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ON THE LEARNING OF ARITHMETICAL PROCESSES

BY SECOND-GRADE CHILDREN

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ARITHMETICAL PROCESSES BY SECOND GRADE CHILDREN

A dissertation submitted to

The Graduate Faculty of the Teachers College
of the University of Cincinnati

in partial fulfillment of the
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CHAPTER I

THE PROBLEM AND ITS SIGNIFICANCE

Purpose of the Study

The purpose of this study, broadly stated, is to investigate the effectiveness of a method of teaching arithmetic in which children's immature, inadequate procedures in dealing with number are accepted as normal and valuable steps toward their achievement of competent, mature behavior with reference to number. More narrowly, the objective of the investigation is to determine the effect upon the arithmetical development of children of their temporary use of certain immature procedures, when careful guidance is given by teachers. These procedures are designated in this report as "intermediate," since they occur between the initial awareness of number and the achievement of mature modes of dealing with number relationships. The study is limited to the development of skill and understanding in the four fundamental processes, on the part of children enrolled in six second-grade elementary-school classes in the public schools of Cincinnati, Ohio.

Justification for the Study

This investigation provides evidence in two significant areas. In the first place, it provides evidence that should influence instruction in arithmetic. The methods of instruction used by the six teachers in the study, and the types of experiences engaged in by pupils which resulted in changes in the maturity of the thought processes of pupils, should be of value to other teachers. In the second place, the study makes a contribution to the existing psychological literature on the learning of concepts by children. It offers a detailed study of the

thought processes of children in the early stages of learning the four fundamental processes in arithmetic. It should therefore serve to extend and to supplement earlier studies which have dealt with concept formation in the area of arithmetic.¹

Investigators have pointed out that intermediate procedures exist, and are used by children when they are left to their own devices in solving number situations. They agree that learning begins on an immature level and progresses gradually toward mature reaction patterns. Just what these intermediate stages of development are, which are of value in learning, how the child progresses from one to the next, how rapidly the child progresses through the stages, and whether all children need to engage in activities at each level, are still matters of some conjecture.

1. W. A. Brownell, The Development of Children's Number Ideas in the Primary Grades. Chicago: Department of Education, The University of Chicago, 1928. Pp. xiii / 241.

W. A. Brownell, Learning as Reorganization. Durham, North Carolina: Duke University Press, (Durham, North Carolina,) 1941. Pp. x / 87 / xx.

W. A. Brownell and C. B. Chazal, "The Effects of Premature Drill in Third Grade Arithmetic," Journal of Educational Research, 29(September, 1935), 17 - 28.

T. R. McConnell, "Recent Trends in Learning Theory, Their Application to the Psychology of Arithmetic," Arithmetic in General Education, New York: Bureau of Publications, Teachers College, Columbia University, 1941. Pp. 268-269.

Ben A. Suelz, "Recent Trends in Arithmetic," Mathematics Teacher, 33 (October, 1940), 270 - 275.

R. H. Wheeler, "The Psychology of Learning," The Teaching of Arithmetic, Pp. 233-250. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935.

The present investigation provides evidence on these points. It represents a first attempt to devise teaching procedures specifically designed to guide children in the use of increasingly more mature methods of thinking about number, to keep detailed records of progress, and to arrive at generalizations with regard to how changes in thought processes are accomplished.

Definition of Terms

Intermediate Procedures.--- For the purposes of this investigation the term, "intermediate procedures," designates those relatively immature ways of thinking which children use prior to the time when they have achieved abstract and direct methods of thinking about number.

Children commonly use a variety of such methods during the early stages of their number learning. To illustrate from addition, a child may use any of a number of methods in arriving at the answer to the number fact

3 . Among them are the following:

5

a. He may guess, in which case his answer, in general, does not appear to be related to the number fact presented. It is likely to be far afield and may be smaller than either of the numbers in the number fact.

b. He may count both numbers in the number fact, as, "1, 2, 3 - 4, 5, 6, 7, 8;" or he may do partial counting, as, "3, - 4, 5, 6, 7, 8;" or "5, - 6, 7, 8."

c. He may recall the answer by relating it to other facts he already knows, as, "3 and 3 are 6 and 2 more are 8," or, "5 and 5 are 10 and 2 less are 8," or, "6 and 2 are 8 and so 5 and 3 are 8, too."

Intermediate procedures are usually indirect or round-about methods of arriving at answers. When the method is direct, as it may be

in guessing, there is evidence of lack of understanding. Illustrations for each of the three remaining fundamental processes will be found in subsequent chapters. Those given for addition, however, may provide some immediate understanding of the term.

Maturity level.--- In the present study the term "maturity level" is used to designate the level of understanding which a child has reached with regard to a fundamental process. For example, a child who can add by using abstract number with complete understanding has reached a high maturity level in his use of the process of addition. On the other hand, the child who obtains answers only by guessing performs at a low maturity level. It should be understood that a correct answer is not necessarily indicative of a high maturity level. Temporary efficiency can be the result of rote memorization, a procedure in which the level of maturity is low.

Between these two extremes are other maturity levels, the use of which indicates varying degrees of understanding. A relatively low level is represented by the performance of the child who finds it essential to count all numbers in an addition fact. Some progress toward a higher level is evident when the child realizes that the answer can be obtained by starting with one number and counting the other, thereby making a substantial reduction in the amount of counting necessary to arrive at an answer. The child who is able to use known facts for solving unknown ones has made even greater progress toward maturity, since greater understanding is required to transfer learning from known to unknown situations than for the methods of counting which have been described.

A differentiation between the terms "intermediate procedure" and "maturity level" may lend clarity to the present discussion.

"Intermediate procedure" is the term used to refer to any method used by a child other than immediate recall of the number fact. In contrast the term "maturity level" refers to an arrangement of these intermediate procedures into groupings which indicate amounts of understanding varying from none at all to understanding of an exceptionally high order.

Concrete, semi-concrete, and abstract.-- The terms "concrete," "semi-concrete," and "abstract" are used to denote different types of media or materials. Concrete materials are movable objects such as blocks and sticks. Semi-concrete materials are pictured materials such as dot or stick drawings, and number picture cards designed to represent the number combinations. The term abstract number materials is used to refer to written symbols.

In this study the use of the terms concrete, semi-concrete, and abstract is limited to materials or media. These terms are not to be confused with "maturity level," though there is undoubtedly a relationship between the types of materials used and the level of maturity. A high maturity level is accompanied by abstract thinking or the ability to use abstract materials, while a low maturity level usually indicates a tendency to rely on concrete materials.

Specific Problems

Preliminary problems.-- Two preliminary problems, which are basic to the major purpose of the study, may be stated as follows:

1. The discovery of the intermediate procedures that were being used by children at the beginning of the study.

2. The selection of those intermediate procedures which showed promise as aids to more mature understanding of number ideas by pupils.

The first preliminary problem necessitated setting up adequate techniques and procedures for securing valid evidence regarding the intermediate procedures used by the subjects in the study. The second preliminary problem required the formulation of criteria by which the worth of the intermediate procedures as teaching aids might be judged.

Sub-problems.--- In the evaluation of a teaching technique in which the immature or intermediate procedures of children are regarded as of central importance, a number of pertinent questions arise. Such questions, which may be regarded as sub-problems, include the following:

1. To what extent must experience be provided for all children on each of the selected intermediate procedures?

2. Is it possible for some children to bypass some of the intermediate procedures without jeopardizing understanding?

3. Do some children tend to move to more advanced procedures before understanding is present?

4. How can readiness for a more advanced intermediate procedure be determined?

5. If children are taught certain of the selected intermediate procedures which are relatively immature, and find them to be useful aids in securing correct answers, will they willingly discontinue them for more mature procedures?

6. Under what circumstances are less mature methods dropped and replaced by more mature methods?

7. If less mature intermediate procedures are not dropped

voluntarily, what methods can be used to encourage children to discontinue to use them?

8. As children are taught intermediate procedures, to what extent is their later thinking characterized by increasingly higher levels of maturity in dealing with number situations?

9. What problems do teachers encounter as they attempt to guide children through succeeding maturity levels in dealing with number in a regular classroom situation?

CHAPTER II

SURVEY OF RELATED LITERATURE

The purpose of the present survey is to review investigations and professional discussions closely related to the topic selected for study, namely, the determination of the effect of certain selected intermediate procedures on the learning of fundamental processes by second grade children.

Evidence from early studies as summarized by Buswell and Judd¹ will first be noted to show that even before 1925 a few investigators were turning their attention to such problems as the development of number concepts, the complexity of counting, and the difficulty of the number processes for young children, as well as to the learning process itself. The remainder of the summary centers around selected topics which bear a close relationship to the problem of this investigation. The list of topics follows:

1. Number abilities of young children.
2. Grade placement.
3. Incidental versus planned instruction.
4. Number learning at the concrete and semi-concrete levels.
5. Number and learning theory.

Summary of Early Studies

The summary by Buswell and Judd² of studies of counting pointed toward counting ability as an essential in building number concepts. In

1. G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic. Supplementary Educational Monographs, No. 27. Chicago: Department of Education, University of Chicago, 1925. Pp. viii / 212.

2. Ibid., pp. 61-62.

contrast to the popular idea that counting is a simple ability, investigation proved this to be a complex process marked by immature methods in early attempts and later by gradual growth toward mature performance.

The analysis by the same authors ³ of studies relating to addition further indicated that research problems centered around such topics as the relative difficulty of the number combinations and the desirability of upward or downward addition. Only a few investigators attacked the problem of the complexity of the fundamental processes or attempted to analyze methods used by pupils as they worked with the processes. The authors pointed out the need for further research on the methods by which children learn.

Before 1925 diagnostic studies ⁴ dealt largely with analysis of pupils' errors as revealed by tests. The interview technique, though its use prior to this time had been limited, was recommended by Buswell and Judd as a promising means of gaining insight into the mental processes used by children in working arithmetic examples. The source of difficulty, they said, is not to be found in isolating errors, in listing them or even in discovering the degree to which they persist, but rather "in a detailed analysis of the mental process which caused the error."

Studies of methods of teaching ⁵ indicated that number concepts were best developed by "purposeful experience with concrete objects" and that "meanings should be taught before the word or symbol is given to the

³ G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic, pp. 62-70. Supplementary Educational Monographs, No. 27. Chicago: Department of Education, University of Chicago, 1925.

⁴ Ibid., pp. 115-127.

⁵ Ibid., pp. 141-145.

child." The influence of studies of drill was evident in the emphasis on repetition for learning and retention of number facts, though the need was recognized for reconstruction of the arithmetic curriculum at the primary level to give consideration to "the psychological demands that grow out of a study of pupils' mental abilities." The tendency away from abstract teaching of the number combinations was noted, but evidence was not sufficient to recommend teaching wholly by incidental methods.

These authors ⁶ found majority opinion in favor of early instruction in number work, with the major emphasis on the concrete aspects at the beginning level. They summarized investigation in which rigorous attacks were made upon the common practice of teaching abstract number which was beyond the level of understanding of the young child. Evidence was provided to show that children already possess much number knowledge before they enter school.

The evidence from transfer studies as reported by Buswell and Judd ⁷ was conflicting. The authors tended to favor these conclusions:

...transfer exists to the extent that the same skills are used in the new situation. Transfer is often small because we fail to realize that the same skills could be used. If we want transfer or the application of skills acquired in one situation to operate in another situation, we must train in the ability to look for uses of old skills.

The authors concluded that studies which dealt with the nature of arithmetical processes were fragmentary and inconclusive, due to the fact that they were undertaken without sufficient understanding of arithmetic as a whole. For example, counting and addition were studied

⁶ G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic, p. 19. Supplementary Educational Monographs, No. 27. Chicago: Department of Education, University of Chicago, 1925.

⁷ Ibid., pp. 150-152.

in isolation, without reference to related processes. The authors saw the few attempts to analyze mental processes of pupils as promising trends, however, and as indicative of the need for further studies in this area. The need for adapting teaching methods to the level of ability of the child was recognized, though practice was characterized by the prevalence of drill methods and by emphasis on repetition.

Number Abilities of Young Children

Buckingham and MacLatchy⁸ found that children on entering grade one had a surprising ability to count both rationally and by rote, to reproduce and identify number groups, and to solve verbal and concrete number problems. There were wide differences among individuals, and the extent of information was dependent upon the experiences of the children. The authors concluded, however, that children are ready for some form of systematic work with numbers when they enter grade one.

Mott's⁹ findings largely substantiate those of Buckingham. One additional point is noted, however. Mott found that about half of the five-year olds tested were counting objects beyond their rote counting ability, and concluded that rote counting was not a necessary pre-requisite to object counting. Buckingham and MacLatchy, testing children one year older, found very little difference in the abilities of children in rote and in rational counting.

⁸ B. R. Buckingham and Josephine H. MacLatchy, "The Number Abilities of Children When They Enter Grade One," Report of the Society's Committee on Arithmetic, Part II, Research in Arithmetic, pp. 508-509. Twenty-ninth Yearbook of the National Society for the Study of Education, Bloomington, Illinois: Public School Publishing Co., 1930.

⁹ Sina M. Mott, "Number Concepts of Small Children," Mathematics Teacher, 38 (October, 1945), 291 - 301.

In 1930 Buckingham¹⁰ asserted that convincing evidence had been provided to justify some form of planned number work in the first grade. He based his conclusions upon the long period of time needed for number ideas to develop and upon information on the amount of number knowledge already in possession of children when they enter grade one. Buckingham¹¹ states:

Their teaching should begin where they are and proceed gradually -- not hurriedly--toward the building up of skills and attitudes which will make their child life and their adult life more abundant.

In the same discussion Buckingham questioned the value of studies which claim as much arithmetical learning for the child when systematic study of arithmetic is postponed until third grade or later, as for the child whose instruction is begun in kindergarten or in grade one. Buckingham believes that a preconceived curriculum in the minds of the teachers fostered a leveling process. Instead of continuing on to more mature learnings the child's progress stopped when a certain accepted level was achieved.

Other investigations¹² are fairly consistent in pointing to the readiness of the first grade child for some form of number work. The crux of the matter seems to center around an interpretation of the child's readiness in terms of psychological maturity, and the adaptation of content material to the child's level of ability.

10. B. R. Buckingham, "The Social Value of Arithmetic," Report of the Society's Committee on Arithmetic, Part I, Some Aspects of Modern Thought on Arithmetic, pp. 48-51. National Society for the Study of Education. Twenty-ninth Yearbook. Bloomington, Illinois: Public School Publishing Company, 1930.

11. Ibid., p. 51.

12. Anita Riess, Number Readiness in Research. Chicago: Scott Foresman and Co., 1947, p. 67.

Grade Placement

Closely allied with the work on arithmetic readiness has been the research relating to grade placement of arithmetic topics. Extensive work on grade placement was done by Washburne¹³ and the Committee of Seven. The committee made recommendations for placement of topics on the basis of the level at which successful achievement could be attained for each process. Successful achievement was measured in terms of the percentage of correct responses. Brownell¹⁴ attacked the work of the committee at several points. In particular he indicated that the method of teaching used was a drill method, and that results of a different nature might have been obtained under more meaningful teaching methods. He also questioned the committee's emphasis upon grade placement rather than upon the more fundamental issue, the learning process itself. He claimed further that Washburne's study failed to give evidence of the opportune time at which a successful beginning might be made in learning the processes.

Sueltz¹⁵ also disagreed with Washburne's findings on grade placement. He claimed that his own research did not bear out the results

¹³ Carleton W. Washburne, "Work of the Committee of Seven on Grade Placement in Arithmetic," Child Development and the Curriculum, pp. 299-324. Thirty-Eighth Yearbook of the National Society for the Study of Education, Part I. Bloomington, Illinois: Public School Publishing Co., 1939.

¹⁴ W. A. Brownell, "Critique of the Committee of Seven Investigation on the Grade Placement of Arithmetic Topics," Elementary School Journal, 38 (March, 1938), 495 - 508.

¹⁵ Ben A. Sueltz, "Recent Trends in Arithmetic," Mathematics Teacher, 33 (October, 1940), 273.

Ben A. Sueltz, "Curriculum Problems - Grade Placement," Arithmetic in General Education, pp. 37-38. Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1941.

found by Washburne, and concluded that "so many psychological factors enter into learning that it is hazardous to be too specific about how and when learning can or will take place."

Willey's ¹⁶ findings tend to substantiate the view that number processes should be spread throughout the grades, with the easier learnings in a process placed in the earlier grades and the more difficult learnings postponed until the later elementary grades. Such a program calls for articulation between all the grades of the elementary school, a problem many schools are unable to solve as yet.

Results of the investigations centering around grade placement point to the need for psychological maturity, if progress in number understanding is to be attained. Stress is placed on the need to adapt content material to the child's level of ability. There is need for further research which will take into account beginnings of learning as well as finished performance, and different methods of teaching and types of content as these relate to grade placement.

Incidental Versus Planned Instruction

The reaction against formalized number teaching at the lower grade levels led to the attempt on the part of many teachers to teach all numbers incidentally in connection with units of work or other phases of the school program. The idea that number learning is a concomitant of other experiences was advanced by Mayhew ¹⁷ in her description of Dewey's experimental school. The author explained that the children

¹⁶ R. D. Willey, "Arithmetic Processes Needed by Children," Elementary School Journal, 42 (March, 1942), 524 - 527.

¹⁷ Katherine C. Mayhew, The Dewey School, pp. 83-85; 120-122; 339-346. New York: Appleton-Century Co., Inc., 1936.

used number processes in solving problems presented to them by the activities in which they were engaged.

Macquire ¹⁸ favors correlating arithmetic with units of work and gives the impression that most of the arithmetic needed by young children can be learned in this way, if careful selection of units is made. Harap and Barrett ¹⁹ agree with Macquire and state, "The fundamentals can very satisfactorily be learned in a program of arithmetic units based on real situations in which arithmetic is learned as it is used -- not before it is used." Salt ²⁰ describes the social phases of arithmetic which grew out of a grocery store unit at second grade level. Through functional use in a real situation, social and arithmetical learnings resulted from such experiences as writing checks, using a charge account, and borrowing money. Hanna and others ²¹ made a survey to determine the extent to which opportunities for arithmetic were possible in an activity program. Six teachers at third grade level were asked to record situations, number problems, and number computations over a period of four months. Situations arose out of large units of work, out of incidental happenings in the classroom, and out of other school activities of children. From an analysis of the data the authors

¹⁸ Hattie Macquire, "Can Arithmetic be Correlated with the Unit of Study?" Mathematics Teacher, 36 (May, 1943), 219 - 225.

¹⁹ H. L. Harap and U. Barrett, "Experimenting with Real Situations in Third Grade Arithmetic," Educational Method, 16 (January, 1937), 188 - 192.

²⁰ Edna Salt, "Functional Mathematics in the Second Grade," Educational Research Bulletin, 21 (November, 1942), 217 - 226.

²¹ Paul R. Hanna and Others, "Opportunities for the Use of Arithmetic in an Activity Program," The Teaching of Arithmetic, pp. 90-120. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935.

concluded that the functional experiences of children alone are not adequate to develop arithmetical skills.

In a recent book, Brueckner ²² has attempted a synthesis of mathematical and social number needs of children through the concurrent use of experience units and instructional units. According to Brueckner, an instructional unit is made up of a large experience unit plus smaller subject-matter units. The subject-matter units are carefully selected and designed to make the number ideas presented by the experience unit more meaningful.

Brownell ²³ sees the activity unit as a valuable means of motivation for learning arithmetic and an excellent opportunity for children to make applications of number learning, but inadequate as a means of providing sufficient understanding for number ideas.

In general, those who have advocated incidental learning of numbers as a sole method of teaching have been heralds of the activity movement and of the "progressive" in education. Serious students of the whole field of arithmetic, while granting many advantages to wise use of incidental learning, do not recommend it as sufficient for developing sequential, orderly number understanding such as is necessary if concepts are to be meaningful and usable. Similarly, those who have given major attention to the problem of how children learn, view the incidental method as inadequate if used in isolation from planned, systematic instruction.

²² L. J. Brueckner, How to Make Arithmetic Meaningful, pp. 97-153. Chicago: John C. Winston Co., 1947.

²³ W. A. Brownell, "Psychological Considerations in the Learning and the Teaching of Arithmetic," Arithmetic in General Education, pp. 15-19. Sixteenth Yearbook of the National Council of Teachers of Mathematics, The Final Report of the National Council Committee on Arithmetic. New York: Bureau of Publications, Teachers College, Columbia University, 1941.

Sueltz ²⁴ seems to summarize the thinking of many investigators when he takes the position that it is not necessary for the teacher to choose between systematic number or number taught in relation to units of work. Rather, both should be used cooperatively. He states:

...the wise teacher is the one who knows when and how the social situations involving arithmetic should be employed to give significance to work with numbers, and when and how the more abstract processes should be employed to give greater meaning and understanding to social situations.

Number Learning at the Concrete and Semi-Concrete Levels

In 1928 Brownell ²⁵ pointed out the need for instruction in concrete number in the early stages of number learning as an aid toward bridging the gap between the concrete and the abstract. Support has been given to this point of view by Wheat, ²⁶ who formulated a plan for helping children discover the combinations by the use of concrete objects. According to Wheat, as the child engages in a process of analysis and synthesis, he discovers the various arrangements of the combinations that go to make up a given number. MacLatchy ²⁷ sees value in the use of markers (uniform objects such as sticks, beads, and blocks) as aids in helping children understand numbers. When the child uses markers he is not forced to use imagination alone. Those aspects of number which are difficult in the abstract are made clear through concrete representation.

²⁴ Ben A. Sueltz, op. cit., p. 275.

²⁵ W. A. Brownell, The Development of Children's Number Ideas in the Primary Grades, pp. 216-218. Chicago: Department of Education, The University of Chicago, 1928.

²⁶ Harry Grove Wheat, The Psychology and Teaching of Arithmetic, pp. 184-189. Boston: D. C. Heath and Co., 1937.

²⁷ Josephine H. MacLatchy, "Seeing and Understanding in Number," Elementary School Journal, 45 (November, 1944), 144 - 153.

Rosenquist ²⁸ also recommends the use of objects which she refers to as "representative materials." She devotes an entire chapter of her recent book to experiences with such materials.

Spitzer ²⁹ advocates wide use of semi-concrete representations or drawings as aids to pupils in the solution of number problems. When pupils are confronted with a problem situation, each child makes a simple, stylized drawing to represent the number ideas in the problem and the solution as he sees it. Pupils first experiment to find possible means of solution to the problem. They are then led to discover the best solution to the problem by analysis of the solutions provided by the group.

Sauble ³⁰ also stresses the advantages of using concrete materials when any new learning in number is attempted. The manipulation of concrete materials is only a first stage in the development of number ideas. She therefore sees the need for "teacher direction and guidance to help them (children) to move from immature to more mature ways of thinking about numbers."

Riess ³¹ deplores the fact that using the fingers has fallen into ill repute because of "a reaction against the habitual, thoughtless

²⁸ Lucy Rosenquist, Young Children Learn to Use Arithmetic, pp. 63-79. Boston: Ginn and Company, 1949.

²⁹ Herbert Spitzer, The Teaching of Arithmetic, pp. 125-127; 173-180. Boston: Houghton Mifflin Company, 1948.

³⁰ Irene Sauble, "Enrichment of the Arithmetic Course: Utilizing Supplementary Materials and Devices," Arithmetic in General Education, p. 163. Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1941.

³¹ Anita Riess, "The Meaning of the 'Meaningful' Teaching of Arithmetic," Elementary School Journal, 45 (September, 1944), 28.

counting on fingers tolerated in schools." She considers the number concept to be fundamentally a "manual concept" which cannot be satisfactorily built without sensory experience. The fingers are the child's most natural aid, Riess says, and he should, therefore, not be denied their use.

McSwain ³² recommends an arithmetical library composed of a variety of concrete and visual materials available to teachers at all times. Wise use of such materials will serve to make the arithmetic program more functional.

Authors who advocate the use of materials in the early stages of number instruction insist that the materials are means to ends rather than ends in themselves and should be so considered by the teacher. Materials aid the child in thinking by providing him with a basis from which to build ideas.

The contribution of grouping to the understanding of the processes has not been a topic for wide experimentation. In 1912 Freeman ³³ experimented to discover the number of objects that could be grouped together "by a single act of attention." The investigation led him to conclude:

...organization of objects into regular forms is very closely related to number processes, and ...learning in this form of perception is one of the best means of giving the child a grasp of the simple number operations.

³² E. T. McSwain, "A Functional Program in Arithmetic," Improving the Program in Arithmetic, p. 16. Papers presented at the Conference on Arithmetic held at the University of Chicago, 1946. Compiled and Edited by G. T. Buswell. Chicago: The University of Chicago Press, 1946.

³³ F. N. Freeman, "Grouped Objects as a Concrete Basis for the Number Idea," Elementary School Teacher, 12 (March, 1912), 310.

While recognizing the value of counting in developing number ideas, Freeman considers counting alone to be inadequate, and recommends that counting experiences be supplemented by a consideration of number groups. Understanding both of the group as a whole and of the component parts of the group is important, according to Freeman.

Carper ³⁴ tested the ability of beginning first grade children to recognize small groups of three, four, and five objects in regular pattern formations. She found that more than half the 300 children tested were able to recognize the five-group, that two-thirds could recognize the four-group, and that four-fifths could recognize the three-group. An interesting finding of Carper's was that children who were successful in dealing with groups were also successful in dealing with verbal problems and with abstract number. On the other hand, those children who needed to count groups with a one-to-one counting process were unsuccessful in dealing with abstract number situations.

In a survey of the literature on number readiness Riess discusses the nature of concreteness, and expresses doubt as to whether experiences with concrete number directly result in abstract understanding. A thorough study of the literature leads her to conclude that there is a need for research to give further consideration to the theory of concreteness, and to the relation between concrete and abstract levels of thinking. Riess ³⁵ says,

It is often assumed that number is concrete merely because of a reference to objects seen or handled and that the child can make the

³⁴ Doris Carper, "Seeing Numbers as Groups in Primary Grade Arithmetic," Elementary School Journal, 43 (November, 1942), 166 - 170.

³⁵ Anita Riess, Number Readiness in Research, p. 51. Chicago: Scott Foresman and Co., 1947.

"transition" to abstract number by generalizing from concrete data. The fact that the numerical aspect of a group, such as its fiveness, does not necessarily change with the simplification or the growing abstractness in the representation of things (such as changing from pictures of real objects to pictures of dots) is often overlooked.

Brownell ³⁶ agrees with Freeman that the experience afforded by counting is insufficient for developing an understanding of number combinations, and suggests a period of instruction devoted to a study of grouping. Brownell expresses his point of view in the following statement:

The purpose of this period of instruction is to provide for the child activities and experiences which will carry him by easy stages from enumeration to meaningful ideas of numbers as groups. The child begins with concrete number--with objects which he can see and handle. He makes groups of objects, compares groups of objects, estimates the total in given groups of objects, learns to recognize at a glance the number of objects in small groups and in larger groups when the latter are in regular patterns. Eventually he comes to think of concrete numbers in terms which are essentially abstract.

Wheat ³⁷ advises the teacher to have children study concrete groups by counting them, by reproducing them, by comparison with larger and smaller groups, and by taking them apart and putting them together again.

In contrast to the writers who see value in a study of number groups, Stern ³⁸ is opposed to teaching grouping, since she would have children think of all number as measurement along a number line, as advocated by McLellan and Dewey. ³⁹ According to Stern, grouping numbers

³⁶ W. A. Brownell, op. cit., p. 22.

³⁷ H. G. Wheat, op. cit., pp. 163-210.

³⁸ Catherine Stern, Children Discover Arithmetic, An Introduction to Structural Arithmetic, pp. 17-21. New York: Harper and Brothers, 1949.

³⁹ J. A. McLellan and John Dewey, The Psychology of Number, pp. 44-51. New York: D. Appleton-Century Co., Inc., 1895.

destroys the number sequence, while measurement preserves it. She has devised a series of experiences with materials designed to help children develop systematic, meaningful number ideas. Children are encouraged to experiment and discover the addition, subtraction, multiplication, and division combinations by the manipulation of materials structured according to the number system itself, i.e., along a number line or sequence of objects arranged by tens as represented on a counting frame or similar device. The child is taught to measure as well as to count, to compare the numbers he wishes to learn about with a fixed sequence of objects. Thus 3 and 4 are 7, because they stretch to the 7 point along the number line from one to ten.

While there is general agreement that concrete and semi-concrete materials are of help to children in the early stage of number learning, the evidence on methods of using materials to provide maximum growth in number understanding is conflicting.

Number and Learning Theory

McLellan and Dewey, ⁴⁰ as early as 1895, discussed wholes as units of teaching and the relation of parts to wholes:

The teacher should from the first keep in view the importance of forming the habit of parting and wholing. This is a fundamental psychical activity; its goal is to grasp clearly and definitely by one act of mind a whole of many and defined parts.

Drummond ⁴¹ makes a plea for a more free and flexible program of teaching arithmetic by which it will be possible to pay more attention to the individual needs of pupils and to the differences in learning

⁴⁰ J. A. McLellan and John Dewey, The Psychology of Number, pp. 44-51. New York: D. Appleton-Century Co., Inc., 1895.

⁴¹ Margaret Drummond, Psychology and Teaching of Numbers, p. 10. New York: World Book Co., Yonkers-on-the Hudson, 1922.

ability. She points out that learning arithmetic by young children is not a mechanical but an individualized process. She recognizes six stages of development in counting ability and lists these as follows: ⁴² The first stage is characterized by no definite ideas of number or of counting. In the second stage the child uses and understands the meaning of one, but refers to all other numbers as two, or many, or some other term by which he means the aggregate of objects. In the third stage the child recognizes small groups and knows the order of the first few number names. In the fourth stage the child has a knowledge of number series up to ten or twenty. In the fifth stage the child shows increasing facility in counting, and in the sixth stage there is perfect familiarity with the number series.

MacLatchy ⁴³ analyzes a series of stages through which children tend to pass as they learn addition combinations. There is first the realization that two numbers can be combined to give a third. In the second stage one number of a combination is likely to be recognized and the other counted. In the third stage one number is recognized and the other is broken into parts and added on to the number. In the fourth stage the sum is given at once. The average six year old, according to MacLatchy, is likely to know some addition combinations (fourth stage), is learning others (second and third stages), and is discovering or making acquaintance of still others (first stage).

⁴² Ibid., p. 35.

⁴³ Josephine H. MacLatchy, "How Number Learning Develops," Childhood Education, 15 (January, 1939), 211 - 212.

McConnell ⁴⁴ considers learning to be a process of discovery and insight. He bases his opinion upon the results of a controlled group experiment designed to determine the relative effects of teaching by a mechanical method and by a method stressing discovery and insight. Group A was taught by mechanical, repetitive methods, while Group B was taught by methods designed to foster discovery and insight. With respect to maturity in dealing with number facts, differences for the two groups were not statistically significant. Group B, however, was superior in transfer to untaught combinations. Group A was definitely superior in ability to give immediate response to number facts. McConnell concludes that, if immediate recall of the answer is the goal, repetition is the method by which to achieve that goal, but if the goal is understanding, a method of discovery and insight is to be recommended.

Wheeler ⁴⁵ develops principles of the new psychology of learning (Gestalt approach) and points out applications to the teaching of arithmetic. In his opinion learning is not a process of forming bonds. "It is an orderly and organized process of differentiating general grasps of situations with respect to experience." Learning does not proceed by trial and error, according to Wheeler, but is directed by the purposes of the learner toward certain goals. It is a process of discovery and of seeing relationships. "The whole purpose of arithmetic is to discover number relationships and to be able to reason with number."

⁴⁴ T. R. McConnell, Discovery Versus Authoritative Identification in the Learning of Children, pp. 56-59. Studies in the Psychology of Learning, Part II., Vol. 9, No. 5. Iowa City, Iowa: Studies in Education, University of Iowa, 1934.

⁴⁵ R. H. Wheeler, "The New Psychology of Learning," The Teaching of Arithmetic, pp. 233-250. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1935.

In a later discussion McConnell ⁴⁶ views learning as a process of "integration and reorganization." Learning is rightfully conceived, he says, as a meaningful rather than a mechanical process. Generalization rather than specificity is stressed, and understanding rather than mere repetitive drill. In this connection McConnell quotes Judd as follows: ⁴⁷

...the end and goal of all education is the development of systems of ideas which can be carried over from the situation in which they were acquired to other situations. Systems of general ideas illuminate and clarify human experiences by raising them to the level of abstract, generalized, conceptual understanding.

The new trend in learning arithmetic, Suelztz ⁴⁸ states, is toward a greater emphasis upon concepts, ideas, information, and mathematical principles and relationships. The key words as he sees them are insight, meaning, significance, resourcefulness, ingenuity, and social and economic importance. He says: ⁴⁹

If we could observe the informal, untutored and ingenious ways in which children discover by themselves, we could learn a great deal about natural approaches to child learning.

The values of a meaningful method over a drill method of teaching arithmetic are stated by Mossman. ⁵⁰ Facts taught by drill suffer a high

⁴⁶ T. R. McConnell, "Recent Trends in Learning Theory, Their Application to the Psychology of Arithmetic," Arithmetic in General Education, p. 272. Sixteenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1941.

⁴⁷ Ibid., p. 272.

⁴⁸ Ben A. Suelztz, op. cit., p. 271.

⁴⁹ Ibid., p. 272.

⁵⁰ Edith L. Mossman, "Which Shall it be; Mechanical Drill or Development in Understanding all the 'Why's'?" Mathematics Teacher, 38 (March, 1945), 103 - 107.

loss through forgetting, and necessitate more drill at repeated intervals to keep them on the threshold of memory. Teaching for meanings, while increasing the time required at the beginning stage of learning, decreases the amount of drill necessary for retention of number facts.

Brownell ⁵¹ presents a study which supports "the conception of learning as the organization and reorganization of behavior." Children who were taught to use a "crutch" device in the early stages of instruction in borrowing were found to reorganize their learning on a higher level and gradually to abandon the crutch in favor of a more mature method of performance. Brownell ⁵² states:

...In types of learning which should result in understanding ...the final reaction pattern cannot be given at once ...It develops gradually, over a long period of time, and it develops in extent and depth to the degree that experiences with it are varied and so require the continuous reorganization of behavior patterns.

Wheat ⁵³ would have meanings taught as early as the child is able to build any number concepts whatever. He sees meaning as leading to meanings on increasingly higher levels.

Brownell and Chazal ⁵⁴ present evidence to show that immature methods of reacting to number situations persist under drill methods of teaching and that drill alone is not sufficient to raise the level of performance to increasingly higher levels.

⁵¹ W. A. Brownell, Learning as Reorganization. Durham, North Carolina: Duke University Press, (Durham, North Carolina,) 1941. Pp. x / 87.

⁵² Ibid., p. 83.

⁵³ Harry Grove Wheat, "Why not be Sensible about Meaning," Mathematics Teacher, 38 (November, 1945), 99 - 102.

⁵⁴ W. A. Brownell and C. B. Chazal, "The Effects of Premature Drill in Third-grade Arithmetic," Journal of Educational Research, 29 (September, 1935), 17 - 18.

A recent discussion by Brownell and Hendrickson ⁵⁵ lends extensive support to the theory that the child goes through a series of levels of development proceeding from less mature methods to abstract, or more adult ways of reacting to learning situations. The authors ⁵⁶ make the following statement:

The responses made at the beginning, ...are crude, clumsy, uneconomical, inept, inexact, undifferentiated, and lacking in meaning. The responses made in the end, ...are refined, neat, expert, economical, precise, and full of meaning.

The authors view learning as a step-wise procedure. As applied to the teaching of number combinations, the child is at first encouraged to arrive at answers by whatever methods are most sensible to him. He advances to higher levels as he is ready for them. Old responses are not eliminated, but remain in the learner's repertoire to be used if needed. Guidance by the teacher is necessary to assure continual advancement toward more mature responses.

The advantage of a generalization method over a drill method of teaching the addition combinations was shown by Thiele ⁵⁷ in a comparative study of two methods. The instructional program for the generalization group emphasized building generalizations by discovery of relationships among the addition combinations. The organization of combinations for presentation to the generalization group was planned to foster the

⁵⁵ W. A. Brownell and Gordon Hendrickson, "How Children Learn Information, Concepts, and Generalizations," Learning and Instruction, pp. 109-111. Forty-Ninth Yearbook of the National Society for the Study of Education. Chicago: The University of Chicago Press, 1950.

⁵⁶ Ibid., p. 109.

⁵⁷ C. L. Thiele, The Contribution of Generalization to the Learning of the Addition Facts. New York: Bureau of Publications, Teachers College, Columbia University, 1938. Pp. viii / 84.

discovery of relationships. Teaching procedures for the generalization group included concrete and semi-concrete number experiences with gradual progress toward ability to deal with abstract number speedily and accurately. For the drill group number facts were presented in random order. Number relationships were not stressed, and each number fact was learned as a separate entity.

Pupils were tested by the administration of group tests at the beginning and at the end of the instructional period. The testing procedure consisted in giving four-second exposures of cards to which answers were recorded by the children. The children who were taught to generalize in learning the combinations excelled both in knowledge of combinations known and in their ability to transfer their learning to combinations which had not been specifically taught.

General Summary

The evidence and theoretical discussions which have been reviewed seem to justify the following conclusions:

1. Children on entering school are ready for some type of number experience.
2. Incidental number experience alone is not sufficient to develop meaningful number ideas for all children. Some planned number experience seems to be essential.
3. The level at which a given topic is to be taught depends upon the types of experiences presented to the learner. If experiences are largely concrete, or semi-concrete in nature, they can be comprehended at an earlier age than if experience is abstract in the main.
4. The abilities to see relationships and to generalize are important aspects of number learning. These abilities are developed

through activities which lead toward discovery and insight.

5. Children tend to go through a series of stages in gaining mature number ideas.

The Relation of the Present Study to Research and Theoretical Literature

In the present study an attempt was made to carry out a more thorough and detailed analysis of the thinking of children as they begin to learn the four fundamental processes in arithmetic, than has been made heretofore. This has been accomplished by means of an extensive use of the interview technique with all children in the study. A weakness of Thiele's investigation was his failure to use a technique by which the level of maturity of pupils' thinking could be determined at the outset and again at the end of his investigation. Just how Thiele's subjects made the transition from immature thought processes to the use of the generalization method is not clear. The present study provides information on the level of maturity of thought processes for all pupils at the beginning and at the end of the investigation. It also throws some light on the problem of how children progress from immature to mature methods of thinking.

Buswell ⁵⁸ makes the proposal that "more attention should be given to descriptive studies of how pupils think" and gives definite suggestions for ways of studying the thought processes of pupils. Among the methods suggested by Buswell are the following:

1. Keeping verbatim reports of pupil's responses as they "think aloud" in carrying out arithmetical operations.

⁵⁸ G. T. Buswell and Maurice L. Hartung, "Arithmetic 1949," pp. 55-63. Supplementary Educational Monographs, No. 70. Chicago: The University of Chicago Press, November, 1949.

2. Asking pupils to illustrate arithmetical operations with the use of manipulative materials in order to determine the thinking which lies back of the procedure used by the pupil.

3. Having pupils make drawings or diagrams for the purpose of determining their understanding of arithmetical operations.

These three methods suggested for studying the thought processes of pupils were used in the present study. During interviews pupils were encouraged to talk aloud and to tell how they arrived at answers to combinations. Manipulative materials and drawings are also used to demonstrate the understanding of number ideas and as further means of interpreting the thought processes of pupils. The value of these techniques as instructional aids is also given consideration in the study.

Carper's study pointed out that children who recognize groups of numbers with ease also experience little difficulty with abstract number. A question may be raised as to whether concrete and semi-concrete number experience is necessarily represented by an immature level, or whether the same range of levels is found as children work with concrete materials and with abstract number. The wide use made of concrete and semi-concrete materials both in the instructional program and in interviews with individual children offers the opportunity to study the relationship between learning at the concrete and at the abstract levels.

The decision to include both incidental and planned number experience in the instructional program of the study is in line with recent research findings which indicate that functional number experience alone is not sufficient to build sequential, meaningful number ideas. The present study offers further evidence on the extent to which each of these two phases of number experience reinforces and contributes to the other.

Throughout the literature on learning theory the importance of helping children see relationships and build generalizations is stressed. Most of the studies which are reported place emphasis on relationships among addition combinations only. The plan of the present study, to teach all the four fundamental processes together or in close relationship in the beginning stages when children are using concrete and semi-concrete materials, provides additional information on the extent to which children are able to see relationships among the processes as well as within a given process.

The evidence from studies relating to the number abilities of young children when they enter school indicates that children have amassed a surprising amount of number knowledge by the time they are five or six years of age. If children have found functional uses for number before entering school to such an extent that they have learned much number already, the inference is that they are ready for a more systematic planned type of number after they have had one or two years of school experience. The decision to place the study at the beginning second grade level was therefore made with due consideration for the findings of research.

CHAPTER III

THE PLAN OF THE STUDY

In order to answer the questions raised by the investigation, teaching situations were set up, an instructional program was planned, and procedures for collecting data were designed. The present chapter has a two-fold purpose: (1) to orient the reader by providing a brief introductory summary of the general plan of the investigation, and (2) to present a detailed description of the various phases of the plan including pertinent information on pupil and teacher personnel, and procedures used for collecting data.

Overview of the Plan of the Investigation

The first step in the study was to determine at the beginning of the school year the level of competence of certain selected second-grade children in dealing with number combinations. In order to obtain this information group tests and interviews with individual pupils were arranged. An instructional program of fourteen weeks' duration was then initiated, with teaching procedures designed to improve methods of thinking by helping the children to build meanings for number. Progress of the children was checked midway through the instructional period by means of interviews and again at the end of the instructional period, when group tests and interviews were repeated.

The data collected were analyzed in order to discover to what extent growth had occurred in rate, accuracy, and level of maturity in dealing with number concepts relating to the four fundamental processes during the period of instruction.

