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I hereby recommend that the thesis prepared under my supervision by Steven K. Kreider

entitled Hedging the Interest Rate Risk of GNMAs

be accepted as fulfilling this part of the requirements for the degree of Doctor of Philosophy

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HEDGING THE INTEREST RATE RISK OF GNMAS

A dissertation submitted to the
Division of Graduate Studies and Research
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by

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ABSTRACT

This dissertation is an empirical study of alternative strategies for hedging the interest rate risk of GNMAs. Four hedging strategies are considered: 1) the short sale of Treasury note futures, 2) the short sale of Treasury bond futures, 3) the simultaneous short sale of Treasury bond futures and Eurodollar certificate of deposit futures, and 4) the purchase of put options on Treasury bond futures. Each of these strategies is evaluated in simulations constructed with daily price series for seven different GNMA coupons covering the period from January 1985 to March 1987. Hedge ratios were computed using modified Macaulay duration and six different models of GNMA price sensitivity.

Statistical analysis of the simulation results shows that efforts to hedge GNMAs may in many circumstances increase, not reduce, interest rate risk. This finding was especially true for GNMAs priced above par and for hedge strategies using puts on Treasury bond futures. In addition, hedge results indicated that two option-adjusted GNMA duration models developed in this study were unable to outperform the implied GNMA duration technique advanced by Pinkus and Chandoha (1985). Finally, monthly adjustment of hedge ratios to account for changes in GNMA durations did not significantly improve hedge effectiveness.

Other findings of interest emerged in this study. In particular, the influence of many technical factors in both the GNMA and futures markets, recognized as basis risk, served to vitiate hedge effectiveness and cloud the interpretation of results. This study also revealed that a parametric analysis of cash and hedge instruments which includes both duration and convexity can add value to the hedge effort. This type of analysis recommends hedging GNMAs with Treasury note futures; simulation results for the period under study indicate that on average this strategy was superior to the others tested. And finally, this study introduced K, a measure of hedge effectiveness which evaluates hedges in terms of variation about the initial portfolio value instead of the traditional method which considers variation about the mean. Simulation results indicate that K provides useful information on hedge effectiveness and exhibits some desirable statistical properties.

Future research on this topic should confirm the results reported in this research against a data base covering the entire history of the GNMA market. In addition, the application of Arbitrage Pricing Theory to hedge strategies using these and other hedge instruments - such as interest-only mortgage securities and over-the-counter GNMA options - is worthy of study.

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This study has benefitted from the support, assistance, and advice of many, many people. It is fitting that their efforts be acknowledged here.

This study would not have been possible without the aid of the Office of Policy and Economic Research at the Federal Home Loan Bank Board. In this regard, I wish to offer thanks to Dr. Joseph A. McKenzie, Director, Policy Analysis Division, whose suggestion that I take careful note of both academic and Wall Street research has been particularly helpful. In addition, the counsel, suggestions, and perspective of Dr. Edward A. Hjerpe have influenced this work in a very positive way.

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The Chairman of my dissertation committee, Dr. Charles Barngrover, merits thanks words cannot convey. I am indebted to him for the observations and criticisms he has made, for the hard work he has invested, and for the example he has provided. His dedication to his work and his unfailing consideration for others have left a strong impression on me. I hope I can live up to the standards he has established.

Finally, I wish to thank my wife Nancy, who supports me in whatever I do. I am humbled that she knew before it was conceived, and before Dr. Barngrover mentioned it in our first meeting, what I am only just beginning to recognize: this study is a beginning.

TABLE OF CONTENTS

LIST OF FIGURES AND TABLES

FIGURES

TABLES

CHAPTER I: INTRODUCTION

I. INTRODUCTION

The fundamental postulate of the theory of relativity as first propounded by Einstein (1905) may be stated as,

"It is physically impossible to detect the uniform motion of a frame of reference from observations made entirely within that frame."[1]

The wisdom in this postulate may be applied outside the realm of physics. In particular, it serves as a proviso to researchers in the social sciences. The point of application emerges when a distinction is made between the assumption laden settings used to develop theories and the complex, unconstrained settings in which theory is applied. When these two settings are viewed as independent frames of reference, one may consider the "real world" frame as necessary to validate progress in the theoretical frame. Thus, advances in theory are accepted as advances when their merits are duly confirmed in a separate frame of reference, the real world.

In this context, the present research was conceived as an effort to evaluate the application of financial theory to the task of hedging the interest rate risk[2] of GNMAs.[3] It accomplishes this task by conducting empirical tests of current, state of the art theoretical approaches to designing hedge strategies and to computing hedge ratios.[4]

II. IMPORTANCE OF THE TOPIC

Fundamentally, the topic of hedging GNMAs is important because it relates to a large number of institutional and individual participants in the economy. This relation is better appreciated in the context of the growth and structure of the secondary mortgage market.

The recent growth of the secondary market has proceeded at a phenomenal rate. Although it has its roots in the post-Depression era, this market was not important until the past decade. A few milestones emphasize the point. In 1975, the total principal amount of mortgages sold to investors amounted to approximately $8 billion; in 1984, the amount was $60 billion; in

1985, $110 billion; and in 1986, over $225 billion. This growth has been astounding in terms of both its speed and magnitude. A once obscure market has exploded to the point where it now rivals the size of the trillion dollar market for U.S. government bonds. Perhaps more importantly, the growth is likely to continue. Residential mortgages alone represent over $1 trillion in assets; the figure rises to $1.8 trillion when commercial and farm mortgages are added.[5]

The structure of this market finds savings institutions, commercial banks, and homebuilders involved in the origination of the loans which underlie the pass-throughs. Three quasi-governmental agencies are involved in the exchange of pass-through securities for mortgages: the Government National Mortgage Association (GNMA), the Federal National Mortgage Association (FNMA), and the Federal Home Loan Mortgage Corporation (FHLMC). Purchasers of the pass-through securities include pension funds, life insurance companies, mutual funds, as well as savings institutions and commercial banks. Investment banks play an intermediary role: they identify willing originators, structure the mortgage for pass-through swaps, and place issues with investors. Their roles as market makers are crucial.[6]

The growth of this market has combined with the size and sophisticated nature of its players to drive the market toward efficiency. These factors, when added to the influence of the volatile interest rates of the last decade, have concentrated the attention of many market participants on the management of interest rate risk. Specifically, more and more institutions have become involved in efforts to hedge large portfolios of pass-throughs. On the origination side, banks, thrifts, and mortgage banks have increasingly moved toward selling originations. They are therefore quite concerned about protecting the value of their mortgages from adverse changes in interest rates during the time when loans are accumulated for sale. The market makers, of course, generally seek earnings from transactions, not portfolio holdings. They too have great interest in hedging pass-throughs. On the investment side, portfolio managers are increasingly concerned about protecting the value of their pass-throughs during bear markets. They share this interest in hedging pass-throughs. And finally, the traders - the speculators and arbitrageurs -are interested in hedging pass-throughs as part of a broad spectrum of trading strategies.

Thus, the topic of hedging mortgage pass-throughs, virtually unheard of a decade ago, is now a central concern of many institutional lenders and investors. Upon reflection, it is easy to see that the concern of these entities is driven by the needs of three groups of individuals. First, on the lending side, consumers have strong preference for low cost, fixed rate residential mortgage financing. Lenders better able to control the rate risk associated with issuing fixed rate mortgages are better able to meet consumers needs.[7] Second, on the investment side, institutional money managers are charged with serving pensioners, policy holders and stockholders. An improved grasp of how to hedge mortgage pass-throughs allows them better to control performance in terms of risk and return. And third, from the regulatory standpoint, which is to represent the interests of society, knowledge of the problems associated with hedging mortgage pass-throughs is essential to responsible deposit insurance for and effective regulation of savings institutions.

Taken from this broad view, the volatile interest rate environment of the last decade created problems for many institutional and individual participants in the American economy. Trends in financial research developed in response to the far reaching effects of these problems. A great deal of theoretical and empirical research was addressed to issues connected with the topic of controlling interest rate risk.

In fact, it is difficult to identify another area in finance in which the application of theory to practice proceeded at a greater rate or with such a wide audience of academics and practitioners. These efforts notwithstanding, there presently (Spring 1987) exists no published systematic empirical study of the application of financial theory to the task of controlling the interest rate risk of GNMA pass-throughs with financial futures and options on futures. The research presented in this dissertation is designed to help fill this void.

III. OVERVIEW

The primary purpose of this research is to study the task of hedging GNMAs by exploring three hypotheses:

1. THE INTEREST RATE RISK OF GNMA PASS-THROUGHS IS REDUCED BY THE USE OF DURATION-BASED HEDGING STRATEGIES WHICH EMPLOY FINANCIAL FUTURES AND OPTIONS ON FUTURES.
2. EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY INCORPORATE THE EMBEDDED CALL OPTION ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT INCORPORATE THIS COMPONENT.
3. STRATEGIES WHICH INCORPORATE DYNAMIC REBALANCING OF HEDGE POSITIONS OUTPERFORM STATIC APPROACHES.

Studying these hypotheses involves the application of research from a broad spectrum of subjects, including the term structure of interest rates, bond portfolio management, option pricing, financial futures, valuation of mortgage-backed securities, and hedging. Thus, following this introductory chapter, the dissertation begins with a review of relevant theoretical and empirical literature in Chapter II. This review is necessary to establish firmly the theoretical underpinnings of the work to follow.

The literature reviewed in Chapter II holds several important implications for this research effort:

DURATION: Empirical results, especially those of Lau (1983) and Bierwag, Kaufman, and Toevs (1983), recommend Macaulay duration as a measure of the sensitivity of bond prices to changes in interest rates.

OPTION PRICING: The quadratic approximation of American option values developed by Barone-Adesi and Whaley (1986) is well-suited to the task of estimating values for futures options.

The Black-Scholes European option model can be used to value the call option embedded in GNMAs. This model is recommended chiefly because the binomial model fails put-call parity for debt options and because there presently exists no persuasive empirical evidence to justify the expense of using finite difference methods to evaluate American formulas for long term debt options.

VALUATION OF MORTGAGE-BACKED SECURITIES: The Pinkus Chandoha (1987) approach and a modified version of the Clayton Goldstein (1986) model can be used to compute GNMA duration without explicitly accounting for the option component.

The option explicit approach to computing GNMA duration should rely on a combination of the Black-Scholes European model and an annuity valuation determined under the assumption of a flat term structure.

None of these approaches to calculating mortgage duration employ the stochastic model of the term structure advanced so successfully in the theoretical literature by Cox, Ingersoll, and Ross (1978, 1985b). This choice is warranted for two reasons. First, two extremely broad immunization studies -Lau (1983) and Bierwag, Kaufman, and Toevs (1983) - have recommended Macaulay duration in favor of the CIR measure. Second, there is a lack evidence demonstrating that the term structure is driven by a clearly identifiable, stationary stochastic process.

HEDGING: Empirical techniques which have dominated the literature on interest rate hedging are inconsistent with rational, arbitrage-free markets. Consequently, this research relies on a "fundamental" approach to hedge ratio computation based on Macaulay duration.

Following the literature review, Chapter III presents the specific research hypotheses to be examined in this study. In addition, it describes the characteristics of the institutional setting constructed for the purpose of specifying and evaluating hedging strategies. In particular, emphasis is given to the development of a new measure of hedge effectiveness, denoted by K, which evaluates hedging performance in terms of the average daily deviation of portfolio value from its initial value. This measure does a better job of measuring how well a given strategy performs in relation to the hedge objectives than variance based measures do. When supplemented with a few simple ratios describing the maximum observed single day deviations, K provides a great deal of meaningful information about hedging performance.

The penultimate section of Chapter III explains the rationale underlying each of the four hedge strategies chosen for testing:

1. Strategy 1: short T-note futures
2. Strategy 2: short T-bond futures
3. Strategy 3: short T-bonds and Euros
4. Strategy 4: long T-bond futures puts

Rules used to select futures contracts delivery months are explained in the context of liquidity constraints and the related needs of consistency and clarity in the structure of the research. In addition, this section presents the rationale for dynamic hedge adjustment and describes the method used to simulate this process in the present research. And finally, a brief discussion identifies other potential hedging strategies which are not selected for testing and explains the reasons for their exclusion.

Chapter IV is presented next. As noted there, the most important aspect of any hedging strategy is the hedge ratio used to implement the strategy. Consequently, from a conceptual standpoint, Chapter IV represents the core of this study. It explains how Macaulay's duration can be modified and used to calculate hedge ratios for a variety of GNMA hedge strategies. In the process, several different views of the price behavior of GNMAs are noted and reflected in the numerator of the hedge ratios. These alternative views produced four approaches to modeling GNMA duration:

1. Clayton-Goldstein inferred duration: In this model, it is assumed that expected prepayments are reflected in GNMA market prices. Iterative numerical search techniques can be used to "infer" the expected prepayment rates (adjusted for maturity considerations using the PSA schedule) which produce the observed market price for a given discount rate.[8] Modified Macaulay duration can then be computed on the basis of the sum of anticipated prepayments and scheduled principal and interest.

2. Option-adjusted duration: This formulation conceptualizes GNMAs as the sum of two components: an annuity and a related call option representing the borrowers' rights to prepay the mortgages underlying GNMA securities. This dissertation employs two variations of this model in the hedge simulations:
a. Option-adjusted (market implied) method: In this case, the value of the embedded calls is determined as the difference between the observed market price for the GNMA and the computed value of the underlying annuity. Call values determined in this way are used to calculate implied volatilities; implied volatilities are in turn used to calculate the call deltas. GNMA

duration is then computed as,

$$D_{GNMA} = D_A P_A * (1 - d_c) / P_{GNMA}$$

where D_A, P_A are the duration and price of the underlying annuity and d_c is the call delta.

b. Option-estimated method: This model computes the value of the embedded calls using the Black Scholes model. Volatility estimates for the price of the underlying annuity are made on the basis of observed interest rate volatility as captured in the daily yield on the ten year Treasury during the preceding sixty trading days.

3. Implied (regression) duration: This model is based on two assumptions: 1) GNMAs trading at par exhibit price volatility relative to the current ten year Treasury which is stable over time, and 2) GNMAs trading at price levels other than par exhibit price volatility relative to the par GNMA which is stable over time. These two sets of relative price volatilities are estimated using regression models. The volatility factors which emerge from this process permit GNMA duration to be computed according to,

$$D_{GNMA} = B_1 * B_2 * D_{10\ yr}$$

where B_1 is the price volatility of the GNMA price range under study relative to the price volatility of the par GNMAs;
B_2 is the price volatility of the par GNMA relative to the price volatility of the ten year Treasury; and
$D_{10\ yr}$ is the duration of the ten year Treasury.

To these four models are added two benchmark duration models:

1. Twelve year "bullet" duration: This model computes modified Macaulay durations for GNMAs assuming only scheduled interest and principal payments are made for 143 months. The entire outstanding principal is assumed to be repaid in month 144.
2. "True" duration: this measure is an ex post computation:

$$D_{TRUE} = <>P/(P*<>y)$$

where <>P and <>y represent changes observed in the GNMA price and yield over the three month simulation period.

Each of these six measures of GNMA duration are used to calculate hedge ratios for use in the simulation experiments. The output of these simulation experiments is a collection of K values: hedge effectiveness is measured for every unique combination of GNMA duration measure, hedge strategy, hedge type, and GNMA coupon (ranging from 8% to 14%).

As explained in Chapter V, the output of these simulations is used in statistical tests of the research hypotheses. The process of exploring and testing these hypotheses produces several contributions to the body of knowledge in finance. The principal contributions emerge in the results of the statistical tests:

1. Efforts to hedge GNMAs with futures and options on futures do not necessarily reduce interest rate risk. They may in fact increase interest rate risk, especially if the GNMA cash position is priced in the super-premium range.

In terms of the four alternative strategies, simulation results for the period under study produced the following effectiveness rankings, beginning with the most effective approach:

1. Strategy 1: T-note futures
2. Strategy 3: T-bonds and Euros
3. Strategy 2: T-bond futures
4. Strategy 4: T-bond futures puts

This ranking was consistent across all three data sets, although differences among strategies 1, 2, and 3 were generally insignificant. Strategy 4, on the other hand, was significantly inferior in all cases.

2. The option-estimated GNMA duration model developed for use in this study performed significantly better than a duration model based on the twelve year bullet prepayment assumption. However, this model did not outperform either the implied or inferred GNMA duration models. From best to worst, these three measures were in every period ranked as follows:

   1. Implied (Regression) Duration
   2. Inferred Clayton-Goldstein Duration
   3. Option-Estimated Duration

This ranking should be viewed with caution, however, as differences among these models were for the most part not significant.

3. The dynamic adjustment of hedge ratios to account for changes in the interest sensitivities of the cash and futures positions does not significantly improve hedge effectiveness.

Several other contributions of interest emerged in the process of investigating these hypotheses:

1. Basis risk appears to be a very important factor in hedge effectiveness. Duration-based hedges, which ignore this factor, suffer accordingly.
2. Accounting for convexity in the hedge design process adds value during periods of volatile interest rates.
3. The use of a variance minimizing measure of hedge effectiveness does not alter the conclusions of the hypothesis tests.
4. K demonstrated some attractive statistical properties: it was not asymmetrically distributed, and it was negatively correlated with large daily losses and extreme portfolio returns.

These findings have implications for future research and for practicing financial managers. These implications are explored in Chapter VI, which concludes this dissertation.

CHAPTER I: FOOTNOTES

[1]Corson, J. and Lorraine, W. Electromagnetic Fields and Waves John Wiley and Sons (Chicago) 1976.

[2]Generally, interest rate risk refers to variability in holding period returns to a security due to changes in interest rates.

[3]GNMA refers to thirty year Government National Mortgage Association pass-throughs. A pass-through security is created when one or more mortgage holders sells shares or participations in a "pool" or collection of mortgage loans. The security entitles the holder to a pro rata portion of the principal and interest payments made against the mortgages in the pool. In effect, the cash flow from the mortgages is "passed through" to the security holder. Pools consist of as few as one or as many as several thousand loans. Generally, each loan is serviced by its originator; a trustee is assigned to hold the titles of all mortgages in the pool and to ensure that all mortgages and properties are in acceptable form and all payments are properly made.

Pass-through securities were created to enable lenders to convert existing loans to "re-lendable cash," hence increasing the availability and decreasing the cost of mortgage credit. See Senft (1983) for a lucid introduction.

[4]Technically, the term "cross-hedging" is used to describe hedging methods in which the securities underlying the financial futures and options on futures are not the same as the security being hedged. In this study, the securities underlying the futures and options on futures are U.S. Treasury notes and bonds and a ninety day Eurodollar Certificate of Deposit index. They are not GNMAs. For purposes of expositional convenience, this dissertation uses the term hedges generally, whereas in reality the hedges considered are cross hedges.

It is appropriate to hedge GNMA pass-throughs with the futures and options on futures noted above because GNMA futures contracts have historically proven very ineffective in this regard. Their shortcomings derive primarily from technical problems associated with delivery and are perhaps best evidenced by the dramatic decline in average daily volume which has occurred over the past three years:

	1984	1985	1986
Avg. Daily Volume (thousands of contracts)	862	156	45

Source: The Chicago Board of Trade

Finally, it should be noted that Ederington (1979) studied the problems of hedging GNMA pass-throughs with GNMA futures contracts.

[5]The pass-through volumes quoted for 1985 and 1986 are taken from Mortgage Finance Marketplace New York: Shearson Lehman Bros., December 1986 - January 1987: p. 15.

The other mortgage security volumes in this paragraph are taken from F. Fabozzi, The Handbook of Mortgage-Backed Securities Chicago, Ill: Probus Publishing Co., 1985: p. 1-2. More exact data on the size of Treasury and Agency debt is presented in the following table ($billions, U.S.):

Issuer	Dec 1984	Dec 1985	June 1986
Federal and federally sponsored agencies	271.2	293.9	296.2
Marketable public debt, U.S. Treasury	1247.4	1437.7	1498.2
Total Treasury and Agency	1518.6	1731.6	1794.4

Source: Federal Reserve Bulletin Washington D.C.: Board of Governors of the Federal Reserve System 73 (2) (February 1987): A-24.

[6]Mortgage for pass-through swaps is a phrase which refers to the process through which mortgage holders convert loans into securities. Investment bankers perform investment analysis to illustrate to lenders the returns to be earned in securitizing and swapping loans. They often follow up by helping mortgage holders select loans for inclusion in pools and by finding purchasers for the resultant security.

[7]It should be noted that strictly speaking, the problem of hedging GNMA pass-throughs is not identical to the problem of hedging loans to be made and warehoused for securitization. The latter problem is complicated primarily by the difficulty involved in predicting the

proportion of loans offered which will close, whereas the former, as noted in the text, is concerned with projecting mortgage prepayments. In addition, the loans underlying GNMAs are FHA and VA loans, whereas warehousing efforts can be broadened to include other types of mortgage loans.

[8] The PSA schedule (PSA stands for Public Securities Association) utilizes information in the FHA survivorship tables to generate benchmark CPR (conditional prepayment rate) values for MBSs (mortgage-backed securities) as a function of age. MBSs are then described in terms of "percent of PSA." See Chapter IV for further discussion.

CHAPTER II: LITERATURE REVIEW

I. INTRODUCTION

The purpose of this chapter is to review financial research germane to the topic of hedging GNMAs. It also discusses the implications of this research as they relate to the development of the theoretical models presented in Chapters III and IV.

Fundamentally, the topic of hedging GNMAs involves the application of research in a broad spectrum of subjects, including the term structure of interest rates, bond portfolio management, duration analysis, valuation of financial futures, option pricing, valuation of mortgage-backed securities and hedging. As a result, the scope of this literature review is broad, and the task of integrating the advances made in each of these areas is imposing.

A common approach to the synthesis of a diverse collection of concepts is to represent them schematically. Figure

2-1 applies this technique to the problem of presenting in a cohesive manner the seemingly disparate subjects involved in hedging GNMAs. The supporting structures and interlocking joints shown in the illustration are intended to convey - as best it can be done in graphical terms - relationships between the various areas of relevant research.

As shown, the foundation for the financial theory applied in this study is the theory of the term structure of interest rates. Research advances in bond portfolio management and debt option pricing are built directly from this base. In like fashion, theoretical work on the valuation of mortgage-backed securities rests on the pillars of option pricing and bond portfolio management. Futures valuation, at the next level, is supported principally by the work in bond portfolio management, while the pricing of futures options rests atop achievements in option pricing. Finally, the mantle of this structure represents the topic of hedging GNMAs. As indicated, it bears heavily on each of the topical areas mentioned.

