was zero. Five hundred sixty-eight pupils took the test. Twenty-seven percent of 568 pupils is 152 pupils. Since this percentage had been chosen as the basis for dividing the good and the poor problem solvers from the group as a whole, steps were taken to determine which pupils belonged in the group making the 152 highest scores and in the group making the 152 lowest scores. A frequency table was made which listed the number of pupils who made each score from twenty-four to zero. Cumulative frequencies were listed in the right hand column, reading from the top downward. Scores of 14 and above occured 151 times in the distribution. The sample of good problem solvers was therefore drawn from the group of pupils who had made scores of 14 and above.

Theoretically, the number of poor problem solvers should equal 152. This number subtracted from the total number of cases leaves 416 as the cumulative frequency below which the poor problem solvers would be chosen. A total score of seven or above included only 408 pupils. The cumulative frequence of 416, then, would be expected to fall among pupils who made a score of six. Since it would be difficult to decide which of these pupils to sliminate from the group of poor problem solvers, all pupils who made a score of six or below were included. Therefore, 160 pupils were included in the group of poor problem solvers. Table 2 shows in detail how this was done.

Table 3 shows by schools the total number of pupils taking the selection test, the number who made scores in the group designated high achievers, and the number in the group designated low achievers. From

TABLE 2

DISTRIBUTION BY SCHOOLS AND THE CUMULATIVE FREQUENCIES OF SCORES MADE BY THE SIXTH GRADE PUPILS IN SIX SELECTED SCHOOLS ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS, TEST D: BASIC ARITHMETIC SKILLS, FORM O, PART III

Test Score	School No. 1	School No. 2	School No. 3	School No. 4	School No. 5	School No. 6	Total	Cumulative Frequency
24	-	-	1	-	-	1	2	2
23	-	-	1	-	-	-	1	3
22	-	1	-	-	-	-	1	4
21	-	-	-	-	1	4	5	9
20	-	-	1	1	-	2	4	13
19	-	-	6	-	-	1	7	20
18	-	1	3	1	1	9	15	35
17	-	3	5	4	1	9	22	57
16	1	3	5	3	2	16	30	87
15	1	3	12	4	3	8	31	118
14	1	9	6	2	5	10	33	151
13	4	5	15	9	3	5	41	192
12	2	7	6	10	5	6	36	228
11	7	8	13	3	3	8	42	270
10	5	4	16	4	4	2	35	305
9	5	5	9	2	3	3	27	332
8	7	4	14	6	3	2	36	368
7	12	8	5	7	6	2	40	408
6	16	2	6	5	4	1	34	442
5	19	6	4	2	2	-	33	475
4	25	2	2	1	1	1	32	507
3	21	-	3	1	1	-	26	533
2	12	2	-	-	1	-	15	548
1	13	-	-	1	1	-	15	563
0	4	1	-	-	-	-	5	568

this table it may be seen that School Number 1 has only three pupils among the high achievers and 110 among the low achievers. Likewise, School Number 6 is disproportionately represented in the two groups, but the situation is reversed with only two pupils as poor problem solvers and sixty good problem solvers. Distribution in the other four schools was somewhat more nearly equal.

The selection of the sample groups.-- After selecting the good and the poor problem solvers groups, forty-eight pupils were chosen arbitrarily from each of these groups. Table 3 shows that disproportionate grouping of good and poor problem solvers had occurred in Schools 1 and 6. This grouping made it unwise to depend upon random sampling, for it was assumed that with forty percent of the good problem solvers from one school and almost seventy percent of the poor problem solvers from another school, random sampling would draw too heavily from these two schools.

The forty-eight good problem solvers were therefore selected as follows: A chart was prepared listing all scores from twenty-four to fourteen, inclusive. Opposite each score, by schools, was listed the name of each pupil who had made that score. From this chart one pupil from each school who had made each score of fourteen or above was chosen. If only one pupil appeared in a group making that score, the name was automatically selected. If more than one name appeared in a group, an arbitrary choice was made. One name from each of the thirty-eight groups produced a total of thirty-eight pupils, ten short of the number previously determined. To secure the remaining ten names, the process was repeated, working downward from the highest score.

TABLE 3

THE NUMBER OF PUPILS BY SCHOOLS WHO TOOK THE IOWA EVERY PUPIL TEST OF BASIC SKILLS, TEST D: BASIC ARITHMETIC SKILLS, FORM O, PART III, THE NUMBER OF GOOD AND POOR PROBLEM SOLVERS BY SCHOOLS AND THE TOTAL NUMBER IN EACH GROUP

	School No. 1	School No. 2	School No. 3	School No. 4	School No. 5	School No. 6	Total
Total Number of Pupils	155	74	133	66	50	90	568
Good Problem Solvers	3	20	40	15	13	60	151
Poor Problem Solvers	110	13	15	10	10	2	160

The forty-eight poor problem solvers were chosen in a similar manner: A chart was made listing all scores from zero to six, inclusive. Opposite each score the names of all pupils, by schools, who had made each of the scores. From this chart, one pupil from each school who had made each score of six or below was chosen. When more than one name appeared in a group, an arbitrary selection was made. There were in all twenty-nine score groups, and one name from each of those groups produced a total of twenty-nine names, nineteen fewer than the arbitrary number. To obtain the remaining names, the process was repeated, working upward from the score of zero.

Throughout the entire process, a conscious effort was made to keep the number of boys and girls approximately equal. In choosing the good problem solvers, it appeared expedient to give some preference to pupils from School Number Six because the small numbers of poor problem solvers in that school would otherwise give disproportionate representation in the total sample. Likewise, in selecting the low achiever group, a similar preference was given to School Number One because of its low percentage of good problem solvers. The pupils who comprised the sample of good and poor problem solvers and the schools from which they were chosen are shown in the tables in Appendix C, pp. 205-6.

Selection of the Problems

Problems limited to whole numbers.-- Practices employed by Cincinnati teachers in presenting arithmetic subject matter at the sixth grade level vary from school to school. Some teachers place considerable emphasis upon common fractions, others stress work in decimals, and many choose to spend much time with denominate numbers. Differences in neighborhoods, in pupil intelligence and in community socio-economic levels apparently justify this variation in emphasis. A common meeting point in the treatment of arithmetic subject matter treatment, however, is the study of whole numbers. Beginning in the primary grades, the pupils have therefore, worked continuously with whole numbers, and their experiences include the basic skills of addition, subtraction, multiplication, and division. Whole numbers, then, represent most nearly the common arithmetical background of the sixth grade pupils in the schools selected for the study.

Sources of information about the problems. Before the problems for use in this study were selected, several sources of information were explored. Textbooks in sixth grade arithmetic were examined for content and for types of problems. Workbooks and tests were surveyed for ideas. The investigator's own experiences as a teacher of sixth grade arithmetic and as a supervisor of elementary teachers were applied in considering and evaluating the problems. Teachers of sixth grade arithmetic in schools participating in the preliminary studies were consulted for suggestions and criticisms. A list of problems prepared by Brueckner and Grossnickle (7:458-9) for use in diagnosing pupils' problem-solving habits, proved useful in making the final decisions.

Development of the problems.-- It is generally accepted that the vocabulary of an arithmetic problem is of importance to the pupil who attempts to work that problem. For this reason, all of the words used in the problems were checked against the word lists for sixth grade pupils prepared by Rinsland (23) in a A Basic Vocabulary for

Elementary School Children. All words used in the problems were found to be within or below sixth grade limits.

An effort was made to equalize the number of times each of the four basic skills in computation was used. Since several of the problems could be solved by more than one method, it is impossible to state definitely the frequency of any one of the basic skills. Five of the problems, however, were one-step problems, and the other three, if solved in the most economical manner, were two-step problems. The subject matter of the problems was such that a pupil with a limited environmental background would likely have had some experience with it. An additional problem was included with the list of eight problems designed for the study. This problem was intended to serve as a sample problem. It enabled the interviewer to explore with the pupil the techniques of the interview without touching the content of the selected list of problems. The list of problems designed for the study may be examined in Appendix D, p. 209.

Steps in the Procedure

Materials used in the study.-- In order to make procedures as uniform as possible, a call list of pupils was prepared and the order of the interviews established. A schedule was arranged to give principals of the various schools advance notice of the day and the hour interviews would be conducted in their respective schools. A mimeographed copy of each problem was placed on a separate four-by-six index card. Remaining space on the cards was left for the pupils' computation and solutions. A sufficiently large supply of these cards was prepared for all of the interviews. A supply of pencils without erasers was provided for the pupils' use in order that faulty starts and incorrect computation could not be erased. A tape recorder and a supply of tapes for recording the interviews were taken from school to school. Both the reading time and the problem solving time of each pupil were checked with a stop watch. a notebook for recording the observations of behavior during the problem solving periods completed the standard equipment used in the investigation.

<u>The mechanics of the interview</u>.-- Standardized procedures insured uniformity of results. All equipment was set up and in readiness before the pupils were called for the interviews. The schools provided a small room, free from noise and disturbance. The tape recorder was put in an inconspicuous place, with the microphone somewhat out of the sight line. The investigator sat across the table from the pupil in a position so that the facial expressions and gestures could be observed. The pupils were summoned either by the school office or by school messengers. All schools had more than one section of sixth grade pupils which made it possible to rotate the sections from which pupils were called for their interviews; a policy which reduced the amount of communication between pupils. Interviews averaged somewhat forty minutes in length. They were begun by a friendly greeting to the child and an informal conversation intended to put the child at ease. The project was explained and the tape recorder discussed briefly. It was made clear that the results of the interview would not be used for or against the pupil, and that the outcomes would be held in confidence. The sample problem was then given with instructions to solve it to the best of his ability. Procedures to be followed in the remaining problems were

discussed after the pupil had finished the sample problem. This allowed ample opportunity to instruct the pupil in what he was expected to do with the succeeding problems.

When the interviewer was certain the pupil understood the procedures to be used, the remaining problems were given him, one at a time, for solution. No attempt was made to hurry the pupil . He was allowed to work the problems as often as he liked. It was only when the problem was apparently too hard for the pupil and he was unable to make further progress toward a solution, that the interviewer recommended that he abandon his efforts. By means of the stop watch, the time required by the pupil both to read and to solve the problems was determined and recorded in notes of the pupils' behavior. If a word blocked the reading of a problem, it was pronounced for the pupil. Other questions were answered in a non-directive manner, and the interviewer avoided all facial expressions and mannerisms that might be interpreted as encouragement or discouragement of his choice of solution. As soon as the pupil had solved each problem, the following questions were asked him:

1. What is your answer? (Unless he had already given it.)
2. How did you work the problem?
3. Why did you work it that way?
4. Do you think your answer is reasonable?
5. Why do you think so?
6. Do you think your answer is right?
7. Why do you think so?

When a pupil failed to work a problem or to complete a solution, the questions were either omitted or adapted to the situation. If the answer to a previous question had included the answer to the next question, that question was withheld rather than annoy the pupil by asking him to answer it twice.

After a day's interviews the tapes were transcribed so they might be used again the next day. Some tapes were preserved in order that they could be used to demonstrate the techniques employed in the interviews. The written solutions were used together with the observations noted by the observer and vocalizations to determine the problem-solving behavior of the pupils. Samples of transcribed vocalizations may be seen in Appendix D, pp. 208 and 210.

Organization of the Data

The classification of the data.-- Eight problems solved by ninety-six pupils produced a total of 768 solutions. These solutions varied in accuracy, clarity of purpose, concreteness, orderliness, reasonableness, and in many other ways. The solution of these problems produced three kinds of data: notes taken by the interviewer of the observed behavior of the pupils, written solutions to the problems, and the transcribed vocalizations of the pupil and the interviewer. In accordance with the expressed purpose of the study as outlined in Chapter I, the data have been organized under the following classifications: data related to the insight, i.e., meaning and understanding shown by a pupil of the situation described by the problem; data illustrating the thinking processes employed in selecting methods of solution, using those methods; and data reflecting the number relationship and computational skills employed by the pupils in working the problems.

Methods used in tabulating the data.-- The written solutions, recorded vocalizations, and the observed behavior notations were then studied and tabulations made of the frequency of the various kinds of behavior. Characteristics of procedure relating to insight were classified clear, partial, or doubtful. Those characteristics pertaining to thought processes involving choice of method were categorized social, mechanical, and doubtful. Characteristics of procedure relating to thought processes involved in applying methods of solution were grouped as abstract, concrete or random. Those dealing with evaluation of results were classified quantitative, parallel, and meaningless. In a similar manner, characteristics of procedure concerned with number relations were arranged in three groups, clear, partial and doubtful.

In accordance with the foregoing classification, final judgments of the actual characteristics of procedure employed by the pupils were made. These judgments are presented in tables, each of which is divided into three groups corresponding to the general classification of the data described above. Tabulations for good and poor problem solvers are presented separately to show more readily the characteristics of each.

Presentation of the data.-- The data are presented in three chapters. The first, Chapter IV, presents those which are related to insight as a factor in arithmetic problem solving. The second, Chapter V, is concerned with the thinking processes employed in selecting methods, using the methods and evaluating the results of the methods employed in solving the problems. The third chapter, Chapter VI, shows the data pertaining to the number relationships and computational skills employed by the pupils. Throughout the three chapters, the investigator has introduced actual transcriptions of vocalizations of the pupils, and when necessary copies of written computations and notes of observed behavior have been included.

Summary

This study is based on the hypothesis that problem solving procedures can be isolated and studied, if observed systematically while pupils are engaged in problem solving. Implementation of this hypothesis requires an adequate sampling of pupils, a systematic plan of observation, and interpretation of results by an investigator trained in observing pupils in an arithmetic problem-solving situation.

Preliminary investigations were conducted in seven elementary schools in Cincinnati, Greenville, and Hamilton, Ohio. Methods of procedure, sample problems, and evaluative techniques were explored during this preparatory phase. After the preliminary investigations had been completed, six elementary schools in the Cincinnati, Ohio Public Schools were chosen for the final investigation. Authorities of the Cincinnati Public Schools considered these schools to be representative of the city schools as a whole.

From these six schools forty-eight good problem solvers were selected from the pupils who made the highest twenty-seven percent of the scores on a standardized test in arithmetic problem solving, and forty-eight pupils poor problem solvers were selected from those who made the lowest twenty-seven percent of the scores on the same test. All pupils were sixth grade children, regularly enrolled in the schools used in the study.

The eight problems chosen by the investigator for this study were developed after consulting current textbooks, workbooks, tests, and teaching materials used in the public schools. These problems, involving whole numbers only, were placed on individual cards with sufficient work space for computation, and given to each pupil to solve.

Each of the ninety-six pupils chosen for the study was interviewed singly, and all vocalizations by these pupils while they worked the eight problems recorded on a tape recorder. Together with the written solutions and notes of observed behavior, these vocalizations provided the chief source of information about the problem solving behavior of the pupils.

The data gathered through the investigation were summarized by classification into three major areas. These areas included: those items which were related to the insight, i.e., meanings and understanding shown by the pupil into the situation described by the problem; data that illustrated with the thought processes used in choosing a method of solution, carrying out that method, and evaluating the results of that method; and those which showed the number relationships and computational skills employed by the pupils. Behavior of the pupils was evaluated by the investigator and placed in tables. These tables were placed in appropriate chapters, and anecdotal references drawn from the transcribed recordings, written solutions and observed behavior introduced to explain and support the data contained in the tables.

CHAPTER IV

INSIGHT AS A FACTOR IN ARITHMETIC PROBLEM SOLVING

The Purpose of the Chapter

The stated objectives of this study are to investigate the characteristics of the procedures of good and poor problem solvers while they were engaged in solving verbal problems in arithmetic. The study attempts to isolate and describe the characteristics of the procedure which show evidence of thoughtful and meaningful understanding, as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error.

When the data from the interview were reviewed, differences in the ways pupils attacked the problems were observed. Some pupils read a problem, indicated complete familiarity with the situation described by the problem, and proceeded with complete awareness of the significance of each step. They evaluated the answer in terms of their own experiences, and recognized the importance of their newly-found information. Others read the problems, recognized certain number combinations, and attempted the computation with little or no consideration of the content of the problem or of the social significance of the problem situation. Although their answers were many times correct, they were announced abstractly, with little evidence that it represented more than a number--the product of a computational process. In still other cases, complete bewilderment as to the meaning of the problem, random calculation, or perhaps sheer guesswork gave evidence that the pupil was performing only perfunctory tasks in working the problem; no evidence of any real insight could be observed, not only between pupils, but also in the performance of individual pupils as they attacked the different problems.

The degree to which the pupil is able to recognize significant aspects of a problem, to associate meaning and understanding with those aspects, to apply logical computation in the solution of the problem, and to evaluate the final results in terms of his own experiences may be defined as insight. Evidence of insight appears in at least four aspects of arithmetic problem solving.

First, the general evaluation made by the pupil of the entire problem situation is a clue to the total insight into the problem. This clarity of insight is reflected in the analysis of separate aspects of the problem, the relation of the computational functions to the problem as a whole, and the judgments shown by the pupil, not only of the situation described by the problem but also of the appropriateness of the solution to the problem.

Second, the utilization of vocabulary clues in problem solving furnish additional information about the insight experienced by the pupil through his association with the problem. Some pupils dwell upon a word or a phrase, others place inflection on specific terms, and still others place emphasis upon numerical values as each sought clues to the meanings involved in the problem.

Third, the extent to which pupils apply social and economic information offers supporting evidence of the insight experienced by the pupils. The extent of this can only be surmised, for silent interpretation and association with known social and economic facts might have been a common occurrence. There is, however, much evidence that vocal recognition of background information aided pupils in the growth of insight into the problems.

Fourth, the ability of pupils to label the answer to a problem, once solved, was evidence of the insight of the pupils into the problem. The look of understanding on the face of the pupil, or an appropriate vocalization often gave the investigator a clue to the recognition by pupil as to the meaning of the answer in terms of the social situation involved in the problem.

It is the purpose of this chapter to show the function of insight as a factor in problem solving in arithmetic. This has been done through frequency tables which show how often good and poor problem solvers made use of the procedures which indicate insight. The tables are supported by anecdotal accounts of actual solutions which describe and illustrate the data contained in the tables.

Variations in Pupil Insight

Three degrees of insight.-- As indicated in Chapter III, the degrees of insight shown by pupils in solving problems were classified as clear, vague, and doubtful. Clear insight is characterized by effective reading, direct attack upon the problem based upon understanding of the problem situation, judgments of the problem situation, critical evaluation of the final answer, and reasonable confidence in the appropriateness of the answer. Vague insight is characterized by faulty reading, obscure number relations, imperfect or hesitant selection of methods of attack, neglecting significant words or ideas, inadequate notions of the meaning of the answer, and lack of confidence in the appropriateness of the answer. Doubtful insight is characterized by inability to attack the problem, choice of a method for no apparent reason except a random effort to compute, fragmentary understanding with complete bewilderment as to how to apply those fragments, and incoherent, meaningless attempts to explain the solution.

A comparison of the behavior of good and poor problem solvers.--Table 4 shows that good problem solvers demonstrated clarity of insight four times as often as poor problem solvers, and vagueness of insight about the same number of times. On the other hand, poor problem solvers showed doubtful insight about six times as often as good problem solvers.

The data in Table 4 show about four times as many good problem solvers with clear insight into the problem situation as poor problem solvers. The solutions of Peggy H. and Ronald R., both good problem solvers illustrate what is meant by clear insight into the problems.

Peggy: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?

Well, - - - - - I'll multiply 75 by 2.

Inv: Well, go ahead.

Peggy: 2 x 5 are 10 - - - - - 2 x 7 are 14 - - - - - and 1 are 15 - - - - - 150 feet.

Inv: How did you get it?

Peggy: Well, - - - - - the pool is 75 feet long - - - - - and he will swim twice the length of the pool - - - - - swim - - - - - 2 x 75 equals 150 feet.

Inv: Do you think your answer is reasonable?

Peggy: Yes, I do.

Inv: Why?

TABLE 4

THE NUMBER OF TIMES CLEAR, VAGUE, AND DOUBTFUL INSIGHT WERE OBSERVED WHILE GOOD PROBLEM SOLVERS AND POOR PROBLEM SOLVERS WERE ENGAGED IN SOLVING EIGHT SELECTED PROBLEMS IN SIXTH GRADE ARITHMETIC

Classification	Number of Pupils	Total Number of Problems	Cases of Clear Insight	Cases of Vague Insight	Cases of Doubtful Insight
Good Problem Solvers	48	384	212	132	26
Poor Problem Solvers	48	384	50	154	158

Peggy: Well, - - - - - I sometimes swim in the swimming pool that far.

Inv: Do you think it is right?

Peggy: Yes.

Inv: Why?

Peggy: I think I worked it right - - - - - 2 x 75 is 150 feet.

****

Ronald: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How much warmer had it become?

7 and 9 is 6 - - - - - carry 1 is 36 - - - - - 36 degrees.

Inv: How did you get it?

Ronald: I added 7 and 29.

Inv: Why?

Ronald: 7 degrees - - below - - and then he had 29 degrees above - - - - - add 7 and 29 and get your answer.

Inv: Is it reasonable?

Ronald: Yessir.

Inv: Why?

Ronald: Why - - - - - add a small number to a large - - - - - it will go up that fast - - - - -

Although poor problem solvers were not so likely to do so, they did on certain occasions demonstrate clear understanding comparable with that of good problem solvers. John S. and Jim S. both poor problem solvers read the problems carefully, used the materials given by the problem, solved it correctly, and evaluated their answers intelligently and with understanding.

John: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?

- - - - - 2 x 75 - - - - - - is 150 feet.

Inv: Write your answer on your paper - - - How did you get it?

John: I took 2 times 75 - - - -

Inv: Why?

John: Why - - - - - cause I wanted to find out how far he'd swim if he swimmed twice the length of the pool - - - - - two times - - - - - It's easier to multiply - - - - - The 30 feet don't have nuthin to do with it - - - - -

Inv: Is your answer reasonable?

John: Yes.

Inv: Why?

John: Cause the length of the pool - - - - - twice is 150 feet.

****

Jim: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

It became 36 degrees warmer.

Inv: How did you get it?

Jim: I added 29 plus 7.

Inv: Why?

Jim: Well, - - - - - - if it was 7 degrees <u>below</u> zero - - - - - you'd have to bring it to zero - - - - - and then add.

Inv: Is your answer reasonable?

Jim: Yes.

Inv: Why?

Jim: Well, - - - - - 29 and 7 would just add up to 36.

Inv: Is your answer right?

Jim: Yes.

Inv: Why?

Jim: I don't know - - - - - I just think it is right.

Vagueness in insight, partial understanding, or difficulties in establishing number relations occured with about the same frequency among good problem solvers as among poor (See Table 4). As in the case of clear understanding, individual cases of partial understanding looked much alike, whether they were produced by good problem solvers or by poor. In solving Problem 4, neither Joyce H., a good problem solver, nor Daniel C., a poor problem solver was able to visualize clearly all of the implications of the problem. Ignorance of the actual value of the five-digit numbers and failure to associate the real meaning of the final answer with its numerical value were factors in both solutions. Although Joyce had the correct answer and Daniel the wrong answer, an accident of computation might have reversed the answers without either showing much concern over the values. Joyce avoided the original error made by Daniel of putting the smaller number above the larger, but Daniel soon found that mistake. No attempt was made by either to associate any social meaning to the problem or its answer.

Joyce: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read 2 million 8 thousand 9 hundred 65 (28965). At the finish of the trip it read 2 million 9 hundred 11 (29011). How many miles did he drive on one trip?

I'd subtract 2 million 8 thousand 9 hundred 65 - - - - - from - - - - - 2 million 9 hundred and 11 - - - - - (vocalizes her computation) - - - - - - and the answer is 46 miles.

Inv: How did you get it?

Joyce: Well - - - - - I subtracted - - - - - 2 million 8 thousand 9 hundred 65 - - - - from 2 million 9 hundred 11.

Inv: Why?

Joyce: Well - - - - - when it started out it read - - - - - 2 million 8 thousand 9 hundred 65 - - - - - and when it stopped it read 2 million 9 hundred 11 - - - - - so I subtracted.

Inv: Does it sound reasonable?

Joyce: Yes.

Inv: Why?

Joyce: Cause - - - - - I think it is that many times - - - uh - - - - - - that many more.

Inv: Do you think it is right?

Joyce: Yes.

Inv: Why?

Joyce: I think I subtracted it right.

****

Daniel: (Reads) Tom's father drives a city bus. The speed - - uh - - - - uh - - - - - uh - - (gets help) - - uh - - - I can't pronounce it - - - - read 2-8-9-6-5. At the finish of the trip it read - - - 2-9-0-1-1. How many miles did he drive on one trip?

Works problem:

```
 28965
 29011
  9954
```

Inv: How did you get it?

Daniel: I subtracted 2-8-9-6-5 from 2-9-0-1-1.

Inv: Why?

Daniel: It said when he started - - - - his - - - - speed - - - (trouble) read 2-8-9-6-5 - - - and when he finished it read - - - - - - - - - 2-9-0-1-1 - - - - - - - - I did it wrong - - - - -

Inv: You did?

Daniel: I took this - - - - from this - - -

Inv: Can you do it right?

Daniel: I think so.

Inv: Well, go ahead.

Daniel: (Works problem, whispering to himself) - - - - I think I got it now - - - - - - - 1-0-4-6.

Inv: Why did you change?

Daniel: Because 2-9-0-1-1 is more than 2-8-9-6-5.

Inv: Why was it necessary?

Daniel: This was bigger than this one, - - - - and you take - - you subtract from the biggest one.

Inv: Is it more reasonable?

Daniel: Yes.

Inv: Why?

Daniel: I don't know.

Inv: Is it right?

Daniel: Yes.

Inv: Why?

Daniel: It's smaller than the other one - - - - - - - (pause) - - that's all I know - - - - -

The solutions of Michael M., a poor problem solver, and of Melvin K., a good problem solver, to Problem 7 were much alike. Both made the same error in interpreting the meaning of the problem, an error which reflected lack of experience with the use of a thermometer. Both were reasonably sure of their answers, giving substantially the same reason for their confidence. Partial understanding of the problem is present in both solutions, but real insight, based upon clear interpretation of all the facts presented is missing.

Michael: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

You subtract 7 from 9 - - - - leaves 2 - - - - the answer is - - - - - - 22 degrees.

Inv: How did you get it?

Michael: I subtracted.

Inv: Why did you do that?

Michael: Because I think that is what the problem calls for.

Inv: Do you think it is reasonable?

Michael: Yes.

Inv: Why?

Michael: Because I think that's what should be done

Inv: Do you think it is right?

Michael: Yes.

Inv: Why?

Michael: Because I think I worked it right.

****

Melvin: 22 degrees warmer - - - - - - - I divided 7 from 29 - - - (vocalizes his subtraction) - - - - leaves 22.

Inv: Why did you work it that way?

Melvin: Cause that was the way to do it.

Inv: Is the answer reasonable?

Melvin: Yessir.

Inv: Why?

Melvin: Because it was 7 degrees <u>above</u> zero at six a.m. - - - - - at noon it was 29 degrees - - - - - - - how much warmer had it become?

Inv: Is it right?

Melvin: Yessir.

Inv: Why?

Melvin: Cause 7 and 22 are 29.

The data in Table 4 also shows that six times as many poor problem solvers demonstrated doubtful insight as good problem solvers. While in these cases there was sometimes fragmentary understanding and ineffective effort to relate the problem situation to their experiential background, the problem solution indicated almost complete bewilderment and confusion. Solutions by Charles T., a good problem solver, of Problems 8 and 5 are examples of the kind of insight which is considered doubtful.

Charles: (Reads) Mary's mother wants to put linoleum on the kitchen floor. (Reads aluminum) Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

Why - - - - - I think you should multiply - - - - - - - 9 x 15 - - - - - - and then - - - - - that's one of that kind of problems that I don't do very well.

Inv: You have trouble with them?

Charles: Yes - - - - -- this is one of the kind I never get.

Inv: Have you studied this kind?

Charles: Yes - - - - - - but I don't understand it very well - - - (tries again) - - - - - - - - I - - - just can't get it.

Inv: You can't?

Charles: I have an idea of how to work it - - - - - - - not the answer - - - - - - but I think it is - - - the - - - - - number of square feet in a square yard - - - - - how many square yards - - - - - - in 145 square feet - - - - - no - - - - - - - - - I can't work it - - - - -

Inv: Is it right so far?

Charles: Yes - - - - - I think I should multiply it - - - - - -

Inv: Why?

Charles: Well - - - - - - - - - - I have to find out - - - how many feet it is in the whole thing.

Charles: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

Well - - - - this is a little bit harder - - - - - Well - - - - I think it should - - - - be - - - I think I should - - - - divide 2 into - - 45 - - - I'm not sure - - - - that's too much - - - - - - I can't get this one - - - -

Inv: Have you any idea?

Charles: I was thinking about - - - I could get these two bars for 45¢ - - - - - I was gonna multiply 6 times 2 - - is 12 - - - double it - - - but I didn't - - - - - - (long pause)

Inv: Any idea?

Charles: Nope.

Doubtful insight, involving fragmentary understanding, and confused applications of inter-problem relations, was more frequent in the work of poor problem solvers. The solution to Problem 8 by Robert B., a poor problem solver, and to Problem 5 by Diana S., also a poor problem solver, shows how this absence of insight defeated the purpose of the pupils in arriving at a correct solution to the problem. The insight shown by these poor problem solvers resembles that shown by Charles T., in the previously cited examples.

Robert: (Reads) Mary's mother wants to put linoleum on the kitchen floor. (Reads aluminum) Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

Is this the last one?

Inv: Yes.

Robert: Yessir, - - - - - it's a pleasure - - - I don't know - - - I can't get this one - - - - - I can't figure this one out - - - - - - this is a hard one - - - - - A square yard would be 3 feet - - - - - 36 inches - - - - - - - - each way - - - - that'd be 12 feet - - - - - - - - - oh - - - - - (sigh) -- - I'll get it somehow - - - - figure around here until I find out - - - - - - - - - 12 feet into 15 - - - - - that wouldn't be right - - - - - oh - - - - - - no - - - (long pause)

Inv: Any idea?

Robert: Nosir.

Inv: Do you know what linoleum is?

Robert: Yessir.

Inv: You don't know how?

Robert: Nosir.

****

Diana: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

(Pause) - - - - - I - - - can't work that one - - - -

Inv: No idea?

Diana: No - - - - (pause) - - - - I might divide - - 2 into 45 - -

Inv: Why?

Diana: Asks you - - - 6 bars for 45¢ - - - - asks you 2 bars for 45¢ - - what Jane would pay for 2 bars for 45¢ - - - - - 2 times 6 are - - no - - - - (long pause)

Inv: Just can't work it?