Pupil and Teacher Personnel

Second-grade pupils were selected as appropriate subjects for the present study. This selection was based on the following considerations:

1. The subjects selected should be pupils who had not yet been inducted into a systematic program of number instruction. Inasmuch as first-grade children in the city of Cincinnati, the locale of the study, have experienced little systematic work in arithmetic, second-grade level was considered appropriate.

According to the Cincinnati course of study for the primary grades,¹ no definite outcomes for any of the processes are expected at first-grade level. Suggestions are given to help teachers take advantage of number opportunities which occur in connection with units of work and other classroom activities. No time allotment is given for number alone, but twenty minutes per day are allotted for numbers and writing combined. Each teacher develops her own techniques and methods according to her interpretation of the general principles set forth in the manual.

Instructional practice, as observed by the author in the first grades of the city, places a divided emphasis on counting and on the serial idea of number. Practice varies in the extent to which incidental experiences are used as a basis for teaching number ideas. A limited amount of teaching emphasizes the meaning of the number processes. The conclusion may be drawn that a systematic presentation of the number processes from a meaningful standpoint is not accomplished to any great extent before the second grade.

¹ The Primary Manual, pp. 482-488. Curriculum Bulletin, No. 95. Cincinnati, Ohio: Cincinnati Public Schools, 1947.

2. The subjects should have a background of familiarity with number adequate to assure a readiness for the number ideas to be presented. It was assumed that second-grade children would have such familiarity with number concepts. This assumption is substantiated by research of Buckingham and MacLatchy,² who offer findings to indicate that even children entering first grade have already acquired some understanding of the processes. They found wide differences among children, and incomplete concepts due to the varied experiences which the members of a particular group had previously encountered. The research of these authors seems to justify the inference that casual number contacts, such as the out-of-school environment affords a child, together with continued experiences with incidental number situations throughout first grade, should provide an adequate background for beginning a systematic study of the simple combinations at the beginning of second grade.

It is generally accepted that any grade level typically represents a spread of at least three grades in both achievement and capacity of children.³ This being true, methods of thinking found in second grade will show considerable overlapping with those found in any first or third grade. An intensive study of second grade pupils should offer information concerning thought processes of pupils of value to teachers of both younger and older children.

2. B. R. Buckingham and J. J. MacLatchy, "The Number Abilities of Children When They Enter Grade One," Research in Arithmetic, pp. 508-509. Twenty-Ninth Yearbook of the National Society for the Study of Education, Part II. Bloomington, Illinois: Public School Publishing Co., 1930.

3. Leo J. Brueckner and Foster E. Grossnickle, How to Make Arithmetic Meaningful, pp. 398-399. Chicago: The John C. Winston Company, 1947.

Description of pupil groups.--- No attempt was made to select groups of children according to any one factor. Both homogeneous and heterogeneous groups were included. There was a wide range of ability as measured by the standards used. Some groups have a fairly stable enrollment and some had a rapidly changing enrollment. Suburban and basin areas,⁴ as well as large and small schools, were represented.

It was impossible to work with an unlimited number of subjects, in view of the amount of conference work with teachers and individual work with pupils which the study demanded. Since methods of thinking were to be classified according to maturity level represented, it was felt that the total experimental group should be of sufficient size to include a representative group of children at each maturity level. This was necessary in order to determine the variations in growth patterns that would undoubtedly occur as the instructional period progressed. Six groups of children in regular classrooms, numbering approximately thirty pupils each, appeared to offer a sufficient number of subjects to meet these requirements.

The various factors which affect instructional procedure are summarized in Table 1. It will be noted from Table 1 that two groups were from schools located in the outlying or suburban areas of the city, while four groups were selected from downtown or basin areas.

In the primary grades of the Cincinnati schools the method used in grouping children varies according to the location of the school. In suburban areas, where schools are comparatively small and there are few

⁴ The term "basin area" is used to refer to schools in thickly populated, downtown sections of the city in contrast to the outlying suburban schools.

groups of children at each grade level, the prevalent practice is to group heterogeneously. In the basin area of the city, where large schools make it necessary to have several groups of children at each grade level, children are grouped homogeneously on the basis of the Metropolitan Readiness Test and teacher judgment. Reference to Table 1 indicates that the six groups in this study illustrate both of these practices. The children of the two suburban groups, B and D, were heterogeneously grouped, while those of the basin area, A, C, E, and F, were grouped homogeneously.

In general the suburban groups represent a more stable population than the basin groups, though Group F, one of the basin groups, had only a slight fluctuation in enrollment. Group D had the heaviest and the most stable enrollment, with no changes during the experimental period. Group A had the least stable enrollment of any of the groups. There were eleven entrants into this group after October first, and nine pupils dropped out or were lost by transfer between October first and February first.

Not included in the table, but of interest to the present discussion, is the number of second grades in each school and the position of each second grade group selected for study in relation to the total range of second grades within the particular school. Of the two suburban groups, Group B was one of three heterogeneously grouped second grades, while Group D was one of four such groups. Of the four basin groups, Group C was the highest of five second grades, Group E was the highest of three, Group A was second from the highest of four, and Group F was the lowest of eight second grades in a very large school.

TABLE 1

DATA ON LOCATION OF SCHOOL, TYPE OF GROUPING, AND CHANGES
WITHIN THE PUPIL GROUPS FOR THE SIX SECOND GRADES INCLUDED
IN THE STUDY *

Factors Influencing Instruction	Group					
	A	B	C	D	E	F
Location of School	Ba	Su	Ba	Su	Ba	Ba
Type of Grouping	Ho	He	Ho	He	Ho	Ho
Enrollment as of Oct. 1, 1948	33	29	36	38	35	28
Enrollment as of Feb. 1, 1949	35	28	36	38	31	29
Number entering between Oct. 1 and Feb 1	11	2	7	0	4	3
Number transferred and dropped between Oct. 1 and Feb. 1	9	3	7	0	8	2

* Ba. - basin area; Su. - suburban area; Ho. - homogeneous grouping;
He. - heterogeneous grouping.

Teachers for the pupil groups.-- First year teachers who were graduates of Teachers College, University of Cincinnati, and who had second-grade teaching assignments in the city for the school year 1948-1949 taught the six groups of children. All teachers were members of a practicum class taught by the investigator throughout the period of the study. The opportunity for frequent supervision and for guidance in the university course made this group well adapted to the purposes of the study. These inexperienced teachers realized their need for guidance in the arithmetic program and therefore gave whole-hearted support to the cooperative study in arithmetic.

Procedures for Collecting Group Data

Construction of the group test.-- The group test used was constructed by the investigator.⁵ Selection of items was made to represent a sampling of examples from the range of numbers below ten. None of the answers in addition or multiplication exceeded nine. All division examples were obtained from numbers less than ten.

The test had two divisions. In Part I the items were presented partially in pictured form. For each number fact, pictures were provided which could be used by the child in obtaining answers. An example from each process shows the exact form.

Addition: oo and $\begin{smallmatrix} oo \\ o \end{smallmatrix}$ are
 Subtraction: $\begin{smallmatrix} ooo \\ ooo \end{smallmatrix}$ take away 3 are
 Division: How many 2's in $\begin{smallmatrix} oo \\ oo \end{smallmatrix}$?
 Multiplication: Three 2's are $\begin{smallmatrix} ooo \\ ooo \end{smallmatrix}$

In Part II the items included in Part I were repeated, with the exception that abstract number symbols were used throughout in place

⁵ See Appendix A for a copy of the group test.

of pictured representations. Symbols denoting process were not used in any part of the test. Instead, statements or questions were written out so that words were used to describe the process. Representative illustrations for each process follow:

Addition: 2 and 3 are ____

Subtraction: 6 take away 3 are ____

Division: How many 2's in 4? ____

Multiplication: Three 2's are ____

The purpose of the group test was to determine the initial status of the pupils at the beginning of the instructional program with regard to rate and accuracy, and to obtain like measures at the end of the instructional period. The division into two parts was made in order to determine whether scores in rate and accuracy were affected by the type of test materials presented to the child.

Administration of the Group Test.--- The group test was tried out during the spring of 1948 with two groups of second-grade children not included in the study. No major revisions were made, though minor revisions to clarify directions were necessary. The test proved sufficiently simple for the children to understand, and easy to administer.

During the second week of school in the fall of 1949, the group test was administered to the experimental groups. The test was repeated at the end of the instructional period of approximately fourteen weeks.

A time limit of twelve minutes was placed on each part of the test. Timing was found to be advisable in order to compare rates of work at the beginning and at the end of the instructional period, and for semi-concrete and abstract materials.

Four practice items were presented on the cover page of Part I of the test. These items were first explained from blackboard representations. Later each child received help as needed in working through the four practice items.

Students from the kindergarten-primary department of Teachers College volunteered to serve as recorders as the test was being administered. Each recorder observed from eight to ten children. Recorders were responsible for helping children with words, explaining directions individually when responses on the cover page showed misunderstanding, and encouraging children to continue to the next page when one page was completed. In addition to assisting with the administration of the test, recorders made note of any apparent evidence relative to ways of securing answers. Such methods as moving lips, counting on fingers, and copying from a neighbor were noted. Records were also made of observable behavior which reflected attitudes toward the test, such as inattention, remarks made by children, and excessive requests for help. Appropriate forms for recording information and a list of directions were provided for each recorder. ⁶

Procedure for Collecting Interview Data

Following the initial group test, individual interviews were held with all children in the six groups. The purpose of the interviews was to determine methods of thinking on arithmetical problems which were being used by each child at the beginning of the investigation. Interview items were selected from those most frequently missed on the group test, the assumption being that the items which had proved most difficult, or

⁶ See Appendix B for samples of the recorders' blanks and instruction sheets.

had been missed most often, would offer more information on methods of thinking than items well known to the children. Six addition, six subtraction, three division, and three multiplication items were selected, making a total of eighteen interview items. Items were arranged in random order, to discourage the solution of any one item by reference to those immediately preceding.

Training of interviewers.--- Selected students from the incoming senior class in the kindergarten-primary department of Teachers College assisted in conducting interviews. Interviewers participated in a preliminary program designed to provide an understanding of the interview technique and practice in interviewing. The purpose of the interview was explained. A few pertinent principles relative to successful interviewing were presented. This brief introduction was followed by three demonstrations with children of different abilities, in order to acquaint interviewers with as wide a range of responses as possible within a short period of time.⁷ After a general discussion of the three demonstrations, each student interviewed two children, under supervision. Each interviewer was provided with a kit of materials including the following:

- an interview booklet
- two practice items
- interview blanks for recording interview responses
- sharpened pencils
- ten paper picnic forks or spoons for demonstrating number combinations in concrete form
- paper and crayon (one color) for drawing dot or stick pictures
- paper clips for clipping all material for each child together to prevent loss.

⁷ Practice interviews were given during the first week of school. Subjects were second-grade children not included in the study.

Results from practice interviews were discussed and suggestions given for improvement. In early attempts, interviewers found it difficult to refrain from teaching the child during the interview. They were inclined to help the child by means of a hint or by the type of question asked, e.g., "You know that isn't right; think again," or, "If you think a little longer, maybe you'll get it." When interviewers clearly understood that the purpose of the interview was to determine methods of thinking, they realized that wrong answers could be as useful as correct ones in obtaining the desired information. They quickly understood that inclination to push children for correct answers was not only unnecessary but would actually defeat the purpose of the interview; and that, furthermore, it was desirable for the child to think out loud if he could be induced to do so, or if not, to report what he had said to himself.

Typical questions which seemed to bring the most successful results were: "How did you think it?" "Can you say it out loud just the way you said it to yourself?" "Can you think this one out loud as you do it?"

Interviewers were cautioned to avoid questions which would suggest a method to a child. To say, "Did you start with one and count up?" is likely to encourage the child to answer in the affirmative, even though he may have used an entirely different method.

Interviewers found it necessary to watch lips and fingers of the children, since these overt forms of behavior were aids to interpretation of thought processes. Information was recorded between items rather than immediately following the presentation of an item. The interviewer was therefore free to observe the child carefully as he was getting his answer. Interviewers were urged to record the exact methods used in counting, in all cases where such information could be obtained.

Administration of initial interviews.--- Arrangements were made with the principal of each school to provide a suitable place for interviewing children. The auditorium or the lunchroom was usually available. Either location provided ample space for interviewer and interviewee to be at a sufficient distance from others to avoid distraction. With eight interviewers working at one time, it was possible to interview all the pupils of a grade in a half day, thereby avoiding prolonged interruption of class work.

Practice interviews had revealed that half an hour allowed sufficient time for most children to complete the interview items. A longer period of time also seemed undesirable when consideration was given to the degree of concentration required on the part of the interviewee. In cases where it was not possible to complete all items, interviewers were instructed to make a sampling of items sufficient to furnish the desired information on thought processes being used by the child.⁸

At the beginning of each interview, the interviewer was faced with the problem of establishing rapport with the child. The purpose was to effect a comfortable relationship before the interview was begun. With few exceptions, this was easily and effectively accomplished by reference to a pretty dress, new shoes, or some activity in progress in the child's classroom.

The children were then told that they were going to do some work with numbers, but they would not need to write anything down. They would just talk out loud and tell how they thought about the numbers.

⁸ Detailed instructions to interviewers for initial, interim, and final interviews may be found in Appendix C.

Following these preliminaries, the child was presented with two practice items for which responses were not recorded, in order to assure that the procedure was understood. Items from the test booklet were then administered. The items were printed in manuscript on strips of tag board which were fastened together to form a booklet. Below are samples illustrating the form used for each of the processes: ⁹

Addition: "2 and 3 are ____"

Subtraction: "5 take away 2 are ____"

Multiplication: "Two 2's are ____"

Division: "How many 2's in 4?"

All addition items were presented first, since addition was the most familiar of the four fundamental processes for the children. Addition items were followed in turn by the subtraction, the division, and the multiplication items. When a new process was to be presented, the child was told that these examples were of a different kind. To avoid any chance of misinterpretation due to reading difficulty, each item was read to the child as it was presented to him. Responses of the child were recorded on an interview blank prepared for the purpose. ¹⁰ The answer, whether right or wrong, was recorded. Other entries included any lip movement or finger movement which furnished a clue to thought processes, and answers to all questions posed by the interviewer.

All items were first presented to the child in abstract form. If he was unsuccessful with abstract materials, he was asked to carry out some demonstration of the number fact, either by making a picture or

⁹ See Appendix D for a complete listing of the items included in the interview booklet.

¹⁰ See Appendix E for sample interview form.

by using concrete materials to tell the number story. No differentiation was made between semi-concrete number materials (simple crayon drawings made by the child) or concrete materials. For variety and interest both semi-concrete and concrete materials were used interchangeably. In order to save time it was necessary, however, to limit drawings to simple "dot" or "stick" pictures. When the child was asked to demonstrate the combination with concrete materials, he was given ten objects and told to use as many as he needed. Semi-concrete or concrete materials were used when the interviewer wished to determine, (1) the extent to which a child who gave an incorrect response to an abstract combination was successful with semi-concrete or concrete materials, and (2) whether an answer which could not be adequately explained by the child was meaningful or merely memorized. It was assumed that, if a clear demonstration could be made, the child understood the process by which he obtained the answer.

Administration of interim interviews.--- Following the initial interviews and an instructional period of approximately six weeks, interim interviews were administered. The procedure for the interim interviews remained the same as for initial interviews, with the following significant changes:

1. Small wooden cubes replaced the picnic forks and spoons as concrete materials in the interim interviews.
2. Interviewers were instructed to present all addition examples in abstract form first. They were then to go back and ask for a demonstration of one or more examples with concrete or semi-concrete materials. The same procedure was followed for the other three processes. It was found on initial interviews that some children tended to learn from the use of concrete materials and were able to apply this learning to subsequent

examples when the demonstration immediately followed the abstract presentation. When the abstract examples in a process had been presented, semi-concrete or concrete demonstrations followed for as many combinations as time allowed.

3. The language form "from" was used to supplement the form "take away," since one of the interviewers discovered that a few children could give correct responses when the former term was used. In cases of failure in subtraction on the original presentation, therefore, the example was re-read to the child, using the language form "from" instead of "take away."

Administration of final interviews.--- Final interviews were given approximately four months later, at the end of the teaching period. The procedure was the same as that for the initial interviews with the following exceptions:

1. In the final interview, brightly colored discs were used for concrete materials.

2. Interviewers were instructed to go through all the items for the four processes in abstract form before having any concrete or semi-concrete demonstrations by the child. This change was made primarily to save time on the final interview.

3. If a relatively immature method of arriving at an answer was used on the final interview, the child was asked if he could think of any other way to find the answer. This change in technique on the final interview was made to determine whether the child's first response represented his only method of attack, or whether he could perform at a higher level than was indicated by his first response. The final interview offered the last opportunity to find the different ways by which the child was able to obtain the answer and the extent of his ability.

Teachers' Records

Teachers were asked to keep two types of records: (1) daily logs or diaries describing the planned number program, and (2) records of the incidental number experiences of the children. All records were kept as a part of the regular requirement for the university course.

The daily log or diary of the planned number program provided information on the time devoted to systematic arithmetic instruction and an account of what took place during the instructional period each day. Diary reports also showed the teacher's interpretation of her success, or lack of success, in helping children progress from lower to increasingly higher maturity levels in the four fundamental processes.

Samples from the diaries were read and discussed at weekly meetings of the teachers. Ideas were shared and evaluated informally. Progress or lack of progress was noted, and plans for the next week were formulated.

As an aid to adequate understanding of incidental number experiences, illustrations were provided in several different areas, such as social studies, reading, music, and classroom routines. At early meetings of the teachers there was some misunderstanding as to the difference between incidental and planned number experience. Distinctions between concrete, semi-concrete, and abstract number materials were also difficult at first. Ample illustrative material, much of which was provided by the teachers themselves, served to clear up misconceptions.

A special form was developed to assure uniformity in recording incidental number experiences in anecdotal form and other pertinent information relative to the experience, such as date, size of group

participating, process (addition, subtraction, division, multiplication), method (guessing, counting, solving, etc.), materials (concrete, semi-concrete, abstract), type of follow-up work planned, and significant comments.

Summary

The subjects in this study were pupils in six second-grade classes in Cincinnati. The selected classes represented many diverse factors affecting learning. Among these were school location, type of grouping, ability of children, and fluctuations in enrollment. The children were taught by beginning teachers in the first semester of their teaching experience. These teachers acquired understanding of the experiment and received guidance through the instructional program, by means of a university course required of all beginning teachers in Cincinnati.

For a period of approximately four months, the children in the study received instruction specifically designed to encourage the use of more mature thought processes than they were using at the outset of the investigation. Data for the study were gathered by means of a group test, individual interviews with children, and teachers' records. The group test was designed to provide measures of rate and accuracy on thirty-six simple combinations selected from the four fundamental processes. It was administered preceding the instructional period and was repeated at the end of this period.

Interviews were conducted with all children in the study on eighteen selected combinations representing the four processes. Interviews provided a means for determining methods of thinking employed by the

children at the beginning of the investigation, midway through the teaching period, and at the end of this period.

Teachers' logs and other records provided information on the planned instructional program and on incidental number experiences.

CHAPTER IV

CLASSIFICATION OF THE TYPES OF CHILDREN'S RESPONSES

Some system of classification of the kinds of responses was necessary for the simplification of data in order to show improvement in the understanding of the processes. To accomplish this purpose, the methods reported by the children in the initial interviews were analyzed, arranged in categories, and classified at four maturity levels. The main topics to be treated in this chapter, therefore, may be stated as follows:

1. To provide descriptions and illustrations for each method used by the children in the initial interviews.
2. To present the procedure followed in arriving at final categories for methods.
3. To describe the procedure for classifying methods according to maturity levels represented.

Certain assumptions upon which material in this chapter is based are as follows:

1. The interview technique revealed the intellectual processes by which the children arrived at answers to a sufficient degree to permit the classification of most responses.
2. Some types of responses are more mature than others. It is possible to set up criteria on the basis of which levels of maturity can be distinguished.
3. Certain admitted difficulties present barriers to accurate classification of all responses. These hazards do not outweigh the values to be gained from the total plan of classification.

Descriptions and Illustrations of Types of Responses

The first step in discovering the variety of methods used by children was to study the responses made by children on the initial interviews. A list of all the methods found as a result of this study is provided to indicate the range of methods in each of the four fundamental processes. Each method is labelled by means of a descriptive title. A code symbol used to designate the method in tabulations of data appears after each method. A variety of counting and recall methods were found with some miscellaneous methods, and other types of responses. With three exceptions, indicated in the list, each method was found in all four processes.

In subsequent sections of this chapter, the methods are described in their order of presentation in the list which follows below. A sufficient number of illustrations are provided to show the variations in the use of the method within a given process. When differences occur in the application of the method from process to process, illustrations are given which point out these differences. The list of methods with appropriate code symbols follows.

Counting methods:

Counting all numbers (Ca)

* Counting to add and adding (CA)

Counting variation (Cv)

Partial counting (Cp)

** Counting in multiples (Cm)

Counting (method unknown) (Cu)

* Used in all processes except addition.

** Used in all processes except subtraction.

Recall methods:

Delayed recall (Rd)
 Recall by problem situation (Rps)
 Recall by logical inference (Rli)
 Immediate recall (RI)

Other types of methods and responses:

* Estimating (E)
 Visual grouping (Vgr)
 Guessing (G)
 Partial omission (Po)
 No response (NR)
 Item omitted (O)

Counting all numbers (Ca).--- When this method was used, the child counted both numbers in the example by ones. At times counting was continuous, with no break or pause in between the numbers. At other times there was a pause which tended to indicate the recognition on the part of the child that more than one was represented in the example.

Addition:

2 and 6

S: "I think of the sticks; I put out 6 and 2. Then count them out--- 1, 2, 3, 4, 5, 6, 7" ¹

5 and 4

Counted 1, 2, 3, 4, 5 on one hand.
 Then 6, 7, 8, 9 on the other.

* Not used in the process of multiplication.

¹ In all the illustrations which follow, the symbol S indicates the subject. The symbol I designates the interviewer.

3 and 4

S: "----7." When asked to describe the method used, the child said, "1,2,3," pointing to the 3 on the card; and, "4,5,6,7," pointing to the 4 on the card. ²

Subtraction:

9 take away 3

S: "----6." Counted on fingers.

9 take away 3

S: "I counted 9 and then took 3 away." Counts 9 first; pulls down 3 fingers; counts the rest by ones. Answers 6.

8 take away 6

S: "That's the hardest one. I'm 8 years old." Counts 8 fingers; pulls down 6 fingers.

S: "Are 2."

Division:

How many 2's in 6?

S: "----5."

I: "How did you know?"

S: "I counted."

I: "How?"

S: "1,2,3,4,5,6. 5 2's."

How many 3's in 9?

S: "----3 (fingers) 1,2,3, - is 1; 4,5,6, is 1; 7,8,9 is 1."

How many 2's in 6?

S: "3." Used fingers to count out 6. Then touched mouth with groups of 2 fingers to count how many 2's.

Multiplication:

Three 2's

S: "----6. I said 1,2, - 3,4, - 5,6." Pauses in between groups of two's indicate that this child is aware of the groups though she counts them by ones.

Two 4's

S: "8. 1,2,3,4, - 5,6,7,8." Says he counts in bunches. Child's remark indicates recognition of groups.

² Spacing indicates a pause before the response is made by the child.

Counting to add and adding (CA).--- In using this method the child counted the numbers together or added without any apparent counting.

Subtraction:

- | | |
|---------------|--|
| 7 take away 3 | S: "----10. I said 7; then I counted 3 to 10." |
| | S: "----14." Began with 1,2,3,4,5,6, 7,8, - paused - 9,10,11,12,13,14. |
| 8 take away 6 | S: "I got \$2.00 for passing last year. Mother learned me how to be good in school. Hard to do lessons in school." |

Multiplication:

- | | |
|-----------|---------------------------|
| Three 2's | S: "3 and 2 more make 5." |
| Three 2's | S: "2 - 3,4,5." |

Division:

- | | |
|--------------------|--|
| How many 2's in 6? | S: "Let's see. --Mm - 7, because 2 and 6 more make 7." |
| How many 3's in 9? | S: "Here goes a hard one." Counted on fingers. |
| | S: "Would it be 12?" |

Counting variation (Cv).--- Children who used this method of counting seemed to realize that counting could be relied upon to get answers, but they had little understanding of how to count in order to arrive at a correct answer. They had hit upon the scheme of counting one more or, in some instances, one less than either of the numbers represented.

Addition:

- | | |
|---------|-------------------------------------|
| 2 and 4 | S: "5 - Because the 4 is before 5." |
| 5 and 3 | S: "6 - Because 6 comes after 5." |

Subtraction:

- | | |
|---------------|---|
| 7 take away 2 | S: "----6." Counted 1,2,3,4,5,6. Counts one less than total for each example. |
|---------------|---|

9 take away 3

S: "10; because after 9 is 10." All answers in subtraction for this child are one higher than the larger number in the example.

Division:

How many 2's in 6? S: "7." Put up 6 fingers and then put up her seventh finger and said 7.

How many 4's in 8? S: "There goes another hard one. If I have 4 balls in 8 and take away one, I have just 3."

Partial counting (Cp).--- In "partial counting" the child counted either one or the other of the numbers in the addition example, but not both. Most often, he began with the larger number and counted the smaller number only by ones. Occasionally the reverse process was used. A few children started with the top number consistently, a procedure which resulted in counting the smaller or the larger number according to its placement in the example.

Addition:

5 and 3

S: "I counted that in my mind. Had 5 and counted up 4. Got 9."

3 and 4

S: "3 and 4 is --- oh --- 3 and 4 is --- wait a minute -- 3,4,5,6,7 (said numbers on his lips) "I would have known it if it was 4 and 3."

I: "Why?"

S: "That's the way we had it."

2 and 4

S: "Wait a minute ----6." He wrote 2 with his finger and then counted 4 more.

An interesting variation of the method of partial counting is seen in the following illustrations:

5 and 4

S: "6 is 1; 7 is 2; 8 is 3; 9 is 4; Answer is 9."

2 and 4

S: "Let's see ---- I'm going to think it out ---- It's 6." Answer comes very fast after a long pause.

S: "I said, 3 is 1; 4 is 2; 5 is 3; 6 is 4."

For the process of subtraction, the children used the partial counting method in two ways: (1) They began with the lower number and counted up to the top number. (2) They began with the top number and counted backwards until they had counted the number to be taken away.

Subtraction:

8 take away 6	S: "----(long pause) ----2. Took the 8 and took away 6." I: "How did you do that?" S: "6-, 7, 8. Have 2 more left."
8 take away 5	S: "8 take away 5 are ----3. Take 5 off it, it leaves 6, 7, 8 and that's 3."
9 take away 3	S: "----6. If you take 3 away, you have 6 left; because you have 9, 8, 7, and 6 left."
9 take away 3	S: "Counted 9; 1 off leaves 8; 1 more off leaves 7; 1 more which is 3; leaves 6."

Another variation of the partial counting method is seen in this example:

8 take away 6	S: "7 is 1; 6 is 2; 5 is 3; 4 is 4; 3 is 5; 2 is 6; leaves 2."
---------------	--

Division:

How many 3's in 9?	S: "All I know is 2. Two times 3 is 6; one more time is 6 - 7, 8, 9; I count 3 times, so 3 times in 9."
How many 3's in 9?	S: "3 is half of 6; 6 - 7, 8, 9, so there's 3 3's in 9."
How many 2's in 6?	S: "2 and 2 are 4 - 5, 6."

Counting in multiples (Cm).--- Certain examples lend themselves to counting by multiples.

Addition:

2 and 4	S: "Counted it up - 2, 4, 6."
---------	-------------------------------

2 and 4

S: "6." Said in his mind - 2,4,6.

2 and 6

S: "8 - 2,4,6,8."

Multiplication:

Three 2's

S: "6." She said 2,4,6 and then counted them. Evidently counted the 2's to be sure she had three of them.

Counting (Method Unknown: Cu).-- This category was used when

it was evident that counting was being done, but the exact method of counting could not be determined. This method therefore overlaps with other methods of counting, since the counting may have been done by any one of those methods.

Addition:

2 and 6

S: "8 - I just count in my head."

I: "Tell me how you do it."

S: "No, sir, no sir; that's my secret!"
Unable to determine method used.

2 and 4

S: "----Could you do it by your fingers?"

I: "Yes, if you need to."

S: "2 on one hand and 4 on other makes 6." No indication by interviewer as to exact method of counting fingers.

Subtraction:

8 take away 6

Counted by ones to 8 and then couldn't hear what he said.

S: "----5 left." Said he counted to 10; took away 6 and had 5 left.

7 take away 3

S: "----4. Used my fingers. This was easy." No evidence as to how fingers were used.

Division:

How many 3's in 9?

S: "----5." Counted 1,2,3,4,5,6,7, 8,9. Then counted 1,2,3, and thought a little bit and got 5. How counting helped to arrive at the answer he gave is not clear.

How many 3's in 9?

Asked if she could use her fingers.
Answered 3 3's. No indication by
interviewer as to how fingers
were used.

Multiplication:

Two 4's

S: "----8." Lips were moving but
was unable to find out how she
was counting.

Three 2's

Child frowned, moved lips; repeated
numbers, used fingers; answered 6.

Delayed recall (Rd).-- Delayed recall designates a recall

response that is not immediate. It usually follows a pause during which
the child is "deep in thought." Questioning, however, fails to reveal
any other process in operation than an attempt to recall the answer.

The assumption could be made that the delay gave the child time
to count and that in reality delayed recall responses are obtained by some
method of counting. Since the evidence is lacking, it seems wise to use
a category which will allow for this possibility but will not exclude
other methods, such as repeating the example, remembering after a time,
and similar methods.

Addition:

5 and 4

S: "----6." Her mother said 5 and
4 are 6 and she tried to remember
it for the teacher.

2 and 4

S: "----6." Didn't know how she
figured it out - just knew it
because they practiced these in
first grade.

5 and 4

S: Shuts eyes "----I saw 8. Then
I looked at it hard and saw 9."
I: "What made you see 8 the first
time?"
S: "Didn't look at it hard enough,
I guess."

2 and 4

S: "2 and 4 are ----6. These things
just come to me. I learn."

2 and 6

S: Repeats example "----8. Had it in first grade but I just forgot. I went like this: 6 and 2 are 8; 6 and 2 are 8."

Subtraction:

8 take away 6

S: "----2." long pause
S: "I just kept thinking."

7 take away 2

S: "----5. I just figured it out."

8 take away 6

S: "----2. I take away at home. My mother learned me."

Division:

How many 2's in 6?

S: "----3. Cause my mother told me when anyone asked me how many 2's in 6, to say 3; cause it never can change."

How many 2's in 6?

S: "Don't know." Long pause. Then answered 2.

Multiplication:

Two 3's

S: "----6. Teacher told us that."

Two 3's

S: "----6. I can think how much 2 3's are."

Recall by problem situation (Rps).--- The child formulates a problem situation using money, balls, and the like. When the problem situation has been described, he gives the answer.

Addition:

5 and 3

S: "----5 and your mother gives you 3 more, you have 8."

7 and 2

S: "Gosh! I forget that! Can I figure it up? If you put 7 balls and 2 more, it'd be 9."

5 and 4

S: "I thought if I had 5 apples and 4 more I'd have 9."

Subtraction:

9 take away 3

Tells story as follows: If she had 9 balls in her hand and took away 3, she would have 3 balls left in the center of her hand.

Multiplication:

Two 4's

S: "I don't hardly know that one. I think it would be 13 or 14, because if you have 4¢ and your mother gives you 4 more, your grandma gives you 4 more, and your uncle gives you 4 more, you'd have that much."

Three 2's

S: "I think it would be ----. If you have 4¢, 2 more makes 6¢."

Recall by logical inference (RLi).--- In using this method the child thinks the answer to the unknown example by recalling other known answers. In other words, he infers, by a process of logical thinking, that if a certain fact he knows is true, the answer he is seeking must follow directly.

Addition:

5 and 4

S: "9. 5 and 5 are 10, so 1 less is 9."

5 and 3

S: "Know 4 and 4 is 8; and 1 more would be 5; and one less would be 4; so must be 8."

5 and 3

S: "----8."
I: "How did you get it?"
S: "I tried to guess it up."
I: "What did you say to yourself when you tried to guess it up?"
S: "I went like this; 5 and 2 is 7 and 1 more is 8."

3 and 4

S: "----7. 4 and 4 is 8; then 3 and 4 must be 7; cause 1 more would be 8."

Subtraction:

6 take away 4

S: "----2; because 3 and 3 are 6." The inference is that, since 3 from 6 is 3; then 4 from 6 is 2.

9 take away 3

Repeats example

S: "Would be 6; I just know."

I: "How do you know?"

S: "Well, I knew 5 and 4 are 9; take away 4, it's 5; take 3 away, it's 6."

8 take away 5

Repeats example

S: "Would be 3. I say 5 and 3 is 8, so take away 5 is 3."

8 take away 5

S: "----3; cause 4 and 4 are 8 and if you take away 5 it would leave 3."

Division:

How many 2's in 6? S: "3; 2 3's are 6; the other 3 take away 1 are 2; 3 take away 1 are 2; the 1's that are left over make another 2; so, there are 3 2's."

How many 3's in 9? Thought and said 3.

S: "One 3 is 3; 3 and 3 are 6 and 3 more are 9."

How many 2's in 6? S: "3. There are 2 3's in 6; so there are 3 2's."

Multiplication:

Two 3's

S: "6. I just knowed that 3 and 3 are 6; I always did know that."

Two 4's

S: "8. Last time I was on 8, and how many 4's are in 8. I was thinking that 2 4's was in 8." Referring to previous division example $4\overline{)8}$.

Two 4's

S: "8 - 4 and 4 are 8. Some of these are easy."

Three 2's

S: "----6. 2 and 2 are 4, and 4 and 2 are 6."

Recall by logical inference was used as the category for all responses in subtraction and division which the child remembered from corresponding multiplication or addition facts.

Immediate recall (RI).--- Immediate recall is used to designate an answer that is given immediately. Children using this method were able

to demonstrate combinations intelligently with semi-concrete or concrete materials when called upon to do so.

Addition:

5 and 4	S: "9" Answer given immediately
2 and 4	S: "6. We had them in school."
7 and 2	S: "9. Just knew it."

Subtraction:

7 take away 3	S: "4. Did it about 3 days ago in school."
---------------	--

Division:

How many 4's in 8?	S: "2. Just looked up and thought of it."
--------------------	---

Multiplication:

Two 4's	S: "What do you mean by 2 4's?" Suddenly said 8 and knew it was right.
Two 3's	S: "6. Just knew it. Didn't have to figure it out."

Estimating (E).--- This method is represented by a response given in approximate rather than in absolute terms. An answer was not considered an estimate unless it was in the neighborhood of the correct one.

Addition:

7 and 2	S: "Make about 8."
2 and 4	S: "7 - I mean 6; 4 and put 2 more with it and it would be 5 or 6."

Subtraction:

8 take away 6	S: "----2?" When asked how he got it he said, "I just guessed." I: "Why didn't you guess 10?" S: "I thought it (the answer) was pretty low to take away 6."
---------------	---

Even though this child refers to his answer as a guess, his statement shows that it was not a pure guess. There is reasoning present.

8 take away 5 S: "----3." Said he guessed. Then said, "I thought it was 3 or 2. I didn't know; so I said 3."

Division:

How many 3's in 9? S: "Those I kinda forget. I can guess 3 or 4."

The total pattern in this case indicates that the child has a fairly good understanding of numbers. These answers are therefore classified as estimates rather than as guesses.

Visual grouping (Vgr).--- The child visualizes groups of dots or groupings of fingers, but there is no evidence of counting.

Addition:

3 and 4 S: "7. At home I got a rug with 7 flowers on it."
 2 and 4 S: "8. Because there'd be 2 dots and 4 more dots."
 2 and 4 S: "Just looked at it and said 6."
 I: "How did you get 6?"
 S: "This here (4 fingers) and this (2 fingers) are 6." No counting.

Subtraction:

9 take away 3 S: "----6." Puts up 9 fingers immediately; put down 3 and said 6. Did not count them individually.

Division:

How many 3's in 9? S: "3 of em. Cause I have blocks that have 3's like that."

How many 4's in 8? S: "2. I play dominoes at home."

Multiplication:

Two 3's Puts up 3 fingers and 3 more fingers immediately without counting and says 6.

Guessing (G).--- "Guessing" is the category used for an answer which is usually far from correct and which seems to have no reasoning back of it. If a guessed answer is questioned, the guesser will give another equally erroneous answer with as much assurance as the first.

At times the guesser may pause before giving his answer. In this case the pause does not indicate a method of thinking, but rather is used to impress the interviewer, since the child knows that others succeed by thinking.

The guesser often shows a strong preference for certain numbers which tend to occur frequently throughout the interview.

Sometimes the guesser may even attempt to count, because he knows other children count. The counting bears no relation to the answer he gives, however.

It is often necessary to consider the total pattern of the guesser in order to differentiate guessing and recall responses. If the total pattern for all of the addition examples seems to indicate guessing, one correct response out of the six is also classified as a guess. If one incorrect response has occurred in a pattern which seems to indicate recall, the incorrect response is also classified as recall.

Addition:

2 and 4

S: "2 and 4 is 29. Am I right? I'm right. I can see." She saw the interviewer write down what she had said and assumed that she was right. Other addition answers for this child ran: 29, 28, 29, 26, 28, 29.

2 and 4

S: "5. I just guessed." Went back and asked her again. She said 8.

7 and 2

S: "----6. I guess it's 6. I just said to myself it's 6 after counting on my fingers.

Subtraction:

- 9 take away 3 S: "Comes back down to 28. Am I right? I'm right." Other responses for subtraction examples were 25, 29, 24, 28, 29.
- 9 take away 3 S: "11,12." When asked how she got her answer, she replied, "Something must have told me."
 I: "Who?"
 S: "God." Other responses were 11, 12, 13, 14, 8.
- 8 take away 5 S: "----6." She just counted wildly on her fingers and then said 6. Other responses were 24, 13, 13, 30, 14, 13. When asked a second time answers were just as wild.

Division:

- How many 4's in 8? S: "I guess it's 2 or 3. I think to myself how many fingers have I got and I guessed it up right."
- How many 2's in 6? S: "Maybe 4 or 2. I just guessed it up in my head."

Responses on all other items for this pupil indicate that the total picture is that of a guesser. Otherwise these two responses might be classified as estimates.

- How many 3's in 9? S: "About 6. I just counted. Is that right?" Thought again and said 8.
 S: "Is that too much? Maybe it ought to be 7. I'm sure it's not 1. You know why I said that? I'm a minister's son and I know!"

Again the total pattern indicates a guesser. A rapid shift to another number when he has reason to doubt the correctness of an answer previously given is indicative of a guesser.

Responses of other guessers to the three division examples were: 5,6, and 7; 30, 50 and 70; 4, 9, and 5; 2, 2, and 2.

How many 4's in 8? S: "There's a lot in 8 - huh?"
 (Lips moving) "I counted and found only one in it."

This child counted each time but came out with one as his answer after counting.

How many 2's in 6? S: "2. I see the numbers."
 I: "Which numbers do you see?"
 Child points to 2 on card. Said he did the other two division examples the same way. His response to $3\overline{)9}$ was 3, and to $4\overline{)8}$ 4, which indicates that he merely called off the number of the divisor in each case.

Multiplication:

Three 2's S: "8. I don't know how I thought. I just said it right off. I'm doing that to all the numbers. It's right, ain't it?"

Two 3's S: "I hope I get it right. ----8."
 I: "Are you sure?" (He said 9.)

Two 3's S: "60." Asked her again after talking to her and she said 80.

Two 3's S: "6." Because when he was downstairs he counted to 100 and when he sees 2 3's he says 6.

The general pattern for this child was guessing. His explanation does not make sense, and the number 6 is a frequent guess throughout the interview. Response is therefore classified as a guess, even though it is correct.

Partial omission (Po).--- Partial omission is the category used when the child looks at one or the other of the numbers in a subtraction example and gives that number as the answer. It was found that some of the children tended to think of physically covering up, erasing, or otherwise getting rid of the number to be taken away.

The method of partial omission was very rarely used with addition, probably due to the fact that the children had a better understanding of the process of addition.

Subtraction:

- 9 take away 3 S: "Leaves 9."
 I: "How do you take away the 3?"
 S: "You erase it off."
- 8 take away 6 S: "8. I thought 8 was 8, and 6 was 6; and then I thought 8 was just 8, see, and 6 was just 6."
- 6 take away 4 S: "4 left, cause, if you take the 6 away (placed his hand over the 6) you'd have 4 left." Pointed to 4 on card.
- 7 take away 2 S: "----7. See, in take aways, I don't count. I just throw the last number away and use the first number."

Division:

- How many 3's in 9? S: "One - I only see one 3 on the book." (number card)

This child gave "one" as the answer for $4\overline{)8}$ and for $2\overline{)6}$ for the same reason as given above. She only saw one.

- How many 3's in 9? S: "One. Daddy taught me that. I didn't think I'd get so many right. I bet I get A on my report for this!"

Responses on all division examples were the same. Since he sees only one 4, 3, or 2 on the card, he assumes that his answer is one.

No response (NR).--- The category of "no response" applies when a child attempted an example, but was unable to arrive at an answer after a reasonable amount of time.

Item omitted (O).--- Sometimes it was necessary for the interviewer to omit some examples in each process in order to complete the interview in the time allotted. If the same method was used consistently throughout the first four examples, there seemed little reason to ask

for responses on the remaining two examples, particularly if the child was working laboriously. Rarely items were omitted through an oversight on the part of the interviewer.

Procedure for Arriving at Categories for Methods

The process of arriving at final categories for methods and the reasons for changes are described in the following section.