Like any schematic, the model in Figure 2-1 is based on many abstractions and simplifications. In particular, details of the complex interrelationships among these topics have been sacrificed in favor of parsimony. This

Topical Relationships in GNMA Hedging

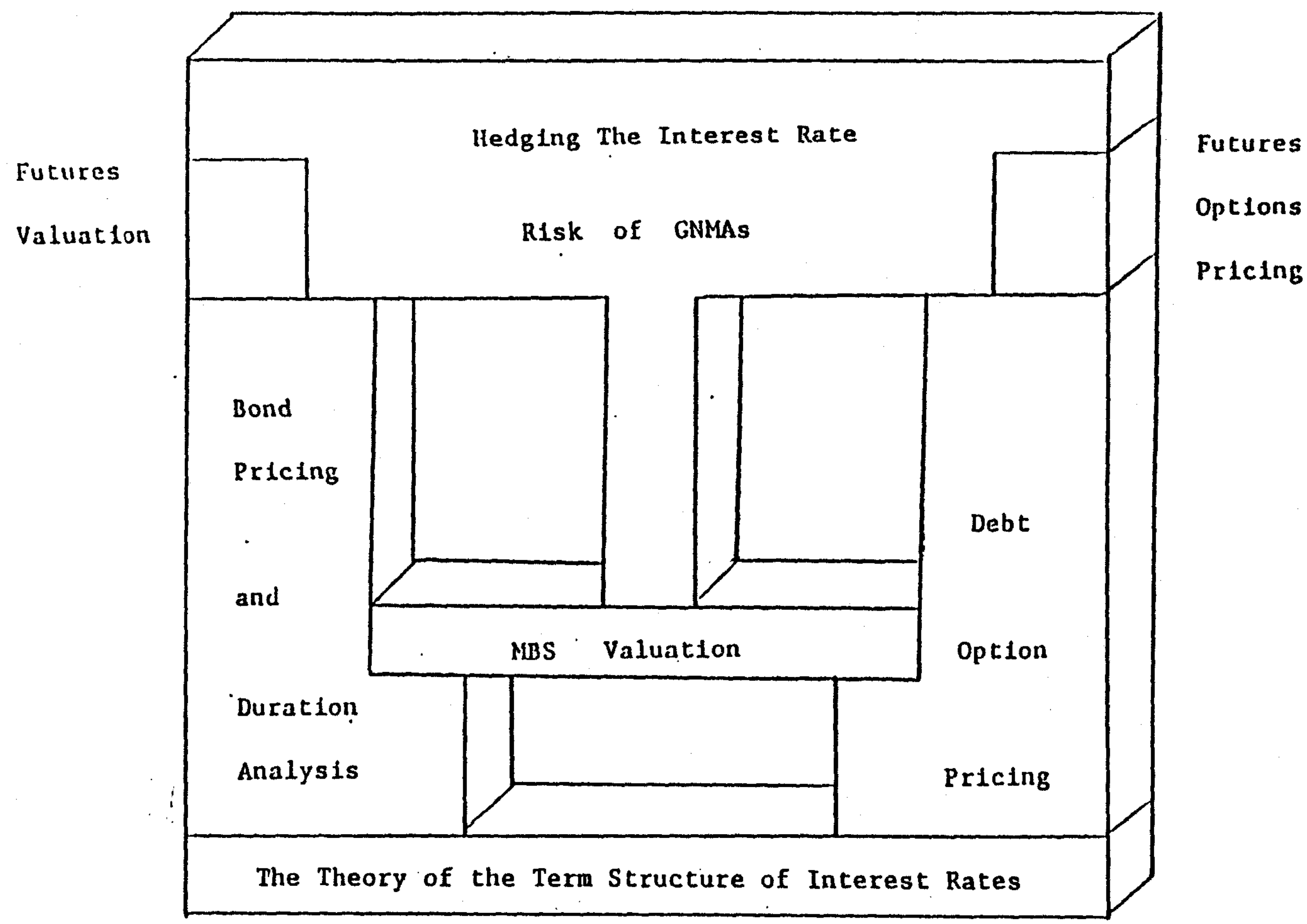


Figure 2 - 1

qualifier not withstanding, the schematic represents a useful way of thinking about the diverse elements brought together in the present research.

The development of this literature review proceeds in a manner which parallels roughly the form of the schematic. Sections II and III discuss accomplishments in duration analysis and the pricing of debt options. Section IV then considers valuation of mortgage-backed securities. Section V reviews the hedging literature and explains why the present study has elected a hedge method based on duration. Section VI concludes the chapter with a summary of the implications of the literature for the present research, thereby completing the foundation for the theoretical models examined in later chapters.

II. DURATION

This section begins with a review of the historical development of the duration concept (see Figure 2-2). It proceeds to explain the various theories of the term structure of interest rates and their implications for choosing a duration measure. A review of empirical tests of duration is presented next. The section concludes with a discussion of how these studies relate to the use of duration in this research.

DURATION RESEARCH

A Timeline

1938 — Macaulay

1. Flat yield curve
2. Parallel shifts

$$D = \sum_{t=1}^{N} \frac{(CF)(t)/(1+y)^t}{CF/(1+y)^t}$$

1971 — Fisher and Weil

1. Sloped yield curve
2. Parallel shifts

$$D_{FW} = \frac{CF_1/(1+r_1) + (2)CF_2/(1+r_1)(1+r_2) + \ldots}{CF_1/(1+r_1) + CF_2/(1+r_1)(1+r_2) + \ldots}$$

1983 — Bierwag, Kaufman, Toevs

1. Correct choice of a duration measure is a function of the stochastic process according to which interest rates change

1983 — Bierwag, Kaufman,

1. Empirical test of five duration specifications: Result: Use Macaulay duration

Figure 2 - 2

A. Historical Development

In 1938 Frederick Macaulay coined the term "duration" to refer to the "average longness" of a bond. His intention was to use this measure, which was computed as the present value weighted average of the maturities of each of the payments made to a bondholder, in place of maturity, which he felt was inadequate. His measure was defined analytically as,

$$D = E\ (t)*s(t)*(1+i)^{-t}\ /\ E\ s(t)*(1+i)^{-t} \qquad (1)$$

where s(t) is the income payment at time t, i is the yield to maturity, and E represents the sum of the terms from n=1 to n=N, where N stands for the number of periods until the bond matures.

Working independently on a study of how changes in interest rates affect asset prices, Hicks (1939) computed an elasticity for the value of an income stream with respect to rate changes. He called this measure "average period", yet its form was identical to Macaulay's duration. Redington (1952) replicated this derivation in a study which found the net worth of a firm to be unaffected by changes in interest rates if the first derivative of the firm's "asset-proceeds" was equal to the same measure of the firm's "liability outgo". Like Hicks, he was unaware of previous development of this measure which he called "mean term".

These advances were generally ignored until an immunization study reported in 1971 by Fisher and Weil in which they showed that investors could construct a portfolio of default-free coupon bonds, the returns from which could be "immunized" against changes in interest rates. FW noted that Macaulay duration, which employed but a single discount rate, namely the yield to maturity, assumed a flat (horizontal) term structure of interest rates. Accordingly, using it to immunize portfolios provides protection against against only one type of rate change: parallel shifts of the flat yield curve. FW advanced the concept of duration by noting that each of the cash flows in the duration equation, denoted by s(t) in equation (1), should be discounted by the spot interest rate corresponding to a zero coupon bond of the same maturity and default risk as each payment.[1] With this improvement, FW showed that duration could be used to immunize portfolios in the presence of sloped yield curves. However, the validity of their analysis is weakened considerably because, like Macaulay's, it is valid only for parallel shifts in the term structure.

Increasingly volatile interest rates present in the middle

1970's stimulated academic interest in duration and immunization research. Bierwag and Kaufman (1977) and Bierwag, Kaufman, and Toevs (1983) showed that the proper specification of a duration measure was dependent upon the proper specification of the stochastic process according to which interest rates change. Cox, Ingersoll, and Ross (1979) applied this concept to the case where the instantaneous compounding risk-free rate, r, follows a first order autoregressive process,

$$dr = k(R_{avg} - R_{t-1})dt + \sigma(R_{t-1})^{1/2}dz \qquad (2)$$

where R_{avg} is the long term, steady state mean interest rate;

R_{t-1} is the value of r in the previous instant;

k is a parameter describing the rate at which r converges to the steady state mean;

σ is the standard deviation of r; and

dz represents the standard Wiener process (a continuous time analog for a random walk process).

This process has two very attractive features: it is mean-reverting, which makes it fairly consistent with historical observations, and it is mathematically tractable, owing primarily to the use of Ito's lemma in handling the partial differential equations generated by the Wiener process. The parameters k, R_{avg}, and σ are readily estimated from historical data. (For example, CIR (1979) estimated k = .692, R_{avg} 5.623%, and σ = .078). Other estimates are discussed in Buser, Hendershott, and Sanders (1984) and Kau, Keenan, Muller, and Epperson (1986).)

Given the realization that the appropriate selection of a duration measure is contingent on the stochastic process describing changes in the term structure, it is appropriate to digress briefly to consider the various theories of the term structure of interest rates and the related procedures for estimating the term structure.

B. The Term Structure of Interest Rates: Theories and Problems of Estimation

The yield curve is commonly constructed by plotting yields to maturity for recently issued Treasury bonds against time to maturity. Treasury bonds are chosen to attempt to factor out differences in bond prices which owe to factors other than maturity, particularly default risk. Recently issued bonds are used to adjust for differentials in taxability and liquidity. Specifically, in the government bond market, old Treasuries tend to be "put away" by investors with the passage of time, hence they become less liquid. At the same time, older bonds tend to deviate from par as rates change, creating the possibility that a portion of the bond's yield is taxed as capital gain (loss). Using recent issues, particularly those marketed in the Treasury's quarterly refunding operation, reduces yield differences owing to these two factors.

In considering the merits of a yield curve plotted according to this method, however, it is important to recognize that yields to maturity are calculated by means of a convenient, but arbitrary, averaging of the spot rates (i.e. zero coupon equivalent rates) which apply to each of the payments for each bond. Information is lost in this averaging process.

The term structure of interest rates, on the other hand, consists of a plot of spot rates against maturity. It preserves information contained in these spot rates regarding future levels and shapes of the term structure. This information can be extracted by estimating implied future forward rates and using them to construct future term structures. (See Caks (1977)). Construction of future term structures, however, is an arduous process involving two major tasks.

The first task concerns the mathematics of fitting curves (i.e. discount functions) to the plotted spot rates. Currently popular techniques involve the use of exponential and polynomial spline functions. These functions are estimated in piecewise fashion, generally governed by conditions requiring equality of function values and first and second order differentials at the points where the pieces are joined. These techniques leave much to be desired in that they involve some rather complex mathematics and sometimes produce erratic results, as noted by Shea (1984).

The second task to be completed in translating the plot of spot rates against maturity into future term structures concerns the selection of a framework for interpreting the present structure. Four theories commonly used to explain the term structure are the pure expectations theory, the liquidity preference hypothesis, the market segmentation hypothesis, and the inertial theory (the current term structure will persist indefinitely). All of these theories are familiar and have been discussed at length elsewhere, hence they will not be belabored here.[2] However, several important points regarding these theories are relevant in estimating the term structure and selecting a duration measure:

1. Although liquidity preference seems to explain short term Treasuries fairly well and pure expectations is an acceptable description of some other short term markets, none of these theories is well accepted as a general explanation of the term

structure. This conclusion is based on a great deal of empirical work, most of which, unfortunately, is restricted to the Treasury market. Toevs and Dyer (1986) summarize and support these findings concisely.

2. Problems associated with estimating risk premia to account for differentials in liquidity and credit risk are very troublesome; adjusting these premia simultaneously for variation with term is even more so.

The combination of the difficulties associated with curve fitting and choosing a theory of the term structure is imposing. A relevant question, given the dependence of the appropriate duration measure on the term structure, concerns the relative value which may be added by the various levels of sophistication reflected in alternative specifications of duration. Fortunately, empirical research which sheds light on this question has recently been reported in the literature.

C. Empirical Tests of Duration Measures

Bierwag, Kaufman, and Toevs (1983) generated results which are fairly representative of several studies which used a variety of data sources and a variety of time periods to test single factor (and even two factor) duration models.[3] Beginning with the proposition that the best test of single factor duration models is to evaluate their effectiveness in immunizing default-free, option-free bond portfolios, BKT evaluated five different duration measures in immunization experiments tested with ten year horizons against bond data spanning the years 1925 - 1978. This study is appealing because the data base includes a wide variety of economic conditions: a major war, a severe depression, and several complete business cycles. In addition, it tested five interesting duration specifications:

D1: the Macaulay duration measure, assuming flat yield curves and parallel shifts;

D2: Fisher-Weil duration, assuming parallel (i.e. additive) shifts of curves of any shape.

D3: Fisher-Weil duration, assuming multiplicative shifts of curves of any shape.

D4: a stochastic model which assumes changes in short term rates are 1.3 times as great as those in long term rates; and

D5: a stochastic model akin to D4, modified to ensure that conditions of equilibrium in financial markets are satisfied.

The primary result of these tests is instructive. The evidence suggests the simple, easy-to-compute Macaulay duration is "at minimum, as cost effective as its more complex and theoretically pleasing counterparts.[4]"

These findings were supported by a similar study reported by Lau (1983). Lau employed three different duration measures in a series of simulations which utilize Treasury bonds to immunize a single payment liability due in eight years. The measures chosen by Lau are those developed by Macaulay, Fisher and Weil, and Cox, Ingersoll, and Ross (1979). Following BKT (1983), Lau reported that the CIR and Macaulay duration measures perform about equally well, but, in light of its relative simplicity, the Macaulay measure seems preferable.

D. Implications For Present Research

The empirical results reported by Lau and BKT (not to mention several others) are illuminating. They reveal that no strong evidence exists to indicate that the complexities introduced into the theoretical development of duration measures provide added value relative to Macaulay duration. At the same time, these studies indicated that the Macaulay measure was reasonably effective in immunization applications. These two results are useful from the standpoint of developing duration-based hedge ratios because they suggest that maximum effectiveness and simplicity are together achieved in the same measure: Macaulay duration.

II. OPTION-PRICING THEORY

A. Introduction

The theory of option pricing is related to the task of hedging GNMAs in two important ways. First, it may be helpful in accounting for the value of call options implicit in the mortgage contracts. And second, option pricing theory is needed to calculate appropriate hedge ratios for those strategies which employ options on financial futures. Accordingly, this section reviews the theory and practice of using option pricing models in these applications.

B. The Theory of Option Pricing in Continuous Time

1. The Black-Scholes Option Pricing Model

Fischer Black and Myron Scholes authored the seminal work in option pricing theory. Published in 1973, their work was distinguished from earlier efforts by Sprenkle (1961), Boness (1964), Samuelson (1965), Ayres (1966) and others because it produced a valuation formula which contained no arbitrary parameters. Unlike some of these earlier efforts, BS ignored the temptation of warrants and instead derived their model by focusing on simple call options. The key theoretical insight involved in this derivation was the recognition that the returns to a risk-free portfolio, constructed by selling a determined number of calls against each share of stock held, were necessarily realized at the risk-free rate of interest to preclude arbitrage.

Given this foundation, BS employed the fundamental theorem of stochastic calculus, known as Ito's lemma, to expand the expression for the change in the value of the call given a change in the price of the stock. The significant aspect of this expansion is that it defined the change in the value of the call in terms of variables all of which were deterministic save one, the stock price. Taken together with simple boundary conditions describing the expiration value of the call, BS were able to show that the price of a European call option is,

$$c(s,t) = S*N(d1) - Xe^{rT}*N(d2) \qquad (3)$$

where $d1 = (\ln(S/X) + r*T)/(o*(T^{1/2}) + o*(T^{1/2})/2;$

$d2 = d1 - o*(T^{1/2});$

S is the stock price;

X is the option exercise price;

r is the risk-free rate;

T is the time remaining until expiration;

o is the instantaneous standard deviation of S; and
N represents the cumulative normal distribution function.

Numerous empirical tests of the BS model have shown that it is fairly robust in terms of its ability to establish economic prices for options. This point is underscored in research reported by Whaley (1982). His test examined the pricing performance of three call valuation models in a sample of 15,582 CBOE call options. The three models he chose to evaluate are distinct in terms of the manner in which they modify and apply the BS formula to the case of a dividend-paying stock. One of these models was developed by Roll (1977). It adjusts for dividends by treating the stock price as the sum of a risky portion - a non-dividend paying stock - and a riskless portion -a certain escrowed dividend from which dividends prior to expiration will be paid.

A second model tested by Whaley was suggested by Black (1975). It adjusts for dividends by valuing calls at the higher of:

(1) the price produced by the BS model, where time to the next dividend is substituted for time to expiration; or
(2) the price produced by the BS model, where the stock price is reduced by the present value of the escrowed dividends.

The third model tested consisted of (2), above, in isolation.

Whaley's results are impressive. They indicate that these dividend adjusted models perform very well, apparently vulnerable only to a bias which underprices low volatility stocks and overprices high volatility stocks. Whaley attributes this bias to varying stock volatility and the imperfect assumptions of certain dividends and equivalent income and capital gains taxes. On an aggregate level, Whaley observes that the average price predicted by Roll's model was within one cent of the actual prices in his CBOE sample. Even the simplest model, described above as (2) operating in isolation, generated an average pricing error of less than three and one-half cents. Both of these average errors are smaller than the bid-ask spreads on these options.

2. Extensions and Applications of the Black Scholes Model: Problems and Assumptions

Empirical successes similar to those reported by Whaley have encouraged wide-spread application of the BS model in both theoretical and "real-world" finance. A reasonable question, given the extensive use of this model, concerns the issue of deciding when it is appropriate to use the model and when it is not. This decision can only be made by comparing the assumptions required in the Black Scholes derivation with conditions prevalent in existing financial markets.

BS began with seven important assumptions, most of which relate to the description of ideal conditions in the stock and option markets.

1. The short term risk free rate is known and constant.
2. Stock price movements are characterized by a random process, wherein the variance is proportional to the square of the price. This is tantamount to assuming that the stock price (x) is lognormally distributed. Importantly, BS assumed the variance rate of returns on the underlying stocks is constant.
3. Stocks pay no dividends.
4. The call options are European; videlicet, they cannot be exercised except at maturity.
5. No transaction costs or taxes are present.
6. Borrowing can be done without restriction at the risk free rate.
7. There are no penalties for short sales activities. Specifically, no supply problems arise from an excess of short positions in the market.

An interesting observation is that three conventional assumptions are not required:

1. Securities are fairly priced.

2. The market holds homogeneous expectations regarding expected security returns.
3. The market holds homogeneous expectations regarding the distribution of security returns.

On a theoretical level, most of these assumptions can be relaxed without significantly impairing the model. However, in practical terms, generalizing the BS model to the case of the American option which pays dividends - the dominant genre in actual financial markets and instruments - has presented serious computational difficulties. These problems are especially significant in the case where the underlying security is a debt instrument. A brief review of the BS assumptions clarifies this assertion:[5]

1. BS assumed the underlying security makes no payouts (assumption 3, above) during the life of the option. However, when the underlying security is a debt instrument, it will almost always produce a payout in the form of a coupon payment (zero coupon bonds and Treasury bills are exceptions). It is important to note that, even if the coupon payment does not occur during the life of the option, the debt security may still be regarded as producing a continuous payout because of the continuing accrual of interest.

2. BS assumed that proportional changes in the price of the underlying security are described by a lognormal distribution. In the case of debt securities, this assumption obviously fails. The full price of coupon debt reflects daily interest accrual. Upon payment of coupon interest, the full price of the security falls to reflect interest accrual toward the next coupon. This pattern will be repeated, generating a saw-toothed function for price plotted against time.

Interestingly, Sharfman (1986) noted that even the flat price (full price less accrued interest) of a debt security cannot be expected to strictly follow the conventional lognormal process. He illustrated the point with an example which shows that a ten year, 8% note priced above 180 implies a negative yield to maturity. This upper boundary on the price of the debt security gave Sharfman reason to reject the lognormal process as appropriate for valuing debt options.

3. The evolution of a saw-toothed price pattern over time for coupon debt also implies the BS assumption of a continuous price process is violated.
4. BS assume that the risk-free rate of interest is constant over the life of the option. This

assumption is extremely troublesome for debt options, given that the major influence on bond prices over time is changing interest rates. To assume a stochastic process for debt security prices in a world where interest rates are assumed constant is inconsistent.

3. Pricing Debt Options in Continuous Time

Although Whaley's results for options on dividend paying stocks seem to recommend the ready extension of the BS model to debt options, the theoretical problems posed by the violation of these four assumptions have stimulated significant research in methods for valuing debt options in continuous time. This branch of inquiry has been integrated with modified BS models by Brennan and Schwartz (1983). They showed that the BS model adjusted for continuous payout and early exercise (i.e. the American feature) may be obtained as a special case of a two interest rate model they developed in a study of savings bonds (1977). The fundamental assumption supporting this model is that prices on default free bonds are a function of an instantaneous rate, r, (the "short rate"), a long term rate, l, and the term to maturity. This assumption implies that the entire term structure of interest rates is determined by its endpoints, r and l.

Brennan and Schwartz supplemented this assumption with another stating that r and l follow a joint stochastic process such that:

$$dr = B_1 dt + n_1 dz_1 \qquad (4)$$

$$dl = B_2 dt + n_2 dz_2 \qquad (5)$$

where dz_1, dz_2 are increments to standard Wiener processes and B_1, B_2, n_1, n_2 are parameters dependent upon r and l.

From this foundation, Brennan and Schwartz invoked arbitrage arguments and Ito's lemma to solve for the value of a call option on a debt security.

Having developed this model, Brennan and Schwartz compared it to Courtadon's (1982) model. They demonstrated that Courtadon's model, which is based on the assumption that a short interest rate, r, follows a univariate Markov process, is also a special case of their general two interest rate model, specified in (4) and (5) above.

Armed with these three models, each of which computed debt option prices using different assumptions for the stochastic process governing interest rates, Brennan and Schwartz proceeded to compare their ability to price options. Two types of debt options were considered. The first type of option confers on its holder the right to purchase a specified bond (e.g. the 6%'s of 1990). It is noted that the maturity of the bond decreases as option expiration approaches. The second type of option refers to a bond the maturity of which is fixed (i.e. the bond maturity is the same at each point in time before the option expires.)

The result of these comparisons is instructive. The three models generate significantly different prices. These differences suggest that model prices are dependent on the choice of an assumption for a stochastic process which generates the term structure. This point is discouraging because there presently exists no well-accepted view of what that process is. Moreover, no strong evidence exists to indicate that the process is stationary, which is an a priori requirement for each of these models.

A recent paper by Buser, Hendershott, and Sanders (1986) refuted the conclusion of Brennan and Schwartz (1983). BHS contended that debt option values are affected only by the general level of uncertainty regarding future rates. They argued that the Brennan and Schwartz study fails to control properly for an uncertainty factor. Specifically, they pointed out that there is inherently less uncertainty reflected in both one factor models than there is in the two factor model. Since there is less uncertainty reflected in the one factor models, it is not surprising to find Brennan and Schwartz reporting systematic and substantial undervaluations by the one factor models when compared to the two factor models. On this basis, BHS concluded that debt option values are independent of whether the term structure is driven by risk aversion, expectations, or a combination of the two. Furthermore, the number of interest rate factors is unimportant. These results are significant because they suggest that "seemingly diverse models of the interest rate process are observationally equivalent with respect to the pricing of bond options.[6]"

4. Real-World Continuous Time Option Pricing Models

The purpose of this subsection is to discuss computational problems associated with continuous time option pricing models. Before proceeding, it is useful to summarize the material presented in the preceding three subsections. Subsection B1 presented the Black Scholes model and empirical evidence to support its validity. Subsection B2 reviewed that assumptions underlying the BS model and discussed those which were violated for the case of debt options. Subsection B3 discussed theoretical developments in the pricing of debt options. In particular, it concluded that continuing disappointments in the effort to identify interest rate processes do not vitiate the use of continuous time debt option models.

With this background, the task of establishing economic prices for real world options should be straightforward. Unfortunately, it is not. The analytic solutions developed to the partial differential equations (PDEs) describing debt option prices create significant computational problems. The conventional response to these computational problems is to use a numerical method for approximating the solution. Generally, the numerical techniques employed for this purpose fall into one of two categories: those which approximate the PDE and those which approximate the process describing changes in the price of the underlying security.

a. Finite Difference Methods

Numerical techniques which approximate the PDE include finite difference methods and numerical integration. The essence of the finite difference method is to replace partial derivatives with finite differences (i.e. the familiar discrete incremental deltas of elementary calculus). This substitution transforms the option pricing equation from a differential equation into a difference equation.

Although frequently used in mathematics, physics, and engineering - not to mention finance[7] - finite difference methods are computationally expensive. They require that all possible price paths for security prices through time be mapped out on very fine grids (price vs. time). This task is cumbersome and can only be accomplished on main frame computers. On the other hand, counterposed against the high cost of computation is the benefit of familiarity. Extensive use of finite difference methods in other fields makes them a known quantity.

b. Numerical Integration

A second numerical technique popular in approximating the PDE is called numerical integration. It involves the calculation of discrete summations as approximations of integrals which cannot be evaluated analytically. Geske and Johnson (1984) rely on this technique in computing option values with their compound approximation technique. Their approach is much more efficient than finite difference methods, but it requires the evaluation of cumulative bivariate, trivariate, and sometimes higher order multivariate normal density functions. Such evaluations require the use of specialized software and are unwieldy.

C. Binomial Option Pricing

1. The Binomial Model

The problems encountered in using continuous time option pricing models stimulated the search for a more tractable approach. This search produced the binomial option pricing method. Originally developed by Cox, Ross, and Rubinstein (1979) and Rendleman and Bartter (1979), this method discards complex continuous time processes. In their place it substitutes a simple two-state, or binomial, model. The essence of the model is that security prices are to be observed at discrete points in time. From one point in time to the next, the price may either increase by a constant proportional amount or decrease by a constant proportional amount. The likelihood of a price increase or a price decrease from one state to the next is dependent upon assigned probabilities. Given an initial price, it is possible to generate a binomial decision tree or "price lattice" which displays security prices at each node. From these security prices, terminal (expiration) values can be computed for call options. The current value of a call option can then be calculated by forming risk-free hedge portfolios (ala Black and Scholes) and discounting their value backward through time, beginning with the expiration values. This recursive procedure is terminated when the value of the call in the current state is determined.