Diana: No.

From the preceding discussion it may be seen that good problem solvers solved their problems with clear insight more frequently, with partial insight about as often, and with doubtful insight much less often than poor problem solvers. Illustrations used in explaining the various kinds of insight show that both good and poor problem solvers are capable of all three kinds of insight, and that when clear insight did occur among poor problem solvers, there was little real difference in the procedures of the individual pupils. Examples of partial, or vague insight on the part of good and poor problem solvers revealed that individual performance of pupils from both groups was much the same. While doubtful insight was much more infrequent among good problem solvers, the examples studied indicate that these pupils performed in much the same manner as poor problem solvers when they were unable to grasp the essentials of the meanings of the problems.

Utilization of Vocabulary Clues in Insight

Insight through word and number clues.-- Examination of recorded vocalizations revealed that pupils would dwell on a word or a phrase, emphasize a certain part of the problem, linger over a term or a number, or in some other manner indicate that they were paying special attention to the vocabularly content of the verbal problem in their search for clues to the solution. Such behavior is illustrated in the solution of Problem 1 by Joan T., of Problem 7 by Carl B., and Problem 8 by Ronnie G., all good problem solvers. Ronnie's solution is especially interesting in that it shows how recognition of certain words and number clues permitted him to make progressive changes in his work, and finally arrive at a correct solution to the problem.

Joan: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?

Well - - - - - - 75 feet is - - - - - length - - - "long" the answer is 150 feet - - - - - -

Inv: How did you work it?

Joan: I multiplied 2 x 75 - -

Inv: Why did you do that?

Joan: Because - - - - - "long" is - - - the length of the pool - - - and he swims twice the length of the pool - - - - - - so I multiplied - - - - - - - - -

Inv: Is your answer reasonable?

Joan: - - - - - - uhuh

Inv: Why?

Joan: Cause twice the length - - - - - - - - - - - 2 x 75 - - - - is 150.

Inv: Is your answer right?

Joan: Yes.

Inv: Why?

Joan: Well - - - - - - because the pool is 75 feet long - - - and 30 feet wide - - - - - - - - - - - "long" - - is the length of the pool - - - - - -

****

Carl: (Reads) At six a.m. the thermometer reads 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

Is that - - - - 29 degrees - - - below zero?

Inv: You'll have to figure that out, Carl.

Carl: Below - - - that's colder - - - - - I guess it would be - - - - - - - - 7 - - plus 29 - - - 36 degrees - - -

Inv: Put your answer on your paper - - -

Carl: It said - - - - - 6 a.m. - - - - the thermometer read - - - - - 7 degrees - - - - - that's colder - - - - - - and find out - - - - how many degrees - - -

****

Ronnie: (Reads) Mary's mother wants to put linoleum on the kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

\- - - - - - - - - I think the answer is 135 square yards - - -

Inv: Can you tell me how you got it?

Ronnie: I multiplied 15 x 9 - - -

Inv: Why did you do that?

Ronnie: That's the way I've learned how to find square feet - - - to multiply length times width - - - - - -

Inv: Do you think it is reasonable?

Ronnie: Yes - - - - I think so - - - -

Inv: Why?

Ronnie: (No reply - - - studies the problem intently)

Inv: Do you think it is right?

Ronnie: I - - - - think so - - - - - I see where I made a mistake - - - - - - - - - it is 3 - - - - - - - - I think the answer is 45 square yards - - - - - -

Inv: Why did you change it?

Ronnie: This is only - - - <u>feet</u> - - - and the question is - - - - how many square <u>yards</u> - - - - will she need - - - -

Inv: I see - - - Can you tell me why you - - - -

Ronnie: Well - - - - I just figured - - - - one - - - no - - - wait a minute - - - - - I still made this one - - - - (sigh) - - - - - Let's see - - - - Oh wait - - - - now let's see - - - - where I made a mistake - - - - I think it's 15 - - - - cause there's 9 square feet in a square yard - - -

Inv: There is? How did you know that?

Ronnie: That's what I learned in arithmetic.

Inv: You mean you memorized it?

Ronnie: Yessir - - - our arithmetic teacher told us that and I remembered it.

Poor problem solvers, likewise, sometimes gave special attention to the vocabulary of the problems. Familiarity with the vocabulary of the problems and realistic application of the meanings of the words and ideas contained in the problems are evidenced in the solution of Sylvia N. to Problem 1, Tom M. to Problem 4, and Ronald B. to Problem 3. These pupils, all poor problem solvers, readily found and interpreted verbal clues to the problems and proceeded to present careful solutions to the problems. Ronald had some trouble expressing his terms, but he seemed to know what they meant and gave every indication that he knew what he was doing when he worked the problem.

Sylvia: (Reads) A swimming pool is 75 feet long and 30 feet wide How far does Bill swim in swimming twice the length of the pool?

I want to add 75 and 30 feet - - - - - - - - twice - - - and - - - I'll get - - - - - - 95 feet - - - -

Inv: Why?

Sylvia: Cause - - - he'll swim twice - - - across the pool - - - and it is 75 feet long - - - and 30 feet wide - - - - - and you add - - - - -

Inv: Is your answer reasonable?

Sylvia: Yes (nods).

Inv: Why?

Sylvia: (Pause - - - - no answer).

Inv: Is it right?

Sylvia: Yes (nods).

Inv: Why?

Sylvia: Because - - - - - it is just how "long" - - - - the "length" - - - of the pool is - - - -

****

Tom: (Reads) Tom's father drives a city bus. Before starting on a route, the speedometer read 28 - 9 - 65. At the finish of the trip it read 29 - 0 - 11. How many miles did he drive on one trip.

Um - - - - - (mumbles) - - - - - - what does that "one trip" mean?

Inv: What do you think it means?

Tom: When it starts at McMicken and comes back to McMicken?

Inv: Go ahead and try to work it. - - - - - How did you get it?

Tom: Subtract 28 - 9 - 65 from 29 - 0 - 11

```
  9 1
29071
28965
-----
  046
```

Inv: Why?

Tom: It says how many miles did he drive on one trip?

****

Ronald: I divided 48 into 3552 - - - - - - (vocalizes his division) The answer is 74 - - - - - pounds a bushel - - - - (pause) - - - - - - - that's my answer.

Inv: How did you get it?

Ronald: There's 48 pounds in a bushel (problem said basket) of apples - - - - and so it says how many bushel would it be - - - - - it would be 74 bushels -

Inv: How did you work it?

Ronald: Divide.

Inv: Why?

Ronald: It says how many bushels.

Inv: Is it reasonable?

Ronald: Yes.

Inv: Why?

Ronald: Cause it said - - - - how many bushels.

Patient and painstaking recognition of word and number clues is seen in the solution to Problem 6 by Ray H., a poor problem solver. He reflected his clear insight into the problem, not only by the careful reading and identification of ideas, but also by the methodical checking procedures offered in proof of his accuracy.

Ray: (Reads) Jean had \$3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left?

55¢ - - - - - she spent - - - - - - - - - - and she had - - - - - - \$3.50 - - - - - - and she spent 55¢ altogether - - - - and she had - - - - - - - \$2.95 left.

Inv: Put it down on your paper. How did you get it?

Ray: Well, - - - - see - - - - - - - she spent 35¢ for a movie - - - - - see - - - - - - - and 20¢ for a soda - - - - see - - - that makes 55¢ - - - - - see - - - - - - and 55¢ from \$3.50 - - - - - - - see - - - - - - - makes - - - \$2.95 - in her purse.

Inv: Why did you work it this way?

Ray: Well - - - - - - - - - I could work it this way, too - - - - (works \$3.50 minus .35 equals \$3.15 - - - - - \$3.15 minus .20 equals \$2.95) It gives the same answer.

On some occasions failure to recognize important word clues to a problem rendered the problem solver unable to see the significant key which would unlock the real meaning of the problem. This was true in the case of Nancy T., a good problem solver, in her attempt to work Problem 7.

Nancy: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

You have to subtract - - - - - 7 from 29 - - - - - 22 degrees - - - - -

Inv: How did you get it?

Nancy: I subtracted 7 from 29 - - - - - -

Inv: Why?

Nancy: If you know how - - - - - - uh - - - warm - - it was in the morning - - - - - - and want to know how - - - - uh - - - how warm it was later - - - - - - - uh - - you'd subtract - -

Inability to find the verbal clues to the problems could likewise be observed in the work of poor problem solvers. Julia P., a poor problem solver, made an apparent effort to find the key to unlock the secret of Problem 4. She "went by the question", and attempted two random computations, but was unable to complete a correct solution. Simon C., also a poor problem solver, found his vocabulary totally inadequate for solving Problem 3. The words in parentheses are words that he was unable to pronounce.

Julia: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive on one trip?

Subtracts:

```
 29011
 28965
   946
```

The answer is 946 miles.

Inv: How did you get it?

Julia: Well - - - - I went by the question - - - - The question was - - - - - how many miles did he drive? - - - - Wait a minute - - - - - - that isn't right - - - - - - - - - -

Inv: Want to work it again?

Julia: Yes! - - - - - - - - - (vocalizes as she adds)

```
 29011
 28965
 57976
```

How I got it - - - - - - uh - - - - - - I added 28965 and 29011.

Inv: Why?

Julia: The reason I did it - - - - - - I went by the question - - - - how many miles did he drive by the trip - - - - - - the first time I - - - - - - I'd better read it again - - - - - - - (reads)

- - - - - - - - well - - - - - - - I add - - - - - - 28965 - - - and 29011 - - - - - - - and I got - - - - 57976 - - - - -

Inv: Why?

Julia: Well - - the question said - - how many miles did he drive on one trip - - - - - -

****

Simon: (Reads) A (basket) of apples (contains) about 48 (pounds). A large (truck) has (on it) a load of 35 - 52 pounds of apples. How many (baskets) would that be?

Can't find - - - - - how many - - baskets that would be - -

Inv: Can you start the problem?

Simon: Can't see nuthin' to start - - - - -

As might be expected, poor problem solvers were more often deficient in understanding of the vocabulary than good problem solvers. When, however, good problem solvers experienced vocabulary difficulty their procedures in the solution of the problems were in most respects similar to those of the poor problem solvers.

Insight Through Social and Economic Information

Sources of information about insight.-- Vocalizations of pupils during problem solving frequently revealed that both good and poor problem solvers used social and economic information which they thought applied to the problem situation. The effects of this practice upon problem solving were not always apparent, for associations might have taken place without audible or written evidence. Such vocalizations, together with the answers to three questions, asked routinely of all pupils after they had solved a problem, provided evidence of the application of social and economic information in problem solving. The questions were:

1. Why did you work the problem that way?

2. Why do you think your answer is reasonable?

3. Why do you think your answer is right?

In the analysis of the data answers to these questions were examined and vocalizations were reviewed for evidence of the use of social and economic information in problem solving. From this analysis it was obvious that three kinds of problem-solving behavior were practiced by good and poor problem solver in associating social and economic background with the solutions of arithmetic problems. Individual solutions have, therefore, been classified as follows: Social solutions characterized by the pupil's identifying himself with the problem situation, directly or vicariously, or in some other manner indicating that he has experienced some aspect of the problem or the social behavior related to it. Mechanical solutions characterized by reference only to mechanical or computational ideas while the problem is being solved and dependence upon that type of reasoning for evaluation and justification of the procedures used. Doubtful solutions characterized by failure to give any sensible reason such as "I don't know," "I just worked it that way," or by obvious or admitted ignorance of their procedures.

Comparison of the frequency of the use of social and economic information by good and poor problem solvers.-- Analysis of individual solutions shows that good problem solvers applied social and economic information to their solutions about twice as often as poor problem solvers. It was somewhat surprising, however, that they gave mechanical justification for their solutions slightly oftener. Poor problem solvers gave doubtful interpretations of their solutions fifteen times as often as the poor problem solvers. The basis for these statements has been obtained from Table 5.

Good problem solvers depended upon social and economic information more frequently than poor problem solvers. In other words, good problem solvers, more often than poor problem solvers associated their own experiences with the solutions to their problems. An example of this can be seen in the work of three good problem solvers, Robert C., who reassured himself of the accuracy of his solution by citing his own experiences with money, Jo T., who indicated clear understanding of the meaning of a load of apples, and Phyllis G., who became involved in interpreting her answer to Problem 7 because she thought the thermometer might be rising too rapidly.

Robert: (Reads) Jean had $3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much did she have left?

I can tell right away - - - - she had $2.95 (two ninety-five) left - - - - - but if you want me to tell you how I got it - - - - - - - - -

Inv: Put it on your card. How did you work it?

Robert: Well - - - - - you add 35¢ and 20¢ together - - - - and subtract from $3.50 and get $2.95.

Inv: Why?

Robert: That's the way to work it.

Inv: Is it reasonable?

Robert: I spend almost that much every time I go to town.

Inv: Is it right?

Robert: Yeh - - - I could see that right away.

TABLE 5

THE NUMBER OF SOCIAL, MECHANICAL, AND DOUBTFUL SOLUTIONS BY NINETY-SIX SIXTH GRADE PUPILS, CLASSIFIED GOOD AND POOR PROBLEM SOLVERS, IN WORKING EIGHT SELECTED PROBLEMS IN ARITHMETIC, BASED UPON THE VOCALIZATIONS OF THE PUPILS AND UPON THEIR ANSWERS TO THREE QUESTIONS: WHY DID YOU WORK THE PROBLEM THAT WAY? WHY DO YOU THINK THE ANSWER IS REASONABLE? AND WHY DO YOU THINK THE ANSWER IS RIGHT?

Classification of the Pupils	Number of Social Solutions	Number of Mechanical Solutions	Number of Doubtful Solutions
Good problem Solvers	130	251	7
Poor problem Solvers	64	122	106

****

Jo: (Reads) A basket to apples weighs about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be?

Well - - - divide 3552 - - by 48 - - - - to see how many baskets that will be - - - - (vocalizes her long division) - - - that would be 74 baskets on the large truck - - -

Inv: How did you get it?

Jo: Well - - - I found out how many pounds - - - how many 48's will go into 3552 - - - - because each - - - - contains 48 pounds of apples - - - and after I found out - - - - it was 74 - - - and that was how many baskets a large truck will hold - - -

Inv: Is it reasonable?

Jo: Yes, I think so - -

Inv: Why?

Jo: Well - - - - - - if there were 3552 pounds - - - - and you divide by 48 - - - - - it would come out 60 or 70. A large truck could hold 74 baskets - -

Inv: Is it right?

Jo: Yes.

Inv: Why?

Jo: I just think it is a reasonable answer.

****

Phyllis: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

Well - - - - 7 degrees would be below zero - - - - 29 degrees would be above - - - - - - zero - - - - - - You would add - - - - - to 29 - - - - you would add - - - - - - 7 - - - - - - - 36 degrees.

Inv: How did you work it?

Phyllis: Well - - - - I added how many degrees below it was - - - - and how many degrees it became above - - - - - and I got how many degrees it became warmer.

Inv: Is it reasonable?

Phyllis: I don't know whether it could become that much warmer - - -

Inv: Is it right?

Phyllis: I'm not sure - - - - - - -

Inv: Do you know any other way to work it?

Phyllis: No.

The frequency of social solutions to problems by poor problem solvers was not so great as by good problem solvers. But examples of such solutions were apparent, and in such cases resembled the work of the good problem solvers. Julia P., a poor problem solver, showed excellent social insight into Problem 6 which she worked efficiently and correctly. On the other hand, Connie D. also a poor problem solver, became so concerned over the social and economic implications of her answer that she lost her confidence in the answer to Problem 6.

Julia: (Reads) Jean had $3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much did she have left?

(Vocalizes) - - - - - - my answer is $2.95 (two ninety-five)

The way I did it is - - in the problem it says - - - 35¢ for a movie - - and 20¢ for a soda - - - I added - - - 55¢ - - and then I subtracted from three-fifty - - - after I subtracted - - I had two ninety-five.

Inv: Is it reasonable?

Julia: Yessir.

Inv: Why?

Julia: Cause - - well - - If I was Jane - - I'd do the same thing - - because I'd want to find out how much I'd spent for the movie and soda - - then I'd want to find out how much I had left - - - - - so I'd subtract - - -

Inv: Is it right?

Julia: Yes.

Inv: Why?

Julia: Then if I wanted to check it - - - I'd just - - - - I'd add 55¢ to two ninety-five - - (mumbles) oh - - oh - - three-fifty.

****

Connie: (Reads) Jean had $3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much did she have left?

No - - - she couldn't - - - - - - - um - - - - (whispers) - - - - - (mumbles) - - - - - - - - (long wait) - - - - - - - - - 2.95 (two ninety-five).

Inv: How did you get it?

Connie: She had $3.50 in her purse - - - - - and she took 55¢ - - - - - 55 - - - - - - - - and she had - - - - - - - $2.95 - - - - - - - That's still wrong - - - - - - - - If she had three-fifty - - - - and spent 55 - - - - - - she couldn't possibly have two ninety-five left - - - - - could she? - - - - - She might - - - - - - - why if she had three-fifty - - - and she up and spent 55¢ - - - - - she couldn't possibly have two ninety-five left.

Inv: You don't think it is right?

Connie: No sir!

Examples of social thinking in which the wrong method of solution was adopted were discovered among good as well as poor problem solvers. Although the problem was incorrectly worked, the pupil continued to justify the solution through social and economic experiences. For example, John G., a good problem solver, worked Problem 7 incorrectly, but he tried to explain his solution by reference to his knowledge of a thermometer. In another instance, Roland H., a poor problem solver from an impoverished home, unconsciously revealed economic pressures when he commented, "She must not have had the money to buy six bars."

John: Cause I wanted to find out how many degrees warmer and I thought it would be easier to subtract.

Inv: Is it reasonable?

John: Yes - - - - - but - - - it would have to warm up pretty fast - - - - or the sun would have to shine - - - - - - cause 22 degrees - - - - - seems like it would take a long time to get up that high.

Inv: Is it right?

John: Umhum.

Inv: Why?

John: Cause 22 plus 7 is 29.

****

Roland: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

Take 2 into 45 - - - - - (vocalizes) - - - - - $22\frac{1}{2}$ - - -

Inv: How did you get it?

Roland: It wants to know how much 2 bars would cost if it sells 6 for 45¢.

Inv: Why did you work it that way?

Roland: It's all that I could think of.

Inv: Is it reasonable?

Roland: Yessir.

Inv: Why?

Roland: Why, - - - if she wanted to buy 2 bars of soap - - - it was on sale at 45¢ - - - - - she must not have had the money to buy six bars - - -

Inv: Is it right?

Roland: Yes.

Inv: Why?

Roland: If six bars cost 45¢ - - - - then if she bought 2 bars - - - - - - it would be 45¢ divided by 2 - - - - which is $22\frac{1}{2}$¢.

As shown in Table 5, mechanical solutions were somewhat more frequent among good problem solvers than among poor problem solvers. These solutions are characterized by adherence to mechanical or computational reasons for the justification of their problem solving behavior. Lois S. and Jack R., both good problem solvers, worked Problem 1 and Problem 8, respectively, with only token reference to concrete ideas or social and economic experiences. Their solutions were correct, but they were based entirely upon mechanical thinking.

Lois: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far will Bill swim swimming twice the length of the pool?

Well - - - I would multiply 75 times 30 - - - no - - - I would take thirty times 3 - - - - that would be 60 feet - - my answer is 60 feet wide - - - - Oh - - - - I don't think that's right - - - -

Inv: Do you know how to work it?

Lois: The answer is 150 feet - - - It's 75 feet long - - - if you'd swim it twice - - - it would be 75 times 2 equals 150 feet.

Inv: Is it reasonable?

Lois: Yes.

Inv: Why?

Lois: Why - - uh - - (laughs) 2 times 75 is 150 - - that's all I can figure out - - - -

Inv: Is it right?

Lois: Yes

Inv: Why?

Lois: Cause 2 times 75 - - - I think I multiplied it right - - -

****

Jack: (Reads) Mary's mother wants to put linoleum on her kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

Multiply 15 x 9 - - - - - (vocalizes his multiplication) is 135 - - - - - There are 9 square feet in a square yard - - - - - so I'd divide 9 into 135 - - - - -

Inv: What is your answer?

Jack: 15 square yards.

Inv: Why did you work it that way?

Jack: That's the way to work it.

Inv: Is it reasonable?

Jack: Yes.

Inv: Why?

Jack: I think that's the way to get it.

Inv: Is it right?

Jack: Yes.

Inv: Why?

Jack: That's the way we learned to work these problems.

Mechanical solutions, likewise, were employed by poor problem solvers. Herbert A., a poor problem solver, solved Problem 1 incorrectly, found his mistake and then worked it correctly. At no time was there any evidence that his solution was based upon social or economic information. As far as could be observed his thinking was purely mechanical, and the reasons given for his procedures and evaluations of those procedures were also mechanical in nature. This is again illustrated in the work of John S., a poor problem solver who used only mechanical thinking in the spoken description of the work he did in solving Problem 4.

Herbert: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool?

110 feet.

Inv: Work it with your pencil.

Herbert: (Works problem)

Inv: How did you get it?

Herbert: I added 75 and 30 - - - - -

Inv: Why?

Herbert: I had to add - - - -

Inv: Why?

Herbert: (No answer)

Inv: Is it reasonable?

Herbert: No sir - - -

Inv: Do you know another way?

Herbert: Yessir, - - - - - - 150 feet - - - -

Inv: Why did you - - - -

Herbert: Cause it said twice the length - - - -

Inv: Is it reasonable?

Herbert: Yessir.

Inv: Why?

Herbert: Cause it said twice the length - - -

Inv: Is it right?

Herbert: (No answer)

****

John: (Reads) Tom's father drives a city bus. Before starting on a route, the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive?

(Pause) - - - - - - He'd drive 46 miles.

Inv: How did you get it?

John: I subtracted.

Inv: Why?

John: To find out how many miles he drove.

Inv: Is it reasonable?

John: Yessir.

Inv: Why?

John: Well - - - - - - - (no answer)

Inv: Is it right?

John: Yessir.

Inv: Why?

John: I think I subtracted correctly.

Sometimes it was impossible to tell what was in the mind of the pupils as they discussed their solutions to the problems. Although the ratio of such incidence was almost ten to one with low achievers using doubtful reasons more often, a good problem solver showed in some cases the same kind of ambiguous reasoning as that characteristic of many poor problem solvers. In his solution to Problem 8, Billy A., a good problem solver, floundered about in an indefinite method of solution, then was unable to give any logical reason for an answer that he said was right. Herbert A., poor problem solver, seemed incapable of taking the first constructive step toward solving Problem 5, and then defended his solution by vague, indefinite reasoning.

Billy: (Reads) Mary's mother wants to put linoleum on her kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

15 is - - - - - - - - - - - uh - - - - (long pause) - - -
uh - - - well - - - there is 9 square feet in 1 square yard
- - - - - - - - and so - - - - - - - - 9 feet that would be 1
square yard - - - - - - 9 feet - - - - that would be 1 - -
square yard - - - - - - - 9 feet - - - that would be 1 - -
square yard - - - - - - - 9 feet and 15 feet would be 1 foot
and 6 left over - - - - would be 2/3 of a square yard - -
would be 6 square feet - - - - - - - that would be 2 square
yards and 6 feet.

Inv: Put that down on your card - - How did you work it?

Billy: Well - - - - - - - 1 square yard is 9 feet - - - - - - and
so 9 in the question there is - - - - - - - 9 feet - - - so
there is 1 yard - - - - - - and there is 15 - - - - - - and 9
is going into 15 - - - - - - 1 - - - - and 6 over and so that
would be 2 square yards, and 1 foot - - -

Inv: Why did you work it that way?

Billy: I hadn't thought of any other way.

Inv: Is it reasonable?

Billy: Yes.

Inv: Why does it sound reasonable?

Billy: It just does.

Inv: Is it right?

Billy: Yes.

Inv: Why?

Billy: I don't know.

****

Herbert: (Reads) A grocery had a special sale on soap at 6 bars for
45¢. At that rate, how much would Jane pay for 2 bars.?

(mumbles - - not audible) Take 18 - - - - how many - - -
in 45 - - - - - - - - would be 9 - - - - - 2 x 9 is 18 - - - -

Inv: Is it a reasonable answer?

Herbert: Yessir.

Inv: Why?

Herbert: Cause I think that is the way to do it.

Inv: Is it right?

Herbert: Yessir.

Inv: Why?

Herbert: Cause I think that's the way to do it.

Lois H., a poor problem solver, obtained a correct answer to Problem 7, but at no time did her answers to the questions indicate the slightest understanding, either social or mechanical, of what she had done. It might be assumed that a fortunate guess gave her the correct method of computation. There was nothing in her vocalizations that explains how she might have arrived at a correct solution other than by chance.

Lois: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How much warmer had it become?

(Pause) - - - - - - - add - - - - - - - 29 plus 7 - - - - 9 and 7 is 16 - - - - - - 2 and 1 is 3 - - - - - - - That's 36 - - - - - - - -

Inv: How did you get it?

Lois: I added.

Inv: Why?

Lois: I don't know.

Inv: Have you no idea why?

Lois: (Pause) - - - - - I wanted to find - - - - how - - - - - much more - - - - - -

Inv: Is your answer reasonable?

Lois: Yes.

Inv: Why?

Lois: Why - - - - - - hm - m - - - - - I don't know - - - - - - -

Inv: Is the answer right?

Lois: I don't know.

Inv: Might it be?

Lois: It might be.

Inv: Is there a better chance that it is right than wrong?

Lois: Yes.

Gary S., a good problem solver, solved Problem 5 correctly. At no time was he sure of his answer or the method employed. He felt no security in his answer and was unable to give any logical answers to the questions asked him.

Gary: (Reads) A grocery has a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

Well - - - - - - - - - I'd divide - - - 45 by 2 - - - and that would give me - - - - (vocalizes) 22½¢ for two bars of soap.

Inv: How did you work it?

Gary: Well - - - I think - - - (seems doubtful) - - if 6 bars cost 45¢ - - - 2 bars would cost - - - 2 into 45¢ - - - or 22½¢ -

Inv: Is your answer reasonable?

Gary: I'm not sure - - - -

Inv: Want to try again?

Gary: (long wait) Well 45 - - - is - - - - 45 divided by 6 - - - is 7½ - - - - 7½ plus 7½ - - - - is 15¢ (doubtful) - - -

Inv: Is your answer reasonable?

Gary: I'm not sure - - - - -

6 divided by 2 is 3 (doesn't know what to do with the 3) (long pause)

Inv: (Finally) Which answer do you think is right?

Gary: I think it would be 15¢

Inv: Why?

Gary: I'm not sure - - - - -

Shirley C. was completely lost in her solution to Problem 4. She used three mechanisms: incoherence, evasiveness, and silence in response to questions about her procedure.

Shirley: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read - - - - - 2 million - - - - - - 2 thous - - - - - - 8 - - - 8 - - - (long pause).

Inv: Go ahead if you can't read it.

Shirley: (Continues) At the finish of the trip it read - - - - - 29 thousand 11. How many miles did he drive on the trip?

Inv: Go ahead.

Shirley: (Proceeds to add the two numbers)

```
 29965
 29011
 58976
```

The answer is - - - 5 - - - million - - - - 89 - - - - oh - - 5 - - thousand - - - 8 - - - hundred - - - uh - - - - 5 - - - - million - - - - 89 - - - - thousand - - - - - - 76.

Inv: How did you get it?

Shirley: I subtracted.

Inv: Why?

Shirley: No - - - - I added - - -

Inv: Is it reasonable?

Shirley: Yessir.

Inv: Why?

Shirley: Because - - - - uh - - - - because - - - - you can't work those two - - - - it won't come out - - - - - - - you can't work it - - - - -

Inv: Is it right?

Shirley: I read it wrong - - - - - - - I can't get those fractions - - - - - - - - - - - - - - - - (puts it down and adds again)

$$\begin{array}{r} 29011 \\ \underline{28965} \\ 58976 \end{array}$$

The reason I wrote it upside down was to see if I'd get the same answer - - -

Inv: Is it right?

Shirley: Yes.

Inv: Why?

Shirley: (Long pause - - - - no answer) - - - - -

Association of social and economic information with the problem solution was more likely to occur in the work of good problem solvers than in that of poor problem solvers. Poor problem solvers, on the other hand, were much more likely to attempt the solution to a problem without any idea of relating meanings to the ideas expressed in the problems. Solving the problems by mechanical reasoning, however, was about as frequent among good as among poor problem solvers. Individual efforts of good problem solvers resembled that of poor problem solvers in examples of the work of pupils from each group.

Insight through Labeling of Answers

Further evidence of the insight a pupil had gained through his experiences with the problems was found in the frequency of correct labeling of the answers. The answer to the problems used in the study were labeled in two ways: The pupil might write the name of the answer on the card containing the written solution or might say the name of the answer aloud when he read or announced his results. Sometimes a pupil labeled only his written answer; sometimes he labeled only the oral answer; on some occasions he labeled both and on some, he labeled neither the written or spoken answer. Table 6 shows the frequency of written and oral labeling by good and poor problem solvers in working the eight arithmetic problems used in the study. Good problem solvers labeled their answers both orally and written, about twice as often as poor problem solvers. Poor problem solvers had about twice as many incorrect labels as good problem solvers.

Labeling of answers was twice as frequent among good problem solvers as among poor problem solvers. The work of June C., Irene T., and Richard C., all good problem solvers, shows the significance of labeling to these pupils. To them it was a necessary and important part of the problem-solving procedure.

June: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read 28965. At the finish of the trip it read 29011. How far did he drive on one trip?

Put down 29011 and take from it 28965 - - - - - - - 46 miles - - -

```
 29011
 28965
    46 miles
```

****

Irene: I'd multiply - - - - (1345 x 15) - - - - 20,175. - - - um - - - - this was supposed to be dollars and cents - - -

Inv: What is your answer?

Irene: $201.75.

****

Richard: (Reads) In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay?

TABLE 6

THE NUMBER OF CASES OF CORRECT ORAL AND WRITTEN LABELING OF ANSWERS BY GOOD AND POOR PROBLEM SOLVERS, IN WORKING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification of Pupils	Correct Oral Labels	Correct Written Labels	Incorrect Oral and Written Labels
Good Problem Solvers	300	203	13
Poor Problem Solvers	153	87	24

That'd be - - - - - - 15 x 1345 - - - - - - (vocalizes his multiplication) - - - - - - - - 20 - - uh - - thous - - - uh - - - (changes answer) - - - - - $201.75.