Counting to add and adding (CA).--- At first "counting to add and adding" were considered two separate categories. Analysis showed that neither of these methods was used frequently. Furthermore, there seemed to be little difference in level of understanding represented. When either of the two methods was used, there was a complete misconception of subtraction, division, and multiplication as processes. Counting to add and adding failed to give correct answers in any of these processes. With these facts in mind, one category was thought adequate for these two methods.

Partial counting (Cp).--- An early decision was made to include several categories for the different methods of partial counting. In the process of addition, these methods were originally described as "counting the larger number," and "counting the smaller number." In subtraction, they took the form of "counting upwards" from the subtrahend to the minuend, or "counting backwards" from the minuend to the subtrahend. While the choice of one of these partial counting methods over the other, in either process, is often more expedient in time and effort, the child who selects one method consistently necessarily has an indiscriminate method of attack on combinations. In adding, he starts some combinations with the left hand number and some with the right hand number. In subtracting, he must

count some upwards and some backwards. It is doubtful if the child loses in understanding by being consistent in his method of attack, and probably there is little difference in the reasoning required for the various methods of partial counting which have been described. The child may lose slightly in accuracy, simply because of more counting involved in certain combinations when a consistent method of attack is used. The child who consistently starts with the left-hand number, for the combination "2 and 7," has more counting to do than in the combination "6 and 2." He has a greater chance of making errors with the former combination than with the latter. For the purpose of simplifying the method, and because it was desirable for consistency within the four processes, it seemed advisable to use one inclusive category, "partial counting."

Counting in multiples (Cm).--- At an early stage in analysis of methods, counting by two's, counting by three's and counting by four's were regarded as separate categories. Further analysis revealed that the choice of multiple used in counting depended largely on the extent to which a given combination lent itself to counting by two's, three's, or four's. To arrive at the answer for the combination, "three's in 9," the child was likely to count by three's, saying "3,6,9," whereas he was more likely to count by two's for the combination "two's in 6." A similar analysis held true for the corresponding multiplication combinations. The addition combinations "2 and 6," and "4 and 2" were often counted by two's. Since differences in level of understanding, accuracy, or reasoning ability were not apparent in the choice of multiples, one category was used for all such methods of solution.

Delayed recall (Rd) and immediate recall (RI).--- In the preliminary analysis, an effort was made to distinguish meaningful recall answers

from those which had no meaning for the child. Since interviewers did not ask for proof by explanation or demonstration in all cases, it was difficult to make accurate differentiations. Consequently, another category, "delayed recall," was introduced to take care of all recall responses which were not immediate and for which the kind of thinking could not be determined.

When tangible proof by explanation or demonstration was not present, several of the child's recall responses were analyzed to determine his total method of response. If his answers were immediate, if errors appeared to be only chance errors, and if reasons given indicated that round-about procedures were not used in arriving at answers, the response was placed in the "immediate recall" category. If there were frequent errors, and an inability to explain the methods of thinking, the category, "delayed recall," was used for the response.

Dual responses.--- A problem also arose as to the best way to classify dual responses for a single combination. The following is an illustration of a dual response:

8 take away 6	S: "4. If I had 7 apples and 3 people took away 3 apples, I'd have ----7,6,5, - I'd have four apples left."
---------------	---

In this example the child used two methods. He formulated a problem situation and then used partial counting in arriving at the answer. In this, as in all similar methods, a dual classification was made. In other words, in the tabulation of methods this example fell under the two categories, "recall by problem situation," and "partial counting."

Counting (method unknown, Cu) and delayed recall (Rd).--- The means by which "counting (method unknown)" and "delayed recall" were differentiated from other methods of counting and recall have been described.

It is advisable to discuss certain remaining factors which are applicable to both of these methods.

Some children lacked the facility to express their ideas clearly. Explanations were not sensible. Some were unable to describe their thought processes, either because of meager language ability, or because they did not know what they were thinking or doing to arrive at solutions. After two years in school, or more in the case of repeaters, some of the children undoubtedly had preconceived notions as to acceptable ways of obtaining answers. Such ideas may have caused these children to feel reluctant to describe their thought processes, even when they were aware of them. These are among the factors which are applicable to the two methods of "counting (method unknown) and delayed recall." Responses in which factors tended to operate against accurate classifications have been relegated to these two methods.

Procedure for Classifying Methods as to Maturity Level

Final classification of methods into maturity levels was made by the writer after analyses of all three interviews for each child were completed. Methods were placed into four maturity levels for purposes of analyzing and studying the growth made throughout the instructional period.

Maturity levels and methods comprising those levels, in this final classification, were the same for each process, though all methods were not used by every group of children. The final listing is found in Table 2.

In order to make definite decisions on the maturity of a method, three criteria for judging the worth of a method were set up. The immaturity of a method was considered from these three standpoints: (1) the promise

TABLE 2

SUMMARIZATION OF METHODS ACCORDING TO LEVELS OF MATURITY

Level *	Method or Type of Response
I	Item omitted No response Guessing Partial omission
II	Counting to add and adding Counting variation Counting (method unknown) Counting all numbers
III	Partial counting Counting in multiples Visual grouping Estimating Delayed recall Recall by problem situation
IV	Recall by logical inference Immediate recall

* I is lowest and IV is highest level

of accuracy of response when the method was applied, (2) the reasoning necessary to apply the method, and (3) the promise offered by the method for understanding a process. The use of these criteria as a basis for decision on maturity levels is described for the methods comprising each maturity level in turn.

Methods for Level 1.-- "No response" indicated a lack of any method of attack and therefore represents a low level of performance. Omissions were usually due to the use of time-consuming methods which were so immature that a number of items had to be omitted in order to arrive at an adequate sampling of methods used in all processes. Level 1 includes all items for which no response was obtainable as well as all omitted items.

Guessing was considered one of the most immature methods, since it rarely gave correct answers, required no reasoning on the part of the child and showed no promise as a means of helping the child understand a process. "Partial omission," the method of calling off one of the two numbers in a combination as the answer, also represents a low level of maturity on the part of the child. While this method calls for a systematic procedure, there is no evidence that the method requires reasoning or promise for understanding a process. Level 1 methods, therefore, are characterized by a high percentage of inaccuracy. Reasoning, if used at all, is not in the direction of the correct answer, because there is no understanding of the meaning of the process when these methods are applied.

Methods for Level 2.-- Level 2 responses are characterized by a systematic approach to numbers which extends beyond mere calling off the numbers as in Level 1. In "counting variation," for example, the child consistently adds one number or takes away one number. In using this

method, children evidently reasoned that, if 4 and 1 are 5, 3 and 1 are 4, etc., the answer to all addition examples could be obtained merely by counting one unit beyond one or the other of the numbers represented in the example. Some children applied this generalization to all addition examples in the interview. Since none of the six examples required that a single unit be added, the children who applied this method missed all six examples. The same kind of reasoning was used in applying this method to other processes. It is therefore obvious that "counting variation" is inaccurate in most cases, requires little reasoning on the part of the child, and offers limited understanding to addition and subtraction, and no understanding to division and multiplication.

In the method of "counting to add and adding," another Level 2 method, the child added all examples regardless of the process indicated. Sometimes he counted the numbers together and sometimes he merely gave an addition answer.

Neither in the method of "counting variation," or "counting to add or adding," does the procedure selected influence the child to obtain correct answers. His understanding of the process is extremely limited in the case of "counting variation" and entirely erroneous in the case of "counting to add and adding." The reasoning applied in either situation is unprofitable, because understanding of the process is lacking.

From the standpoint of the criteria set up, "counting all numbers" is undoubtedly a more mature method than the two methods already described for Level 2. If accurately applied, "counting all numbers" results in the correct answer. Some understanding of the process is necessary to determine the direction in which counting is to be done. Inaccurate results often occur, however, when the child attempts to count both

numbers in an example, but does not understand how to count in order to come out with the correct answer. Difficulties seemed to arise from (a) a lack of understanding of the groups represented in the example, and (b) failure to recognize the place to stop, i.e., to know when both groups had been counted. This method represents a low maturity level, since inaccuracies are prevalent due to misconceptions as to how counting may be used to arrive at correct answers. Reasoning is on a low level, because numbers are conceived as aggregates of units rather than as groups.

"Counting (method unknown)" is included in Level 2 rather than in Level 3, on the assumption that this procedure is more often immature and likely to lead to error than those methods listed for Level 3.

A wide range of maturity is apparent in the methods described for Level 2. The range did not appear sufficiently extensive to warrant the subdivision of this maturity level.

Methods for Level 3.-- "Partial counting" requires more reasoning ability than previously described methods of counting. In "partial counting" only one of the two groups is counted. The child is able to grasp one group as a whole. Furthermore, he has reasoned that he can save himself time and effort by counting only one number. More often the child counts only the smaller of the two numbers in addition. If he is consistent in his method of attack, however, he will sometimes count the smaller and sometimes the larger number.

The two types of partial counting in subtraction, which correspond to counting the larger or smaller number in addition, are "counting upwards" and "counting backwards." Sometimes "counting upwards" is the more expedient way for the child to arrive at his answer. For instance, in the example, 6 take away 4, it is more expedient for the child to count

up, saying 5,6. He counted two numbers to arrive at six; therefore his answer must be 2. To count backwards for this example involves more labor on the part of the child. He must say 6,5,4,3 (count four numbers away); since he has counted four numbers, the next number, 2, gives the answer.

If the reverse example is presented, 6 take away 2, it is simpler to count back 6,5. Since two numbers have been counted away, the answer is 4. To count upwards for this example involves starting with 2 and counting 3,4,5,6; since four numbers have been counted, the answer is 4.

The child who can do partial counting appears to have a greater understanding of the process to which the method is applied. The method of "partial counting" was decidedly superior in accuracy to the method of counting all numbers.

Other Level 3 methods, "counting in multiples," "visual grouping," "estimating," and "recall by problem situation," though used to a limited extent, seemed to indicate better understanding of the processes than Level 2 methods. The ability to use the fingers as groups without counting rather than to count them in units, or to visualize other groupings without the need of counting, shows much more understanding than is necessary to count each unit separately. Estimating an answer, as here defined, and the ability to count in multiples of two, three, and four also call for a higher level of thinking than counting by ones.

By the very nature of the method, "estimating" results in some wrong answers, or the method would not be true to the meaning of the term. The child gives an estimate in approximate terms. Nearness to the correct answer, or the child's reasoning concerning it, indicates an understanding of number, though not an exact knowledge of the answer.

"Delayed recall" is the term used to indicate that there was a pause before the answer was given, but that, so far as could be determined by interviewers, no counting took place. The child merely tried to remember the answer, which he had acquired at some earlier time. It often took him a perceptible period of time to produce the response. It varies from guessing in that guessing is not based on prior learning. "Delayed recall" is the result of some kind of former learning, though the exact nature of the learning is not evident. Rating as to maturity is difficult for this method, since there is no adequate explanation of the kind of thinking by which the answer is reached. It is definitely not so mature as the methods listed in Level 4, because it is less accurate and there is less assurance on the part of the child using it.

On the whole, greater accuracy was obtained by Level 3 methods than for those of Level 2, though some glaring inaccuracies occurred when the children attempted to use Level 3 methods with inadequate understanding.³

Methods for Level 4.--- The highest level of maturity is characterized by understanding which is based on the possession of several abilities which give evidence of accuracy, reasoning ability, and understanding of processes. If the child knows the answer immediately without recourse to any round-about procedures whatsoever, granted that his answer is accompanied by meaning, he has completed his task of abstract learning of the number fact. Such a response can be immediately called to mind when needed. It is abstract in nature. It can be applied to situations when the need

³ See Appendix F for data showing percentage of error for the method of "counting all numbers" as compared with "partial counting," and for percentage of error at levels 2, 3, and 4.

arises. It is economical in time and effort expended. It is given with confidence. This characterization applies to the Level 4 method labelled "immediate recall."

For many children, the method which immediately precedes "immediate recall" in growth toward maturity is that of "recall by logical inference." If a child can reason an answer from other known facts without having to resort to counting, he has a high level of understanding of the process. Though answers are not immediate, certainly more reasoning is required for this method than for any described for previous levels. Accuracy is high when the method is understood sufficiently for accurate applications to be made, though, as is true of the beginning use of any method, some inaccuracies occur in the learning process.

To summarize, the three criteria of accuracy, reasoning ability, and understanding of the processes were considered in relation to each category of responses in determining its proper classification as to maturity level. Level 1 is made up of methods almost completely lacking when these criteria are applied. Level 2 methods are slightly more accurate, but require only limited reasoning ability and offer little to the understanding of the processes. Level 3 methods represent a higher level, since these methods meet the criteria more adequately. Level 4 methods exact a high level of accuracy, reasoning ability, and understanding of the processes on the part of the child and therefore merit the highest maturity rating.

Summary

This chapter provides descriptions and illustrations of each of the methods children used on initial interviews, the difficulties encountered in the process of classification of methods, and the procedure used to arrive at maturity levels. The responses made by the children to combinations

presented in the initial interview were classified under some fifteen categories.

The general procedure for arriving at categories resulted in condensation in some instances. Four different methods of partial counting were reduced to one simple category applicable to all processes. A similar procedure was used to simplify other categories.

For other methods, such as "delayed recall" and "immediate recall," clear differentiations were only possible as the process of analysis served to further clarify these methods. As the process of analysis continued, and reasons for changes in classifications became apparent, it was obviously necessary to reclassify each response to which changes applied.

The classification of method permitted a further analysis of responses in terms of four levels of maturity. The criteria by which decisions on maturity were made were, (1) accuracy, (2) reasoning ability, and (3) understanding of the process. It was found that these criteria had little application to Level 1 methods, such as "guessing" and "partial omission." Neither were they adequately met by the immature counting methods of Level 2, such as "counting all numbers," and "counting variation." The criteria are more successfully fulfilled by Level 3 methods which contain more mature counting methods and less mature recall methods. The most stringent application of the criteria may be made with Level 4 responses of "recall by logical inference" and "immediate recall."

CHAPTER V

THE INSTRUCTIONAL PROGRAM

It is the purpose of this chapter to describe the instructional program as it was carried out by the teachers who cooperated in the study. This program had two phases: the planned number activities, and the incidental number experiences. Preliminary sections of the chapter deal with the general objectives of the second grade arithmetic program and with the children's background of experience. An account is then given of the principles and practices which were proposed by the investigator as guides to the teachers for the planned number program. A discussion of the program as actually carried out includes illustrations from the teaching situations which were studied. This analysis is followed by a similar account for the incidental experiences which paralleled the planned activities. Both the guidance given by the investigator to the teachers as to incidental experience and the actual experiences of the children in the classes are reviewed.

General Objectives for the Instructional Program

The instructional program as described in this chapter was designed to meet two major objectives in arithmetic which may be stated as follows: (1) to help children acquire number meanings so that the quality of their thinking will show continuous improvement in the direction of maturity; and (2) to help children become more proficient in solving the day-by-day problem situations which arise in the classroom.

Two major aspects of the teaching program were planned specifically to meet these objectives. First, a systematic program of selected activities was designed to promote mature ways of thinking. Second,

incidental number experiences which arose in connection with daily activities were used as a basis for helping children grow in the ability to solve number situations.

There is no sharp dividing line between the two objectives. In actual practice there was some merging of the two phases of the program. Incidental experiences were often extended beyond the immediate solution of the problem. They were often used as a base around which systematic procedures were planned. Systematic procedures, on the other hand, frequently had carry-over value for subsequent incidental experiences. The interest and curiosity aroused in one phase of the program were definitely reflected in the other.

Throughout the investigation the teachers assumed major responsibility for the instructional program. Each teacher played a creative role by developing plans for those instructional activities which she felt would promote the growth of her own pupils.

Interviews With Previous Teachers of Children in the Study

Analysis of initial interviews with the children indicated the desirability of making contact with the teachers who during the previous year had taught the children in the study. The purpose of these contacts was to determine the extent to which certain practices used by children were due to previous instruction. For example, children almost invariably strung out objects and pictures instead of grouping them. They used a variety of methods in working out answers to abstract combinations. They had a better understanding of addition than of any other process. It was obviously of interest to find out to what extent emphasis had been placed on grouping during the previous year, whether or not teachers were aware

of the methods being used by their children, what methods were encouraged or discouraged, and to what extent processes were taught together. Since the program followed by the second-grade teachers in the study included a combination of planned and incidental experience, it was appropriate to find out whether previous experience varied from this plan. In order to obtain answers to these problems, questions were listed as a basis for interviews with teachers.¹

It was possible to locate twelve teachers, each of whom had taught six or more of the children in the study. Individual interviews were held with each of these teachers.

Findings on planned and incidental work.-- Seven of the twelve teachers reported a greater emphasis on planned than on incidental work. Three teachers used an incidental approach altogether, and two claimed that about half of their procedures were planned and the other half incidental. Comments made during interviews, however, led to the conclusion that planning for some of the teachers meant planning the independent work of the children, rather than group activities carried out under the guidance of the teacher. Some of the comments ran as follows:

Discussed work to be done with children in a group. It was carried out as independent work.

Drew pictures and crossed off so many (subtraction).

Children didn't like copying numbers from the blackboard and writing answers.

Children wrote numbers incorrectly, reversed numbers.

Comments such as these bear out observations of the investigator on frequent visits to first grade classrooms in the city. Outside of

¹ See Appendix G for a copy of interview questions.

independent work, involving following directions from the blackboard, assigned to be carried out while reading groups are in session, there is little planned directed number activity. Three teachers reported that they worked with children in small groups, using concrete or semi-concrete materials to help children develop number ideas. It was of interest to note that these teachers had recently taken a course on teaching arithmetic or had attended an arithmetic workshop.

Findings on the processes.-- The processes of addition and subtraction were taught together by six teachers, and separately by five. One teacher, who used an incidental approach, said she taught these processes in the way they happened to come up. The teachers agreed that subtraction lagged behind addition. One teacher said she tried to show the relationship of the two processes toward the end of the year.

The only experiences in multiplication and division were incidental in nature, considered by the teachers as readiness activities, such as counting by two's or three's for games.

Some teachers did not remember having any awareness as to how children obtained answers in addition and subtraction. Some had observed that certain children counted all the numbers in a combination. Some teachers recalled that a few of the more advanced children used partial counting and recall by logical inference. The following comments were made in response to questions on the method of obtaining answers:

It depended on the child.
 Different children did different things.
 Taught group as a whole so didn't notice how they worked out answers.
 Didn't teach enough subtraction to say.

When questions were asked as to which methods were encouraged, comments ran:

None - took them as they came.
 Discouraged counting.
 Encouraged partial counting toward the end of the year.
 Different methods encouraged with different children.
 Encouraged recall by logical inference during group work.

Six teachers reported using the term "take away" exclusively in subtraction. Six teachers said they taught both the terms "take away" and "from," using them interchangeably.

Findings on grouping of numbers.--- The consensus of opinion seemed to be that children tended to string out objects when they carried out blackboard directions for independent work. Three teachers reported having children make number booklets showing groupings for numbers below 10. Comments made by individual teachers concerning grouping were:

Left it up to the children.
 Encouraged to group, but when left alone they strung them out.
 If paper was blocked into squares, children tended to group.
 If they put their work into rows, they strung them out.
 Did not group as much in subtraction as in addition.

Miscellaneous findings.--- Finger counting was discouraged by nine of the twelve teachers. Three teachers paid little attention to it. Comments by teachers were:

Mamas didn't like it.
 Wasn't allowed.
 Offered objects to count instead of fingers.
 Danger that children will use finger counting as a crutch.

Only two teachers said they had attempted to have children memorize the combinations. Some teachers used abstract drill cards with children during the latter part of the year.

All twelve teachers said that the majority of their children liked arithmetic. Qualifying statements were:

Liked it if successful; disliked it if unsuccessful.
 Most children liked it. A few were indifferent.
 Some liked it. Wanted to count everything.
 Attitude varied with different children.

All children had learned to write numbers to ten, some as far as twenty-five, and some to one hundred and above. Some children had also written numbers to one hundred by two's, five's and ten's.

A variety of incidental number experiences, centering around attendance, money, the calendar, thermometer, and the like, were reported.

Teachers stated their most difficult problems in teaching arithmetic as follows:

Some children have no number sense, no concept of numbers.
Counting fingers.
Learning wrong concepts at home from brother and sisters.
Breaking away from the use of concrete materials.
The tendency to write numbers incorrectly - reversing numbers.

Summary of findings from interviews with teachers.--- The major findings from interviews with teachers may be summarized as follows:

1. Both planned and incidental experiences were provided for children. With few exceptions, planned work was limited to planning for the independent work carried out while reading groups were in progress.
2. The process of addition was given greater emphasis than other processes.
3. Some teachers were unaware of any variation in methods of working out answers by different children.
4. While some teachers were aware of various methods being used, no organized procedures for teaching any method were reported.
5. In general, counting of fingers was discouraged.
6. In general, teachers made no attempt to have children memorize the combinations.
7. The subtraction terms "take away" and "from" were both used.
8. Experiences with grouping numbers were largely of the independent work type. With few exceptions, children were not successful in learn-

ing to group objects when working independently.

9. Arithmetic was well liked, except by children who were unsuccessful.

10. Major difficulties in teaching arithmetic centered around concept development, reversals in writing numbers, and dependence on concrete materials.

Principles and Practices for the Planned Arithmetic Program

A systematic number program was planned and carried out by each of the teachers in the study. While certain general principles and practices on which procedures were based were uniform for all teachers, each teacher was encouraged to plan her own specific teaching procedures in the light of the needs and interests of her own group of children. The teachers cooperating in the study met frequently with the investigator to discuss progress and future plans. In addition, there were occasional supervisory visits to each classroom for the purpose of helping each teacher to solve the specific problems presented by her situation.

The general principles and practices which served as guides to the teachers in setting up their instructional programs may be stated as follows:

1. Some grouping of children is necessary for effective number instruction.
2. An appropriate time allotment is needed for planned number instruction.
3. Number experience proceeds from the concrete to the abstract.
4. The sequence of number presentation follows the arrangement of the numbers in the number system. In general, a span of several numbers

is selected over which instruction is concentrated for a period of time rather than a single number.

5. The teacher tries to be aware of the thought processes of her pupils, and of means of helping them evaluate their own progress.

6. Processes are taught together in order to enable relationships to be seen readily.

7. The teacher strives to know the attitudes of her children, to build favorable attitudes, and to develop a growing sensitivity to number situations and possibilities.

8. Incidental number experience often becomes a basis from which planned experience stems.

9. Any type of number experience selected for independent work follows previous group work of a similar type carried on under the guidance of the teacher.

10. The introduction of signs and symbols is delayed until meanings for numbers have been well established.

11. Teachers learn by sharing ideas and experiences. Practicum sessions (weekly meetings of the teachers in the study) serve as a clearing house for ideas, future planning and evaluation.

In succeeding sections each of these principles and practices is amply illustrated by excerpts from teachers' diaries. Selections from the diaries have been made to show: (1) variations in the practices of different teachers, (2) the extent to which teachers were successful in carrying out the principles and practices, and (3) the difficulties they encountered in doing so.

The Planned Number Program

Grouping children for number instruction.-- As the teachers approached the problem of meeting individual differences of the children

in arithmetic, they realized that some grouping of children within each grade was necessary. Plans for grouping children were developed in accordance with the needs of each group. Data from group tests and interviews were helpful in deciding upon appropriate groups. The most general practice was to organize two groups selected on the basis of number understanding.

Teacher A began by dividing her children into two groups. She soon realized the need for three groups for most purposes and she continued to work with three groups for the duration of the instructional period.

The usual plan for Teachers B and C was to have two groups alternating on subsequent days throughout the week. While one group worked under the direction and guidance of the teacher, the other group was engaged in some type of independent work, consisting of number games or review activities. Teachers A, B, and C made use of smaller groupings on occasion. Toward the end of the semester these teachers were able to run two number groups concurrently. Teacher A describes her experience as follows:

While I worked with one group with blocks, etc., I divided the more independent children into groups of three children in each group. They worked with combination cards (abstract). Each child took a turn at being teacher, showing the others the card, and having them answer. I walked around for a while to see how things were going. It seemed to work fine.

Teacher B also made use of pupil helpers with games, and small practice groups.

Used number picture cards and games by themselves. I went over to see how they were coming along and checked with them while S used number cards with her group for a while.

All teachers found smaller groups advisable on occasion. Often a temporary group was arranged to provide specific help on a given process, causing confusion for certain children.

Teacher D worked with her group as a whole for about one week. She then divided them into two groups, but continued to use the whole group for certain types of activity, such as acting out number combinations with the children themselves. After the interim interviews were given and Teacher D had an opportunity to study the children's responses, she divided her children into three groups. She made some shifts in the original two groups on the basis of responses on the interview data, and placed six children who seemed to have very little understanding of numbers into a group for special help.

Teacher E used the two-group plan during the entire semester, working with each group on alternate days. She rearranged her groups after the interim interviews were given.

Teachers A and C made some use of convenient chance groupings in which children were not divided on the basis of number ability. While the boys went to showers, the girls sometimes worked with numbers, and vice versa. When a group went to church school, the group which remained worked with numbers on occasion. If regular number groups were crowded out of the program on a particular day because of more pressing activities, the reading group served as a number group. These practices were discouraged, however, since many children were not working on their own level of understanding.

In general, grouping was flexible, with each teacher having the freedom to employ whatever grouping techniques she felt would best meet the needs of her particular groups of children.

Time devoted to planned number instruction.-- The teachers in the program were requested to spend an average of thirty minutes each day on planned number activities. Each teacher was free to apportion the time

among her groups as she felt best. Teacher A preferred short periods of time and therefore usually worked with two groups each day, spending about fifteen minutes with each. In general, Teacher D also favored short fifteen-minute periods of guided number activities. Teachers B, C, and E preferred longer periods, around thirty minutes in length, with each group. This procedure necessitated alternating groups on successive days and planning independent activities for one group each day.

On some days planned number work was omitted altogether because of an excursion, holiday festivities, unexpected occurrences, and the like. The following explanations for omitting number work are typical of those offered:

Oct. 15. On picnic.

Oct. 21. I had to spend a lot of time with a parent. Did not have number work.

Dec. 13. Made Christmas tree ornaments. Spent most of the day on them. Also had church school and a movie. No numbers.

Nov. 17. Since we had interviews in the morning, I had to use my afternoon for other subject matter. Also had to take time today to have a discussion as to how we conduct ourselves during reading groups.

Nov. 24. The afternoon was taken up with Thanksgiving Festival, so could not get any number work in.

Omissions of number work from the program were rare, however.

On the whole, diary reports of the teacher indicated rather strict adherence to the plan of approximately thirty minutes per day for planned number work, though the division of time varied with different teachers.

Progress from concrete to abstract number experiences.-- Early number experiences of the second-grade groups in the fall centered around activities which could be carried out with concrete materials, i. e., objects that could be manipulated by the children. Each teacher was provided with at least one set of materials in sufficient quantity to make it possible for a group of ten to fifteen children to be supplied

with materials for simultaneous directed work under the guidance of the teacher. Small wooden squares, oblongs, or circles, and inch cubes were among the concrete materials provided. Each teacher was also supplied with a counting frame of twenty beads, and a felt-o-gram (large piece of felt with smaller pieces of contrasting color) to be used for demonstrating groups of numbers. In addition, the teacher used other available materials for variety and interest.

A report of an early lesson plan for Teacher E is typical of the types of concrete experience provided for the five groups during their early stages of number instruction.

Sept. 29. The children sat on the floor and each was given a 9" by 12" sheet of paper and ten wooden squares. They each counted their squares. All had ten. I asked the children to take away 4 because we were not going to use those right now.

"Now, how many do you have on your paper?" J. answered, "6." P. answered, "7." I took 4 from my flannel-o-graph, and asked how many I had left. B said, "6." I then asked P. to count her remaining blocks again. She did so by pointing to them one at a time. She counted 6.

J. had his squares on his paper like this:  I asked him to tell about his squares. He said,

"There are 2 here and 4 here. Two and 4 are 6."

The squares on my flannel-o-graph were arranged like this:  I asked B. to tell us how she saw the picture. She replied,

"There are 2 in the first row, 2 in the second row, and 2 more. That makes 6."

I asked all the children to make the same pattern I had. All except P. did it correctly. Her pattern was . I asked how many were in her first row. She counted by ones, and said four. She counted her second row and said two. I asked her what she would have to do to make her pattern look like mine. She pointed her finger at my pattern, and counted three and three. She then took away one from the first row and put it in the second row. I used my materials to explain to the children what P. had done, and how she had solved her problem.

I then asked the children how many three's they saw on their paper. They hesitated and looked from one to another. M. said, "2." I asked her

to show us the two three's.

"Here is 3 and here is 3," she said. After all had pointed out the two three's with their squares, I asked how many two's they could see. R. said, "3," and showed us how he saw his three's.

The children then made different patterns for the number six and each told how he saw his pattern.

Variations in the use of concrete materials may be illustrated by the following excerpts from the diaries of different teachers.

Teacher A. Oct. 9. Had general discussion about grouping. Used the children themselves. The children move around to make number groups. Then we tell the story, as,

"Two children and 2 children are 4 children."

Teacher D. Oct. 1. The whole class came up in front of the room and sat on the floor. On a small table in front of them I had placed seven acorns. We talked about the number seven. First we found how many groups of "two" were in seven. Then how many groups of "three," etc. We always noticed how many were left. I then called on different children to come up and group them as I had done. All seemed to have an understanding.

We then took so many away to see how many were left. The same procedure was used for other processes.

This report was written during the first week of the instructional period, before Teacher D had placed her children into groups for number work.

Teacher B describes a unique way of giving out and putting away materials, such that this routine becomes a worthwhile learning activity.

Oct. 3. We used wooden squares today. I gave each child 2,3,4, or 5 blocks. Each was then asked to tell how many he needed to have five in all. The child's estimate was checked as he got his blocks. The children made groupings of 5,6,7, and 8, getting more blocks as needed. They put their blocks away by counting by 2's.

Referring to a later occasion when 8 blocks were to be put away, Teacher B says,

Each child tried to think up a different way to put his blocks away. Some counted by 2's, starting with 2. Some counted by 2's starting with 1. P. put in 3 and 3 and 2. J. put hers in by saying, "4 and 4 are 8."

The following experiences with semi-concrete number activities were provided. These were usually in the form of pictured materials and were differentiated from the concrete in that they were fixed, or set, in contrast to the movable quality of the concrete materials. Passages from teachers' diaries provide illustrations of semi-concrete number experiences.

Teacher B. Used picture cards for rapid recognition of number groups. Most of these children are quick in answering, but still counting.

Used playing cards with the corners cut, so that the large central number groups are clearly visible. The purpose was to encourage the children to see the groups as wholes quickly without counting.

Compared two number picture cards, such as $\begin{smallmatrix} \circ \circ \circ \\ \circ \circ \end{smallmatrix}$ and $\begin{smallmatrix} \circ \circ \\ \circ \end{smallmatrix}$. Children looked at the pictures. Told the number in each group. Then compared groups to see which group had more, and how many more. Also used them to find which group had less, and how many less. Used cards with groups from one to ten.

Made up number picture cards for a lotto game, using number groups from four to ten. (Number groups on a board with duplicate groups on separate cards for matching. Games were later used independently by the children while the teacher worked with a number group.)

Teacher D. I had a small toy catalogue with toys grouped according to kinds. We made up number stories about them. There were not more than six toys in any group. The children enjoyed this.

Teacher E. Worked with addition and subtraction. Had cards with groups of different farm animals. Each child selected a card and made up a number story. I made up a story first. My card had pigs on it grouped in three and four. I said,

"Here are 7 pigs. Three went away to eat. (I covered up 3 with my hand.) How many were left?"

Twelve of the eighteen children in this group told a story about their farm animals and derived much pleasure from it.

Experiences of a semi-concrete nature were followed by problem-solving situations and abstract experiences, although a child who was unable to give abstract answers was encouraged to return temporarily to the use of semi-concrete or concrete materials. All children were not

expected to achieve the same end results. Instead, the purpose of the teacher was to help each child progress from a given level to a higher level of understanding. Examples of problem-solving experiences as provided for the children follow:

Teacher C. I made up word problems which the children seem to like, as,

"You have 8 children. You want to put them in 2 lines. How many will be in each line?"

We often dramatize the problem to check on a child's understanding or to help a child who is having difficulty solve the problem.

We opened the grocery store. The children had good experience in deciding what they wanted to buy and how much they could afford. Toy money was passed to the customers. They all got different amounts up to ten cents. The customer and the other children in the group had to be sure the grocer did not make a mistake in change. When each child went up, the rest of the group watched what he bought, and figured it out, too. Some of the problems were:

C. spent 3¢ and 5¢. How much did he spend? He had 9¢. How much did he have left?

H. spent 5¢ and 1¢ and 2¢. How much did he spend? He gave the grocer 10¢. How much change did he get back?

W. spent a penny more than he had, so he had to return a penny.

The children play in the store during their free time.

Teacher B. The children arranged toys in a toy shop window. Each child made up his own number story. Some of them were:

"I bought 2 dolls. They cost 3¢ apiece. How much did I pay for them?" J. called on someone to give the answer to his number problem.

The children were helped to work out the answers and did not lose their turn if they could not give the answer immediately.

Teacher D. Worked with the better group. We went on shopping tours and bought Christmas presents. Practiced with the larger numbers, 8, 9, and 10.

Purely abstract types of number experience came late in the semester and were usually limited to the more advanced groups of children. Illustrations follow:

Teacher D. Today I worked with the children who are ready to work out answers from combinations they already know. We practiced this and it worked very well. Most of the children caught on.

Teacher D. Worked with the more advanced group that is ready to start memorizing some of the combinations. We worked with flash cards and next week we are going to start making our own.

Subsequent reports explain that this group completed making addition and subtraction cards for all the numbers up to ten and used them in partners or small groups for memorization of the number facts.

Teacher B. Dec. 10. Children in the fast moving group used abstract cards for the first time with no check with concrete or semi-concrete materials.

As would be expected, many number experiences were not exclusively concrete, semi-concrete, or abstract, but rather a combination of more than one type. The following experiences combine concrete and semi-concrete experiences:

Teacher A. We made pictures with blocks and told what they said. A few drew a picture on the board to show us how we could make the same number of blocks into other number stories. A child's block story showed two groups, 5 and 2. Blackboard drawings showed 6 and 1, and 3 and 4.

Teacher B. We worked with blocks. Started with six and went on to eight. I showed number pictures which children had made. (Pictures cut from magazines and arranged into number groupings for a number scrap book) The children worked them out with blocks if they did not know the answer.

When the children began to show a readiness for the abstract understanding of numbers, the teachers worked out combinations of concrete and abstract experience or semi-concrete and abstract, to help the children make an easy transition to the abstract. The teachers found at times that the concrete, semi-concrete and abstract levels seemed to merge to some extent. When a group of children were nearing understanding of a number span expressed in abstract terms, there seemed to be a period during which concrete and semi-concrete procedures were used only as means of

determining the correctness of the answer. If the answer was correct, and if the teacher felt convinced that understanding was present and that mature processes had been used, it was assumed that the child had reached the abstract level. If, on the other hand, his answer was incorrect, or if for any reason he was asked to furnish proof, he resorted to either the concrete or semi-concrete level to justify his answer. Experiences of this type may be illustrated by the following excerpts:

Teacher A. "I have 10 sticks. J. will take some away. He will not tell how many he takes. I have this many left. (shows children number left) How many did he take?"

Some children had to work it out from the beginning with their own sticks. Some could think the missing number without having to work it out.

Teacher A. Played a game of "give and take," which we made up ourselves.

"I give you 5; now I take back 3. How many do you have left?"

When numbers were large (7, 8, and 9), we looked at the sticks to find out each time. When numbers were small (4, 5, and 6), we play this way: The children closed their eyes. I made a group. They opened their eyes, named the number group, and closed their eyes again. Then I took away some. They opened their eyes again, and told how many were gone.

Teacher B. The children sat in a circle, each with ten blocks and a pencil and piece of paper. I asked C. to put two blocks into the box and S. to put in four.

"Write down the number that tells how many blocks are in the box."

I checked to see if all answers were correct. Everyone got this without using blocks to count. Next S. came and took two blocks out.

"Write the number that tells how many blocks are left."

Then S. took two more away.

"Now how many are left? Write the number."

"That's easy," said J. "C's two blocks are left!"

Teacher B. I used buttons, starting with nine. I put four in one hand, and the others in the other hand.

"If I have four in one hand, how many do I have in the other hand?"

Worked on subtraction in the same way.

"I have nine buttons. I will take two away and put them in my other hand. How many are left in this hand?"

Children divided into small groups and continued to work with the button game in small groups. This was purely abstract except for the child who presented the problem. But, of course the buttons could be used for checking whenever a child gave a wrong answer or guessed. Later the same game was used in this way:

"I have nine buttons. Guess how many I have in each hand."

This brought out many variations, but I found several children using fingers. Next time I will provide them with blocks to use.

Teacher C. I wrote some number combinations on the board. One child would read the story and we would work with blocks to show it. We got extra addition and subtraction when adding or subtracting blocks to start the next problem. In the first problem we needed two and four. In the next one, we needed 4 and one, so we took one away. Sometimes we called attention to this. Other times it was done without presenting the problem. For the combination, 7 and 2, we checked the answer by counting, 7, 8, 9.

Teacher D. I worked with the lower group on number picture cards (abstract combinations on one side and pictured groups on the other) using numbers up to five. If the child could not get the answer, I turned the card over to show the story with pictures. Only one child had difficulty. They wanted to go farther, so we went on to sums up to seven.

Sequential plan for teaching the combinations.-- The selection of combinations to be taught followed the serial order of the number system. The first teaching unit for each teacher consisted of a span of about three numbers. One teacher found it necessary to begin with the number span containing the numbers two, three, and four, while another teacher, whose children were already familiar with these numbers, selected a higher span composed of the numbers five, six, and seven. Children worked within the selected number span on a concrete level, doing the

types of activities which were suggested in the preceding illustrations. The group moved to the semi-concrete level with a given series of numbers, when the informal day-by-day evaluation of the teacher revealed a readiness for semi-concrete materials. At the same time, another series, overlapping the first and extending beyond it, was selected for exploration and discovery on the concrete level. The two or more groups within a given grade worked on different number spans selected to meet the level of understanding of the children within each group.

Eventually, a child was working on approximately three number series concurrently--one series on the concrete level, one on a semi-concrete, and one on the abstract. The teacher's plan was to keep the work at the concrete level well in advance of semi-concrete and abstract number experiences. To illustrate, if the concrete number series at a given time consisted of the numbers from six through nine, the semi-concrete series for that particular period would probably be four through seven, and the abstract level would be represented by a series of even smaller numbers, perhaps two through five.

As problems arose, teachers solved them creatively in the light of their understanding of the general principles underlying the teaching plan. To the extent that goals were fully comprehended and kept in mind, the specific teaching procedures that emerged seemed consistent with these goals. As would be expected, the teachers varied greatly in their ability to comprehend and to carry out the idea of sequence as described. Quotations such as the following give evidence that the teachers had the general plan of a number span in mind as they worked with their children:

Teacher B. Worked with blocks. Started with five blocks and worked up to seven. Played a game to see how many different groupings we could make. As the children made their groupings, I drew them on the board.

Used wall chart (phrase card holder generally used for reading, but adapted for number work), blocks and circles. Called upon individual children to build number stories, while others watched to give the answer. Started with seven and worked up to ten. We did addition stories first, and then subtraction. Using blocks, each child made up his own take-away story, told it to the rest of the group, and called upon some one for an answer.

Awareness of thought processes.--- The teachers cooperating in the investigation made continuous study of the methods used by children throughout the semester. Preliminary decisions as to the maturity of various methods were made in group conferences with the teachers after the initial interviews had been completed. Teachers studied the responses of children on interview sheets and were then asked to help decide which of the methods used by the children showed the least promise for helping children understand number, and which showed the greatest promise. Implications for teaching were continuously drawn as the maturity of a method was considered.

In general, instruction was designed to move the children from whatever stage of maturity they had reached at the time of the interview to a higher stage. Methods used by children which showed no promise for further growth in accuracy or understanding of the processes were discouraged. Pupils were encouraged to discard them as soon as possible. "Guessing" and "counting all numbers" are illustrations.

While decisions on maturity of levels of thinking were being made, questions continually kept in mind were: (1) If a child is at a given level, what constitutes growth for him? In other words, what seems to be the logical next step in the direction of growth toward maturity for this child? (2) If the method he is using is inaccurate, what analysis of the method, as the child is applying it, can be made to help him become more accurate in the use of the method? What difficulties is

he having with the method? How can he be helped to overcome those difficulties? (3) How can readiness for a next level be determined?

A thorough study of interview data enabled teachers to become aware of thought processes and of helpful teaching techniques. With the methods of "counting variation" and "counting to add and adding," the teachers recognized their task as one of helping the child differentiate clearly those number situations to which these generalizations could be accurately applied. The recognition of situations to which "counting variation" was not applicable involved a study of the relative sizes of groups. When children had a clear understanding of groups, they could no longer be satisfied to add or to subtract one unit indiscriminately in all combinations.

Certain instructional procedures with regard to the method of counting all numbers were apparent to the teachers. They realized that those children who were aware of the two groups in an addition example were more accurate than those who counted continuously with no apparent realization of the groups as groups. They therefore encouraged children to pause, or in some way to indicate recognition of the two groups. Children were helped to recognize groups readily - to see a group of "five" as five, without the need to count each single object represented. Making groups in various patterns and designs with different types of materials helped children arrive at generalizations with regard to groups which were helpful in learning to combine groups accurately.

As one step beyond "counting all numbers," children were helped to use the method of "partial counting" whenever they showed a readiness for it. Readiness was considered present if a child could recognize groups with facility, and if he was adept at counting both numbers in the example.