This approach to option pricing has become quite popular. Its relative simplicity and intuitive appeal make it attractive for "real world" applications. Importantly, as demonstrated by Cox, Ross, and Rubinstein (1979) and Jarrow and Rudd (1983), when the discrete time intervals between nodes in the price lattice are made sufficiently small, the limiting value of the binomial model converges to the continuous time Black Scholes solution. This convergence is crucial to assuring that the binomial model is well-behaved and will produce prices consistent with the continuous time approach.

The binomial model has been widely used. Pitts (1985) demonstrated its utility in valuing debt options and in valuing options on debt futures. In the case of debt options, the decision tree is used to generate values for interest rates at future points in time. These interest rates are then used to compute bond prices at future points in time. The familiar hedge portfolios are then constructed and used to solve recursively for current option values.

2. Criticisms of the Binomial Model

Criticism of the binomial approach to pricing debt options have conventionally been addressed to the interest rate values assigned to the nodes of the decision tree. This criticism parallels that made against continuous time models which, as noted earlier, are open to criticism because of difficulties arising from identification of a stochastic process which explains how the term structure changes through time. The binomial model is vulnerable first because it summarizes the term structure with a single rate - the one period risk free rate. In addition, the use of constant change proportions and constant state probabilities through time has little basis in empirical fact. And finally, a major criticism of the use of the binomial model to generate interest rates is based on a comparison to observed rates. Historically, interest rates have demonstrated some degree of mean-reversion, presumably in response to "corrective" monetary and fiscal policies. The binomial model, by contrast, relies on a lognormal process which produces a variance of interest rates which increases proportionally with time. This implies an ever-increasing range of rates, which is inconsistent with historical interest rate experience.

These criticisms notwithstanding, the binomial model has gained a substantial following with institutional investors and Wall Street research groups alike. This practice has recently been called into question by Bookstaber, Jacob, and Langsam (1986).

BJL have shown that the binomial model exhibits fundamental inconsistencies in pricing debt options. Specifically, put-call parity violations may occur for option values derived from any binomial model which considers options with more than one period remaining until expiration. These violations result in arbitrage opportunities using puts, calls, the underlying bond, and bonds of other maturities. BJL showed that the severity of this problem increases proportionally with the number of periods to expiration.

The source of these inconsistencies is the binomial model. BJL showed that an arbitrage-free interest rate process can only be assured in a multinomial model where the number of possible rate states emanating from a given node is equal to the number of periods remaining to option expiration.

These results are very troubling. They show clearly that a rational option pricing model - i.e. one where no arbitrage opportunities exist - demands the enumeration of a tremendous number of possible interest rate states. For example, consider the valuation of a debt option with only three months to expiration, where time intervals are denoted in days. The number of interest rate states emanating from the first node would be 89. The number emanating from each of these 89 nodes would be 88, or 7,832 altogether. Each of these nodes corresponds to a possible state on the second day. The number of states on the third day would be 87 states for each of the 7,832 nodes, or 681,384 states. Clearly, the number of states which must be enumerated is extremely large even for an option with only ninety days remaining to expiration. Hence it is apparent that the computation of option prices in an arbitrage-free discrete time framework is an imposing task.

D. Efficient Analytic Approximation of American Option Values: Pricing Debt Options and Financial Futures Options in Continuous Time

It is instructive to reflect on the progress of financial research in pricing debt options. Through early 1986, theoretically satisfactory solutions had been obtained in both continuous and discrete time. Unfortunately, the use of either of these approaches has been hindered by computational problems.

In this context, the contribution of Barone-Adesi and Whaley (October 1986) assumes added importance. They have developed an accurate, inexpensive method for pricing American options on debt and debt futures. The derivation of the model they produced is interesting and worthy of review.

Barone-Adesi and Whaley began with three assumptions:

1. The short term interest rate, r, and the cost of carry, b, are assumed to be constant proportional rates. The cost of carry is defined as the riskless rate, r, less the yield on the debt instrument, y.

2. The price of the underlying security, S, is described by the stochastic differential equation,

$$dS/S = a*dt + o*dz \quad (6)$$

where a is the expected instantaneous relative price change of the security;
o is the instantaneous standard deviation of S; and
z is a Wiener process.

3. A riskless hedge may be formed between the option and the underlying security. This hedge implies that the partial differential equation governing the option price, P is,

$$1/2o^2 S^2 P_{ss} + bSP_s - rP + P_t = 0 \quad (7)$$

where subscripts denote partial differentials. This is the same equation developed by Merton (1973).

The key insight in the approximation developed by Barone-Adesi and Whaley is that equation (7) may be applied to the early exercise premium inherent in the price of an American option. That is, the price of an American option may be decomposed into two parts. The first part is a premium paid for the right of early exercise. Because PDE (7) applies to the American option and the European option in isolation, it must also apply to their difference, the early exercise premium.

Building from this insight, Barone-Adesi extended the work of MacMillan (1986) to show that the price of an American option may be efficiently approximated. The expression for the approximation is fairly lengthy and hence is relegated to the Appendix. However, despite its expositional length, the approximation may be calculated on electronic spreadsheets available for microcomputers.

The theoretical appeal of the quadratic approximation developed by Barone-Adesi and Whaley is supported by simulation results which show that their technique generates values which are extremely similar to those produced by finite difference methods. The major caveat to the use of this approximation is that it fails to perform well relative to finite difference methods when option maturities increase beyond one year. Barone-Adesi and Whaley, apparently unaware of the work of Bookstaber, et. al. (1986), recommend the use of the binomial model in these applications.

E. Implications For Present Research

This section has reviewed the literature on the theory of option pricing. (Major advances considered are summarized in Figure 2-3.) The purpose of this review has been to develop a framework within which to select models for estimating the rate sensitivity of both options on financial futures and the long term debt options embedded in GNMA securities. Based on the review, the quadratic approximation developed by Barone-Adesi and Whaley seems to be the best approach for studying the futures options. As will be explained in more detail in Chapter III, none of the futures options to be used in the hedge simulations have expirations greater than nine months. Hence, the caveat advanced by Barone-Adesi and Whaley regarding the diminished accuracy of the quadratic approximation for options with maturities greater than one year is of little consequence.

Pricing the debt options implicit in GNMAs, however, is another matter. Since these options could easily have

OPTION PRICING THEORY

Some Important Advances

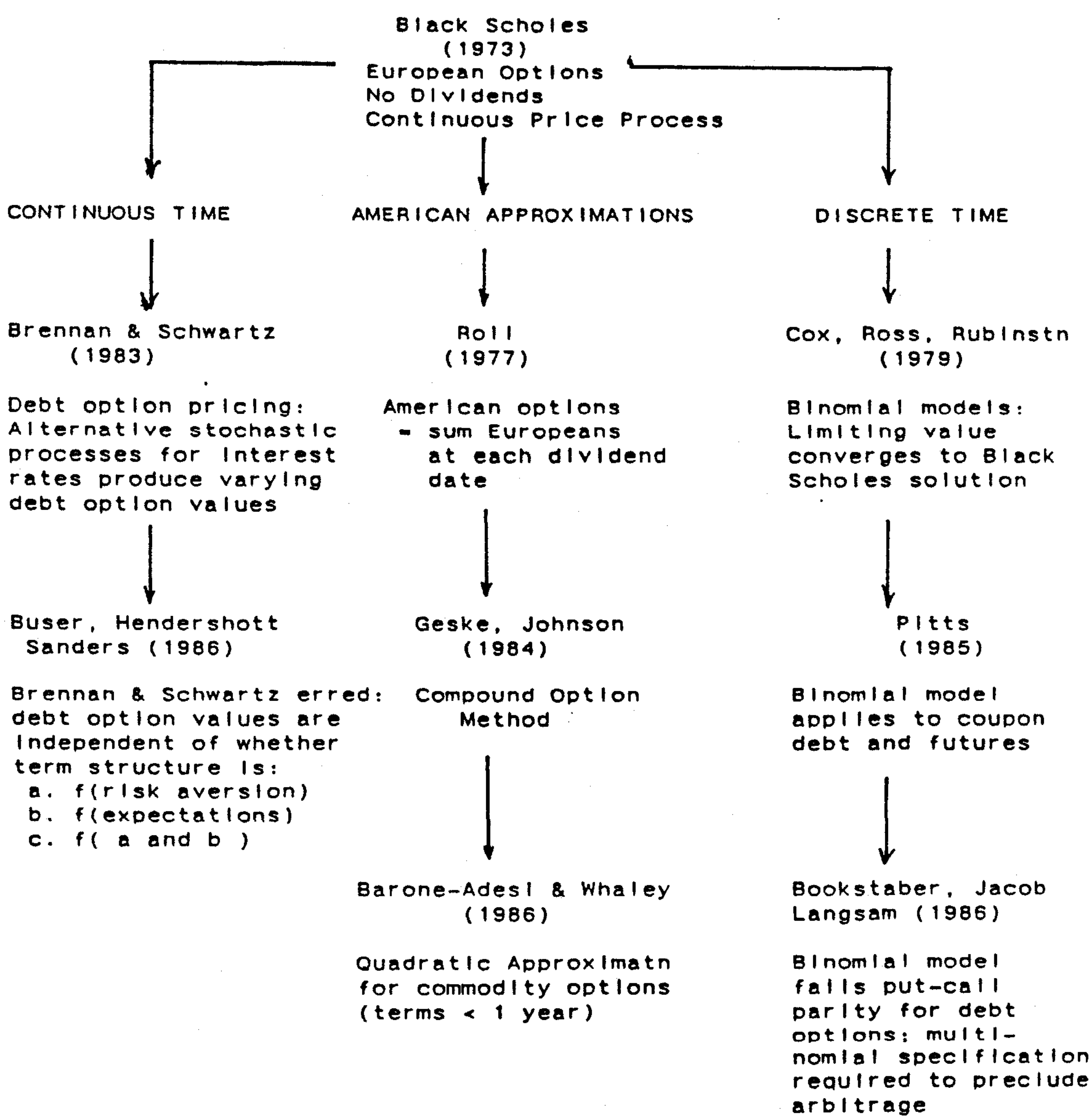


Figure 2-3

maturities of over ten years, the quadratic approximation is clearly inappropriate. The recommendation of Barone-Adesi and Whaley, as noted above, is to use the binomial model in these applications. However, the findings of Bookstaber, Jacob, and Langsam (1986) indicate that the use of the binomial model to price debt options in a multiperiod framework provides for arbitrage opportunities and is equivalent to assuming irrational markets. Therefore, the binomial model seems equally inappropriate for pricing long term debt options.

The remaining alternatives for pricing long term options seem to be limited to the use of the simple Black Scholes European model or to the use of finite difference methods to evaluate an expression for American options (perhaps one of the three tested by Whaley (1982).) The present research has chosen to employ the Black Scholes European model, citing two important reasons. First, empirical evidence on the accuracy of option pricing methods for long options is not complete. Many studies have found that the Black Scholes model is biased for long term options, including the famous MacBeth Merville (1979) study. Although Geske and Roll (1984) make convincing arguments that these biases are due to failures to address properly the American feature, there presently exists no empirical evidence to validate more sophisticated approaches. Indeed, we are unlikely to have such results in the near future. The study of long term options is complicated because they are generally not traded explicitly but instead are embedded in other financial instruments.

The second reason to use the Black Scholes European formula is that there is no evidence to indicate that the more costly finite difference method will produce better results. Kennedy (1986) states:

> "To anyone, but particularly economists, the extra benefit associated with chosing one estimator over another must be compared with its extra cost, where cost refers to the expenditure of both money and effort. Thus, the computational ease and cost of using one estimator rather than another must be taken into account whenever selecting an estimator."

This rationale weighs heavily in the decision to use the Black Scholes European model in the present research. Review of the literature indicates that similar thinking has likely motivated the same choice in other studies. The best that can be said for the present research is that the shortcomings of this model are duly identified and noted, whereas some others have tended to apply the model with less than complete qualification. The latter practice is sure to impede eventually both theoretical and empirical progress.

IV. MORTGAGE-BACKED SECURITIES: PREPAYMENT ESTIMATION AND VALUATION

A. Introduction

The purpose of this section is to review briefly the financial literature concerning mortgage-backed securities (MBS). This body of research may be conveniently divided into two sub-topics: prepayment estimation and valuation models. Accordingly, after a brief background discussion of the characteristics of GNMAs, a review of the prepayment articles is presented, followed by a consideration of various pricing models. The section concludes with a discussion of the implications of these studies for the present research.

B. Characteristics of GNMA Mortage-Backed Pass-Through Securities

GNMA mortgage-backed pass-through securities (henceforth GNMAs), as their name suggests, are securities backed by mortgage loans. In particular, the mortgage pools underlying GNMAs are made up of mortgage loans insured by the Federal Housing Administration (FHA) or guaranteed by the Veteran's Administration (VA). The Government National Mortgage Association (GNMA) is a wholly-owned U.S. government corporation. Consequently, GNMAs are backed by the full faith and credit of the U.S. government. The security holder is thus assured of receiving full and timely payment of principal and interest.

GNMAs are issued by many kinds of mortgage originators, including mortgage bankers, thrift institutions, and commercial banks. The underlying loans are fully amortizing, and each month the issuer is required to "pass-through" to the security holder all scheduled interest and principal, regardless of whether or not the issuer has actually received payment from individual borrowers. In addition, the issuer is required to pass through any additional principal received as loan prepayment or settlements from FHA or VA foreclosures. An additional noteworthy feature of the FHA and VA loans is that they are transferable. This means the sale of the property does not require the loan to be paid in full.

From an investor's standpoint, there is a fundamental difference between GNMAs and more conventional government and corporate debt. Unlike borrowers using conventional debt instruments, GNMA borrowers have the option to prepay (i.e. call) debt with little or no penalty. In effect, the purchase of a GNMA is equivalent to the simultaneous purchase of a pool of thirty year annuities and sale of call options against each annuity. Accounting for this implicit call feature greatly complicates any attempt to value GNMAs.

C. Prepayment Estimation

The economic function of the secondary mortgage market is to provide liquidity to the primary market through the purchase and resale of mortgage loans. Although it has its roots in the post-Depression era - FNMA (Federal National Mortgage Association) was chartered in 1938 - the secondary mortgage market was not a major contributor to capital market flows until after the creation of GNMA in 1968 and FHLMC (Federal Home Loan Mortgage Corporation) in 1970. Consequently, it is not surprising to find the academic literature devoid of prepayment studies prior to this time.

Curley and Guttentag (1974) were the first to publish research on estimating prepayment rates for MBS. They employed an ordinary least squares regression technique (OLS) to explain variation in the logarithm of the loan termination rate, where termination rate was defined as the sum of the number of loans which were prepaid or defaulted (and hence were prepaid) during the year divided by the number of loans outstanding at the beginning of the year. Three independent variables were used. The first variable accounted for the average difference between primary mortgage rates and the rate on the loan being observed over the year. The second variable was defined as the logarithm of the ratio of the age of the mortgage to its original term, and the third variable accounted for the average buyer discount points charged over the current year. CG reported coefficients for all three of the explanatory factors which were significant in the expected direction.

Peters, Pinkus, and Askin (1984) employed a similar method in work performed for FHLMC in the Merrill Lynch MBS Research Department. They used an OLS procedure to estimate prepayment rates on conventional (not FHA and VA loans, hence not transferable) mortgages purchased by FHLMC. PPA assigned mortgages to cohorts based on similarities in origination year, geographic region, and contract interest rate. Given this classification scheme, PPA estimated a linear regression model for the conditional prepayment rate (CPR) in year t for mortgage cohort j. Using the three variables chosen by CG (1974) plus nine others, they produced a model which explained 85% of the variation in cohort CPRs. (This model appears in the Appendix.)

Arak and Goodman (1984) estimated CPRs for GNMAs in a linear model expanded to include variables for the unemployment rate and the volume of home sales. Their results showed that these variables add little explanatory power to CPR projection models.

Navratil (1985) criticized the use of linear models in estimating prepayment functions. He noted that the financial incentives to increase prepayments are much larger when the current market rate is less than the current rate than are incentives to decrease prepayments when the reverse is true. In addition, Navratil observed that prepayment rates must be constrained to the range between zero and one. Linear and logarithmic models could produce predicted values outside this range. And finally, Navratil pointed out that mortgages whose contract rate is near the current market rate are likely to be much more sensitive to rate changes than are those which are distant.

On the basis of these arguments, Navratil proceeded to model the relationship between GNMA prepayment rates and interest rates in a nonlinear, asymmetric model which employs a logistic curve. He assumed factors other than interest rates exert a constant influence on prepayment rates. Although Navratil found, as expected, an inverse relationship between the contract to market rate differential and interest rates, his results are limited. The data base used in his study lumped together mortgages of varied ages and covered only one year.

In addition to OLS regression and logistic curves, proportional hazards models have been applied to the prepayment estimation problem. In a study of over 4,000 California mortgages, Green and Shoven (1986) computed a survival table for mortgages analogous to mortality tables computed by demographers. Specifically, GS computed the conditional probability of prepayment in year n+1 for a mortgage which has been outstanding n years. The sensitivity of this probability series to a proxy for the change in the contract to market rate differential was then estimated for the years 1975 - 1982. GS concluded that market interest rates are a significant determinant of prepayment probabilities. In addition, GS echoed the sentiments of Peters, Askin, and Pinkus (1984) and Navratil (1985) in stating that a major impediment to solving the prepayment estimation problem is a lack of data.

D. Valuation Models

Like the literature on prepayment estimation, studies on MBS valuation have a short history. Curley and Guttentag (1977) were the first to address the problem of pricing

GNMAs. They presented a model which incorporated the results of their work on prepayment estimation. At the time their work was published, and in fact for several years thereafter, it was common practice in the mortgage markets to price GNMAs assuming they generated level cash flows for 143 months (based on thirty year amortization) and then paid off completely in the 144th month. CG discarded this twelve year balloon cash flow stream. In its place they assumed cash flows produced by the prepayment probabilities they had estimated. They used simulation and sensitivity analysis to show that their model produced price estimates at variance with prices produced by the twelve year life assumption.

Dunn and McConnell (1981) extended the work of CG (1977) in a study designed to compare three alternative pricing models. The three models chosen for study included the twelve year average life model, the CG model modified to reflect aggregate FHA prepayment experience, and a contingent claims model based on the work of Cox, Ingersoll, and Ross (1978).

To construct the contingent claims model, DM assumed that the risk-free interest rate follows a stationary, mean-reverting Markov process described by the stochastic differential equation,

$$dr = k(R_{avg} - R_{t-1})dt + \sigma(R_{t-1})^{1/2}dz \qquad (8)$$

where R_{avg} is the long term, steady state rate mean;
R_{t-1} is the value of r in the previous instant;
k is a parameter describing the rate at which r converges to the steady state mean;
σ is the standard deviation of r; and
dz represents the standard Wiener process.

In addition, DM employed a Poisson process to model a suboptimal prepayment function (i.e. prepayments made when the contract rate is below the market rate.)

Given these assumptions, DM produced a numerical solution for the value of a GNMA. Using simulation and sensitivity analysis, they compared the performance of this model to the twelve year life and CG/FHA models under various assumptions regarding the shape of the yield curve. Generally speaking, DM concluded that the models produce different values, where the magnitude of the difference is larger for GNMAs priced farther from par and for GNMAs with a longer remaining term.

Although DM are to be commended for the theoretical elegance of their contingent claims model, their effort has two significant weaknesses. First, they concluded, on the basis of theoretical simulation results, that "the closer the security's price to par, the less sensitive is the price to changes in interest rate....[8]" This conclusion is inconsistent with the work of CG (1977) and recent market experience. Pinkus and Chandoha (1987) and others have shown in clear statistical terms premium GNMAs are the least sensitive to changes in interest rates and that the price volatility of GNMAs is **most** sensitive to changes in interest rates for GNMAs near par.

A second criticism of the DM (1981) study is that it fails to test the theoretical models against market prices. Lacking such a test, it is not possible to judge whether the contingent claims approach adds value.

The same observation applies to the theoretical advances reported by Brennan and Schwartz (1985), Hall (1985), and Kau, Keenan, Muller, and Epperson (1986). Brennan and Schwartz apply the results of their 1983 paper on pricing debt options to the GNMA valuation problem. As noted in Section III, the 1983 study examined the implications of three alternative stochastic processes for valuing debt options. In the latter study Brennan and Schwartz simulated the effects of these three process assumptions on the options implicit in GNMAs. Not surprisingly, they once again concluded that the choice of process is important. Recalling the criticism of Buser, Hendershott, and Sanders (1986), however, these conclusions do not seem irrefragable. Brennan and Schwartz concluded by suggesting that the next step is to compare price predictions of these models with actual GNMA market prices.

Clayton and Goldstein (1986) provided market evidence on the ability of a deterministic model to estimate FHLMC prepayments and durations. They valued FHLMCs in a simple present value framework, where expected cash flows from scheduled payments and prepayments are inferred from recent market prices. These flows are discounted at the current FHLMC thirty day forward commitment rate for thirty year fixed rate mortgages. Clayton and Goldstein justified this assumption by reasoning that all FHLMC MBSs have durations longer than one year and hence are priced on that part of the yield curve which is relatively flat.

The results of the Clayton and Goldstein study are illuminating. In tests of 125 FHLMC MBSs from July 1983 to June 1984, they were able to generate useful estimates of prepayment rates (CPRs) and Macaulay durations based on inferences made from market prices. Their major disappointment was the apparent inability of the market's expectations to account for seasoning (i.e. mortgage age).

Pinkus and Chandoha (1987) present an alternative approach to GNMA valuation. Their method considers the price volatility of GNMAs relative to the price volatility of the current ten year Treasury note. Specifically, in their model, GNMAs are assumed to exhibit price volatilities relative to a parity series of GNMA prices which are relatively stable over time when GNMAs are stratified by price level.[9] That is, PC assume that the regression,

$$\%<>G_n = B_1 \ \%<>G_p \qquad (30)$$

where $\%<>G_n$ refers to the percentage change in price of the GNMA being analyzed;

B_1 is a stable slope coefficient;

$\%<>G_p$ refers to the percentage change in price of the parity series.

accurately describes the relative price volatility of a given GNMA with the parity series. Further, PC assume that the parity series likewise exhibits a stable price volatility relative to the ten year Treasury. This relationship is expressed in the regression,

$$\%<>G_p = B_2 \ \%<>T_{10}$$

where $\%<>T_{10}$ refers to the percentage change in price of the ten year Treasury; and

B_2 is a stable slope coefficient.

When relative price volatility is taken as a proxy for duration, this framework may be used to compute the duration of a GNMA in a given price range as a function of the duration of the ten year Treasury, which is readily determined.

An example clarifies the method. An investor analyzing a GNMA priced at 87.5 refers to the regression slope coefficients estimated by Pinkus and Chandoha. The price volatility of this GNMA relative to the parity series, denoted by B_1, has been estimated at 1.14.[10] B_2, on the other hand, is estimated at 1.45. If the duration of the ten year Treasury is currently five years, then the implied duration of the GNMA is,

$$\begin{aligned} D_{G87.5} &= B_1 D_{GP} = 1.14\ D_{GP} = \\ &= 1.14\ (B_2 D_{T10}) = \\ &= (1.14)(1.45)(5.0) = 8.265 \text{ years} \end{aligned}$$

This method has recently gained popularity with practitioners. The regressions underlying the technique have, on the whole, generated relatively high r^2 values and T statistics. As a result, it holds promise for effective use in GNMA hedging applications and therefore is a natural candidate for use in this research.

E. Implications For The Present Research

The present research employs three approaches to GNMA valuation in estimating hedge ratios. The first approach is based on the model of Clayton and Goldstein (1986). It infers expected prepayments from current market prices. As described in Chapter III: Model Development, this model has been improved to account explicitly for mortgage age (seasoning). The results of Clayton and Goldstein suggest that this approach will provide useful duration estimates.

The second approach to GNMA valuation in this study assumes that the price sensitivity of a GNMA is equal to the sensitivity of its constituent parts. These parts are considered to be a long position in a default-free annuity and a short position in a call option written on the annuity. The sensitivity of the annuity is determined under the assumption of a flat term structure; the sensitivity of the call is valued using the Black Scholes European formula, as noted in Section III.

The third approach to GNMA valuation used in this study is based on the Pinkus-Chandoha method. It assumes a flat, horizontal term structure and constant credit and liquidity risk premia between the various GNMA price ranges and the ten year Treasury. Statistical testing of the PC specification provides encouragement that this method will provide accurate duration estimates.