That'd be 15 cents - - - a child would pay. You'd want to find out how much each child would pay - - - - - - - You'd multiply 15 cents times 1345 to find out how much they all pay - - - -

Although the poor problem solvers were only about half as likely to label their answers correctly, correct labeling did occur. The work of Tom M. and Carl J., both poor problem solvers, was typical of that of other pupils in their group. These pupils recognized that the answer had to have a name if it were to be reasonable.

Tom: I multiplied 15¢ times 1345.

Inv: Why did you do that?

Tom: I wanted to find out the answer.

Inv: Is it reasonable?

Tom: Huh - Uh - - - - - I made a mistake - - I don't have no places - I don't have no decimal points - - - you'd have $201.75.

****

Carl: (Vocalizes his multiplication) I multiplied - - - - twenty thousand, one hundred, seventy-five - - - I multiplied - - - there's 1345 children at 15¢ each - - -

Inv: Is your answer reasonable?

Carl: Uh-huh - - - -

Inv: Why?

Carl: Um - - it should be dollars - - - now - - that's $201.75.

In some cases there was evidence among high achievers that abstract methods were followed, answers announced, and no steps taken to translate the abstractions into concrete answers. Ronnie G., a good problem solver, labeled only the answers that were concerned with money. All others were abstractly given, but the interviewer believed that understanding was such that any of them could have been translated without difficulty if there had been pressure to do so. His work in all eight problems provides the basis for this conclusion.

Problem 1

Ronnie: I figured - - be twice the length - - - be 150.

Problem 2

Ronnie: I figured 1345 times 15¢ - - - - be $201.75.

Problem 3

Ronnie: The answer is 74.

Inv: How did you get it?

Ronnie: I divided 48 into 3552 and got 74.

Problem 4

Ronnie: (Puts down problem - quite aware of the zeros)

29011
28965
ØØ46

I don't need those - - - (zeros in front)
The answer is 46

Inv: How did you get it?

Ronnie: I subtracted 28965 from 29011 - - - - and got 46.

Problem 5

Ronnie: The answer is 15¢

Inv: How did you get it?

Ronnie: Well I figured 2 bars is 1/3 of six bars - - - - - I divided 3 into 45 - - - -

Problem 6

Ronnie: The answer is $2.95.

Inv: How did you get it?

Ronnie: Well - - I added 35¢ and 20¢ and got 55¢ - - - then I subtracted 55 - - from $3.50 - - - -

Problem 7

Ronnie: I think the answer is 36.

Inv: How did you get it?

Ronnie: Well - - I figured - - - I added - - - 29 and 7 and got 36 - -

Inv: Why did you do that?

Ronnie: Well I figured it would be 7 degrees up to - - - zero - - and then 29 more would be 36.

Problem 8

Ronnie: (Was unable to work it the first three attempts. Finally he located the source of his trouble.)

I think it is 15 - - - because there's 9 square feet in 1 square yard.

Solutions by George W., a poor problem solver who omitted all labeling, reflect complete lack of understanding. His eight solutions were all wrong except that to Problem 6, and he was unable to read that answer correctly.

Problem 1

George: You add - - - - 75 plus 30 - - - - is 105. (wrong process)

Problem 2

George: You multiply - - - - 13 x 13-45 - - - - and get - - - - - 20 thousand - - 1 hundred and 75. (figures correct)

Problem 3

George: You divide - - - - - 48 into - - - thir - - - thir - - - 35 - - 52 - - and - - - (all mixed up in his computation) - - - - - the answer is 51 - r 5.

Problem 4

George: Answer is 9 million 9 thousand 51.

```
 28965
 29011
  9954
```

Inv: How did you get it?

George: I subtract the lowest from the highest.

Problem 5

(Answer 15¢)

George: (Multiplies 2 x 45) 2 times 5 is 10 - - - 2 times 4 is 8 - - and 1 is 9 - - - - - - The answer is 90.

Problem 6

(Answer $2.95)

George: I'd subtract - - - - - The answer is 2 hundred 95. (2.95)

Problem 7

(Answer 36 degrees)

George: I - - - - - I - - - - - - I - - - - - - - I - - - - - I'd addem. - - - - - - - (long pause)a a a - - - No, I'd subtract - - - - - - - my 7 from 29 - - - - - bring down my 2 - - - - - 22.

Problem 8

(Answer 15 sq. yds.)

George: She needs - - - - she needs - - - - - she needs - - - - she needs - - - - - about - - - - - 1 square - - - - - -

Inv: How did you work it?

George: I've got it right - - - - 15 is 1 - - - - floor is 15 ft. long - - - - - - and 9 ft wide - - - - - - - - - - Yessir. she needs 1 square - - - - - 1 - - - - - - -

Carol D. and Nancy S., both good problem solvers, failed to visualize the quantitative value of the answer to Problem 2, left it unlabeled and omitted the decimal point.

Carol: (Vocalizes her multiplication) 1345 times 15 is - - - two thousand, one hundred seventy-five - - - no - - - wait a minute - - - twenty thousand, one hundred, seventy-five - -

****

Nancy: (Vocalizes her multiplication) 1345 times 15 - - answer 20,175.

Inv: How did you get it?

Nancy: I multiplied.

Inv: Why?

Nancy: Cause there's 1345 children and each child paid 15¢ - - you'd have to find out how much they paid in all - - - -

Inv: Is it reasonable?

Nancy: Yes.

Inv: Why?

Nancy: Well - - - 1345 is a large number - - - and it comes out 15 times it and I think it would be - - - -

The work of Robert D. and Carolyn S., both poor problem solvers, resembles that of the two pupils just shown.

Robert: I'd multiply - - - (vocalizes his multiplication) twenty thousand, one hundred, seventy-five.

Inv: How did you get it?

Robert: I multiplied.

Inv: Why?

Robert: Cause 1345 children wanted to go to the zoo and it cost 15¢ each.

Inv: Is it reasonable?

Robert: Yessir.

Inv: Why?

Robert: Cause 1345 children would cost a lot of money to go to the zoo.

****

Carolyn: I'd multiply - - - - - (pause - no vocalization) - - - answer - - - twenty - thousand - - one hundred - - seventy-five.

Inv: How did you get it?

Carolyn: I multiplied - - -

Inv: Why did you do that?

Carolyn: Cause I wanted to find out how much they all paid - - -

Labeling of answers occurred in two ways. Pupils labeled in writing the answers to their written solutions and they spoke the name of the answer in announcing or discussing their results. Good problem solvers were more likely to label correctly than poor problem solvers. However, the illustrations show frequent cases of unlabeled answers among good problem solvers and frequent instances of proper labeling among poor problem solvers.

Summary

Insight has been defined in this chapter as the degree to which a pupil is able to recognize the significant aspects of a problem, to associate meaning and understanding with those aspects, to apply logical computation to the solution of a problem, and to evaluate the final results of his solution in terms of his own experiences.

Four patterns of behavior indicative of insight into the problems emerge as the result of the analysis of the data: the general evaluation made by the pupil of the entire problem situation, the utilization of vocabulary clues in problem solving, the use of social and economic information to develop insight into a problem, and the extent of labeling of answers as an indication of insight into the problem.

Good problem solvers, in their general evaluation of the problems, showed clear insight about four times as often, vague insight somewhat less frequently, and poor problem solvers showed doubtful insight six times as often as good problem solvers. Clear insight into a problem occurred when a pupil read the problem effectively, attacked the problem directly using the information gained through the reading, showed proper judgment of the problem situation, demonstrated a critical evaluation of the answer and reasonable confidence in the appropriateness of the answer. Vague insight into a problem was characterized by faulty reading, obscured number relations, imperfect or hesitant selection of methods of attack, inadequate notions of the meaning of the answer, and lack of confidence in the appropriateness of the answer. Doubtful insight was characterized by the inability to begin a solution, choice of method for no apparent reason other than a random effort to compute, fragmentary understanding but complete bewilderment as to how to apply those fragments, and incoherent, meaningless attempts to explain the solution.

Vocabulary was a factor in the solutions of the problems and in the insight experienced by both good and poor problem solvers. Illustrations from the work of both good and poor problem solvers show the importance of word and number clues to the pupils from each group.

In applying social and economic information to problem solving, good problem solvers applied social information to their problem situations twice as often and mechanical or computational reasoning to the same extent. Poor problem solvers showed doubtful thinking fifteen times as often as good problem solvers. Social solutions to problems are those in which the pupil identified himself with the problem situation, directly or vicariously, or in some other manner indicated that he had experienced some aspect of the problem or of the behavior related to it. Mechanical solutions to the problems were those in which pupils referred only to mechanical or computational ideas while the problem was being solved, and depended upon that type of reasoning for evaluation and justification of the methods or procedure used. Doubtful solutions to the problems were those in which pupils failed to give any sensible reason, such as "I don't know", "I just worked it that way", some incoherent reason, unintelligible reply, or silence in answer to the questions.

Good problem solvers labeled their answers in the written solutions and in their vocalizations twice as often as poor problem solvers. Labeling was done by placing the name of the answer on the written solution, or by saying aloud the name of the answer when the results were announced after completion of the problem. Illustrations given in support of the data show that in many cases labeling was an essential part of the problem solution. Other cases revealed that abstract solutions, completely without labeling, did occur.

In general, good problem solvers were more proficient in all aspects related to insight into a problem situation. They more often were able to see clearly the meanings of the problems, they used vocabulary more expertly, they applied social thinking more frequently, and labeled their answers more often. They were not as likely to show doubtful insight, to reflect vocabulary failures, to become bewildered over social and economic implications of problems or fail to label their answers correctly.

The group data in the chapter show in relation to insight that good problem solvers are more likely to show greater clarity of insight than poor problem solvers. On the other hand, individual examples clearly indicate that when both good and poor problem solvers are successful, they employ similar procedures. Likewise, when they are unsuccessful, the good problem solvers employ procedures like those of the poor problem solvers.

CHAPTER V

THOUGHT PROCESSES AND THEIR RELATION TO ARITHMETIC PROBLEM SOLVING

The Purpose of the Chapter

The importance of insight into the situation presented by an arithmetic problem was discussed in Chapter IV. It was shown that pupils achieve varying degrees of insight into the meanings of problems. Regardless of the degree of insight, however, the pupil must select a method, apply that method and evaluate the results of his efforts. The thought processes related to these activities are many and diverse. For the purposes of this study they are divided into three main groups: the thought processes involved in the choice of a method, those connected with the use of that method, and the thought processes related to the evaluation of the results of the use of the method.

It is the purpose of this chapter to present the data that show the frequency with which good and poor problem solvers employed the various kinds of thought processes in selecting, applying and evaluating methods. Data related to choice of a method are discussed in the first section of the chapter, those concerning the application of the method in the second, while data related to the appraisal and evaluation of the results of the method are presented in the third section.

Thought Processes of Pupils Used in Choosing Methods of Solving Problems

The data, as shown in Table 7, indicate that good problem solvers chose correct methods more than twice as often as poor problem solvers. Those data were derived by careful scrutiny of each problem, after which the method used in working the problem was classified

TABLE 7

THE NUMBER OF CASES IN WHICH GOOD AND POOR PROBLEM SOLVERS, CHOSE RIGHT AND WRONG METHODS FOR SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Pupils	Number of Pupils	Right Methods	Wrong Methods
Good Problem Solvers	48	384	297	87
Poor Problem Solvers	48	384	145	239

right or wrong. When a pupil chose a method that would produce a correct answer to the problem if all computation and interpretation of results were accurately done that method was considered a correct method. All others were designated as incorrect methods.

The frequency of social, mechanical, and doubtful reasons for choice of a method of solution is shown in Table 8. Here it is shown that good problem solvers gave social reasons twice as often as poor problem solvers, and mechanical reasons about the same number of times. Poor problem solvers, however, gave doubtful reasons eighteen times as often as good problem solvers. A social reason was one in which pupils referred to some social or economic information, or in which he associated some social behavior of his own with his choice of method. Mechanical reasons were those in which the pupil referred only to the manipulative or computational procedures used in the problem. Doubtful reasons were unintelligible, incoherent, unrelated to the procedures, or based upon obvious or admitted ignorance of the reason for the choice. All decisions as to the kind of reason given were based on vocalizations of the pupil and the answer to the third of seven questions asked the pupil following each solution. This question as shown on page 39 was "Why did you work the problem that way?"

Table 8 shows that social reasons were given three times as often by good problem solvers as by poor problem solvers for the choice of method for solving the problems. The solutions of Jo T. and Robert G., both good problem solvers illustrate this kind of thinking. Both pupils seemed to be able to visualize the social implications of the problem and to disucss the meanings of the numbers as they made their choice of a method.

TABLE 8

THE NUMBER OF SOCIAL, MECHANICAL, AND DOUBTFUL REASONS GIVEN BY GOOD AND POOR PROBLEM SOLVERS, FOR THEIR CHOICE OF METHOD IN WORKING EIGHT SELECTED ARITHMETIC PROBLEMS

Classification	Number of Pupils	Number of Problems	Number of Social Reasons	Number of Mechanical Reasons	Number of Doubtful Reasons
Good Problem Solver	48	348	105	254	6
Poor Problem Solver	48	348	39	204	105

Jo: (Reads) Jean had \$3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left?

I will add 35¢ and 20¢ to see how much she will spend in all - - - That makes 55¢ - - - - Then I subtract 55¢ from \$3.50 and find out - - - get - - - \$2.95 - - -

Inv: Why did you do that?

Jo: First I found out how much she had with her - - - then I found out how much she spent - - - then I subtracted - - -

****

Robert: (Reads) In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay?

Multiply 15 x 1345. - - - - (vocalization correct) \$201.75.

Inv: How did you get it?

Robert: Multiplied 15 x 1345.

Inv: Why?

Robert: Because there's 15¢ to get in to see the animals and there's 1345 children. The question is to see how much they all paid - - - - - - \$201.75 - - -

Inv: Is the answer reasonable?

Robert: Yeh.

Inv: Why?

Robert: Well, - - - because there's about 1345 children that goes to this school (his own) here, and we have to pay 10¢ apiece - - - - - - if they paid 15¢ apiece - - it'd be reasonable.

Although it was only one-third as frequent, poor problem solvers sometimes gave clear social reasons for their choice of method. Examples of this kind of thinking may be seen in the work of Pat L. and Lois H., both poor problem solvers. They chose the correct methods and solved the problems correctly. Their reasons for doing this apparently were based principally upon some previous experiences of their own.

Pat: I'd multiply 1345 x 15¢ - - - - - I'd do that because 1345 is how many children is there - - - - - and 15¢ is how much they spend - - - - - (vocalizes her multiplication) draw a line and add - - - - - They'd - - - $201.75 - - - - -

Inv: How did you get it?

Pat: I multiplied it because there's 1345 children and they spent 15¢ there - - - - - and I got my answer - - - -

Inv: Is it reasonable?

Pat: Yes.

Inv: Why?

Pat: So many people - - - - so many children - - - would cost that much - - - -

****

Lois: (Reads) Jean had $3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left?

Jean had $3.50 - - - - you add 35 and 20 - - that is 55¢ - - - and you subtract 55¢ from $3.50 - - - - (vocalizes the subtraction) - - - the answer is $2.95. - - -

Inv: How did you get it?

Lois: I added and - - and - - multiplied - - I added and subtracted -

Inv: Why did you do that?

Lois: Why she bought two different things at two different prices. She would have to see how much that was and then she would have to subtract from the other - - - -

Mechanical reasons were given by good problem solvers about as often as by poor problem solvers. Both good and poor problem solvers depended upon their computational processes in much the same way for justification of the methods chosen. Irene K., a good problem solver and Shirley C., a poor problem solver, both referred to the computational processes they had used when they were asked to explain their methods.

Irene: (Reads) In one day 1345 children visited the zoo. They each paid 15¢ to get in to see the animals. How much in all did they pay?

1345 children - - - - paid 15¢ - - - - - (vocalizes her multiplication) two - o - one - seventy - five - - - - altogether -- -

Inv: How did you get it?

Irene: Multiplied 15¢ and 1345 and got two - o - one - seventy - five - - - -

Inv: Why?

Irene: Because it gives me the answer if I multiply 15¢ x 1345.

Inv: Does it sound reasonable?

Irene: Umhum.

Inv: Why?

Irene: It just does.

Inv: Is it right?

Irene: Umhum.

Inv: Why?

Irene: 15¢ x 1345 equals $201.75.

****

Shirley: (Reads) In one day 1345 children visited the zoo. They each paid 15¢ to get in to see the animals. How much in all did they pay?

I'm dividing - - - that's wrong - - - - I'll multiply - - - -

Inv: What is your answer?

Shirley: $201.75 - - -

Inv: How did you get it?

Shirley: I multiplied

Inv: Why?

Shirley: Cause it asked how much in all did they pay? Um - - (places decimal) $201.75.

Inv: Is it reasonable?

Shirley: Yessir.

Inv: Why?

Shirley: (no answer)

Inv: Is it right?

Shirley: Yessir.

Inv: Why?

Shirley: Cause - - I - - added that - - - - - cause - I - multiplied first - - and then added - - -

Problem 2 was solved incorrectly by both Wally H., a good problem solver, and by Donald H., a poor problem solver. Their reason for selecting their methods was the same, a reference to their mechanical processes.

Wally: I think I'll multiply - - - 15 times 1345 - - - (stumbles a little as he vocalizes his multiplication)

Inv: What is your answer?

Wally: 20175

Inv: How did you get it?

Wally: Multiply 15¢ each times 1345 - - -

Inv: Why?

Wally: To find how much all of 'em cost - - - -

Inv: Is it reasonable?

Wally: Yes - -

Inv: Why?

Wally: I just think it is - - - -

Inv: Is it right?

Wally: Yes - -

Inv: Why?

Wally: Cause that's the only way I could work it - -

****

Donald: Multiply 1345 times 15¢ - - - - (vocalizes)

Inv: What is your answer?

Donald: 20,175 - - -

Inv: How did you get it?

Donald: If there were 1345 children - - - and they each paid 15¢ - - - - - I'd have to multiply - - - - -

Inv: Is it reasonable?

Donald: Yes.

Inv: Why?

Donald: Well - - - the figures - - - - where I multiply - - - - they come out right - - - - -

Inv: Is it right?

Donald: Yes.

Inv: Why?

Donald: Well - - - if they each spent 15¢ - - - - - and there were 1345 children - - - - I multiplied and got that - - -

Table 8 shows that good problem solvers were much less likely to give a doubtful reason when asked why they solved the problem as they did. They did, however, sometimes give reasons that are hard to evaluate in terms of any significant reason on the part of the pupil. The solution of Ellen H., a good problem solver is an example of this kind of thinking.

Ellen: (Reads) Mary's mother wants to put linoleum on the kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

Well - - - - - add 15 and 9 together - - - - that would be 24 feet - - - 27 square feet in a square yard - - - divide 24 by 27 - - - but - - - - - - (long pause) I'm stuck - - -

Inv: How did you work it?

Ellen: I said 15 feet plus 9 feet - - - - are 24 feet - - - -

Inv: Are you right so far?

Ellen: I'm not so sure - - -

Inv: Do you know how to work it?

Ellen: I don't think so - - - we work them all the time - - but I don't think I can work it - - - -

As shown in the solution of Thomas J., a poor problem solver, the same type of behavior occurred in the work of poor problem solvers. In this problem Thomas shows no workable plan for solving the problem, has done some random selection, and cannot justify his choice.

Thomas: (Reads) Mary's mother wants to put linoleum on the kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

She needs - - uh - - - (multiplies 15 x 9) Oh - - - I think I got it wrong - - - -

Inv: What should you have done?

Thomas: I should have added - - - - 29 feet - - - - - -

Inv: Why?

Thomas: Because it said - - - if a floor is 15 by 9 feet - - - how many square yards will she need - - - - it'll have to be more than 15 plus 9 - - - so I added - - -

Immediate and delayed choice of methods of solution.-- The time spent by good and poor problem solvers in choosing a method of working a problem varied. Some pupils chose a method immediately, beginning computation immediately after reading the problem and continuing to the announcement of the answer without further consideration of the fitness of the method. Others delayed their choice; re-reading the problem; exhibiting deliberate thinking, false starts and reconsideration; beginning computational procedures and then returning for another evaluation of the method. Table 9 shows the frequency with which good and poor problem solvers chose a method without hesitation or delayed their choice of a method for additional thinking. Good problem solvers were somewhat more prompt than poor problem solvers in choosing a method.

The pupils' own evaluation of their work and the questioning that followed each solution sometimes prompted both good and poor problem solvers to re-evaluate their methods of solution. Table 10 shows the frequency with which good and poor problem solvers reconsidered their solutions to the problems they had worked, as well as the frequency with which both groups accepted their first solution without any change. Poor problem solvers somewhat less frequently reconsidered the solution and the method they had adopted in working the problem. They more often allowed their original choice of method to stand without re-evaluation.

Table 9 indicates that many pupils reached immediate decisions in their choice of a method for working the problems. Others lingered before deciding upon a method. Good and poor problem solvers employed both practices, sometimes in much the same way. Good problem solvers, somewhat more frequently than poor, made prompt decisions in choice of a method. Poor problem solvers were more likely to delay that choice. Actually, when individual pupils were studied, good and poor problem

TABLE 9

THE NUMBER OF CASES OF IMMEDIATE AND DELAYED CHOICE OF A METHOD OF SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC BY GOOD AND POOR PROBLEM SOLVERS

Classification	Number of Pupils	Number of Problems	Immediate Choice of a Method	Delayed Choice of a Method
Good Problem Solvers	48	384	348	36
Poor Problem Solvers	48	384	295	73

TABLE 10

NUMBER OF TIMES GOOD AND POOR PROBLEM SOLVERS RECONSIDERED THEIR METHOD OF SOLUTION IN SOLVING EIGHT SELECTED ARITHMETIC PROBLEMS

Classification	Number of Pupils	Number of Problems	Reconsidered Their Method of Solution	Did Not Reconsider Method of Solution
Good Problem Solvers	48	384	56	328
Poor Problem Solvers	48	384	69	299

solvers selected their methods in much the same manner, as shown in the following examples. In these examples prompt choices of the correct method were made by two good and two poor problem solvers in solving Problem 7.

Jack R.: (good) I'd add 29 plus 7 equals 36 degrees.

****

Richard C.: (good) You'd add 29 plus 7 equals 36 degrees.

****

Jim S.: (poor) It became 36 degrees warmer.

Inv: How did you get it?

Jim: I added 29 and 7.

****

John S.: (poor) 36 degrees - - If it were 7 degrees below - - it would have to go up 7 degrees to get to zero - - and 29 more would be 36.

Sometimes good results were obtained by both good and poor problem solvers by delaying the choice of the method. The following examples show successful solutions to problem 7, when the method of solution was chosen deliberately.

Gary S.: (good) Above or below - - (long pause) - - (mumbles) - - 36 degrees warmer - - (pause) - - had to work from zero to 7 below - goes up to zero - - then on up to 29 - - I added 29 and 7 - -

****

Jani R.: (good) Well - - - (pause) 7 degrees below zero - - (pause) that'd be 7 - - (pause) and 29 - - (pause) that'd be 36 degrees - -

****

Paul D.: (poor) M-mm - - (pause) - 7 degrees below zero - - (pause) is 7 below zero - - and put down 7 - - (pause) and add 29 - (pause) - 9 and 7 are 16 - - and 2 - - and I carried 1 and I get 3 - - - 36 degrees.

Both good and poor problem solvers made the wrong first choice of a method. Table 7 showed that good problem solvers were almost twice as likely to choose correct methods as poor problem solvers. When individual pupils are considered, however, little difference could be seen in the behavior of good and poor problem solvers in cases where these pupils had chosen the incorrect method and had not altered their choice.

Salena T.: (good) You'd have to say 7 from 29 - - 7 from 9 is 2 - - 2 from 0 - - is 2 - - - - 22 degrees - -

****

Helen G.: (good) 29 degrees - - and subtract 7 degrees - - 7 from 9 is 2 - - - - leaves 22 degrees - - -

****

Carl J.: (poor) 29 degrees - - and subtract 7 degrees - - - is 22 degrees -

****

Gertie P. (poor) Answer 22 degrees - - -

Inv: How did you get it?

Gertie: I took 7 from 29 and I got 22.

Sometimes pupils delayed their decisions and still chose the wrong method of working the problems. When this occurred, there was little difference in individual performances of good and of poor problem solvers. The following examples show similar thinking by good and by poor problem solvers.

Marilyn D.: (good) At six a.m. - - (long pause) - I don't think I can - - - - - - - (pause) - - - - - - answer is 22 degrees - - -

****

Jim H.: (good) 7 degrees below zero is - - - - (pause) is 29 degrees - - (pause) well - - - 30 degrees - - if 29 at noon - - and 7 at six a.m. - - - - (pause) so you'd take 7 from 29 - - and get 22 - - - -

****

Tom M.: (poor) Subtract 7 from 29 - - - umm - mm - - can't do that - - - hmm - boy that would make it lower - - ummm - - - Oh - - I found my mistake - - 22 degrees - - now - - uh - - I subtract 20 - - first it was - - - at noon it was - - 29 - - above - - so I'd subtract - -

****

George W.: (poor) I'd - - I - - I - - I - -I - - I'd addem - - No, I'd - - I'd - - - - (long pause) - - - Oh - - - I subtract - - - I subtract my 7 from 29 - - - and bring down my 2 - - is - - 22 - -

Slightly more poor problem solvers than good problem solvers made changes in their method of solution. When the individual cases in which pupils did change their methods are considered, certain similarities can be seen in the reasons for doing so. Jacqueline B., a good problem solver, and Thomas J., a poor problem solver, chose incorrect methods of working Problem 7. Each considered the significance of the answer produced by that method and discarded it in favor of another solution to the problem.

Jacqueline: 22 degrees - - - - -

Inv: How did you work it?

Jacqueline: I subtracted 7 from 29 and got 22 - - - -

Inv: Why?

Jacqueline: Because it was 7 degrees below and at noon it went up - 29 degrees - - - so I subtracted 7 from 29 - - -

Inv: Is it reasonable?

Jacqueline: Yes.

Inv: Why?

Jacqueline: (pause)

Inv: Is it right?

Jacqueline: No, I don't think it is - - - - - I don't think you should subtract 7 from 29 - - -

Inv: What should you do?

Jacqueline: I think you should addem - - - - -

Inv: Why?

Jacqueline: Because it was 7 below - - and it was getting higher - - and warmer - - - and then it was 29 degrees - - and I put them together to see how much higher and warmer it got - - - -

****

Thomas: Uh - - - I multiplied 29 by 7 and got - - - 203 - - -

Inv: Why did you work it that way?

Thomas: I think - - - - - - - it is wrong - - - - I'm gonna - add 29 and 7 - - - -

Inv: What is your answer?

Thomas: 36 degrees.

Inv: How did you get it?

Thomas: I added 29 and 7 - - - - -

Inv: Why?

Thomas: Because it read 7 degrees below zero - - - and at noon it read 29 degrees - - - - in order to get that high you'd have to add up - - - -

Thomas J., a poor problem solver, and Diane N., a good problem solver, took the wrong course in beginning the solution to Problem 1. Reconsideration brought a change in method in both cases. Diane's evaluation is clear and her reasons for the change apparent. Thomas moved from his incorrect method to the correct one without showing any evidence of why he made the change. Yet, one can see that evaluation of the results had taken place and that the first answer had been rejected.

Diane: He had to swim 420 feet if he swam twice the length of the pool - - - there's two sides - - 150 feet - I had to find out why both lengths are - - it's 60 feet - - - and I add both answers together and got 210 - - - and 2 times that is 240 - feet -

Inv: Is it reasonable?

Diane: I don't know - - - I think I did it wrong after all - - I think I did it wrong - - - If he swam twice the length - - - he swam 2 times 75 feet - - - he swam 150 feet - -

Inv: Why?

Diane: Well, first I was thinking something else - - I don't think I read very carefully - - - -

****

Thomas: I'd multiply 75 times 30 - - - - it'd give me how far he swam it - - - - - - 2250 feet - - - -

Inv: Is it reasonable?

Thomas: Uhuh - - -

Inv: Why?

Thomas: Why he swam it twice - - so I'd add 75 and 75 - - -

Inv: What is your answer?

Thomas: 150 feet - - - -

Choice of methods involving "type solutions".-- Table 11 shows the frequency of "type solutions" in the methods chosen by good and poor problem solvers. While these solutions appeared in only one problem out of ten, such a proportion is some indication that they were of some significance. Good and poor problem solvers employed the "type solution" to approximately the same extent. A "type solution" is one in which the problem solvers did one of three things: (1) They adopted the method used in the preceding problem without considering its appropriateness to the present solution, (2) They jumped to conclusions about the objectives of the problem when they recognized verbal or number clues and began computation without verifying their procedures. For example, in Problem 1 (A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?) the extraneous "and thirty feet wide" prompted some to solve for area and others for perimeter. (3) They followed a rule or formula with little more than a mechanical reason for its adoption.

A study of individual cases revealed that good and poor problem solvers made approximately the same errors in respect to "type solutions". The tendency to follow a method established by the preceeding problem may be illustrated in the solutions to Problem 3 by Wally H., a good problem solver, and Alfie C., a poor problem solver. The preceding problem, Problem 2, was solved by multiplying a four-digit number by a two-digit number (1345 by .15). Seemingly, without considering the meaning of

TABLE 11

THE FREQUENCY OF "TYPING" IN CHOOSING A METHOD OF SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC BY GOOD AND POOR PROBLEM SOLVERS

Classification	Number of Pupils	Number of Problems	Typing of Solution Evident	Typing of Solution Not Evident
Good Problem Solvers	48	384	35	349
Poor Problem Solvers	48	384	44	340

Problem 3, both pupils multipled (3552 x 48) instead of dividing and accepted the product without question as their answer.

Wally: I think I'll multiply - - - 48 x 3552 - - - - (vocalizes his multiplication) one hundred sixty-eight-million - - - four hundred ninety-six - - - (168,496) (computation right)

Inv: How did you get it?

Wally: Multiplied 48 times 3 hundred - - - 3 thousand five hundred fifty-two - - -

Inv: Why?

Wally: It's the only way I could - - - -

****

Alfie: (Reads) A basket of apples weighs about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be?

Subtract - - - No - - I think I would multiply - - - (vocalizes her multiplication) - - - - 17 thousand 4 hundred 96 - - (170,496) (computation incorrect)

Inv: Why did you work it that way?

Alfie: They want to find out how many baskets - - - -

Inv: Is it reasonable?

Alfie: Yes.

Inv: Why?