Instruction in "partial counting" involved helping children who were ready for this method to work out the answers from materials in the most expedient way. Instruction also involved helping them arrive at such generalizations as the following from their work with concrete materials: (1) If only a few are taken away, the answer will be high in relation to the total. (2) If many are taken away, the answer will be low in relation to the total. (3) If any are taken away, the answer must be smaller than the original number. Similar generalizations apply in the process of addition. Some of these are: (1) If many are added to a group, the answer will be large. (2) If few are added, it will be only a little larger than the original group. (3) If one is added, the answer is only one larger than the original group.

As the children progressed toward more abstract types of methods such as "recall by logical inference" and "immediate recall," teachers were faced with the problem of determining if the use of these methods was accompanied by understanding. Techniques most often used for checking understanding were (1) informal discussion in which the children were encouraged to give reasons for the correctness of the answer, and (2) some form of concrete or semi-concrete demonstration of the number fact.

An instructional problem in subtraction caused the teachers much concern in the early weeks of teaching. On the initial interviews certain children had preconceived notions which completely blocked their understanding of subtraction as a process. For these children "6 take away 2" meant they were to put six and two together first, then take away the two, and have six left. With some children the ideas were so firmly fixed that much reteaching was necessary to redirect their understanding

of subtraction as a process. Instructional procedure included a wide use of concrete materials accompanied by oral discussion.

Variations in instructional procedure were made to help children understand the processes of multiplication and division. The children learned to group objects in multiples of 2's, 3's and 4's and to discuss these groupings in terms of multiplication and division.

It should be understood that any method can be used without understanding on the part of the child. Methods can be learned as mere feats of memory or as tricks of achievement. For this reason there is no substitute for the intelligent teacher and for her constant check and evaluation to make sure that understanding is present.

The following excerpts furnish evidence that the teachers were aware of the ways of thinking of their pupils and were consciously trying to help them use more mature methods:

Teacher B. The children were asked how many were 5 and 4. Everyone but S. and J. used blocks to find out the answer. I asked J. how he knew the answer.

"Well, 4 and 4 are 8, and one more are 9."

Then asked S. how she got her answer.

"Five and 5 are 10 and take one away, you have 9." Then asked S. how she knew that 3 from 9 would leave 6.

"Because 3 3's are 9," she said.

I put five blocks and then 3 blocks into the box. How many did I put in? J. said nine. I asked how she knew. She said,

"I counted 3,4,5,6,7,8,9." Asked her to put the blocks down and count them again. This time she got the right answer. J. said,

"I know an easier way. You start with 5 and say, 6,7,8."

I held up various number stories for the numbers 5 and 6 and then asked the children how they arrived at their answers. The children used blocks only when they could not think the answer. More of this group is counting by starting with the larger number.

Teacher B. made a comment which shows the reluctance of some children to go through the effort necessary to use concrete materials to arrive at answers:

There was some guessing at first by those who didn't want to work the problem out with their blocks. I encouraged them to use blocks when they were unsure of the answer. More of this group are beginning to see numbers in groups and though they are still counting, they start with the larger number. Several have solved problems by relating the answer they wish to find, to one they already know, as, 4 and 4 are 8, so 3 and 4 are 7.

Some of this group showed greater understanding of their numbers today. B. was called upon to give the answer for "6 and 3." He answered nine very quickly.

"You really knew that answer. How did you know?" I asked.

"Well, Virginia just had 7 and 2 and she said 9. So I took one away from 7 and put another one on 2."

Involved explanation, but he knew.

Teacher C. I find that children most often think a subtraction answer from an addition answer they know. When asked the combination "7 take away 4," R. said,

"I thought 3 and 4 are 7."

K. knew that 3 and 5 are 8; therefore 3 and 4 are 7; therefore 7 take away 4 are 3.

Teacher D. (picture of nine trees grouped in three's) P. had dabbed at the air with her finger counting each one. C. counted by saying 3, 4, 5, 6, 7, 8, 9. B. said,

"I know three 3's are 9."

Teacher B realizes by conversation with a pupil that, when number picture cards are used with a child who already knows his combinations, he is slowed up unduly by the necessity of reading the pictured groups:

Used cards to check each pupil. M. was slow which surprised me as he has been very quick at other times. He said he could do it quicker with numbers. When he stopped to figure out that $\begin{smallmatrix} 000 \\ 000 \end{smallmatrix}$ and 00 was the same as 7 and 2, he knew the answer.

Teacher C. describes the following procedure for helping children see relationships between numbers:

Worked with the doubles and related numbers; 4 and 3 as related to 3 and 3; 2 and 3 as related to 2 and 2; 4 and 3, and 4 and 5, as related to 4 and 4; etc. Did the same with subtraction at a later period; 8 take away 3 as related to 8 take away 4; 6 take away 2, and 6 take away 4, as related to 6 take away 3, etc.

Many times during these two weeks (report of Jan. 25th) I found that the children might know two 4's are 8 immediately, but if I turned the combination around and asked them what four 2's were, they usually stopped to figure or count. So I've shown them with blocks that a rearrangement of groups is necessary though the result is the same. Some children are not able to see this relationship by means of abstract thinking alone.

Teaching processes in interrelation.--- Teachers were instructed to teach the four processes together during the beginning stages of learning. This method was found feasible during the stages in which children used concrete and semi-concrete materials. To illustrate with concrete materials, the child can do the following with eight blocks:

1. He can take away and given number from eight blocks and find out how many are left, thus finding one of the relationships between the sub-groups with which he is dealing, and gaining some understanding of subtraction as a process.

2. Each time he takes away from eight blocks, he can put them back together again and thereby engage in the process of addition. As different relationships between the groups are recognized, addition comes to have meaning for him.

3. He can divide eight into four two's or into two four's. With such operations he can answer either multiplication or division questions. As he engages in these activities, he has the opportunity to discover the meaning of multiplication and of division as processes, and to see other existing relationships within the sub-groups for the number

eight. In like manner, when presented with a group of six balls, such as $\begin{smallmatrix} \circ \circ \circ \\ \circ \circ \circ \end{smallmatrix}$, possibilities for addition, subtraction, division and multiplication combinations may be seen by the child. By working with the four fundamental processes at once, the relationships between them may be more easily seen and used as a basis for making generalizations.

Teacher A. We used the felt-o-gram and blocks. We worked on all processes together. Addition is rather easy for this group (middle group). Most of the children did not need to use blocks. Subtraction is harder and blocks came in handy.

Teacher B. Used number picture cards with the numbers six and seven. Worked with all processes together. Some have difficulty in the last two processes (division and multiplication), and I concentrated on these children.

While the general procedure was to work on all processes together, teachers often found it necessary to isolate a process temporarily for concentrated practice to clear up misconceptions:

Teacher A. I worked with a group of children using blocks and sticks. This group was having difficulty with problems involving "how many 2's in 6?" etc. Some got the idea after a concentrated ten minutes on this alone.

Had a group of children who were having trouble with subtraction. Tried to straighten out the children who were saying "9 take away 6 are 9," "7 take away 3 are 7," etc. It helped. Some lost their over-confident attitude that "this is easy - all you do is just say the first number."

When the children began to use symbols freely (second half of the instructional period for the more advanced groups), processes were not mixed indiscriminately where written forms were required. As long as concrete and semi-concrete materials were used, experiences could involve all processes without confusion. The written forms seemed to bring new difficulties of reading and interpretation. Combinations were usually grouped by process, therefore, when written forms were introduced.

Attitudes toward arithmetic and sensitivity to number situations.--

Teacher E tells of a mother who came to school to inquire what the child

was learning in numbers, since the child insisted that the ornaments on the Christmas tree must be arranged in groups!

The teachers reported other instances of carry-over of school number experience into the home, such as making lotto games, number picture cards, and the like. Teachers also mentioned a wide use of number materials during free time at school.

In the course of interviews with twelve teachers who had taught the children the previous year, nine teachers said they had discouraged the children from counting on their fingers. It is therefore not surprising to find that certain children sensed such strong disapproval that they felt it necessary to hide their fingers when counting them. Teacher C describes how she handled such a situation:

Somehow when they use their fingers they try to hide them. Then I have to say, "If you want to use your fingers, you don't have to hide them." R. uses his fingers on problems like 9 take away 6, but if I just say, "What is 10 take away 6?" and follow with "Then, what is 9 take away 6?" he can get it very fast, so I guess he just needs practice in thinking along these lines.

Teacher C first of all accepts the child's counting on his fingers, but at the same time takes steps to help the child toward ways of thinking which are more mature and which will eventually eliminate the need for counting fingers.

Certain spontaneous remarks of the children as they worked with numbers showed enjoyment. While working with concrete materials, H. was heard to remark, "I didn't think this would be this much fun." T. volunteered, "I'm going to practice this at home." E. showed very evident pleasure when he progressed beyond the need for concrete materials.

Teacher E. Dec. 8. Put three written problems on the board for the better group to do. Example: Mary went to the toy shop. She had 10¢. She bought a top for 6¢. How much did she have left? Most of the children

understood what to do after I explained that they were to figure out whether it was an adding problem or a take away problem. They all seemed to enjoy thinking it out. P. said,

"This is more fun than that other stuff (working with concrete materials). It gives you something to do."

The children often showed ingenuity in working out a rather difficult problem. Teacher C tells the following incident which occurred as the children were counting money for a trip:

One child made the comment that there were ten fifty-cent pieces in \$5.00. I asked how he got it. He had counted on his fingers, tapping two times on each finger, once below the knuckle, and once above!

On occasion the activity under consideration took precedence over the number ideas involved, as Teacher E discovered when she attempted to have the children place animals on a farm frieze to show number groups:

We worked out places for the farm animals. By sticking them on temporarily with Scotch tape, we made many number groups. When the children pasted the groups in permanently, they didn't make the groupings so prominent.

"There is too much space left that way," they said. "This sheep can eat more here!"

Trees in an orchard was a more natural situation and the children raised no objection to putting the trees in regular groupings:

We looked at the trees painted on our farm frieze and J. said, "They really look like the picture of the orchard because they ain't in one line."

"Show us on the board how they are grouped." J. went to the board and made lines to show three 3's.

"How many trees are here?"

"Nine," he said.

Incidental experience and needs of the group used as a basis for planned number experience.-- The planned number program was not limited to work on the four fundamental processes with numbers below ten. Also included were number experiences dealing with money, telling time,

measuring, Roman numerals, meanings for numbers larger than ten, writing numbers larger than ten, fractional concepts, and tax stamps. The teachers planned number activities centering around these topics to meet the needs of the various groups for more understanding, and to fulfill the requirements of the course of study in arithmetic for second grade. Experiences dealing with money concepts were similar to the following:

Teacher B. Since we have been counting money so much, I used this as a basis for our group work. I drew pictures on the board to show ten pennies in two groups of five each, two nickels, and one dime. Pennies were counted by one's and by five's. Ideas brought out were: The worth of each piece of money in cents. Five pennies will buy the same as one nickel. If there are two nickels in a dime, there are two five's in ten, because there are five pennies in a nickel and ten pennies in a dime.

"There are six tens in sixty," said M., "because when F. brought fifty cents he got five savings stamps. Let's tell some other number stories. If I had 20¢, how many stamps could I buy?"

"Two," said T. "You count 10, 20."

"How many nickels in 20¢?"

J. said, "Four, because two and two are four. There are two nickels in each dime, so in two dimes, you get four nickels."

Teacher D. M. was quite upset at noon time that I gave her a dime instead of the two nickels she had given me in the morning. When I handed the children the blocks at number time, I said, "I am giving each of you ten pennies." We found how many five's were in ten. We let two big blocks stand for nickels. Five little blocks (pennies) were the same as one big block (nickel). Two nickels (or ten pennies) were the same as a dime.

Planned activities in measuring are described as follows:

Teacher E. Measuring was needed to see how long our dowel sticks should be for the wheels of our food train. Two dowel sticks were longer than the ruler, but not so long as the yardstick. I cut a dowel stick into twelve one-inch pieces to show the twelve inches that made up a foot. We decided to make the sticks nine inches long. Paul measured a dowel stick twelve inches long and sawed it off. Then he measured nine inches and cut off the remaining three inches. We put the nine separate parts and the three-inch piece together to show twelve inches again. Worked some addition and subtraction problems with the parts.

Teacher B. We talked about measuring things. Last week I measured each child for height. We talked about the difference between a ruler and a yardstick, pointing out that a yardstick is always three feet long. I drew a ruler on the board, and explained inches and half inches, showing two halves to a whole inch. Then I measured several objects to show that we measure both length and width. Then each child measured some objects that he had access to, such as his pencil, eraser, his foot, etc.

Teacher A. We measured ourselves and made a chart to show how many inches tall we were. As each child was measured I wrote the number on the board and asked someone to tell me how tall the child was. Then I would put on another child's height and ask which was tallest or shortest. Some knew how much taller and shorter, too.

The following accounts illustrate the type of experience provided to help the children learn to tell time:

Teacher A. We started telling time today. Had a general discussion on how many 5's there were around the clock. How many 5's in 60? How many 5's in 30? In 15? We counted by 5's to find out. How much is half of 60? Worked it out by counting by 5's. When the long hand is on six, it is called "thirty," as 6:30, or 7:30.

Teacher B. Everyone made a clock. I discussed the clock and ways to make one - dividing the circle first in half, then in fourths, explaining that six numbers are in each half. Discussion included: How many hours are shown on the clock; how many minutes in an hour; the long hand designates the minutes while the short hand tells the hour. Children had an opportunity to manipulate the clock, and ask others what time it was. We watched our own clock in the room, and noted that the little hand moved more slowly than the long hand.

The need for understanding numbers larger than ten came about when children had difficulty recognizing page numbers in their reading books. The following activities were designed to help children reach a better understanding of large numbers:

Teacher C. Planned work on the number sequence from one to one hundred. We counted by ones, by tens, and by twos. Then I pointed to different numbers and we went around the group taking turns. There were three girls who could not recognize the names of the tens - 20, 40, 50, etc. So I tried to explain it. I then asked the children to find the number that followed or came before the number I called. I was real happy because they seemed to get it.

Teacher B. Worked with numbers from one to fifty to establish meanings of tens and ones. Also brought in sequence:

"What number comes before 44? After 35?"

In a later period extended this work to 100. A hundred chart was presented to the group. Discussion followed these lines:

"How many in each column? How many altogether? How many tens in 100? Count to find out. Who can count by fives to 100? Look at these numbers (numbers pointed out by teacher). What does 55 mean? 35? 12?"

This was to bring out the idea of tens and ones and they had no trouble at all. Later, I erased several numbers and had different children write in the missing numbers. Then,

"If we count by tens, what number would follow 20? 10? 15? 40? What number means 10 and 7? 10 and 2? 10 and 1? etc."

This group catches on very quickly and seems to have the concept of ten (better group).

Work with sales stamps involves bundling stamps so as to make tens. For example, five two's, or three three's and a one, or ten one's, could be bundled together to make a ten. These were then put into hundreds. Group work was devoted to this activity as often as necessary until the children became sufficiently independent to carry it on without guidance.

Teachers B, C, and D taught the meanings for Roman numerals to 12 and how to write them. Teacher B planned some work on halves and fourths for her more advanced group. The fast-moving groups in each grade learned to write numbers beyond ten.

Number symbols and signs.--- Number symbols were introduced only after meanings for the number symbols had been developed. Of course, the majority of the children had learned to read and write the number symbols in first grade. In any event there was a short teaching period during which symbols were omitted altogether, and meanings for numbers were stressed in order to assure understanding for the symbols.

Teacher A describes introducing the addition sign (+) on January 22. Signs for other processes are not mentioned. Teacher C's

fast-moving group began to use signs for addition and subtraction on December 3. No mention is made of multiplication or division signs being used by any group. With the exception of groups A and C, signs were not used at all, but processes were described by means of descriptive language phrases, such as, "Two 3's are;" "How many 4's in 8?" "9 take away 6 are;" and "3 and 4 are."

Types of number work used as independent work.--- Any type of work which the children were expected to accomplish independently was first engaged in by the group to assure understanding. Afterwards independent work was checked with the children to correct mistakes and clear up misunderstandings.

Teacher A. We had written examples from the board. Each child was given blocks, sticks, or discs to work out his examples if it was necessary. When I was free, each child came up to me with his paper. We went over the work. Even if he had them all right, I still had him use materials to work out a few. If he had some wrong, we worked those out together.

Teacher B. The children cut out pictures and pasted them on paper to make number stories. They then wrote their number stories on separate pieces of paper. They did this while I worked with another group. Later I looked over each one, and held up the pictures for individual children to give the answers.

Teacher B. Draw pictures to tell these number stories:

1. Jane had 10 oranges. How many different ways can she group them?
2. She gave five oranges to Jim. Show how many she had left.
3. Show how many two's are in ten.
4. Show how many five's are in ten.

I had different children demonstrate to show how they could make different groups. Then the children worked at their tables.

The results on the above led the teacher to realize the need to plan more group work of this type before independent work was again attempted. Her comment was as follows:

Eventually this work was accomplished, but needed a lot of explanation. I plan to follow up with group work.

Teacher C. Draw 4 houses, one red, one blue and two green. Put one tree in the yard. Draw five children in the street jumping rope. Have two cars in the street. There are three dogs. We counted these things together to see how many things would be on our paper. We got 15. Therefore, when the children finished, they checked by counting to see if anything had been left out.

Teacher C. Each child had ten blocks. They worked independently to find all the different groupings they could find for the number ten. The next day we brought our papers up and took turns writing something that we had found on the board, as 2 and 2 and 2 and 2 and 2 are ten. We related this to five 2's in 10, and to five 2's are 10.

Sharing ideas and experiences.-- The teachers shared materials and experiences which they had found to be successful with their own children. One teacher brought in pages of the number scrap book her children were making and described how the children worked out the number groups and accompanying stories. Ways of using commercial games for independent number periods were explained. The teachers also described games they had prepared themselves and found successful. A tally system for keeping score was reported. Ways of making adaptations to other groups of children were discussed.

When initial interview sheets were analyzed, there seemed to be a definite need for help in understanding groups, since a large majority of the children tended to string out objects rather than to group them. Grouping activities were discussed. Later diary entries give the following illustrations:

Halloween cut-outs (pumpkins, apples, etc.) used to make groupings of different numbers.

We talked about things that come in fives. J. mentioned a star with five points. H. remembered five dots on a tie. B. had five buttons on his shirt. R. mentioned five pennies in a nickel.

The children had a tendency to spread their objects all over the paper when working with concrete materials. They scattered them over the paper to such an extent that groups lost their essential grouping quality. We worked on making groups clear and simple, - working near the center of the paper rather than toward the edges.

Early misunderstandings of some teachers which were cleared up by means of illustrative material were differentiations between incidental and planned number experience, and between concrete, semi-concrete, and abstract number experience.

At one time concern was expressed that the children were finding work with concrete materials monotonous and repetitious, though they were not ready for more advanced work. Teachers pooled ideas to see how much variety they could bring into concrete number experience.

The value of the interview technique as a teaching device was repeatedly brought up for discussion. Questions suggested were: "How did you think it?" "Can you think it another way?" "Suppose you couldn't remember it some day, how would you work it out?" "Which way is quicker?" Such questions were recommended as workable ways of helping children become aware of thought processes and of beginning an evaluation of different methods.

The principles and practices listed at the beginning of the chapter were continually discussed and evaluations were made as to the success or failure of the teachers in achieving them. Weaknesses were pointed out and plans made for overcoming them. For example, a frequent error was to move ahead to larger numbers before the children had developed the understanding necessary to cope with them successfully. One teacher had a very bright group of children who were very eager to advance rapidly. It was evident that the teacher was in conflict as to how to satisfy the children's desire to go on, and at the same time be sure that concepts of small numbers were adequate.

Another weakness seemed to be a greater stress on addition than on other processes. Teachers were encouraged to incorporate all processes except when isolation of a given process seemed feasible for a particular

group of children. Efforts were made to relate the processes, and, on the whole, the teachers reported gratifying results in teaching the processes in interrelation. The independent work of the children tended to over-emphasize addition throughout the semester, however, probably due to the traditional idea that addition should be taught first, and to the fact that the language form for addition involved less writing and was therefore easier to prepare.

Principles and Practices for the Incidental Number Program

Certain of the principles and practices previously stated as guides for the planned number program are also applicable in the case of the incidental program. Others, such as those dealing with time allotment, grouping of children, and a defined sequence for number presentation, do not apply. To a great extent these were chance factors in the incidental program.

In incidental as well as planned number experience, the teachers tried to be aware of the thought processes of pupils, to point out relationships to other processes than the one most immediately involved, to use concrete and semi-concrete representations when needed, and to build favorable attitudes toward number.

In addition to the principles and practices which were the same for the two phases of the program, certain others are of importance for the incidental program alone. These are:

1. Incidental experience offers an opportunity to meet needs of the children for solving real problems which occur throughout the day's activities.
2. Incidental experience provides a setting in which applications of number may be made. There is a real use for number understandings which have been acquired.

3. Incidental experience often points out a need for more understanding of numbers, and offers the incentive to acquire greater skill.

4. The teacher who guides children through incidental number experience is faced with problems such as the following: (a) wise selection of incidental experience so that children will not practice number abilities which offer no further growth for them, (b) adapting more difficult number experiences so that a readiness for later work in arithmetic will be promoted, (c) judicious selection of individuals to engage in a particular experience, so that children needing this type of experience will profit, (d) developing within herself and within her group of children a sensitivity to number possibilities which offer opportunities for number growth, and (3) becoming aware of adaptations that can be made to promote further growth.

Weekly meetings of the teachers were again to serve as a place for sharing ideas, making clear differentiations between incidental and planned number experience for purposes of recording, and working out difficulties with regard to incidental experience.

The Incidental Number Program

The first three weeks of the fall term were spent in administering group tests and interviews. A definite program of instruction was of necessity delayed until the beginning of the fourth week of school. In the meantime, teachers were urged to make use of incidental number experiences in teaching and to record them for discussion at the first meeting of the university class.

The problem posed by the first set of incidental number experiences turned in by teachers was that of making clear differentiations between

planned and incidental experience. The conclusion was reached that any experience which was planned as group work and decided upon ahead of the time of meeting of the group would be included in the diary as planned experience. Experiences which occurred in the course of the day's activities, at the instigation of either teacher or pupils, but were not pre-planned, would be considered incidental.

A total of 236 incidental number experiences were reported during the semester. These were distributed among the five pupil groups, as follows:

Group A	- 40
Group B	- 71
Group C	- 42
Group D	- 71
Group E ²	- 12

It is obvious that the teachers varied greatly in the number of incidental experiences they recorded. Whether Groups B and D were actually having almost twice as many experiences as Groups A and C, or whether Teachers B and D were more conscientious in keeping records, is a question. The teachers agreed that certain experiences were carried on almost daily, or periodically, though they failed to record them over and over, and concentrated on those which offered variety. The experiences recorded are therefore more accurately thought of as a sampling of experiences which occurred rather than an exhaustive list. Teacher B was particularly adept at creating original ways for carrying out routine tasks so as to vary the number aspects involved.

Following is a list of the varieties of incidental number experience reported by the six teachers. Categories are selected according

² Illness in the family and personal illness were responsible for the poor showing of Teacher E with respect to recording incidental number experiences.

to the situations out of which the number experience arose. A total of 236 experiences are represented. The various items are classified by topics.

Experiences with money

Collected and counted money for Weekly Reader
 Counted lunch money
 Counted money collected for Red Cross
 Collected and counted money for ice cream for Halloween
 and Christmas parties
 Bought lunch - checked on change
 Counted lost money which had not been claimed
 Counted money for Christmas tree
 Collected and counted money for Art League
 Decided on number of stamps to buy with stamp money
 Checked on cost of Mother's Christmas gift
 Checked on money needed for overdue library books
 Learned that lunch money can be received in different
 denominations, and that there will still be the same
 amount of money
 Decided how much money to spend for regular lunch and how
 much for dessert
 Collected and counted money for trips
 Bought stamps in play post office
 Made change in post office
 Put different denominations of tax stamps into groups
 totaling ten each

Experiences with time

Found out how many minutes before recess, time to go home,
 lunch time, etc.
 Found out number of minutes in half hour, quarter of an hour
 Found out how many minutes it took to clean up
 Used play clock to set time
 Used a library book on telling time
 Examined clock to find out how many little spaces (minutes)
 there were from one number to the next
 Stayed fifteen minutes after school
 Estimated time needed to complete a story

Experiences with temperature and the calendar

Kept weather report by half days - found two and a half
 rainy days in week, one and a half cloudy days, etc.
 Read temperature; compared with previous days
 Made thermometers with movable tape for recording temperature
 each day
 Kept a record of rainy days, sunny days, cloudy days, etc.,
 in a week; used for comparisons

Counted days until a child's birthday, or until a holiday
 Discussed days in a week; days we come to school; days we
 do not come to school
 Counted days in the month of December
 Counted number of school days in December
 Brought out idea that months do not all have the same
 number of days
 Compared temperatures; discussed terms "more" and "less"

Number experiences in classroom routines

Compared large and small pieces of paper
 Counted strips of paper needed to make pumpkin men (arms
 and legs, body, head, etc.)
 Counted squares when paper was folded for construction work
 Checked on library books to be returned
 Gave out paper
 Checked on materials ordered
 Checked on materials needed for each table
 Checked on number of extra items needed when passer did
 not have enough; number left over when too much was
 given; total in each case
 Drew numbers to decide place in line
 Checked on number of children present and absent; number
 staying for lunch
 Counted children at each table; compared with other tables
 Decided on number of rows needed in auditorium if twelve
 children sat in each row (38 children)
 Found changes in total number of children when some left,
 more were added, etc.
 Decided how to space three pictures among seven windows
 Decided what to do when there were four blackboard spaces
 to be washed and only three rags
 Decided how many more desks were needed when we had thirty-
 five children and only thirty-three desks
 Divided paper into fourths

Number experiences in subject matter areas exclusive of arithmetic

Compared meaning of girl and girls, boy and boys, etc.
 Checked on number of books needed for reading group
 Put scrap books together; checked on missing pages
 Found number stories in content of reading material or in
 pictures in books
 Changed size of reading groups; discussed changes
 Guessed size of group; then counted to check accuracy of
 guess
 Used one-inch graph paper for keeping record of spelling
 work
 Placed words in groups of three for spelling game
 Found how many words start alike on a given page
 Found six-letter words, four-letter words, etc.
 Read half a page

Arranged reading group so that half the children would be
 on each side of the teacher
 Found the middle of the group of children
 Found one-half of eight pages
 Children read one and a half of four pages and found they
 had two and a half pages left to read
 Called attention to a line in reading book (third, fourth,
 etc.)
 Discussed what it means to find the seventh line

Number experiences in play and game situations

Divided children so as to have an even number on each side
 Kept score in bean bag and ring toss games
 Counted by three's to get ready for "Squirrel in Trees"
 Divided thirty-two children into five squads for play room
 Divided box of thirty marbles between two boys, each taking
 five at a time until all are gone (home experience described
 at school)
 Decided on number of children to play in playhouse at one
 time; how total number should be divided among boys and
 girls
 Counted gym shoes by two's

Experiences in measuring

Found number of inches in a foot
 Found number of feet in a yard
 Measured materials for Father's gift (14 inches by 12 inches)
 Measured linoleum to go under easel
 Measured heights of children and made comparisons
 Used rulers to make mats for desks

Miscellaneous number experiences

Discussed odd and even numbers (house numbers)
 Gave out lollipops at a party
 Read dental cards to see how many teeth needed attention
 Counted napkins and spoons to be used at a party
 Counted votes
 Counted lights on Christmas tree
 Decided on number of children needed for a committee
 Noticed that cars in parking lot were arranged in groups
 Counted by two's to get partners for trip
 Found how many dozens in six

Analysis of incidental number experiences.--- Forms prepared for
 recording incidental number experiences provided for an account of the
 number experience, the date, the teacher's name, the process or processes
 involved, the kinds of materials used in carrying out the experience, and

the size of the groups taking part in the experience. Figure 1 illustrates the procedure used in analysis.

The description cited in Figure 1 is an account of an experience of Group A which occurred on October 11. Two processes, addition and subtraction, were involved. The total group of children watched the teacher make the demonstration with concrete materials. The experience was also partially abstract. Checks were made in appropriate blanks to indicate these facts.

Some of the incidental number experiences which were reported involved no process as such, but rather called for the use of number abilities such as counting, comparing, estimating, and the like. This type of experience is illustrated by an account from Teacher C which was dated October 26. The solution of the problem necessitated ordinal counting, but no process was needed.

We had trouble with our girl's line coming in. There were always fights. So one girl wrote the numbers on little pieces of paper for each girl. Another girl checked her. Then we put them face down and each girl drew one. It was to stand for her place in line. I wrote the names on the backs of each piece. Then one boy put them in order to 25. Then we had the order of the line, and each girl knew who she came after in the line.

An analysis such as has been described was made for each of the 236 incidental number experiences. Table 3 gives the distribution of experiences according to process. The number and percentage of experiences involving no process at all, a single process, two processes, and more than two processes is given. Fifty-four experiences, or 23 per cent of the total number, needed no process for solution. These were largely counting experiences, or experiences dealing with measuring, fractions, reading and writing numbers, and the like. The process of addition was the most widely used of any of the single processes. The most widely used combination of

INCIDENTAL NUMBER EXPERIENCE

Teacher A Date: S O N D J F
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Process

A ✓

S ✓

M _____

D

Type of Material

Concrete ✓

Semi-concrete

Abstract

Description of Number Experience

Girls and boys brought in money for Red Cross. We needed 50¢ to become members. We had 42 cents. One boy said, "If I give my nickel, will we have enough?" I got out blocks and called big blocks nickels and little ones pennies. Add enough blocks to make 40¢. Add one more to make 45 (his extra nickel), then added our two pennies. Found out we had 47¢. Knew 5 pennies were a nickel, so we saw we needed three more cents to make a nickel.

Comments:

Size of Group:

Total	Small	Individual
1		

Figure 1.-- Pattern for Describing and Labeling Incidental Number Experiences.

TABLE 3

DISTRIBUTION OF INCIDENTAL NUMBER EXPERIENCES AS TO PROCESS
OR PROCESSES REPRESENTED, WITH PERCENTAGES OF TOTALS AND SUBTOTALS

* Process or Processes	Number of Experiences	Percentage of Subtotal	Percentage of Total
No Process Subtotal ...	54 54		23
Single Process			
A	43	63	
S	16	24	
D	1	1	
M	8	12	
Subtotal ...	68		29
Two Processes			
AS	44	49	
AM	18	20	
AD	17	19	
SM	4	4	
SD	4	4	
DM	3	3	
Subtotal ...	90		38
Three or Four Processes			
ASD	8	33	
AMD	9	38	
SMD	1	4	
ASM	4	17	
ASMD	2	9	
Subtotal ...	24		10
Total	236		100

* Process Code: A = Addition; S = Subtraction;
D = Division; M = Multiplication

two processes was addition and subtraction. Forty-nine per cent of the total number of experiences for two processes fell into this classification. More than two processes were used only rarely (10 per cent of the total).

Data are assembled in Table 4 to furnish a distribution by pupil groups of the types of materials used in the solution of problems arising in incidental number experiences. Seventy-four problems, or 31 per cent, were solved abstractly. For a slightly larger percentage, a combination of materials was used. Sixty experiences, or 25 per cent of the total, were solved by means of concrete materials. Semi-concrete materials were employed for only a small number of experiences (10 per cent).

There was a wide use of combinations of materials, such as concrete and abstract, semi-concrete and abstract, or semi-concrete and concrete. The combination of materials usually meant that some children in the group could solve the problem abstractly; others needed some type of representation for clarification.

In Table 5 a distribution by pupil groups is provided, showing the size of group participating in incidental number experience. The data show that the size of group varied greatly with the individual teacher. In Group A, experiences were usually individual. Teacher B made a wide use of combinations of one child and the total group, a small group and the total group, or a small group and an individual. The total group was commonly involved in Group C, while for Teacher D the most prevalent practice was to engage the total group, though small groups and individuals were also used.

Illustrations of Incidental Number Experience

Solving day-by-day problems of individuals or groups---

Incidental experience provides an excellent opportunity for children to

TABLE 4

FREQUENCY WITH WHICH VARIOUS TYPES OF MATERIALS
WERE USED IN INCIDENTAL NUMBER EXPERIENCES

Group	Type of Material				Total
	Concrete	Semi-concrete	Abstract	Combina-tion	
A	5	0	18	17	40
B	18	16	16	21	71
C	14	5	10	13	42
D	21	3	29	18	71
E	2	0	1	9	12
Total	60	24	74	78	236
Per cent of Total	25	10	31	33	

TABLE 5

FREQUENCY WITH WHICH GROUPS OF VARIOUS SIZES
PARTICIPATED IN INCIDENTAL NUMBER EXPERIENCES

Group	Size of Group				Total
	Ind. Child	Small Group	Total Group	Combi- nation	
A	21	7	11	1	40
B	12	4	9	46	71
C	5	10	25	2	42
D	21	19	30	1	71
E	5	2	1	4	12
Total	64	42	76	54	236
Per cent of Total	27	18	33	22	

solve the real problems that occur from day to day. The following accounts of incidental experience show how the children were helped to solve problems of concern to the total group:

Teacher D. We had to figure how much money we needed to buy ice cream for our party. There were 38 children and we all needed to bring a nickel. The children decided that would be 38 fives. So I put 38 fives on the board and we counted by fives to figure it out. We found it was \$1.90. Then two children said they would not be able to come. So they said two fives off. Ten off altogether. This meant we would need \$1.80 in all.

The teacher helps the children solve a difficult problem by adapting it to their level of understanding. They are unable to multiply 38 by five, but they can count by fives. In addition to solving their problem successfully, they are also building a readiness for understanding the meaning of multiplication as a process.

Teacher B. How many cupcakes will we need for each table if each person gets two? I had a boy count the boys at his table, and a girl count the girls. M. was at the first table. He said,

"Three boys, so we need six - two, four, six."

C. gave the number needed for the girls, saying, "Four, because there are two girls. I counted by two's."

The teacher simplifies this problem for the children, helping them to work it out by tables to keep the numbers small.

Teacher B. Problem: To find out how much mother's gift would cost. Each child brought 10¢ for the gift. Each child was to trim two coat hangers. The coat hangers cost 2¢ each, and the ribbon for trimming cost 5¢. How much would each child need to pay? Several of the children said, "seven." I went to the board and wrote, "two coat hangers - 2¢ each," and asked,

"How much is that?"

The children knew. "Now put 5¢ more to it." Some of the children knew it was 9¢. "You give me 10¢. How much change do you get?"

A problem involving several processes can be solved by second-grade children when they pool their knowledge and work under the direction of the teacher.

The experiences which follow illustrate how an individual child was helped to solve his problem:

Teacher D. A child brought \$1.00 to school. He was supposed to pay me 89¢. He wanted to know how much change he should get back. I told him to help me find out. I asked him how much he should get back to make 90¢. He said 1¢. Immediately after this, he said,

"I get 11¢ back." After he got to 90, he knew there was ten left right away.

Teacher E. J. was collecting lunch money. He had 5¢ too much in the boy's pile. He was minus 5¢ in the girl's pile. He had only a dime, however. He couldn't figure out how to get it right. I helped him use the counting frame and showed him five pennies in a nickel and two nickels in a dime. He saw that he could take two nickels from one of the piles, put in a dime and still come out right.

Teacher D. After reading group was over W. asked me if it was recess time yet. I said,

"No, it would be recess at 2:20." I asked him how much time he had to work yet. He counted by five's around the clock and said "25 minutes" which was correct.

It is significant that in each of these illustrations the child was given what help he needed to solve his problem. In none was he merely told the answer.

Awareness of thought processes of children.--- In incidental experience as well as in planned number experiences, the teacher demonstrated a consciousness of the thought processes of pupils, and often tried to help them raise the level of their thinking by pointing out more mature ways of solution, or having another child do so.

Teacher D. While decorating the Christmas tree, the children wanted to know how many lights we would have on our tree. We found we have three strands of lights with twelve lights on each strand. I asked if any one could tell how many altogether.

W. said, "Thirty-six." I asked how he got it.

He said, "Three tens are thirty, and two three's are six, so that makes thirty-six."

Teacher D. We were recording the thermometer readings on a chart. The temperature was 45 degrees. The time before it had been 38 degrees. I asked the children how many degrees difference there was. Finally M. said 7 degrees. I asked how he had arrived at his answer and he said,

"Thirty-eight to forty is two, and you know 40 to 45 is five. So 5 and 2 are 7."

Teacher E. One person from each table was to get two pieces of paper for each person at the table. P. started counting her table by one's. J. stopped her and said,

"Count by two's. It's easier." There were six people at the table. I put circles in groups of 2's on the board to show six, and counted by two's.

Teacher A. We had two 15¢ tax stamps. Some one said if those were 5¢ stamps, we would have six stamps to turn in instead of two. I said for him to explain what he meant.

"Well," he said, "there are three nickels in 15¢, and a nickel is five cents. There are two 15¢ stamps, so three nickels from each put together are six."

Teacher B. In reading group, we counted to see how many books we needed. There were eight in the group. Two children already had books. T. started to count all over again to find out how many books he needed to get, so I said,

"Just take two away from eight." He counted 8, 7, 6, and knew he must get six books.

Such illustrations demonstrate that the teachers were as concerned with the method by which the child arrived at his answer as with the correctness of the answer itself. The teachers often asked children to explain how they thought out an answer. There is evidence of children helping each other, and of teachers pointing out more mature ways for children to try.

Sensitivity to number situations.--- The following excerpts are an indication that the children were sensitive to number situations, and were able to see applications for numbers within the level of their understanding.

Teacher A. Got a new boy and lost one. We added one boy to our list and we had 15. We took one away and we had 14 boys - the same as before. D. said,

"Yep, that's right. One take away one leaves zero, so we ain't got nothing to add. It all stays the same."

Teacher A. C. had a birthday. He was eight years old. He got eight swats, one to grow on, and one to be good on. Some one said,

"When he's nine, he'll get eleven swats, 'cause when he was eight he got ten. He'll always get two more than the number of years old he is."

Teacher A. Had nine books for a reading group of twelve children. Therefore we had to share. Three children were absent one day. Some one said,

"Six people are absent because no one has to share." We figured out then that six people have to share but only three are missing books.

Teacher A. We moved our desks into blocks of four. After we had put all the fours together, we had two left over (34 children). Some one said,

"We'll have to make three and three are six, instead of two and two are four, somewhere in this room."

The two illustrations which follow show that these particular children were curious about numbers and eager to find opportunities to use them. They also indicate the readiness on the part of the teachers to grasp the opportunity afforded by the number experience to extend number understanding in the direction of further learning for the children.

Teacher A. We were finishing a book. We had eight more pages to go. Some one said,

"If we each read a page we can finish the book today."

I said, "Yes, but if we each read a half of a page, how many days will it take us to finish?" Some knew. Some counted one, two, on each page, and concluded two days. Some didn't respond at all.

Teacher D. While in reading group, a boy had finished his reading before the rest. He looked around at the group who were still reading and said,

"I wonder how many groups of three are in this group." I told him to find out. He went around the circle and pointed out three groups.

I made one extra. I said, "How many are three groups of three?"

He said, "Nine, because three 3's are 9." I asked what one extra would make. He replied "ten."

Adapting number experience to the level of the individual or group.-- Teachers often carefully selected certain children to carry out a number activity, because they knew the children's need for the particular experience. This was true in the case of the following incidents.

Teacher B. Counted money for Red Cross. Had V. collect from children and hand me the money, telling me how many pennies each child gave her. V. is slow in numbers. I thought this would help her. Later, she counted the pennies by two's after saying,

"Oh, I can't do that." She stopped to think before picking up every two pennies, counting to herself.

Teacher B. Counting attendance. One child counted people at his table.

"How many boys? How many girls? Show us at the board."

The children then drew groups at the board to represent four girls and three boys. I chose children who have obviously been counting by one's so they could begin to count in groups.

The following accounts show how teachers helped children clear up misunderstandings by making adaptations of the number experience necessary to clarify the ideas involved.

Teacher C. P. came up and asked about a combination on the board. It was, "How many are five 2's?" I asked her how much were two five's. She said ten. I said,

"Five 2's are the same." Then I showed her with pictures on the board that you have the same number but the groups are different. Then showed her that two 3's and three 2's made the same number.

Teacher C. W. was making up subtraction combinations. He had 7 - 10, and 4 - 6, etc. I asked him if he could do that. He said, "Yes." I asked him,

"If we had only four boys in the room, could we send six boys on an errand? He said, "No." Then I asked,

"If you had four pieces of candy, could you give me six pieces?" He said, "No."

"Then if you have four, can you take six away?" He understood and went back and changed his work.

When children were faced with the need to work out a difficult problem with large numbers, the teachers often helped the children think it out by means of numbers they already knew.

Teacher D. We were going to see a movie in the auditorium. We talked about how we should sit and act in the auditorium. We were talking about how we should file into our seats. One child asked how many rows we would take up, since there were so many of us. I asked how many were in our room. They said thirty-eight. I said,

"There are twelve seats in a row." This was too much for them so I said,

"How many tens in thirty?" W. said, "Three."

"How much do we have to add to them to make twelve?" They said, "Two." So I said,

"Ten in each row, and two added onto each row." They said three rows of two in each row would be six. Six children would go on the three rows. There would be two children left over for another row.

Late in the semester (December and January), the teachers found that responsibilities which they had themselves assumed in the early part of the semester could now be delegated to the children, if a scheme could be worked out by which the experience was simplified for the children.

Teacher B. Assigned one child to collect money for savings stamps. His job is to put down purchaser's name and the amount given him. Each child tells the number of stamps he wants to buy. The collector counts by 5's to find out how much he has. If he has a dime, he counts it as two 5's. A quarter as five 5's, etc.

Teachers were often aware of the fact that the children were capable of carrying out certain phases of a number experience, but that other phases needed to be done for them by the teacher. In the following experience, the teacher added the total amount after the children had worked up the smaller amounts.