None of these approaches to estimating mortgage duration employ the stochastic model of the term structure advanced so successfully in the theoretical literature by Cox, Ingersoll, and Ross (1978, 1985b). This choice seems warranted for two reasons. First, two extremely broad immunization studies - Lau (1983) and Bierwag, Kaufman, and Toevs (1983) - have recommended Macaulay duration in favor of the CIR measure. Second, we lack evidence demonstrating that the term structure is driven by a clearly identifiable, stationary stochastic process.

These three approaches to estimating GNMA rate sensitivity involve a second simplification. They both assume no variation in prepayments will occur as a result of change in factors other than interest rates. This assumption is supported by the precedents of Navratil (1985) and Green and Shoven (1986), who shared this focus on a single variable, interest rate differentials, and by the work of Peters, Pinkus, and Askin (1984) and Curley and Guttentag (1974), who found this factor to be very significant. This assumption also seems reasonable in the context of the relatively short hedging horizons used in the tests. More detailed consideration of these issues is presented in later chapters.

V. HEDGING

A. Introduction

The purpose of this section is to review that part of the hedging literature which is relevant to the present research. The section begins with an overview of the historical development of hedging theory (see Figure 2-4). Building from this base, it discusses the contributions of Ederington and others in what are classified as technical approaches to interest rate hedging. It is argued that these empirical techniques are inconsistent with rational, arbitrage-free markets, and are therefore not valid. These shortcomings suggest a fundamental approach to hedging, which is presented next. The section concludes with a summing up of the implications of the hedging literature for the present research.

B. Hedging Theory: Early Development

The establishment of the St. Louis Merchants Exchange in 1836 is generally regarded as the beginning of organized commodity trading in the U.S. The practice became more widespread with the opening of a corn exchange in Buffalo

HEDGING THEORY

An Overview

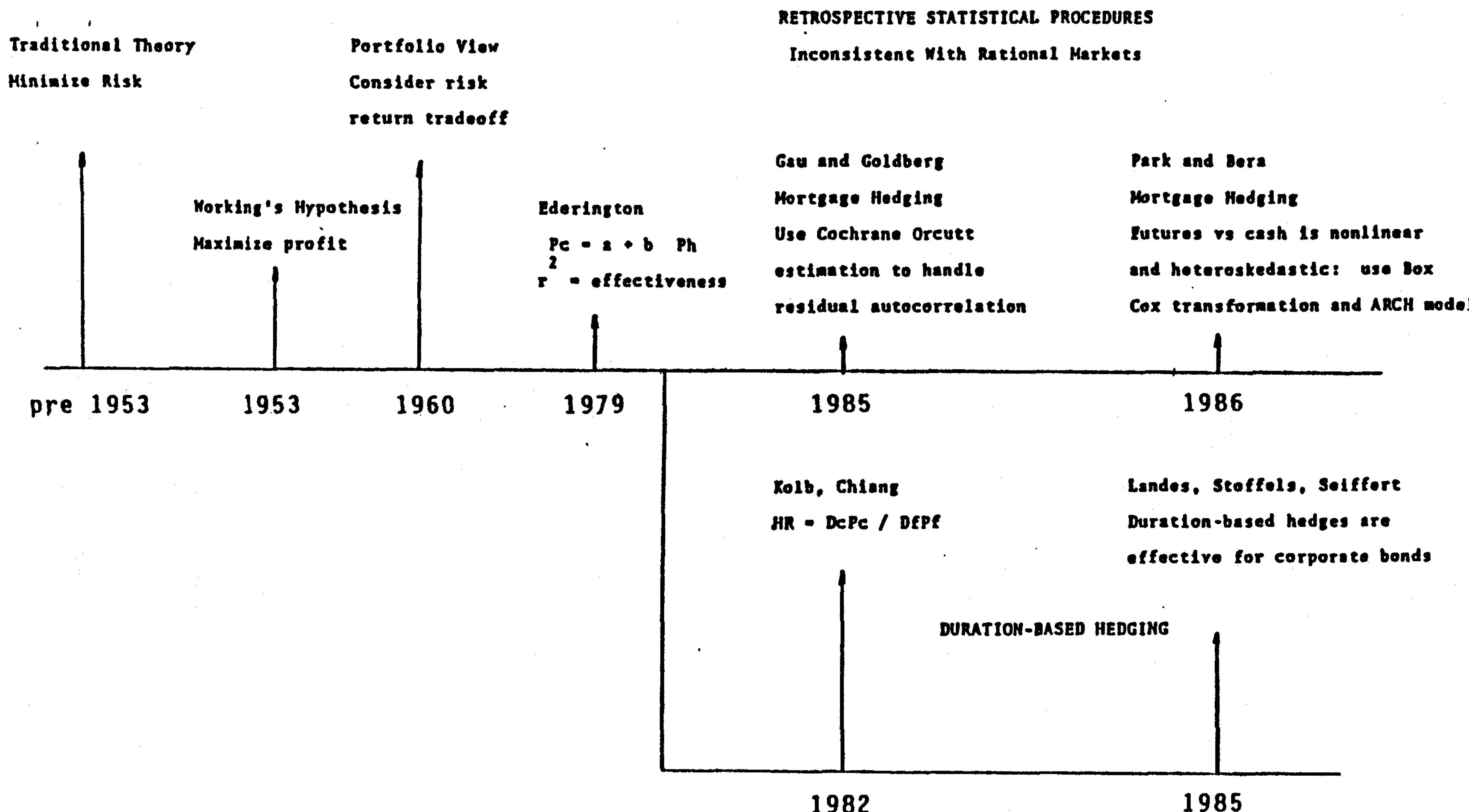


Figure 2 - 4

in 1844 and the Board of Trade in Chicago in 1848. The first trading in forward commodity contracts, precursors to commodity futures contracts, appeared at the Board of Trade in March 1851. [11] Trading in these contracts expanded rapidly as bankers began accepting them as collateral and as commodity prices became increasingly volatile as a result of the Crimean and Civil Wars.

Forward contracts were developed to serve a specific economic purpose. Farmers trying to sell crops which they had harvested and brought with them to Chicago found themselves increasingly at the mercy of commodity merchants. Forward contracts provided a means for farmers to transfer the risk of volatile crop prices to another party at a relatively low cost. Technological advances such as the McCormick reaper (1834), the Chicago - New York telegraph (1848), and the Illinois and Michigan canal (1848) were a boon to agricultural trade in Chicago in the second half of the 19th century. The volume of trading in forward contracts grew as a natural part of the exchange process.

The objective of many early participants in these markets was to hedge completely the risk of adverse commodity price changes by transferring this risk to other parties. As a result, traditional hedging theory, which grew from these roots, emphasized risk avoidance. It was based on the idea that spot and futures prices generally move together. A commodity owner could therefore protect the value of his holdings by selling them short in the futures market. Any subsequent loss (gain) on the commodity holdings would be exactly offset by a corresponding gain (loss) on the futures position.

The major assumption of the traditional theory is that the basis (i.e. the cash price less the futures price) does not change. The result is that hedgers are in effect substituting basis risk for price risk. Because basis risk is usually much smaller than price risk, the traditional theory was generally accepted as a reasonable explanation of the behavior of hedgers.

Holbrook Working (1953) challenged this view. He argued that hedgers were not pure risk minimizers but instead were rational profit maximizers. He pointed out that because hedgers held positions in the cash market, in reality they were concerned with relative (cash versus futures) prices, not absolute prices. In his view, hedgers do not assume an unchanging basis. Instead, they formulate expectations regarding the direction of the basis change and establish positions accordingly. For example, holders of a long position in the cash market would hedge if the basis is expected to fall (i.e. expected gains on the short futures position exceed expected losses on the cash) and would not hedge if the basis were expected to increase. According to Working, hedgers speculate on basis moves instead of price changes.

Johnson (1960) and Stein (1961) are credited with integrating the risk avoidance perspective of the traditional theory with the profit maximization framework of Working (1953). They argued that futures contracts are assets like any other and therefore are traded on the basis of the same risk and return considerations as other assets. This means that hedgers need not be fully hedged, as suggested by the traditional model. It suggests that hedgers make decisions regarding the extent to which their holdings should be hedged on the basis of expectations for price levels in the future and on the basis of their preferences for assuming risk. Thus, this approach, which has come to be known as the portfolio theory of hedging, offers an explanation for the existence of partial hedges, which were not admitted by the traditional view. At the same time, the portfolio approach allows the concept of risk avoidance to be used in describing the hedger's outlook, whereas Working's theory explained hedging behavior strictly in terms of profit maximization.

C. Interest Rate Hedging: Technical Methods

1. Review of Significant Research

Ederington (1979) applied the portfolio theory of hedging to derive a measure of hedging effectiveness and an optimal hedge ratio. His derivation began with expressions for the expected returns on a portfolio, E(R), and the variance of the portfolio, V(R):

$$E(R) = X_s E(P2_s - P1_s) + X_f E(P2_f - P1_f) - K(X_f) \qquad (9)$$

$$V(R) = X_s^2 \sigma_s^2 + X_f^2 \sigma_f^2 + 2X_s X_f \sigma_{sf} \qquad (10)$$

where $X_{s,f}$ represent the proportions of the portfolio in spot and futures holdings;

$K(X_f)$ represents all cost of futures transactions;

σ_s^2, σ_f^2, represent the variance of the spot and futures prices between times one and two;

$P2_s, P1_s, P2_f, P1_f$ represent the spot and futures prices at times one and two; and

σ_{sf} is the covariance of spot and futures prices over the period.

Defining $b = -X_f/X_s$, holding X_s constant, and differentiating V(R) with respect to b leads to an expression for the minimum variance hedge ratio,

$$b^* = \sigma_{sf}/\sigma_f^2 \qquad (11)$$

Using this framework, Ederington also developed a measure of hedge effectiveness,

$$e = 1 - V(R^*)/V(U) = \sigma_{sf}^2/\sigma_s^2\sigma_f^2 = p^2 \quad (12)$$

where $V(R^*)$ represents the minimum variance of the hedged portfolio and the variance of the unhedged cash holdings; and p^2 equals the coefficient of determination between the change in cash price and the change in futures price.

Ederington employed these expressions for optimal hedge ratios and hedge effectiveness in a study designed to compare futures markets for GNMA Collateralized Depository Receipts (CDRs) and Treasury bond futures with older futures markets for wheat and corn. Among other things, Ederington concluded that the older commodity markets were more effective in hedging price risk than the new markets in financial futures. In addition, Ederington demonstrated that the lynchpin of the traditional theory - the assumption of a constant basis - was entirely indefensible in hedging with these two financial futures contracts.

Ederington's work was published in March 1979. It soon became the fountainhead for a profusion of research in interest rate hedging. In October of 1979, the Federal Reserve revised its operating policy and began to focus on targets for monetary aggregates as critical policy variables. The result was a tremendous increase in the volatility of interest rates, which in turn led to a new emphasis on interest rate hedging in financial institutions of all types. Researchers responded to these circumstances with a wide variety of developments in duration, bond price volatility, and hedging. It is worth noting that most of the hedging studies which came out of this new research orientation applied the work of Ederington in calculating optimal hedge ratios and in measuring hedge effectiveness.

Franckle (1980), for example, showed that the hedging efficiency of the T-bill futures market exceeded that reported by Ederington when the optimal hedge ratio was properly adjusted to account for changes in the maturity of the cash instrument during the hedge period. Anderson and Danthine (1981) expanded Ederington's approach to consider multiple futures markets. Where Ederington had suggested the hedge ratio, b, be estimated from the simple regression,

$$<>P_s = a + b<>P_f \qquad (13)$$

where <>, delta, stands for "the incremental change in,"
a is a constant; and
b is the regression coefficient,

Anderson and Danthine suggested the estimation of multiple hedge ratios with the OLS multiple regression,

$$<>P_s = a + b_1 <>P_{f1} + b_2 <>P_{f2} + \ldots \quad (14)$$

This formulation enables the hedger to take advantage of more than one futures market. As is the case with the simple regression model, hedge effectiveness can be represented by the sample coefficient of determination, r^2, for the multiple regression.

Gau and Goldberg (1983) applied the methodology of Anderson and Danthine to the problem of hedging the interest rate risk of residential mortgages. The motivation for their study was provided by a dramatic increase in the volatility of mortgage interest rates in the U.S. From 1971 to 1975, the standard deviation of monthly interest rates on newly originated mortgages was .64%; during the period from 1976 to October 1979, it was 1.44%; and from October 1979 (i.e. the month when the Fed policy change was announced) through 1981, it was 1.87%.[12] GG noted that much of this increased rate risk had been shifted to reluctant households through the development of adjustable rate mortgage vehicles. They suggested that the futures markets might be a more efficient alternative for transferring this risk from the balance sheets of mortgage lenders.

Applying the insight of Anderson and Danthine, GG determined to hedge the rate risk of mortgages using the nearby futures contracts for the GNMA CDR and the Treasury bond. Initially, they estimated hedge ratios using the OLS regression,

$$<>P_m = a + b_1 <>P_{T\text{-}bond} + b_2 <>P_{GNMA} \qquad (15)$$

where $<>P_{m,T\text{-}bond,GNMA}$ refer to price changes for mortgages and the T-bond and GNMA futures;
a is a constant; and
$b_{1,2}$ are regression coefficients

Unlike some other hedging studies, GG tested these results for autocorrelation in the residuals, using Durbin Watson statistics. Finding it present in substantial measure, they repeated the regressions using the Cochrane-Orcutt iterative least squares technique. This approach eradicated the autocorrelation problem and produced more reliable hedge ratios. GG reported that only half of the rate risk present in mortgages could be eliminated with futures hedges.

While the application of Cochrane-Orcutt estimation to the hedge regressions was a significant contribution, the results reported by Gau and Goldberg had a major weakness. The mortgage "prices" used in their hedge simulations were not actual market prices. They were artificial estimates based on the twelve year life assumption (which was shown to be extremely deficient by Curley and Guttentag (1974)) and weekly FNMA auction rates. As such, they failed to reflect the true complexity of the mortgage market and cast considerable doubt on their results.

Gau and Markese (1985) repeated the Gau and Goldberg (1983) study using weekly FHLMC thirty year fixed rate mortgage quotes and futures data through 1983. They demonstrated again the utility of the Cochrane-Orcutt procedure. Unfortunately, this study is also limited by a lack of actual mortgage prices for use in the hedge simulations.

Park and Bera (1986) provided a more recent study of mortgage hedging. Their study began with a frontal attack on the conventional hedge regression estimation technique promulgated by Ederington (1979). Park and Bera noted that this technique has been prominent, having been used in important studies by Franckle (1980), Maness (1981), Cecchetti, Dale, and Vignola (1981), Hegde (1982), Gau and

Goldberg (1983), and Gau and Markese (1985). Notwithstanding the widespread use of the technique, Park and Bera advanced two significant criticisms.

At the heart of the first criticism is the work of Leibowitz (1984), who stated that while the cash price of a bond is dependent only on the yield level at one point on the yield curve (i.e. the point where the bond is located), the price of a futures contract is also a function of the cost of carry, which is dependent on the shape of the yield curve. Therefore, futures prices and cash prices may be expected to exhibit different volatilities. These differential volatilities produce heteroskedasticity in the regression residuals and reduce the reliability of the estimated hedge ratios.

The second criticism levied by PB is also rooted in the view that futures prices depend on the shape of the yield curve while cash prices do not. Specifically, PB suggested the possibility of basis risk and hence nonlinearity in the cash to futures price relationship. Thus, the linear relationship expressed in the conventional regression misrepresents the true hedge ratio.

PB suggested that the best approach to dealing with heteroskedasticity is to use an ARCH (autoregressive conditional heteroskedastic) model formulation. This model utilizes past errors to forecast changes in the variance of the disturbance term. In hedging simulations involving spot GNMA prices and GNMA futures prices, PB showed that the ARCH model produces more efficient estimates than the conventional OLS technique.

Similarly, PB asserted that a better approach to capturing the nonlinearity present in the futures to cash price relationship is to employ a Box-Cox transformation. Tests for the GNMA spot-futures relationship demonstrated significant nonlinearity over the period from September 1979 to January 1985.

In summing up, Park and Bera made a strong case in favor of replacing conventional OLS regression techniques with a combined ARCH, Box-Cox transformed formulation when estimating hedge ratios.

2. Estimating Hedge Ratios With Retrospective Statistical Procedures: A Criticism

Each of the studies examined in the previous subsection employed retrospective statistical procedures to estimate hedge ratios. These techniques may be appropriate in exploring the maximum amount of variance reduction which could be obtained with the use of futures contracts in a given futures market during a given period. They may also be appropriate for hedging securities which are uniquely deliverable into a given futures contract. However, they seem unsuited to the numerous institutional hedging applications where hedgers must construct hedges on the basis of information available at the outset of the hedging period. In particular, each of the retrospective statistical procedures discussed previously considered hedge effectiveness assuming the hedger knew the optimal hedge ratio which prevailed over the hedging period. The information necessary to determine the optimal hedge ratio is the price series for the cash and futures instruments over the hedging period. Since this information is not available at the beginning of the hedge period, the hedger is forced to choose a price series from a prior time period and assume that it will describe price movements in the hedging period ahead.

Taken from a broader perspective, these retrospective techniques seem very similar to what is commonly referred to as technical analysis in the valuation of equity securities.[13] Technical analysis in equities assumes that information contained in past price series can be used to predict future prices. It implies that superior investment returns can be earned through proper analysis of past price information. Unfortunately, weak-form market efficiency, which was advanced by Fama (1966) and has been subsequently supported by a large body of empirical research, contends that superior returns cannot be earned on the basis of information present in past prices.

Returning to the realm of interest rate hedging, the use of retrospective statistical procedures (i.e. technical hedging methods) requires the assumption that optimal hedge ratios are stable through time (i.e. the future will be like the past), an assertion widely refuted in the hedging literature. The assumption that the optimal hedge ratio is stable over time is equivalent to assuming that futures to cash price relationships can be accurately forecast in the future. This means that retrospective statistical procedures can be used to earn arbitrage profits whenever market price relationships deviate from predictions. To accept this assumption is to accept the idea that futures markets are irrational.

Viewed in this light, the regression and ARCH models used in the hedging literature seem unattractive. Indeed, much empirical evidence has been produced to show that futures markets are relatively efficient.[14] Fortunately, an alternative approach, which - to extend the analogy from equity valuation -is more fundamental in nature, has been developed.

D. Interest Rate Hedging: Fundamental Methods

Kolb and Chiang (1982) proposed the use of a hedge ratio based on Macaulay duration. They suggested that the ratio of the price sensitivity of the cash position to the price sensitivity of the futures position could be estimated as,

$$HR = D_c P_c / D_f P_f \qquad (16)$$

where D_c, D_f, represent Macaulay durations for cash and futures; and P_c, P_f represent prices for cash and futures positions.

This expression develops directly when the incremental change in the price of a cash or futures position is taken to be,

$$<>P_i = -D_i P_i <>y/(y) \qquad (17)$$

where y equals the yield to maturity plus one;
D equals the Macaulay duration of the position; and
P represents the price of the security.

Simple division of the change in the cash price caused by a change in interest rates, $<>y$, by the change in the futures price produces equation (17).

This method has been successfully employed in several hedging studies. Landes, Stoffels, and Seifert (1985) used it in a series of hedge experiments where Treasury bond futures contracts were used against several different portfolios of industrial and utility bonds. In the majority of their experiments, the ending value of the hedged position was within five percent of the initial value. They concluded that the duration-based technique provided substantial protection. Gay, Kolb, and Chiang (1983) reached similar conclusions in hedge experiments where Treasury bond futures positions were used against a random sample of corporate, utility, and transportation bonds.

E. Implications For The Present Research

The theoretical appeal and empirical support in favor of the Macaulay duration measure described in Section II combined with the favorable evidence provided by Gay, Kolb, and Chiang (1983) and Landes, Stoffels, and Seifert (1985) for its use in hedging applications recommend the use of a duration-based hedge technique. More detail concerning the estimation of hedge ratios is provided in Chapter IV, Model Development.

The hedging literature surveyed above represents but a small part of the whole. Many other studies have influenced important aspects of the present research. For the sake of clarity, these contributions are summarized and presented in relation to specific aspects of the hedging process in the following paragraphs.

1. Hedging Philosophy

The preceding subsection described the logic behind the choice of a duration-based hedge in fundamental terms. This choice is supported and refined by observations made elsewhere. Ahmadi, Sharp, and Walther (1986) noted that regression approaches require the arbitrary selection of a sample period. In research addressed to hedging foreign currency risk, they were careful to classify different sources of risk according to whether they involved contingent or obligatory claims. This classification has important implications for hedging strategy:

> "Furthermore, while options contracts and futures contracts are substitutable in the case of non-contingent claims, the use of futures to hedge contingent claims is speculative since the obligatory nature of the futures contract leaves the corporate treasurer exposed to exchange risk should the contingent claim not materialize. This lack of

> differentiation between different types of claims is usually not fully addressed in practical guides..[15].

Goodman (1986) made the same point in recommending that asymmetric risks be hedged with asymmetric payoffs (i.e. options positions) and symmetric risks be hedged with symmetric payoffs (i.e. futures positions).

2. Hedge Objectives

Overdahl and Starleaf (1986) pointed out that the correct choice of a measure of hedging effectiveness is dependent upon the hedge objective. Proper definition of the hedge objective is therefore important. From a broad perspective, Wardrep and Buck (1982) identified four fundamental approaches for reducing interest rate risk:

1. A maturity diversification strategy, designed to protect portfolio value against changes in the shape of the yield curve, but vulnerable to changes in the level of the yield curve;
2. A short term maturity concentration strategy, designed to protect portfolio value against upward shifts in the yield curve, but vulnerable to significant reinvestment risk when the yield curve shifts downward;
3. Immunization, a strategy which matches portfolio duration to the investment horizon, but which is

vulnerable to unanticipated changes in the shape of the yield curve; and

4. Hedging.

Toevs and Jacobs (1986) improved on this presentation with a scheme which classified hedges into one of four categories:

1. Weak Form Cash Hedge (Inventory Hedge): The cash position to be hedged is currently held, but the length of the hedge horizon is uncertain. The hedge objective is capital preservation on a day to day basis. The recommended strategy is a short position in financial futures contracts.
2. Strong Form Cash Hedge (Immunization): The cash position to be hedged is currently held, and the length of the hedge horizon is certain. The hedge objective is to assure returns equivalent to those which would be produced by a zero coupon bond of maturity equivalent to the hedge horizon. The recommended strategy is to modify portfolio duration, through the use of either a long position or a short position in financial futures contracts, to match it to the hedge horizon.
3. Weak Form Anticipatory Hedge: The cash position to be hedged is not currently held, but anticipated, and the length of the hedge horizon is uncertain. The hedge objective is to lock in current returns or prices at an

uncertain future date. The recommended strategy is to buy the futures contract expiring nearest to the estimated inflow date.

4. Strong Form Anticipatory Hedge: The cash position to be hedged is not currently held, but anticipated, and the length of the hedge horizon is certain. The hedge objective is to lock in current returns or prices at a known future date. The recommended strategy is to buy the futures contract expiring nearest to the inflow date.

This framework is well conceived. It makes plain that many of the debates in the literature have arisen from varying conceptions of the hedge objective. For reasons which are considered more carefully in Chapter III, the present research is classified as a study of an inventory hedge. The seven cash positions to be hedged are assumed to be $25 million GNMA portfolios with coupons of 8%, 9%, 10%, 11%, 12%, 13%, and 14%. The definition and simulation of the hedging periods is discussed at length in Chapter III.

3. Other Aspects

Other aspects of the hedging process integral to the present research include the specific techniques used in estimating hedge ratios and evaluating effectiveness. Discussion of these issues is deferred until later chapters because they are more closely related to the specific models used in this study than they are to the hedging literature in general.

VI. SUMMARY

The topic of hedging GNMAs involves the application of research from a broad spectrum of subjects, including the term structure of interest rates, bond portfolio management, option pricing, financial futures, valuation of mortgage-backed securities, and hedging. The studies reviewed in this chapter have several important implications for the present research:

DURATION: Empirical results, especially those of Lau (1983) and Bierwag, Kaufman, and Toevs (1983), recommend Macaulay duration as a measure of the sensitivity of bond prices to changes in interest rates.

OPTION PRICING: The quadratic approximation of American option values developed by Barone-Adesi and Whaley is well-suited to the task of estimating deltas for futures options.