Alfie: 3552 pounds of apples - - - there - - - - 48 pounds in a basket - - and 48 times 3552 - - - gives 17 thousand 4 hundred and 96 - - -

Problem 1 provided an invitation to the second "type" of solution, that of accepting clues offered by words and numbers in the problem itself. The presence of extraneous figures dealing with width suggested perimeter to some students and area to others. In Problem 1, Mike L., a good problem solver, and Pat L., a poor problem solver made approximately the same error in choice of a method, that of solving for perimeter instead of twice the length of the pool as the problem had asked.

Mike: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool?

That would be 2 x 75 and 2 x 30 - - - - 210 feet - - -

Inv: How did you get it?

Mike: I multiplied - - - - no - - - I added - - then multiplied.

Inv: Why?

Mike: There's two ways of working it - - - I could have added them together and then multiplied - - - I took the easiest way - - -

****

Pat: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool?

I multiplied 75 times 2 - - - - 150 - - - I do the same with 30 - - add - - - -

Inv: What is your answer?

Pat: 210.

Inv: How did you get it?

Pat: I put 75 - - and multiplied by 2 - - and I got 150 - - I did the same with 30 - - - - and got 60 - - - then I put 60 under 150 and added - - - and got - - 210 - - - -

Solving for area was another "type" error sometimes found in solutions of both good and poor problem solvers. This was suggested by the presence of both dimensions in the problem data. Irene T., a good problem solver, and Robert D., a poor problem solver, followed the same verbal clues in deciding upon a method for solving Problem 1.

Irene: (Reads) - - - - you multiply - - - 2250 feet - - -

Inv: How did you get it?

Irene: I multiplied - - -

Inv: Why?

Irene: Because I wanted to find out how many feet he swam - - - -

****

Robert: (Reads) Multiply - - - -

Inv: What is your answer?

Robert: 2250

Inv: How did you get it?

Robert: I subtract - - - - I mean I multiply - - - -

Inv: Why?

Robert: Cause the problem asked how far he'd seim if he swim twice the length of the poor -

The third technique, that of following a formula or rule in solving a problem, was used by both good and poor problem solvers. Problem 8 was sometimes solved without the pupil seeming to be aware of anything except a rule he had learned for similar problems. Richard C., a good problem solver and Jim S., a poor problem solver, arrived at a solution to Problem 8 in much the same manner, obviously following a formula learned for such problems.

Richard: (Reads) Mary's mother wishes to put linoleum on her kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

15 times 9 is - - - - - 135 - - - - There are - - - let's see - - - 9 square feet in a square yard - - - - 135 divided by 9 is - - - - (vocalizes) 15 square yards - - -

Inv: Why did you work it that way?

Richard: Well - - - that's the way we learned to do this kind of problems.

****

Jim: (Reads the problem) 15 square yards - - - -

Inv: How did you get it?

Jim: I - - - uh - - - multiplied - - 15 by 9 and got 135 - - - square feet. I - - - - divided by 9 and got - - - 15 square yards - - -

Inv: Why did you work it that way?

Jim: That's the way we learned it.

Inv: Is your answer reasonable?

Jim: Yeah - - - -

Inv: Why?

Jim: I don't know.

Inv: Is it right?

Jim: Yes - - I think so - -

Inv: Why?

Jim: Cause I can prove it - - -

The choice of method by good problem solvers was more often based upon social reasons, than those of poor problem solvers, and upon mechanical reasons about as often as poor problem solvers. Poor problem solvers gave doubtful reasons for choice of a method eighteen times as often as good problem solvers. Good problem solvers chose their methods somewhat more promptly and once having chosen them, changed less frequently. Use of "type solutions" was about as frequent in the choice of method by good and poor problem solvers, with about one in ten pupils using type solutions. In the matter of choosing a method, good problem solvers, as a group, were more proficient than poor problem solvers. When individual cases, as shown by the illustrations given, are considered the work of pupils of the two groups was much alike.

Thought Processes of Pupils While Working the Problems

One of the purposes of this investigation was to study the thought processes of good and poor problem solvers while engaged in solving problems in arithmetic. In order to establish a basis for classifying such thought processes three descriptive terms have been adopted. The term "abstract" was given to thought processes characterised by selection of a method of solution with little or no reference to the subject matter content of the problem, and by calculation based purely upon the mechanical relations of the numbers. In this type of thinking, the method was of primary importance; social and economic information of little significance. The term "concrete" was applied to thinking processes characterized by social and economic references made during the solution, by weighing the importance of the subject matter content of the problem, and by associating the problem solver's own experiences with the situation described in the problem. The term "random" was used in relation to thought processes characterized by illogical or dissociated thinking, cloudy or distorted reasoning, by guesswork, and by obvious or admitted ignorance about how to preceed with a solution. All data concerned with the solutions of the problems were reviewed and the thinking processes accompanying each solution classified as indicated into the three categories: namely abstract, concrete, and random. These data are presented in Table 12 which shows that abstract thinking was more than twice as frequent among good as among poor problem solvers, and that concrete thinking was almost twice as common among good problem solvers. Random

TABLE 12

THE FREQUENCY OF ABSTRACT, CONCRETE AND RANDOM THINKING EMPLOYED BY GOOD AND POOR PROBLEM SOLVERS IN WORKING EIGHT SELECTED PROBLEMS IN SIXTH GRADE ARITHMETIC

Classification	Number of Pupils	Number of Problems	Abstract Thinking	Concrete Thinking	Random Thinking
Good Problem Solvers	48	384	234	117	33
Poor Problem Solvers	48	384	108	65	211

thinking, as shown by the same table, occurred six times as often among poor as among good problem solvers.

As stated above, abstract thinking was twice as frequent among good problem solvers as among poor problem solvers. The work of Carol D. and Vera C., both good problem solvers, illustrates the manner in which these two pupils applied abstract thinking to the solution of the problems. In each solution reference was made only to the computational procedures and the mechanics of problem solving. Both had correct answers, and both pupils worked the problem effectively.

Carol: (Reads) Jean had $3.50 in her purse. She spent 35¢ for a movie and 20¢ for a soda. How much did she have left?

(Solves the problem) - - - answer is two - ninety - five

Inv: How did you get it?

Carol: Well I took 35 and 20 and added them together - - and I got a - - - 55 - - - and I took 55 from 3 - 50 and got - - 2 - 95.

Inv: Why?

Carol: Well - - - well you gotta find how much 20 and 35 are to begin with - - - - and if you take that from three fifty you find how much you got left - - -

****

Vera: (Reads) 35 plus 20 is 55 - - - - I take 55 from three - fifty two ninety five - - - -

Inv: How did you get it?

Vera: I added 35 and 20 and got 55 - - - and so I took 55 away from three fifty and - - -

Inv: And that gave you what?

Vera: two - ninety - five - - -

Inv: Why did you work it that way?

Vera: Wanted to know how much she had left and you'd have to know how much more she spent to find out - - - -

Poor problem solvers, likewise, applied abstract thinking to the use of the methods they had chosen. This is illustrated in the case of Ronald B. and Michael M., both poor problem solvers, who referred only to computational processes in their solutions to Problem 6.

Ronald: You add 35 and 20 and get 55 - - - and after - - you take 55 you divide it by 3 - 55 - - - - and it will go - - - 35 - - 35 - - is going to be 6 - - 6 x 5 is 30 - - - (pause) - - may I cross this out? - - - He had 2.95 left - - (two ninety five)

Inv: How did you get it?

Ronald: Well - - you take - - you add - - - 35 and 20 - - - that makes 55 - - - - you subtract from three-fifty and you get two-ninety-five - - - -

Inv: Why did you do that?

Ronald: You had to find out how much she had left so you'd subtract.

****

Michael: (Reads) You add these - - - - (20 and 35) and then - - - - subtract 55 from three-fifty - - - the answer is two-ninety-five

Inv: How did you work it?

Michael: First I added - - - then I subtracted - - - -

Inv: Why did you do that?

Michael: Because that's what I think I should have done - - -

Inv: Is it reasonable?

Michael: Yes - -

Inv: Why?

Michael: Cause that's what the problem calls for - - -

Concrete thinking, like abstract thinking, occurred about twice as often among good problem solvers as among poor problem solvers. This kind of thinking, applied to a problem solution, may be seen in the work of Peggy H. and Phyllis G., both good problem solvers. Both pupils were aware of the significance of the numerical values, and mentioned them in connection with the problem solutions.

Peggy: (Reads) I added 35¢ and 20¢ to see how much she spent - - she spent 55¢ - - take that from $3.50 - - - and you have $2.95 left - - -

Inv: How did you get it?

Peggy: I added 35¢ and 20¢ to see how much she spent - - - then I subtracted that from three-fifty to see how much she had at the beginning.

Inv: Is it reasonable?

Peggy: Well - - after she had paid that 55¢ she would have $2.95 left - - -

****

Phyllis: (Reads) Well - - - - you would take 35¢ and add 20¢ to it - - - - - - that would be 55¢ - - which is what she spent - - - Then you take $3.50 and subtract 55¢ from it and - - - she had $2.95 left - - -

Inv: How did you get it?

Phyllis: I added the two amounts she spent together to find out how much she spent altogether - - I subtracted that from what she had to see how much she had left - - -

Inv: Is it reasonable?

Phyllis: Yes - -

Inv: Why?

Phyllis: I think movies and sodas cost about that - -

Although the probability of its occurrence was only about half that of good problem solvers, poor problem solvers sometimes used concrete thinking in much the same manner as good problem solvers. The work of Roland H. and Carl J., both poor problem solvers, illustrates evidence of concrete thinking. Both pupils made frequent references to meanings and associated the values of the numbers with the results of the computation.

Roland: (Reads) Add 35¢ and 20¢ and get 55¢ - - - for a soda and a movie - - - Take 55¢ from $3.50 - - - and - - - left $2.95.

Inv: Why?

Roland: She spent 35¢ for a movie and 20¢ for a soda - - - you had to add 35¢ to 20¢ to get 55¢ - - - all she spent - - - Then you had to subtract 55¢ from $3.50 to see the amount she had left - - -

****

Carl: (Reads) I'd add - - - - 35¢ and 20¢ and get 55¢ - - - - - Then I'd take $3.50 - - which is what she had in her purse - - - and subtract 55¢ - - - which is what she spent - - - and you get $2.95 - - - -

Inv: Is it reasonable?

Carl: Yessir - - -

Inv: Why?

Carl: Cause if she spent 35¢ for a movie and 20¢ for a soda - - - she'd have about $2.95 left - - -

Paul D., a poor problem solver, was not able to perform the computation necessary to solve Problem 6 correctly. Application of concrete reasoning enabled him to present a correct solution, even though he was unable to subtract $3.50 - .55 in the usual manner. By concrete reasoning based upon an obvious familiarity with monetary values, he solved the problem satisfactorily.

Paul: (Reads) Jean had $3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much did she have left?

(He has added 35¢ and 20¢ and has as the sum, 55¢. This he subtracted from \$3.50 - - - - - \$3.50
.55
2.05

Inv: Is it reasonable?

Paul: Yes

Inv: Why?

Paul: Because 55 from - - three-fifty - - would be two - o - five -

Inv: Is it right?

Paul: M - mmm -- - (pause) (no figures on paper) I got two - ninety-five there - - - - - I don't think it is right -- -

Inv: Do you want to correct it?

Paul: No - - - - - I don't think I can - - - - -

Inv: Why do you think - - - -

Paul: Well - - - if I took three-fifty - - and took 50¢ away - - - that would be fifty cents out of there - - - that would be three dollars - - - - then I'd take five cents out of three dollars - - that would be \$2.95 - - -

Inv: Is that answer right?

Paul: Yes - -

Inv: Why?

Paul: Well - - - - because - - - - - (pause) - - - - - I just know it's right - - - - -

Random thinking was much less frequent among good problem solvers than among poor problem solvers, but, examples of undesirable kinds of thinking were noticeable among the solutions of good problem solvers. Such thinking took place in the solutions of Irene T. and Diane N., both good problem solvers. Irene attempted a random solution, but apparently was unable to support her decision with sound thinking. The thinking shown by Diane was apparently disorderly as well as inefficient.

Irene: (Reads) Mary's mother wishes to put linoleum on the kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

You multiply - - - - its supposed to be yards - - - - 135 square yards - - - -

Inv: How did you work it?

Irene: Uh - - - well - - - I worked it that way because - - - I wanted to - - - - I wanted - - - - - know how much she needs - - -

Inv: Is it reasonable?

Irene: Yes -

Inv: Why?

Irene: Well - - - because I multiplied - - - 15 times 9 - - -

Inv: Is it right?

Irene: (Laughs) - - - - - In a way I do - - - - and - - in a way I don't - - - - I think maybe - - - - oh I guess it's right - -

Inv: What were you going to do?

Irene: I was going to add 135 and 135 - - - - -

****

Diane: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

7 from 29 and you'll find how many it raised - - - - it raised 22 degrees - - - - - since six a.m.

Inv: How did you get it?

Diane: At noon it was 29 - - - in the a.m. it was 7 degrees - - - - - - - - to find - - - - I had to take my 7 from 29 and get 22 - - - -

Inv: Why?

Diane: Couldn't add for she'd get more - - - - Couldn't multiply for she'd get more - - - - couldn't divide for she'd get less - - - - So you'd have to subtract - - - -

Inv: Is it reasonable?

Diane: Yes - - - - - - if you had 29 and took away - - - you'd get less - - - - -

Poor problem solvers much more frequently depended upon random thinking. This may be illustrated in the work of Lois H. and Joann B., both poor problem solvers. It is difficult to see any constructive steps toward solving the problem in their thought processes.

Lois: (Reads the problem) Multiply 9 x 15 - - - (vocalizes her multiplication) is 135 square yards - - - -

Inv: How did you get it?

Lois: I multiplied - - - To find how many square feet - - - square yards she would need - -

Inv: Is it reasonable?

Lois: I don't know.

Inv: Is it right?

Lois: I don't know.

****

Joann: (Reads) A basket of apples weighs about 48 pounds. A large truck has on it 3552 pounds of apples. How many baskets would that be?

(She has divided 3552 by 48) Answer - - 89 r 10

Inv: How did you get it?

Joann: Divided - - -

Inv: Why?

Joann: (laughs - - - no answer) - - - - - just thought I'd divide - -

Inv: Read your answer.

Joann: Is it dollars - - - or children - - - no - - - it should be cents - -

Inv: Is it reasonable?

Joann: (no answer - - shakes head)

Three kinds of thinking, abstract, concrete, and random, were employed by good and poor problem solvers in applying the method of solving a problem. Good problem solvers were about twice as likely to use both abstract and concrete thinking. Poor problem solvers were much more likely to employ random thinking. The illustrations given in the preceding discussion showed that good and poor problem solvers as individuals made use of all three kinds of thinking in applying methods of solving problems.

Thought Processes of Pupils in Evaluating Their Answers to Problems

Two questions were asked of all pupils who completed a solution to a problem. These questions were: Why do you think your answer is reasonable? and Why do you think your answer is right? In answering these questions, good and poor problem solvers depended upon three general methods of evaluation in deciding upon the accuracy of an answer. These methods are the quantitative method--characterized by identification of the results with a known numerical value, reworking the computational process, or the association of the answer with some social or economic information; the parallel method--characterized by use of a reverse process such as multiplication to check division, addition to check subtraction, substitution of approximate values, and using a reversed set of numbers, as in changing multiplier and multiplicand in multiplication; the meaningless method--characterized by the inability of the pupil to give a plausible reason and by vague and evasive replies. Results of this classification are shown in Table 13. This table indicates that

TABLE 13

THE FREQUENCY OF QUANTITATIVE, PARALLEL, AND MEANINGLESS METHODS OF EVALUATING RESULTS OF SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC USED BY GOOD AND POOR PROBLEM SOLVERS

Classification	Number of Pupils	Number of Problems	Quantitative Method of Evaluation Used	Parallel Method of Evaluation Used	Meaningless Method of Evaluation Used
Good Problem Solvers	48	384	235	68	81
Poor Problem Solvers	48	384	103	12	243

good problem solvers gave quantitative reasons twice as often, parallel reasons five times as often, while poor problem solvers gave meaningless reasons three times as often as good problem solvers.

Table 13 shows that quantitative methods of checking were used twice as often by good problem solvers as by poor problem solvers. Good problem solvers employed recognition and application of quantitative values of numbers in confirming the accuracy of the solutions to the problems. John S., a good problem solver, referred to a known value (2 x 75) to illustrate that he was sure of his answer. Marilyn D., also a good problem solver, appeared to visualize a large number of pupils in relation to the approximate value of $201.75.

John: Why - - - - it's a logical answer because I know that 2 75's are - - - - -

Inv: Is it right?

John: Yes.

Inv: Why?

John: Because the pool is 75 feet long - - - he swam it twice - - and I multiplied it by 2 - - - -

****

Marilyn: You'd have to put the number of children down - - - - - - and beings they all paid 15¢ - - - you'd have to multiply that by 15 - - - - (vocalizes her multiplication) two thousand - - one hundred and seventy-five - - - no - - - - $201.75

Inv: How did you get it?

Marilyn: Well - - - beings there was 1345 children - - - - that went to the zoo - - - - and each cost 15¢ - - - - you'd have to find out how much they all paid by multiplying - -

Inv: Is it reasonable?

Marilyn: Yes - - - - there'd be a lot of children - - - - and they'd cost that much - - - -

Inv: Is it right?

Marilyn: Yes - - - - for that many chil ren - - - -

Poor problem solvers also gave quantitative reasons for believing their answers to be right or wrong. Connie D., a poor problem solver, almost duplicated the statement made by John S., a good problem solver, when she insisted that 2 x 75 is 150 feet. Paul D. also a poor problem solver, was incorrect in his answer to Problem 2, and something in his answer made him aware of it. That awareness appears to be the result of a quantitative evaluation.

Connie: Because 2 x 5 is 10 - - - - 2 x 7 is 14 and 1 is 15 - - - 150 feet - - - that's how I got it - - - -

Inv: Is it right?

Connie: (nods)

Inv: Why?

Connie: Cause I - - - - cause 2 x 75 is 150 feet - - - -

****

Paul: 20,175 doesn't sound reasonable for a zoo to make that much money from 1345 children.

Barbara R., a poor problem solver, considered her answer of 613 baskets to be correct. The illustrations, however, show that both based their reasoning upon quantitative thinking

Jo: A large truck would hold about that many baskets.

****

Barbara: The truck's big enough - - I think that many baskets would go on it - - -

Parallel methods of evaluating the accuracy of an answer were used somewhat less frequently by both good and poor problem solvers. Table 13 shows a frequency of 68 cases among good problem solvers and only 12 among poor problem solvers, a ratio of more than five to one. Almost identical parallel answers were given by June C., a good problem solver, and Daniel C., a poor problem solver.

June: Well - - - - 55¢ is almost a half-a-dollar - - - - and it is $3.50 - - - - If it is a half-a-dollar - - - she would have $3.00 left - - and she has 5¢ more than that - - - - so it would be $2.95 - - -

****

Daniel: Cause 55¢ from $3.50 - - - - - - that's 50 away - - - - and $3.50 off that - - - - is $3.00 - - - - and 5¢ is $2.95 - -

Billy B., a good problem solver, and Ronald B., a poor problem solver, used parallel computation in verifying the results to Problem 6. Both added the remainder and the expenditures, thus confirming their belief that their answer was right.

Inv: Is it right?

Billy: Yessir.

Inv: Why?

Billy: Why, cause I added 55¢ and $2.95 and got $3.50 - - -

****

Ronald: Well - - - if you ain't sure it's right - - - - - you add two-ninety-five - - - and fifty-five - - - and it comes out three-fifty - - - - and you know it's right - - - -

As shown on Table 13, poor problem solvers gave meaningless reasons three times as often as good problem solvers. An example of the use of meaningless reasons for believing a problem to be correct may be seen in the following problem. The answer to Problem 3 was 74 baskets, but Mike E., a good problem solvers, submitted an answer of 68 "and 16 left over." He declared that his answer was reasonable and that it was right. As shown in the illustration, there was no meaning in the reasons he gave for his belief in his answer.

Inv: Is your answer reasonable?

Mike: Uhuh- - - -

Inv: Why do you think so?

Mike: I don't know - - -

Inv: Is it right?

Mike: Yes - - -

Inv: Why do you think so?

Mike: I don' know - - - -

Donald H., a poor problem solver, had an incorrect answer to the same problem, 613 "and 6 left over." He, too declared his answer was reasonable and that it was right. The actual phrasing of his reasons was somewhat different, but no more meaningful than those of Mike .

Inv: Is your answer reasonable?

Donald: Yes - - -

Inv: Why do you think so?

Donald: I just do - - -

Inv: Is your answer right?

Donald: Yes.

Inv: Why do you think so?

Donald: I just do - - - -

Leslie M., a good problem solver, and Patricia P., a poor problem solver, both solved Problem 5 incorrectly, dividing 45¢ by 2. They both submitted $22\frac{1}{2}$¢ as the correct answer and declared that that answer was both reasonable and right. While the reasons they gave for these opinions were not alike, neither supported these reasons by meaningful thinking.

Inv: Is your answer reasonable?

Leslie: Yes.

Inv: Why do you think so?

Leslie: Why - - - - cause you'd have to divide to - - - get the price - - you couldn't subtract or divide - - - that'd come out too much - - - or too less - - - -

Inv: Is it right?

Leslie: Yes - - - -

Inv: Why do you think so?

Leslie: Cause that's the way to work it - - - -

Patricia showed a little annoyance at the repeated questioning.

Inv: Is it reasonable?

Patricia: Yes - - -

Inv: Why?

Patricia: I - - - - - - I - - - - - - I don't know - - - -

Inv: Is your answer right?

Patricia: Yes - - - -

Inv: Why do you think so?

Patricia: I think it is right - - or I wouldn't have put it down - - -

The preceding discussion points out that three methods were used by good and poor problem solvers to evaluate their results in working the problems. Good problem solvers were more likely to use quantitative methods or parallel methods than poor problem solvers. The poor problem solvers, more frequently than good problem solvers, used meaningless methods for checking their results. Individual solutions of problems that were used as illustrations of the three methods of checking and evaluating results, however, show that good and poor problem solvers employed similar reasoning when they chose the various methods they employed to evaluate their work.

Summary

A correct method for solving a problem was defined as a method that will produce a right answer if all computation and interpretation of results are properly done. When the frequency of correct choice of method was surveyed, it was found that good problem solvers chose correct methods twice as often as poor problem solvers. Upon being asked to give reasons for choosing their methods, three times as many good problem solvers as poor problem solvers gave social reasons, and about an equal number from both groups gave mechanical reasons. Poor problem solvers, on the other hand, gave doubtful reasons eighteen times as often as good problem solvers.

A social reason was characterized by reference to social or economic information or association of an experience with the choice of method. A mechanical reason was characterized by reference to the manipulative or computational processes used in the solution. A doubtful reason was one that was unintelligible, incoherent, unrelated to the procedures, or such an answer as "I don't know."

Good problem solvers were somewhat more prompt in choosing a method, and they were less likely to change that method, once chosen. Good and poor problem solvers employed "type solutions" to about the same extent, about once in ten problems. A type solution employed one of three techniques of choosing a method of solution: choosing a method because it was the method used in the previous problem without verifying its applicability; jumping to conclusions about a method of solution as the result of a word or a number clue; or following a rule or formula in working the problem without understanding the actual procedures.

Applying a method of solution resulted in three kinds of thinking: abstract, concrete, and random. Good problem solvers were twice as likely as poor problem solvers to use abstract thinking and about twice as likely to think concretely. Poor problem solvers, however, were six times as likely to employ random thinking as good problem solvers. The term abstract is applied to thinking procedures which are characterized by little or no reference to the subject matter and meanings of the problems, and by calculations based purely upon the mechanical relations of the numbers. Concrete thinking is that in which social and economic references are made while solving the problems, in which the importance of the subject matter content is weighed, and in which the problem solver associates his own experiences with the situation described in the problem. Random thinking is characterized by illogical or dissociated thinking, cloudy or distorted reasoning, guesswork, and by obvious or admitted ignorance about how to proceed with the solution.

Evaluation of the results of the methods of solution employed by good and poor problem solvers was done in three ways. Some of the pupils employed quantitative methods; some used parallel methods, and others meaningless methods. Quantitative evaluation was characterized by identification of the results with a known numerical value, reworking the computational processes, or association of the answer with some social or economic information. Parallel evaluation was characterized by use of a reverse process, substitution of approximate values, and reversing the terms of a computational process. The meaningless method of evaluation was characterized by the inability of a pupil to give a plausible reason, by vague and evasive replies. Good problem solvers gave quantitative reasons twice as often, and parallel reasons five times as often as poor problem solvers. Poor problem solvers, in contrast to these figures, gave meaningless reasons three times as often as good problem solvers.

In general, good problem solvers were more proficient in all of the skills related to the selection, application, and evaluation of methods for solving the problems. They used thought processes based upon social information more frequently in choosing their methods, were more likely to apply abstract or concrete thinking, and evaluated their results by quantitative or parallel methods more frequently than poor problem solvers. Poor problem solvers were more likely to employ questionable methods of thinking in relation to the selection, application, and evaluation of methods of solving problems. Measured by group methods alone, in respect to choice, use, and evaluation of methods, good problem solvers might be assumed to be in every way superior to poor problem solvers. Illustrations, however, demonstrate that in these respects the behavior of good problem solvers was duplicated in many instances by similar procedure on the part of poor problem solvers. In some cases, poor problem solvers demonstrated the ability to work a problem as efficiently and as meaningfully as the good problem solvers. In other cases, good problem solvers showed ineffective and confused thinking, in fact, no better than that of undesirable thinking procedures by poor problem solvers.

CHAPTER VI

NUMBER RELATIONS AND COMPUTATION AS FACTORS IN PROBLEM SOLVING

The Purpose of the Chapter

Insight, as an essential process in establishing thoughtful and meaningful understanding of a problem situation, was considered in Chapter IV. The thought processes accompanying the selection, application, and evaluation of the results of methods used in solving the problems have been discussed in Chapter V. As indicated in Chapter III, the third aspect of problem solving to be considered is that of interpreting the number relations in the problems and applying the computational processes to the solutions of the problems.

The understanding of number relations demonstrated by pupils varied in clarity and scope. Some pupils read a problem and immediately saw the relationships of the numbers to the ideas, processes, and subject matter content. Others saw little or no significance in the number relationships inferred by the problems. Likewise, in computation the practices varied as good and poor problem solvers attempted to apply their mechanical skills to the solutions of the problems. Significant aspects of computation that are discussed in this chapter include: the ability to read and manipulate four and five-digit numbers, the extent of vocalization of computational processes by good and poor problem solvers after they were directed to do so, the prevalence of the use of computational aids by pupils in working problems, and the effect of incorrect computation on the general accuracy in problem solving.

It is the purpose of this chapter to show the number relations and the computational skills employed by pupils in associating numbers with the problem situation. Basically, this chapter stresses two somewhat contradicatory positions. The superiority of good problem solvers, as a group, in understanding of number relationships appears in relative contrast to the approximate equality of good and poor problem solvers in applying the basic computational skills.

Number Relations in Problem Solving

Comparison of the frequency of clear, partial, and doubtful number relations by good and poor problem solvers.-- In attacking an arithmetic problem, some pupils examined the numbers and immediately associated them effectively with the ideas, processes, and subject matter content. Others showed inadequate knowledge of numbers, inability to read numbers and lack of skill in their use. Still others were unable to associate any ideas, processes, or subject matter content with the numbers in the problems.

Three categories of number relations were identified in the solutions of good and poor problem solvers to the problems used in this study. Clear number relations were those in which the pupil identified the numbers with other ideas in the problems, manipulated the numbers readily and logically, and considered their reasonableness and evaluated the accuracy of their results. Partial number relations were characterized by cloudy number identification, incomplete recognition of the number values, partial solution without clear understanding of the answer, and inability to evaluate the solutions by means of usual techniques of checking. Doubtful number relations were those in which the relationships of numbers were obscured, the computation random and without purpose, and the results without significance.

Table 14 shows that good problem solvers, more than four times as often as poor problem solvers, clearly understood the numbers they used and that good problem solvers were partially clear in their number relations somewhat less often than poor problem solvers. On the other hand, poor problem solvers were doubtful of number relationships nine times as often as good problem solvers.

The data in Table 14 indicate that good problem solvers showed clear understanding of number relationships more than four times as often as poor problem solvers. Clear understanding is illustrated in the solutions of Patti P. and Carl B., both good problem solvers.

Patti: (Reads) A grocery has a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

She would pay - - - - 2/6 - - equals 1/3 - - 1/3 of 45¢ is - - - uh - - - (vocalizes) 15¢ - - - - she would pay for the two bars - - - -

Inv: How did you get it?

Patti: I multiplied 1/3 times 45¢ - - - - 2/6 is 1/3 - - - -

Inv: Why did you do that?

Patti: 2/6 equals 1/3 - - - - you want to find 1/3

Inv: Is it reasonable?

Patti: Yes - - -

Inv: Why do you think so?

Patti: Well - - - 15 into 30 - - - goes 2 times - - - and 15¢ more is 45¢ (checking her 15's into 45)

Inv: Is it right?

Patti: Yes

Inv: Why?

Patti: Because 3 will go into 45¢ 15 times - - -

TABLE 14

THE NUMBER OF CASES IN WHICH CLEAR, PARTIAL, AND DOUBTFUL NUMBER RELATIONS WERE SHOWN BY GOOD AND POOR PROBLEM SOLVERS IN SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Pupils	Number of Problems	Clear Understanding of Number Relations	Partial Understanding of Number Relations	Doubtful Understanding of Number Relations
Good Problem Solvers	48	384	273	90	21
Poor Problem Solvers	48	384	62	129	194

****

Carl: (reads the problem)

Well - - it - - - I would say 6 into 45 - - - to see how many - - how much one - - - no - - - to see how much one - - - yeah - - - that'd be - - - (mumbles) - - - about 7 - - - (mumbles) - - that'd be about 15¢ for two bars - - - - (writes the answer down - - all previous work done in his head) - - - -

Inv: How did you work it?

Carl: First I would find out how much one bar costs - - - then I found out how much two bars cost - - - -

Inv: Is it reasonable?

Carl: Yes - - -

Inv: Why?

Carl: Two cost 15¢ - - - - 6 bars cost 45¢ - - 2 x 3 is 6 - - -

Inv: Is it right?

Carl: Yes - - -

Inv: Why?

Carl: Cause that's the way I worked it - - -

Poor problem solvers did on occasions, however, indicate clear understanding of the number relationships. The solutions of Jim S. and Carl J., both poor problem solvers, show that they, as well as Patti P. and Carl B., understood the significance of the numbers in Problem 3.