Teacher B. We counted our money for the Red Cross box again. We had seven dimes. S. counted them by 10's. We had two quarters. We

had six nickels. Six nickels made thirty cents. (I made pictures on the board to help them figure out the amounts.) Also talked about how many dimes would be the same amount as six nickels. Then we counted our pennies by 2's, in piles of ten. We had three piles of ten and three left over. I added the total amount for them.

It is of interest that the teacher was also capitalizing on this experience as an opportunity to help children begin to develop some understanding of the tens idea upon which our number system is based. The following experience gives another way of putting meaning into the idea of ten.

Teacher B. We were voting for names for our fish. I tabulated votes and showed the children that tallying was an easy way to keep score.  means "five" because four up and down marks and one mark going across makes five.

"Two   would be how many?" "Ten," said K., "because five and five are ten." We had twelve votes for one name.   // "Twelve is ten and how many more?" They could see the ten and two.

Extensions and variety in routine experiences.--- The teachers handled school routines, such as passing materials, collecting money, and taking attendance, in a variety of ways. The teachers were aware that almost any of the problems arising from such routines could be solved by counting ones. They were also conscious of the fact that there came a time when counting ones offered no further growth for certain children, or for the group as a whole. They were, therefore, constantly on the alert for variety in such experience. Some of the previous illustrations also give evidence of this fact. Others follow:

Teacher B. Attendance: I placed pictures on the board to show how the children sat at the tables, thus:     

Then I called on a child at each table to look and see if he could find his table among the pictures. The children found their tables.

"How many boys at your table, M.?" "Three," he answered promptly, "and three girls, but one is absent."

Draw a circle around the person who is absent at your table.
 "How many are left?"

"Five," said M., "because if you take one from six, there are five left."

L. went up to find his table. "There are seven - four boys and three girls, but one boy is absent, so six are left." "How many 2's in six, L.?" "I don't know."

"Look at the picture and find out." "Oh, yes, three, because you say 2, 4, 6."

By checking the two tables with three pupils at each table, we found there were two 3's also.

Teacher C. I used this method of giving out paper. I dropped a small pack on each table. Then one child at each table passed it. Naturally, none was exactly right, so the children came up and said, "I have four too many," or "I need three more," etc. Then we reviewed the problem, asking how many they had given out, or how many they would need in all, how many they had at first, etc.

Teacher C. We are making spelling graphs and keeping track of our progress on one-inch squared paper. There is addition or counting in finding how many they have right. Then they subtract from ten (the number of words and the number of squares) to find the number of blocks to color. They can also count by 2's or ones.

Naturally, it was still possible for children to obtain answers by counting ones. As other methods were continually pointed out, however, more children tried to use them.

Relationships among the processes.--- More than one process was often involved in the solution of a number problem. The teachers were alert to the possibilities for showing relationships between the processes, and for using such experiences wisely for helping individual children gain a better understanding.

Teacher B. We have begun a post office unit. The children are mailing letters to another class. We bought three-cent stamps and sold them in the room. We chose one child to be the postal clerk each day. J. had ten cents. B., who was postal clerk, said he did not have enough change. He had only six cents.

"How much more will you need?" "Another penny," he answered.

I told him to wait on someone else first. Then he would have enough change. Later, we needed more change again. Someone had five pennies to exchange for a nickel, so we continued our business. B. counted the amount of money he collected. He had a dime and eleven pennies. We brought out that this was two tens and one more. Then we found out how many children had to buy stamps to give us 21¢. We did this by making stick pictures on the board, marking them into groups of three, and counting the 3's.

Teacher B. We changed seats. We counted the spaces at each table. We decided there would be no more than seven at each table. Asked who wanted to sit at Table 1. Seven people only. If eight stepped to the table, we took one away. If only five went, we found how many more were needed. After everyone was seated, we found out the number of boys and girls, and the total number of children at each table. Then we found how many 2's at each table. How many 3's, etc. How many left over.

Illustrations which have been cited show that children as young as second-grade pupils can deal with all processes in solving daily problem situations presented to them. Even more complicated phases of number, such as division of fractions, column addition, remainders, and the like, are often attempted and solved successfully with the help of the teacher to simplify the number aspects of the situation.

Summary

The overall objectives for the classroom program during the instructional period of this study were (1) to help children acquire meanings toward the end that their thinking would continually show improvement in the direction of maturity, and (2) to help children solve their daily problems involving number.

Eleven principles and practices were proposed as guides to the teacher in developing the planned number program. These dealt with time allotment, grouping of children for number instruction, progress from the concrete to the abstract, sequence for presenting numbers, awareness of thought processes of pupils, number relationships, attitudes toward number, and the consideration of independent number work.

Illustrations revealed much variation in practice. Many difficulties in understanding and interpretation were experienced early in the semester, though, in general, the teachers were successful in meeting the requirements as set forth in the principles and practices.

Some of the principles and practices for the planned number program did not apply to the incidental program. These dealt with a specified time allotment, grouping of children according to number understanding, and a preplanned number sequence. Others applied equally well to both programs. Teachers were aware of children's thought processes, of the need for showing relationships between the processes, and of the need for variety in experience, in the incidental as well as in the planned number program.

In addition, incidental number experience served the distinct purposes of helping children solve their immediate number needs and of pointing out applications for number.

CHAPTER VI

ANALYSIS OF PUPIL GROWTH

The purpose of this chapter is two-fold. In the first place, data from group tests will be reported and analyzed in order to indicate changes in rate and accuracy from the initial test given at the beginning of the study to the final test which was administered at the end of the instructional period. In the second place, data from interviews will be presented in order to show changes in methods of thinking which occurred from the initial to the interim interview, and from the interim to the final interview.

The present analysis is based upon data for 131 children for whom complete records are available. These children took both initial and final group tests, and were interviewed at the beginning, middle, and end of the instructional period. The distribution of these children in the various classes was as follows: Class A, 20; B, 23; C, 27; D, 34; E, 27. Data for all other children in the five classes were incomplete and are not presented.

Growth as Revealed by Results of Group Tests

Group tests were given to each of the five pupil groups for the purpose of determining rate and accuracy of performance at the beginning and at the end of the instructional period. In Part I (semi-concrete) dot pictures were provided, to be used by the child in obtaining answers to examples. Part II (abstract) was a duplicate of Part I with pictures omitted.¹ The two parts of the test made it possible to determine differences in accuracy and rate when working with semi-concrete and with abstract materials.

Each child had an opportunity to respond to nine examples in each process, or to a total of thirty-six examples in the four processes.

Data are presented in this section for the total group of 131 pupils. Since the trend for all groups was essentially the same, data for separate pupil groups are in Appendix H (Tables 36-40) rather than in the body of this report.

Accuracy on Part I (semi-concrete phase) of the group test. --

Data relating to the accuracy of responses on the semi-concrete phase of the group test are presented in Table 6. The data reveal improvement in performance from the initial to the final test in two respects:

- (1) On the initial test the number of omitted items was negligible.
- (2) The final test showed a marked reduction in the number and percentage of errors.

On the initial test few addition items were omitted. There was

1. Details on the group test were provided in Chapter III. A sample of the group test will be found in Appendix A.

TABLE 6

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART I: SEMI-CONCRETE) FOR 131 PUPILS ON THIRTY-SIX EXAMPLES, SHOWING NUMBER OF ITEMS OMITTED, NUMBER ATTEMPTED, NUMBER OF ERRORS, AND PERCENTAGE OF ERROR FOR EACH PROCESS

	Items Omitted		Items Attempted		Errors		% Error	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Addition	17	0	1162	1179	90	27	7.7	2.3
Subtraction	82	1	1097	1178	297	32	26.2	2.7
Division	398	1	786	1178	423	147	55.2	12.5
Multiplication	696	7	483	1172	174	126	36.0	10.7

an increase in the number of omitted items in the process of subtraction and a decided increase for the processes of division and multiplication. The differences between the processes may be explained by the following facts: (1) Previous instruction for these pupils had stressed addition more than any other process. (2) While subtraction had received some attention in earlier training, the children had experienced only casual contacts with the processes of division and multiplication. (3) The multiplication items appeared on the last page of the test booklet, and were not completed by all pupils within the period of time allotted for the test. On the final group test only nine items were omitted. Of these, seven were in multiplication. These appear to be chance omissions, except for one child who used the maximum amount of time and completed only four of the nine multiplication items.

There was a marked reduction in the number of errors and percentage of error in each process from the initial to the final test. A greater number of errors in division as compared to multiplication may be attributed to the following facts: (1) Since fewer multiplication items were attempted, there was less chance of error in this process. (2) A feature of the semi-concrete phase of the group test made it possible for children to obtain correct answers by chance. Multiplication items were presented in the following: "Two 3's are 000 ____"
000 ____

The child who merely counted the number group obtained the correct answer in multiplication, whereas in all other processes some manipulation of the group or groups was necessary to arrive at correct answers.

A comparison of errors for the four processes indicates that more errors were made in the processes of division and multiplication

than in addition and subtraction. A partial explanation may be found in the instructional procedure. The test included items requiring multiplication and division of "ones", though the teachers found these items confusing to children and therefore either omitted them altogether, or postponed them until late in the instructional period. While other combinations occurring in the group test were taught during the instructional period, dividing and multiplying "ones" were not stressed.

Accuracy on Part II (abstract phase) of the group test. ---

In order to discover the pupils' accuracy in dealing with identical test items without the aid of illustrative objects, the number combinations were also presented in abstract form, e.g., "2 and 2 are ____." Data relative to performance on the abstract phase of the test are given in Table 7. The same general observations made in the preceding section for Part I of the group test may also be made for Part II. First, there were few omitted items on the final test as compared with the large number on the initial test. Second, the number of errors and percentage of error dropped markedly on the final test. As with Part I, omissions, errors, and percentage of error increased from process to process. On the final test there were few omissions except for two children who failed to complete the test in the time allotted.

TABLE 7

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART II: ABSTRACT) FOR 131 PUPILS ON THIRTY-SIX EXAMPLES, SHOWING NUMBER OF ITEMS OMITTED, NUMBER ATTEMPTED, NUMBER OF ERRORS, AND PERCENTAGE OF ERRORS FOR EACH PROCESS

Process	Items Omitted		Items Attempted		Errors		% Error	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Addition	5	0	1174	1179	205	50	17.5	4.2
Subtraction	140	3	1039	1176	446	98	42.9	8.3
Division	413	2	766	1177	440	180	57.4	15.3
Multiplication	643	16	536	1163	306	230	57.1	19.8

Comparison of percentage of error in semi-concrete and abstract phases of the test. -- In the previous sections it has been indicated that improvement in accuracy occurred in both the semi-concrete and abstract phases of the group test. In order to determine if accuracy was affected by the abstract quality of the materials presented to the child, a comparison of percentages of error in the responses to the two parts of the test was made. The data presented in Table 8 indicate a smaller percentage of error in the semi-concrete phase of the test (Part I) than in the abstract phase (Part II) for every process. In general the children were decidedly more accurate on the initial test when they had access to pictured groups. At the end of the instructional period, when the processes were better known, the children were still more accurate in every process when they had access to pictured materials.

Significance of differences between scores on initial and final group tests. -- A t-test for significance of difference was applied to the group test data to determine if a significant difference existed between (1) the number of errors on Part I (semi-concrete) of the initial group test and on Part I of the final group test, (2) the number of errors on Part II (abstract) of the initial group test, and Part II of the final group test, (3) the number of errors on Part I (semi-concrete) and on Part II (abstract) of the final group test. These data are presented in Tables 9 and 10. The formula used for testing significance of difference was the following: $t = \frac{\sum D}{\sqrt{\frac{N \sum D^2 - (\sum D)^2}{N - 1}}}$

Reference to Table 9 shows that a "t" of 8.61 was obtained when the t formula was applied to determine the significance of

TABLE 8

A COMPARISON OF THE PERCENTAGE OF ERROR ON PART I (SEMI- CONCRETE) AND
PART II (ABSTRACT) OF INITIAL AND FINAL GROUP TESTS

Process	Part I		Part II	
	Initial	Final	Initial	Final
Addition	7.7	2.3	17.5	4.2
Subtraction	26.2	2.7	42.9	8.3
Division	55.2	12.5	57.4	15.3
Multiplication	36.0	10.7	57.1	19.8

TABLE 9

TEST FOR SIGNIFICANCE OF DIFFERENCE IN NUMBER OF ERRORS, BETWEEN INITIAL
AND FINAL GROUP TESTS, FOR ALL PUPIL GROUPS COMBINED

Section of Test	N	ΣD	D^2	$\sqrt{\frac{N \Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t *
PART I	131	623	8159	72.36	8.61
PART II	131	840	17682	61.13	13.74

* A t of 2.62 is significant at the one percent level for
125 cases.

TABLE 10

TEST FOR SIGNIFICANCE OF DIFFERENCE IN NUMBER OF ERRORS, BETWEEN PART I (SEMI-CONCRETE) AND PART II (ABSTRACT) OF THE FINAL GROUP TEST, FOR ALL PUPIL GROUPS COMBINED

	N	ΣD	D^2	$\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t*
	131	233	2403	44.76	8.60

* A t of 2.62 is significant at the one percent level for 125 cases.

difference between the number of errors made on the initial and final group tests (Part I: semi-concrete). The obtained ratio of 8.61 indicates a highly significant difference, since it exceeds the t of 2.62 required for significance at the 1 percent level.

An even greater difference of 13.74 is apparent between the number of errors made on the initial and final tests for Part II (abstract). It may be stated with assurance, therefore, that the differences obtained in the reduction of errors from the initial to final group tests were not chance differences, but were produced by the instructional procedures.

In previous presentations of group test data, a consistent increase was noted in the number of errors on the abstract phase of the group test as compared with the semi-concrete phase. The same t -formula was used to test the significance of the difference between the number of errors made on the semi-concrete and abstract phases of the final group test. The t -score of 8.60 (Table 10) indicates that a highly significant difference also exists between errors on semi-concrete and abstract material.

Comparison of rates of work for Part I and Part II of the group test. -- From Table 11 it may be seen that the average number of minutes per child on Part I (semi-concrete) was reduced by 3.4 from the initial to the final test.² On Part II (abstract) the reduction in time was slightly greater.

2. Data on time according to pupil groups are presented in Appendix I, Tables 40-41.

TABLE 11

AVERAGE NUMBER OF MINUTES PER CHILD AND AVERAGE NUMBER OF MINUTES PER COMBINATION REQUIRED FOR THE GROUP TEST (PART I: SEMI-CONCRETE AND PART II: ABSTRACT)

	Part I (semi-concrete)		Part II (Abstract)	
	Initial	Final	Initial	Final
Ave. no. of min. per child	11.5	8.1	11.2	7.2
Ave. no. of sec. per comb.	25.8	13.5	25.0	12.0
Total no. of attempted items	3528	4707	3515	4694

According to this comparison the time does not appear greatly reduced from the initial to the final test. The time limit of twelve minutes penalized many children on the initial test. Few children were able to complete either phase of the test in the time allotted. In both phases of the test, therefore, the average number of minutes per child should be considered in relation to the total number of items attempted. It is evident that the reduction in time is accompanied by an increase in the number of items attempted. A comparison of the average number of seconds per combination indicates that the time was reduced approximately by half from the initial to the final test.

Evidence from analysis of pupils' written responses and from recorder's notes. -- Careful analysis of the responses on the group test provided evidence on the kinds of errors made by children, which supplements information already obtained on accuracy. On the initial test some children merely copied a number (abstract) or a number group (semi-concrete) for the answer. A tally made from an examination of test papers disclosed the following facts concerning the extent to which children used this method on the initial test: (1) Only three out of the total of 131 children used this method for addition. (2) In the process of subtraction the method was applied by twenty-three children on Part I (semi-concrete) and by twenty-two children on Part II (abstract). In division it was applied by twenty-four children on Part I and by twenty-two on Part II. The number of children using the method was smaller for the process of multiplication, probably because few children completed the last page which was made up of multiplication items. Analysis of the final test responses showed that the method of copying a number or a

number group had disappeared except in the case of two children.

A few children with no understanding of any process except addition tended to add in all processes on the initial test. On Part I (semi-concrete), four children applied this method in subtraction, five in division, and three in multiplication. On Part II (abstract), it was applied by thirteen children in subtraction, seventeen in division, and eighteen in multiplication. On the final test, Part I, addition for all processes was used by only two children, but seven children continued to use it on Part II. The method was more widely used with abstract symbols than with pictured materials on both the initial and final tests.

Analysis of the group test also gave evidence on the number of children who reversed numbers in writing or made their numbers incorrectly. On the initial test (Part I) thirty-nine children reversed one or more numbers or made their number incorrectly. On Part II seventeen children fell into this category. On the final group test eleven children reversed or made numbers incorrectly in Part I, and eight in Part II.

Recorder's notes made while the initial and final tests were administered gave evidence on counting, copying from one another, and requesting help from recorders. Information on these areas is offered with the recognition that it is limited only to what recorders were able to see and record during the twenty-four minutes of work on the group test. It is therefore fragmentary rather than inclusive. It is intended to show a few of the many observable factors which indicate immaturity, lack of assurance and security, and the extent to which these evidences decrease or shift when greater efficiency is achieved.

Recorders observed counting in many forms as children were taking the abstract phase of the test. Some tapped with pencils, counting the number of taps they made; some found imaginary spots on the number symbols themselves; many counted fingers; some whispered numbers to themselves. Almost half of the children were observed to be counting on one or more processes. Specifically, counting was noted by recorders for sixty children on Part I of the initial test and for fifty-five children on Part II. In the absence of pictured materials, the children were versatile in devising their own counting materials. On the final test the numbers of children observed counting were approximately the same (fifty-six on Part I, and fifty-eight on Part II). Again, almost half of the children were doing some counting. Two factors should be remembered, however, in the interpretation of these figures: (1) The less mature method of copying off a number, or a number group, had practically disappeared on the final test. Counting was probably used by most of these children in lieu of these less mature methods. (2) Analysis of the recorder's comments for the final test frequently stated that the child was counting on a single combination, whereas in the initial test the comment was general, or had reference to an entire process. "Used fingers on 8 less 2," "counted for 9 less 3," "counted one number in 4 and 3", and the like were frequent comments concerning the final test.

There was a decrease in the number of children observed copying from each other and requesting help on the initial test (Part I, thirty-one; Part II, twenty-one), as compared with the final test (Part I, twenty-three; Part II, fifteen). This decrease probably gives some

evidence of growth in independence as efficiency increased. Such comments as "works independently," "whizzing right along," "pays no attention to anyone," "answers immediately," "works quickly," and "sure of self," were frequently found on the records for the final test. Comments on the records for the initial test more often indicated that children were puzzled, confused, and generally bewildered.

Requests for help on the final test were often requests for assistance in the recognition of number words which appeared on the final page of the test booklet in multiplication items. On the initial test children who tended to have difficulty with reading worked so slowly that they failed to reach the last page of the booklet and therefore made few requests for help on the number words which they found baffling on the final test.

Growth as Revealed by Interviews

The primary purpose of this section of the chapter is to report the improvement made by pupils as revealed by data from initial, interim, and final interviews. First, the method of tabulating and arranging data for purposes of study and analysis is described. Next, growth is presented in terms of changes in maturity level as determined by responses to abstract examples on successive interviews.

Finally, improvement is noted in the success with which concrete materials were used to demonstrate or to prove abstract answers on successive interviews.

Tabulation of interview data. -- Data from the three interviews (initial, interim, and final) were summarized on a single five by eight inch card for each child. A sample card is reproduced in Figure 2. The

SUMMARY OF INTERVIEW DATA																			
Child's Name				M.S.			Group C												
Addition	Initial			Interim			Final			Subtraction	Initial			Interim			Final		
	Cu 6			RI 3 Rd (1) 1 Pc 1 RI (1),Pc 1			RI 5 Rd 1				Cu 6			RI (1) Cu 1 R Li 1 RI 1 Cu 1 Rd (2) Cu 1			RI 4 R Li 2		
Concrete Materials	Att	S	U	Att	S	U	Att	S	U	Concrete Materials	Att	S	U	Att	S	U	Att	S	U
	2	2	0	3	3	0	3	3	0		2	2	0	3	3	0	2	2	0
Division	Initial			Interim			Final			Multipli- cation	Initial			Interim			Final		
	Cu 2 RI 1			RI 3			RI 3				NR 1 RI 2			Rd 1 RI 2			RI 3		
Concrete Materials	Att	S	U	Att	S	U	Att	S	U	Concrete Materials	Att	S	U	Att	S	U	Att	S	U
	1	1	0	1	1	0					1	1	0	1	1	0			

Fig. 2. A sample of the form used for recording the methods used by children in arriving at answers to interview items.

Code: Cu (Counting - method unknown); RI (immediate recall); Rd (delayed recall); Pc (Partial counting); R Li (recall by logical inference); NR (no response); Att. (attempted); S. (Successful); U. (unsuccessful).

figure can best be understood if the data for one child are reviewed. M. A. counted all of the addition examples on the initial interview, but her method of counting was not indicated by the interviewer (Cu). On the interim interview she knew three of the six examples immediately (RI - immediate recall); she remembered one after a time (Rd - delayed recall), but missed this one (misses are indicated by the numbers within parentheses), using partial counting (Pc - partial counting); she used immediate recall on one (RI), missed it with the recall method, and resorted to partial counting (Pc), with which she was successful. On the final interview, she knew five out of the six addition examples immediately (RI), and remembered the sixth one after a time (Rd). There were no mistakes on the final interview.

On the initial interview M. A. was given two trials at using concrete materials to demonstrate addition examples. She was successful at both attempts. On the interim interview and in the final she was given three trials at concrete materials, all of which were successful. Data for other processes are read in a similar manner.

The next step in arriving at a tabulation of methods for each group of children was a summarization of methods for each group. This was accomplished by tallying the number of times each method was used in initial, interim, and final interviews. Data from the summarization of methods was used to compile tables showing (1) growth made by each pupil group in all processes combined from initial to interim interview, and from interview to final interview, and (2) growth made in each process by all groups combined. Improvement is shown in the number and percentage of responses found at each of four maturity levels on successive interviews.

Later analyses enable a comparison between pupil groups to be made and present results from tests of significance of difference as applied to Level 4 responses on successive interviews.

A compilation was also made of successes and failures with concrete materials on successive interviews. These data were used to show progress in the successful use of concrete materials during the instructional period.

Growth made by each pupil group. -- As growth toward maturity occurs, the responses may be expected to become less frequent at the lower levels of maturity (Levels 1 and 2), and to be increasingly concentrated at the higher levels (Levels 3 and 4). The most obvious evidence of growth for each pupil group is seen in a comparison of the number and percentage of responses in Levels 1 and 4 on initial, and final interviews. In general Levels 2 and 3 also indicate growth in the direction of maturity, though growth is not as consistent as in Levels 1 and 4. Responses in Level 2 increase in number and percentage as responses in Level 1 drop out of use. Likewise responses in Level 3 increase as the two less mature levels decrease. While the total number of responses in Levels 2 and 3 does not change markedly, growth takes place in the shifts which occur within these levels.

Growth for pupil group A is reported in Table 12.³ Significant observations from a study of Table 12 may be summarized as follows:

3. Six addition items, six subtraction, three division, and three multiplication give a total of 360 possible single responses for the twenty pupils of Group A. Dual responses increased the total to 377 for the initial interview, to 373 for the interim, and to 380 for the final interview. Irregular totals for other tables are explained in a similar manner.

(1) Level 1, which includes the largest percentage of responses in the initial interview, practically disappears in the final interview. (2) A considerable increase at Level 4 responses is realized from the initial to the final interview. (3) Of the total number of responses, 72.7 per cent are found in the two most mature levels (3 and 4) in the final interview. Data for pupil group B (Table 13), pupil group C (Table 14), pupil group D (Table 15), and pupil group E (Table 16) reveal essentially the same findings. In each case the number and percentage of responses in the two lower levels decrease from the initial to the final interview, while the two higher levels show a corresponding increase. ⁴

⁴. Data for each pupil group according to separate processes are found in Appendix J, Tables 42-46.

TABLE 12

PER CENT OF RESPONSES FOR ALL PROCESSES COMBINED IN EACH MATURITY LEVEL
IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR TWENTY PUPILS OF GROUP A

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	141	37	59	16	1	.3
Level 2	86	23	134	36	102	27
Level 3	76	20	73	20	129	34
Level 4	74	20	107	29	148	39
Total	377	100	373	101	380	100.3

TABLE 13

PER CENT OF RESPONSES FOR ALL PROCESSES COMBINED IN EACH MATURITY LEVEL
IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR TWENTY-THREE PUPILS OF
GROUP B

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	101	22	45	10	17	4
Level 2	148	33	79	18	64	15
Level 3	81	18	119	27	109	25
Level 4	115	26	197	45	245	56
Total	445	99	440	100	435	100

TABLE 14

PER CENT OF RESPONSES FOR ALL PROCESSES COMBINED IN EACH MATURITY LEVEL
IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR THIRTY-FOUR PUPILS OF
GROUP C

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	160	31	85	15	15	4
Level 2	193	37	159	29	35	7
Level 3	64	12	120	22	134	25
Level 4	106	20	190	34	345	65
Total	523	100	554	100	529	101

TABLE 15

PER CENT OF RESPONSES FOR ALL PROCESSES COMBINED IN EACH MATURITY LEVEL
IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR TWENTY-SEVEN PUPILS OF
GROUP D

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	161	25	64	10	12	2
Level 2	147	23	128	20	81	12
Level 3	171	27	190	30	251	38
Level 4	162	25	254	40	315	48
Total	641	100	636	100	659	100

TABLE 16

PER CENT OF RESPONSES FOR ALL PROCESSES COMBINED IN EACH MATURITY LEVEL
IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR TWENTY-SEVEN PUPILS OF
GROUP E

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	90	19	39	8	16	3
Level 2	163	31	159	31	112	21
Level 3	130	25	112	22	184	35
Level 4	138	26	199	39	213	41
Total	521	101	509	100	525	100

Analysis of growth according to process. -- In the present analysis data from the various classes are combined according to each of the four arithmetical processes. Results are again indicated by means of the number and percentage of responses in each of four maturity levels. Data for each of the processes are found in tables as follows: addition, Table 17; subtraction, Table 18; division, Table 19; multiplication, Table 20. Two general observations may be made concerning all of the four processes. These are: (1) The lowest level (Level 1) almost completely disappears in each. (2) The two most mature levels (3 and 4) show a decided increase in number and percentage of responses from initial to final interviews.

Other observations than the two cited above apply only to certain processes. The small percentage of addition responses in Level 1 (Table 17) in the initial interview indicates that most of the children had some understanding of addition at the beginning of the instructional program. In contrast, the process of subtraction was relatively unfamiliar to many of the children at the time of the initial interviews. This fact is apparent from the large percentage of responses in the two lower levels when initial interviews were given, (Table 18). Progress in subtraction was slow in the early stages of the instructional period, due to an inadequate understanding of the process and to certain preconceived notions and false generalizations. As a result only slight gains were evident at the interim interview.

TABLE 17

PER CENT OF ADDITION RESPONSES AT EACH MATURITY LEVEL IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR ONE HUNDRED THIRTY-ONE PUPILS

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	78	10	40	5	10	1
Level 2	258	32	134	16	99	12
Level 3	294	36	349	42	376	45
Level 4	192	23	302	37	347	42
Total	822	101	825	100	832	100

TABLE 18

PER CENT OF SUBTRACTION RESPONSES AT EACH MATURITY LEVEL IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR ONE HUNDRED THIRTY-ONE PUPILS

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	281	33	126	15	32	4
Level 2	289	34	364	42	202	23
Level 3	157	18	185	22	295	34
Level 4	126	15	183	21	341	39
Total	853	100	858	100	870	100

TABLE 19

PER CENT OF DIVISION RESPONSES AT EACH MATURITY LEVEL IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR ONE HUNDRED THIRTY-ONE PUPILS

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	164	38	65	16	12	3
Level 2	100	23	89	21	57	14
Level 3	45	11	53	13	83	20
Level 4	118	28	209	50	268	64
Total	427	100	416	100	420	101

TABLE 20

PER CENT OF MULTIPLICATION RESPONSES AT EACH MATURITY LEVEL IN INITIAL, INTERIM, AND FINAL INTERVIEWS FOR ONE HUNDRED THIRTY-ONE PUPILS

	Initial		Interim		Final	
	Resp.	%	Resp.	%	Resp.	%
Level 1	130	32	61	13	7	2
Level 2	90	22	72	17	36	9
Level 3	26	6	27	6	53	13
Level 4	159	39	253	62	310	78
Total	405	99	413	98	406	102

The processes of division and multiplication were also unfamiliar to the children at the beginning of the instructional period (Tables 19 and 20). Level 3 responses were used sparingly by these children for the processes of division and multiplication. Once the meaning of these processes was understood, the children tended to infer the division and multiplication facts from addition answers already known to them. They, therefore, tended to jump from immature to mature methods without the same need for in-between methods as seemed necessary for other processes.⁵

Growth indicated in each maturity level in successive interviews. -- A study of each of the four maturity levels was made for the purpose of comparing pupil groups as they progressed through the instructional period. Evidence is presented in graphic form showing a downward trend for the two lower maturity levels (Figures 3 and 4), and an upward trend for the two higher maturity levels (Figures 5 and 6).

Level 1 responses (Figure 3), which are extremely immature, ranged from below twenty per cent to above thirty-five per cent at the beginning of the instructional period. The general downward trend indicates that these immature responses almost completely disappeared by the close of the instructional period. Progress for Groups B, D, and E was more marked during the period between the initial and interim interviews, while Groups A and C tended to make gradual growth throughout.

Progress through Level 2, which represents somewhat more mature responses than Level 1, lacks consistency in the five groups, (Figure 4).

5. Data according to process for each pupil group are found in Appendix K, Tables 47-50.

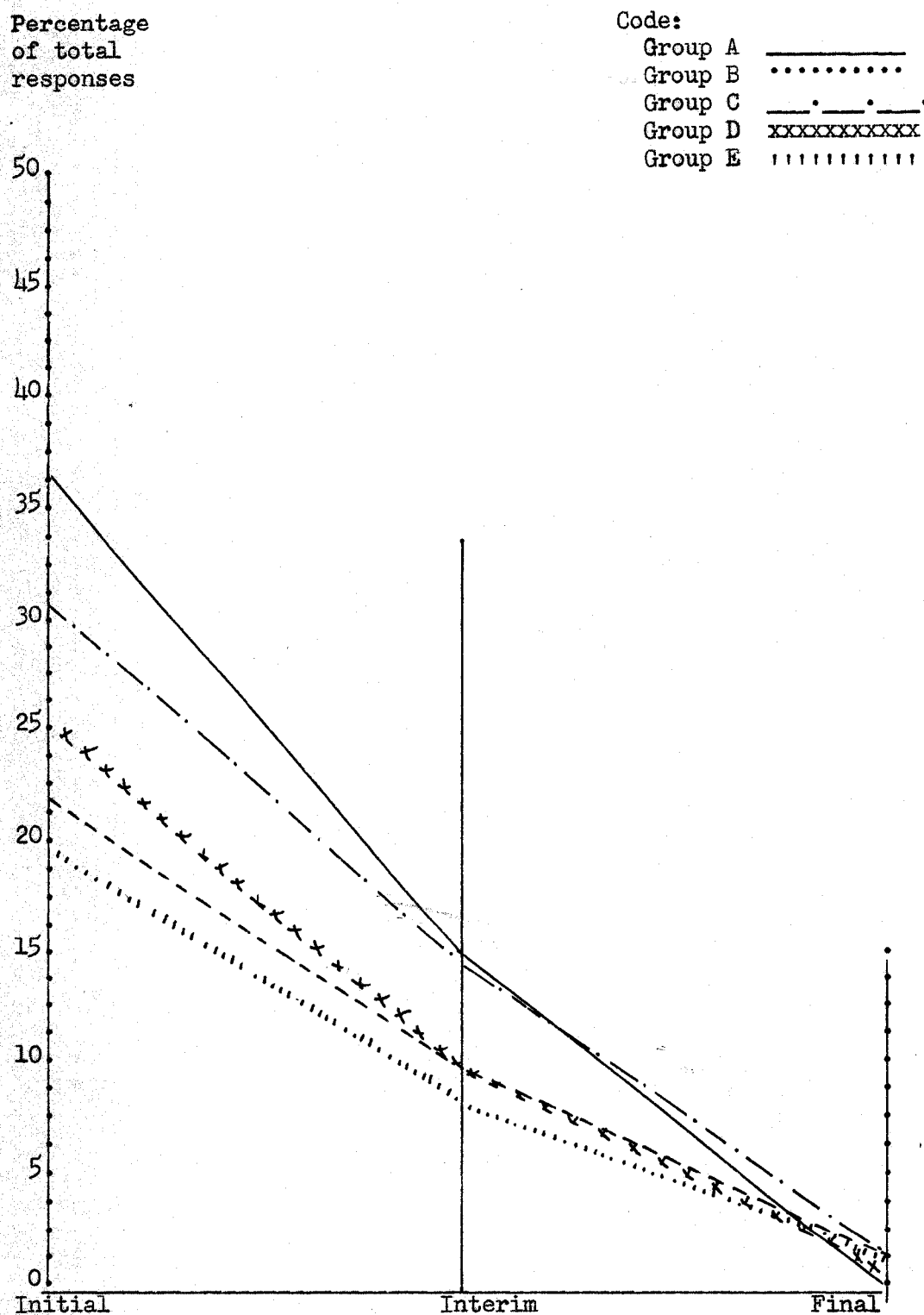


Fig. 3. Percentage of total responses at level 1 for each pupil group on initial, interim, and final interviews.

Percentage
of total
responses

Code:

Group A —————
Group B - - - - -
Group C — . — . — .
Group D xxxxxxxxxxxxxx
Group E !!!!!!!!!!!!!

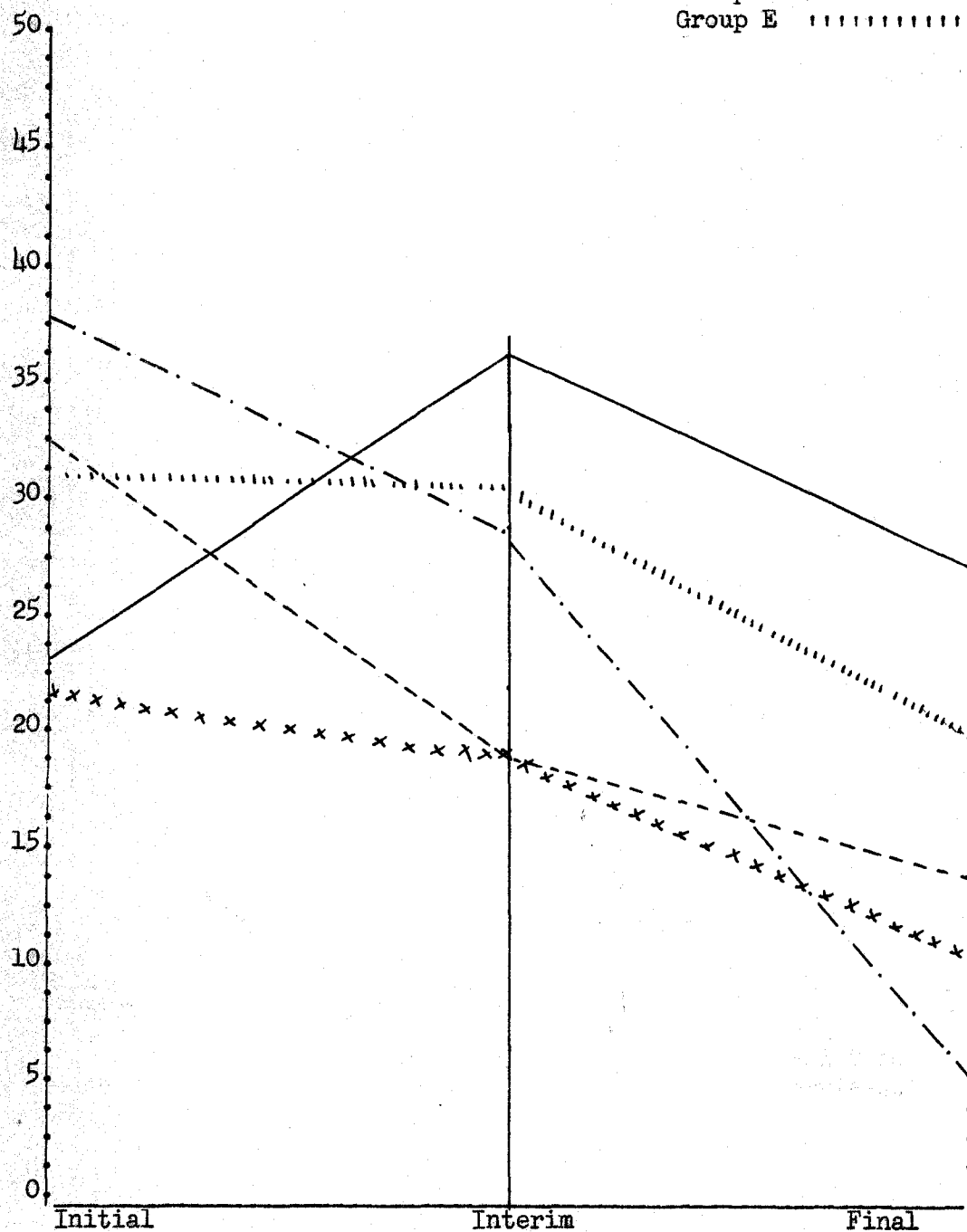


Fig. 4. Percentage of total responses at Level 2 for each pupil group on initial, interim, and final interviews.

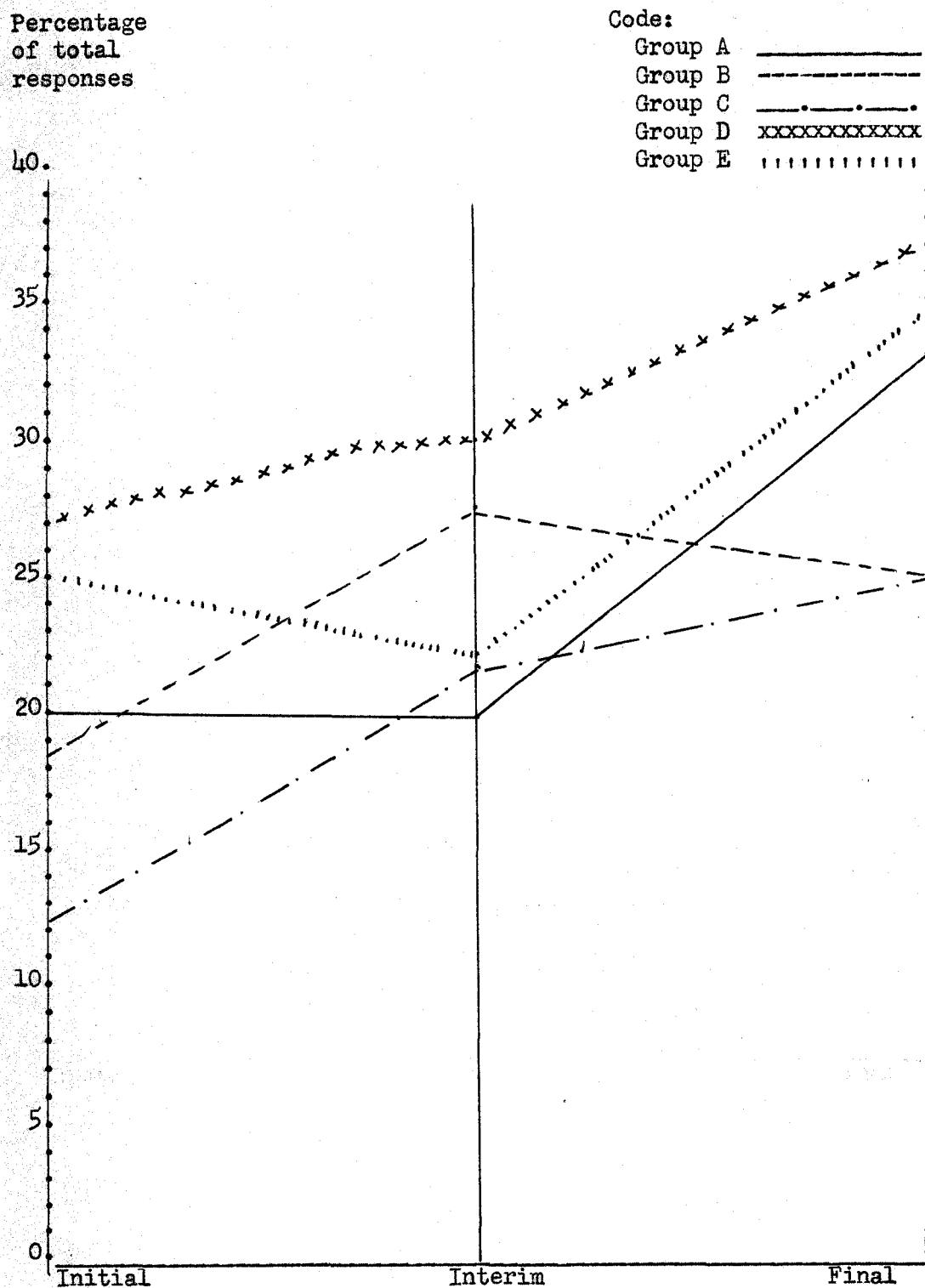


Fig. 5. Percentage of total responses at Level 3 for each pupil group in initial, interim, and final interviews.