The Black-Scholes European option model will be used to value the call option embedded in GNMAs. This model is recommended chiefly because the binomial model fails put-call parity for debt options and because there presently exists no persuasive empirical evidence to justify the expense of using finite difference methods to evaluate American formulas for long term debt options.

VALUATION OF MORTGAGE-BACKED SECURITIES: The second research hypothesis to be tested in the present study states:

EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY INCORPORATE THE EMBEDDED CALL OPTION ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT INCORPORATE THIS COMPONENT.

The Pinkus-Chandoha relative price volatility approach and a modified version of the Clayton Goldstein (1986) model will be used to estimate GNMA price sensitivities without explicitly accounting for the option component.

The option explicit approach will rely on a combination of the Black-Scholes European model and an annuity valuation determined under the assumption of a flat term structure.

None of these approaches to estimating mortgage duration employ the stochastic model of the term structure advanced so successfully in the theoretical literature by Cox, Ingersoll, and Ross (1978, 1985b). This choice seems warranted for two reasons. First, two extremely broad immunization studies - Lau (1983) and Bierwag, Kaufman, and Toevs (1983) - have recommended Macaulay duration in favor of the CIR measure. Second, there is a lack evidence demonstrating that the term structure is driven by a clearly identifiable, stationary stochastic process.

HEDGING: Empirical techniques which have dominated the literature on interest rate hedging are inconsistent with rational, arbitrage-free markets. Consequently, this research will rely on a "fundamental" approach to hedge ratio estimation based on Macaulay duration.[16]

These five conclusions serve as the conceptual foundation for the theoretical models developed in the following chapters.

CHAPTER II: FOOTNOTES

[1] A popular convention in the finance literature denotes authors by the first initial of their surname after the first appearance of their name in the text. The current study adopts this convention for expository convenience.

[2] See Van Horne (1984).

[3] A notable exception is the study reported by Ingersoll (1983).

[4] See Bierwag, Kaufman and Toevs (1983), page 30.

[5] These observations were made by Sharfman (1986).

[6] See Buser, Hendershott, and Sanders (1986), page 13.

[7] See Brennan and Schwartz (1977).

[8] See Dunn and McConnell (1981a), page 478.

[9] The GNMA parity series created by Pinkus and Chandoha was defined as a series of price changes of GNMAs trading in the range between 99.5 and 100. Because this narrow range produced a limited number of observations, the original series was augmented with price changes for the GNMAs trading closest to par on a given day and adjusted with simple regressions.

[10] This value is taken from the Pinkus Chandoha results.

[11] See Lebeck (1984), page 12.

[12] See Gau and Goldberg (1983), page 446.

[13] See Levy (1966).

[14] Kamara (1984) provides a concise review of the evidence.

[15] See Ahmadi, Sharp, and Walther (1986), page 174.

[16] The Pinkus-Chandoha method is actually a hybrid of the fundamental and technical approaches.

CHAPTER III: RESEARCH HYPOTHESES AND INSTITUTIONAL HEDGING CONSIDERATIONS

I. INTRODUCTION

The primary purpose of the present research is to analyze alternative strategies for hedging the interest rate risk of GNMAs. This chapter presents the specific research hypotheses to be explored in this study along with institutional hedging considerations relevant to testing the hypotheses. The hypotheses and the hedging considerations flow directly from the financial research reviewed in Chapter II.

These topics are considered in sequence, beginning with a description of the research hypotheses in Section II. Section III addresses the characteristics of the institutional setting used to frame the research. The details of this setting are important because they heavily influence the choice of the hedge objective and the choice of the measure used in evaluating hedge effectiveness. Section IV discusses the strategies to be used in testing the hypotheses. The chapter concludes in Section V with a summary which lays the groundwork for the development and application of the theoretical models in Chapter IV.

II. HYPOTHESES

The three research hypotheses to be tested in this study are:

1. THE INTEREST RATE RISK OF GNMA PASS-THROUGHS IS REDUCED BY THE USE OF DURATION-BASED HEDGING STRATEGIES WHICH EMPLOY FINANCIAL FUTURES AND OPTIONS ON FUTURES.
2. EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY INCORPORATE THE EMBEDDED CALL OPTION ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT INCORPORATE THIS COMPONENT.
3. STRATEGIES WHICH INCORPORATE DYNAMIC REBALANCING OF HEDGE POSITIONS OUTPERFORM STATIC APPROACHES.

Each of these hypotheses represents a generalization which is widely accepted by financial managers and institutional investors participating in the secondary mortgage market. The primary purpose of this study is to provide empirical evidence documenting or disproving their reliability.

III. SETTING

A. Introduction

To gather the empirical evidence sought in this research, certain choices are necessary to frame the analysis. This section describes and explains the choices which have been made in constructing an institutional setting suitable for testing the research hypotheses.

B. Motivation For Choosing An Institutional Setting

As noted in Chapter I, the topic of hedging GNMAs is relevant for financial managers across a broad range of institutions as well as for the consumers, pensioners, policy holders, and stockholders they serve. The interests of these financial managers, and therefore of the individuals they serve, are best represented in a real-world, institutional view of the hedging process. As a result, this study utilizes an institutional point of view in designing and evaluating hedging strategies.

Such a view differs significantly from the framework employed by Ederington (1979) and the many other financial researchers who followed him. As noted in Chapter II, many of these hedging studies relied on retrospective statistical procedures for estimating hedge ratios. While these techniques may be appropriate in exploring the maximum amount of variance reduction which can be obtained with the use of futures contracts in a given market during a given period, they seem decidedly less appropriate for institutional hedging applications. In a real-world institutional setting, hedges must be constructed only on the basis of information available at the start of the hedge period. In addition, hedgers must be concerned about the stability of the hedge ratio through time. Indeed, in real world applications, the value of a hedge is generally greatest when expected, historical price relationships are changing. These reasons have motivated the adoption of an institutional setting for designing and evaluating hedging strategies.

C. Characteristics of the Assumed Institutional Setting

The institutional setting used in this study is specified by six essential characteristics: 1) the cash position to be hedged, 2) the financial instruments used in the hedge position, 3) the hedge costs, 4) the hedge period, 5) the hedge objective, and 6) the technique used to measure hedge effectiveness. Each of these characteristics is discussed in turn.

1. The Cash Position

Seven cash positions are defined for testing of hedge strategies. Each of these positions is assumed to contain \$25 million of GNMAs of a single coupon rate. The seven coupons represented in this collection are the 8%, 9%, 10%, 11%, 12%, 13%, and 14% coupons.

The size of the cash position has been set at \$25 million to facilitate comparison of hedging strategies. Small cash positions require significant rounding of hedge ratios, which complicates the comparison of strategies. On the other hand, positions larger than \$25 million further reduce the effect of rounding, but are less common in real world portfolios, hence less representative of actual institutional problems. Thus, a cash position of \$25 million in size represents one which is small enough to make this research relevant to a wide range of practitioners yet at the same time is large enough to prevent undue and unrealistic influence of rounding errors.

An alternative to assuming a specific size for a cash position is to assume fractional hedge contracts are available. Such an assumption eliminates hedging errors due to rounding, and consequently seems attractive. In keeping with the intention to evaluate hedging strategies in a practical institutional setting, the present research rejects this alternative because it seems to represent an unacceptably large abstraction.

2. The Hedge Instruments

The hedging strategies employed in this study make use of financial futures contracts and options on these contracts. The specific futures contracts used are those written on U.S. Treasury notes, Treasury bonds, and ninety day Eurodollar certificates of deposit; the options used are puts written on T-bond futures. These instruments have been chosen because they are highly liquid and because they have acquired popularity as hedging tools in the financial markets.[1]

Other hedging vehicles, such as interest rate swaps and over the counter options on GNMAs, have been excluded. These vehicles are often unattractive as hedging instruments, for three reasons:

1. They are illiquid, meaning that transaction costs are high and flexibility consequently scarce;
2. They require direct, party to party negotiation, necessitating considerations of credit risk; and
3. Due to the private nature of the market, reliable pricing information is sometimes scarce. This lack of information implies that a relatively heavier commitment of resources is required for the

tasks of research and analysis.

Similarly, exchange-traded options on actual Treasury debt ("physicals") are excluded for a lack of liquidity. Taken together, these concerns reinforce the selection of the futures and futures options contracts described above.

3. Hedge Costs

Hedge costs are an important consideration in the management of hedging programs in institutional environments. Traditional **futures** hedges usually involve three major types of costs. The most important of these costs can be collected under the classification of transaction costs. **Transaction costs** include cash payments to cover brokerage commissions and the bid-ask spread on futures contracts. A less visible cost of futures hedges is the **opportunity cost** associated with posting performance margin (approximately $2,000 per contract). Although hedgers can post this margin in T-bills, opportunity costs are incurred to the extent that these assets may not be employed elsewhere. And finally, because futures contracts are marked to market on a daily basis, the hedger may experience **settlement costs** in the form of payments required **to settle** the hedge position accounts **daily**. Potential interest income on the funds used for this purpose is of course foregone.

Hedging with futures options likewise involves costs. The transaction costs arising from options hedges are similar to those for futures: they include commissions and bid-ask spreads. However, the margin costs involved with options hedges are quite different from those involved with futures. Market participants owning short positions in futures options are required to post initial margin and variation margin as required by the exchanges.[2] Participants holding long positions of course have no margin costs to consider: option premiums are paid in full when the positions are established.

Table 3-1 presents the numbers in the transaction cost assumptions assumed in the present research.[3] Other hedge costs are assumed to be negligible. This assumption is quite acceptable for the options hedges considered, since none of them entail short positions. This assumption is less appealing in reference to the futures hedges, however, where daily margin flows and the concomitant loss of interest income can be material for large positions. Some comfort is provided by the realization that reporting systems in institutional settings generally do not account for opportunity costs and interest income foregone. This realization and the fact that the measure of hedge effectiveness used in this study (explained below) accounts for daily margin flows makes this assumption seem fairly realistic.

TABLE 3-1: TRANSACTION COST ASSUMPTIONS
(Opening and closing transactions, quoted per contract in fractions of one percent of face value)

Instruments	Commissions	Bid-Ask
T-Note Futures	$12.50	2/32
T-Bond Futures	$12.50	4/32
Eurodollar CD Futures	$12.50	2/32
T-Bond Futures Puts	$12.50	4/64

4. The Hedge Periods

The hedge periods used in this study begin on the eighth business day of successive months and extend for three months, ending on the seventh business day of the fourth month. The eighth day was chosen as the starting point because "most" GNMA factor information is first available on the seventh business day of each month.[4,5] Using this format, twenty-one hedging periods are extracted from the available data, where the first period extends from April 11th through July 10th, 1985; the second period extends from May 10th through August 9th, 1985; and the remaining periods are defined in like manner, with the last (twentieth) period extending from December 10th, 1986 through March 10, 1987. The minimum number of trading days occurring in a hedge period is fifty-nine (periods ten and twenty), while the maximum is sixty-three. Table 3-2 presents a detailed listing of the hedge periods.

5. The Hedge Objective

In the terminology of Toevs and Jacobs (1986), a hedge against a currently held cash position for an uncertain period is a weak form cash hedge, or an inventory hedge. In this view, the proper objective for an inventory hedge is capital preservation on a daily basis.[6] This thinking applies to the present research, where the cash positions currently held are the $25 million GNMA portfolios described earlier. Following Toevs and Jacobs, the hedge objective used in this study is to preserve the initial value of the portfolio for an indeterminate number of days.

Describing the hedge period as uncertain seems contradictory to the choice of the fixed, three month hedging period described earlier, but the paradox is easily resolved. The length of the hedging period is assumed uncertain from the **standpoint of the hedger** initially constructing a hedge. The hedge period is artificially terminated after three months to simplify the process of measuring hedge

HEDGE PERIODS: April 1985 through March 1987

Hedge Period	Date (Start)	Day No.	Date (End)	Day No.	Period Length
1	4/11/85	68	7/10	130	62
2	5/10	89	8/9	152	63
3	6/12	111	9/11	174	63
4	7/11	131	10/9	194	63
5	8/12	153	11/12	216	63
6	9/12	175	12/10	235	60
7	10/10	195	1/10	256	61
8	11/13	217	2/11	277	60
9	12/11	236	3/11	296	60
10	1/13/86	257	4/9	316	59
11	2/12	278	5/9	338	60
12	3/12	297	6/10	359	62
13	4/10	317	7/10	380	63
14	5/12	339	8/11	402	63
15	6/11	360	9/10	423	63
16	7/11	381	10/9	444	63
17	8/12	403	11/12	466	63
18	9/11	424	12/9	484	60
19	10/10	445	1/12	506	61
20	11/13	467	2/10	526	59
21	12/10	485	3/10	545	60

TABLE 3 -2

effectiveness and comparing strategies. The intent of this artifice is to prevent effects arising from differential hedge period length from obscuring the relative performance of the strategies tested.

Therefore, because the hedger is unaware that the hedge is to be lifted after three months, he truly perceives the hedge period as uncertain. Consequently, the hedges studied in the present research are accurately described as inventory hedges, and the appropriate hedge objective is capital preservation on a daily basis.

It is worth noting that the emphasis on the uncertain hedging period distinguishes the present research from immunization experiments. Some "hedging" studies reported in the literature ignore this distinction.[7] They tend to employ fixed hedging periods and measures of effectiveness which depend primarily on cash/futures price relationships extant on the hedge termination date. As a result, reported effectiveness provides little information on the day to day performance of the hedge.

This point is illustrated in Figure 3-1. Plotted on the vertical axis is total portfolio value, which includes the initial price of the cash bonds times the face value of these bonds, the cumulative sum of subsequent changes in the value of the cash position, and the cumulative sum of

Measuring Hedge Effectiveness:

Implications of Fixed Horizons

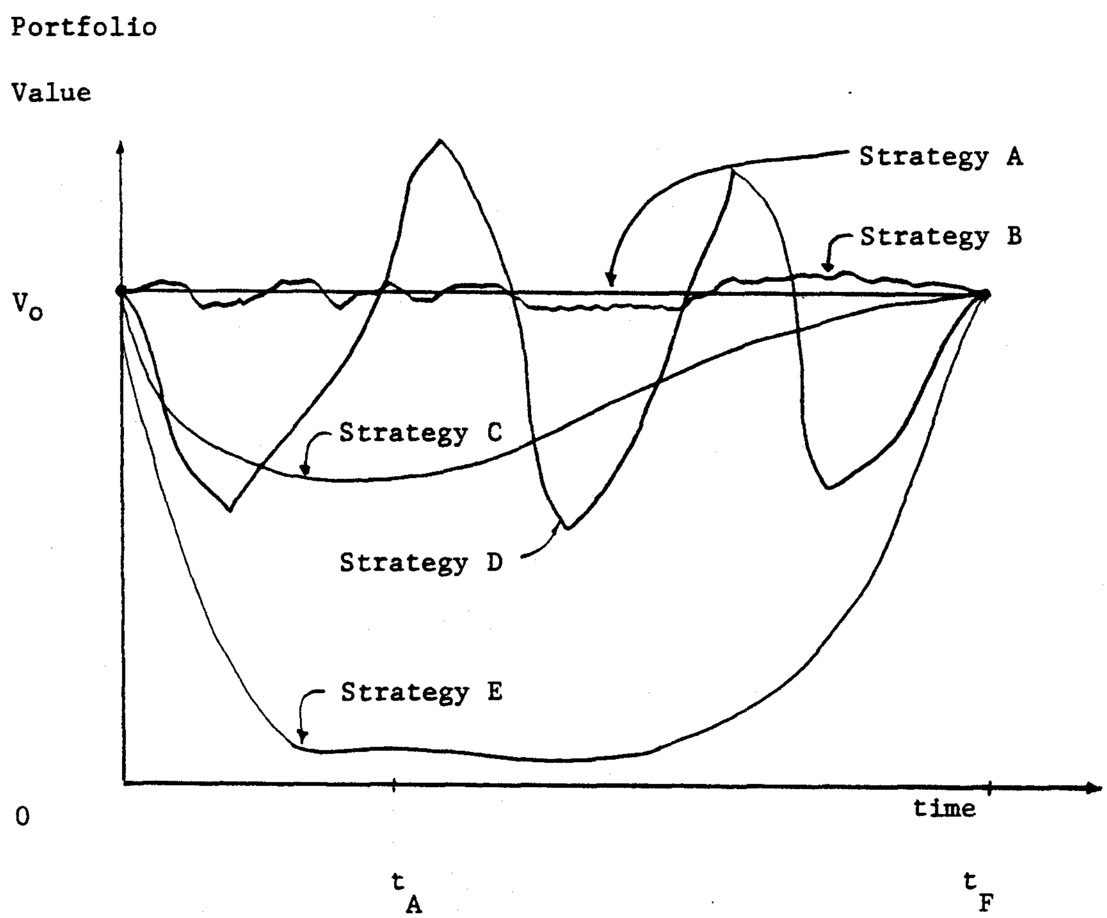


Figure 3 - 1

changes in the value of the futures position. Plotted on the horizontal axis is time, beginning at the start of the uncertain hedge period. Strategy A, as shown, represents a perfect hedge: the initial value of the portfolio is perfectly preserved on a day to day basis.

An interesting result develops when the five strategies identified in this figure are evaluated from the standpoint of a hedge horizon fixed at time t_F. In this framework, strategies A,B,C,D, and E are equivalent: each succeeds in "preserving" the initial value of the portfolio on the hedge termination date.

Clearly, in an institutional setting with an uncertain hedge period, these strategies are not equivalent. Were the hedge to be terminated at time t_A, for example, strategies D and E would produce large losses. In general, it is apparent that these strategies are materially different in terms of how they perform on a day to day basis. Thus, the assumption of an uncertain hedging period has implications for choosing both a hedge objective and a measure of hedging effectiveness which differ markedly from those associated with immunization applications where the hedge period is certain.

6. Measuring Hedge Effectiveness

Overdahl and Starleaf (1986) correctly note that the appropriate choice of a measure of hedge effectiveness is dependent on the choice of a hedge objective. Given the hedge objective of preserving the initial value of a cash position on a daily basis, the present research introduces a measure of effectiveness defined as,

$$K = [SDD(U) - SDD(H)]/SDD(U) = 1 - SDD(H)/SDD(U) \quad (1)$$

where SDD(U), SDD(H) represent standard daily differences, calculated for the hedged and unhedged portfolios, respectively, as follows:

$$SDD = \{ \sum_{t=1}^{N} (V_t - V_0)^2)/ (N-1) \}^{1/2} \quad (2)$$

where E refers to the sum of the terms following from t=1 to N;

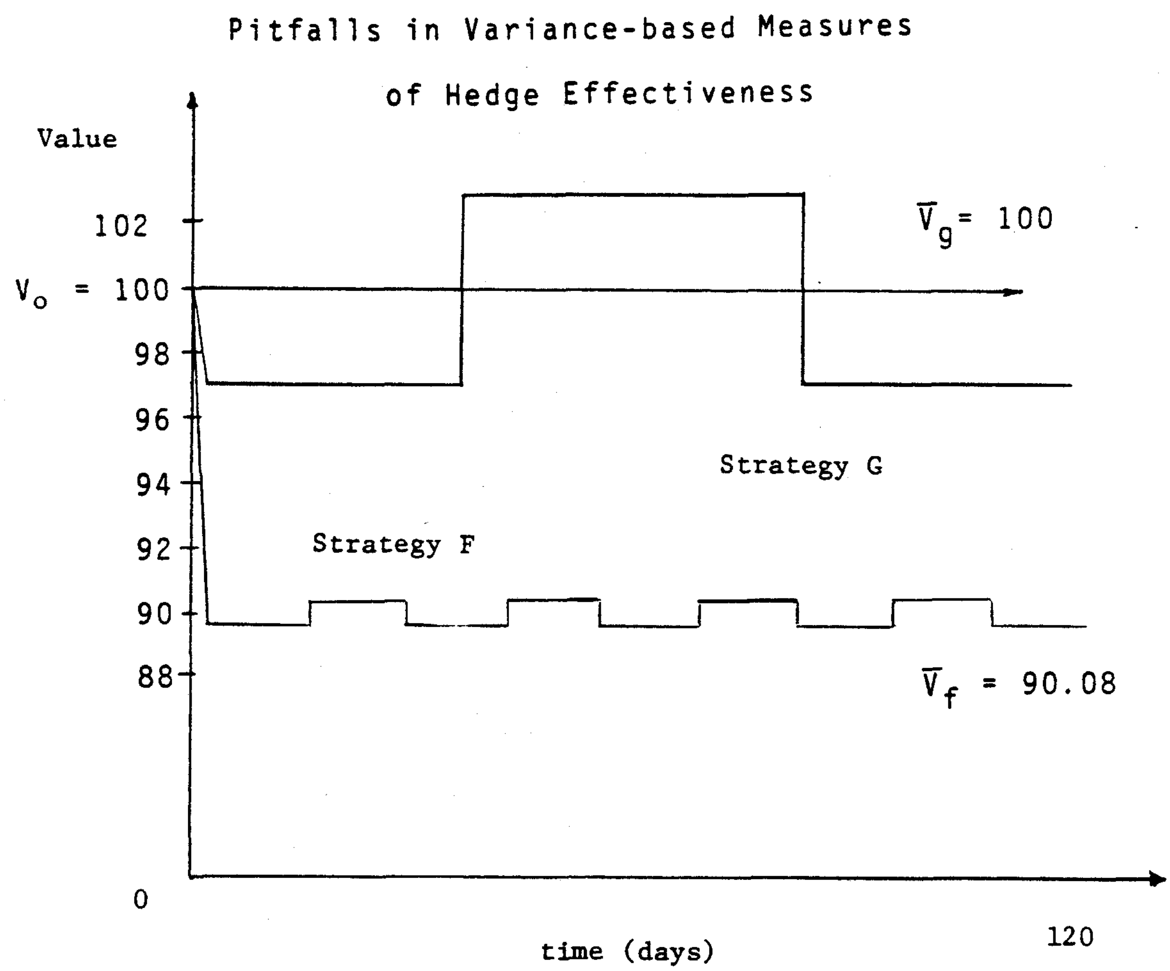
V_t , V_0 refer to the value of the portfolio at time t,0; and

N refers to the number of days in the hedging period.

Intuitively, this measure of effectiveness (henceforth K) evaluates hedges in terms of their ability to minimize the standard deviation (and correspondingly, the variance) of daily portfolio values from the **initial** portfolio value. The term standard daily difference has been coined to emphasize the distinction between this measure and others which are based on the traditional concept of minimizing variance. The root of the difference in these two approaches lies in the fact that variance-based measures consider deviations from the **mean** portfolio value, whereas K considers deviations from the **initial** portfolio value. The two approaches are equivalent only in the case where the mean value of the portfolio over the hedging period is identical to the initial value.

This distinction can be clarified with an example. Figure 3-2 shows results for two hypothetical hedging strategies, F and G. An effectiveness measure based on variance would favor strategy F, despite the fact that, from an institutional point of view, strategy G exhibits dominance with respect to the stated hedge objective for each day in the hedge period.

In adopting the measure K, the present research discards the strict variance-minimization approach popular in the literature.[8] The main advantage in using K is that strategies which minimize variation about the initial portfolio value are distinguished from those which minimize variation about the average portfolio value. This arrangement is appropriate because, in an institutional setting, average portfolio values substantially below the initial value are undesirable. Hence, conventional



$$Var(G) = \sum_{1}^{120}(V_t - \bar{V})^2/N = 9.00$$

$$Var(F) = \sum_{1}^{120}(V_t - \bar{V})^2/N = (\sum_{1}^{119} .4^2 + 10)/120 = .24$$

Figure 3 - 2

variance minimization is not as important as minimizing variation from the initial portfolio value. This result follows directly from the stated hedge objective.

The K defined for use in this study has other virtues. Foremost among them is its use of squared deviations from the initial portfolio value (V_o). In contrast to alternative measures based on mean absolute deviations from V_o, K stresses outlier values. Once again, this emphasis is appropriate for institutional hedging applications where hedge reliability is important.

On the other hand, at least three criticisms are likely to be advanced against K. First, it penalizes strategies which increase portfolio value in comparison to those which simply maintain it. While this may seem to be undesirable in an institutional setting, it is nonetheless appropriate. This result follows because hedgers by definition and stated objective have determined to avoid speculation.