Jim: (reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

Well - - - - uh - - - it was 6 bars for 45¢ - - - - and it wanted to know how much Jane would pay for 2 bars - - - - so I divided 6 into 45¢ to find how much is the cost of 1 bar of soap - - - - and then I multiplied by 2.

Inv: What is your answer?

Jim: 15¢

Inv: Is it reasonable?

Jim: Uhuh.

Inv: Why?

Jim: Well - - - - if you'd put $7\frac{1}{2}$ by 2 - - - you could just tell it would be 15¢ - - - -

Inv: Is it right?

Jim: Yes.

Inv: Why?

Jim: I could prove it - - - - - - - I could multiply $7\frac{1}{2}$ by 6 - - - - - - and 2 into 15¢ would go $7\frac{1}{2}$¢ - - - -

****

Carl: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much would Jane pay for 2 bars?

First - - - I'd divide - - - - 45 divided by 6 - - - (vocalizes) 7 remainder 3 - - - - Then I'd multiply - - - 7 remainder 3 by 2 - - - and get 14¢ and a little over - - - 15¢

Inv: Why did you do that?

Carl: Well - - - to find out how much one bar costs - - - I'd divide - - Then I'd multiply to find how much 2 bars cost - - - -

Inv: Is your answer reasonable?

Carl: Yes - - -

Inv: Why?

Carl: Two bars would cost about that much - - -

Inv: Is it right?

Carl: I'm pretty sure it is - - -

Partial understanding of number relationships between numbers was somewhat more frequent among poor problem solvers than among good problem solvers, in individual cases the actual behavior of pupils in both groups was much alike. In the illustrations which follow, Mike L. and Helen G., good problem solvers and Julia P. and Marlene R., poor problem solvers, all show evidence of confusion in regard to the number relationships involved in Problem 5.

Mike: (Reads the problem) - - - (sighs) - - - uh - - - I just can't seem to get this - - - - - - - - - (writes down $7\frac{1}{2}$)

Inv: How did you get that?

Mike: I divided - - - I don't think that's right - - - um - - uh - - shall I do it over? - - - - first I divide 2 into 6 and goes 6 times - - and then I divide 3 into 45 and got 6 times -

Inv: Why did you do that?

Mike: Well - - - 2 into 6 goes 3 times - - - and 3 into 45 is 15 - - - - - - what is that 3 - - - - I just don't know - - - -

Inv: Is your answer reasonable?

Mike: Yes - -

Inv: Why?

Mike: Cause 3 into 45 is 15 - - -

Inv: Is your answer right?

Mike: Yes - - -

Inv: Why?

Mike: I - - don't know - - - it just seems right - - -

****

Helen: (Reads the problem)

6 into 45 - - - - - - 6 will go into 45 - - - - 6 7's are 42 remainder 3 - - so you'll say 2 into 7 - - 2 will go into 7 - no - - - I did that wrong - - - it's 2 into 45 - -(vocalizes) in our arithmetic book our teacher - - - in the book when you say - - - 23 - - -(checks) I think so - - - - -

Inv: How did you work it?

Helen: I said 2 into 45 - - - I think that is wrong - - - I think 2 into 22 - - the answer is 11¢ - - -

Inv: Is it reasonable?

Helen: Yes - - 6 bars are 45¢ - - - 2 bars can't be 22¢ - - - 11 is more reasonable - - -

Inv: Is it right?

Helen: It's not quite right - - -

Inv: Can you work it another way?

Helen: No sir - - - I think that's about the only way I could work it - - you could say 2 into 6 goes 3 times - - - then 3 into 45 is 15 - - - - no - - - I think this is right - - - 11¢ - - I think so - - -

****

Julia: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much will Jane pay for 2 bars?

(Mumbles - - inaudible)

I think she would pay 13¢ - - - I think she would either pay 14¢ for 2 bars for they want to know how much you would pay for 2 bars - - - - On the sale they had 6 bars for 45¢ - - - - So I divided - - - - and 7¢ with 3 remainder - - - - So I multiplied by 2 and - - - got 14¢.

Inv: Is it reasonable?

Julia: I think it is a reasonable answer.

Inv: Why?

Julia: If I would do it the other way - - - - make that 3/6 into 7 and ½ - - - - - which would be another penny - - - - - she would either pay 14¢ or 16¢ - - for 2 bars - - - -

****

Marlene: 6 bars into 45¢ is 7 - - - - 3/6 is ½ - - - - equals - - - 8¢ - - - - -

Inv: How did you get it?

Marlene: I divided.

Inv: Why?

Marlene: Cause it asked how much she paid for 2 bars at 6 bars for 45¢ - - - - -

Inv: Is it reasonable?

Marlene: Umhumm - - - Because - - - 6 into 45 is 7 - - - and 3/6 is $\frac{1}{2}$ - - - and when the half is over - - - - you have to pay another cent - - - - -

Doubtful understanding of number relationships or the inability to grasp a workable understanding of the numbers in relation to the problem solution, was nine times as frequent among poor problem solvers as among good problem solvers. There were, however, numerous cases in which both good and poor problem solvers were unable to interpret satisfactorily the numbers in the problem. The s lutions of Billy B. and Ellen H., both good problem solvers clearly illustrate their inability to interpret the number relationships involved in Problem 5. Likewise, Sylvia N. and Charles B., both poor problem solvers, showed doubtful understanding of the number relationships involved in the same problem.

Billy: (Reads the problem) A grocery has a sale on soap at 6 bars for 45¢. At that rate, how much will Jane pay for 2 bars?

Um - - - Oh - - - - Um - - - - (no vocalization) - - - 5¢

Inv: How did you work it?

Billy: I don't know - - - -(mumbles)- - - no - - - be 7¢ - - yes - -

Inv: How did you get it?

Billy: Divide - - - 6 bars for 45¢ - - 6 into 45 - -

Inv: Is it reasonable?

Billy: Yes - - - 6 bars for 45¢ - - - 2 bars for 7¢

Inv: Is it right?

Billy: Yes - - - 6 goes into 45 - - 7 times and 3 over - - - 6 x 7 are 42 and 3 over - - -

****

Ellen: I think you'd say 2 from 6 is 4 - - - - - - and 4 into 45¢ is 11¼¢

Inv: Why?

Ellen: It says - - - - - how much is - - - - paid for - - (Whisper) 2 bars - - - - - - I think - - - I'm not sure - - - -

Inv: Is it reasonable?

Ellen: I don't know.- - - -

Inv: Is it right?

Ellen: I'm not so sure - - - -

****

Sylvia: Subtract - - - (another pause) - - - - - (no vocalization) I subtracted 45 from 6 and my answer is 39 - - - - - - - -

Inv: Why?

Sylvia: Because it was six bars for 45¢ - - - and if I subtract it is - - - it would be - - - 2 bars for 39¢ - - - - -

Inv: Is it reasonable?

Sylvia: I think so - - - - -

Inv: Why?

Sylvia: Because 2 bars would cost as much as 6 bars - - - - -

****

Charles: 6 bars were 45 - - - - so you say 6 x 5 are 30 - - - - - 6 x 7 are 30 - - - - so you pay 7¢ for 6 bars - - - - - - 2 bars you would pay about 14¢ - - - - - - -

Inv: How did you get it?

Charles: Yessir - - - - you want to find out how much - - - - how much was - - - - - - um - - - - that times 9 is 45 - - - - - 5 times 9 is - - - - - - - um - - - - it couldn't cost 5¢ - so it had to cost about 6

Inv: Is it reasonable?

Charles: I don't think so.

Inv: Is it right?

Charles: No.

The foregoing discussion indicates the superiority of good problem solvers in understanding the relationships of numbers. Illustrations were given to show the function of this understanding in the problem solving procedures. The illustrations reveal, however, that when individual solutions are considered, good problem solvers and poor problem solvers show comparable understanding as well as similar lack of understanding.

Computation as a Factor in Problem Solving

Reading and manipulation of larger numbers.-- Both good and poor problem solvers experienced little difficulty with numbers composed of three digits or less. Three of the problems, however, namely, Problems 2, 3 and 4 involved numbers four or five digits in length. In these problems, pupils in both groups indicated some difficulty in reading and in manipulating these larger numbers. As shown in Table 15 good problem solvers were able to read the numbers correctly more than twice as often as poor problem solvers. Correct manipulation of the numbers by good problem solvers occurred three times as often as by the poor problem solvers.

TABLE 15

THE NUMBER OF CASES OF CORRECT READING AND OF CORRECT MANIPULATION OF THE NUMBERS IN PROBLEMS 2, 3, AND 4 BY GOOD AND POOR PROBLEM SOLVERS

Classification	Number of Pupils	Number of Problems	Numbers Were Read Correctly	Numbers Were Manipulated Correctly
Good Problem Solvers	48	144	130	112
Poor Problem Solvers	48	144	62	35

Correct reading and manipulation of larger numbers occurred more often in the work of good problem solvers than in the work of poor problem solvers. Individually, however, their work looked much alike and pupils from both groups performed in a similar manner. The work of Jerry K., a good problem solver, and that of Carl J., a poor problem solver, shows that both boys read and manipulated the numbers in all three problems correctly.

Problem 2

Jerry: You take one thousand, three hundred, forty-five and multiply it by 15¢ - - - (vocalizes his computation) the answer is $201.75 - - -

****

Carl: I multiplied thirteen hundred forty-five - - (vocalizes) twenty thousand one hundred seventy-five - - - there's 1345 children at 15¢ each - - - um - - it should be dollars - - (places dollar sign and decimal point)

Inv: Now what do you have?

Carl: $201.75

Problem 3

Jerry: First I'd take 48 and divide it into three thousand five hundred fifty-two - - - - - the answer is 74 baskets - - -

****

Carl: I'd divide 48 into three thousand five hundred fifty-two - - - - (vocalizes his division) - - - comes out even - - the answer is 74 baskets - - -

Problem 4

Jerry: I'd subtract - - - twenty-nine thousand eleven - - take away twenty-eight thousand nine hundred sixty-five (vocalizes) 46 miles - - -

****

Carl: (Reads) (miscopied his minuend - - 99, 011 when putting it down to subtract) (Shows concern over the pronunciation of the word "route" - - finally decides to pronounce it "root")

$$\begin{array}{r} 99011 \\ \underline{28965} \\ 70046 \end{array}$$

Inv: How did you get it?

Carl: I subtracted - - - twenty-eight thousand nine hundred sixty-five from ninety-nine thousand eleven - - - oh, no - - - I put 99 - - it's 29 - - - (reworks the problem) - - the answer is 46 miles

In partial contrast to Jerry K. and Carl J., Irene K., a good problem solver and Thomas H., a poor problem solver, were unable to read the numbers correctly, but manipulated the numbers efficiently and accurately.

Problem 2

Irene: Thirteen - - forty-five children - - - paid 15¢ - - - and got $201.75 - - (two - oh - one - seventy-five)

****

Thomas: Answer - - - 20175 - - - (no vocalization)

Inv: How did you work it?

Thomas: 15 times one-three-four-five - - -

Inv: Is it reasonable?

Thomas: Huh-uh - - - - I made a mistake - - - I don't have no places - - I don't have no decimal points - - - You'd have $201.75.

Problem 3

Irene: 48 into 3552 (thirty-five -- fifty-two) (vocalizes her division) 74 - - - -

****

Thomas: Divide 48 into 3 - 5 - 5 - 2 -- -- (no vocalization)

Inv: What is your answer?

Thomas: 74 baskets - - - -

Inv: How did you get it?

Thomas: I divided - -

Problem 4

Irene: Take 2 - 9 - 0 - 1 - 1 - - - and subtract - - 2 - 3 - 9 - 6 - 5 - - - and (vocalizes her computation) I got 46 - - - -

****

Thomas: Twenty-eight-nine-sixty-five - - - 29 - oh - eleven (vocalizes his subtraction) 46 miles - - - -

Inv: How did you get it?

Thomas: Subtract 29 - 9 - 65 from 29 - 0 - 11

Still further contrast is indicated in the work of Charles T., a good problem solver, and Robert B., a poor problem solver, who were neither able to read the numbers correctly nor to manipulate them in the proper manner.

Problem 2

Charles: Well - - - - you multiply thirteen-forty-five - - - - 18 x 1345 - - - - thirteen-forty-five - - multiply that by 15 - - - be 211.75 - (two eleven seventy-five)

****

Robert: 15 times - - one-three-four-five - - - (vocalizes - - has trouble with the sums of his partial products) - - - I'm a little excited - - - answer is 8070 - - (eight -oh - seven - oh)

Problem 3

Charles: Divide 48 into - - three - five - five - two - - - (vocalizes - - - - - - - - I think the answer is 74 and 9/48 baskets

****

Robert: I'd divide 48 into three - five - five - two - - - (vocalizes) that's the answer - - -

Inv: What's the answer?

Robert: 444 (four - forty-four)

Problem 4

Charles: (Reads - - - 28 million - - 965 - - 29 million 11 thousand) Well I figure you should subtract what the thing read before he started from what it is at the finish of the trip - - - I think I do it - - - - this is too high a number - - - (pauses and figures) I think he drove 8 thousand and 46 miles - - -

Inv: How did you get it?

Charles: I subtracted - - - but I don't think it's right for sure - - -

****

Robert: 28 hundred and 9 hundred sixty-five - - - (stammers) I don't see how that is - - - - 29 hundred and eleven take away twenty-eight and nine hundred sixty-five - - - - be 946 miles - - -

<u>Average time spent by good and poor problem solvers in reading and in working the problems.</u>-- Poor problem solvers spent more time both in reading and in working the problems than did good problem solvers. Good problem solvers required an average of sixteen seconds to read a problem before beginning a solution, while the poor problem solvers spent an average of twenty-five seconds in reading the problems. Poor problem solvers spent an average of sixty-one seconds reading the problems, whereas the good problem solver averaged forty seconds. Table 16 shows the average time of good and poor problem solvers in reading and in working the problems.

TABLE 16

AVERAGE TIME SPENT IN READING EIGHT SELECTED PROBLEMS IN ARITHMETIC AND IN SOLVING THEM BY GOOD AND POOR PROBLEM SOLVERS

Classification	Number of Pupils	Number of Problems	Average Time Spent In Reading the Problems	Average Time Spent In Working The Problems
Good Problem Solvers	48	384	16 seconds	40 seconds
Poor Problem Solvers	48	384	25 seconds	61 seconds

The frequency of vocalization by good and poor problem solvers in solving arithmetic problems. -- As indicated by the anecdotal illustrations, pupils talked about their problems, discussed their solutions, their reasons for working the problems as they did, and the evaluation of their answers. Each pupil was requested before he began his interview, and when necessary, once during the interview, to vocalize or say aloud all of the computational processes used in solving the problem. Many pupils readily gave step-by-step analyses of computational procedures; others did not do so, even though they were reminded by the interviewer. Good problem solvers, about twice as often as poor problem solvers, vocalized their computational processes, as is shown on Table 17.

The frequency of computational aids in problem solving by good and poor problem solvers.-- Computational aids, such as carrying marks in addition, extra figures in subtraction, trial multiplication of the divisor in division, and written addition sequences for the recall of multiplication facts, appeared somewhat more often in the work of poor problem solvers than in the work of good problem solvers. Only fifteen cases of use of such devices as counting on the fingers, extra marks on the paper, or tapping or pecking with the fingers or other movable parts of the body were observed among the 768 solutions, and twelve of the fifteen were by poor problem solvers. Only seven pupils, all good problem solvers, attempted to draw a diagram of a problem, although the content of Problems 1 and 8 might have suggested such drawings. These were not pencil and paper drawings, as might be expected but rather were traced with the finger in the air while solving Problem 1, indicating the act of "swimming down and then swimming back." The frequency of the use of extra-computational aids is shown in Table 18.

TABLE 17

THE NUMBER OF CASES OF VOCALIZATION OF COMPUTATIONAL PROCESSES BY GOOD AND POOR PROBLEM SOLVERS, IN SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Pupils	Number of Problems	Used Vocalizations in Computation
Good Problem Solvers	48	384	196
Poor Problem Solvers	48	384	82

TABLE 18

THE NUMBER OF CASES OF THE USE OF COMPUTATIONAL AIDS, COUNTING, AND DIAGRAMS BY GOOD AND POOR PROBLEM SOLVERS IN SOLVING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Pupils	Number of Problems	Used Computational Aids	Used Counting	Used Diagrams
Good Problem Solvers	48	384	100	3	7
Poor Problem Solvers	48	384	132	12	0

<u>The accuracy of good and poor problem solvers in computation</u>.--In the analysis of the data it was apparent that two procedures are necessary if an arithmetic problem is to be solved correctly: the right method must be chosen and all computations must be correct. According to Table 19, good problem solvers were generally more accurate than poor problem solvers. Wrong choice of method was the most frequent cause of difficulty, both among good and poor problem solvers. About sixty-five percent of the mistakes made by good problem solvers were made because of wrong choice of method, while eighty-one percent of the errors of poor problem solvers was caused by wrong choice of method. Computation, which included placing of the decimal point, accounted for thirty-one percent of the errors of good problem solvers and twenty percent of the errors of poor problem solvers.

Table 19 shows that wrong choice of method was responsible for three times as many incorrect solutions as inaccurate computation. While this table proves interesting, it does not show the complete situation regarding the computational competencies of good and poor problem solvers. If a pupil chose an incorrect method, he obviously could not work his problem correctly, no matter how his computation might be. To determine the extent of accuracy of good and poor problem solvers in computational skills, the matter of method was overlooked and the actual number processes were studied. For example, if the correct method for working a problem required division and pupil multiplied, if he multiplied correctly his computation was considered correct. Table 20 shows that good problem solvers achieved ninety percent accuracy and poor problem solvers about seventy percent accuracy, when only the basic number processes were studied.

TABLE 19

THE NUMBER OF PROBLEMS MISSED, THE NUMBER MISSED BECAUSE OF WRONG CHOICE OF METHOD, AND THE NUMBER MISSED BECAUSE OF FAULTY COMPUTATION, BY GOOD AND POOR PROBLEM SOLVERS, IN WORKING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Problems Missed	Missed Because of Wrong Choice of Method	Missed Because of Incorrect Computation
Good Problem Solvers	133	87	42
Poor Problem Solvers	305	239	62

TABLE 20

THE NUMBER OF CASES OF CORRECT COMPUTATION, REGARDLESS OF METHOD EMPLOYED BY GOOD AND POOR PROBLEM SOLVERS IN WORKING EIGHT SELECTED PROBLEMS IN ARITHMETIC

Classification	Number of Pupils	Number of Problems	Number of Correct Computations
Good Problem Solvers	48	384	340
Poor Problem Solvers	48	384	266

Table 17 shows the frequency of vocalization of computational procedures by good and poor problem solvers. Table 18 indicates the extent to which pupils of the two groups relied upon computational aids of various kinds. In Table 19 may be seen the frequency of incorrect computation and its effect upon errors in problem solving. The accuracy of computation, when the method of working the problem was not a consideration, is revealed in Table 20. To illustrate the foregoing procedures of good and poor problem solvers, anecdotal accounts, both written and oral are presented. The solutions of Marilyn D., a good problem solver and Jane C., a poor problem solver, show how vocalizations of the number processes appear when a pupil completed the problem as directed. Extra-computational aids in the way of additional marks are evident in both solutions.

Marilyn: Take 29011 and take 28965 from it - - -

```
 8910
 29011
 28965
 00046
```

Five from 1 won't go - - - have to go to the 9 and borrow one - - makes that an 8 - - makes the 0 a 9 and the 1 a 10 - - - 5 from 11 - - - 6 - - - - 6 from 10 - - - 4 - - zero - - zero - - zero - -

The answer is 46 miles

****

Jane: Put down

```
  8 910 1
2 9 0 1 1
2 8 9 6 5
0 0 0 46
```

5 from 1 you can't do - - - - you have to go way over and borrow from your 9 - - - - make that an 8 - - - that makes it a 10 - - cross that out and that makes 9 - - makes your 1 a 11 - - cross it out and make it 10 - - - make you 1 - - 11 - - 11 from 5 is 6 - - 10 from 6 leaves 4 - - 9 from 9 leaves 0 - - 8 from 8 leaves 0 - - 2 from 2 leaves 0 - - - he goes 46 miles - - - -

In some cases there were no extra marks and computation was completed with simple, but complete vocalization. Mike S., a good problem solver, and George G., a poor problem solver illustrate these skills in solving Problem 4.

Mike: You subtract 28965 from 29011

```
 29011
 28965
 00046
```

5 from 11 are 6 - - 6 from 10 are 4 - - - - - - - - - 0 - - 0 - - 0 - - - He went 46 miles

****

George: Subtract - - - - - - - 29011

```
 29011
 28965
```

11 take away 5 are 6 - - - 10 - - from 6 are 4 - - - answer - - - 46

Dick G., a good problem solver, and Pat L., a poor problem solver, did all of their borrowing without extra marks and worked the problem incorrectly. Dick made two errors in subtraction and Pat made one. Pat also read her answer incorrectly.

Dick: You subtract 29011 by 28965 - - - - -

```
 29011
 28965
  1036
```

****

Pat: I subtract - - - - - - 28965

```
 28965
 29011
  9954  (crosses out)

 29011
 28965
  1046
```

The answer is one hundred forty-six

John ?., a good problem solver, and Leon B., a poor problem solver, tried to use extra marks in borrowing, but both were unable to perform the computation with accuracy. Leon had his numbers reversed, but he made a subtraction error even with the terms reversed.

Joan: Take 29011 from 28965 - - - - - - - 29011 (crosses out)
28965

I mean the other way - - - - - 29011
28965
9146 mi.

****

Leon: I add - - - 28965 answer 57 thousand 9 hundred 76
29011
9854

Inv: How did you get it?

Leon: I added - - -

In solving Problem 3, John G., a good problem solver, and Tom M., a poor problem solver, worked their division correctly and evaluated their results in similar manner.

John: Divide 48 into 3552 - - - answer - - - - 74 baskets

74 baskets
48) 3552
336
192
192

****

Tom: Divide 48 into 3552 - - - - - 74
48)3552
336
192
192

Inv: What is your answer?

Tom: 74 baskets - - - - -

An error in copying the divisor stemmed from an error in interpreting the numbers of the problem and Mike B., a good problem solver, got the wrong answer to Problem 3. All of his computation, however, was correct.

Mike: I would divide 48 into 3552 - - - - - 52 wouldn't go into 35 - - - - - 52 will go into 355 - - - -

```
       8
      6̶4̶
52) 3552
    312
     432
     416
      16
```

6 time - - - - 6 x 2 are 12 - - -
6 x 5 are 30 and 1 are 31 - - - -
2 from 5 are 3 - - - 1 from 5 are 4
bring down 2 - - - goes 4 times
4 times 2 are 8 - - - - 4 times 5
no - - - - it won't go - - - about
8 times - - - etc. - - -

That would be 68 and 16 left over - -

Inv: How did you get it?

Mike: I divided

Inv: Why?

Mike: If there were 3552 pounds - - - and you'd want to find out how many baskets there'd be - - - - you'd divide 48 into 3552. (48 was underlined to show change in divisor)

Rita L. and Simon C., both poor problem solvers, chose incorrect methods but worked all computation correctly in solving. Rita added instead of subtracting in Problem 4. Simon attempted to find the number of square yards in a kitchen floor 15 feet by 9 feet by adding 15 and 9.

Rita: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read 2 million 9 thousand 8 hundred 65. (28965) At the finish of the trip it read - - - - (unable to read at all).

Inv: Go ahead if you can't read it.

Rita: I added:

```
 28965
 29011
 57976
```

(The second digit from the right was made somewhat like a 4 which confused her in reading her answer.)

I added - - - - 2 million 7 thousand - - - 9 hundred 46. (There is no explanation for reading the five as a two.)

****

Inv: Why did you work it that way?

Simon: Why - - if I work it like that - - - I get how many feet it is - - - -

Inv: Is it reasonable?

Simon: Yessir.

Inv: Why?

Simon: (No answer)

Inv: Is it right?

Simon: Yessir.

Inv: Why?

Simon: I don't see how to get it - - - - that's the only way - - -

Computation on Problem 2 was incorrect in the solution to that problem by Jane C., a poor problem solver. At least four errors can be detected in her work. Some of the errors are computational, some reading and some are in the interpretation of the problem. In telling what she did in working the problem she spoke of 1354 instead of 1345-- but she put it down right. There are two errors in adding the partial products, and the decimal points are entirely missing. The indefiniteness of her answers reflects little confidence in her solution.

Jane: (Reads) In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much did they pay in all?

You can multiply 13 - - - hundred - - - times 54 - - - 13 hundred fifty-four - - - - times 15 - - - - or you can - - subtract it - - - - -

Inv: Which?

Jane: Multiply it - - - -

```
 1345
   15
 6725
1345
19675
```

nineteen - - - 1 hundred and 98 thousand - - - and 75.

Inv: How did you get it?

Jane: Multiplied 13 hundred and 54 by 13 - - - -

Inv: Why?

Jane: I don't know.

Inv: Is it reasonable?

Jane: Yes.

Inv: Why?

Jane: I don't think he could swim that far.

The foregoing discussion of computation in problem solving has revealed that good problem solvers are more proficient than poor problem solvers in reading and manipulating larger numbers. They were somewhat more able to vocalize their computational processes and were slightly more accurate in computations. Good and poor problem solvers, to about the same extent, used computational aids, but about two-thirds of the problems were solved without any indication of the use of computational aids. Individual pupils performed in much the same manner regarding the procedures listed above. Examples of both desirable and undesirable practices and procedures appeared both in the work of good and poor problem solvers.

Summary

Two aspects of numbers are considered in this chapter: the ability of pupils to experience and interpret the number relationships in solving arithmetic problems, and their skill in the application and use of computational processes.

Good problem solvers, more than four times as often as poor problem solvers, understood clearly the numbers they used, somewhat less often were only partially clear in their understanding. Poor problem solvers, nine times as often as good problem solvers, were doubtful about the number relations of the problems they attempted to solve. Clear number relations are those in which pupils identifies the numbers with the ideas, processes, and subject matter content of the problems, manipulate the numbers readily and logically, consider the reasonableness and the accuracy of their results. Partial understanding of number relations are characterized by cloudy number identification, incomplete recognition of all of the number values, partial solution without clear understanding of the answer, and inability to evaluate a solution through usually accepted techniques of checking. Doubtful understanding of number relations are those in which the meanings of numbers are obscured, the values represented by numbers are vague and indefinite, computations random and without purpose, and results without significance to the pupils.

Understanding of number relations were reflected in the ability of good and poor problem solvers to read four and five digit numbers correctly and to compute accurately when numbers of this size were involved. An analysis of the work of good and poor problem solvers on the three problems involving numbers of this size shows that good problem solvers read the larger numbers correctly twice as often as poor problem solvers, and they performed the computational functions three times as accurately as the poor problem solvers.

Poor problem solvers spent about fifty percent more time reading and about the same amount more time in working each of the problems used in the study. Vocalization of the computational processes, done at the suggestion of the interviewer, were about twice as frequent among good problem solvers as among poor problem solvers. Good and poor problem solvers showed little difference in the use of computational aids. Both used extra figures on the paper frequently, but counting was not observed to any great extent. No drawings appeared on the papers of either the good or the poor problem solvers. Seven good problem solvers drew imaginary figures in the air as aids in thinking through their problems.

Problems were missed most frequently because of the failure to choose the right method. Errors in computation were the cause of about one-third of the mistakes of good problem solvers, but only one-fifth of the errors of the poor problem solvers. Regardless of the methods selected, the accuracy of computation was high among both good and poor problem solvers. Nine-tenths of the good problem solvers and seven-tenths of the poor problem solvers were accurate in their computation.

Group techniques of evaluating the number relations and computational skills show that good problem solvers are superior to poor problem solvers. Good problem solvers showed clear number relations more often, could read and manipulate large numbers more efficiently, were faster in reading and in solving problems, could vocalize computational processes more effectively. They were little more accurate in all of their computational processes. Poor problem solvers were more often doubtful in their number relations, less proficient in reading and manipulating larger numbers, slower in reading and in working the problems, and made somewhat more errors in computation. It is of interest to note, however, that when individual performances of good and poor problem solvers were presented in the illustrations of the particular kinds of behavior being discussed, that the work of some of the poor problem solvers often matched and sometimes exceeded in skill that of some of the good problem solvers.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The Problem

__The purpose of the study.__-- The purpose of the study was to investigate the characteristics of procedures of good and poor problem solvers while engaged in solving verbal problems in arithmetic. Specifically, this study attempted to isolate and describe the characteristics of the procedure of good and poor sixth grade problem solvers which show evidence of thoughtful and meaningful understanding, as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error.

__The significance of the study.__-- The study not only outlines for teachers of arithmetic hitherto unreported information about the characteristic procedures of good and poor problem solvers, but it also suggests a technique that will be more productive of information about the individual problem solving behavior of both good and poor problem solvers. Group methods that have commonly been used to study arithmetical problem solving by pupils have not been effective in isolating the individual performances of pupils, nor in determining exactly what pupils do when they solve an arithmetic problem.

__The scope of the study.__-- The conclusions of this study are based on the performance of good and poor problem solvers in the solution of eight selected problems in sixth grade arithmetic. These problems, of one and two-step complexity, involve only whole numbers and utilize the four basic skills, namely addition, subtraction, multiplication, and division. Two characteristics of procedure were observed. One was that of thoughtful and meaningful understanding; the other that of mechanical and almost meaningless attack. The study describes those characteristics of procedures of good and poor problem solvers in arithmetic which show evidence of thoughtful and meaningful understanding, as well as those which show adherence to purely mechanical manipulation, sheer guesswork, or trial and error.

Sources of data.-- The Iowa Every Pupil Test in Basic Skills, Test D: Basic Arithmetic Skills, Form O, Part III, was used as a selection test for choosing the good and the poor problem solver groups. Evidence of thoughtful and meaningful understanding, mechanical manipulation, guesswork, and trial and error was furnished by the solutions to eight selected problems in arithmetic worked by each of ninety-six sixth grade pupils. Tape recordings of the vocalizations of pupils while they were working the problems were an important source of information about the problem solving behavior of pupils. Pencil and paper solutions on individual work sheets provided additional evidence of thought processes and mechanical activity. Notes taken during the interviews were useful in the recall of emotional behavior, physical reaction, and the extra-computational aids used in the solutions.

Methods of research.-- The basic method of research used in this study was the modified case study method. Individual pupils were studied through a systematic observational technique designed to localize and record each pupil's problem solving procedures. Total patterns of behavior were observed, classified, evaluated and tabulated. Anecdotal excerpts from individual case records were introduced to explain and support the tabular evidence introduced.