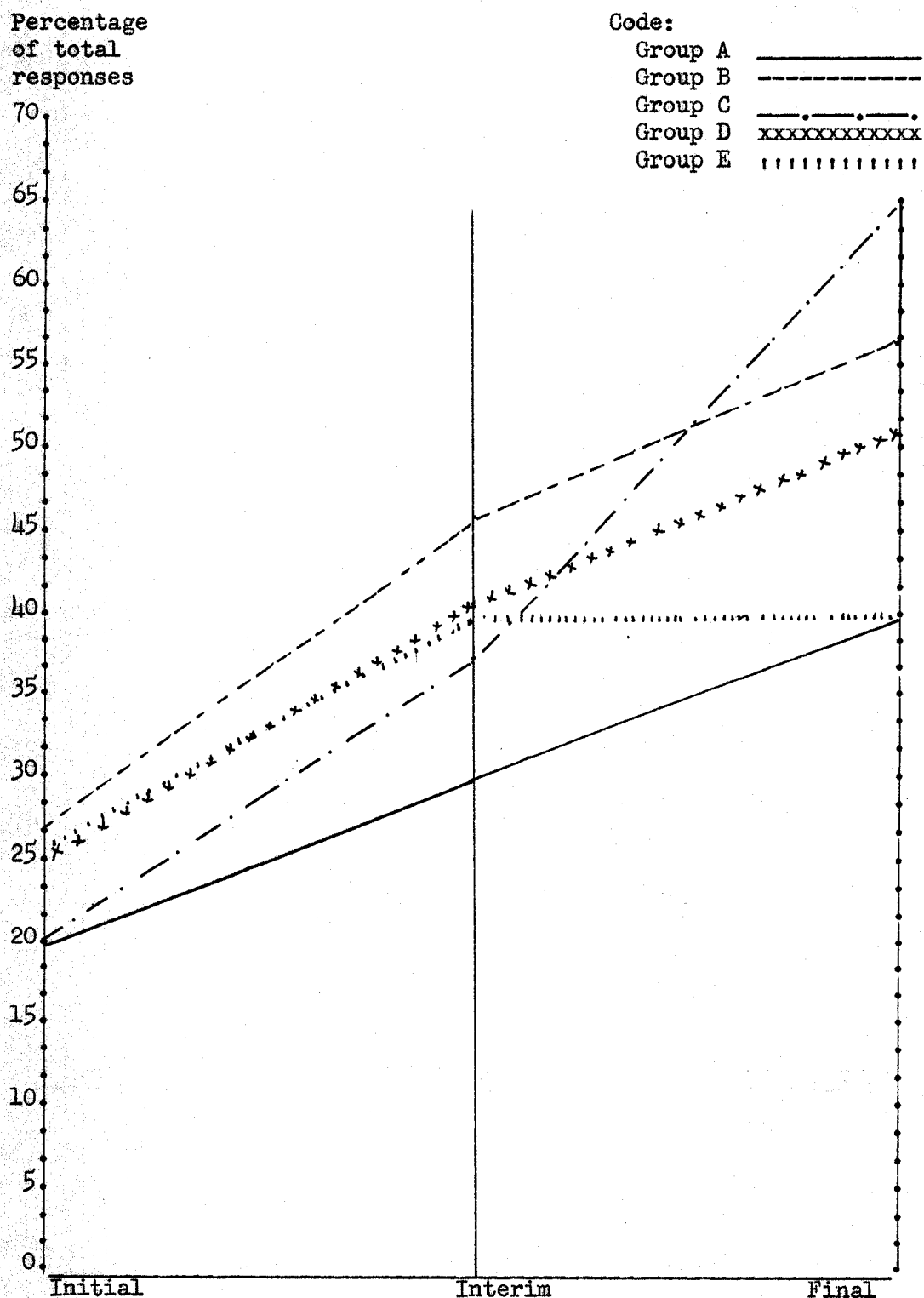


Fig. 6. Percentage of total responses at Level 4 for each pupil group in initial, interim, and final interviews.

Groups D and E appear to make little progress through the first half of the instructional period with slight gains during the second half. Group B, on the other hand, made rather rapid progress through the first half and leveled off during the second half. Group C showed some progress during the first half with a marked reduction in the second half. In Group A Level 2 responses were more numerous at the end than at the beginning of the instructional period.

Levels 3 and 4 represent relatively mature methods of obtaining answers. Reference to Figure 5 shows that the general trend is upward and therefore in the direction of more mature responses. Two groups, A and E, show no growth in the first half of the instructional period, but make rapid gains in the second half. Growth is more marked for Groups B and C during the first half and less so in the second half, while Group D appears to make gradual growth throughout the instructional period.

Level 4 responses (Figure 6) indicate a much wider use of mature methods by all groups at the end of the instructional period than at the beginning. The most marked progress was realized by Group C.

These figures, considered together, show improvement in all classes in the maturity of responses to the interview items. In general, the less mature responses were supplanted by responses at higher levels of maturity. Progress through Levels 2 and 3 is correctly interpreted only as the previous levels are considered. A leveling off of responses gives the impression that no progress has been made, though different responses are represented due to the dropping out of responses at less mature levels. A plateau may therefore mean that progress is being

made which is not apparent from a consideration of one level in isolation from others.

Significance of differences between number of responses found in Level 4 in initial and final interviews. -- The t-test for significance of difference was applied to Level 4 responses to determine if a significant difference existed between the number of responses found at Level 4 in the initial and final interviews. It was of interest to know if a difference of sufficient size took place to justify the inference that the difference was due to the instructional procedures used. The test was used for Level 4 responses, since this level denoted the highest degree of maturity. If a significant difference was found at this level, it might be assumed that significant changes had taken place at other levels as well.

The t-ratios required for each pupil group for significance at the one per cent level are listed in the footnote to Table 21.⁶ A comparison of ratios in the footnote with those listed in the column labeled "t" shows that Groups B, C, and E surpass the t-ratios indicating significance at the one per cent level. In other words, not more than once in over a hundred trials could such a difference be obtained by chance. Group D fails to meet the score of 2.78 required for significance at the one per cent level, but the obtained difference of 2.52 is significant at the two per cent level. The difference for Group A of 1.45 misses the score of 1.73 needed for significance at the ten per cent level.

6. Henry E. Garrett, Statistics in Psychology and Education, p. 190. New York: Longmans, Green and Company, 1947.

TABLE 21

TEST FOR SIGNIFICANCE OF DIFFERENCES IN SCORES BETWEEN INITIAL AND FINAL INTERVIEWS, FOR LEVEL FOUR RESPONSES ON ADDITION, FOR GROUPS A-E

Group	N	ΣD	$\sqrt{\frac{N \Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t*
A	20	15	10.3491	1.45
B	23	36	9.8949	3.64
C	27	50	14.8867	3.35
D	34	33	13.1164	2.52
E	27	19	5.8821	3.23

* The t-ratios required for significance at the one per cent level are as follows: Group A (19 degrees of freedom), $t = 2.86$; Group B (22 d.f.), $t = 2.82$; Group C (26 d.f.), $t = 2.78$; Group D (33 d.f.), $t = 2.73$; Group E (26 d.f.) $t = 2.78$.

TABLE 22

TEST FOR SIGNIFICANCE OF DIFFERENCES IN SCORES BETWEEN INITIAL AND FINAL INTERVIEWS, FOR LEVEL FOUR RESPONSES ON SUBTRACTION, FOR GROUPS A-E

Group	N	ΣD	$\sqrt{\frac{N \Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t *
A	20	18	7.8013	2.31
B	23	44	6.5477	6.72
C	27	81	11.2557	7.20
D	34	51	10.2764	4.96
E	27	11	9.2571	1.19

* The t-ratios required for significance at the one per cent level are as follows: Group A, t = 2.86; Group B, t = 2.82; Group C, t = 2.78; Group D, t = 2.73; Group E, t = 2.78.

TABLE 23

TEST FOR SIGNIFICANCE OF DIFFERENCES IN SCORES BETWEEN INITIAL AND FINAL INTERVIEWS, FOR LEVEL FOUR RESPONSES ON DIVISION, FOR GROUPS A-E

Group	N	ΣD	$\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N-1}}$	t *
A	20	25	5.4406	4.60
B	23	28	4.3159	6.49
C	27	47	5.6823	8.27
D	34	34	6.4007	5.31
E	27	15	4.6326	3.24

* The t-ratios required for significance at the one per cent level are as follows: Group A, t = 2.86; Group B, t = 2.82; Group C, t = 2.78; Group D, t = 2.73; Group E, t = 2.78.

TABLE 24

TEST FOR SIGNIFICANCE OF DIFFERENCES IN SCORES, BETWEEN INITIAL AND FINAL INTERVIEWS, FOR LEVEL FOUR RESPONSES ON MULTIPLICATION, FOR GROUPS A-E

Group	N	ΣD	$\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t *
A	20	18	6.3079	2.85
B	23	25	4.0170	6.22
C	27	51	5.6432	9.04
D	34	33	8.4297	3.92
E	27	26	6.5298	3.99

* The t-ratios required for significance at the one per cent level are as follows: Group A, t = 2.86; Group B, t = 2.82; Group C, t = 2.78; Group D, t = 2.73; Group E, t = 2.78.

For the process of subtraction Groups B, C, and D. again go beyond the differences required for significance at the one per cent level (Table 22). The obtained differences for these groups are therefore highly significant. Group A, with a t of 2.31, exceeds the 2.09 needed for significance at the five per cent level. Group E, with a t of 1.19, fails to meet the t of 1.71 necessary for significance at the ten per cent level. Highly significant differences were obtained for all pupil groups for the processes of division and multiplication (Tables 23 and 24).

Success with concrete materials in the four processes. --

The plan for using concrete materials as a part of each of the three interviews was described in some detail in Chapter III. The purposes for using concrete materials were: (1) to determine whether a child who was unsuccessful with abstract numbers could be successful when using concrete materials, and (2) to indicate to the interviewer whether the child understood the correct answer he had given.

Data are presented in Table 25 to show the success of the children (all groups combined) on each of the processes in the use of concrete materials. ⁷ On the initial interview the children were more successful with concrete materials on the processes of addition and subtraction than with division and multiplication. They were successful in slightly over half of their attempts in division and multiplication on their first trial with concrete materials, while they achieved more than six successes out of ten trials in addition and subtraction.

7. Data for each pupil group on the use of concrete materials are found in Appendix L, Tables 51-54.

TABLE 25

NUMBER AND PERCENTAGE OF SUCCESSFUL RESPONSES IN THE USE OF CONCRETE MATERIALS FOR EACH PROCESS IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Process	Initial		Interim		Final	
	Total	% of successful responses	Total	% of successful responses	Total	% of successful responses
Addition	171	63.8	217	89.9	313	92.7
Subtraction	213	63.4	275	75.3	321	90.1
Division	121	50.4	134	81.4	176	88.1
Multiplication	85	55.3	106	80.2	134	90.3
Total	590	59.7	732	81.4	944	90.6

Improvement is noted on the interim interview for all processes. The lowest gains were made in subtraction, with approximately three successes out of four trials. The highest gains were made in addition, where the children were successful in nine out of ten attempts. There is further growth in success for all processes on the final interview, and a general leveling to approximately nine successes out of ten trials for each process.

Success with concrete materials when the abstract answer was correct and when the abstract answer was incorrect. -- Data assembled in Table 26 give the relation between success with concrete materials and previous success or failure with abstract answers. On the initial interview the children were asked to demonstrate sixty-eight addition examples for which the previous abstract answer had been correct and 155 for which the abstract answer had been incorrect. A comparison of successful demonstrations for correct and for incorrect abstract answers shows much greater success when the abstract answer was correct than when it was incorrect. Even on the initial interview, almost half of the children were able to correct errors they had made with abstract materials when they were given the opportunity to use concrete materials to demonstrate combinations. As the children progressed through the instructional period they became more able to correct previously made errors with the use of concrete materials.

TABLE 26

PERCENTAGE OF SUCCESSFUL DEMONSTRATIONS WITH CONCRETE MATERIALS ON
SUCCESSIVE INTERVIEWS FOR EACH PROCESS FOR CORRECT AND INCORRECT ABSTRACT
ANSWERS

Process	Interview	Number of Trials		Percentage of Successes	
		Abstract Correct	Abstract Incorrect	Abstract Correct	Abstract Incorrect
Addition	Initial	68	155	77.9	48.4
	Interim	158	142	91.8	77.5
	Final	260	120	97.7	80.0
Sub- traction	Initial	35	226	88.6	46.0
	Interim	139	215	87.0	58.6
	Final	224	163	93.7	73.0
Division	Initial	21	129	71.5	42.7
	Interim	82	96	92.7	56.2
	Final	140	78	97.8	61.5
Multi- plication	Initial	29	76	79.3	44.7
	Interim	54	79	94.4	64.6
	Final	124	46	92.7	71.7

Summary

The purpose of the chapter has been to show pupil growth from an analysis of group test and interview data. Analysis of group test data revealed the following facts:

1. Almost no items were omitted on the final group test as compared with a large number of omissions for all processes except addition in the initial test.
2. The number of errors and the percentage of error showed a marked decrease on the final test as compared with the initial test. The application of a t-test for significance of difference resulted in highly significant differences between errors made on initial and final tests.
3. The children were decidedly more accurate with semi-concrete materials than with purely abstract materials. Again, highly significant differences were found on the application of the t-test.
4. Analysis of responses on the group test revealed reductions from initial to final tests in the number of responses in which the child (a) merely copied a number group, or a number, (b) added for all processes, and (c) reversed, or otherwise made numbers incorrectly.
5. Evidence from recorders' notes revealed that: (a) almost half of the children were observed to count to some extent on both the initial and final tests, though recorders' comments on the final test more often referred to counting a single combination than to counting during the entire process; (b) there was evidence of greater independence on the part of the children on the final test.

Results from analysis of interview responses in initial, interim and final interviews indicated general growth for all pupil groups toward

more mature methods of arriving at answers. Analysis of growth by processes also showed growth towards maturity. The greatest growth was realized in the process of addition. A comparison of the changes made by each pupil group through each of the four maturity levels showed: (1) almost complete elimination of Level 1 responses, (2) some fluctuations through Levels 2, 3, and 4, but in general growth toward maturity throughout each of these levels.

A test for significance of difference between Level 4 responses in the initial and final interviews showed: (1) highly significant differences for all groups on the processes of division and multiplication; (2) highly significant differences for Groups B, C, and E on the process of addition, and for Groups B, C, and D on subtraction. The remaining groups met requirements for significance at two per cent, five per cent, or ten per cent levels.

An analysis of the use of concrete materials in initial, interim, and final interviews showed: (1) steady improvement in the ability of the children to demonstrate combinations successfully on each successive interview, (2) greater success with concrete materials when the previous abstract answer was correct than when it was incorrect, and (3) decided growth in the ability to correct previous errors made on abstract combinations when given the opportunity to use concrete materials.

CHAPTER VII

PATTERNS OF ARITHMETICAL THINKING OF SELECTED GROUPS OF CHILDREN

The purposes of the present chapter are (1) to reveal patterns of arithmetical thinking of three selected groups of children whose responses in initial interviews were extremely immature or relatively so, and (2) to determine the progress made in maturity of thought processes by the children of an atypical group (Group F).

The responses for two groups of children are analyzed to determine patterns of thinking over a period of time for a group of children who were performing at the lowest level at the beginning of the investigation. The first group consists of six children who guessed at the answers to all eighteen items in the initial interview. The second group is made up of fifteen children who used the method of partial omission for all subtraction examples in the initial interview. Twelve children consistently used the method of counting all numbers in initial interviews. The responses of these children are studied to determine patterns of thinking for a method which is slightly more mature than the methods of guessing and partial omission.¹ The reports on these three groups are intended to furnish answers to the following questions:

1. Did children who started out with the same method of attack continue to use similar methods over a period of time? What differences in growth pattern occurred as instruction proceeded?

1. See Appendix M for a complete record of methods used by the children of these three groups in successive interviews (Tables 55-57).

2. What factors tended to affect the learning of these children?

3. Did the children in a given group tend to reach the same level of maturity at the end of instructional period?

The study of an atypical group is made to determine (1) the progress made by these children, (2) factors affecting the progress of such a group, and (3) differences from, or likenesses to, other groups in the study.

Patterns of Thinking of Six Children Who Guessed in the Initial Interviews

Throughout this study guessing has been considered the most immature response a child can make to a number situation. It is perhaps even more immature than no response at all, since the child who knows he does not know is probably more aware of what is involved in a number situation than one who merely guesses.² It is therefore of interest to see how children who guess progress through an instructional period. Do they remain guessers? If they change, what characterizes their thinking as they do so?

Naturally, there were many other children who guessed all of the answers within a given process, or had guesses intermingled with other types of responses. On the whole, these children had a better understanding of numbers and progressed more rapidly than the children whose only method was guessing. This report is, therefore, limited to

2. The personality of the child is undoubtedly a factor in determining whether he guesses or gives no response to a number situation.

the six children whose only method of attack on combinations was guessing.

Responses to abstract examples in the initial interview. --

The children who guessed at answers in the initial interview showed a preference for certain numbers which tended to recur frequently during the interview. On the whole, answers fell within a rather limited range of numbers. The answers of three different children ran as follows for the six addition examples: 8, 10, 12, 12, 11, and 10; 10, 11, 13, 11, 10, and 13; 29, 28, 29, 26, 28, and 29. The frequent recurrence of the numbers 6, 7, 8, 9, and 10 was noted for one child. At no time did she give an answer larger than ten and only once was her answer smaller than five. She was unable to reason out her answers and was therefore unsuccessful in all processes except addition, the process within which her chosen answers happened to be true answers. If the interviewer returned to the same examples after a few minutes had elapsed, answers were different though still obviously guesses.

Five of the children who guessed responded to all examples without hesitation. One child varied from this pattern. The interviewer felt that she paused deliberately in order to make it appear that she was trying hard to arrive at an answer. For the example "6 take away 4," she gave five different guesses, 8, 6, 7, 4, and 8.

Explanations by the children for answers they had given showed no evidence of understanding. Such comments as the following were typical:

"I just say the numbers and see if it's right."

"God tells me these words (numbers), but I can't hear them out loud."

One child preceded each subtraction answer with the statement

"Comes back down to ____:"

"8 take away 6" "Comes back down to 28." (Very pleased with herself.)

"I'm going to tell Mr. _____ (principal) that I know all of these words."

"7 take away 3" "Comes back down to 29. I'm still right again."

She assumed that her answers were correct if the interviewer wrote them down. Her answers for division were more nearly correct, (2, 2, and 1), but were evidently guesses, since she could give no reasonable explanation or proof for them. She expressed her answers for the three multiplication examples in terms of money (15¢, a dime, and 15¢). While the range of numbers varied according to process for this child, she had no idea of the meanings which differentiate one process from another.

Use of concrete materials on the initial interview. -- None of the six children who guessed was able to demonstrate examples successfully with concrete or semi-concrete materials in the initial interview. All attempts showed a complete lack of understanding of the meanings of the processes. In response to requests for semi-concrete representations, the following responses were obtained:

"3 and 4" "Should I just draw circles? I don't know how to do these numbers. Can I make another round circle?
(After making three circles) Can I put a 3 and a 4?

"6 take away 4" Drew a picture of 6 balls and 4 balls. Put a box around two balls to indicate the number she thought was to be taken away. When the interviewer reminded her she was to take away 4, said 6 were left.

Concrete demonstrations were equally unintelligible. The children failed to understand what to do with the materials. They often refused to make any attempt beyond laying out all of the objects they were given and

counting the total number. The comments of one of the interviewers concerning the child she interviewed seems to characterize all of the six children who guessed in initial interviews:

Doesn't have the slightest inkling of what numbers are, or any concept of numbers.

Responses to abstract examples in the interim interview. --

Guessing was still a preferred method for two children in the processes of addition and subtraction, and of three children in the processes of division and multiplication, when interim interviews were administered. One of these children seemed definitely to be groping for a generalization that would eventually be of use to her. Her guesses were relatively more accurate than in the initial interview, since they were larger or smaller as the number situation called for more or fewer numbers to be put together. Another of these children coupled inaccurate counting with guessing, which showed a realization that another method than guessing was possible, though she was unable to use the method successfully.

Some of the children seemed to substitute one immature method for another in the interim interview. While they had guessed in initial interviews, they merely called off a number as the answer, or added for all processes in the interim interview. Sample responses are:

"8 take away 5" (pause) "8." When asked 5 'from' 8, he said
"Let 8 stay right there?"

"Two 4's "6. I don't know how I got it."

The children began to experiment with immature counting methods in the interim interview, though, on the whole, they were inaccurate and uncertain in their use of counting. Errors in counting resulted from failure to recognize the groupings in the example, and therefore counting too many or

too few, as in these illustrations:

"2 and 7" (pause) 10. Counted her fingers. Said 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

"3 and 4" Counted his fingers - 1, 2, 3, 4, 5, and said 5 was his answer.

"7 and 2" Counted his fingers - 1, 2, 3, 4, 5, 6, 7, 8?

"How many 2's in 6?" Counted on fingers to 6, but said 4 was the answer.

"How many 4's in 8?" "1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Gee, I can't hardly. I think there are 2." Counted to 8 on her fingers.

The children had no idea of the groups represented in these illustrations. They did not pause between groups, but merely counted until they thought they had counted far enough. Sometimes the answer was stated with a question as if the child doubted the correctness of his answer.

More mature methods such as partial counting, visual grouping, delayed recall, and immediate recall were used to some extent by three of the children in the interim interview. Illustrations of these methods follow:

"5 and 3" After a pause he replied 8. When asked what he said to himself, he said, "5, - 6, 7, 8."

"3 and 4" (pause) "7. I think about it when I go home at night. Like 4, - 5, 6, 7."

"2 and 6" First answered 10 immediately. When asked to explain how he got the answer, he said, "I said, 1, 2, 3, 4, 5, 6, -7, 8: it's 8."

He evidently tried to recall the answer first, but when he checked by counting, he found his error.

"2 and 4" (pause) "6. I just thinked my fingers. Just looked at 2 and 4." Interviewer noted that he merely looked at his fingers and did no counting.

"9 take away 3" "5. (Answered immediately) "Counted in my house with crayons. Had 9 and took away 3 and had 5 left."

Some of the recall responses of these children were non-meaningful, since the children were merely attempting to remember answers, and were unsuccessful when called upon to demonstrate examples with concrete materials.

Use of concrete materials in the interim interview. -- These six children had made decided progress in their ability to demonstrate addition examples successfully with concrete materials by the middle of the instructional period when interim interviews were given. There was evidence that the children had improved in their understanding of the groups, and in their ability to arrive at correct answers by the use of materials.

"5 and 3" 000 Counted 8.
 00000

"2 and 6" 0 000 "Makes 8. 1, 2, 3, 4, 5, 6, 7, 8."
 0 000

One child was confused on the arrangement of his groups, but he clearly had the idea that there were two separate groups, and that he should do something about them.

As a group, these children were unsuccessful in the use of concrete and semi-concrete materials with the processes of subtraction, division and multiplication. The following attempts illustrate the difficulties and confusion experienced by these children:

"7 take away 2" Laid out 7 blocks. Then added on the 3 remaining blocks she was given and said her answer was 10.

"7 take away 3" 00000000 Answered 7.
 000

"8 take away 5"

00000000
00

"Ain't got but 5."

By his expression, "ain't got but five" this child means that he wants more blocks, so that when he takes 5 away, he will have 8 left.

Responses to abstract examples in the final interview. --

With the exception of one child who was still guessing at answers in the final interview, the children made progress in their ability to use more mature methods of thinking as they progressed through the instructional period. Immature counting methods persisted to some extent. The more mature counting methods (partial counting, and counting in multiples) were used much more extensively and with more assurance than in interim interviews. Illustrations are provided to show the type of thinking involved in the use of these methods;

"5 and 3" (pause) "8. I said it in my mind. Said 5, and put 3 more on - 6, 7, 8."

When asked by the interviewer if she could think of another way to get the answer, she said,

"I know if you leave 2 more out (from 10) you'd have 8."

"2 and 4" "Can I use my hands?" When assured that she could, she looked at one hand, recognizing without counting that she had 2 and 3 there, and said, "2 and 3, - 1 more up makes 6."

"2 and 4" (pause) "6. I said 2 and 4 are 6. I knew it couldn't be 5, so I said 6."

"7 take away 2" (pause) Held up 5 fingers on one hand, and 2 on the other. Put down 2 fingers and said 5 was his answer.

"9 take away 3" (pause) "6." Looked at his fingers. No counting.

One child experienced difficulty in making the transition from

addition to subtraction examples. On the first subtraction example, the interviewer says,

"9 take away 3" Took a long time to get him started.
Used fingers. Counted out 9 several
times. Finally took away 3 and said 6.

The use of concrete materials on the final interview. -- On the whole the six children who guessed were able to correct errors they had previously made with abstract examples by the use of concrete materials. In the following illustrations the answer was wrong on the abstract phase of the interview, but the child was able to correct his error with his materials:

"2 and 4" 00 00 "6."
 00 ,
"6 take away 4" 00Ø "2."
 00Ø

The interviewer made repeated comments to the effect that one child was very quick on abstract answers. At the conclusion of the interview, however, the interviewer added, "I think she knows them from memory." The fact that concrete demonstrations were accurate in every case for this child would tend to indicate that she had some understanding of the combinations and of the processes represented. Some illustrations follow:

"2 and 6" 000 00 "8"
 000
"7 take away 3" 0000000 "4."
"How many 3's in 9?" 000 "Three 3's in 9."
 000
 000

It is possible that the child had memorized procedures with materials as carried on in instructional periods in the classroom. Even so, it is

doubtful if she could have been successful on all of her concrete demonstrations without some understanding. The correct demonstrations may indicate that her previous immediate recall answers were meaningful to her.

Personal data for the six children who guessed. -- Two of the six children were of normal age for the second grade and four were over-age. Three of the four overaged children had repeated first grade and the fourth was repeating second grade at the time of the investigation. Absences incurred by these children during the instructional period averaged fourteen days, or more than twice the average number of days absence for children in the five regular classrooms.³ Scores on the Metropolitan Readiness Test were available for three of the six children in the group. Two of these three children were in the lowest quintile range in both readiness level and in learning rate. One was in the fourth quintile range in readiness level and the third quintile range in learning rate. Comments made by the teacher of these six children were as follows:

Can read very well. That is why she is in the A class (highest of five second grades in this particular school). She does not seem to be good in anything else, however. She does unusual things such as spitting all over her dress and pulling out her hair.

Child is alone at home much of the time. Mother works. Appeared at Juvenile Court on charges of burglary (1/20/49) /. He is mean and nasty to other children. Cannot take correction.

Mother left home. Child lives with grandmother. Father beats child.

3. The average number of days absent for the children of typical groups was 6.9.

Good disposition. One day she knows things. The next day she doesn't.

Always has a headache. Absent a lot. Partly because of headache and partly because her mother lets her stay home. Rather dull and slow. Almost lazy.

Summary and interpretation. -- Detailed reports have been presented to show progress toward maturity of thought processes for six children, all of whom guessed throughout the initial interview. Except for one child whose performance was somewhat erratic, all of the children showed progress in one or more processes. Three children showed growth in all processes. One child made gains in the processes of addition and subtraction, but not in division or in multiplication. Progress was noted in all processes except division for another child.⁴

Figure 7 provides a summary of the responses of the six children who guessed in the initial interview. All responses were at Level 1 when initial interviews were given. The figure shows that 22.3 percent of responses were at the two higher levels midway through the instructional period. When the final interviews were given, more than half of the responses (56.5 %) were found at the two higher levels.

Children's responses as they used concrete materials indicated that they had made growth in the area of the concrete, even though they continued to use immature methods with abstract examples.

4. Diary notes for Teacher F indicated that Group F had no planned experience with the processes of division and multiplication. Four of the six children who guessed were in Group F.

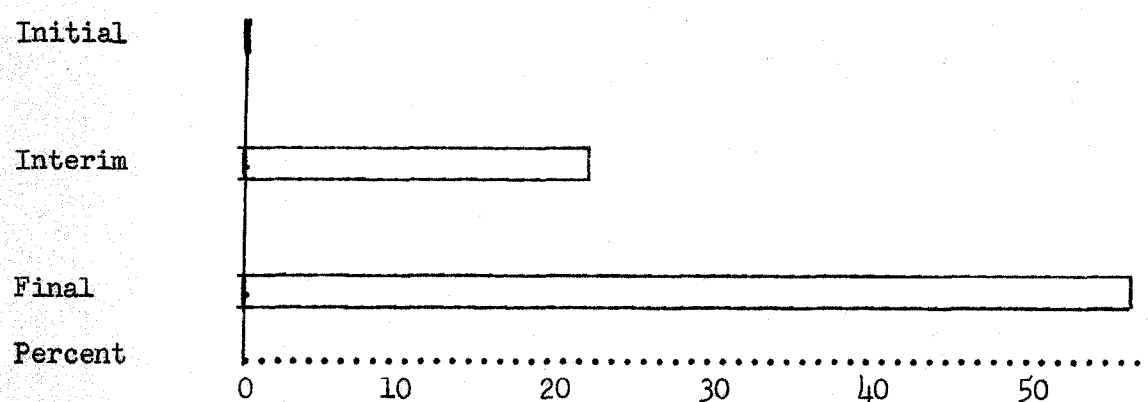


Figure 7. Percentage of responses at the two higher levels for all processes in successive interviews for six children who guessed in the initial interview.

It appears that guessers tend to go through certain other immature exploratory stages before they begin to perform at a level that shows understanding. When counting was first attempted by these children, immature counting methods were usually employed, or the method used was blundering and unsuccessful for a time. For a while these children tended to substitute one immature method for another in subtraction, division, and in multiplication. By so doing they remained on the same level instead of making progress toward a higher level. A longer period of time than four months is apparently necessary to enable these children to gain complete understanding of the number facts sampled by the interviews. It should also be noted that these six children were at a disadvantage from the standpoint of (1) irregularity of attendance in school, (2) unfavorable home conditions, even neglect in some cases, (3) retardation (four of the six children were repeaters), (4) a relatively low readiness level and learning rate, and (5) behavior difficulties.

It is outside the province of this investigation to draw far-reaching conclusions on the relation of such factors as home background, social behavior, and the like, to the success of the child in arithmetic. With so many undesirable factors weighing heavily against them, the gains these children made are even more surprising.

It is promising to realize that children can progress from completely unsuccessful methods to those which bring reasonable success in a rather short period of time, if teachers are making a conscious effort to know how children are thinking and are continually attempting to raise the level of their thought processes.

Patterns of Thinking of Fifteen Children Who Used the Method of Partial
Omission with Subtraction Examples in the Initial Interview

A method which is almost as immature as guessing is partial omission, or calling off one or the other of the numbers represented in ten examples as the answer. The fact that it is systematic is perhaps indicative of a small amount of understanding.

Responses to abstract examples in the initial interview. --

In using the method of partial omission, the child seemed to think merely of getting rid of one number and of leaving the other which was the answer. Little mental activity was involved in this process, in contrast to the prolonged efforts of some of the children to arrive at answers to addition examples immediately preceding. They approached the subtraction, therefore, with such comments as:

"I like take aways. Take aways are easy."

One child voiced her feeling of inadequacy with subtraction on the first example presented to her by saying:

"I haven't been doing this because I've been out of school for the summer."

After this comment she proceeded to call the subtrahend the **answer** for this, and for each successive subtraction example.

Other children planned to get rid of the "take away" number by covering it up with their hands, erasing it, or merely pretending that it was not there. The more usual plan was to do away with the subtrahend and call the minuend the answer, though the reverse procedure was used occasionally.

The interviewer's comment concerning a child's use of this

method was;

"6 take away 4" He is merely taking the last number he hears. When I turned it around and said, "You take 4 from 6," he said 6 was his answer.

The interviewer discovered that one child obtained the correct answer when the language form, "from", was used, though she had resorted to the method of partial omission when the form, "take away", was presented to her. The interviewer made this report:

"8 take away 6" First answer was eight. Read it to her, 5 from 8, and she said three.

This child was equally successful with two other examples, but in general the children realized little success by the change in language form.

Use of concrete materials in the initial interview. -- An analysis of the children's responses as they used concrete materials gave many clues to their ways of thinking. Some of the unsuccessful attempts with concrete materials indicated the types of errors made by the children. Errors were of three general types. In the first type the child thought of the remainder as meaning what was left of all the objects given him after he had taken away one or both of the groups represented in the subtraction example:

"8 take away 5" Counted out 8 forks; took away 5 and then 3 more. "2 left," he said, as he pointed to the two left of the ten forks he had to start with.

"7 take away 2" Arranged forks so that she could count the points: 1,2,3 4,5,6 7,8,9
"Here's 7; here's take away 2", she said referring to the two points left on the third fork.

In the second type of error, the child represented both of the numbers in the subtraction example with concrete materials, and then took one group away, leaving the remaining group as the answer.

"7 take away 3" Drew 7 circles with 3 more underneath.
Crossed off 3 and said, "It leaves 7".

"8 take away 5" Drew 8 pennies and then 5 more. Crossed
off 5 and counted the rest. Said 8.

The third type of error resulted when the child merely represented the minuend and referred to this number as the answer.

Two children were successful in all attempts with concrete materials in the initial interview, though they had used the method of partial omission with abstract examples earlier in the interview. Some responses, though successful, indicated a lack of assurance with concrete materials:

"7 take away 2" "I don't know how to play the game with
spoons." She laid out 7; took away 2, and
said, "Only 5."

The majority of the fifteen children were as unsuccessful with concrete materials as they had previously been with abstract examples.

Responses to abstract examples in the interim interview. --

Eight of the fifteen children continued to use the method of partial omission in the interim interview. Thinking seemed to follow the pattern which was previously described, and is well illustrated by one child who showed how he brushed one number away with his hand and had the other number left.

Seven of the fifteen children, however, had abandoned the method of partial omission almost entirely and had substituted a variety of other methods:

"7 take away 2" "5 - because you have 5 fingers on this hand and 2 fingers on this hand."

"8 take away 6" "2 - because 6 - 7, 8." Counted up from 6 to 8 to get this one.

In each of these illustrations, the method of immediate recall was used first, and then the example was counted as proof.

Two children used a combination of visual grouping and counting for four of the six examples:

"8 take away 6" (pause) "Leaves 2." Puts up 8 fingers immediately without counting; counts 6, which she takes away, and reads the group that is left."

Another child used a combination of recall and recall by logical inference:

"8 take away 5" "3 - 4 and 4 are 8."

She evidently used the addition example, 4 and 4, as a basis for her thinking, and inferred that, if 4 from 8 is 4, then 5 from 8 is 3.

"9 take away 3" "6 - because 3 and 3 and 3 are 9."

Her inference was that if one three was taken away, two three's or six would be left.

Difficulty in making the transition from addition to subtraction examples was noted for some of the children:

"9 take away 3" Added. When I repeated the example to be sure he understood, he used his fingers; counted out 9; took away 3, and had 6 left.

Occasionally a child referred to the method he had used to arrive at an answer as guessing, when his true method was not guessing:

"7 take away 2" "You only got 5. Sometimes I guess. Sometimes I don't."

He was not guessing in this instance, for he had only three incorrect answers out of eighteen examples, and two of these errors were corrected

by the use of another method.

Use of concrete materials in the interim interview. -- The same general types of errors which were illustrated for the initial interview persisted on the interim interview. The children realized greater success on the interim than in the initial interview, however, and there was a noticeable improvement in grouping their concrete materials.

Some of the children who felt inadequate when the abstract example was presented, insisted they could do it with blocks. One child gave up blocks and resorted to fingers with which she evidently felt more secure.

Some of the children appeared to be on the verge of understanding how to represent the example with concrete materials. They tended to forget some little detail or became confused with another process:

"9 take away 3"	Arranged his blocks in groups of three. Apparently misunderstood and thought the example said three 3's are how many. I tried again with blocks later on in the interview, but he did the same thing again.
-----------------	---

"7 take away 3"	Used all ten of the blocks handed to her instead of counting out the seven she needed.
-----------------	--

This child had made one successful demonstration for a subtraction example immediately preceding this one. She evidently became confused and forgot to count out her total number before taking away.

Responses to abstract examples in the final interview. -- In the final interview not a single response was obtained by the method of partial omission. Nine of the fifteen children were using rather mature methods, except for occasional lapses to an immature method on an example

which proved to be especially difficult.

Five of the children used immature methods of counting, though some visual grouping was apparent, as they were able to put up the beginning number immediately without counting. One child knew four answers immediately, but reverted to an immature method of counting for the two answers she did not know:

" take away 6" Uses fingers. Says 2. When asked to show how she did it, she said,
 "Well, here's my fingers, - 1,2,3,4,5,6,7,
 8; then 1,2,3,4,5,6; 2 left."

More mature methods which were used by the majority of the children in the final interview were visual groupings, and various recall methods:

"9 take away 3" Put up 9 fingers immediately; took away 6; 3 left. No counting.

"9 take away 3" "6. If your mother gave you 9¢, and you spent 3¢, you'd have 6¢ left. I started to use my fingers but changed my mind."

This child's last statement is illustrative of the attitudes of most of the children with regard to the use of their fingers. They knew there were better ways of getting answers, but had no hesitancy in using them if other methods were unsuccessful.

"8 take away 5" "3. 4 and 4 is 8; and 8 take away 5 is 3."

"7 take away 3" "Leaves 4; have 6 and take away 3, you have 3; so 7 take away 3 leaves 4."

When asked if she could find the answer to this last example in any other way, she said, "3 and 4 are 7." In other words, she inferred that she could obtain the answer from the corresponding addition example.

As in interim interviews the children often corrected errors they had made with one method by the use of another method.

Use of concrete materials in the final interview. -- No errors were made by any of the five children who had used immature counting methods in the interim interview. These children grouped their objects consistently and worked with confidence:

"8 take away 6"		Picked up discs by 2's to place them. Pulled away 6 and said 2 were left.
"7 take away 3"		Pulled away the 3 through the middle and said, "4 left."

In the final interview the children were more expert in their ability to correct errors they had previously made with abstract examples through the use of concrete materials. The use of materials, in most cases, showed an excellent understanding of grouping, with the repeated comment, "no counting", on the part of interviewers. One comment was:

Seems as if she knows them already and is merely showing them with discs. Usually does not count, but used groups as groups, unless the first number is large, when she does some counting.

Personal data for fifteen children who used the method of partial omission with subtraction examples. -- Five of the fifteen children who used the method of partial omission with subtraction examples had repeated first grade and one child was spending his second year in second grade at the time of the investigation. The average number of days absent from school for this group (7.0) was only slightly higher than the average for the five typical groups. Metropolitan Test ratings, which were available for twelve of the fifteen children, indicated a wide range in readiness level and in learning rate. According to both measures four

children were in the highest quintile range, two were in the lowest and three were in the third. Scores for the three remaining children varied on the two measures. Readiness level for two of these children fell into the fourth quintile range and learning rate in the third. Readiness level for one child was in the second quintile range and learning rate was in the first.

Comments by the teachers of these children were as follows:

Always complaining about aches and pains which are imaginary. Seeks attention. Does good work. Could do even better with more favorable home conditions.

Good worker.

Very slow in all subjects. Scattered absences due to illness. Mother busy taking care of grandchildren. Child receives little attention at home.

Varies in ability. Is going through a slump right now. This is especially true of reading. I feel that she will come out of it.

Youngest in class. Has been discipline problem for three years in school. No father. Mother keeps him inside at home. Fears bad company. Goes wild at school. Did have a dog but got rid of it.

Slow in nearly everything. Likes to see what others are doing. Has difficulty concentrating.

Cannot read even a preprimer. Knows numbers quite well.

Steals; Uses filthy language. Annoys others. Does work well and rapidly. Improving in reading. Is having a hard time with numbers.

Summary and interpretation. -- Only twelve errors were made by the fifteen children in the final interview, in contrast to a total of ninety errors in the initial interview. All of the twelve errors were corrected by another method or by the use of concrete materials. In

general, more errors were corrected in the final interview than in previous ones, due to the fact that interviewers purposely asked children if they could get the answer in more than one way. Moreover, the children were more aware of the mistakes they made, and therefore corrected themselves more frequently than in former interviews.

A summary is provided by Figure 8, which gives the status of the group of fifteen children in the three interviews for responses on abstract examples. Only two per cent of the total number of responses were at the two higher levels when initial interviews were given. Approximately a fourth of the responses were at the two higher levels when interim interviews were given. In the final interview, however, slightly more than two-thirds of the responses were at the two higher levels. It appears, therefore, that this group of fifteen children had made definite growth toward maturity of thought processes during the course of instruction.

This group of children did not vary considerably from the five typical groups in readiness level, learning rate, or in attendance at school. There was a difference, however, in percentage of retardation. Forty per cent of the group of fifteen children were retarded as compared with fourteen percent for the five typical groups. It is evident from the comments of teachers that some of the group of fifteen children had personality factors and home conditions which made adjustment to the school situation difficult. It is not unreasonable to assume that these factors influenced all aspects of the child's school life, including his progress in arithmetic.

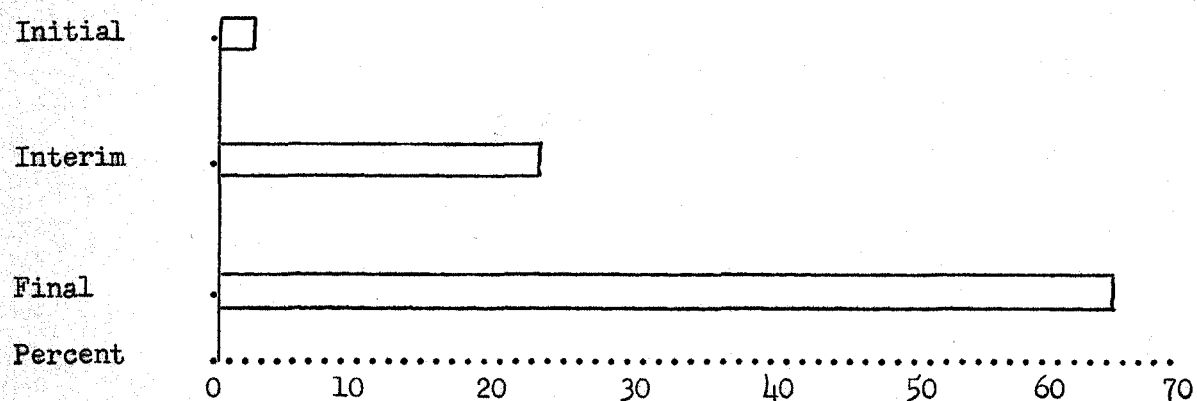


Figure 8. Percentage of responses at the two higher levels in successive interviews for fifteen children who used the method of partial omission for subtraction examples in the initial interviews.