A second criticism evolves because K may in some cases favor a strategy for which ending portfolio value, V_f, is further from V_o than for an alternative. This possibility is illustrated in Figure 3-3. Although V_{Lf} is identical to V_o while V_{Kf} is significantly below it, strategy K is preferred when K is used to measure effectiveness. This

Using K To Measure Hedge Effectiveness:

An Anomaly

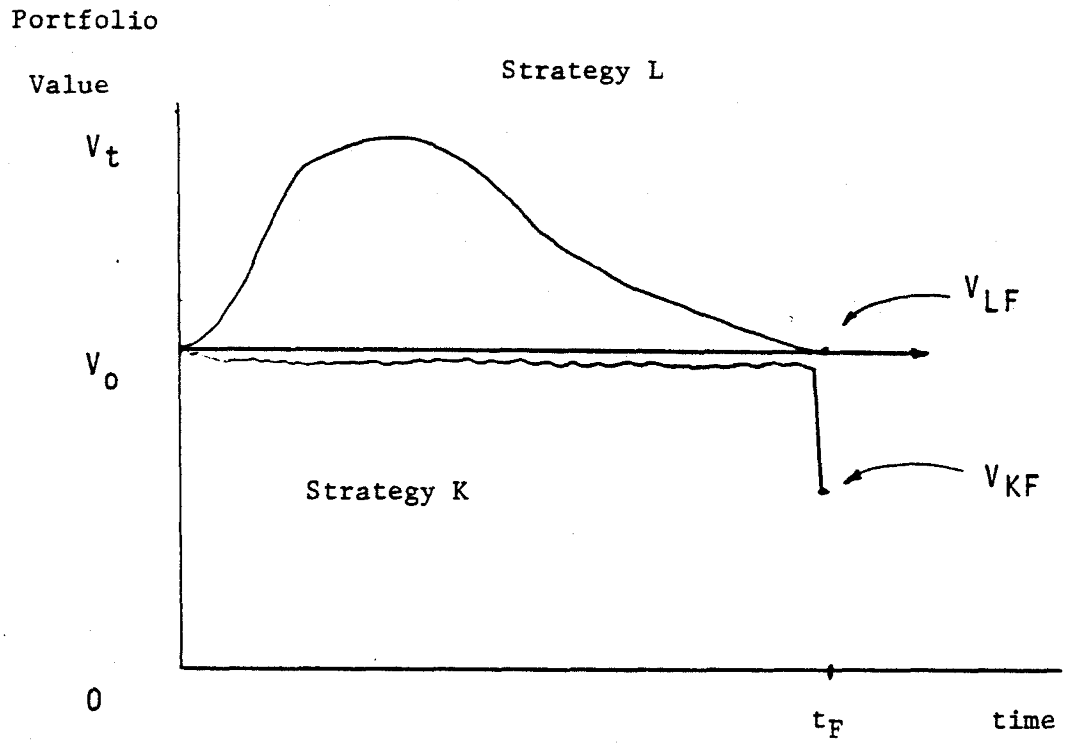


Figure 3 - 3

result seems incongruous.

Careful consideration of the hedge objective reveals why this second criticism is inconsequential. The hedge objective emphasizes capital preservation on a daily basis. This choice is made because the hedge horizon is uncertain. As a result, the logical preference for the hedger lies in favor of the strategy which on **average** results in a portfolio value closest to V_o. This approach unfortunately admits the possibility that occasionally a strategy producing a V_f further from V_o may be preferred to one producing a V_f close to V_o. Nonetheless, it is correct.[9]

Finally, the use of K may be criticized because it is not simple and intuitive. To mitigate this problem, and to provide a sense of perspective, the present research supplements the use of K with five other measures: Gmax, Lmin, G/Vo, L/Vo, and E. Gmax is simply the largest single day gain in portfolio value which occurs during the hedge period under study. Lmin, in parallel fashion, is the largest single day loss occurring during the period. G/Vo and L/Vo are simple ratios of these extreme changes to the initial value of the portfolio, $25 million. E is the coefficient of determination for the regression of cash and futures price changes as developed by Ederington (1979). These five measures, taken together with K, provide much information on the effectiveness of the hedging strategies under study.

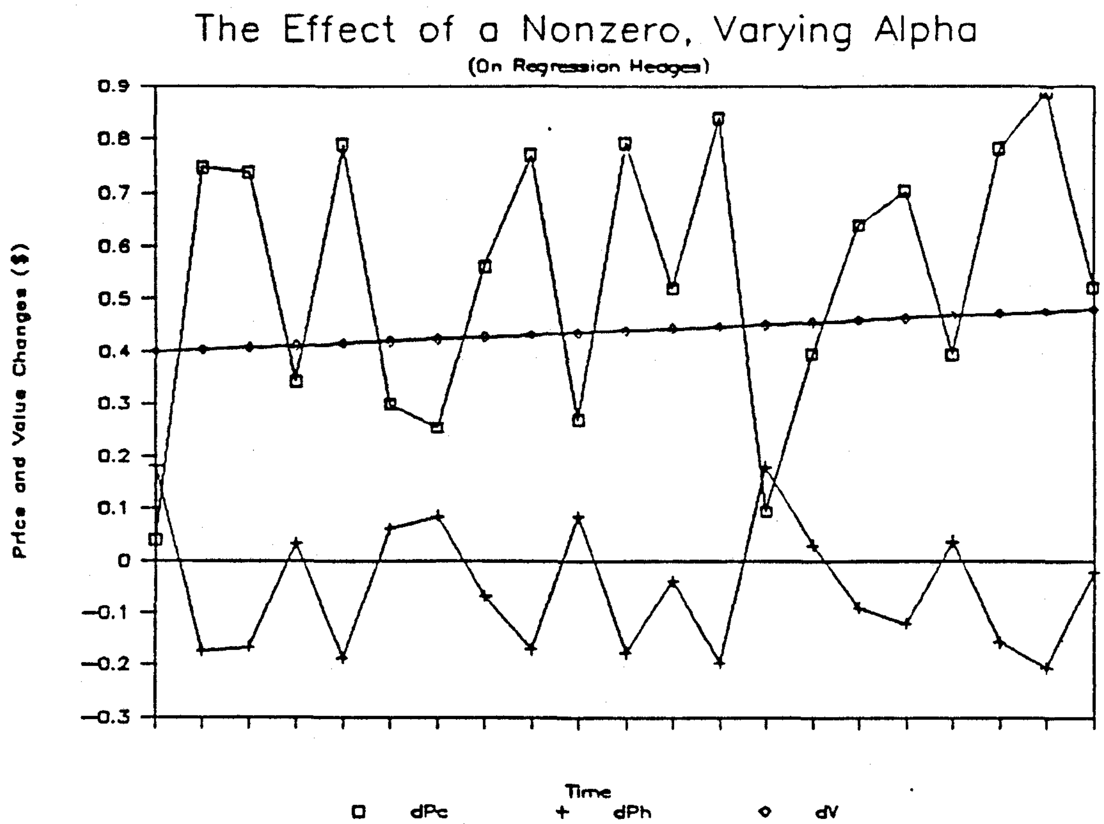
Before moving to a discussion of the hedging strategies, a final word of perspective on measures of hedging effectiveness is warranted. As noted in Chapter II, many studies since Ederington (1979) have estimated variance minimizing hedge ratios by calculating the sample coefficient of determination, r^2, for the regression,

$$<>P_c = a + b_1 <>P_h \qquad (3)$$

where $<>P_c$, $<>P_h$ represent the daily price changes in the cash and hedge positions.

This measure is plagued by two problems. Already noted is the difficulty arising from the implicit assumption that the historical price series represent accurately the price relationships expected for the (future) hedge period. A more subtle problem can develop when a, the alpha term, varies or is nonzero.[10]

Table 3-3 presents an artificial data set constructed to highlight this problem. Twenty-one random numbers between zero and one have been generated to represent the daily price changes in the cash position, $<>P_c$. Daily price changes in the hedge position, $<>P_h$, are **calculated**



Regression Hedging: The Effect of a Nonzero, Varying Alpha Term

dPh = a +(-.5)dPc (The linear equation DEFINING dPh)

dPc = a +(b)dPh (The artificial regression equation)

Value Change = dPc + 2dPh

Growth Index	Pcash Change	Alpha (a)	Phedge Change	(dV) Value Change
1.00	0.2760	0.2000	0.0620	0.4000
1.01	0.7780	0.2020	-0.1870	0.404
1.02	0.9597	0.2040	-0.2759	0.408
1.03	0.3550	0.2060	0.0285	0.412
1.04	0.0536	0.2080	0.1812	0.416
1.05	0.1001	0.2100	0.1600	0.420
1.06	0.2587	0.2120	0.0826	0.424
1.07	0.3389	0.2140	0.0446	0.428
1.08	0.5851	0.2160	-0.0766	0.432
1.09	0.3469	0.2180	0.0446	0.436
1.10	0.8849	0.2200	-0.2225	0.440
1.11	0.1131	0.2220	0.1655	0.444
1.12	0.2839	0.2240	0.0821	0.448
1.13	0.5439	0.2260	-0.0459	0.452
1.14	0.8273	0.2280	-0.1857	0.456
1.15	0.6131	0.2300	-0.0765	0.460
1.16	0.8684	0.2320	-0.2022	0.464
1.17	0.9907	0.2340	-0.2614	0.468
1.18	0.7105	0.2360	-0.1192	0.472
1.19	0.8292	0.2380	-0.1766	0.476
1.20	0.8400	0.2400	-0.1800	0.480

Regression Output:

(a)	Constant	0.2175
	Std Err of Y Est	0.0126
	R Squared	0.9947
	No. of Observations	21
	Degrees of Freedom	19
(b)	X Coefficient(s)	-0.49458
	Std Err of Coef.	0.008252

Table 3 - 3

according to,

$$<>P_c = a + (-.5)<>P_h \qquad (4)$$

The alpha term, a, in this linear equation is defined to be one on the initial day of the twenty-one day hedge period. Thereafter, it grows by .01 per day.

Simple regression analysis of the price series related by (4) produces an r^2 very near to one (the actual value is .9947), indicating a near-perfect hedge. However, a review of the daily changes in portfolio value, dV, and the total changes in portfolio value, DV, indicates that the hedge is far from perfect. As revealed by Table 3-3, a nonzero alpha term can result in substantial deviation from initial portfolio value. Furthermore, these deviations may vary in proportion to the variation in alpha.

This finding is important. It means that the regression hedging technique is flawed even in cases where the assumption of a stable price relationship is satisfied. The use of E (r^2) as a measure of effectiveness has nonetheless been included in this study to provide perspective on the value added by the use of the less familiar K.

D. Summary

The present research evaluates hedging strategies from a real world, institutional point of view. The principal characteristics of the assumed setting are:

1. Cash position: Seven different GNMA portfolios are constructed for independent testing. Each portfolio is assumed to contain $25 million of a single coupon, where the coupons represented are the 8%, 9%, 10%, 11%, 12%, 13%, and 14%.

2. Hedge position: The hedge positions are constructed using the T-note, T-bond, and ninety day Eurodollar CD futures contracts and T-bond futures put options.

3. Hedge costs: All hedge costs are assumed to be included in transaction costs.

4. Hedge period: The hedge period is assumed to be uncertain from the standpoint of the hedger. However, from the standpoint of measuring effectiveness, alternative hedging strategies are evaluated over similar three month hedging periods. The intent of this regimentation is to prevent effects which might arise from differential hedge period length from obscuring the relative performance of the strategies tested.

5. Hedge objective: The objective of the hedges constructed in this study is to preserve the initial

constructed in this study is to preserve the initial value of the cash position.

6. Measuring hedge effectiveness: Based on the hedge objective, the appropriate measure of hedge effectiveness is,

$$K = [SDD(U) - SDD(H)]/SDD(U) = 1 - SDD(H)/SDD(U)$$

This measure is appealing because, unlike conventional, variance-based measures which consider variation about the average value of the portfolio, this measure evaluates hedges in terms of their ability to minimize the standard deviation and variance of daily portfolio values from the initial value. In short, it does a better job of measuring how well a given strategy performs in relation to the hedge objectives than variance based measures do.

IV. STRATEGIES

A. Introduction

This section describes and explains the hedging strategies tested in this research. The development of the strategies is closely related to the hypotheses presented in Section II. In addition, this section discusses a number of alternative strategies which were not selected for testing, along with the reason for their exclusion.

B. Hedging Strategies

Specifying hedge strategies can be reduced to the problem of specifying which contracts should be bought (or sold) and in what quantity. The decision regarding quantity is generally framed in terms of the hedge ratio, which is discussed in detail in Chapter IV. The decision regarding which contracts are to be used (long or short) is a matter of conceptualizing the pricing characteristics of the cash position. That is, to hedge the interest rate risk of a cash position, one must simply build a hedge position with a price elasticity (interest sensitivity) equal in magnitude but opposite in sign to that of the cash position. Consequently, the design of a hedging strategy reflects assumptions about the manner in which the cash position responds to changes in interest rates.

The four strategies to be tested in this study are:

1. Assume a short position in Treasury note futures contracts.
2. Assume a short position in Treasury bond futures

contracts.

3. Assume short positions in Treasury bond futures and Eurodollar CD futures contracts simultaneously.

4. Assume long positions in put options on Treasury bond futures contracts.

Each of these four strategies is based on a different view of the pricing characteristics of GNMAs.

Strategy 1 is an approach which has been widely used by market participants. It assumes that Hypothesis 1 (the interest rate risk of GNMAs can be reduced) is true and that Hypothesis 2 (explicit consideration of the options embedded in mortgages improves hedge effectiveness) is false.

Strategy 2 is identical to Strategy 1 in terms of the assumptions it makes regarding Hypotheses 1 and 2. It differs only in the use of Treasury bond futures instead of the Treasury note futures. Generally speaking, T-bond futures have longer durations and more convexity than the T-note futures. Testing this strategy and comparing the results to those generated in testing Strategy 1 sheds light on the value added by hedging strategies which consider convexity.

Strategy 3 is built on similar assumptions regarding

Hypotheses 1 and 2. It is modified in two important respects. First, the combination of Euro futures and T-bond futures permits the construction of a hedge which has a duration which closely approximates the duration of the cash position. This matching stands in contrast to Strategies 1 and 2, where unequal cash and hedge durations must be addressed in adjustments to the hedge ratio. (Chapter IV provides more detail on this subject.) Second, the combination of Euros and T-bond futures permits a partial diversification of basis risk.[11] This diversification arises because Treasury yield and Eurodollar CD rates (LIBOR) are not perfectly correlated - each market has unique sensitivities to a wide variety of fundamental and technical factors. This modification seems warranted because the experience of some market participants has shown that basis risk can have a material, adverse effect on hedges employing Strategies 1 and 2.

Strategy 4 is based on assumptions which differ from those underlying Strategies 1, 2, and 3. It assumes that Hypothesis 2 is correct. In effect, it recognizes the short call positions implicit in the GNMA securities. Following Goodman (1986), Ahmadi, et. al. (1986), and Jones (1987), Strategy 3 seeks to hedge this asymmetric risk, or contingent claim, with an asymmetric, or contingent payoff: a long position in put options on

T-bond futures.

The logic behind this strategy is straightforward. The short calls embedded in the GNMAs remove much of the potential for price appreciation in premium GNMAs. As a result, a hedge using futures - which are fully obligatory claims, not contingent claims - will fail in circumstances where market rates fall significantly. This failure will develop as losses arising from the short futures position during a rally are not fully offset by gains in the cash position (i.e. the short calls place a cap on cash position gains). A better approach, according to this view, is to hedge the GNMAs by purchasing puts. These puts will produce gains in the hedge position needed to offset losses in the cash during bear markets. At the same time, the puts will produce no losses in the hedge position during market rallies. Hence, the failure of the GNMA cash position to generate significant gains beyond par in bull markets does not destroy the hedge.

C. Delivery Month Selection

The four strategies enumerated in the previous section have been defined with respect to the type of contract to be used in each hedge. Further definition is necessary, however, because there exists a multiplicity of delivery months for each futures and option contract cited. In addition, for Strategy 4, choices must be made among the various strike prices available in each option expiration month.

Table 3-4 shows the delivery months which have been chosen for each of the four strategies. These choices reflect several concerns. First and foremost, these choices reflect a decision to employ stack hedges (all contracts used in the hedge are "stacked" in one delivery month) in favor of strip hedges (contracts used in the hedge are dispersed among available delivery months.) The primary motivation for this decision is to avoid further complicating an already complex set of issues. In particular, by stacking the hedge contracts in a delivery month beyond the hedge horizon, the necessity of rolling over the hedge contracts during the simulation of hedge periods is avoided.[12]

A second important consideration in selecting delivery months is liquidity. While the choice of distant delivery months obviates the task of rolling over hedges, it raises the possibility that a low degree of liquidity may be encountered for some contracts. This possibility is of special concern in a simulation. By construction, in a simulation, hedge transactions are assumed to occur at the last reported settlement prices on each day in the data

FUTURES DELIVERY MONTHS

Hedge Period	T-note Futures	T-bond Futures	EURO CD Futures	T-bond Futures Puts	Strike Price
1	9/85	12/85	12/85	9/85	68
2	9/85	12/85	12/85	9/85	70
3	9/85	3/86	3/86	12/85	76
4	12/85	3/86	3/86	12/85	76
5	12/85	3/86	3/86	12/85	74
6	12/85	6/86	6/86	3/86	72
7	3/86	6/86	6/86	3/86	72
8	3/86	6/86	6/86	3/86	78
9	3/86	9/86	9/86	6/86	80
10	6/86	9/86	9/86	6/86	82
11	6/86	9/86	9/86	6/86	84
12	6/86	12/86	12/86	9/86	96
13	9/86	12/86	12/86	9/86	102
14	9/86	12/86	12/86	9/86	98
15	9/86	3/87	3/87	12/86	90
16	12/86	3/87	3/87	12/86	98
17	12/86	3/87	3/87	12/86	98
18	12/86	6/87	6/87	3/87	94
19	3/87	6/87	6/87	3/87	94
20	3/87	6/87	6/87	3/87	96
21	3/87	9/87	9/87	6/87	98

TABLE 3 - 4

base. Hedging results are valid to the extent that trades could have actually been consummated at the reported settlement prices. In markets or instruments with a low degree of liquidity, violations of this assumption can create serious problems.

This line of reasoning has played an important role in the selection of delivery months for T-note futures and selection of T-bond put strike prices. The point is clarified with an example. The third hedge period in this study spans the period from June 11, 1985 to September 11, 1985. A hedge using T-note futures against $25 million GNMA 8's involves over 200 contracts. The number of December T-note futures contracts traded on June 11 was only 38, while the number of September contracts was 8,246.[13] Rather than assume that the market could have absorbed a five- to six-fold increase in volume in the December contract at the settlement price, a better choice is to elect to use the September contract in the hedge.

This type of informal liquidity analysis has been applied in developing a systematic set of rules for selecting delivery months and strike prices for all hedge instruments. The rules can be summarized simply:

T-notes: select the second earliest delivery month available at the outset of the hedge period.

T-bonds and Euros: if the hedge period begins in the delivery month, select the fourth earliest delivery month available; otherwise select the third earliest delivery month.

T-bond puts: if the hedge period begins in the delivery month, select the third earliest delivery month available; otherwise select the fourth earliest delivery month; and select the strike price closest to the at-the money level of the most recent cheapest to deliver bond.[14]

In addition to meeting the aforesaid requirements for liquidity and roll-over avoidance, these rules add a measure of consistency helpful in comparing the performance of the various hedging strategies which would not be present had the delivery months been selected in a less uniform manner.

D. Dynamic Hedging

The preceding sections are addressed to the problem of deciding which contracts to use in the hedge and in what quantity. Implicit in each of the strategies considered is the notion that the hedge is established on the first day of the hedge period and allowed to remain in place, untouched, for the entire period. Accordingly, these hedges may be termed "static hedges."

Dynamic hedges, by contrast, involve strategies which undertake to adjust the hedge position when necessary. This approach to hedging derives from market experience which has shown that the relative price sensitivities of the cash and hedge positions is likely to change with time and with interest rates, resulting in an imperfect or unbalanced hedge. This effect can be especially pronounced in GNMA hedges, where unexpected prepayments can significantly alter the rate sensitivity of the cash position.

Motivation to rebalance hedges is restrained by two factors: transaction costs and whipsaw risk. Transaction costs naturally increase in proportion to the number and size of the adjustments made. Whipsaw risk refers to the possibility that the hedger will sustain losses both during the period in which he is rebalancing and in the period immediately after rebalancing. These kinds of losses can materialize if interest rates move against the exposed cash (hedge) position during the rebalancing effort and then reverse course and move against the newly established hedge (cash) position thereafter. They arise because rebalancing cannot be implemented instantaneously.

Hypothesis III expresses the issue suggested by the opposing costs and benefits of dynamic hedging:

3. STRATEGIES WHICH INCORPORATE DYNAMIC REBALANCING OF HEDGE POSITIONS OUTPERFORM STATIC APPROACHES.

The present research tests this hypothesis with the same four strategies described earlier. The hedge simulations of these strategies are modified in only one respect: each hedge is rebalanced the eighth business day of each month during the hedge period.[15,16] The hedge ratios used to make these adjustments are derived and estimated from the same theoretical models and market data employed for the static hedges.

E. Excluded Strategies

There are many alternative views of the price behavior of GNMAs and many views of the price behavior of futures and futures options. Most of these views can be translated into one or more hedging strategies. Two strategies of special interest in this regard hedge GNMAs with T-note put options and with a portfolio of T-bond and Eurodollar futures puts. These strategies are of special interest because they parallel the futures-only strategies discussed in the previous section. They are not considered in this study because the markets for these options are simply too thin at the present time, even for at-the-money puts.

Two other genre of hedging strategies have been ignored to restrict the scope of the present research to manageable breadth. They include the strip hedging strategies mentioned previously and a range of strategies using futures options the strike prices for which are chosen to reflect the distance of the current GNMA price from the at-the-money level. Although the present research hypotheses can be explored without reference to these strategies, they hold promise for future study.

V. SUMMARY

This chapter has presented the specific hypotheses to be examined in the present research. In addition, it has described the characteristics of the institutional setting constructed for the purpose of specifying and evaluating hedging strategies. In particular, emphasis has been given to the development of a new measure of hedging effectiveness, denoted by K, which evaluates hedging performance in terms of the average daily deviation of portfolio value from its initial value. This measure does a better job of measuring how well a given strategy performs in relation to the hedge objectives than variance based measures do. When supplemented with a few simple ratios describing the maximum observed single day deviations, K provides a great deal of meaningful information about hedging performance.

The penultimate section of this chapter has explained the rationale underlying each of the hedge strategies chosen for testing. Rules used to select futures contracts delivery months were explained in the context of liquidity constraints and the related needs of consistency and clarity in the structure of the research. In addition, this section presented the rationale for dynamic hedge adjustment and described the method used to simulate this process in the present research. And finally, a brief discussion identified other potential hedging strategies which were not selected for testing and explained the reasons for their exclusion.

To conclude, this chapter has utilized a portion of the literature reviewed in Chapter II in formulating research hypotheses and in developing hedging strategies. Chapter IV, which follows, is devoted to the completion of the process of translating the hedging strategies into concrete terms. It accomplishes this task through the derivation of theoretical models based on the literature reviewed in Chapter II and applied to the strategies and settings developed in Chapter III.

FOOTNOTES TO CHAPTER III

[1] A review of Jones (1987) and historical transaction data for these instruments (provided by the CFTC) supports this statement.

[2] These standards vary frequently in response to changes in market volatility. Several exchanges set these standards using modified Black-Scholes option pricing models.

[3] The transaction costs used in this study have been developed through discussions with market participants at Goldman Sachs, Morgan Stanley, Shearson Lehman, Putnam Investment Management, and Lind-Waldock during the Spring of 1987.

[4] This information was obtained in a telephone interview with Charles Clark of GNMA (March, 1987). It is compatible with the convention used by Dunn and McConnell (1983). The imprecise term "most" is used because revisions and adjustments may be occasionally made in the following days.

[5] The release of factor information affects GNMA prices. (Factor information tells market participants the fraction of principal outstanding for a given GNMA pool in terms of the original amount.) In the context of a dynamic approach to hedging where adjustments are made on a monthly basis, the intuitive choice for adjusting hedge ratios is the first opportunity following the release of factor information, or at closing prices on the seventh business day of each month. To facilitate comparison of static and dynamic approaches, this study has defined the eighth business day of each month as the start of a new hedge period and as the "adjustment day" for dynamic strategies.

[6] In keeping with Toevs and Jacob, value is defined in terms of the capital in the cash position. It is calculated as the current price of one dollar of GNMA principal multiplied by the amount of outstanding principal in the cash position ($25 million).