Related Literature

Contribution of the literature to the study.-- The literature made three important contributions to the study: the importance of meaning and understanding in problem solving in arithmetic, reports of research studies associated with the purpose of this investigation, and techniques for studying the arithmetic problem-solving behavior of pupils.

The significance of the term "meaning and understanding" was explored through its interpretation by recognized writers in the field of arithmetic. Although writers expressed opinions concerning the importance of meaning and understanding in arithmetic, few attempted to define specifically what is meant by the term.

Research studies associated with the purpose of this investigation indicated the complex and diverse nature of problem solving in arithmetic. General procedures, specific objectives, and the results reported in those investigations were especially helpful in organizing and developing this study.

For studying individual differences in pupil procedures during problem solving, the diagnostic interview was shown to be of significant value. It was generally agreed that diagnostic interviews on an individual basis, with well-defined objectives in mind, may give valuable clues regarding procedures employed by pupils in working arithmetic problems.

Procedures Used in the Study

Basic hypothesis of the study.-- The investigation was based on the hypothesis that problem solving can be isolated and studied if observed systematically while pupils are engaged in working problems. Systematic observation required an adequate sample, uniform procedures, problems typical of those worked by sixth grade pupils in the public schools, selection of the group by approved research methods, accurate and complete records of problem solving behavior, and the analysis of the behavior by an investigator trained in observing pupils in arithmetic problem solving.

Preliminary investigations for establishing techniques and developing procedures.-- Seven preliminary investigations were conducted in elementary school in Cincinnati, Hamilton, and Greenville, Ohio. These preliminary investigations contributed to the development of the procedures of the investigation, and in the inte pretation of the data derived from it. Problems for use in the investigation were tested, methods of interviewing investigated, and ways and means for recording and reporting the procedured of the pupils during problem solving were explored.

The plan of the study.-- Preliminary studies helped suggest and ultimately supported the following plan of investigation: Six schools, believed to be typical of the elementary schools of the Cincinnati Public Schools, were selected. From the sixth grade pupils in those schools, forty-eight good problem solvers and forty-eight poor problem solvers were selected as a sample of the pupils to be used in the study. Good problem solvers were those making scores on a standardized test of problem solving in the upper twenty-seven percent of the sixth grade pupils in the six schools. Poor problem solvers were pupils who made scores that were in the lowest twenty-seven percent on the same test. Eight verbal problems in arithmetic were prepared and presented to each of the ninety-six pupils in the total sample. The pupils were instructed to work each problem and to say aloud the procedures used in working it. Each solution was followed by a question and answer period in which seven standard questions were asked each pupil who completed the solution. All vocal utterances of both the pupil and the investigator were recorded on a tape recorder. Notes of the behavior of the pupils while working the problems were made. Written solutions, tape recordings, and the notes taken of the observed behavior of the pupils served as the source of the data used in the study.

Organization of the data.-- From the data listed above, three classifications were made of the characteristic procedures of good and poor problem solvers while solving problems in arithmetic. These procedures were grouped as follows: the procedures of good and poor problem solvers which show insight as a factor in problem solving; those which show the thought processes related to the choice, employment, and evaluation of the methods used in working the problems; and those which indicate the number relations and computational skills employed by pupils in associating number ideas with the problem situation.

Reporting the data.-- Data relating to each of these characteristics of procedure were tabulated as follows: characteristics of procedure relating to insight were classified clear, partial or doubtful. Characteristics of procedure relating to thought processes fell into three groups as follows: thought processes concerned with choice of method were grouped social, mechanical and doubtful; those dealing with applying methods of solution abstract, concrete, and random; those related to evaluation of results quantitative, parallel and meaningless. Characteristics of procedure dealing with understanding of number relationships were classified clear, partial and doubtful.

Other tabular data supporting these classifications and anecdotal excerpts from actual problem solutions were introduced to illustrate and to explain the characteristics of procedure presented.

Findings of the Study with Regard for Insight as a Factor in Problem Solving

Definition of insight.-- Insight was defined as the degree to which a pupil is able to recognize significant aspects of a problem, to associate meaning and understanding with those aspects to apply logical computation in the solution of a problem, and to evaluate the final results in terms of his own experiences. Clear insight was characterized by effective reading of the problem, direct attack upon the problem based upon understanding of the problem situation, accurate judgments of the problem situation, critical evaluation of the final answer, and reasonable confidence in the appropriateness of the results. Vague insight into a problem was characterized by faulty reading, obscured number relations, imperfect or hesitant selection of methods of attack, neglecting significant words or ideas, inadequate notions of the meaning of the answer, and lack of confidence in the appropriateness of the answer. Doubtful insight was characterized by inability to attack the problem, choice of a method for no apparent reason except a random effort to compute, fragmentary understanding with complete bewilderment as to how to apply those fragments, and incoherent, meaningless attempts to explain the solution.

Insight into meaning and understanding of the problems by good and poor problem solvers.-- Good problem solvers showed clear insight into the problems they solved about four times as often as poor problem solvers, and they showed vague insight about as often as poor problem solvers.

Poor problem solvers, however, showed doubtful insight six times as often as good problem solvers.

Vocabulary as a factor in the insight of a pupil into a problem.-- Illustrations from the work of good and poor problem solvers show the importance of word and number clues to pupils in solving problems. As might be expected, poor problem solvers were more often deficient in understanding the vocabulary than the good problem solvers. When, however, good problem solvers did experience difficulties with vocabulary, their procedures in the solutions were much like those of the poor problem solvers.

Social and economic information as sources of insight.-- Evidence of the use of social and economic information in the solution of problems was observed in the solutions of both good and poor problem solvers. The absence of this evidence was likewise apparent in the work of pupils from both groups. Solutions to the various problems were classified according to the extent of the use of social and economic information in working the problems. Three general groupings were made: Social solutions--characterized by a pupil's identifying himself with a problem situation, directly or vicariously, or in some other manner indicating that he has experienced some aspect of the problem or of the social behavior related to it. Mechanical solutions--characterized by reference only to mechanical or computational ideas while the problem was being solved, and dependence upon those ideas for evaluation and justification of the procedures used. Doubtful solutions--characterized by failure to give any sensible reason, such as "I don't know," or "I just worked it that way," or by obvious or admitted ignorance of their procedures.

Use of social and economic information by good and poor problem solvers.-- Use of social and economic information and the use of mechanical solutions by good problem solvers occurred twice as often as by poor problem solvers. On the other hand, poor problem solvers produced doubtful solutions, in which little or no evidence of social or economic information or of clear mechanical solutions could be seen, fifteen times as often as good problem solvers.

Labeling of answers as a factor in insight.-- Labeling the answers to the arithmetic problems occurred in two ways. The pupil might label the answer in the written solution, or he might say it orally as he announced the results of his solution. Sometimes one of these situations was evident, sometimes both, and sometimes neither kind of labeling could be found in the work of the pupils. Good problem solvers, however, were twice as likely to label orally and twice as likely to write the labels on their written solutions.

Generalizations from the data.-- In general, good problem solvers were more proficient in all of the skills related to insight into a problem situation. They more readily perceived the problem situations, they used word and number clues more expertly, they applied social and mechanical thinking to their problem situations more often, and they labeled their answers more efficiently. They were not so likely as poor problem solvers to show doubtful insight, reflect vocabulary failures, to become bewildered over social and economic implications, or to label inadequately. The generalizations above are based upon group data rather than upon analysis of individual efforts in problem solving. In fact, good problem solvers are superior to poor problem solvers in all aspects of problem solving dealing with insight into the problems. Individual pupils, however, both good and poor problem solvers, show similar procedures in successful solutions to problems. Unsuccessful solutions, likewise, show comparable procedures and lack of insight into the problem situation both by good and poor problem solvers.

Findings of the Study with Regard for Thought Processes and Their Relation to Problem Solving

Selection of methods of solving problems by good and poor problem solvers.-- A correct method of solving a problem was defined as one that would produce a correct answer if all computation and interpretation of results were properly done. Twice as many good problem solvers as poor problem solvers chose the correct methods for solving the problems.

Reasons given by pupils for their choices of methods for solving the problems were classified social, mechanical, and doubtful. A social reason was characterized by reference to some social or economic information, or by association of some personal experiences with the choice of method. A mechanical reason was one that offers manipulative or computational justification for the method used in solving the problem. A doubtful reason was one that is incoherent, unintelligible, unrelated to the procedure, or one that expressed lack of knowledge of the processes really used.

Reasons given for choice of methods by good and poor problem solvers.-- Good problem solvers gave social reasons for their choice of method three times as often as poor problem solvers, and they gave mechanical reasons about as often as poor problem solvers. On the other hand, poor problem solvers gave doubtful reasons for choosing their methods eighteen times as often as good problem solvers.

Promptness in choosing methods and extent of re-evaluation of methods once chosen.-- Good problem solvers were somewhat more likely to choose a method for solving problems immediately, and with little or no reflection. Once a method was selected, good problem solvers were somewhat more likely to retain that method without reconsideration of its appropriateness.

The use of "type solutions" by good and poor problem solvers.-- About one out of ten good and poor problem solvers chose their methods by means of procedures which have been called "type solutions." These procedures included: choosing a method because that method had been used in the preceding problem, jumping to conclusions because of a word or a number clue, and following a rule or a formula with little or no awareness of the meaning of what they were doing.

Thought processes used in applying the methods of solving the problems.-- Three kinds of thought processes were observed in the procedures of good and poor problem solvers in applying the methods they had chosen to work the problems. These three classifications of thought processes were abstract, concrete, and random thinking. Abstract thinking was characterized by little or no reference to the subject matter content and meanings of the problems, and by calculations based purely upon the mechanical relations of the numbers. Concrete thinking was characterized by social and economic references made during the solutions of the problems, by weighing the importance of the subject matter content, and by associating the problem solver's experiences with the situations described in the problems. Random thinking was characterized by illogical or dissociated thinking, cloudy or distorted reasoning, by guesswork, or by obvious or admitted ignorance about how to proceed with a solution.

<u>The use of abstract, concrete, and random thinking by good and poor problem solvers in applying methods of solution.</u>-- Good problem solvers were twice as likely to use either abstract or concrete thinking in applying methods of solving the problems used in the study. Poor problem solvers, however, were six times as likely to use random thinking in applying the methods of solving these problems.

<u>Thought processes used in evaluating the results of methods of solving the problems.</u>-- After solving the problems, pupils evaluated their results in three different ways. Some used quantitative methods, some parallel methods and others meaningless methods of appraising their solutions. A quantitative method was characterized by identification of the results with a known numerical value, by reworking the computational processes, or the association of the answer with some known social or economic values. A parallel method was characterized by use of a reverse procedure, such as multiplication to check division; substitution of approximate values; and using a reversed set of number, as in exchanging the terms in multiplication. A meaningless method was characterized by the inability of the pupil to give a plausible reason, and by vague and evasive replies.

<u>The use of quantitative, parallel, and meaningless methods of evaluating the answers to problems.</u>-- Good problem solvers used quantitative methods twice as often as poor problem solvers in evaluating their answers to problems, and they used parallel methods five times as often as poor problem solvers. Poor problem solvers, however, used meaningless methods about fifteen times as often as good problem solvers.

<u>Generalizations from the data.</u>-- In general, good problem solvers

chose correct methods more often, used social reasons for choice of method more often, were more likely to use abstract and concrete thinking, and more frequently used quantitative or parallel methods of checking and evaluating the answers. On the other hand, poor problem solvers were more likely to choose wrong methods, to give doubtful reasons for their choices, to employ random thinking, and to evaluate their results by meaningless methods of appraisal. If, however, the anecdotal illustrations are examined, it may be clearly seen that poor problem solvers were capable of desirable problem solving procedures and that good problem solvers sometimes demonstrated inefficient procedures.

Findings of the Study with Regard for Understanding of Number Relations and Computation as Factors in Problem Solving

<u>Number relations as a factor in problem solving</u>.-- Differences were noted in the manner in which various pupils exhibited an understanding of the relationships of the numbers within a problem. These differences were classified, and the understanding of the number relations which pupils demonstrated in the problems they solved was divided into three groups. The groups were: clear number relations, partial number relations, and doubtful number relations. Clear number relations were characterized by identifying the numbers with other ideas in the problems, manipulating the numbers readily and logically, and by considering the reasonableness and evaluating the accuracy of the results. Partial number relations were characterized by cloudy number identification, incomplete recognition of number values, partial solution without clear understanding of the answer,

and inability to evaluate the solutions by means of the usual techniques of checking. Doubtful relations were those in which the meanings of numbers were obscured, computations random and without purpose, and the results without significance.

Understanding of number relations by good and poor problem solvers.-- Good problem solvers understood the relations of the numbers they used in solving the problems four times as often as poor problem solvers. Somewhat less frequently than poor problem solvers, they were partially clear in their understanding of number relations. On the other hand, poor problem solvers demonstrated doubtful understanding of number relations in working the problems nine times as often as good problem solvers.

Computational procedures by good and poor problem solvers in solving problems.-- A procedure that gave some evidence of the ability of pupils to perform computation necessary for problem solving was that of reading and manipulating numbers four or five digits in length. Good problem solvers were able to read numbers of this size twice as often as poor problem solvers, and they could manipulate them accurately three times as often as poor problem solvers. Good problem solvers vocalized their computational processes, when requested to do so, twice as often as poor problem solvers. Poor problem solvers used one and one-half times as much time, both in reading the problems and in performing the computation. Very little difference was apparent in the use of computational aids, and counting was almost negligible in frequency. Of all the computational aids, the use of extra marks on the paper was most often used. The frequency of extra marks of this kind among good and poor problem solvers was about the same.

The most common cause of failure to get a problem right was wrong choice of method. Surprisingly, incorrect computation was responsible for only about one-third of the errors of good problem solvers and about one-fifth of those of the poor problem solvers. When only the computation was considered, and the work of the pupils was surveyed without reference to their accuracy of choice of method, good problem solvers were ninety percent accurate in computation and poor problem solvers were seventy percent accurate.

Generalizations from the data.-- In general, good problem solvers were superior in understanding of number relations used in solving the problems. Their superiority in computation is not as marked as in other aspects of problem solving, but there is some indication that in this respect good problem solvers likewise were more proficient. Presentation of illustrations which explain the various computational procedures reveal, however, that the work of individual good and poor problem solvers was much alike in the employment of desirable and undesirable procedures.

Conclusions

Procedures of good and poor problem solvers.-- After careful consideration of the procedures of good and poor problem solvers in working arithmetic problems, the following conclusions were reached:

1. Good problem solvers, as a group are superior to poor problem solvers in the insight shown into a problem situation. They understand the meanings of the terms of the problems more frequently, use word and number clues to better advantage, apply social and economic information to a greater extent, and demonstrate greater familiarity with results by more efficient labeling of the answers.

2. Both good and poor problem solvers are capable of clear, partially clear, and doubtful insight when individual solutions to problems are considered. Thus, a poor problem solver may present a solution to a problem which shows equal or greater insight than that shown by a good problem solver to the same problem. On the other hand, a good problem solver may produce a solution which indicates bewilderment equal to or greater than any shown by a poor problem solver.

3. Insight into a problem is closely related to the ability of a pupil to recall background experiences and to apply those experiences in the problem solving operation.

4. Good problem solvers, as a group, are more likely to employ social reasons for choosing their method of solving a problem, to make use of abstract or concrete thinking in applying that method, and to evaluate their results through quantitative or parallel techniques. On the other hand, poor problem solvers are more likely to use undesirable thinking procedures.

5. Individual good and poor problem solvers deviate from the normal behavior of the group. When these deviations occur, poor problem solvers may show efficient use of desirable thinking processes and good problem solvers may give little evidence of the employment of effective thought processes.

6. Thinking processes employed in solving arithmetic problems are likewise an individual accomplishment, based upon the experiences of the pupils with number ideas and quantitative meanings. It may be reasonably expected that good problem solvers will produce more desirable responses, however, it cannot be assumed that poor problem solvers are

incapable of clear thought processes nor that good problem solvers are incapable of utter confusion and bewilderment in their thinking processes.

7. Understanding of relationships between numbers is more likely to occur in the solutions of good problem solvers as a group, than in those of poor problem solvers.

8. Poor problem solvers sometimes demonstrate that they understand clearly all of the number ideas and quantitative values involved in working a problem. On the other hand, good problem solvers may show in individual solutions that they are unable to understand these relationships.

9. Understanding of number relationships by either good or poor problem solvers is evident when the pupil shows familiarity with the ideas presented by the problem and with the quantitative values expressed by the numbers in the problems.

10. Good problem solvers are more likely to be efficient in computational skills than poor problem solvers, although superiority in this respect is not so pronounced as in the procedures discussed above. Reading four or five-digit numbers and manipulating these numbers are skills more often demonstrated by good problem solvers. Perhaps because of group superiority in such procedures as insight, thought processes, and understanding of number relationships, good problem solvers are more often able to vocalize their computational processes when they are requested to do so.

11. Poor problem solvers are slower as a group, both in reading and in solving arithmetic problems, than good problem solvers.

12. When individual performances are considered, poor problem solvers show that they too may be competent in such computational tasks as reading larger number, manipulating these numbers, and vocalizing

computational processes. Good problem solvers, on the other hand, show that in individual solutions to problems they are sometimes inadequate in these computational functions.

13. Computational aids do not figure excessively in the work of either good or poor problem solvers. The most common of these devices, that of using extra marks in borrowing or carrying, is a practice encouraged by some teachers of arithmetic in the intermediate grades. The infrequent occurrence of counting, both among good and poor problem solvers, indicates approaching maturity in computation and abandonment of the immature processes employed in the lower grades. The absence of drawings and diagrams was surprising, a fact which suggests either lack of instruction in the use of such devices or the premature dependence upon abstract reasoning in problems involving dimensional data.

14. As compared with wrong choice of method, computational error accounts for a relatively small percentage of the incorrect solutions to problems. If method is partialled out, accuracy in calculation by both good and poor problem solvers reaches approximately eighty percent. This accuracy tends to promote complacency on the part of pupils, who check their computation and if no errors are detected, conclude that their answer is right. This may be the result of over-emphasis upon computation and the performance of excessive mechanical arithmetical busywork. This is supporting evidence that the desire to compute may gloss over the elements of understanding and render them inactive.

Group and individual evaluation of problem solving procedures.--The employment of group data alone in such a study tends to emphasize only the probability of occurrence of certain behavior on the part of good and poor problem solvers. Such data fails to point out that on an individual basis, a poor problem solver may perform equally as well as a good problem solver or that a good problem solver may be as inept in working certain problems as any poor problem solver. Although the probability of desirable performance always favors the good problem solvers, individual good and poor problem solvers searched for the same verbal clues, recognized the same extraneous materials, and associated their own experiences with the situations described in the problems. Individual pupils from both groups failed to see the relationships between the ideas of the problems, followed ambiguous procedures, and were unable to relate the results of their computation with the problem situations. Good and poor problem solvers both labeled their answers and failed to label their answers as pupils from the two groups completed their solutions. Pupils from both groups chose methods thoughtfully and with care, or they jumped to conclusions and chose at random the first idea that occurred to them. Their thinking was abstract, concrete and random, computation was careless and careful, accurate and inaccurate, simple and involved. With the exception of the ability to solve more problems accurately, individual characteristics of good and poor problem solvers bear close resemblance. Group data, then, tends to show the frequency of occurrence of desirable, efficient problem solving procedures of good and poor problem solvers. Individual problem solving procedures of pupils, however, must be studied by techniques other than those which produce evidence of a pupil's behavior as a part of a group. In order to understand exactly what a pupil does when he works a problem, it is necessary to examine the individual work of good and poor problem solvers for evidence of similarity and dissimilarity of procedure as well as for the motivation behind that procedure.

Implications for Teachers

Procedures of good and poor problem solvers.-- Outcomes of this study may be interpreted by teachers of arithmetic in the following manner:

1. Insight into arithmetic problems has been shown to be an individual matter, based upon the experiences of pupils. Although superior performance may be expected from good problem solvers as a group, there is reason to expect extremes of insight, either desirable or undesirable, from pupils of either group.

2. Greater success in problem solving will occur as pupils develop the ability to relate ideas presented by problems, quantitative values represented by the numbers involved, and their own experiences. Achievement of this skill is entirely within the reach of either good or poor problem solvers.

3. The experiential backgrounds of pupils and the knowledge of quantitative values are important factors in the thinking processes employed by these pupils in choosing, applying, and evaluating the methods used in solving arithmetic problems. Whether a pupil is a good or a poor problem solver, thinking takes place when insight into a problem stimulates a desire to apply experiences and quantitative values to the solution of that problem. Definite growth in thinking processes can be expected only as the result of arithmetic experiences which are rich in meaning and are based upon the real interests of the pupils.

4. Teachers can expect good problem solvers to experience less

difficulty in understanding number relationships than poor problem solvers. Understanding of number relationships, however, may occur in the work of either good or poor problem solvers, and extremes of performance in this respect, both desirable and undesirable, may be expected in the work of pupils from either group. Understanding of number relationships apparently comes from repeated and varied experiences with numbers as well as with quantitative values. The fact that a poor problem solver may also be a slow learning child emphasizes the need for many and diverse experiences as well as repeated iteration of quantitative subject matter. The successful teacher of either good or poor problem solvers must build on present knowledge and must make arithmetical experiences meaningful to the pupils.

5. The slowness of poor problem solvers, both in reading and in working problems, suggests a lighter work load and carefully selected vocabulary. On the contrary, good problem solvers may be expected to do more work than poor problem solvers and to profit from enriched vocabulary experiences.

6. Computational aids do not present a serious problem, either in the work of good or of poor problem solvers. The use of carrying or borrowing marks represented most of the cases observed; it is likely that some sixth grade pupils have not yet achieved sufficient computational maturity to warrant abandonment of these practices. Gradual elimination of these practices is evidenced by the fact that more than two thirds of the pupils did not use any of them. A surprising absence of diagrams and drawings indicated that teachers have failed to impress upon pupils the value of these practices.

7. The fact that good problem solvers were able to vocalize

computation more often than poor problem solvers indicates that relaxation and freedom from strain might accompany greater familiarity with the vocabulary and the number relationships of the problems.

8. Accuracy of computation must be considered desirable procedure in arithmetic problem solving, both by good and by poor problem solvers. The same accuracy, however, may provide false security for both good and poor problem solvers, particularly when the method is chosen at random or by some other undesirable method. Pupils are likely to examine the computation in such cases, and if no error is discovered in the mechanics, declare that the problem is right. Confidence in their ability to perform a certain computational process might lead to the selection of that process rather than one they consider more difficult. Over-emphasis of computation by teachers might restrict the evaluation of results of problem solving to superficial checking of computation alone.

Group and individual evaluation of problem solving procedures.--Group methods have served teachers for many years as devices for measuring class achievement, diagnosis of pupil attainment, and testing instructional materials. They have not, however, given teachers much information about what a pupil actually does when he works an arithmetic problem. This study isolated certain significant procedures employed by pupils while working arithmetic problems: namely, insight, thought processes, number relationships, and computational skills. The study further outlined certain techniques that may be used in examining these procedures. These techniques of studying the procedures of individual pupils during problem solving may be employed by teachers of arithmetic, not only to substantiate the findings of this study but also to conduct further investigations along the same line.

The teacher of arithmetic may learn from this study that either good or poor problem solvers are capable of desirable, partly desirable, or undesirable problem solving procedures. Desirable procedures are most often associated with meaning and understanding. Undesirable procedures are most frequently associated with gussswork, trial and error, or random calculation based upon lack of understanding. Purposeful study of the problem solving behavior of good and poor problem solvers may be profitable to the teacher who would like to know more about how a pupil goes about working an arithmetic problem.

Suggestions for Further Research

The following studies were suggested while the writer was preparing and executing the present investigation. Each of them presented an area of research which might have produced significant results. This report opens these fields for a study of individual procedures of pupils while working arithmetic problems. The following topics are suggested as being significant for anyone wishing to make a study of a nature similar to the present investigation:

1. A study of the characteristics of the procedures of good and poor problem solvers, using pupils selected at random from an elementary school population.

2. A study of the characteristics of the procedures of good and poor problem solvers, using intelligence rather than arithmetical achievement as the basis for selecting the sample.

3. A study of the characteristics of problem solving procedures of pupils while their ability limits are being explored.

4. A study of the characteristics of the procedures of good and poor problem solvers, using subject matter content other than whole numbers, for example, fractions or decimals.

5. A study of the effects of directive assistance upon the characteristics of the procedures of good and poor problem solvers.

6. A study of the effects of frustration upon the characteristics of the procedures of good and poor problem solvers.

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APPENDIX

A. TESTS

B. PROBLEM LISTS

C. SELECTION TABLES

D. TRANSCRIPTIONS OF PROBLEM SOLUTIONS

APPENDIX A

TESTS

FUNCTIONAL EVALUATION IN MATHEMATICS

William A. Brownell, Editor

Number Right

Test 2 — Problem Solving

by

BEN. A. SUELTZ

lementary Level | **Grades 4, 5, and 6** | **Form A**

AME .... Boy .... Girl .... Grade ....

acher .... Date .... (Year / Month / Day)

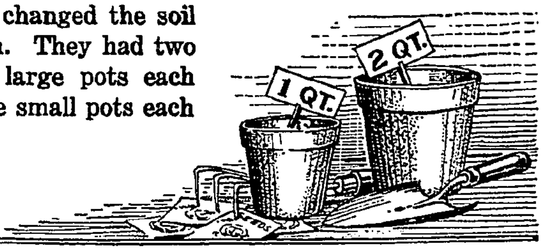
hool .... Born .... (Year / Month / Day)

ty .... State .... Age .... (Years / Months / Days)

RECTIONS — Read each problem carefully and be sure to do just what it asks. Show your work in the work ace and write your answer on the dotted line at the side of the page. The sample problems show you how to this.

SAMPLE PROBLEMS

The pupils in one room changed the soil for the plants in their room. They had two sizes of flower pots. The large pots each held 2 quarts of soil and the small pots each held 1 quart of soil.



	Space for Work	Answers
How many quarts of soil were needed for 7 of the large-size pots?	7 × 2 = 14	A. ...14... qt.
For new soil for the plants 1 quart of sand was used for each 2 quarts of soil from the garden. How much sand was used with a bushel (32 quarts) of soil from the garden?	32 ÷ 2 = 16	B. ...16... qt.

EDUCATIONAL TEST BUREAU
EDUCATIONAL PUBLISHERS, Inc.

Philadelphia 3433 Walnut St. | Minneapolis 720 Washington Ave. S. E. | Nashville 2106 Pierce Ave.

THE NEW PARK STREET SCHOOL

Next year the pupils from the old Hill School will go to the new school on Park Street. The new Park Street School will have large pleasant rooms and a fine new playground. There will be two classrooms for each of the grades one through six.

	Space for Work	Answers
1. How many classrooms will there be for all the grades, one through six?		1. ....
2. The new school will have seats for 480 pupils. That is how many more than the 368 pupils now in the old Hill School?		2. ....
3. The new school has a total of 6 acres for playgrounds. This is divided into three equal spaces. How many acres has each of the spaces?		3. ....
4. Of the 368 pupils enrolled in the school, 179 are girls. The rest are boys. How many are boys?		4. ....
5. One room has six rows of seats with 6 seats in each row and another row with 4 seats. How many seats are in this room?		5. ....
6. The new school will have 2 teachers for each of the six grades. There will be 3 other special teachers, also. How many teachers will the school have?		6. ....
7. One day Sally was asked to divide a package of 500 sheets of drawing paper equally among four rooms. How many sheets did she give to each room?		7. ....
8. To prevent waste, the new school will allow each pupil 2 paper towels a day. How many towels will be needed for a week of 5 school days for the 368 pupils?		8. ....
9. Pupils are in school from 8:45 to 11:50 each morning. How many hours and minutes is this?		9. ......hr. ......mi
10. To decorate for a party, one class needs 105 silver stars. If 7 stars can be cut from one sheet of silver paper, how many sheets of paper are needed?		10. ....

Go on to Page

IN THE FOOD MARKET

Ellen went shopping with her mother. They took a list of things to buy but did not buy until they could see the quality of the foods in the market. The prices that day were:

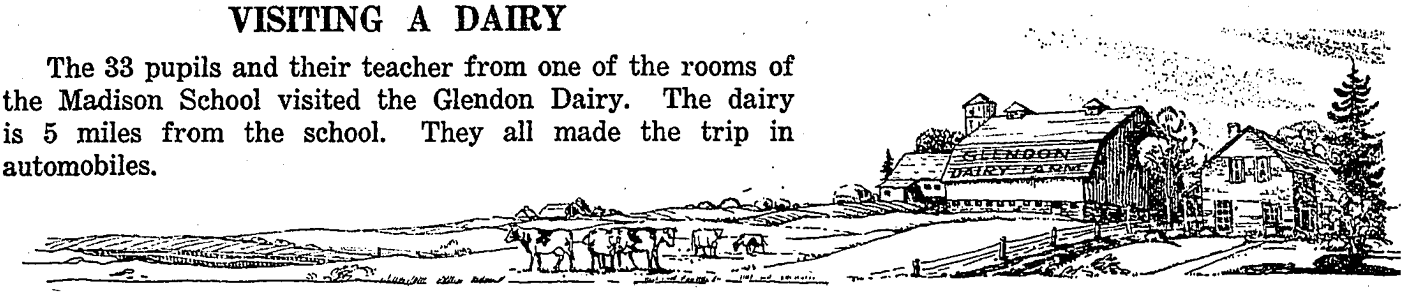
CANNED GOODS		PRODUCE		MEATS	
omatoes, 20 oz. can	21¢	Oranges	37¢ doz.	Roast	58¢ lb.
eaches, 30 oz. can	27¢	Celery	2 for 17¢	Bacon	62¢ lb.
ilk, tall can	3 for 39¢	Spinach	10¢ lb.	Steak	94¢ lb.
eas, 20 oz. can	22¢	Eggs	68¢ doz.	Hamburger	2 lb. for $1.15

	Space for Work	Answers
1. How much did 6 cans of milk cost?		11. ....
2. How much did a dozen oranges and two dozen eggs cost?		12. $ ....
3. How much change did Ellen's mother get from a dollar when she bought a can of tomatoes and a can of peaches?		13. ....
4. How much cheaper was 2 pounds of hamburger than 2 pounds of steak?		14. ....
5. Ellen's mother chose a 4-pound roast and a half pound of bacon. How much did these meats cost?		15. $ ....
6. A 3-pound package of frozen lima beans sold for 75¢. One-pound packages sold for 30¢. How much was saved by buying one large package rather than three small one-pound packages?		16. ....
7. Ellen's mother spent $7.78 for groceries and meats. She also paid 25¢ for delivery. How much change did she have left from a ten-dollar bill?		17. $ ....
8. The 4-pound roast needed to be baked 20 minutes for each pound of weight. If it was placed in the oven at 11 o'clock, when was it ready?		18. ....
9. How many 20-ounce cans would be filled with the tomato juice from a dozen 30-ounce cans?		19. ....
0. How much did $3\frac{1}{2}$ pounds of steak cost?		20. $ ....