Patterns of Thinking of Twelve Children Who Counted All Numbers in The Initial Interviews

Counting all numbers has been described as the method by which the child counts both numbers in an example by ones, beginning with one. This method was used with all processes, but more liberally with addition. The present analysis deals with addition only.

Responses to abstract examples in the initial interview. --

Though all of the twelve children used the same method, there were variations in the way in which the numbers were counted. Some children consistently started with the larger number, some with the smaller, and some were inconsistent in their method of attack. Some openly counted their fingers, while some counted to themselves, counted imaginary objects, or counted as they pointed to the numbers on the card presented to them. For some children the pattern was rather firmly fixed, so much so that the same procedure was used in practically the same way with each of the addition examples. Typical patterns of counting follow:

"5 and 3"	Counted 1, 2, 3; then 4, 5, 6, 7, and 8 all on one hand.
-----------	---

In other words, this child used the fingers of one hand only.

She first counted 1, 2, 3; then started over with her fingers, but continued counting, 5, 6, 7, 8.

"5 and 3"	Answered 8 after a long pause. When questioned as to how he arrived at his answer, he pointed to five on the interviewer card, saying, "1, 2, 3, 4, 5;" he then pointed to three and said, "6, 7, 8."
-----------	---

In a slight variation of this method, a child pointed with her finger in the air as if she were pointing to imaginary objects as she counted.

"2 and 6" (pause) "8. 1, 2, 3, 4, 5, 6, -
7, 8."

This child consistently counted both numbers, but started with the larger of the two numbers each time.

Many children found counting all numbers a laborious method of getting through six examples. When boredom or fatigue set in, they tended to resort to guessing, or to some other more immature method than was typical of their pattern of work.

Use of concrete materials in the initial interview. -- The children were able to correct some of their previous errors by the use of concrete materials, though a correct answer did not always indicate an understanding of grouping:

"2 and 6" Gave her forks and she got answer 8.
Counted them out in one stack and did
not divide them into groups, but was
successful.

Four of the children who were successful with concrete materials gave no evidence of grouping objects beyond the mere separation of the two groups in the addition example.

"5 and 3" Made a picture of two balls and then 6 more
balls strung out in a row. Stopped to
count after making five balls in the second
group, and then added another one to make 8.

One child tended to use his fingers first, when he was asked to demonstrate with materials. He would then match fingers and materials:

"5 and 4" Uses fingers first; then forks; gets 10
on first trial with fingers; then gets
9 when he rechecks with forks.

Errors in the use of concrete materials occurred when the children became confused on the number of objects needed to represent the groups.

"2 and 6"

Drew 7 balls. At first was unaware that he had only 7. Finally realized it and drew the last one.

Responses to abstract examples in the interim interview. --

Three children were still counting all numbers in the interim interview. Two of these children made no errors in either interview. One child was more inaccurate in the interim than in the initial interview. She counted her fingers each time, but was inaccurate in her counting. Greater accuracy in the initial interview may have been due to the carry-over effect when concrete materials were used immediately following the presentation of the abstract example. It will be remembered that in the interim interview all abstract examples in a given process were completed before concrete materials were used.

Three children were still using counting all numbers as a preferred method, but they were beginning to experiment with other methods, such as counting in multiples, partial counting, delayed recall, visual grouping, recall by logical inference, and immediate recall. The remaining six children in the group of twelve had either abandoned the method of counting all numbers altogether or were using it less frequently than other methods. Some children used the same method with all six addition examples, while others varied considerably in the methods used.

A few children were observed in the initial stages of using a new method. One child had the idea that breaking up numbers into 2's or 3's would help him to arrive at answers. His techniques were clumsy, however, and were applied without any definite end in view.

"3 and 4"

(pause) "7. 4 and 3 are 7." When asked to tell more about how he had thought out the example, he volunteered, "3 and 2 and 2 are 7."

"5 and 4" (pause) "8. Cause 4 and 5 are 8; 2 and 2 and 2 and 1 more are 8."

The interviewer commented that this child tried to break up numbers whether he needed to or not. He was attempting to apply some of the methods he had been using in instructional periods, but was unsure of the method he was attempting to use, and therefore used it in a manner which was not profitable.

Use of concrete materials on the interim interview. -- All twelve of these children demonstrated the ability to use concrete materials successfully. They clearly separated the two groups in the example each time. Some showed excellent ability to group materials:

"7 and 2"	000		"5 and 4"	0	
	0	0		0 0	00
	000	0		0 0	00

A child who failed to count correctly in five out of six of the abstract addition examples, counted her blocks accurately each time, and was able to make successful semi-concrete drawings as well. The interviewer made the following general comment about her work:

Always a pause before giving an answer to abstract examples. Always starts right, but seems to get stuck. Has the right idea about counting her fingers but usually miscounts somewhere along the line. Very good with blocks and pictures.

Responses to abstract examples in the final interview. -- With the exception of one child, growth was evident from the interim to the final interview on responses to abstract example. Although this child had used more mature methods for four of the six examples in the interim interview, she regressed in the final interview and counted all numbers for five examples at this time. It is difficult to say why she should revert to a less mature method. It would appear that she did not

actually need to count all numbers, since she had demonstrated an ability to use more mature methods. It may be that she found it easier to count all numbers than to use the more mature methods with which she was experimenting in the interim interview.

A child who used partial counting for all six examples in the final interview was more accurate than on previous interviews, though she was still slightly confused on how to start, in partial counting, in order to come out with the correct answer:

"2 and 6"	Looks at fingers and says, "6, 7." Tries again. This time she starts with 2 and says, "2, 3, 4, 5, 6, 7. It might be 8." When asked why, she said, "I don't know."
-----------	--

She made her error by counting the last number of the group she was taking as a whole, rather than counting onto this group.

The consistent use of partial counting showed an improvement in maturity for three children over methods used in the interim interview. These children made no errors. They counted the smaller number first each time.

When a child felt dissatisfied with his answer a less mature method was sometimes used as a check on a more mature method:

"7 and 2"	"8. Wait a minute." Used fingers, saying, "7, 8, 9."
-----------	--

Recall methods were used more extensively in the final than in the interim interview. One child, who had used partial counting consistently in the interim interview, used recall by logical inference for five of the six addition examples in the final interview. He returned to his former method of partial counting for one example:

"5 and 4"	He knew 5 and used fingers to get 6, 7, 8, 9. Merely moved his fingers on his arm and did not look at them.
-----------	---

Use of concrete materials in the final interview. -- In general, the use of concrete materials in the final interview showed that the children had grown considerably in their ability to group materials and to arrive at answers in more mature ways. Two children were exceptions to this general rule. While neither made errors in the use of concrete materials, they strung them out into rows instead of grouping them.

After the children had made an arrangement with concrete materials and had made a response, they were asked if they could think of any other way to get the answer. Responses showed an ability to see relationships between 4 and 2 and 3 and 3, between 2 and 6 and 4 and 4, and the like. Children who were as yet unable to see these relationships with abstract examples could explain them with the aid of materials. The children continued to improve in their ability to correct previously made errors with the use of concrete materials.

Personal data for the twelve children who used the method of counting all numbers in initial interviews. -- The average number of days absent for a group of twelve children who counted all numbers (9.0) was considerably higher than the average for the five typical groups (6.9). Four of the twelve children had repeated first grade and two were repeating second grade during the year in which this study was made.

On the whole, this group of children ranked rather high in readiness status and in learning rate. Eight of the ten children for whom records were available were in the two highest quintile ranges in both measures. Of the remaining two children one was in the third and one was in the fourth in both measures.

Comments made by teachers concerning these children were as follows:

Steady worker; good reader.

Timid child; absences due to ear trouble and colds.

Very shy, but mature and independent. No close friends in school. Sick a lot. Despite absences (26 days) reading and spelling are O.K. In fact, none of her school work has suffered.

Nervous; cocky attitude. Has an older sister who takes him to and from school.

Quiet; daydreams; babied at home.

Only child; quite a favorite with the children.

Has improved greatly. Always tries to do her best. Neat and clean.

Quiet pupil. Favorable home conditions. Has a speech impediment. Is advanced in reading.

Summary and interpretation. -- All of the twelve children experienced some growth from the initial to the interim interviews. Interesting bits of evidence were one child's almost complete failure on abstract examples, but excellent use of concrete materials, and the lack of assurance and temporary loss of accuracy noticed when the children were experimenting with methods that were new to them. In some cases lack of more obvious growth seemed to be due to the hold that this particular method of counting all numbers had on some of the children which made it difficult for them to break away from it. Another interesting observation from the interim interviews was the occasional reversion to immature methods when the child neared the end of the six examples or when he desired greater security than the new method offered him. It was as if he tired of his laborious methods and tended to grasp

again the method with which he felt the greatest confidence and assurance.

Figure 9 shows that the percentage of responses at the two higher levels increased from zero percent to 49.8 percent in interim interviews, and to 76.6 percent when final interviews were administered.

Many of the children in this group of twelve suffered loss of time from school. One third of the group were repeaters in contrast to 14 per cent for the five typical groups. The teachers described four of the children as quiet, shy and timid. Two were dependent and over-protected. In spite of these facts the growth of the children was in the desired direction, though they did not reach the highest level of maturity during the instructional period.

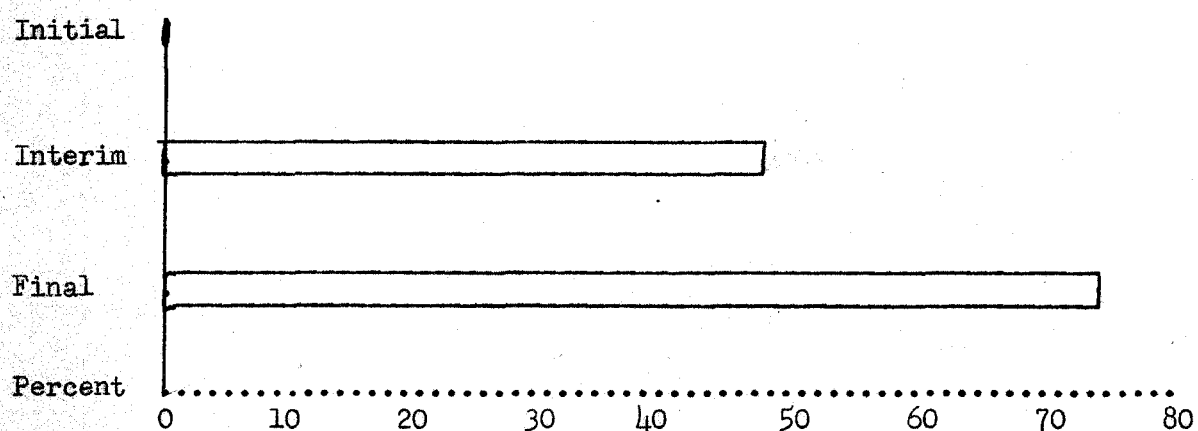


Figure 9 : Percentage of responses at the two higher levels in successive interviews for twelve children who used the method of counting all numbers for addition examples in the initial interviews.

A Study of an Atypical Group (Group F)

In this section of the chapter a study of Group F will be presented, including comparisons with the five typical groups of children on retardation, attendance, readiness status, teaching methods, and findings from group tests and interviews.

Retardation. -- Eighteen of the twenty-six children in Group F had repeated first grade and six were repeating second grade during the year of the investigation. No record of retardation was available for the remaining two children who entered the group from outside the city. Both of these children were past the normal age for second grade, however. Therefore, it is assumed that there was 100 per cent retardation for Group F. The total number of repeaters and percentages of retardation in all other groups were distributed as follows: Group A, 7, 35 percent; Group B, 6, 26 per cent; Group C, 1, 4 per cent; Group D, 3, 9 per cent; and Group E, 1, 4 per cent, or an average of 14 per cent for the total of 131 children.

Attendance. -- Group F suffered a greater loss of time from absences during the experimental period than the children in typical groups. The average number of days of absence per child for the typical groups combined was 6.9, with a distribution for separate groups as follows: Group A, 6.3; Group B, 7.3; Group C, 5.0; Group D, 7.4; Group E, 8.5. The average number of days of absence per pupil for Group F was 9.6.

Readiness status and learning rate. -- Measures on the Metropolitan Readiness Test were available for 114 of the 131 children. A summary of information provided by this test is given in Table 27.

TABLE 27

READINESS AND LEARNING RATE OF TYPICAL AND ATYPICAL PUPILS AS MEASURED
BY THE METROPOLITAN READINESS TEST

Quintile	Readiness Status		Learning Rate	
	Typical	Atypical	Typical	Atypical
1	47	1	49	1
2	32	0	25	0
3	22	4	24	5
4	7	7	12	3
5	6	10	10	13
No Record	17	4	17	4
Total	131	26	131	26

The number of pupils in each quintile range for readiness status and for learning rate is presented for typical and atypical groups. Learning rate is determined by readiness status as related to chronological age. Thus a given score means that a six-year old child learns at a faster rate than a seven-year-old who receives the same score. From Table 27 it is apparent that more than half of the children from typical groups are concentrated in the first and second quintile ranges as against only one child from the atypical group.⁵ This fact holds for both readiness status and for learning rate. A large majority of pupils from the atypical group were in the fourth and fifth quintile ranges, while only a small minority of children from typical groups fell into the two lowest intervals.

Teacher guidance of children's experiences. -- Teacher F described her group of children as follows:

The seventh section of the second grade at W. School is greatly in need of number work. The children lack a background of number experiences. They have limited concepts of numbers. For example, they have no idea that the number "five" denotes five units.

Teacher F was therefore faced with the problem of providing a background of experience for a group of children of low ability. Her task was made more difficult by problems resulting from poor attendance, unfavorable home conditions, behavior difficulties, and undesirable attitudes engendered by retardation.

There were several major differences between the teaching methods of Teacher F and those of the teachers of typical groups. The

5. Placement in Group F was made for this particular child because of excessive absence due to prolonged illness.

special features of the program in Group F may be listed as follows:

1. The processes of division and multiplication were taught only as these processes occurred in the course of incidental number experiences.
2. The processes of addition and subtraction were usually taught separately, with one process or the other receiving major consideration during a given arithmetic period.
3. Due to the short attention span of the children in Group F, planned instruction for small groups lasted only from ten to fifteen minutes at a given time.
4. All abstract work was done with easy access to concrete or semi-concrete materials, which the children were encouraged to use as needed. The majority of children from typical groups tended to work with abstract materials toward the end of the instructional period.
5. Much stress was placed on the number sequence from ten to twenty, since many children could not count accurately to twenty.
6. More planned experiences of each type were necessary to assure understanding for the pupils for Group F than were needed by the children of typical groups. The kinds of experiences did not vary greatly from those which have been described for the children of typical groups. They include, (1) grouping with discs, blocks, sticks, cut-out figures, and the like, (2) counting things by two's and three's, (3) making number groups on the felt-o-gram and discussing them, (4) matching a number group on the blackboard or felt-o-gram with sticks or other concrete materials, (5) starting with the larger of two groups when counting in order to arrive at the total number, (6) reading groups on number

picture cards, (7) drawing pictures from directions (independent work), or arranging pictures from magazines to show the processes of addition and subtraction, (8) finding answers to abstract examples written on the blackboard (2 and 4 are _____, 6 take away 4 are _____, etc.) by using materials to work out the answers, (9) finding a number that comes before or after a given number in the series from one to twenty, and (10) demonstrating addition and subtraction examples with varied types of concrete materials.

The kinds of incidental experiences provided for Group F did not vary markedly from the experiences which have been described for typical groups (Chapter V). Experiences which could be used repeatedly, such as taking attendance, distributing supplies, and counting lunch money, were used daily throughout the instructional period, since Group F did not exhaust the learning possibilities afforded by these experiences during that time. While the teachers of typical groups were constantly searching for variations in the number activity which would provide new learning, Teacher F found it necessary to follow the same procedure for a longer period of time before any variation was introduced, in order to assure understanding for her children.

Teacher F was adept at diagnosing their difficulties and in adapting material to their level of understanding. When she attempted to combine addition and subtraction examples into the same arithmetic period, she found the children confused and unable to make the necessary transition from one process to the other. She said,

This (combining addition and subtraction examples) creates a problem for me. The children get so mixed up when I combine "take away" and "put together" examples in the same group. I guess this is a matter of understanding "take away" thoroughly and being able to read and recognize the working of the problem.

An inability on the part of the children to read the examples intelligently was also seen in this comment:

I went over the work the children were to do and they all seemed to understand. At least they could give the answer and show how they arrived at it. I still don't think they are familiar with expression 'take away'. They don't seem to read it. They look at the two numbers in the problem and, without reading the words, they automatically add the two numbers together.

When Teacher F experienced difficulty with the two processes in combination, she was advised to separate them for emphasis as often as necessary in order to teach clear meanings.

The insight of Teacher F into the needs of her group is again shown in the following illustrations:

I asked the children to find answers to subtraction examples with materials at their tables. They did less copying from each other than they had done on previous addition examples. Perhaps the reason was that the subtraction examples were harder for them and they had to use materials to work them out, whereas many of the children felt they already knew the addition examples and were therefore reluctant to use the sticks.

As a directed number lesson I gave the children a series of "take away" examples. The children themselves acted out some of the examples to illustrate the process of subtraction. The children seemed to understand this, so we did the same with pieces of chalk, with sticks, and with blocks. I then made the mistake of using crayons. When I asked how many were left (4 take away 2) they said, "one green and one orange." They were not thinking in groups at all. I also became aware of the fact that they do not understand what is meant by "4 take away 2" in abstract form, but when I say, "4 boys take away 2 boys," they understand better.

On another occasion when working with subtraction, Teacher F remarked:

I found the children doing this for the example, "4 take away 3". They would put marks on their papers (llll); then instead of taking three marks away from these four, they would add three more marks to it (llll lll); then they would cross off three and write four as the answer. Each child who was making this error worked the examples in a group demonstrating them with sticks under my direction. They seem to understand when working together with me, but when they work alone they become confused.

Teacher F realized the need of her group for extensive experience with each type of materials. She was aware of the limited ability of her children to generalize from one type of materials to another, of the ease with which they were distracted from the number ideas themselves, and of the difficulties this group had in carrying over ideas developed under teacher guidance to independent work. They experienced conflicts with the process of addition, although they had had one more year's experience than most of the children in typical groups. Because of reading disabilities increased difficulty was imposed by the necessity to read and interpret such number terms as, "take away". In spite of these difficulties in learning, Teacher F stated repeatedly throughout her diary that her children were eager to learn and enjoyed their work in numbers immensely.

Results from group tests. -- Results from initial and final group tests for Group F are found in Table 28. Omitted items decreased to a negligible number from the initial to the final tests, while the number of items attempted increased accordingly for both parts of the test. The percentage of error remained high at the end of the instruction period, since at this time the children missed approximately one out of four items on Part I, and approximately one out of three items on Part II. The percentage of error from the initial to the final tests was reduced considerably, however, for both semi-concrete and abstract phases.

Notes taken by recorders during the administration of the group tests provided the following significant facts concerning the differences in performance of the children in initial and final tests:

- (1) The number of children who copied a number as the answer was reduced

TABLE 28

RESULTS FROM GROUP TESTS FOR TWENTY-SIX CHILDREN OF GROUP F, GIVING
 NUMBER OF ITEMS OMITTED, NUMBER ATTEMPTED, NUMBER OF ERRORS, AND
 PERCENTAGE OF ERROR IN INITIAL AND FINAL TESTS

	Items Omitted		Items Attempted		Errors		% Error	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Part I	396	12	540	924	245	241	45	26
Part II	315	25	621	911	366	308	59	34

TABLE 29

TEST FOR SIGNIFICANCE OF DIFFERENCE IN NUMBER OF ERRORS MADE IN INITIAL
AND FINAL GROUP TESTS

	N	D	D ²	$\sqrt{\frac{ND^2 - (\Sigma D)^2}{N - 1}}$	t
Part I (Semi-Concrete)	26	4	2715	36.93	.11
Part II (Abstract)		58	2940	29.66	1.96

approximately by half, though five children out of the total of twenty-six were still using this method on the final test. (2) More children were counting on the final than on the initial test, which may be interpreted as favorable evidence, since counting for this group of children supplants less mature methods. (3) Comments gave evidence of more assurance on the part of the children during the final test.

The t-test for significance of difference between the number of errors made by Group F on the initial and final group tests (Table 29) gave a t of .11. No claims for significance, therefore, can be made for this phase (semi-concrete) of the test. The t of 1.96 obtained for Part II fails to reach the t of 2.06 required for significance at the five per cent level, but surpasses the t of 1.71 required at the ten per cent level.

Results from interviews. -- The number of responses at each level of performance for each of the four processes is found in Table 30. Division and multiplication responses are combined, since each child gave responses to only three division and three multiplication items, but to six items each in addition and in subtraction. Forty-five per cent of the total number of responses are found at Level 1 in the initial interview in contrast to fifteen per cent in the final interview. The percentages for Level 2 responses apparently undergo little change, though shifts in responses are indicated by changes at Level 1. Percentages of responses at Levels 3 and 4 show growth in the direction of maturity in successive interviews.

Level 1 responses, which almost disappeared for typical groups, accounted for fifteen per cent of all responses of Group F on the final

TABLE 30

RESULTS FROM INTERVIEWS FOR TWENTY-SIX CHILDREN OF GROUP F, GIVING
NUMBER OF RESPONSES AT EACH OF FOUR MATURITY LEVELS FOR INITIAL, INTERIM,
AND FINAL INTERVIEWS

Process	Level 1			Level 2			Level 3			Level 4		
	Ini.	Int.	Fin.	Ini.	Int.	Fin.	Ini.	Int.	Fin.	Ini.	Int.	Fin.
Addition	54	32	8	45	49	30	34	46	66	33	45	71
Subtraction	88	64	32	52	63	79	5	26	43	13	10	32
Division and Multiplication	78	56	38	43	44	39	8	8	19	35	62	68
Totals	220	152	78	140	156	148	47	80	128	81	117	171
* per cent	.45	.30	.15	.29	.31	.28	.10	.16	.24	.17	.23	.33

* Total number of responses on successive interviews was as follows: Initial interview, 488; Interim interview, 505; Final interview, 525.

TABLE 31

TEST FOR SIGNIFICANCE OF DIFFERENCE BETWEEN THE NUMBER OF LEVEL FOUR
RESPONSES IN INITIAL AND FINAL INTERVIEWS

N	D	D ²	$\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N - 1}}$	t
26	97	749	20.06	4.85

interview. Similarly, on the final interview the percentage of responses at Level 4 for Group F (thirty-three per cent) was lower than the percentages for typical groups.

Application of the t-test for significance of differences between Level 4 responses in initial and final interviews, however, revealed a t of 4.85 (Table 31), which surpasses the t of 2.79 required for significance at the one per cent level. The methods of thinking being used by Group F at the end of the instructional period were comparatively immature, since forty-three per cent of the total number of responses fell at Levels 1 and 2. Yet this percentage contrasted with seventy-four per cent in the initial interview indicates a considerable amount of growth in thought processes during the instructional period.

Percentages giving the degree of success of the pupils of Group F in the use of concrete materials are assembled in Table 32. The percentages show an increase in ability to demonstrate examples successfully as children progress through the instructional period. The children had more failures than successes for all processes in the initial interview with concrete materials. In the final interview they were able to demonstrate nine out of ten addition examples successfully and approximately three out of four examples for the other processes.

In general the same kinds of errors were made in the use of concrete materials by both typical and atypical children. A number of children from Group F showed ability to think out the answer or to demonstrate the example in more than one way on the final interview. These responses showed developing ability to use more mature methods, such as, counting in groups rather than by ones, and thinking out a solution from

TABLE 32

NUMBER AND PERCENTAGE OF SUCCESSFUL AND UNSUCCESSFUL DEMONSTRATIONS WITH
CONCRETE MATERIALS IN ALL PROCESSES BY GROUP F

	Addition			Subtraction			Division and Multiplication		
	Ini.	Int.	Final	Ini.	Int.	Final	Ini.	Int.	Final
Trials	50	75	64	51	77	66	51	71	77
Suc.	22	61	58	11	40	47	18	38	56
Unsuc.	28	14	6	40	37	19	33	33	21
% Suc.	44	81	90	21	52	71	35	54	73
% Unsuc.	56	19	10	78	48	29	65	46	27

other known combinations.

Results for basin children as compared with suburban children.--

A comparison of basin and suburban children as to maturity level represented by responses in initial and final interviews reveals only slight differences for the two groups. Data relating to the two types of children are found in Table 33. The suburban children appear to have had a slightly better understanding of the combinations both at the beginning and at the end of the study. As compared with the basin children they gave eight percent fewer immature responses and eight percent more responses of a mature character at the time of the initial interview. In the final interviews, however, this difference between mature and immature responses dropped to four percent, thus indicating equal progress for both groups.

It is of interest that the program of work in arithmetic was equally successful with the two groups of children. In general, the same principles and practices were followed by all teachers, though there were some variations in specific procedures.

Table 1 (page 37) shows that children in the basin area were homogeneously grouped, whereas children in suburban areas were grouped heterogeneously. Data provided also indicate that basin groups were subject to greater fluctuations in enrollment than were suburban groups. The conclusion may be drawn, therefore, that progress was continuous for both groups regardless of the type of grouping represented, or of fluctuations in enrollment.

Summary for Group F. -- Group F was composed of twenty-six children who were overage for second grade. Attendance for the group was

TABLE 33

A NUMERICAL AND PERCENTAGE COMPARISON BASED ON THE MATURITY LEVEL OF
 RESPONSES MADE BY BASIN AND SUBURBAN CHILDREN IN INITIAL, AND FINAL
 INTERVIEWS

Levels	Initial				Final			
	Basin		Suburban		Basin		Suburban	
	No.	Percent	No.	Percent	No.	Percent	No.	Percent
1 and 2	833	59	557	51	281	20	174	16
3 and 4	588	41	529	49	1153	80	920	84
Total	1421	100	1086	100	1434	100	1094	100

poor, and the readiness level as measured by the Metropolitan Readiness Test was low. Many of the children of Group F had behavior difficulties in addition to their learning problems. The kinds of experiences provided for Group F were similar to those provided for typical groups, though a few changes in methods of presentation were found necessary to meet the short attention span, lesser ability to see relationships and to generalize, reading difficulties, and conflicts with previously established number ideas. Teacher F found that a wider variety and a greater number of experiences of each kind were necessary to assure understanding for her group than seemed essential for typical groups.

In general there was no significant difference in the number of errors for Group F on initial and final group tests. Changes in thought process from initial to final interviews, however, indicated significant differences in the number of responses found at Level 4 on initial and final interviews.

Summary of the Chapter

Three groups of children having similar immature methods of attack on arithmetic examples and one atypical group were studied in order to determine changes in thought processes as a result of the instructional procedures used.

A detailed study of six children who guessed all examples on the initial interview gave evidence that these children, with one exception, had made considerable growth in the use of more mature methods by the end of the instructional period. The greatest amount of growth took place during the latter half of this period. All children progressed in their ability to demonstrate combinations successfully with concrete materials.

The fifteen children who used the immature method of partial omission with subtraction examples in the initial interview had eliminated this method completely by the final interview. All errors made in attempting to use more mature methods were corrected by the use of another method or were recognized as errors when the children demonstrated the example with concrete materials. Again, the most obvious growth was made in the second half of the instructional period.

The twelve children who counted all numbers in addition examples in the initial interview all made progress in the use of more mature methods.

These studies of groups of children having like methods of attack on combinations at the initial interview indicate that, even though these children started out with the same method, they did not continue to use the same methods as they progressed through the instructional period. Only one child out of the total of thirty-three children in the three groups studied failed to make progress. Some tended to substitute one immature method for another for a time, however, before any real progress was made. Some progressed more rapidly than others. While Level 1 responses were practically eliminated by the end of the instructional period, Level 2 responses were still frequent. Many children were beginning to experiment with Level 3 and Level 4 responses, while some had gained considerable facility with the use of these more mature methods.

Early attempts in the use of new methods were often unsuccessful and marked by an eagerness on the part of the child to prove his answer by using a less mature method with which he had greater assurance, or by demonstrating with concrete materials.

Fatigue or boredom often seemed to cause a child to revert to a less mature method than he was capable of using.

The study of typical and atypical children indicated that the atypical group made as much progress as some of the typical groups, though the level of maturity was lower at the beginning of the instructional period, and the children therefore failed to reach the level of maturity attained by typical groups. Adaptations in teaching methods were made to meet particular needs of the atypical group, though in general the kinds of activities were the same for both types of children.

Results obtained from a comparison of basin and suburban children as to maturity levels represented by responses for the two groups in initial and final interviews reveal approximately the same amount of growth for each group. In general, the same type of program was effective with children in both basin and suburban areas.

It is evident that children who start out with such immature methods as are represented in these studies of small groups require a longer period of time than four months to achieve facility in the use of mature methods. Since the children made progress through this period, the assumption can be made that they would have continued to progress in the use of more mature methods, if appropriate guidance had been given for a longer period of time.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

The purposes of this chapter are (1) to give a summary of the study, (2) to present such conclusions as seem justified on the basis of data secured, (3) to propose certain problems arising from the study which indicate the need for further research, and (4) to draw implications for the teaching of arithmetic.

Summary of the Study

As stated in the first chapter, the purpose of the study was to determine the effect of guiding children through certain selected temporary intermediate procedures in learning the four fundamental processes in arithmetic. The subjects in the study were children representing various types of background and experience from six second grades in the Cincinnati Public School System.

Group tests and interviews were given at the beginning of the investigation for the purpose of determining the number abilities of the children at that time. An instructional program was launched immediately following the testing procedures and was continued for approximately fourteen weeks. The program was designed to encourage children to progress toward more mature methods of thinking than they were found to be using at the beginning of the study.

Six beginning teachers who were graduates of Teachers College, University of Cincinnati, were in charge of the instructional program, which consisted of approximately thirty minutes per day spent in planned number instruction, and of those experiences which occurred incidentally in the course of other school activities. Each teacher kept daily records

of both types of number experiences. Five of the six groups of children represented typical classrooms in Cincinnati, while the sixth group was designated as atypical because of a lower level of ability, excessive amount of retardation, and behavior difficulties.

Interviews were repeated midway through the instructional period. Both group tests and interviews were repeated at the end of this period. Qualified students in the kindergarten-primary department of Teachers College assisted the writer in carrying out the testing program.

Data from group tests gave evidence of greater accuracy of responses on the final than on the initial test. Statistically significant differences were obtained for the children of the five typical groups, but differences for the atypical group were only slightly significant. A comparison of the responses on semi-concrete and abstract phases of the test revealed that children made more errors on the abstract phase.

Analysis of interview data gave responses ranging in maturity from guessing, which was considered the least mature method of arriving at answers, to immediate recall, which was representative of the highest level of maturity. The types of responses found in the initial interviews were classified as representing one of four maturity levels on the basis of (a) the promise the method held for giving the correct answer, (b) the reasoning required on the part of the child to apply the method, and (c) the extent to which the method helped the child to understand the process in question. Level 1 methods included omitted items, items for which no responses could be obtained, and the methods of guessing and of partial omission (using one of the numbers in the example as the

answer). These methods were almost completely inadequate from the standpoint of the criteria set up. Methods placed at Level 2 were immature counting procedures, such as counting all numbers by ones, counting when the exact procedure could not be determined, and adding for all processes. These methods were slightly more accurate, but required only a limited amount of reasoning ability, and had little to offer to the child's understanding of the processes. At Level 3 were classified more mature counting methods such as partial counting (counting one number only in the example) and counting in multiples (by 2's, 3's and 4's), visual grouping which involved no counting estimating, and two recall methods (remembering the answer after a time, and recalling it after posing a problem situation). These methods met the criteria more adequately, since they resulted in more correct answers, required a greater amount of reasoning ability, and gave considerable evidence of understanding of the processes. Level 4 responses, which included two recall methods, recall by logical inference (working out answers from other known facts), and immediate recall, exacted a high level of accuracy, reasoning ability, and understanding on the part of the child.

Analyses of interview data were made in order to determine the amount of growth made on successive interviews in the maturity of thought processes. Responses were analyzed for each pupil group, for each of the four fundamental processes, and for each of the four maturity levels. All three analyses gave evidence of decided growth in maturity of thought processes. A t-test for significance of difference between the number of responses in Level 4 in initial and final interviews revealed that differences were significant for all pupil groups,

and highly significant for four of the five groups. All groups showed evidence of decided improvement in their ability to demonstrate and to prove number facts by the use of concrete materials.

Conclusions

The investigation was planned to furnish data relative to a number of specific questions which were raised at the beginning of the study. Such conclusions as are warranted from the data secured on each of these questions are presented in this section. The questions are considered in the order in which they appear in Chapter I.

1. Was experience provided for all children on each of the selected intermediate procedures? -- Evidence has been provided from teachers' diaries (Chapter V) to show that each teacher grouped her children for number instruction so as to meet their needs as she was able to interpret them. Adaptations in instruction for each group were accomplished by subdivisions of the group for particular purposes, and by giving attention to individuals within the sub-groups. Only those intermediate procedures which offered promise of growth for a given group were specifically taught.

Many of the children were beyond the level of counting all numbers in the examples at the beginning of the investigation. No child was urged to participate in experiences with the use of an intermediate procedure which was less mature than was his customary method of arriving at answers, unless the teacher had reason to believe that he lacked understanding for the method he was attempting to apply.

2. Is it possible for some children to by-pass some intermediate procedures without jeopardizing understanding? -- Obviously the

answer to this question is in the affirmative, since some children had reached abstract understanding of the combinations before group instruction was provided. Diary records showed that the majority of experiences provided during the first half of the instructional period were concrete or semi-concrete. Yet results from interim interviews indicated that there were more responses in Level 4 at interim interviews than when initial interviews were given.

All methods were not used to the same extent with each process. Counting in multiples was more widely used with the processes of division and multiplication than with addition and subtraction. The children tended to adapt those methods most useful for a particular process at a given time, rather than to concentrate on each method for the same length of time. Interview data showed that Level 3 responses were used very sparingly with the processes of division and multiplication. Yet many of the children whose responses were immature at initial interviews had reached mature ways of finding answers to these combinations by the final interviews. They had probably by-passed Level 3 responses in doing so.

3. Do some children tend to move to more advanced procedures before understanding is present? -- Some of the pupils of Group F had memorized the number facts before they had acquired understanding for them. Although their answers were immediate and often correct, pupils were unable to demonstrate number facts with concrete materials or to reason them out in any way. Initial interviews for Group F provide evidence to this effect.

The inaccuracies and misconceptions in the use of more mature methods (small group studies in Chapter VII) gave evidence of attempts

to use methods before a readiness for them was completely established. Teachers reported (diary reports in Chapter V) that some children tried to use a method they had overheard another group using. They also reported that a few children tended to revert to less mature methods for the same reason.

4. How can readiness for a more advanced intermediate procedure be determined? -- In general, teachers found two factors of importance in determining readiness for a more advanced intermediate procedure. These were (a) understanding and facility in the use of those methods immediately preceding the new method in maturity, and (b) an ability to master the new procedure without excessive repetition. For example, a child was considered ready to attempt partial counting when he could recognize small groups with ease, and when he was adept at counting both groups in the example. Teachers tried to keep in mind the goal that no child should fail completely in arriving at an answer to a combination. If he was unsuccessful with a given method, he was either helped to correct his error in the use of the method, or he was encouraged to return temporarily to a less mature method.

5. Do children willingly discontinue the use of immature intermediate procedures? -- The best evidence on this question is provided by the shifts the children made in maturity level on successive interviews. Since the children did not remain at a given level, the inference may be drawn that they would continue to progress until the highest level of maturity was reached, if adequate guidance continued for a sufficient length of time. The teachers of typical groups found it difficult to encourage ambitious children to be satisfied with less mature

methods for a while longer. This presented a problem equally as serious as encouraging elimination of methods no longer useful to them.

On the whole, the children who had progressed to a more mature level reverted to less mature methods when they were unsure of themselves, were bored or tired, or wished to prove the answer by another method.

6. Under what circumstances are less mature methods dropped and replaced by more mature methods? -- It was found that the stimulation of the teacher and of the children themselves, as they worked together in groups, afforded the strongest motivation away from less mature and toward more mature methods of thinking. Evaluation sessions in which teacher and pupils together appraised the merits of a method resulted in the desire on the part of the children to progress to more mature methods.

7. What methods may be used by the teacher to encourage children to discontinue the use of immature methods? -- Much guidance on the part of teachers was necessary in the early stages of using a new method to prevent children from becoming discouraged and to tide them over the faltering period during which they were likely to make mistakes with the new method. When a child proved he was capable of using a more mature method, teachers tried to encourage extensive use of the method, by asking such questions as the following: "Can you think of another way to get the answer? Can you do this one the same way as (some previous answer was obtained)? Can you think of a quicker way to get the answer?"

8. As children are taught intermediate procedures, to what extent is their later thinking characterized by increasingly higher levels

of maturity in dealing with number situations? -- There is evidence from interview data that for the length of time of the investigation (approximately fourteen weeks) the thought processes of the children on the whole were characterized by increasingly higher levels of maturity.

9. What problems do teachers encounter as they attempt to guide children through succeeding maturity levels in dealing with number situations? -- The teachers found that a few children tended to break up numbers in ways that had no relation to the solution of the combinations. For example, when asked to tell how they found the answer for the combination 4 and 5, they would say, "2 and 2 are 4; 2 and 2 and 1 are 5", but would not be able to proceed further without help.

Another problem was presented by those children who apparently had excellent understanding of combinations on the concrete level, but seemed unable to progress beyond that point. Some of the teachers felt that techniques which encouraged the children to estimate first, and then to count, or to check by thinking of other known combinations, were profitable in overcoming this difficulty.

Teachers found it difficult to work with all children at their best level of understanding. Children who had a very elementary understanding of number ideas overheard other groups using more mature methods, and were ambitious to try them also. Conversely, children who were capable of more mature ways of thinking saw other groups engaging in immature counting methods and occasionally reverted to them.

Need for Further Research

There is a need for research similar to the present study which would extend over a longer period of time. In this study there was a

tendency for the less mature responses of children to drop out in favor of more mature processes. Study for a longer period of time would enable more definite conclusions to be made on the question as to whether less mature responses continue to be discarded until the abstract level is reached. The recent trend toward having the same teacher continue with a group of children for two years should make it possible for such a study to be undertaken.

Further research is needed to determine more definitely the relationship between concrete experience and abstract learning. The root of this problem, stated in the words of teachers participating in the present study, runs as follows: "I have given him all kinds of experiences with concrete materials. He understands perfectly. But he is still lost when he attempts to work only with abstract materials." An intensive study could be made of a limited number of children who had reached understanding at the beginning of the investigation on the concrete level, but were unsuccessful at the abstract level. Further information on the means by which the transition is accomplished could thereby be obtained. Some questions involved are: (1) Does the child merely need more concrete materials, or does he need a longer period of time in which to use concrete materials? (2) Are there some partially concrete and partially abstract procedures which may help to bridge the gap? (3) How may estimating followed by checking the correctness of the estimate be used to advantage? (4) What characterizes the thinking of these children after understandings on the concrete level have been acquired, as they progress to the abstract level?

As has been indicated, partial answers to these questions

constitute the findings of the present study. Many of the children in this study, however, had not yet reached an understanding of concrete number when the investigation began. The proposed study could be delimited to include only those children who already had attained an understanding of the concrete level of working with number at the beginning of the investigation, but who were unable as yet to deal satisfactorily with abstract number.

The analyses of the responses of small groups of children with like methods of performance at the beginning of the investigation dealt with thought processes that were exceedingly immature or relatively so (guessing, partial omission, and counting all numbers). In general the children of these groups realized growth in maturity of thought processes. It would be of interest to make a further analysis of children who were using more mature methods at the outset in order to determine if the same trend held for more mature methods. For example, does the child who has reached the level of partial counting, or the level of thinking out answers from other known combinations, continue to progress until these procedures are no longer necessary or does he tend to continue to use these methods? In other words, does the same trend hold for the more mature as for the less mature responses, or does the child tend to become satisfied with relatively mature methods of arriving at answers and to remain at that point?

The children of Group F (the atypical group) appeared bewildered when they attempted to deal with processes interchangeably. It would be profitable to know if the same difficulty would result if such a group were given broad experience at the concrete and semi-concrete levels with the four processes used interchangeably from the beginning of

their systematic study of numbers. Under these circumstances would they experience the same blockings and misconceptions as this group experienced? Such a study could be made in any large city where homogeneous grouping of slow-learning children was practiced.

Implications for Teaching

The effect of heterogeneous and homogeneous grouping on the learning of number. -- The fact that the children in the study represented both homogeneous and heterogeneous groups does not warrant any deductions on the value of one type of pupil classification in preference to the other. Regardless of the type of grouping, each teacher found it necessary to group children within her grade in order to make adaptations to the abilities of her children and to give much individual guidance.