This definition does not include accrued interest, which, in strictest terms, is part of portfolio value, if not capital. Accrued interest has been ignored in the interest of clarity. See Landes. et. al. (1985) for a discussion of the complicating assumptions involved in attempting to hedge total portfolio returns.

[7] Hoeven (1987) and Landes, Stoffels, and Seiffert (1985) are in this category.

[8]Many researchers have employed variance minimizing hedge ratios. This approach has merit for non-institutional applications.

[9]From this example it is clear that Ederington's E is inappropriate for hedging applications where the hedge horizon is known (i.e. immunization applications).

[10]This observation was provided by Bruce Peterson, an analyst at Goldman Sachs, in New York, in an interview in February, 1987.

[11]As noted in earlier chapters, the basis is defined as the difference between the cash and the futures prices. Basis risk refers to potential variability in the basis.

This point is best illustrated in a simple example. Assume $100,000 of A-rated corporate bonds priced at 96 are hedged with one T-note futures contract priced at 92. The initial basis is four (96-92) points. Consider the performance of this hedge in a situation where the basis remains constant. Assume rates have increased, and at the termination of the hedge the corporates are priced at 94. If the basis has remained constant, the futures price is 90 (94 -4) and the hedge has functioned perfectly: a $2,000 ((94-96) x $100,000) loss on the cash position has been offset by a $2,000 (-(90-92) x $100,000) gain on the futures position.

By contrast, consider the performance of this hedge in a similar situation where the basis narrows. For example, assume that the rise in interest rates spurs a flight to quality which reduces the prices of A-rated corporates by a larger amount than the drop in T-note futures prices. If the corporates have fallen to 93 $^{23/32}$ while the T-note futures have fallen to 90, the basis at hedge termination is 3 $^{20/32}$. This narrowing of the basis has a negative impact on the performance of the hedge: the cash loss of $2,281 ((93 $^{23/32}$ -96) x $100,000) is only partially offset by the $2,000 gain on the short futures position. The net result is that the hedged position suffers a net loss of $281.

[12]There are costs associated with the selection of a stacking approach. Generally speaking, stack hedges are more susceptible to convergence than corresponding strips. Stack hedges, by virtue of their concentration at one point on the futures yield curve, exhibit more volatile behavior than strip hedges during periods of volatile interest rates. Hoeven (1987) discusses these and related issues in a study which finds that Eurodollar strip hedges outperformed the corresponding stack hedges during the

period 1982-1985.

[13]Volume data is provided in the CFTC data base.

[14]The assumptions used to identify the cheapest-to-deliver bond are discussed at length in Chapter IV.

[15]This day is chosen for rebalancing hedges because GNMA factors are released on the preceding day. (See footnote 2).

[16]Unfortunately, the nature of a hedge simulation based only on closing prices eliminates the possibility of whipsaw risk. Thus, a potentially detrimental influence on hedge effectiveness is ignored.

CHAPTER IV: METHODOLOGY: DEVELOPMENT AND APPLICATION OF THEORETICAL HEDGE RATIO MODELS

I. INTRODUCTION

The purpose of this chapter is to explain the derivation and application of the theoretical models used to compute hedge ratios in this study. As noted in Chapter II, a fundamental technique based on Macaulay duration has been selected in favor of retrospective statistical procedures popular with many researchers. This technique is derived in the following section. Thereafter, in Section III, it is applied in a variety of ways to the task of approximating the interest sensitivity of GNMAs. In parallel fashion, Section IV explicates the procedures according to which Macaulay durations are computed for each of the four hedging strategies discussed in Chapter III. Section V integrates the results of Sections III and IV: it reviews briefly the hedge ratio calculation. The chapter ends in Section VI with a summary.

II. DURATION-BASED HEDGE RATIO MODELS

A. Introduction

The most important aspect of any hedging strategy is the hedge ratio used to implement the strategy. This section develops the theoretical models underlying the duration-based hedge ratios used in the present research. In addition, it addresses important weaknesses inherent in the choice of this technique for calculating hedge ratios.

B. The Role of Hedge Ratios

The hedge objective used in this study is to minimize the average daily deviation from the initial portfolio value. This objective is perfectly achieved when, for each day in the hedge period,

$$<>P_{c,t} = -<>P_{h,t} \qquad (1)$$

where $<>P_{c,t}$ = the change in the value of the cash position during day t; and
$<>P_{h,t}$ = the change in the value of the hedge position during day t.

This simple equality expresses a fundamental concept noted earlier: to hedge the interest rate risk of a cash portfolio, the hedger must simply construct a hedge position with a price elasticity (interest sensitivity) equal in magnitude but opposite in sign to that of the cash position. The purpose of the hedge ratio is to determine the number of contracts needed in the hedge to offset exactly the rate sensitivity in the cash position.

The need for a hedge ratio (other than 1.0) arises for two reasons. First, the denominations of cash and futures instruments are rarely the same. For example, T-bond futures contracts call for the delivery of $100,000 of Treasury bonds, whereas a GNMA security may be millions of dollars in size. Obviously, hedging a long position of $10 million GNMAs with a short position in a single T-bond futures contract ($100,000) is unlikely to eliminate interest rate risk in the GNMA portfolio. Second, cash and futures prices rarely exhibit perfect correlation in terms of price changes induced by a given change in rates. That is, a hedge consisting of a short position in a single T-bond future, delivery for which is contrained to bonds with remaining terms in excess of fifteen years, is unlikely to offset precisely price changes in a cash position consisting of, for example, a five year corporate note.

A simple example illustrates the role of the hedge ratio.

Assume a portfolio manager currently holding \$2.3 million of five year corporate notes wishes to hedge this holding with a short position in T-bond futures. If he estimates the T-bond future is 1.77 times as volatile as the corporates, he can determine the number of contracts to sell short as follows:

In a perfect hedge,

$$<>P_{c,t} = <>P_{h,t}, \qquad \text{or}$$

$$<>P_{c,t} \;/\; <>P_{h,t} = 1.0 \qquad (2)$$

In the instant case, we have:

$$\$2.3M \; <>P_{c,t} = <>P_{h,t}$$

$$= 1.77\,(<>P_{c,t})\;\$100k, \quad \text{or}$$

$$<>P_{c,t} \;/\; <>P_{h,t} = [\$2.3M <>P_{c,t}]/[(<>P_{c,t})(1.77)\$100k]$$

$$= 13.0 \qquad (3)$$

Expression (3) is known as the hedge ratio. It is defined as the ratio of the dollar denominated interest sensitivities of the cash and hedge positions. In the example, the hedge ratio is 13.0, which indicates that the hedger can expect to offset changes in the value of the cash position by selling short thirteen T-bond futures contracts.[1] Thus, the hedge ratio specifies the number of contracts to be used in the hedge position.

C. Approximating Interest Rate Sensitivity: The Role of Duration

The validity of a calculated hedge ratio is dependent on the validity of the volatility estimates used for the cash and hedge positions. This study computes hedge ratios using modified Macaulay duration to approximate price volatility. The starting point for the derivation of this measure is the standard expression for the price, P, of a bond:

$$P = \sum_{t=1}^{N} (CF_t)/(1+y)^t \qquad (4)$$

where CF_t stands for the cash flow to be received by the investor at the end of period t, y stands for the periodic yield to maturity of the bond, and N represents the number of periods to maturity.

The local sensitivity of price to changes in yield is expressed in the first derivative,

$$dP/dy = -\sum_{t=1}^{N} (CF_t)(t)/(1+y)^{t+1} \qquad (5)$$

Macaulay defined his duration measure, D, as the present value weighted average of the maturities of each of the individual cash flows generated by a bond, or

$$D = \frac{\sum_{t=1}^{N} (CF_t)(t)/(1+y)^t}{\sum_{t=1}^{N} (CF_t)/(1+y)^t} = (-dP/dy)(1+y)/(P) \qquad (6)$$

Substituting (4) for the expression in the denominator in (6) and substituting the result into equation (5) yields, [2]

$$dP/dy = -DP/(1+y) \qquad (7)$$

Rearranging and denoting by D_M modified duration, $D/(1+y)$, we have,

$$dP/P = -D_M\, dy \qquad (8)$$

which states that the percentage change in bond price resulting from an incremental change in yield is equivalent to modified Macaulay duration times the yield increment. Alternatively,

$$dP = -P\, D_M\, dy \qquad (9)$$

the change in bond price resulting from an incremental change in yield is equivalent to the product of the initial bond price, the modified Macaulay duration of the bond, and the incremental yield.

Equation (9) is useful in hedging applications. It suggests that hedge ratios of the form of (2) can be computed with simple duration calculations:

$$<>P_{c,t}/<>P_{h,t} = P_c\ D_{Mc}\ dy_c\ /\ P_h\ D_{Mh}\ dy_h \qquad (10)$$

where $<>P_{*,t}$ is assumed equivalent to dP.

Assuming that incremental changes in yields are equivalent for both cash and hedge positions allows the simplification,

$$\text{Hedge Ratio} = <>P_{c,t}/<>P_{h,t} = P_c\ D_{Mc}\ /\ P_h\ D_{Mh} \qquad (11)$$

The analogy to the example presented in the previous subsection is clear: the ratio of the volatility of the cash and hedge positions is the ratio of their price - modified duration products.

D. Limitations of the Duration-Based Hedge

1. Problems of Parametric Analysis

Before proceeding to the task of computing D_M for the

GNMAs and hedge instruments used in this study, it is instructive to consider the many limitations of this approach. Perhaps the most significant of these limitations owes to the fact that duration is only a local measure. Thus, it is valid only for extremely small changes in yield, dy.

Diller (1984) crystallizes the observations of many researchers in this regard. He notes that the use of proxies for rate sensitivity (such as maturity, average life, duration, and convexity) is actually an effort to summarize the price performance of a fixed income security in one or more parameters.[3] Viewed in this light, the myriad fixed income and derivative securities traded in financial markets are not incomparable entities - apples and oranges - but commodities possessing in varying degrees the same essential characteristics of duration and convexity. He applies this framework to arbitrage analysis using bonds, GNMAs, zero coupon bonds, futures, and options on futures.

Diller's perspective is useful in understanding the shortcomings involved with using duration to summarize price performance of fixed income securities when changes in interest rates are not infinitesimal, but large (on the order of 100 basis points or more). Figure 4-1 displays performance profiles for three hypothetical bonds. Figure

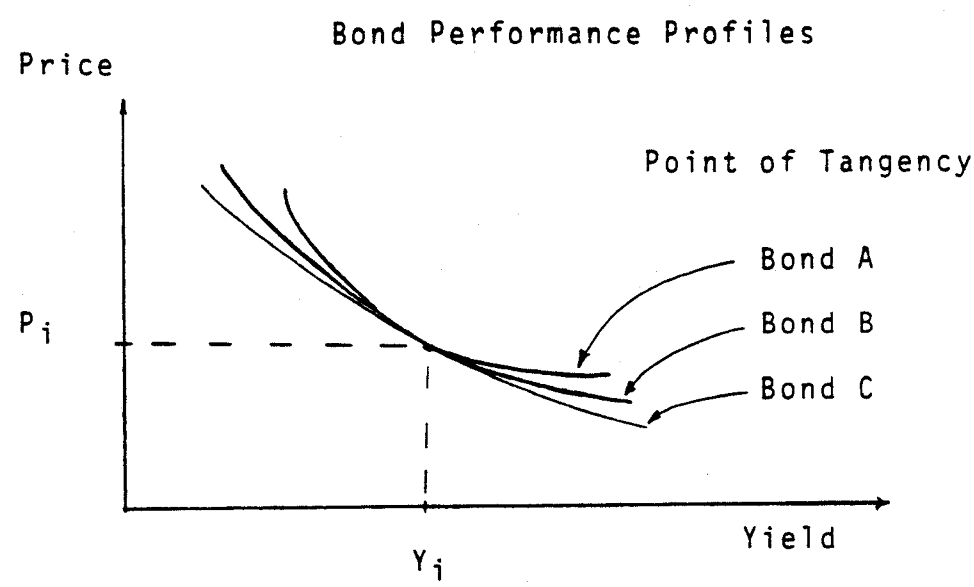


Figure 4 - 1

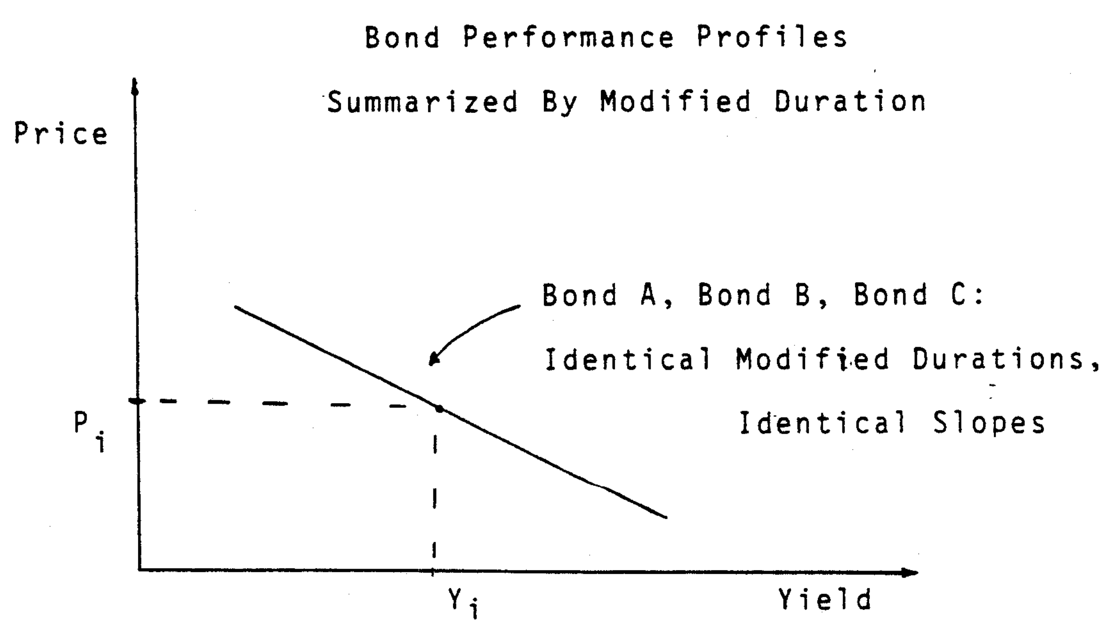


Figure 4 - 2

4-2 shows performance profiles for the same three bonds, where price changes from the initial price, P_i, are summarized in modified duration. While the modified duration measures appear to be effective in approximating price changes caused by infinitesimal changes in yield, they are clearly deficient for "large" yield changes. In fact, it is apparent that the observed deficiencies are greater for those bonds possessing more convexity.

These observations can be expressed in a more concrete fashion using a Taylor series expansion for bond price change:

$$<>P = (dP/dy)<>y + (1/2)(d^2P/dy^2)(<>y)^2 + \ldots \quad (12)$$

It is apparent from (12) that modified duration summarizes price changes in terms of first order effects. The influence of higher order terms is ignored, reducing the accuracy of the approximation.

Important hedging concepts emerge from this simple analytical view of the bond price performance profile. They can be summarized in a concise two-step hierarchy of rules:

1. The first priority in designing a hedge is to match the modified duration of the cash instrument

and the modified duration of the hedge instrument (presumed to be sold short). This action is tantamount to matching the first order terms in the Taylor series expansions of price changes for both positions.

2. The second priority is to match higher order terms, beginning with the second. Matching achieved in these higher order terms ensures that the initially matched duration values remain matched as yield levels change.

This second priority has recently received much attention in Wall Street research publications.[4] Unfortunately, most of these writers have ignored the deterministic relationship between these two rules. That is, the choice of hedge instruments made to satisfy the first order matching conditions **determines** the value of the higher order terms. The hedger does not have the ability to match durations and then independently set about adding instruments to the hedge to match higher order terms; the addition of instruments to modify higher order terms will undo the more important first order matching.

What then is the value of considering convexity? Chiefly this: a hedger able to construct synthetically multiple hedge positions of the same duration is well advised to compare them both in terms of their relative cost and in terms of how closely their convexities match that of the cash position. For example, a hedger attempting to synthesize a hedge position with a duration of six years could consider many alternatives. To name a few:

1. Short a weighted average combination of Euro futures (D = .25 yrs, denominations of $1M) and Treasury bond futures (D = 6+ yrs, denominations of $100k);
2. Short a weighted average combination of T-bill futures (D = .25 yrs, denominations of $1M) and Treasury bond futures (D = 6+ yrs, denominations of $100k);
3. Lengthen the cash duration by purchasing a zero coupon bond (D = 25 yrs, denominations vary) and short futures on Euros and T-bonds or T-bills and T-bonds.
4. Buy puts on Treasury futures (puts exhibit a negative implied duration: rate increases boost prices) in combination with short positions in other Treasury futures.

Viewed from the perspective of parametric analysis, the proper choice of hedge positions reduces to identifying the alternative, the parameters of which best approximate, in a hierarchical way, the parameters of the cash position. In short, the best hedge should be the alternative the duration of which matches the cash position and the convexity of which best approximates the convexity in the cash position.

Unfortunately, application of this theory in real world hedging applications is difficult. No empirical evidence is available to support its use. Indeed, satisfactory methods for computing the duration of simple bonds is still in debate, as noted in Chapter II. This debate seems to suggest that accounting for convexity is presumptuous. In addition, critical problems related to the large diversity of instrument types available impede its use. In particular, accounting for differentials in liquidity and basis stability is an imposing and as yet unresolved problem.

A systematic study of the merits of convexity is well beyond the scope of the present research. However, because of the intuitive appeal of the "parameter-matched" hedges, convexity values are computed and employed in the analysis of the effectiveness of alternative strategies.

2. Intertemporal Drift

The main thrust of "parametric-matching" is to ensure that duration changes occurring in the cash position as a result of interest rate changes are replicated in the hedge. A review of equation (6), however, reveals that duration is also a function of time to maturity. Thus, as time passes, durations "drift" and parametrically matched hedges may become unbalanced. The implication of this observation is that the perfect hedge must also match these duration drift components. In effect, the price performance profile must be mapped into a second dimension (in addition to yield), time, and partial derivatives with respect to yield and time should be equated in the cash and hedge positions.

The present research does not utilize this concept, which, for want of a better term, may be called "planar duration." Instead, it is assumed that duration drift is negligible over the hedge period. This simplification is an assumption made for convenience and to restrict the scope of this study. Yet it represents another shortcoming of the duration-based approach to hedging.

3. The Influence of Exogenous Variables

The use of duration-based techniques to hedge the interest rate risk of GNMAs is hampered by another problem entirely separate from the parametric issues. The technique assumes price changes of the GNMAs and hedge instruments are solely a function of interest rates. While it is generally true that the largest systematic influence on these prices is interest rates, it is shortsighted to exclude other factors. In particular, fluctuations in the supply and the demand for these securities may occur independent of interest rates. These fluctuations arise from many sources, including such widely disparate factors as demographic trends affecting the demand for housing (and hence the supply of GNMAs) and volatility in interest rates (which affects the demand for futures and options). To the extent that such factors are not represented in the level of interest rates, duration-based interest rate hedging suffers.

A related problem in duration-based hedging stems from the influence of investor expectations on the prices of fixed income securities. Use of the popular yield to maturity concept sometimes obscures the fact that bond prices are a function not of interest rates but of investor expectations for interest rates (and other, perhaps less systematic factors) in the future. To the extent that investor expectations change in a manner which deviates from changes in interest rates, (or rate, singular, as is the case in the MacAulay measure), the performance of duration-based hedges will be impaired.

4. Familiar Criticisms

The standard criticisms which apply to Macaulay duration apply also to hedging strategies founded upon it. Chief among these are its assumptions of a flat yield curve and parallel shifts. Other, security-specific shortcomings related to the use of the measure are reviewed in the ensuing sections describing the computation of hedge ratios.

III. Calculating the Numerator of the Hedge Ratio: GNMA Durations

A. Introduction

The previous section presented a derivation of duration based hedge ratios. Clearly, accurate assessment of the hedge ratio is the key ingredient in any hedging strategy. Since the market prices of the cash and hedge instruments are readily observable, this task is essentially a matter of computing modified duration values for the cash and hedge positions.

The purpose of this section is to explain the various methods employed in this research to quantify the duration of the cash position. The crucial role played by these estimates of GNMA durations encourages consideration of a variety of approaches. Accordingly, four different techniques are used for computing GNMA durations. Duration values generated by each of these techniques are then employed in hedge simulations. In later chapters, the results of using different duration calculations will be considered against the results of using the conventional twelve year bullet assumption.[5] This comparison sheds light on the value added to hedging strategies by more sophisticated attempts to measure GNMA duration. In addition, simulations using these alternative specifications of GNMA duration provide evidence required in studying Hypothesis 2:

EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY ACCOUNT FOR THE OPTION COMPONENT EMBEDDED IN GNMAS ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT TREAT THIS COMPONENT EXPLICITLY.

B. Inferred GNMA Durations

1. The Pricing Model: Using Inferred CPRs To Model Prepayment Uncertainty

A popular practice in options markets is to calculate implied volatility from observed prices.[6] This calculation is easily made because the option price and the four other determining factors (exercise price, price of the underlying instrument, time to expiration, and the risk-free rate) are readily observed. Consequently, assuming the market properly prices the option, the volatility of the underlying security can be obtained by simply rearranging the price equation to solve (iteratively) for volatility.

Clayton and Goldstein (1986) applied this inferential approach to FHLMC mortgage-backed securities. The conventional expression for the price of a mortgage is,

$$P = \sum_{t=1}^{N} (CF_t)/(1+y)^t \qquad (13)$$

where the variables are defined as they were for bonds except CF_t, which is,

$$CF_t = f(CPR) = SP_t + (CPR)(B_{t-1}) \qquad (14)$$

where SP_t represents the gross monthly payment less servicing costs in month t and B_{t-1} represents the outstanding principal balance at the end of month t-1. That is, in this case CF_t is taken to be the sum of scheduled principal and interest and prepayments in period t. Using the assumption that the MBSs are fairly priced, (13) is rearranged to solve for expected cash flows in iterative fashion. Expected cash flows are then compared to scheduled cash flows to infer market expectations for prepayment rates (CPRs). Clayton and Goldstein (CG) reported some success with this approach, noting however that market expectations did not seem to account for the relationship between the age of the MBSs and CPRs.

The present research calculates GNMA durations using a modified version of the CG pricing approach. The modification is made to address the relationship between GNMA age and CPR. It consists in assuming that CPRs are related to GNMA age in an amount which is directly proportional to the PSA model.[7] This formulation may be stated in terms identical to those of expression (13) with the exception,

$$CPR = CPR_t = f(t, \text{PSA schedule}) \qquad (15)$$

Following CG, expected future cash flows implied by market prices may be determined as a function of the PSA standard in an iterative procedure. Modified duration of a GNMA may then be computed as,

$$D = \frac{\sum_{t=1}^{N} (CF_t)(t)/(1+y)^t}{P_{GNMA}\ (1+y)} \qquad (16)$$

For convenience of exposition, the measure described by (16) will henceforth be referred to as inferred duration, D_{CG}.

2. Computing Inferred Durations With Market Data

One of the salient features of the present research is that it evaluates alternative hedging strategies with actual market data. As noted in Chapter II, many previous studies were limited because they used artificial prices. This subsection explains in precise terms how market data are used to evaluate D_{CG}. Insights gained here are helpful in qualifying research results.

The variables in (16) are computed from market data taken from the day prior to the construction of the hedge (i.e.the seventh business day of the first month in the hedge period) as follows:

P_{GNMA} is defined as the closing price of the generic coupon under study;

N is interpolated from the weighted average age of outstanding pools in a particular coupon group, in months.[8] This approach is an improvement over the standard assumption of thirty years remaining to maturity and is consistent with the ambition of improving on the Clayton Goldstein age - CPR specification.

y is defined as the posted yield on Federal Home Loan Mortgage Corporation thirty year standard conventional fixed rate mortgages for delivery within thirty days less twenty-five basis points.[9]

The inferred duration values determined in this manner for each of the seven GNMA portfolios are discussed in Chapter V.