Go on to Page 4

VISITING A DAIRY

The 33 pupils and their teacher from one of the rooms of the Madison School visited the Glendon Dairy. The dairy is 5 miles from the school. They all made the trip in automobiles.



Space for Work | Answers

21. They left the school at 8:50 in the morning. If they were gone 2½ hours, what time was it when they returned?

21. ....

22. Each automobile could carry 6 children or 5 adults in addition to the driver. How many cars were needed for the pupils and teacher?

22. ....

23. At the rate of 20 miles per hour, how many minutes did it take to drive the 5 miles from the school to the dairy?

23. .... mi

24. Mr. Glendon has 35 head of cattle on his 315-acre farm. How many acres does this average for each of his cattle?

24. ....

25. One cow gives 7 gallons of milk each day. How much is this worth at 15¢ per quart?

25. $ ....

26. Each day Mr. Glendon fills 28 crates, each holding 24 quart bottles. At 2 pounds per quart, how many pounds of milk are needed to fill the 28 crates?

26. ll

27. Mr. Glendon gets 21¢ per quart for the milk he delivers. The milk itself costs only ⅓ of the 21¢ and the rest is for bottling, delivery, and other services. How much per quart is for bottling, delivery, and other services?

27. ....

28. Mr. Glendon's largest expense is for feed. How much must he pay for 700 pounds of feed when the price is $5.18 per hundred pounds?

28. $ ....

29. Mr. Glendon gave each of 40 persons a 6-ounce bottle of milk. How many quarts were needed to fill the 40 bottles?
(1 quart = 32 ounces)

29. .... q

30. On the average, a milk bottle makes 20 trips out of the dairy before it is lost or broken. To the nearest whole number, how many bottles are needed for making a total of 365 trips per year?

30. ....

 End of Test. Look over your work. *No. Right*..........

ORM A
-8

NATIONAL ACHIEVEMENT TESTS

By PROFESSOR ROBERT K. SPEER, Ph.D., New York University
General Editor and Co-Author
and
SAMUEL SMITH, Ph.D., Research Director

SCORE
Part I ______
Part II ______
Part III ______
Part IV ______
TOTAL ______

ARITHMETIC TEST — REASONING

(For Grades 3 to 8)

NAME ________________ (Last) (Middle) (First) NAME OF SCHOOL ________________

AGE: Years ____ Months ____ GRADE ____ CITY ________________ STATE ________________ DATE ________

GENERAL DIRECTIONS: After doing Part I, read the directions for Part II. Then do Part II. Go on in the same way with Parts III and IV. DO NOT HURRY; BUT DO NOT WASTE TIME. If you do not know the answer to a question, go on to the next one. DO NOT GUESS.

PART I. COMPARISONS

(Key No. AK)

READ THE FOLLOWING SAMPLE:

Mother gave you 1 cent. Father gave you 2 cents. Sister gave you 5 cents. WHO GAVE YOU MOST?

m. mother
z. father
a. sister

In this SAMPLE, sister gave you most. A line was drawn under the word "sister".

PRACTICE EXERCISE:

Bread costs 10 cents a loaf. Milk costs 10 cents a quart. Candy costs 5 cents a bar. WHICH OF THE FOLLOWING COSTS MOST?

c. 2 loaves of bread
e. 2 quarts of milk
o. 5 bars of candy

DIRECTIONS: BELOW, there are 10 problems. For each problem, DRAW A LINE under the right answer. Do all your work on this paper.

1. Mary sells candy for 2 cents a piece. Albert sells the same candy for 3 cents a piece. George sells the same candy for 4 cents a piece. WHO SELLS THE CANDY AT THE LOWEST PRICE?

b. Mary
c. Albert
d. George

2. Henry weighed 95 pounds in January; he gained 3 pounds in February; and he weighed 97 pounds in March. IN WHICH MONTH DID HENRY WEIGH THE MOST?

m. January
n. February
o. March

(GO ON TO NEXT COLUMN.)

3. Three boys sold papers every day, including Sundays. John earned 26 dollars a month. George earned 6 dollars a week. Albert earned 1 dollar a day. WHO EARNED THE MOST IN A MONTH?

i. Albert
j. George
k. John

4. There are three places. It costs 200 dollars to go to the first place, one way. It costs 200 dollars to go to the second place and come back (both ways). It costs 300 dollars to go to the third place, one way. TO WHICH PLACE IS IT CHEAPEST TO GO?

a. the first place
c. the third place
e. the second place

GO ON TO NEXT PAGE.)

PART I. COMPARISONS (Continued)

5. John sells pencils at 5 cents each. George sells pencils at 6 for a half-dollar. Sam sells pencils at 45 cents a dozen. WHO SELLS PENCILS AT THE LOWEST PRICE?
   x. John
   f. George
   g. Sam

6. A pound of meat costs 40 cents. Four pounds of cake costs 80 cents. Six pounds of candy costs 1 dollar. WHICH OF THE FOLLOWING COSTS LEAST TO BUY?
   a. a pound of meat
   b. two pounds of candy
   c. two pounds of cake

7. A nine-dollar coat lasts one year. A seven-dollar coat lasts ten months. A twelve-dollar coat lasts fifteen months. WHICH COAT WEARS BEST FOR THE MONEY?
   e. the seven-dollar coat
   t. the nine-dollar coat
   p. the twelve-dollar coat

8. The first position pays the income from $3,500.00 at 6%, plus a salary of $1,000.00 a year. The second position pays the income from $3,000.00 at 7%, plus a salary of $1,200.00 a year. The third

(GO ON TO NEXT COLUMN.)

position pays the income from $3,500.00 at 5 plus a salary of $1,500.00 a year. WHICH POS TION PAYS MOST?
   h. the first position
   i. the third position
   r. the second position

9. Sam has 2 pieces of candy to share with John a Mary. Sam gives half of one piece to John, a shares the rest of this piece equally with Ma Sam gives half of the other piece to Mary, a shares the rest of this piece equally with Jol WHO GETS THE SMALLEST SHARE OF TE CANDY?
   s. John
   n. Sam
   f. Mary

10. A cake weighing 26 ounces cost 24 cents a pour A cake weighing one-half pound costs 11 cents. cake weighing 5 ounces costs 6 cents. WHIC CAKE SELLS AT THE LOWEST PRICE?
   y. the 26-ounce cake
   z. the half-pound cake
   g. the 5-ounce cake

(GO ON TO PART II.)

NUMBER RIGHT IN PART I ______

PART II. PROBLEM ANALYSIS
(Key No. AL)

READ THE FOLLOWING EXAMPLE:

PROBLEM: You bought 5 pounds of butter for 2 dollars. How much did the butter cost per pound?

THIS PROBLEM TELLS YOU
   d. how much you spent for 1 pound
   b. how much money you had left
   c. how much butter you bought
   a. how old you are

In this SAMPLE, sentence "c" is true. A line was drawn under sentence "c".

PRACTICE EXERCISE:

PROBLEM: You bought 3 loaves of bread for 30 cents. How much did 1 loaf cost?

THIS PROBLEM ASKS YOU TO FIND
   m. how much bread you bought
   n. how much 1 loaf cost
   o. how much you spend for 3 loaves
   p. how much money you had

DIRECTIONS: In this Part, there are 20 questions. For each question, DRAW A LINE UNDER THE ONE SENTENCE *that is true. Read every problem carefully.*

(GO ON TO NEXT PAGE

PART II. PROBLEM ANALYSIS (Continued)

OBLEM A: You had 10 cents. Then you got 5 its from mother, and 3 cents from father. How ich did you have all together?

PROBLEM A TELLS YOU

d. how much you gave away

s. how much you spent

a. how much money your mother has

e. how much money you had at the start

PROBLEM A ASKS YOU TO FIND

f. how much you had at the start

o. how much you got from father

n. how much you had all together

v. how much money you got from mother

IN PROBLEM A, WHICH ANSWER IS MOST NEARLY CORRECT?

y. 10 cents

x. 25 cents

i. 20 cents

t. 30 cents

TO FIND THE ANSWER TO PROBLEM A, WHAT SHOULD YOU DO?

b. add

m. subtract

z. multiply

c. divide

OBLEM B: You save 6 cents every day. How ch will you save in 7 days?

PROBLEM B TELLS YOU

h. how much money you now have

k. how much you can get from mother

g. how much you save every day

p. how much you spend every day

WHAT DOES PROBLEM B ASK YOU TO FIND?

u. how much you save every day

b. how much you will save in 7 days

j. how much you spend every day

o. how much you now have

(GO ON TO NEXT COLUMN.)

7. IN PROBLEM B, WHICH ANSWER IS MOST NEARLY CORRECT?

l. 5 cents

q. 10 cents

e. 45 cents

f. 35 cents

8. TO FIND THE ANSWER TO PROBLEM B, WHAT SHOULD YOU DO?

m. add

r. subtract

v. divide

n. multiply

PROBLEM C: If you save three dollars every week, how many weeks will it take you to save enough to buy a thirty dollar bicycle?

9. PROBLEM C TELLS YOU

c. how much you now have

i. how much you save every week

s. how long it will take you to save 30 dollars

w. how many weeks you have saved 10 dollars

10. PROBLEM C ASKS YOU TO FIND

g. how long it will take you to save 30 dollars

a. how much you save every week

d. how much you now have

t. how much you had when you began to save

11. IN PROBLEM C, WHICH ANSWER IS MOST NEARLY CORRECT?

b. twelve weeks

u. five weeks

f. thirty weeks

r. twenty weeks

12. TO FIND THE ANSWER TO PROBLEM C, WHAT SHOULD YOU DO?

z. add

h. multiply

e. divide

k. subtract

(GO ON TO NEXT PAGE.)

PART II. PROBLEM ANALYSIS (Continued)

PROBLEM D: An airplane travels 120 miles per hour. You must pay 7 cents per mile, to travel by airplane. How much will it cost you to travel, by airplane, 960 miles in 8 hours?

13. PROBLEM D TELLS YOU
    - c. how much it will cost you to travel 8 hours
    - v. how much it will cost you to travel 1 hour
    - i. how much it will cost you to travel 1 mile
    - o. how many hours you can travel for 7 cents

14. PROBLEM D ASKS YOU TO FIND
    - m. how much it will cost to travel 1 hour
    - p. how far you will travel in 1 hour
    - j. how far you will travel in 8 hours
    - n. how much it will cost to travel 8 hours

15. IN PROBLEM D, WHICH ANSWER IS MOST NEARLY CORRECT?
    - x. $100.00
    - q. 120 miles
    - s. $15.00
    - g. $70.00

16. TO FIND THE ANSWER TO PROBLEM D, WHAT SHOULD YOU DO?
    - d. add
    - e. multiply
    - f. subtract
    - y. add, and then divide

PROBLEM E: Three boys have a stamp club. The first boy invested ten dollars for stamps; the second boy invested five dollars for stamps; and the third boy invested five dollars for stamps. They have just sold the stamps for thirty dollars. Each boy must now get his money back; and the profits are to be divided in proportion to the amount each boy invested. How much more profit would the first boy make than the third boy?

(GO ON TO NEXT COLUMN.)

17. PROBLEM E TELLS YOU
    - w. how much more the first boy invested th the third boy
    - h. how much more profit the first boy made th the third boy
    - l. how much profit each boy made
    - n. how much each boy invested in the club, f stamps

18. PROBLEM E ASKS YOU TO FIND
    - o. how much the three boys invested all togeth
    - s. how much more the first boy invested than t third boy
    - i. how much more the first boy gained than t third boy
    - t. how much profit the three boys made together

19. IN PROBLEM E, WHICH ANSWER IS MOS NEARLY CORRECT?
    - d. 12 dollars
    - a. 6 dollars
    - c. 20 cents
    - b. 2 dollars

20. TO FIND THE ANSWER TO PROBLEM WHAT SHOULD YOU DO?
    - e. find the total profits made, and divide so th the first boy will have one-half the tot profits
    - p. find the profit made by the second boy, an divide by 3
    - u. divide the total profit into three equal part
    - k. add the amounts the three boys invested, pl the amount received for the stamps th sold

---

(GO ON TO NEXT PAGE.)

NUMBER RIGHT IN PART II ______

PART III. FINDING THE KEY TO A PROBLEM
(Key No. AM)

READ THE FOLLOWING SAMPLE:

You had 3 cents. Then father gave you 2 cents. BEFORE TELLING HOW MUCH YOU HAD ALL TOGETHER, WHAT SHOULD YOU FIND OUT?

a. how many cents there are in a dollar
b. the sum of money you started with and the money father gave you
m. how much you had after father gave you 1 cent

In this SAMPLE, sentence "b' is true. A line was drawn under sentence "b".

PRACTICE EXERCISE:

You have 1 cent. You want to buy a toy that costs 4 cents. BEFORE TELLING HOW MUCH YOU NEED, WHAT SHOULD YOU FIND OUT?

s. how much you must add to 1 cent, so that you will have 4 cents
h. how many cents there are in a dime
c. how many toys you could buy for ten cents

DIRECTIONS: BELOW, there are 10 problems. For each problem, DRAW A LINE under the one sentence that is true. DO NOT GUESS.

1. You must buy seven books at a price of 2 dollars each. Your father will give you the money, if you tell him how much you need. BEFORE GETTING THE MONEY, WHAT SHOULD YOU FIND OUT?
   k. the cost of 2 books
   a. the number of cents in a dollar
   b. the cost of seven books

2. You have paid five cents for five pieces of candy. You want to sell one piece of candy for what it costs you. BEFORE SELLING THE CANDY, YOU SHOULD FIND OUT
   d. how many pieces of candy you can buy for ten cents
   e. how much you paid for one piece of candy
   f. how many cents there are in a nickel

3. You have four old books for which you paid one dollar each. You could buy two new books at the same price per book. You want to tell how much money you would need to buy the books. YOU SHOULD FIND OUT
   o. how much six books would cost
   u. how much the four old books cost all together
   i. the cost of one new book

4. You earn five dollars every week. You want to tell how many weeks it will take you to earn enough for a bicycle that costs thirty-five dollars. YOU MUST FIND OUT
   g. how many weeks you must earn five dollars a week, to buy the bicycle
   t. how many days there are in a week
   c. how much you will earn in 35 weeks

(GO ON TO NEXT COLUMN.)

5. In each of two stores, there are a coat and hat which you may want to buy. The first store asks five dollars for both the coat and hat. The second store asks one dollar and fifty cents, for the hat alone. You must decide which store has the lowest price for both the hat and coat. BEFORE DECIDING, YOU SHOULD FIND OUT
   n. how much the hat and coat, together, cost in the second store
   r. how much the coat alone costs in the first store
   s. how much the coat in the first store plus the hat in the second store would cost

6. You have \$6.40. You want to keep \$4.50, and spend the rest for a toy that costs \$2.80. You want to decide whether or not you will have enough money to keep \$4.50, and still buy the toy. BEFORE DECIDING, YOU SHOULD FIND OUT
   c. how much money you would have if you did not buy the toy
   d. how much you would have left if you subtracted the cost of the toy from the amount of money you want to keep
   e. how much you would have left if you subtracted the cost of the toy from the amount of money you now have

(GO ON TO NEXT PAGE.)

PART III. FINDING THE KEY TO A PROBLEM (Cont'd)

7. Fred has thirteen apples. He wants to give half the apples to Sam; and Sam wants to give half of what he gets to you. You must tell us how many apples you are going to get. BEFORE TELLING US, YOU MUST FIND OUT

   i. how many apples Sam would give you after he got six and one-half apples from Fred

   j. how many apples Sam would have left if he gave you one-third of all the apples

   k. how many apples you would have if you got one-third of what Sam got

8. You have fifty pencils, of which 32% are green, 58% are blue, and the remainder are black. You want to tell us how many of the pencils are black. BEFORE TELLING US, YOU MUST FIND OUT

   f. how many pencils are blue

   g. what percentage of the pencils is black

   h. the number of green pencils plus the number of black pencils

9. A schoolroom that is perfectly square, is 1600 square feet in area. You want to buy enough wire to go all around the room. BEFORE TELLING

(GO ON TO NEXT COLUMN.)

HOW MUCH ALL THE WIRE WOULD COS[T] YOU MUST FIND OUT

   a. the cost of enough wire to cover one squa[re] foot of the room

   b. how much wire equals the length of one sid[e] of the room

   c. the cost of enough wire to fill the room

10. You have $1,000.00 in a bank which pays you 4[%] interest per year. You could loan $500.00 safel[y] to George for three months at 6% per year. Yo[u] want to tell us how much you would gain if yo[u] took $500.00 out of the bank, and loaned thi[s] money to George for three months. BEFOR[E] TELLING US, YOU MUST FIND OUT HO[W] MUCH INTEREST

   l. the bank would pay you if you kept $1,000.0[0] there for nine months at 4%

   m. you would get from the bank on $1,000.00 fo[r] three months at 2%

   n. George would pay you on $500.00 for thre[e] months at 6%

(GO ON TO PART IV.)

NUMBER RIGHT IN PART III ________

PART IV. PROBLEMS
(Key No. AN)

DIRECTIONS: There are ten problems in this part. Do these problems in order, beginning with the first. Do all your work on this paper. For each problem, WRITE YOUR ANSWER on the line, at the right of the word, ANSWER________.

1. You have fifty cents. Your mother then gives you seventeen cents. Then your father gives you thirty cents. HOW MUCH WILL YOU HAVE ALL TOGETHER?

1. ANSWER________

(GO ON TO NEXT COLUMN.)

2. If you buy 22 tickets at a price of 18 cents each HOW MUCH WILL YOU SPEND?

2. ANSWER________

(GO ON TO NEXT PAGE.)

PART IV. PROBLEMS (Continued)

3. You bought 15 apples, for which you paid 3 cents each. You also bought 12 melons for which you paid 10 cents each, and 27 peaches for which you paid 5 cents each. HOW MUCH DID YOU SPEND ALL TOGETHER?

3. ANSWER__________

4. You pay 18 dollars per month for rent. You also pay an average of 32 dollars per month for food. HOW MUCH WILL RENT AND FOOD TOGETHER COST YOU FOR ONE YEAR?

4. ANSWER__________

5. You spend $6.00 for ten books. HOW MUCH WILL THREE OF THESE BOOKS COST YOU?

5. ANSWER__________

(GO ON TO NEXT COLUMN.)

6. A pound of butter costs twenty cents. A pound of lard costs 12 cents. HOW MUCH MORE MUST YOU PAY FOR A HALF POUND OF BUTTER THAN YOU MUST PAY FOR A QUARTER POUND OF LARD?

6. ANSWER__________

7. HOW MUCH INTEREST MUST YOU PAY IF YOU BORROW $126.00 FOR 90 DAYS AT 6%?

7. ANSWER__________

(GO ON TO NEXT PAGE.)

PART IV. PROBLEMS (Continued)

8. Mr. Jones, who now pays $36.00 per month for rent, has asked for a 25% reduction in rent. IF HE GETS ONE-FOURTH OF THE REDUCTION FOR WHICH HE ASKS, HOW MUCH LESS WILL HIS ANNUAL RENT BE THAN IT IS NOW?

8. ANSWER________

9. You have $500.00 in a bank which pays 3% per year. You could buy a bond for $500.00 paying 6% per year. HOW MUCH WOULD YOU GAIN BY TAKING THE MONEY OUT OF THE BANK TO BUY THE BOND, IF YOU KEPT THE BOND FOR ONE YEAR, AND THEN SOLD IT FOR $550.00?

9. ANSWER________

(GO ON TO NEXT COLUMN.)

10. Last year, the state tax on your property w $1.50 per thousand dollars of assessed valuatio This year the tax is $1.80 per thousand. Last ye you paid a tax on property assessed at $8,000.0 This year you must pay a tax on property assess at $7,000.00; but if you pay promptly this yea you will be allowed a 2% discount. IF YOU PA PROMPTLY, HOW MUCH MORE WILL YO HAVE TO PAY THIS YEAR THAN YOU PAI LAST YEAR?

10. ANSWER________

(END OF TEST. LOOK OVER YOUR WORK.)

NUMBER RIGHT IN PART IV ________

ACORN PUBLISHING COMPANY, Rockville Centre, Long Island, New York

IOWA EVERY-PUPIL TESTS OF BASIC SKILLS

New Edition

TEST D: BASIC ARITHMETIC SKILLS — FORM O

ADVANCED BATTERY — GRADES 5-6-7-8-9

By

H. F. Spitzer, in collaboration with Ernest Horn, Maude McBroom, H. A. Greene, and E. F. Lindquist (General Editor), all of the College of Education, State University of Iowa, with the Assistance of the Faculty of the University Experimental Schools.

Directions: The other side of this page is an *answer sheet* on which you will mark your answers to all of the questions in this est. To use this answer sheet, you will have to tear it off. Do this now, tearing very carefully along the perforation at the ɔft-hand side of this page.

* * * * *

Each question in Part I of the test is followed by four possible answers, only one of which is correct or definitely bettei ıan any of the others. To answer a question, first decide which is the best answer, then look at the rows of boxes undei art I on the answer sheet and find the *row* of boxes numbered the same as the *question*. Then place an **X** in one of these ıur boxes, as follows:

If you think the *first* answer is best, mark the *first* box in the row.

If you think the *second* answer is best, mark the *second* box in the row.

If you think the *third* answer is best, mark the *third* box in the row.

If you think the *fourth* answer is best, mark the *fourth* box in the row.

Mark only one box in each row. If you change your mind about an answer, erase your first mark very thoroughly.

Directions for Parts II and III of the Test will be given to you after you finish Part I.

Answer the questions in all parts of the test in the order in which they are given, but do not linger too long over difficult ıestions or problems. Skip them, and return to them later if time permits. If you do skip any questions, be sure to skip e corresponding boxes on the answer sheet also.

Do not begin work until you are told to do so.

HOUGHTON MIFFLIN COMPANY

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PAGES 1-2. PART I

ANSWER SHEET: ARITHMETIC TEST

Name ______________________ Grade ______
(Last Name) (First Name)

Age on Last Birthday ______ (Years) Number of Months Since Last Birthday ______ Sex ______ (Boy or Girl)

Town or City ______________________

School ______________________

Teacher ______________________ Date ______

SCORES

	Poss. Score	Raw Score	Grade Equivalent*
Part I	(40)		
Part II	(33)		
Part III	(31)		
Total	(104)		

*See page 10 in Examiner's Manual for Conversion Table for Form O.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

PAGES 3-4. PART II

Section A

Samples:

0	3	☒ 4	5	N
00	2	4	5	☒ N
41	1916	2017	1918	N
42	1086	186	86	N
43	4340	42400	43500	N
44	$20\frac{3}{5}$	$2\frac{8}{13}$	21	N
45	130	129	132	N
46	1751	1641	1741	N
47	2437140	262740	2427140	N
48	$40\frac{82}{85}$	38	308	N
49	$\frac{7}{12}$	$1\frac{1}{6}$	$1\frac{1}{3}$	N
50	$36\frac{2}{3}$	$36\frac{1}{3}$	$37\frac{2}{3}$	N
51	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{1}{3}$	N
52	$6\frac{3}{4}$	12	27	N
53	$12\frac{1}{2}$	$13\frac{1}{3}$	$12\frac{2}{3}$	N
54	$\frac{2}{3}$	$1\frac{2}{3}$	$1\frac{1}{3}$	N
55	28	30	60	N
56	$\frac{2}{7}$	7	$3\frac{1}{2}$	N
57	$8\frac{5}{24}$	$8\frac{11}{24}$	$8\frac{3}{8}$	N
58	$7\frac{17}{24}$	$8\frac{2}{3}$	$8\frac{17}{24}$	N
59	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{3}{4}$	N
60	10	$15\frac{1}{2}$	$5\frac{4}{5}$	N
61	$8\frac{3}{4}$	$8\frac{1}{4}$	$8\frac{3}{8}$	N
62	$2\frac{1}{4}$	$1\frac{1}{4}$	$2\frac{1}{2}$	N
63	$10\frac{1}{2}$	$12\frac{3}{4}$	13	N

Section B

64	792	782	.792	N
65	1.22	122	12200	N
66	80	20	8000	N
67	80	800	25	N
68	$500	$50	$100	N
69	$\frac{3}{4}$%	25%	75%	N
70	$\frac{495}{1000}$	49.5%	4.95%	N
71	$\frac{76}{100}$	.076	.76	N
72	.9	$\frac{9}{10}$	$\frac{9}{100}$	N
73	$44.63	$44.60	$4463.00	N

PAGES 5-6. PART III

74	41	52	111	N
75	108	110	112	N
76	6	7	8	N
77	2	4	5	N
78	74	84	147	N
79	47	49	52	N
80	$1.36	$1.62	$1.72	N
81	$.40	$4.00	$6.00	N
82	40%	50%	60%	N
83	$7\frac{1}{2}$¢	11¢	$12\frac{1}{2}$¢	N
84	3	60	120	N
85	12	20	32	N
86	2,500 tons	3,000 tons	6,000 tons	50,000 tons
87	3 times	4 times	5 times	6 times
88	105 per cent	one-half	two-thirds	three-four[illegible]
89	one-third	one-fifth	one-tenth	three-four[illegible]
90	6 hours	$7\frac{1}{2}$ hours	$8\frac{1}{2}$ hours	9 hours
91	40 miles	47 miles	50 miles	N
92	3 miles	6 miles	7 miles	10 miles
93	15%	40%	55%	7[illegible]
94	$132,000	$1,300,000	$1,320,000	N
95	7 miles	20 miles	91 miles	N
96	96	97	99	N
97	51 sq. mi.	120 sq. mi.	240 sq. mi.	480 sq. mi
98	108 sq. ft.	680 sq. ft.	720 sq. ft.	N
99	9 feet	108 feet	20 feet	N
100	$129.20	$387.60	$1162.80	N
101	$\frac{1}{18}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$
102	None	$1.50	$.50	N
103	$185	$260	$285	N
104	$9	$15	$18	$2[illegible]

PART III. PROBLEMS

Directions: Read each problem carefully. Do your work on scratch paper. Compare your answer with those given on the answer sheet and mark the proper box, as you did in Part II.

In some problems, you are asked to give only an *approximate* answer. For these particular problems, no N is given on the answer sheet, but you are to mark the box in front of the answer that is *most nearly* like your own.

At the beginning of the year, there were 13 girls and 18 boys in the third grade, 15 girls and 12 boys in the fourth grade, 11 girls and 16 boys in the fifth grade, and 13 girls and 13 boys in the sixth grade of the Jackson School.

74 How many girls were in the four grades?

75 How many more boys than girls were there in all four grades?

76 At the end of the year, there were 34 children in the fifth grade. How many more children were in the fifth grade at the end of the year than at the beginning?

77 The absences in the fifth grade during one week were as follows: Monday 3, Tuesday 0, Wednesday 5, Thursday 2, Friday 5. What was the average number of absences for each day?

On an automobile trip with his father, Tom kept a record of the speedometer readings as they drove along. At home it read 9209; at Salem the reading was 9217; at Vale City, 9291 miles; and at Greenville, 9356 miles.

78 How far was it from Salem to Vale City?

79 If it took 3 hours to make the trip from home to Greenville, how many miles per hour did they travel?

80 Before he started, Tom's father bought 8 gallons of gasoline at 17¢ per gallon and a quart of oil at 36¢ per quart. What was his bill?

The Girls' Club sold Christmas cards at $1.00 per box. The cards cost them 60¢ per box.

81 How much profit did they make on each box of cards they sold?

82 Their profit was what per cent of the selling price?

---

83 How much would it cost to send a letter weighing 2½ ounces to Australia if postal rates are 5¢ for the first ounce and 3¢ for each additional ounce or fraction of an ounce?

84 How many tons of coal can be stored in a bin 4 feet wide, 10 feet long, and 3 feet deep? (Coal weighs about 50 pounds per cu. ft.)

85 The seventh grade planned to take a trip to an Indian reservation. The teacher said, "Mr. Brown is taking 5 of the children in his car, and I can take 3. That means we have rides for one-fourth of the class." How many children were in the seventh grade?

86 Ship A is rated as of 12,480 tons. If ship B is about one-fourth as large, what is its tonnage? (Note that in this problem no *exact* relationship is stated. Therefore, your answer will be only an approximation.)

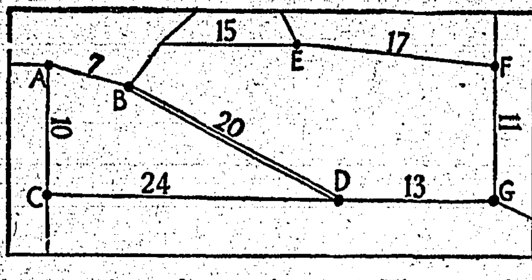
87 A certain airplane has a top speed of 435 miles per hour. The airplane is how many times as fast as an automobile which has a top speed of 90 miles per hour? (Only an *approximate* answer is required.)

88 If a man plants 105 of his 160 acres in corn, about what part of his farm does he plant in corn? (Only an *approximate* answer is required.)

89 A dress in a store window has these two prices marked on it: "Was $12.98 — Now $10.25." The amount that the dress was reduced is what part of the original price? (Only an *approximate* answer is required.)

90 John is waiting for a train that is scheduled to arrive at 9:35 A.M. but has been marked 8 hours late. John looks at his watch and sees that it is 9:00 A.M. About how much longer must he wait for the train? (Only an *approximate* answer is required.)

(Go on to the next page.)



This is a section of a road map. The numbers between points indicate the number of miles between those points. The solid line indicates paved road. The double line indicates gravel road.