It has been noted that the children progressed at approximately the same rate, regardless of the type of grouping. An inference may be drawn, therefore, that factors other than those relating to intellectual ability are perhaps of greater importance in determining the type of grouping. Furthermore, the practice of segregating children of decidedly low ability into one group, as was done with children of Group F, is neither favored nor condemned in this discussion. That the children of Group F made considerable progress has been shown. It is also a fact that they had serious difficulties which may in part have been caused by unfavorable habits and attitudes engendered by retardation. Certainly such children present difficulties in any classroom. In general, the smaller suburban schools in the study met their problems of the slow-learner by attempting to make adaptations to the learning

level of the individual within their own classrooms. Perhaps the most important factor from the standpoint of instruction is that teachers accept children as they get them, with the conviction that all children can learn if the level of understanding is met, and if progress toward more mature goals is adapted to the child's learning pace.

The organization of number facts for presentation. -- The plan of organization used by the teachers in the study was to center instruction of number facts to those within a given span of two, three, or four numbers. The selection of the number span, and the time spent on it at varying levels of performance, varied with the needs of the pupil groups. As children progressed, smaller numbers dropped out from the instructional span, and larger numbers were added to it. In general activities began on a concrete level with gradual advancement toward abstract experience. The teachers realized reasonable success with this plan of presentation. They found it workable, interesting and challenging for the children.

The effect of presenting the processes to children in interrelation. -- The teachers in the study, with the exception of Teacher F, felt that the children profited from the plan of presenting the four processes in interrelation during the early stages of number learning. They felt there was economy of time and effort as the children worked with concrete materials, and a greater variety of possible activities when the processes were taught together. They also found that the practice helped children to see relationships between the facts they were discovering. Teacher F thought her slow-moving group was confused when they were expected to make transitions from one process to another even

with concrete materials. The teachers of typical children experienced similar difficulties with a few individuals within their groups.

On the basis of the evidence from these teachers of these pupil groups, it seems justifiable to assume that the majority of children can profit from working with the processes in a rather close relationship. Caution needs to be taken, however, with regard to the necessity for making adaptations to children of differing abilities. Processes should probably be separated temporarily when abstract symbols are introduced, if children tend to confuse the processes at this time.

The value of teacher guidance in learning number. -- The practice of delegating a large portion of number experience to unsupervised, independent practice periods is in part responsible for the fact that teachers in general learn so little about the methods of thinking of their pupils. When left to their own devices, pupils tend to practice the same immature procedures over and over.¹ It is essential that the teacher work directly with children, listening to them, talking with them, if she is to be in a position to help them raise their level of thinking.

When unsupervised, independent work was used by the teachers in this study, it was preceded by careful teaching with much pupil participation. Group instruction continued until the teacher was assured that the work could be carried out independently by pupils. After the work had been accomplished, teacher and pupils jointly evaluated the results of the independent work. A careful check on the independent work of the children often gave the teacher new leads as to further profitable

1. W. A. Brownell and C. P. Chazal, "The Effects of Premature Drill in Third-Grade Arithmetic," Journal of Educational Research, 29 (September, 1935), 17-28.

group work within a given area.

The children in the study were urged to represent numbers and number facts by drawing groupings of pictured objects, in preference to a serial arrangement which encouraged counting by ones. There was agreement that children learned more about number when they were encouraged to make symbolic forms or geometric shapes in preference to realistic drawings of cats, rabbits, and the like. Geometric shapes were less attractive to the children and therefore less likely to draw attention away from the number ideas themselves. They were more quickly made and therefore permitted more practice in a given period of time. The children did not confuse these forms with art work, which the teachers wished to be large and free. Finally, the symbolized forms were more readily grouped into patterns than were realistic drawings.

Emphasis on success. -- Under proper instruction no child needs to guess or to use the methods which have been designated as Level 1 responses in this study. It is granted that for testing purposes at the beginning of the investigation children were urged to give answers for number facts in processes which had not been specifically taught. None the less, many children showed evidence of intelligent, though immature, methods of arriving at answers, while others gave evidence of no apparent ability to do intelligent thinking under such circumstances. With appropriate instruction, children were helped to recognize the worthlessness of extremely immature methods and to substitute more effective means of arriving at answers.

The relation of planned and incidental number experience in the teaching program. -- In this study the position is taken that planned

number experience is necessary to the development of systematic number ideas, but that valuable supplementary learning may be provided by means of incidental number experience. When used in relation to each other, they represent two phases of a unified whole.

A point in need of clarification is that incidental number, while it is usually purposeful, does not always coincide with the child's level of maturity. For example, incidental number experience is often abstract. If such an experience occurs when the child is able to handle only concrete number satisfactorily, the teacher is faced with the problem of making the experience meaningful by the use of representative materials, or by drawings, or by showing the solution to the children herself, with as much explanation as they can comprehend. Much writing on the subject of incidental number leads to the inference that the purposeful quality of such experience insures understanding. The incidental experiences recorded for the children in this study justify the conclusion that it is rather the sensitivity of the teacher to methods by which the experience can be adapted to the level of the children which makes the experience have meaning for them. Teachers found that when such adaptations were not made, only the brighter children, those most capable of thinking abstractly, and therefore already possessing the most mature understanding of number, were profiting from the incidental experience.

Dependence on concrete materials. -- Some teachers seem to fear that children who use concrete materials will develop a dependence upon such materials which will prevent them from reaching the abstract level of thinking. The section of this report which dealt with the use of

concrete materials by the children in the study provided evidence to the effect that essentially the same range of levels of maturity was represented with concrete materials as with abstract. In other words, the child can use his fingers, blocks, discs, and the like, in very immature ways, such as inaccurate counting of objects, and laborious counting by ones. In contrast materials may be used by the child in such a way as to indicate a good understanding of groups, and mature ways of working with groups in order to arrive at solutions to problems. If the child is to progress to more mature ways of using concrete materials, he must have guidance in the use of them. When left to his own resources, or when denied the use of concrete materials, he will devise his own methods with which he will be completely satisfied as long as they serve his purpose satisfactorily. When self-devised methods fail to meet his needs, he may have no idea of what can be done in the way of improvement unless guidance is provided. Under appropriate guidance the child eventually works himself away from the need for elementary concrete materials.

The need for teachers to be continuously aware of the maturity level of pupils. -- If teachers are to help children progress toward a mature level of thinking, they must themselves have a clear understanding of what constitutes maturity. A high maturity level is characterized by economy in time and effort, by the absence of clumsy and ineffective methods of thinking, and by generalized ideas which have meaning for the individual. When a high level of maturity is reached, the child is no longer dependent upon concrete materials or counting procedures. He is able to see relationships between number ideas and thereby to furnish proof for his answers in more than one way if called upon to do so. He

has assurance and confidence as he attacks number problems.

In contrast, an immature response is marked by inaccuracies, by ineffectiveness, by a dependence on the use of concrete materials and counting methods, by a specificity which requires that each example must be worked out again and again, and by an inability to see relationships or to generalize.

The present study provides evidence on the values to be derived from the continuous exploration of the thought processes of pupils during the process of instruction. In order to help children progress to more mature levels of thinking, teachers must know their initial level. Information on initial status of pupils as revealed by interviews was available to the teachers who cooperated in the study. While they had considerable help in determining the level of ability of their children at the beginning of the year, they continued to gather information on the thought processes of pupils by working cooperatively in small discussion groups in which both teacher and pupils were aware of a need for improvement in methods of thinking.

Knowledge of a present level of thinking implies acceptance of that method by the teacher as representing the child's ability at a given time, because children whose responses are not accepted will not continue to let the teacher know how they are thinking. At the same time that immature procedures are temporarily accepted, the teacher begins to lead the child toward the level for which he seems to have a readiness. It is the continuous study and evaluation of thought processes which accomplish results. The satisfaction teachers receive from working with children over a period of time with such goals in mind is well illustrated

by the elation these beginning teachers felt when subsequent interviews revealed that their children had profited from instruction and that their children were actually thinking in more mature ways about number situations.

Appendix A

The Group Test

The group test booklet presented to the children was prepared in manuscript writing giving one process to a page. Four practice items not included in the booklet preceded the test.

Part I - Semi-concrete Phase of the Group Test

Addition:

0 and 0 are
0

0 and 00 are
0 0

0 and 000 are
000

0 and 00 are
0 00

000
0 and 0 are
000 0

0 0 00 are
0 and 00
0 0

00 and 00 are
0 00

0 and 000 are
0 000

0 0 00
0 and 0 are
0 0

Subtraction:

00 take away 2 are
00

000 take away 3 are
000

000 take away 2 are
0
000

0000 take away 5 are
0000

000 take away 4 are
000

0 0 take away 2 are
0
0 0

000 take away 3 are
0

0000 take away 6 are
0000

000 take away 3 are
000
000

Part I - Semi-concrete Phase of the Group Test - Con't.

Division:

How many 3's in 000?
000

How many 2's in 00 ?
00

How many 1's in 000 ?
000

How many 3's in 000 ?
000
000

How many 2's in 000 ?
000

How many 4's in 0000 ?
0000

How many 1's in 00 ?
00

How many 2's in 0000 ?
0000

How many 6's in 000 ?
000

Multiplication:

Two 2's are 00
00

Three 2's are 000
000

Seven 1's are 000
0
000

Two 3's are 000
000

Two 4's are 0000
0000

Four 1's are 00
00

Three 3's are 000
000
000

Four 2's are 0000
0000

One 6 is 000
000

Part II - Abstract Phase of the Group Test

Addition:

1 and 2 are _____

2 and 3 are _____

1 and 6 are _____

2 and 4 are _____

7 and 2 are _____

5 and 4 are _____

3 and 4 are _____

2 and 6 are _____

5 and 3 are _____

Subtraction:

4 take away 2 are _____

6 take away 3 are _____

7 take away 2 are _____

8 take away 5 are _____

6 take away 4 are _____

5 take away 2 are _____

7 take away 3 are _____

8 take away 6 are _____

9 take away 3 are _____

Division:

How many 3's in 6? _____

How many 2's in 4? _____

How many 1's in 6? _____

How many 3's in 9? _____

How many 2's in 6? _____

How many 4's in 8? _____

How many 1's in 4? _____

How many 2's in 8? _____

How many 6's in 6? _____

Multiplication:

Two 2's are _____

Three 2's are _____

Seven 1's are _____

Two 3's are _____

Two 4's are _____

Four 1's are _____

Three 3's are _____

Four 2's are _____

One 6 is _____

Appendix B
Recorder's Sheet

Name of child	Recorder M. B.	Part II Date 2/15/49 School Sands
11	Does some sort of figuring on table in D. * Does own work quickly.	
12	Does work very quickly. Uses fingers in D.	
13	Reads aloud. Takes in whole sentence with one glance.	
14	Whispers to herself. Very pleased with her work.	
15	Counts on fingers in A. Scratches head - finger in mouth quite a lot.	
16	Slow, deliberate, talks to herself, chews end of pencil. Makes motions in air with pencil. Says, "Hey, teacher, is we goin' to do another book?"	
17	Works very fast and doesn't do much looking around.	
18	Seems to be having trouble with S. Skips around in writing answers.	
19	Asks some S. answers from No. 20. Very slow.	
20	Compares work with No. 19. Counts on fingers under the table.	

* A - Addition; S - Subtraction; D - Division
M - Multiplication.

Appendix B

Instructions to Recorders

1. Check booklets. Eliminate any defective ones.
2. Place a number card at each child's place. (Numbers correspond with children's names.)
3. Fill in blanks at top of recorder's sheet.
4. Number two recorder's sheets (one sheet for each part of the test) with the numbers of the children assigned to you. Use a new recorder's sheet for each part of the test.
5. What to record:
 - a. Any comments of child that reflect attitude toward test or toward arithmetic in general.
 - b. Any observable behavior that indicates ways of thinking or manner of working, such as, counting on fingers, making marks, counting dots by ones, by two's, etc.
 - c. Ability to concentrate, extreme nervousness, excessive moving about, apparent lack of interest, etc.
6. Use code to save time in recording. Indicate by code letter (A S M D) the process on which a child is working when you enter a comment.
7. Watch that each comment is entered beside the appropriate number.
8. Booklets will be collected at the end of the entire test. At the end of Part I, the child will turn his booklet over and be ready to receive Part II.
9. Feel free to tell a child a word, or to indicate how to make a number, but do not tell him What number to make.
10. Watch for the child who stops and waits at the end of each page. Encourage him to continue without any waste of time.
11. Encourage child to look at his own work only. Enter comment if child tends to copy work of others.

Appendix C

Instructions to Interviewers

Initial Interview:

1. Fill out blanks at the top of your interview sheet giving your name, name of child, school and date.
2. Encourage the child to talk out loud as he arrives at his answers.
3. If the answer comes very quickly and is correct, the child usually knows it. To find out if he really understands, have him demonstrate with concrete materials, or draw a picture to represent the number fact. It is advisable to have at least one demonstration for each of the processes.
4. Do not teach ! The purpose of the interview is to find out how children are thinking. A wrong answer should be accepted in the same spirit as a correct one. It can tell you just as much about the child's thought processes. However, you may test the child's understanding at the concrete level, (using paper picnic forks or spoons) or at the semi-concrete level (drawing stick or circle pictures).
5. Watch the child's fingers and lips. Try to do your recording in between items - not while the child is figuring out an answer. If fingers are used record the exact procedure by which the answer was obtained.
6. Indicate methods used in counting, such as counting all numbers, counting large number only, counting small number only, counting by 2's, 3's, and 4's, etc.
7. When materials are used, present all ten objects to the child, saying, "Use as many of these as you need to tell the story of ____". Watch to see if grouping occurs in the use of materials. Indicate this by drawing the materials just as the child lays them out to show his story. Ask the child to describe his drawing or his concrete demonstration.
8. Choose items for which concrete or semi-concrete demonstrations are to be made with consideration of the following:
 - a. incorrect answer given to abstract example
 - b. an example on which the child paused before giving answer, and for which the exact method of thinking could not be determined
 - c. an example known immediately for which you wish to check understanding.

9. Guard against saying, "Did you start with 3 and count up?" or, "Did you do it this way?" Rather, get from the child what he did.
10. Indicate pauses (during which the child is thinking) with a dash or by writing the word "pause".
11. Be sure to enter the answer given by the child. It is best to enter this in the space provided for comments rather than under the example so that pauses can be indicated.

Interim interview:

1. In general the procedure for the interim interview is the same as for the initial interview. (Review all points from initial interview.)
2. So far as feasible each interviewer will interview the same children she interviewed at the initial interview.
3. Changes in technique for the interim interview are as follows:
 - a. If a child says, "8 take away 6 are 8; 9 take away 3 are 8" etc., repeat the example for him using the language form "from" instead of "take away".
 - b. For variety use small one-inch wooden cubes for concrete demonstrations on the interim interview.
 - c. Go through all six of the addition examples in abstract form first. Then go back and have the child demonstrate one or more. Follow the same procedure for the remaining three processes.

Final interview:

1. Review all points from the initial interview.
2. Changes in technique for the final interview are as follows:
 - a. Use small round discs as concrete materials.
 - b. Go through the entire test in abstract form before asking for any demonstrations.
 - c. It is expected that many of the children will complete the test quickly. If time allows ask the child if he can think of another way to arrive at a given answer. This procedure can be used with both abstract examples and with demonstrations. The purpose is to find out the following:
 - (1) Is the first method the only method by which the child can arrive at the answer?

- (2) Is the first method the most mature method he is capable of using?
- (3) What is the limit of his ability with regard to the use of mature methods of thinking?

Appendix D

Complete List of Interview Items Included in Interview Booklet

Addition:

2 and 4 are _____

5 and 3 are _____

2 and 6 are _____

3 and 4 are _____

7 and 2 are _____

5 and 4 are _____

Subtraction:

9 take away 3 are _____

8 take away 6 are _____

7 take away 3 are _____

8 take away 5 are _____

6 take away 4 are _____

7 take away 2 are _____

Division:

How many 3's in 9? _____

How many 4's in 8? _____

How many 2's in 6? _____

Multiplication:

Three 2's are _____

Two 4's are _____

Two 3's are _____

Appendix E
Sample Interview Form

Item	Ch. M. L. R. Int. E. D. Sch. Oyler	Date 9/20/48
Abstract		Concrete or Semi-concrete
2 + 4	5. Can't show how she does it.	Picture: 0000 00 Says 6. Counted by ones.
5 + 3	"Are you allowed to count on your fingers?" She was told she could if she needed to. "1,2,3,4,5,6,7,8."	
2 + 6	1,2,3,4,5,6,7,8. Used fingers	
3 + 4	1,2,3,4,5,6,7. Used fingers	
7 + 2	1,2,3,4,5,6,7,8,9. Used fingers	
5 + 4	Uses fingers for all additions. Counted by ones. Ans. 9.	
9 - 3	Counts 9 first; pulls down 3 fingers; counts rest by ones; ans. 6.	
8 - 6	10? 1,2,3,4,5,6,7,8 fingers; but could not get correct ans. Said 10 again.	Picture: 00000000 Leaves 2.
7 - 3	No response	Picture: 00000000 Leaves 3

8 - 5	No response	Picture: 00000000 Leaves 3
6 - 4	"Try it without a picture first this time." Used fingers. Ans. 3.	Picture: 000000 Ans. 2
7 - 2	7,8,9 Used fingers	Picture: 0000000 Leaves 5.
3)9	No response	Forks: Counts all forks; gets ans. 10.
4)8	No response	Forks: Same as above. "Would it be 10?"
2)6	"7". "Don't know how I got it."	Forks: Counts all again ans says 10.
2 x 3	"4". Guessed	Forks: Counts, but has no idea how to use ob- jects to get her answer. "Would it be 11?"
4 x 2	"6" Guessed	Same. "Would it be 10?"
3 x 2	"4". Guessed	

General comment: Strung balls out into one long row for each picture. Was accurate in counting fingers in A and in counting balls in S.

Appendix F

A Comparison of the Percentage of Error Made with the Methods of "Counting all Numbers" and "Partial Counting" (Table 34), and at Levels 2, 3, and 4 (Table 35)

TABLE 34

A COMPARISON OF PERCENTAGE OF ERROR FOR THE METHODS OF PARTIAL COUNTING, AND COUNTING ALL NUMBERS IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Method	Interview	No. of Examples	No. of Errors	Percent of Error
Counting all numbers	Initial	362	100	27.0
	Interim	234	48	20.5
	Final	135	10	7.4
Partial counting	Initial	332	40	12.0
	Interim	422	52	12.0
	Final	430	23	5.0

TABLE 35

A COMPARISON OF PERCENTAGE OF ERROR FOR RESPONSES AT LEVELS TWO, THREE AND FOUR AT INITIAL, INTERIM, AND FINAL INTERVIEWS

Level	Interview	No. of Examples	No. of Errors	Percent of Error
Level 2	Initial	877	358	40.4
	Interim	818	269	32.9
	Final	542	98	18.0
Level 3	Initial	575	103	17.9
	Interim	701	135	19.3
	Final	947	121	12.8
Level 4	Initial	683	92	13.5
	Interim	1066	129	12.1
	Final	1437	136	9.4

Appendix - G

Interview Questions used with Previous Teachers of children in the study.

- | | |
|--|---------|
| 1. What amount of your teaching procedures came from the primary manual? | A M F |
| 2. For the most part, was your work in number planned or incidental? | P I |
| 3. Did you teach the processes of A. and S. together or separately? | T S |
| 4. Which of these methods of obtaining A. answers did your children use? | 1 2 3 * |
| 5. Which of these methods did you encourage? | 1 2 3 |
| 6. Did you discourage your children from counting on their fingers? | Y N |
| 7. Did you have your children memorize the number combinations? | Y N |
| 8. What experience with S. did your children have? | N P I |
| 9. Which of these methods of obtaining S. answers did you observe your children using? | 1 2 3 |
| 10. Which of these methods did you encourage? | 1 2 3 |
| 11. Did you use A. as seat work? | Y N |
| 12. In carrying out seat work directions in A., did your children tend to string out objects or to group them? | S G |
| 13. Did you use S. as seat work? | Y N |
| 14. In carrying out seat work directions in S., did your children tend to string out objects or to group them? | S G |
| 15. In S. did you use the language form "take away" or "from?" | T F B |
| 16. What experience with M. and D. did your children have? | N P I |

* The numbers 1, 2, and 3, refer to the methods of counting all numbers, partial counting, and recall by logical inference. Examples of these methods were placed on small cards and shown to the teachers as the questions on Addition and Subtraction were asked.

17. What was the attitude of your children toward numbers? L D I
18. About how far did your children learn to write the number symbols? _____
19. What number games did you use frequently with your children?
20. What difficulties did you meet in trying to teach numbers to children?

Appendix H

TABLE 36

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART I: SEMI-CONCRETE, AND PART II: ABSTRACT), SHOWING NUMBER OF EXAMPLES OMITTED, NUMBER OF RESPONSES, NUMBER OF ERRORS, AND PERCENTAGE OF ERROR FOR EACH PUPIL GROUP IN ADDITION

	Pupil Group	No. Omitted		No. Responses		Errors		% Error	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Part I Semi- Concrete	A	11	0	169	180	28	7	16.0	3.9
	B	0	0	207	207	24	1	11.0	0.5
	C	3	0	240	243	6	2	2.5	0.8
	D	3	0	303	306	25	12	8.2	3.9
	E	0	0	243	243	7	5	2.9	2.0
Part II Abstract	A	0	0	180	180	56	7	31.0	4.0
	B	4	0	203	207	32	5	15.7	2.4
	C	1	0	242	243	44	13	18.0	5.3
	D	0	0	306	306	50	11	16.3	3.6
	E	0	0	243	243	23	14	9.5	5.7

Total number of possible responses is 1179. N. for each pupil group is as follows: A, 20; B, 23; C, 27; D, 34; and E, 27.

TABLE 37

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART I: SEMI-CONCRETE, AND PART II: ABSTRACT), SHOWING NUMBER OF EXAMPLES OMITTED, NUMBER OF RESPONSES, NUMBER OF ERRORS, AND PERCENTAGE OF ERROR FOR EACH PUPIL GROUP IN SUBTRACTION

	Pupil Group	No. Omitted		No. Responses		Errors		% Error	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Part I Semi- Concrete	A	33	0	147	180	65	5	44.2	2.8
	B	7	0	200	207	46	6	23.0	2.9
	C	17	0	226	243	83	6	36.7	2.4
	D	19	1	287	305	76	4	26.4	1.3
	E	6	0	237	243	27	11	11.4	4.5
Part II Abstract	A	29	1	151	179	90	7	59.6	3.9
	B	33	0	174	207	57	20	32.8	9.6
	C	28	0	215	243	97	17	45.1	7.0
	D	28	1	278	305	143	25	51.4	8.2
	E	22	1	221	242	59	29	26.7	12.0

TABLE 38

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART I: SEMI-CONCRETE, AND PART II: ABSTRACT), SHOWING NUMBER OF EXAMPLES OMITTED, NUMBER OF RESPONSES, NUMBER OF ERRORS, AND PERCENTAGE OF ERROR FOR EACH PUPIL GROUP IN DIVISION

	Pupil Group	No. Omitted		No. Responses		Errors		% Error	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Part I Semi- Concrete	A	85	0	95	180	61	25	66.3	13.9
	B	51	0	156	207	89	25	57.0	12.0
	C	109	1	134	242	56	35	41.8	14.5
	D	68	0	238	306	128	32	53.8	10.5
	E	80	0	163	243	89	30	54.6	12.3
Part II Abstract	A	56	0	124	180	80	27	64.5	15.0
	B	75	0	132	207	89	29	67.4	14.0
	C	94	1	149	242	79	42	53.0	18.2
	D	81	0	225	306	109	44	48.4	14.4
	E	107	1	136	242	83	38	61.0	15.7

TABLE 39

RESULTS OF INITIAL AND FINAL GROUP TESTS (PART I: SEMI-CONCRETE, AND PART II: ABSTRACT), SHOWING NUMBER OF EXAMPLES OMITTED, NUMBER OF RESPONSES, NUMBER OF ERRORS, AND PERCENTAGE OF ERROR FOR EACH PUPIL GROUP IN MULTIPLICATION

	Pupil Group	No. Omitted		No. Responses		Errors		% Error	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final
Part I Semi- Concrete	A	147	0	33	180	2	31	6.1	17.2
	B	113	0	94	207	40	19	42.6	9.2
	C	150	0	93	243	19	12	20.4	5.0
	D	153	1	153	305	66	47	43.0	15.4
	E	133	6	110	237	47	17	43.0	7.2
Part II Abstract	A	87	0	93	180	51	56	55.0	31.0
	B	87	0	118	207	70	34	59.3	16.4
	C	147	1	96	242	58	35	60.4	14.3
	D	168	0	138	306	72	59	52.2	19.3
	E	152	15	91	228	55	46	60.4	21.0

Appendix I

TABLE 40

TIME RECORD IN MINUTES FOR INITIAL AND FINAL GROUP TESTS (PART I:
SEMI-CONCRETE: PART II: ABSTRACT)

Pupil Group	No. of Pupils	Part I		Part II	
		Initial	Final	Initial	Final
A	20	234'	194'	217'	167'
B	23	264'	156'	250'	148'
C	27	316'	200'	309'	163'
D	34	388'	252'	383'	243'
E	27	317'	263'	307'	219'
Total	131	1519'	1065'	1466'	940'

TABLE 41

AVERAGE NUMBER OF MINUTES PER CHILD

Pupil Group	Part I		Part II	
	Initial	Final	Initial	Final
A	11.7	9.7	10.9	8.4
B	11.5	6.8	10.9	6.4
C	11.7	7.4	11.4	6.0
D	11.4	7.4	11.3	7.1
E	11.7	9.7	11.4	8.1

Appendix J

TABLE 42

SUMMARY OF METHODS USED BY TWENTY CHILDREN OF GROUP A IN EACH OF THE FOUR PROCESSES IN INITIAL, INTERIM, AND FINAL INTERVIEWS

		Initial	Interim	Final
Level 1	A	21	11	0
	S	69	26	0
	D	32	11	0
	M	19	11	1
		141 = 37 %	59 = 16 %	1 = .3 %
Level 2	A	31	33	31
	S	29	71	56
	D	14	21	10
	M	12	9	5
		86 = 23 %	134 = 36 %	102 = 27 %
Level 3	A	55	43	54
	S	10	15	46
	D	6	8	18
	M	5	7	11
		76 = 20 %	73 = 20 %	129 = 34 %
Level 4	A	24	37	36
	S	15	15	33
	D	11	21	35
	M	24	34	44
		74 = 20 %	107 = 29 %	148 = 39 %
Total		377	373	380

TABLE 43

SUMMARY OF METHODS USED BY TWENTY-THREE CHILDREN OF GROUP B IN EACH OF THE FOUR PROCESSES ON INITIAL, INTERIM, AND FINAL INTERVIEWS

		Initial	Interim	Final
Level 1	A	9	8	0
	S	40	27	14
	D	28	5	0
	M	24	5	3
		101 = 22 %	45 = 10 %	17 = 4 %
Level 2	A	57	25	23
	S	58	35	24
	D	17	9	9
	M	16	10	8
		148 = 33 %	79 = 18 %	64 = 15 %
Level 3	A	40	54	48
	S	28	47	41
	D	10	11	14
	M	3	7	6
		81 = 18 %	119 = 27 %	109 = 25 %
Level 4	A	41	56	71
	S	23	45	72
	D	23	48	49
	M	28	48	53
		115 = 26 %	197 = 45 %	245 = 56 %
Total		445	440	435

TABLE 44

SUMMARY OF METHODS USED BY TWENTY-SEVEN CHILDREN OF GROUP C IN EACH OF THE FOUR PROCESSES IN INITIAL, INTERIM, AND FINAL INTERVIEWS

		Initial	Interim	Final
Level 1	A	17	11	6
	S	70	31	8
	D	39	21	1
	M	34	22	0
		160 = 31 %	85 = 15 %	15 = 4 %
Level 2	A	70	23	6
	S	74	100	22
	D	26	22	7
	M	23	14	0
		193 = 37 %	159 = 29 %	35 = 7 %
Level 3	A	42	71	74
	S	13	34	45
	D	5	11	8
	M	4	4	7
		64 = 12 %	120 = 22 %	134 = 25 %
Level 4	A	40	73	98
	S	19	26	105
	D	22	37	67
	M	25	54	75
		106 = 20 %	190 = 34 %	345 = 65 %
		523	554	529

TABLE 45

SUMMARY OF METHODS USED BY THIRTY-FOUR CHILDREN OF GROUP D IN EACH OF THE FOUR PROCESSES IN INITIAL, INTERIM, AND FINAL INTERVIEWS

		Initial	Interim	Final
Level 1	A	24	9	3
	S	68	27	1
	D	38	17	6
	M	31	11	2
		161 = 25 %	64 = 10 %	12 = 2 %
Level 2	A	45	18	13
	S	55	74	46
	D	26	14	13
	M	21	22	9
		147 = 23 %	128 = 20 %	81 = 12 %
Level 3	A	88	101	111
	S	68	69	99
	D	9	15	21
	M	6	5	20
		171 = 27 %	190 = 30 %	251 = 38 %
Level 4	A	53	83	91
	S	33	47	81
	D	32	59	68
	M	44	65	75
		162 = 25 %	254 = 40 %	315 = 48 %
Total		641	636	659

TABLE 46

SUMMARY OF METHODS USED BY TWENTY-SEVEN CHILDREN OF GROUP E IN EACH OF THE FOUR PROCESSES IN INITIAL, INTERIM AND FINAL INTERVIEWS

		Initial	Interim	Final
Level 1	A	7	1	1
	S	34	15	9
	D	27	11	5
	M	22	12	1
		90 = 19 %	39 = 8 %	16 = 3 %
Level 2	A	55	35	26
	S	73	84	54
	D	17	23	18
	M	18	17	14
		163 = 31 %	159 = 31 %	112 = 21 %
Level 3	A	69	80	89
	S	38	20	64
	D	15	8	22
	M	8	4	9
		130 = 25%	112 = 22 %	184 = 35 %
Level 4	A	34	53	51
	S	36	50	50
	D	30	44	49
	M	38	52	63
		138 = 26 %	199 = 39 %	213 = 41 %
Total		521	509	525

Appendix K

TABLE 47

SUMMARY OF METHODS USED FOR SIX ADDITION EXAMPLES BY EACH PUPIL GROUP
IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Group		Initial	Interim	Final
Level 1	A	21	11	0
	B	9	8	0
	C	17	11	6
	D	24	9	3
	E	7	1	1
		78 = 10 %	40 = 5 %	10 = 1 %
Level 2	A	31	33	31
	B	57	25	23
	C	70	23	6
	D	45	18	13
	E	55	35	26
		258 = 32 %	134 = 16 %	99 = 12 %
Level 3	A	55	43	54
	B	40	54	48
	C	42	71	74
	D	88	101	111
	E	69	80	89
		294 = 36 %	349 = 42 %	376 = 45 %
Level 4	A	24	37	36
	B	41	56	71
	C	40	73	98
	D	53	83	91
	E	34	53	51
		192 = 23 %	302 = 37 %	347 = 42 %
Total		822	825	832

TABLE 48

SUMMARY OF METHODS USED FOR SIX SUBTRACTION EXAMPLES BY EACH PUPIL GROUP
IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Group		Initial	Interim	Final
Level 1	A	69	26	0
	B	40	27	14
	C	70	31	8
	D	68	27	1
	E	34	15	9
		281 = 33 %	126 = 15 %	32 = 4 %
Level 2	A	29	71	56
	B	58	35	24
	C	74	100	22
	D	55	74	46
	E	73	84	54
		289 = 34 %	364 = 45 %	202 = 23 %
Level 3	A	10	15	46
	B	28	47	41
	C	13	34	45
	D	68	69	99
	E	38	20	64
		157 = 18 %	185 = 22 %	295 = 34 %
Level 4	A	15	15	33
	B	23	45	72
	C	19	26	105
	D	33	47	81
	E	36	50	50
		126 = 15 %	183 = 21 %	341 = 39 %
Total		853	858	870

TABLE 49

SUMMARY OF METHODS USED FOR THREE DIVISION EXAMPLES BY EACH PUPIL GROUP
IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Group		Initial	Interim	Final
Level 1	A	32	11	0
	B	28	5	0
	C	39	21	1
	D	38	17	6
	E	27	11	5
		164 = 38 %	65 = 16 %	12 = 3 %
Level 2	A	14	21	10
	B	17	9	9
	C	26	22	7
	D	26	14	13
	E	17	23	18
		100 = 23 %	89 = 21 %	57 = 14 %
Level 3	A	6	8	18
	B	10	11	14
	C	5	11	8
	D	9	15	21
	E	15	8	22
		45 = 11 %	53 = 13 %	83 = 20 %
Level 4	A	11	21	35
	B	23	48	49
	C	22	37	67
	D	32	59	68
	E	30	44	49
		118 = 28 %	209 = 50 %	268 = 64 %
Total		427	416	420

TABLE 50

SUMMARY OF METHODS USED FOR THREE MULTIPLICATION EXAMPLES BY EACH PUPIL GROUP IN INITIAL, INTERIM, AND FINAL INTERVIEWS

Group		Initial	Interim	Final
Level 1	A	19	11	1
	B	24	5	3
	C	34	22	0
	D	31	11	2
	E	22	12	1
		130 = 32 %	61 = 13 %	7 = 2 %
Level 2	A	12	9	5
	B	16	10	8
	C	23	14	0
	D	21	22	9
	E	18	17	14
		90 = 22 %	72 = 17 %	36 = 9 %
Level 3	A	5	7	11
	B	3	7	6
	C	4	4	7
	D	6	5	20
	E	8	4	9
		26 = 6 %	27 = 6 %	53 = 13 %
Level 4	A	24	34	44
	B	28	48	53
	C	25	54	75
	D	44	65	75
	E	38	52	63
		159 = 39 %	253 = 62 %	310 = 78 %
Total		405	413	406

Appendix L

TABLE 51

NUMBER AND PERCENTAGE OF UNSUCCESSFUL RESPONSES FOR EACH PUPIL GROUP IN
THE USE OF CONCRETE MATERIALS WITH ADDITION EXAMPLES FOR INITIAL, INTERIM,
AND FINAL INTERVIEWS

Pupil Group	Initial			Interim			Final		
	Resp.	U	% U	Resp.	U	% U	Resp.	U	% U
A	31	8	25.8	26	2	7.7	51	2	3.9
B	27	10	37.0	26	3	11.5	47	0	0.0
C	35	15	42.9	62	7	11.3	60	4	6.6
D	52	20	38.5	66	9	13.6	98	12	12.2
E	26	9	34.6	37	1	2.7	57	5	8.8
Total	171	62	36.2	217	22	10.1	313	23	7.3

TABLE 52

NUMBER AND PERCENTAGE OF UNSUCCESSFUL RESPONSES FOR EACH PUPIL GROUP IN
THE USE OF CONCRETE MATERIALS WITH SUBTRACTION EXAMPLES FOR INITIAL,
INTERIM, AND FINAL INTERVIEWS

Pupil Group	Initial			Interim			Final		
	Resp.	U	% U	Resp.	U	% U	Resp.	U	% U
A	38	24	60.5	32	11	34.4	46	1	2.2
B	30	8	26.6	34	7	20.6	55	2	3.6
C	46	20	43.5	63	15	23.8	64	7	10.9
D	55	18	32.7	90	22	24.4	98	11	11.2
E	44	8	18.2	56	13	23.2	58	11	18.9
Total	213	78	36.6	275	68	24.7	321	32	9.9

TABLE 53

NUMBER AND PERCENTAGE OF UNSUCCESSFUL RESPONSES FOR EACH PUPIL GROUP IN
THE USE OF CONCRETE MATERIALS WITH DIVISION EXAMPLES FOR INITIAL, INTERIM,
AND FINAL INTERVIEWS

Pupil Group	Initial			Interim			Final		
	Resp.	U	% U.	Resp.	U	% U	Resp.	U	% U
A	24	12	50.0	20	2	10.0	28	3	10.4
B	18	4	22.2	9	1	11.1	31	1	3.2
C	21	10	47.6	34	11	32.3	32	5	15.6
D	29	17	58.6	45	7	15.5	45	4	9.1
E	29	17	58.6	26	4	15.4	40	8	20.0
Total	121	60	49.6	134	25	18.6	176	21	11.9

TABLE 54

NUMBER AND PERCENTAGE OF UNSUCCESSFUL RESPONSES FOR EACH PUPIL GROUP
IN THE USE OF CONCRETE MATERIALS WITH MULTIPLICATION EXAMPLES FOR
INITIAL, INTERIM, AND FINAL INTERVIEWS

Pupil Group	Initial			Interim			Final		
	Resp.	U	% U	Resp.	U	% U	Resp.	U	% U
A	17	10	58.8	21	5	23.8	20	3	15.0
B	7	1	14.3	7	1	14.3	26	1	3.8
C	18	11	61.1	27	7	25.9	24	0	0.0
D	23	8	34.8	29	5	17.2	37	3	8.1
E	20	8	40.0	22	3	13.6	27	6	22.2
Total	85	38	44.7	106	21	19.8	134	13	9.7

Appendix M

TABLE 55

A SUMMARY OF METHODS USED IN INTERIM AND FINAL INTERVIEWS BY SIX CHILDREN WHO GUESSED THROUGHOUT THE INITIAL INTERVIEWS

Child	Addition		Subtraction		Division and Multiplication	
	Interim	Final	Interim	Final	Interim	Final
M.S.	Ca (2) 3 (Rd(1) (Cu 1 Rd(2) 2	(Cu (Vgr 1 Cp 2 (Cp (Vgr 2 Ca 1	G (4) 4 (Cu (G (2)2	Vgr (1)	6 G (6) 6	Cv (3) 3 RI 3
A.M.	(Rd(1) (Ca 1 Cp 4 Vgr 1	RI 4 Cp 2	Rd(1)1 Op(3)3 Cu(1)2	RI E (1) Rd(1)	4 Op(4) 4 1 CA(2) 2 1	Op 3 G 1 CA 2
H.M.	G (5) 5 Cu 1	G (5)5 Cu 1	G (5)6	G (5)	6 G (6) 6	G (6) 6
D.N.	G (5) 5 Rd 1	Rd 2 Cp(1)4	CA(6)6	RI Cu Rd	2 RI 1 3 Ca(1) 2 1 Po(3) 3	Cm 4 RI 2
L.C.	G (2) 2 Ca(3) 4	RI(1)2 Rd(1)1 Ca(1)3	Op(5)5 0 1	Cu(1)	6 G (5) 6	NR 4 G (2) 2
M.L.	RI(1) 4 Ca(1) 2	RI(1)6	RI(4)6	Cu RI (2)	1 G (3) 3 5 RI 3	RI (1) 6

Code: Ca Counting all numbers Po Partial Omission
Cu Counting(method unknown) G Guessing
Ca Counting to add or adding RI Immediate recall
Cp Partial Counting RLi Recall by logical inference
Cm Counting in Multiples Vgr Visual Grouping

Errors are indicated by numbers in parentheses.
Dual responses are indicated by brackets.

TABLE 56

A SUMMARY OF METHODS USED IN INTERIM AND FINAL INTERVIEWS BY FIFTEEN CHILDREN WHO OBTAINED ANSWERS TO SUBTRACTION EXAMPLES IN INITIAL INTERVIEWS BY THE METHOD OF PARTIAL OMISSION

Child	Interim	Final	Child	Interim	Final
D.G.	Op (6) 6	Cu 2 Cu Vgr (1) 4	G.A.	Op (6) 6	Vgr (1) 3 RLi 1 Rd 1 RI 1
B.W.	Op (6) 6	Cu (1) 1 Vgr 3 RI 2	B.T.	Ca 5 RI 1	Rps 3 RI 3
C.S.	Cu 5 (Cu (Vgr 1	(Cu (Vgr 4 Ca 1 (RI (1) (Cu 1	D.C.	(RI (Cu 5 (RI (Cp 1	RLi (1) 5 Rd 1
A.K.	Op (6) 6	RLi 2 RI 2 (RI (1) (Cu 1 (RI (Vgr 1	E.S.	RLi 1 RI 4 Cu 1	RI 6
B.S.	NR 1 Cu 4 Op (1) 1	Cu 6	R.N.	Cu 1 (Cu (1) (Vgr 4 Cp 1	Vgr (1) 2 RLi 3 Rd 1
J.K.	(Vgr 4 (C (RI (RLi 1 RI 1	Rd 2 RI 1 Vgr 3	C.M.	Op (6) 6	RI 4 Ca 2
			R.H.	Op (6) 6	RLi 1 Rd (1) Rd 3 RLi 2

TABLE 56 (Cont.)

Child	Interim	Final	Child	Interim	Final
C.D.	Op (6) 6	(Vgr (Cu 1 Cu 5	C.H.	Op (6) 6	Cu (2) 6

TABLE 57

A SUMMARY OF METHODS USED IN INTERIM AND FINAL INTERVIEWS BY TWELVE CHILDREN WHO OBTAINED ANSWERS TO ADDITION EXAMPLES IN INITIAL INTERVIEWS BY THE METHOD OF COUNTING ALL NUMBERS

Child	Interim	Final	Child	Interim	Final
C.H.	Ca 1 Cp 5	Vgr (1) 5 Rd (1) 1	E.S.	R 2 Rd (1) 4	R 4 (Cm 1 (Vgr 1 (Cu 1 (Vgr 1
S.S.	Ca 6	Cp 6	T.W.	Ca (1) 5 Cm 1	RLi 3 Ca 3
B.S.	Ca 3 Cp 3	Cm 1 Cp 5	B.C.	Ca (1) 6	Ca 1 Cm 1 Cp 2 Cu(1) 2
R.M.	RI (1) 3 Ca 2 Cp 1	Ca 5 Cp 1	J.L.	Cp 6	(RLi 1 (Pc 1 RLi 3 Cp 2
D.H.	Cm 2 Ca 1 Rd(1) 2 Cu 1	RI 2 Rd (2) 2 Ca 2	G.C.	RLi 1 Ca 5	Ca 4 Rd(1) 1 Cp 1
J.M.	Ca (5) 6	Cp (1) 5 (RI (1) 1 (Pc 1	V.R.	Ca 3 Rd (1) 3	Cp 6

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