91 What is the shortest road distance from A to G?

92 In going from D to A, how many miles farther is it to go the all paved road than to go over part that is gravel?

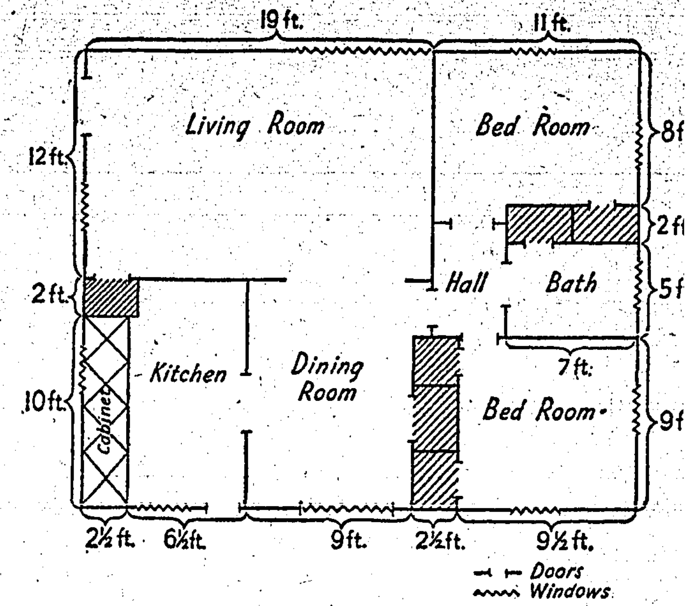
93 About what per cent of the most direct road from B to G is paved? (Only an *approximate* answer is required.)

94 If the cost of building a paved road is $55,000 per mile, what was the total cost of the road from C to D?

95 If the cost of building a gravel road is only $6,000 per mile, how many miles of gravel road can be built for the same amount of money that one mile of paved road costs ($55,000)?

96 On an auto trip, Mr. Brown goes from C to F by way of G. He returns by way of A. How many miles did he drive on the trip?

97 What is the approximate area in square miles of the region enclosed by the road from A to C to D and then back to A by way of B?



This is a simplified floor plan of a house. (You ma consider the dimensions given as the inside dimen sions of the room, and you need pay no attention t the thickness of walls and partitions.) The shade areas represent space used for closets. Problems 9 to 104 are based on this diagram.

98 What is the total area of this floor plan?

99 The kitchen floor and cabinet top are to be co ered with linoleum which comes only in 6-foo widths. How many feet of this 6-foot widt material should be purchased?

100 The floor carpeting for the living room of th house costs $5.10 a square yard. What wi be the cost of this carpeting?

101 About what fraction of the total area is used i closets? (Only an *approximate* answer required

102 The builder of this house thought that the est mate on the cost of doors was too high. Th contractor pointed out that outside doors we $15 each, standard interior doors were $4.5 each, and closet doors were $3.00. The est mate for doors was $70.00. How much *too hig* was this estimate?

103 If the large living room and dining room wi dows together cost $85.00 and the other wi dows cost $25.00 per unit, what was the cost windows in this house?

104 The loan on this house is $5,000, on which th owner pays $30.00 per month. If the rate interest is 5%, what is the approximate amou of the principal that is paid the first month (Only an *approximate* answer is required.)

(Turn your booklet over and wait until the papers are collected.

APPENDIX B

PROBLEM LISTS

TEN PROBLEMS USED AT THE CENTRAL FAIRMOUNT SCHOOL

1. Terry runs a shoe shine stand. He gets fifteen cents for a shine. One afternoon he had 21 shines and received $1.45 in tips. How much did he make in all?

2. Mr. Maxwell, the principal, believes that there should be no more than 35 pupils in a room. How many classrooms would be needed to take care of a school enrollment of 420 pupils?

3. Jack's father runs a gasoline station. In the morning when he opened the station, the meter on his gasoline pump read 39065. In the evening it read 43287. If he sold the gasoline at 26¢ per gallon, how much did he receive from the sale of gasoline that day?

4. A flagpole is thirty feet tall. How much rope would be needed to make a loop that would reach to within four feet of the ground?

5. In one day 1345 children visited the animals at the city zoo. Each child paid 15¢ to get in to the zoo. How much did they all pay?

6. The sixth grade was running the pop stand at the school carnival. Mr. Thomas, the principal, said that last year they had sold 75 cases of pop. If there are 24 bottles to a case, how many bottles would they have sold?

7. Mr. Johnson wanted to build a fence around the back yard. The yard is 90 feet by 54 feet. How many feet of fence will he need?

8. A basket of apples weighs about 48 pounds. A large truck has on it a load of 15696 pounds of apples. How many baskets of apples would that be?

9. At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How much warmer had it become?

10. Kroger's had a special sale on soap at 6 bars for 45¢. At that rate, how much would 2 bars cost?

TEN PROBLEMS USED AT THE WHITTIER SCHOOL

1. When Joe was ill the children in the room gave 15¢ each for flowers. If there were 32 children in the room, how much did they give for the flowers?

2. A basket of apples contains about 48 pounds. A large truck has on it a load of 15,696 pounds of apples. How many baskets would that be?

3. Mr. Maxwell, the principal, believes that there should be no more than 35 pupils in a classroom. How many classrooms would be needed to take care of 420 pupils?

4. In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much did they all pay?

5. A flagpole is 30 feet tall. How much rope would it take to make a loop that would reach to within four feet of the ground?

6. At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How much warmer had it become?

7. Kroger's had a special sale on soap. They sold it at 6 bars for 45¢. At that rate, how much would 2 bars cost?

8. Tom's father drives a city bus. In the morning when he started on his route, his speedometer read 28965. At the end of his first trip it read 29011. If he makes eight trips per day, how many miles does he have to drive each day?

9. A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool?

10. Mary's mother wants to put linoleum on her kitchen. If the floor is 15 feet long and 9 feet wide, how many square yards will she need? (Linoleum is sold only by the square yard.)

TWELVE PROBLEMS TESTED IN THE GREENVILLE AND THE HAMILTON, OHIO, SCHOOLS

1. When Joe was ill the children in the room gave 15¢ each for flowers. If there were 32 children in the room, how much did they all give for the flowers?

2. A basket of apples weighs about 48 pounds. A large truck has on it 3552 pounds of apples. How many baskets would that be?

3. Mr. Maxwell, the principal, believes that there should be only 35 pupils in a classroom. How many classrooms would be needed to take care of 420 pupils?

4. In one day, 1345 children visited the zpp. They paid 15¢ each to get in to see the animals. How much did they pay in all?

5. A flagpole is 30 feet tall. How much rope would it take to make a loop that would reach to within four feet of the ground?

6. At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How much warmer had it become?

7. A grocery had a special sale on soap at six bars for 45¢. At that rate, how much would Jane pay for 2 bars?

8. Tom's father drives a city bux. In the morning when he started on his route the speedometer read 28965. When he returned it read 29011. How many miles had he driven in making one trip?

9. A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool?

10. Mary's mother wishes to put linoleum on the kitchen floor. If the kitchen is 15 feet long and 9 feet wide, how many square yards will she need? (Linoleum is sold only by the square yard.)

11. The sixth grade had a pop stand at the school fair. They sold 75 cases of pop. If there are 24 bottles of pop in a case, how many bottles did they sell?

12. Jean had $3.50 in her purse. She spent 35¢ for a movie and 20¢ for a soda. How much money did she have left?

APPENDIX C

SELECTION TABLES

TABLE 21

SCORES MADE BY INDIVIDUAL PUPILS IN EACH OF THE SIX SCHOOLS IN THE POOR PROBLEM SOLVER GROUP ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS, TEST D:BASIC ARITHMETIC SKILLS, FORM O, PART III, WITH THE PUPILS INDICATED WHO WERE SELECTED FOR THE SAMPLE TO BE USED IN THE INVESTIGATION

Score	School no. 1	School no. 2	School no. 3	School no. 4	School no. 5	School no. 6
0	George W.*	Roland H.*				
	Joan P.*					
	Robert H.					
	Annette P.					
1	Harry C.			Diana S.*	Gary G.*	
	Zestina L					
	Lynne B.					
	Joyce H.					
	Leon B.*					
	Nathaniel T.					
	Wilbur M.					
	Louise K.					
	Willa K.					
	Mary T.					
	Earl B.					
	Lester B.					
	Florence B.					
2	Charles H.	JoAnn B.*			Ronald B.*	
	Rosemary E.	Rita L.*				
	Marguerite C.					
	Virginia S.					
	JoAnn S.					
	Ann M.					
	Marie C.					
	Ralph T.*					
	Carol P.*					
	Laura P.					
	Freddie G.					
	Jessie E.					
3.	Vernon B.		Simon C.*	Barbara R.*	Paul D.*	
	Nedia B.		Jerome W.			
	Richard G.		Patricia P.*			

TABLE 21 (CONTINUED)

SCORES MADE BY INDIVIDUAL PUPILS IN EACH OF THE SIX SCHOOLS IN THE POOR PROBLEM SOLVER GROUP ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS, TEST D; BASIC ARITHMETIC SKILLS, FORM O, PART III, WITH THE PUPILS INDICATED WHO WERE TO BE SELECTED FOR THE SAMPLE USED IN THE INVESTIGATION

Score	School no. 1	School no. 2	School no. 3	School no. 4	School no. 5	School no. 6
3	Thomas L.					
	Richard O.					
	Ray H.*					
	James J.					
	John P.					
	Harold S.					
	Robert D.					
	James J.					
	Elwood M.					
	Willie S.					
	Nancy L.					
	Ruthie D.					
	Connie D.*					
	Maude F.					
	Delores S.					
	Anna M.					
	L. D. F.					
	Maxine J.					
4	Catherine C.	Gertie P.*	Patricia S.*	Louise P.*	Lois H.*	Jim S.*
	Robert M.	Earl R.	Shirley C.*			
	Ralph B.					
	Raymond S.					
	Robert F.*					
	Donald W.					
	Gowen H.					
	Josehpine H.					
	Virgil P.					
	George B.					
	Bernard G.					
	Doris T.					
	Ronald G,					
	Barbara W.					
	Madeliene G.					
	Carrie S.					
	Norma W.					
	Carolyn S.*					
	Donald C.*					
	Betty L.					
	Willie H.					
	Ernie D.					
	Waldo H.					
	Lois K.					
	Rose H.					

TABLE 21 (CONTINUED)

SCORES MADE BY INDIVIDUAL PUPILS OF EACH OF THE SIX SCHOOLS IN THE POOR PROBLEM SOLVER GROUP ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS, TEST D: BASIC ARITHMETIC SKILLS, FORM O, PART III, WITH THE PUPILS INDICATED WHO WERE SELECTED FOR THE SAMPLE TO BE USED IN THE INVESTIGATION

Score	School no. 1	School no. 2	School no. 3	School no. 4	School no. 5	School no. 6
5	Claire M.	Alfie C.*	Herbert A.*	Robert B.*	John W.	
	Sandra F.	Helen K.	Norman W.*	Donald H.*	Daniel C.*	
	Arvinia Q.	Thomas S.*	Archie G.			
	James H.	Carl J.*	William C.			
	Verrona W.	Eugene S.				
	Charles B.*	Judy R.				
	Francis J.					
	Gwendolyn C.					
	Betty J.					
	William M.					
	Sylvia N.*					
	Erlene L.					
	Marian E.					
	Harold M.					
	Fred K.					
	Edith N.					
	William S.					
	Patty O.					
	Gloria A.					
6	Patsy B.	Thomas M.*	Robert W.	Larry R.*	George G.*	John S.*
	Wilbom F.	Vicky K.	Lois M.	Donald S.	Michael M.*	
	Julia P.*		Jane C.*	Pat L.*	Glenda A.	
	Ted D.		Steve A.	Charles A.	Marlene R.*	
	Ernest H.		Joe B.	Parker S.		
	Woodrow P.		Thomas J.*			
	Nathaniel W.					
	Ella C.					
	Dorotha B.					
	Willie M.					
	Gertrude H.					
	Norma H.					
	Willie O.					
	Ronnie S.					
	Donald G.					
	Robert D.*					

* indicates that the pupil was chosen for the sample used in the study.

TABLE 22

SCORES MADE BY INDIVIDUAL GOOD PROBLEM SOLVERS IN EACH OF THE SIX SCHOOLS ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS: TEST D, BASIC ARITHMETIC SKILLS, FORM O, PART III, WITH THE PUPILS INDICATED WHO WERE SELECTED FOR THE SAMPLE TO BE USED IN THE INVESTIGATION

Score	School no. 1	School no. 2	School no. 3	School no. 4	School no. 5	School no. 6
24			Jack R.*			Peggy H.*
23			Richard C.*			
22		Marilyn D.*				
21					Ronnie G.*	Jim H. * Robert F. Dick G.* David L.
20			Phyllis L.	June C.*		Nancy S. * Phyllis G.*
19			Lois S.* Bruce R.			John M. George S.* Mike B.* James L. Jimmy W.
18		Edward D.*	Joseph B.* Jerry L. Jani R.*	Ronald R.*	Irene K.*	Donald S. Gayle P. George B. James B. Ken E. Jo T. * Kenneth K. Bill J. Mike L. *
17.		Salena T.* James B. John R.	Donald S. Gary S. * David W. Helen L.* David S.	Judith T. Nancy T.* Joyce Y. Charles T.*	Carol D.*	Walter R Susan M.* David C. Roland B. John B.* Nedra R. Martha A. Marjorie A. William S.

TABLE 22 (CONTINUED)

SCORES MADE BY INDIVIDUAL GOOD PROBLEM SOLVERS IN EACH OF THE SIX SCHOOLS ON THE IOWA EVERY PUPIL TEST OF BASIC SKILLS: TEST D: BASIC ARITHMETIC SKILLS, FORM O, PART III, WITH THEPUPILS INDICATED WHO WERE SELECTED FOR THE SAMPLE TO BE USED IN THE INVESTIGATION

Score	School no. 1	School no. 2	School no. 3	School no. 4	School no. 5	School no. 6
16	Melvin K.*	Helen G.* Jimmie E. Patricia K.	Jimmy S. Irene T.* David G.* Carl K. Jasper P.	Beatrice F. Wally H.* Jimmy T.	Vera C. Marjorie N.	Ronald P. Nancy H. Richard H. Tina H. Edwin H. William S. Paul N.* Judy F. Joan T.* Judith K. Virginia V. Judy R. Betty H. Ann P. Dale G. Ronald K.
15	Robert C.*	Donald S. Jerry K.* Vernon S.	Bonnie R. Judy T. Helen E. Nancy D. Leslie M.* Melvin S. Bobby A. Sara J. Patti M. Patricia D. Barbara Z. Richard H.	Billy A.* Barbara H. Lois H.	Jacqueline B* Bobby B. Robert B. David S.	Sharon M. Laura H. Kenny H. Mike S.* Robert G. Nancy G. Tom G. Thomas E.
14.	Carl B.*	Mary W. Joyce H. Jimmy W. Melva H. Jerry H. Jerry B. Bobby Z.* Jack G. Wava P.	Clarence H. Dale D. Paul Z. Robert P. Ellen H. Harald S.	Emma G. John G.*	Billy Z. Diane N.* Kenny D. Pete M. Marcia D.	David H. Billy B.* Judith B. Carol V. Patti P.* Julie S. Dorothy W. Janet N. Barbara T. Tom Z.

* indicates that the pupil was chosen for the sample used in the study.

APPENDIX D

TRANSCRIPTIONS OF PROBLEM SOLUTIONS

TRANSCRIBED TAPE RECORDINGS OF EDWARD D., A GOOD PROBLEM SOLVER, OF HIS SOLUTIONS OF THE EIGHT PROBLEMS

Problem 1.

Edward: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far would Bill swim in swimming twice the length of the pool ?

First I add two 75's - - - then two 30's - - then add - - answer is - - - - 210 - -

Inv: How did you get it?

Edward: Well - - if the swimming pool is 75 feet long - - you have to go up - - (draws in the air with his finger) - - and 30 feet wide - - - you have to go up - - then back - - 150 - - and then you have to go over - - 210 feet

Inv: Why did you work it that way ?

Edward: You have to go up - - (Uses hands to show up and over) - - then go over - - then you have to go back - - over on one side - - - - then back over on the other - - -

Inv: Is your answer reasonable ?

Edward: Yes - -

Inv: Why ?

Edward: Cause 75 and 30 is 105 - - and 75 and 30 is - - 210 - -

Inv: Is it right ?

Edward: Yes - -

Inv: Why ?

Edward: Cause that's the size a swimming pool will be - -

Problem 2.

Edward: (Reads) In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay ?

First I multiplied - - - (will not vocalize) The answer is 201.75 (two - o - one - seventy-five)

Inv: How did you get it ?

Edward: I multiplied 15¢ by 1345 - -

Inv: Why did you work it tha t way ?

Edward: Cause - - if there's 1345 children - - went to the zoo - - and paid 15¢ - - - you'd multiply 1345 times 15 - - That'd come out 201.75 - - -

Inv: Is it reasonable ?

Edward: Sounds about right - - -

Inv: Why ?

Edward: Cause if there's 1345 children - - - 15¢ is only - - if you'd divide - - it'd come out about 15¢ - - - 15¢ is about 1/8 of a dollar - - It'd come out pretty close - - -

Inv: Is it right ?

Edward: Yessir - -

Inv: Why ?

Edward: Cause we have to pay about 15¢ - - and if you take that many children - - it costs about that much - - -

Problem 3.

Edward: (Reads) A basket of apples weighs about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be ?

First I'd divide 48 into 3552 - - (no vocalization) - -(gets mixed up in his division)- - the answer is 74 - - remainder 32-

Inv: How did you work it ?

Edward: I divided 48 into 3552 - - -

Inv: Why ?

Edward: Cause there's 48 pounds in a basket - - - - a truck was carrying 3552 pounds of apples - - you'd divide 48 into that and get - - remainder 32 - - -

Inv: Is it reasonable ?

Edward: Yessir - -

Inv: Why ?

Edward: Cause 48 into 3552 would go about 75 times - - -

Inv: Is it right ?

Edward: Yessir - - -

Inv: Why ?

Edward: (long pause) - - - Cause 48 goes into 355 - - 7 times - - subtract - - that leaves 27 - - 48 times 5 is 240 - - that leaves 32 - - --

Problem 4.

Edward: (Reads) Tom's father drives a city bus. Before starting on a route the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive on one trip ?

First I subtract 29011 - - take away 28965 - - - 46 miles on one trip - - (vocalizes his subtraction)

Inv: How did you get it ?

Edward: I subtracted 28965 from 29011 - - - and got 46 - -

Inv: Why ?

Edward: If the speedometer read 28965 when he started - - and when he finished it read 29011 - - you'd subtract to find how much - -

Inv: Is it reasonable ?

Edward: Yessir - -

Inv: Why ?

Edward: 29011 is not much more than 28965 - -

Inv: Is it right ?

Edward: Yessir - - -

Inv: Why?

Edward: Why - - cause I subtracted - - - (vocalizes again) - -

Problem 5.

Edward: (Reads) A grocery had a special sale on soap at 6 bars for 45¢. At that rate how much will Jane pay for 2 bars ?

I'd take 2 into 45 - - the answer is $22\frac{1}{2}$¢ - - -

Inv: How did you get it ?

Edward: Divided 2 into 45 - - -

Inv: Why ?

Edward: Cause 6 bars are for 45¢ - - - divide 2 into 45¢ and get 22 -

Inv: Is it reasonable ?

Edward: Yessir - - -

Inv: Why ?

Edward: (vocalizes the process) - - 2 into 4 goes 2 - - 2 into 5 is 2 and 1 over - - 22 and $\frac{1}{2}$ - - -

Inv: Is it right ?

Edward: Yessir - - - -

Inv: Why?

Edward: Well - - if you'd multiply 2 times 22 - - it'd be about 44¢ - it'd be about 45¢ - - -

Problem 6.

Edward: (Reads) Jean had $ 3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left ?

First - - I'd add 35¢ - - 20¢ - - then I'll subtract it from five dollars - - - three-fifty - - - you get $ 2.95 - - -

Inv: How did you work it ?

Edward: First I added 35 and 20 - - got 55¢ - - then I subtracted the 55¢ from three-fifty and got two-ninety-five - - -

Inv: Why ?

Edward: Well - - if you spend 35¢ for a movie and - - 20¢ for a soda - - you'd add them together and then you'd subtract from three - fifty and you'd get two-ninety-five - - - -

Inv: Is it reasonable ?

Edward: Yessir - - -

Inv: Why ?

Edward: Cause if you added 55¢ and two-ninety-five - - - - it'd come out three-fifty - - -

Inv: Is it right ?

Edward: Yessir - - -

Inv: Why ?

Edward: Cause 20¢ and 35¢ are 55¢ - - - - well - - - fifty cents would be just half of it - - - and you take away 50¢ from 3.50 and you'd get three dollars - - and another 5¢ is $ 2.95.

Problem 7.

Edward: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become ?

I - - - add - - - - 29 plus 7 - - - the answer is 36 degrees -

Inv: How did you get it ?

Edward: Well - - - I added 29 plus 7 - - -

Inv: Why ?

Edward: It was 7 degrees below zero - - - and it was 29 degrees at noon - - - 7 degrees up to zero - - - and 29 would be 36 degrees - - - -

Inv: Is it reasonable ?

Edward: Yessir - - -

Inv: Why ?

Edward: Why - - 7 and 29 are 36 - - - -

Inv: Is it right ?

Edward: Yessir - - - -

Inv: Why ?

Edward: Why - - 7 up to zero - - plus 29 more - - - makes 36 degrees.

Problem 8.

Edward: (Reads) Mary's mother wants to put linoleum on the kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need ?

3 into 15 goes 5 yard - - 3 into 9 goes 3 yard - - then you add 5 and 3 yard - - - the answer is 8 yards - - 8 sq. yds/

Inv: How did you get it ?

Edward: First I divide 3 into 15 and get 5 yards - - 3 into 9 is 3 - - I add 5 plus 3 - - gets 8 square yards - - -

Inv: Why ?

Edward: Cause the floor is 15 feet long - - I divided 15 by three and got 5 - - - then 9 by three and got 3 - then I added - - - -

Inv: Is it reasonable ?

Edward: Yessir - -

Inv: Why ?

Edward: Why - - 8 square yards - - - you added 4 times 8 would be 36 - yards - - -

Inv: Is it right ?

Edward: Yessir - - -

Inv: Why ?

Edward: If you took 3 into 15 goes 5 - - - 3 into 3 goes 9 - - -

Edward Dwelley

Sample: The sixth grade was running the pop stand at the school fair. They sold 75 cases of pop. If there are 24 bottles in a case, how many bottles did they sell?

```
   2
   75
   24
  300
 150
 1800
```

Edward Dwelley

1. A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?

```
  1
  75
  75
  30
  30
 210
```

Edward Dwelley

2. In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay?

```
   1 2
  1345
    15¢
  6,725
 1345
 $201.75
```

Edward Dwelly

3. A basket of apples contains about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be?

48) 3552
468
7

48) 3552 = 75 r 32
328
272
240
32

Edward Dwelly

4. Tom's father drives a city bus. Before starting on a route the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive on one trip?

29011
28965
0046

Edward Dwelly

5. A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much will Jane pay for 2 bars?

2) 45¢ = 22 r ½
4
05
4
1

Edward Dwelly

6. Jean had $ 3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left?

35¢
20¢
55

3.50
.55
$2.95

Edward Dwelly

7. At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

29
7
36

Edward Dwelly

8. Mary's mother wants to put linoleum on her kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

3)15 = 5 yard

3)9 = 3 yard

5 yard
3 yards
8 yards

TRANSCRIBED TAPE RECORDINGS OF ALFIE C., A POOR PROBLEM SOLVER, OF THE SOLUTIONS OF THE EIGHT PROBLEMS

Problem 1.

Alfie: (Reads) A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool ?

multiply - - - - (no attempt to vocalize) 2250 feet - -

Inv: How did you get it ?

Alfie: Well - - - I multiplied - - -

Inv: Why ?

Alfie: Cause they wanted to find how much if they swam it twice - -

Inv: Is it reasonable ?

Alfie: I don't know - - -

Inv: Is it right ?

Alfie: I think so - - - -

Inv: Why ?

Alfie: The pool is 75 feet long - - and 30 feet wide - - -

Problem 2.

Alfie: (Reads) In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay ?

You multiply 15 x 1345 - - (vocalizes) two - - thousand - - one hundred - - - seventy-five - - - (20,175)

Inv: How did you get it ?

Alfie: Multiply - -

Inv: Why ?

Alfie: Cause I want to find out how much they had to pay - - - -

Inv: Is it reasonable ?

Alfie: Yessir - - -

Inv: Why ?

Alfie: Cause if so many children paid that much - - that's what they had to pay - - -

Inv: Is it right ?

Alfie: Yessir - - - -

Inv: Why ?

Alfie: Cause they'd pay about that much - - -

Problem 3.

Alfie: (Reads) A basket of apples weighs about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be ?

Subtract - - - I think I'd - - - multiply (vocalizes her multiplication) - - - 17 thousand - - 4 hundred - - 96 - (170496)

Inv: How did you get it ?

Alfie: Multiplied - - -

Inv: Why ?

Alfie: They wanted to find out how many baskets there was - - - -

Inv: Is it reasonable ?

Alfie: Yes - -

Inv: Why ?

Alfie: 3552 pounds of apples - - there's 48 pounds in one basket - - and - - - 48 x 3552 - - - gives 17 thousand - - 4 hundred 96 -

Inv: Is it right ?

Alfie: Yes - -

Inv: Why ?

Alfie: (No answer)

Problem 4.

Alfie: (Reads) Tom's father drives a city bus. Before starting on a trip the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive on one trip ?

Alfie: (Read numbers incorrectly - - 28 and 9 hundred - 65 - - 29 and 11 - -) Add - - (vocalizes) answer - - 57 hundred and 9 hundred 76 - -

Inv: How did you get it ?

Alfie: Add - -

Inv: Why ?

Alfie : He wanted to find out how many miles he drove that day - -

Inv: Is it reasonable ?

Alfie: Yessir - -

Inv: Why ?

Alfie: Why - - 28 and nine - six - five - - and 29 and eleven - - - - (long pause - - -)

Inv: Is it right ?

Alfie: Yessir - -

Inv: Why ?

Alfie : Why - - on a trip - - they took that many miles - - -

Problem 5.

Alfie: (Reads) A grocery had a special sale on soap, at 6 bars for 45¢. At that rate, how much will Jane pay for 2 bars ?

Subtract 2 from 45 - - - (vocalizes) - - 43 - - -

Inv: How did you work it ?

Alfie: Yessir - - I subtracted - - -

Inv: Why ?

Alfie: You're going to find out how much 2 bars cost - - at 6 for 45 - -

Inv: Is it reasonable ?

Alfie: Yessir!

Inv: Why ?

Alfie: Six bars for 45 - - - that leaves 2 bars for 43 - - -

[illegible]: [illegible] !

Inv: Is it right ?

Alfie: Yessir - -

Inv: Why ?

Alfie: (Long pause - - - - no answer - -)

Problem 6.

Alfie: Jean had $ 3.50 in her purse. She paid 35¢ for the movies and 20¢ for a soda. How much money did she have left ?

Bring down your 5 - - 2 from 3 is 1 - - - bring down three-fifty and subtract your 15 - - - that leaves two- thirty-five - - left-

Inv: How did you get it ?

Alfie: I added 35 and 20 - - - and first - - - and got - - oh - - I got that wrong - - add 35 and 20 - - is 55 - - subtract from three - fifty is - - - - - 5 - - mark that out is 14 - - it'd be 2.95 (two ninety-five)

Inv: How did you get it ?

Alfie: I added 35 and 20 and got 55 - - I subtracted 55 from three-fifty and got two ninety-five - -

Inv: Why ?

Alfie: She wanted to know how much she spent - - so I added - - Then I subtracted to find out how much she had left - -

Inv: Is it reasonable ?

Alfie: Yessir!

Inv: Why ?

Alfie: Cause she wanted to find out how much she had left after she spent that much - -

Inv: Is it right ?

Alfie: Yessir !

Inv: Why ?

Alfie: Cause three-fifty take away 55 is two-ninety-five - - - -

Problem 7.

Alfie: (Reads) At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degsees wammer had it become ?

Add 7 dègrees and 29 degrees - - - (vocalizes) 36 degrees -

Inv: How did you get it ?

Alfie: I added - - -

Inv: Why ?

Alfie: To find out how much warmer it had become - -

Inv: Is it reasonable ?

Alfie: Yessir - - -

Inv: Why ?

Alfie: It was 7 degrees below at 6 a.m. - - - and it raised 29 degrees - I would add - - 29 and 7 - - - no - - you'd subtract - - no - - you'd add - - - it was 7 below at - - - at noon it was 29 - - I added to find out how many degrees warmer - - - it had become -

Inv: Is it right ?

Alfie: Yessir - -

Inv: Why ?

Alfie: If you add 29 and 7 you get 36 - - - -

Problem 8.

Alfie: (Reads) Mary's mother wants to put linoleum on the kitchen floor. Linoleum is sold only by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need ?

15 plus nine is - - and it comes out - - 15 and 9 are 24 sq. ft.

Inv: How did you get it ?

Alfie: I added - - -

Inv: Why ?

Alfie: It wanted to know - - how many square yards she would need - - -

Inv:	Is it reasonable ?
Alfie:	Yessir - - - -
Inv:	Why ?
Alfie:	If her floor is 15 feet long and 9 feet wide - - she'd need 24 sq. yds. - - - -
Inv:	Is it right ?
Alfie:	Yes - - -
Inv:	Why ?
Alfie:	15 feet and 9 feet are 24 sq. yds.

Alfie Creech

Sample: The sixth grade was running the pop stand at the school fair. They sold 75 cases of pop. If there are 24 bottles in a case, how many bottles did they sell?

75
x 24
300
150

Alfie Creech

1. A swimming pool is 75 feet long and 30 feet wide. How far does Bill swim in swimming twice the length of the pool?

75
30
00
225
2250

Alfie Creech

2. In one day 1345 children visited the zoo. They paid 15¢ each to get in to see the animals. How much in all did they pay?

1345
15
6725
1345
2,0175

Alfie Creech

3. A basket of apples contains about 48 pounds. A large truck has on it a load of 3552 pounds of apples. How many baskets would that be?

3552
48
28416
14208
170496

Alfie Creech

4. Tom's father drives a city bus. Before starting on a route the speedometer read 28965. At the finish of the trip it read 29011. How many miles did he drive on one trip?

28965
29011
57976

Alfie Creech

5. A grocery had a special sale on soap at 6 bars for 45¢. At that rate, how much will Jane pay for 2 bars?

45¢
2
43

Alfie Creech

6. Jean had $ 3.50 in her purse. She paid 35¢ for a movie and 20¢ for a soda. How much money did she have left?

35
20
~~15~~
55

$3.50
.55
~~$2.35~~
$ 2.95

Alfie Creech

7. At six a.m. the thermometer read 7 degrees below zero. At noon it read 29 degrees. How many degrees warmer had it become?

29
7
36

Alfie Creech

8. Mary's mother wants to put linoleum on her kitchen floor. Linoleum is sold by the square yard. If the floor is 15 feet long and 9 feet wide, how many square yards will she need?

15 ft.
9 ft.
24 sq yd.