3. Computing Convexities From Inferred Durations

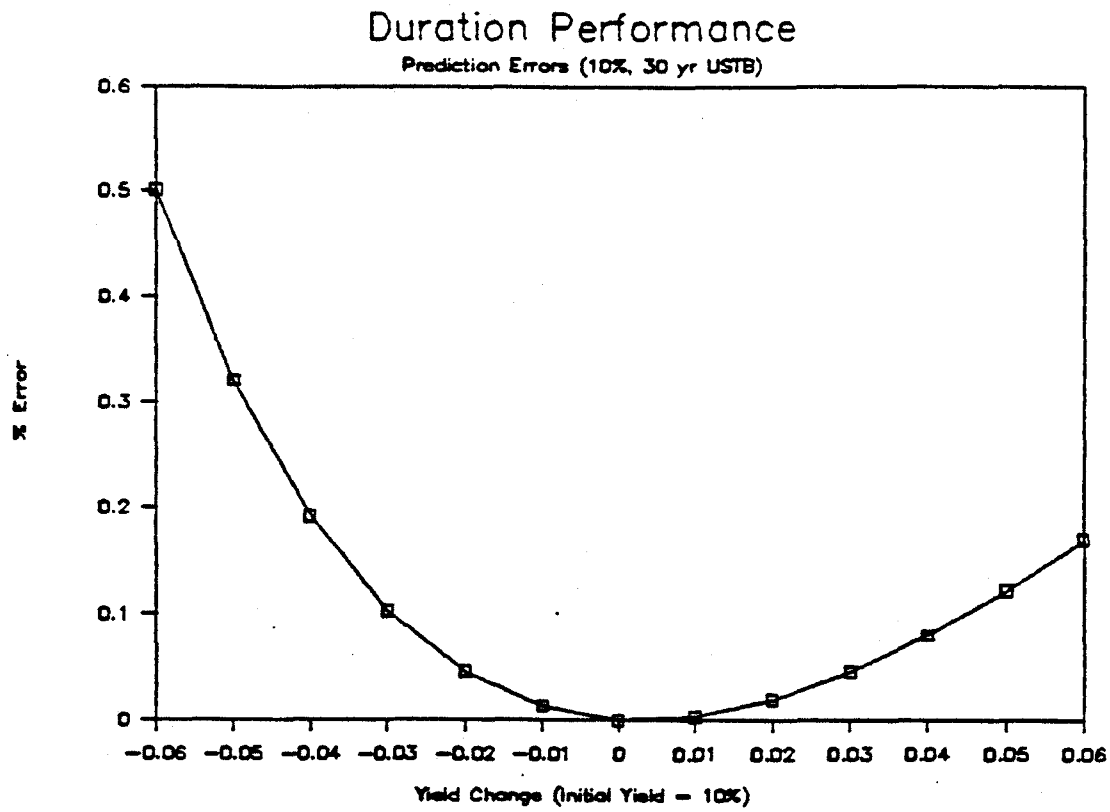
Previous sections of this study have used the term convexity to describe that part of a bond's performance profile not captured in the summary measure, modified duration. From an analytical point of view, convexity could be defined as the sum of the higher order terms in the Taylor series expansion of bond price change with respect to changes in yield, equation (16). It could then be applied by simply calculating the various derivatives required and then evaluating them with appropriate parameter values for the instrument under consideration.

Unfortunately, this type of analytical approach greatly reduces the utility of the measure. The great merit of convexity is its generality - it can be used to compare the performance of seemingly disparate securities such as coupon bonds, options on futures, and derivative mortgage securities.[10] The task of computing in analytical terms the value of higher order terms of securities priced as contingent claims is arduous and unrewarding. This point is made emphatically by a review of the analytical forms of the various option pricing models.

The intractability of the analytical approach suggests a less rigorous but more convenient definition of convexity. This study computes convexity as the percentage change in modified duration occurring for a parallel downward shift in interest rates of 100 basis points.

At first glance, this definition, which is cast in terms of a specific direction for rate changes, seems extremely arbitrary. Figure 4-3 illustrates the logic behind it. The nonlinearity present in the bond pricing equation generates larger duration prediction errors for rate decreases than it does for rate increases. Similarly, percentage changes in duration values for rate decreases of 100 basis points are not as large as percentage changes generated by rate increases of 100 basis points. Since the motivation for computing convexity values is to compare that portion of price performance of various securities which is not explained by duration, it is logical to use the measure where duration errors are largest. Thus, convexity is defined as the change in duration occurring for a rate decrease of 100 basis points.[11]

Duration Performance



Bond Duration Performance Analysis

USTB: 10%, 30 yr   Modified Duration: 9.03

Yield	Actual Price	Rate Change	Dmod Predicted Change (%)	Actual Change	% Error
0.02	279.82	-0.08	72.24%	179.82%	107.58%
0.03	237.83	-0.07	63.21%	137.83%	74.62%
0.04	204.28	-0.06	54.18%	104.28%	50.10%
0.05	177.27	-0.05	45.15%	77.27%	32.12%
0.06	155.35	-0.04	36.12%	55.35%	19.23%
0.07	137.42	-0.03	27.09%	37.42%	10.33%
0.08	122.62	-0.02	18.06%	22.62%	4.56%
0.09	110.32	-0.01	9.03%	10.32%	1.29%
0.1	100	0	0.00%	0.00%	0.00%
0.11	91.28	0.01	-9.03%	-8.72%	0.31%
0.12	83.84	0.02	-18.06%	-16.16%	1.90%
0.13	77.45	0.03	-27.09%	-22.55%	4.54%
0.14	71.92	0.04	-36.12%	-28.08%	8.04%
0.15	67.1	0.05	-45.15%	-32.90%	12.25%
0.16	62.97	0.06	-54.18%	-37.13%	17.05%

Figure 4 - 3

Determining convexities from D_{CG} follows in a straightforward manner. D_{CG} is simply reevaluated at a yield 100 basis points below the initial level. Convexity is then,

$$C_{CG} = (D_{CG}(y_f) - D_{CG}(y_i)) \qquad (17)$$

The convexity values computed in this manner for each of the seven GNMA portfolios are discussed in Chapter V.

C. Option-Adjusted GNMA Durations

1. The Pricing Model:
Explicit Recognition of Borrowers'Prepayment Options

The second research hypothesis to be tested in this study states:

EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY ACCOUNT FOR THE OPTION COMPONENT EMBEDDED IN GNMAS ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT TREAT THIS COMPONENT EXPLICITLY.

Therefore, in contrast to the inferred CPR technique developed in the preceding section, this section accounts for prepayment uncertainty by modeling the GNMA as the sum of two components: a noncallable annuity and a short position on the annuity. This model is expressed as,

$$P_{GNMA} = P_A - P_C \qquad (18)$$

The value of the annuity is determined in a simple yield to maturity framework,

$$P_A = \sum_{t=1}^{N} CF / (1 + y)^t \qquad (19)$$

where all the variables are as defined previously, except CF. In this model, CF is the scheduled principal and interest due to the investor on a monthly basis.

Based on the research reviewed in Chapter II, the call on the annuity may be valued as,

$$P_C(a,t) = aN(d1) - Xe^{-rT}N(d2) \qquad (20)$$

where $d1 = (\ln(a/X) + rT)/\sigma T^{.5} + (1/2)\sigma T^{.5}$;

$d2 = d1 - \sigma T^{.5}$;

a = the annuity value;

X = the exercise price;

T = the time to option expiration;

σ = the standard deviation of the annuity price; and

r = the risk-free rate.

The same argument which prevailed against the analytical formulation of convexity in the preceding section recommends an intuitive approach to computing the duration of the security described by (18). Following Toevs (1985) and Jones (1987), the present research calculates option-adjusted GNMA duration according to,

$$D_{OA} = D_A P_A \ (1-d_C) \ / \ (P_A - P_C) \qquad (21)$$

where dc represents the call delta.[12] The derivation of this expression is straightforward. Remembering that the duration of a portfolio is the weighted average durations of the constituent securities, equation (18) implies,

$$D_{OA} = (D_A P_A - D_C P_C) \ / \ P_{GNMA} \qquad (22)$$

Using the definition of the call delta, and applying equation (11) to interest rate induced changes in the value of the call and the annuity, results in,

$$d_C = dP_C/dP_A = P_C D_C dy / P_A D_A dy \qquad (23)$$

Rearranging results in,

$$P_C \ D_C = P_A \ D_A \ d_C \qquad (24)$$

Substituting (24) into (22) produces equation (21).

2. Computing Option-Adjusted GNMA Durations With Market Data

The present research computes option-adjusted GNMA durations with two distinct methods. In the first method, the value of the embedded call is assumed to be the difference between the market price of the GNMA and the value of the annuity component. In the second method, the value of the embedded call is calculated with the Black Scholes equation, where the variance is estimated from recent volatility of the annuity component. Each of these methods is discussed in turn.

a. Market Implied Method

Calculating D_{OA} from market data is accomplished by first calculating the price of the noncallable annuity component according to equation (19):

$$P_A = \sum_{t=1}^{N} CF_t \;/\; (1 + y)^t \qquad (19)$$

where CF_t is taken to be the cash flow projected for period t on the basis of the weighted average maturity for the particular coupon under study and on the basis of the twelve year bullet assumption for prepayments;[13]

y is assumed to be the yield on the current ten year Treasury plus ten basis points.[14]

The next step in calculating D_{OA} is to approximate the call delta, dc. This calculation is accomplished by first calculating an implied standard deviation of the annuity price using the Black Scholes formula (equation 20), where the five variables are defined as follows:

P_a = the annuity value, as calculated in (19);

X = the exercise price, which is assumed to be 102.5. The two and one-half point premium above par is included to reflect exercise costs (refinancing charges) which face the homeowner and make it seem unprofitable to exercise at par;

T = the time to option expiration (annuity maturity);

r = the risk-free rate, taken to be the yield on ninety day Treasury debt.

Calculation of the call delta follows in a straightforward manner. A marginal change in the price of the underlying annuity is generated by evaluating equation (19) at a discount rate one basis point lower than the initially.

This computation allows,

$$<>P_a = P_a(y) - P_a(y-.0001) \qquad (25)$$

In similar fashion, a marginal change in the value of the call is generated according to,

$$<>P_c = P_c(P_a(y),T) - P_c(P_a(y-.0001),T) \qquad (26)$$

The call delta is then computed,

$$dc = <>P_c/<>P_a \qquad (27)$$

Values for D_{oA} may now be computed from equation (21).

b. Estimated Volatility Method

The second approach used in this study to compute option-adjusted GNMA durations differs from the market implied method in the way it calculates the call delta. Instead of using an implied standard deviation in the Black Scholes equation, this approach computes volatility from recent market data. Specifically, standard deviations are calculated as the standard deviation of the natural logarithm of daily annuity price relatives for the sixty trading days prior to the start of a given hedge period. Annuity prices are generated on each of these days using the same discount rate (the yield to maturity on a constant maturity ten year Treasury plus ten basis points) employed in the market implied method.

The standard deviations calculated in this manner are then used in the Black Scholes formula to compute values for the embedded calls. Following the same steps enumerated for the market implied method, call deltas are determined and substituted into equation (21) to arrive at option-adjusted durations. For expository convenience, GNMA durations developed in this manner will be referred to by the variable D_{OE} in the remainder of this work. This designation serves to distinguish these estimates from those developed earlier and denoted by D_{OA}.

Conceptually, this approach is more fundamental in nature than the market-implied, option adjusted method. Whereas the former relies on market efficiency to accurately value the embedded option, this method admits the possibility of inefficiency but in its place presumes the validity of the Black Scholes model for the embedded calls. To the extent that the market implied method is susceptible to the same types of inefficiencies that could weaken the inferred (Clayton-Goldstein) technique, the estimated volatility method offers the potential of improved hedge performance.

This possibility is considered in the analysis of simulation results in Chapter V.

3. Computing Convexity From Option-Adjusted GNMA Durations

Generally, this study computes convexity according to,

$$C = D(y_f) - D(y_i) \qquad (28)$$

where y_f is 100 basis points below the initial yield. Using this method presents some special problems when applied to option-adjusted GNMA durations. The problems arise in determining the value of the call at the hypothetical lower rate, y_f. For example, in the case of the market implied method, where the call is value as the difference between the market price of the GNMA and an assessed value of the annuity component, valuing the call requires an estimate of the GNMA price at y_f. But the very purpose of computing duration and convexity values is to genearate such an estimate. Thus, to calculate duration and convexity values on the basis of a subjective estimate is circular reasoning and a meaningless exercise.

The estimated volatility method is not subject to this difficulty, although it requires some other strong assumptions. Using the method suggested by equation (28), $D(y_f)$ may be computed according to,

$$D(yf) = \frac{Da(yf)\ Pa(yf)\ (1 - d(yf))}{Pa(yf) - Pc(yf)} \qquad (29)$$

Evaluation of Pa(yf) and Da(yf) is straightforward; d(yf) and Pc(yf) are more complicated. Pc(yf) can be computed from equation (20), the Black Scholes model,

$$Pc(a,t) = aN(d1) - Xe^{-rT}N(d2) \qquad (20)$$

where $d1 = (\ln(a/X) + rT)/oT^{.5} + (1/2)oT^{.5}$;

$d2 = d1 - oT^{.5}$;

a = the annuity value calculated at yf;

X = the exercise price;

T = the time to option expiration;

r = the risk-free rate.

o = the standard deviation of the annuity price, as it was calculated at yi.

dc(yf) can then be computed by repeating the calculation at y = yf - 1 basis point and noting the ratio of the change in the value of the call to the change in the value of the annuity.

At least two unattractive assumptions are implicit in this procedure: 1) r_f is assumed constant through the 100 basis point drop in long term rates, and 2) the annuity volatility is assumed to remain at the same level even though rates have fallen (generally, volatility is observed to decrease with rates). To the extent that these assumptions are not valid, this procedure fails to produce accurate GNMA convexity values.

4. Inferred Duration Versus Option-Adjusted Duration: A Comparison of the Models

The D_{OA}/D_{OE} and D_{CG} models are defined as,

$$D_{CG} = \sum_{t=1}^{N} \frac{(CF_t)(t)/(1+y)^{t+1}}{P_{GNMA}\ (1+y)} \qquad (16)$$

$$D_{OA} = D_A P_A\ (1-d_C)\ /\ (P_A - P_C) \qquad (21)$$

$$D_{OE} = D_A P_A\ (1-d_C)\ /\ (P_A - P_C) \qquad (21)$$

These models are very similar in that each is based on the concept of Macaulay duration and its assumption of a flat yield curve which endures only parallel shifts. The primary difference in these models lies in the way they account for borrowers' rights to prepay the underlying mortgages. D_{CG} assumes the interest rate sensitivity of this right is fully reflected in market expectations. D_{OA}

and D_{OE}, on the other hand, assume the interest rate sensitivity of this right is not fully reflected in market expectations. Instead, explicit consideration of the borrowers' rights to prepay in terms of an option on an annuity is necessary to properly compute interest rate sensitivity. One of the objectives of the empirical tests reported in this study is to shed light on the relative merits of the two concepts.

D. Implied Duration

1. The Pricing Model

Pinkus and Chandoha (1987) posit a very simple approach to computing GNMA durations. In their model, GNMAs are assumed to exhibit price volatilities relative to a parity series of GNMA prices which are relatively stable over time when GNMAs are stratified by price level.[15] That is, PC assume that the regression,

$$\%<>G_n = B_1 \; \%<>G_p \qquad (30)$$

where $\%<>G_n$ refers to the percentage change in price of the GNMA being analyzed;

B_1 is a stable slope coefficient;

$\%<>G_p$ refers to the percentage change in price of the parity series.

accurately describes the relative price volatility of a given GNMA with the parity series. Further, PC assume that the parity series likewise exhibits a stable price volatility relative to the ten year Treasury. This relationship is expressed in the regression,

$$\%<>G_p = B_2 \%<>T_{10} \qquad (31)$$

where $\%<>T_{10}$ refers to the percentage change in price of the ten year Treasury; and
B_2 is a stable slope coefficient.

When relative price volatility is taken as a proxy for duration, this framework may be used to compute the duration of a GNMA in a given price range as a function of the duration of the ten year Treasury, which is readily determined.

An example clarifies the method. An investor analyzing a GNMA priced at 87.5 refers to the regression slope coefficients estimated by Pinkus and Chandoha. The price volatility of this GNMA relative to the parity series, denoted by B_1, has been estimated at 1.14.[16] B_2, on the other hand, is estimated at 1.45. If the duration of the ten year Treasury is currently five years, then the implied duration of the GNMA is,

$$D_{G87.5} = B_1 D_{GP} = 1.14\ D_{GP} =$$
$$= 1.14\ (B_2 D_{T10}) =$$
$$= (1.14)(1.45)(5.0) \quad = \quad 8.265 \text{ years}$$

This method has recently gained popularity with practitioners. The regressions underlying the technique have, on the whole, generated relatively high r^2 values and T statistics. As a result, it holds promise for effective use in GNMA hedging applications and therefore is a natural candidate for use in this research.

2. Computing Implied GNMA Durations

Application of the PC technique is a straightforward matter. The primary inputs are estimates for B_1 and B_2 across the GNMA price ranges represented in the cash positions. This study utilizes the estimates provided by Pinkus and Chandoha for this purpose. The secondary inputs are values for the duration of the current ten year Treasury at the outset of each hedging period. These values are easily computed using the standard modified duration expression and published bond market data. GNMA durations generated in this manner and used in hedge ratio calculations are discussed and analyzed in Chapter V.

3. Computing Implied GNMA Convexities

One shortcoming associated with the use of the implied duration concept is that no specific convexity values can be calculated. This problem develops because estimating changes in GNMA durations in this framework requires an estimate of the price changes of the GNMA under analysis in response to a change in interest rates. Such an estimate would allow selection of the appropriate B_1 value and computation of the new duration value. Unfortunately, the primary purpose of estimating duration is to estimate GNMA price changes. To use such an estimate as an input is circular reasoning and produces meaningless results.

One generalization is appropriate in reference to GNMA convexities in the implied duration framework: regardless of their specific values, they are likely to be negative (duration decreases with interest rates). This trait is apparent from the steady decline in the value of the B_1 slope coefficients which accompanies higher GNMA price levels in the Pinkus-Chandoha results. This observation is encouraging, as it fits recent market experience and intuition: as rates fall, prepayments are more likely, and GNMA durations would be expected to fall as a consequence.

IV. Calculating the Denominator of the Hedge Ratio: Durations of Financial Futures and Futures Options

A. Introduction

As noted in the preceding section, the crucial task in computing hedge ratios is determining modified duration values for the cash and hedge positions. This section explains the techniques used to calculate duration measures for each of the four alternative strategies considered in this study. It also describes the market data used to compute values for these measures.

B. Strategy 1: Short Treasury Note Futures

1. Computing Duration

Following Kolb and Chiang (1982) and Landes, Stoffels and Seifert (1985), the present research computes the modified durations of financial futures contracts as,

$$D_F = D_C/(1+y) \qquad (32)$$

where D_c represents the Macaulay duration of the cash instrument underlying the futures contracts. This formulation permits,

$$dFP = D_c FP\ dy/(1+y) = D_F\ FP\ dy \qquad (33)$$

(where dFP is the change in futures price and FP is the futures price), which is consistent with the approach used in equations (7) and (19) for the cash instruments.

Two informational inputs are fundamental to the application of equation (33) to real world hedging problems: the futures price on the day the hedge is constructed, and specification of the underlying cash instrument. Identifying the futures price to be used is a matter of choosing the contract month for the given futures instrument (e.g. T-notes, T-bonds) to be used in the hedge. The rules for selecting delivery months and strike prices are presented in Chapter III along with a table illustrating the choices made for each strategy and each hedge period to be simulated. With this information, the correct futures price can be easily extracted from the database.

The second informational input required in using equation (33) refers to the cash security to be delivered into the contract. By way of background, there is no single unique Treasury note to be delivered into the T-note futures contract. Instead, the Chicago Board of Trade, which functions as the exchange for T-note futures, describes as acceptable for delivery,

> "U.S. Treasury Notes with a nominal 8% coupon maturing not less than six and one-half, nor more than ten years, from the first day of the delivery month.[17]"

This policy means T-notes of any coupon meeting the maturity specifications may be delivered into the contract. The CBOT publishes a table of adjustment factors to be used in converting non-8% coupons to an equivalent amount of 8% coupon.

The broad specifications for delivery into the T-note contract have implications for market participants. Individuals maintaining short positions search the market for those T-notes which are cheapest to deliver, where the cost of delivery is a function of the CBOT conversion factors. As interest rates change, and as conditions of supply and demand for the various T-notes traded in the cash market change, the identity of the cheapest to deliver security changes also.

These changes present problems for the hedger attempting to compute futures durations. An assumption that the nominal delivery, 8% coupon and ten years to maturity, is to be made produces much different results than the assumption that a 12% coupon with six and one-half years to maturity will be delivered. Three alternative assumptions can be employed in response to this problem:

1. Assume the hedger knows which T-note will be cheapest to deliver over the hedge period.
2. Calculate the cheapest to deliver on the date the hedge is initiated and assume it continues to be the cheapest to deliver over the hedge period.
3. Assume that the coupon and maturity of the instrument which was most commonly delivered into the nearest past delivery month will characterize the notes to be delivered into the present contracts.

Each of these approaches is imperfect. Assumption (1) is the simplest; it also seems to be the most subjective, making it ill-suited to a systematic analysis. Assumption (2) has the merit of identifying the actual cheapest to deliver when the hedge is initiated. However, the data requirements for this approach are quite large.[18] Assumption (3) is consistent with an inertial view of interest rates. Although it also requires a significant amount of data, it has been selected for use in this study on the basis of its practical appeal relative to assumptions (1) and (2).

The preceding paragraphs noted the selection of a contract month and the cheapest to deliver assumption used in this study.[19] A brief example serves to clarify the use of these assumptions in computing a hedge ratio.

Assume the hedger is attempting to evaluate expression (31) for a hedge to be initiated April 11, 1985:

$$D_F = D_C/(1+y) \qquad (31)$$

Referring to delivery data from the CBOT, it appears that only one T-note issue was delivered into the March, 1985 contract: the 11 1/4 % of February, 1995. The yield on this note on April 11 was 11.755%. The Macaulay duration of the cash instrument on April 11th was therefore,

$$D_C = \sum_{t=1}^{N} \frac{CF(t) / (1+y)^t}{P_C} = 6.128 \text{ years}$$

which implies that the modified duration of the futures contract is,

$$D_F = 6.128 / (1 + .11755/2) = 5.788 \text{ years}$$

This result is then used to compute a hedge ratio,

$$HR = D_C P_C / D_F FP$$

$$= \frac{(5.1)\ (97.5)\ (\$2,300,000)}{(5.483)\ (95.00)\ (100,000)} = 20.793$$

where the values for Dc and Pc are assumed to describe a \$2.3 million cash position priced at 97.50, FP is the assumed price of the December T-note future, 95.00, and \$100,000 refers to the denomination of the T-note futures contract. This hedge ratio indicates that 21 futures contracts should be sold short.

2. Computing Convexity

Expression (31) describes the modified duration of the futures position in Strategy 1. It can also be used to compute the convexity of this position. Following the incremental procedure described for calculating GNMA convexities, the convexity of the futures position is computed as the change in duration which occurs for an assumed rate decrease of 100 basis points. Convexities computed in this manner are discussed in Chapter V.

C. Strategy 2: Short Treasury Bond Futures

Strategy 2 is identical to Strategy 1 with the exception that T-bond futures are used in place of T-note futures. The similarities between these two contracts allow the same procedures to be used to calculate durations and convexities.

D. Strategy 3: Short Euro CD and T-bond Futures

1. Computing Duration

Computing the duration of Strategy 3 follows the procedure used from Strategies 1 and 2 directly:

$$D_3 = (1 - x)D_E/(1 + y_E) + (x)\ D_T/(1+y) \qquad (34)$$

where x refers to the portfolio weight of the futures on ninety day Eurodollar CDs (Euros) and (1-x) to the portfolio weight of the T-bonds.[20] Because Euros have no associated coupons , (34) may be simplified to

$$D_3 = (1 - x)(.25)/(1+y_E) + (x)\ D_T/\ (1+y) \qquad (35)$$

where it is assumed that the Euro rates (ninety day LIBOR) and T-bond yields are perfectly correlated.

The weight x is chosen by setting D_3 equal to the duration of the GNMA to be hedged,

$$D3 = (1 - x)\, D_E + x\, D_T = D_{GNMA} \qquad (36)$$

$$\text{or} \quad x = (D_{GNMA} - D_E)/(D_T - D_E) \qquad (37)$$

Initial hedge ratios for the T-bond and Euro futures are calculated according to,

$$HR_E P_E D_E + HR_T P_T D_T = P_{GNMA} D_{GNMA}\ (\text{Principal}) \qquad (38)$$

where HR_E, HR_T represent the number of each type of futures to be sold short (T-bond futures are denominated in units of $100k each, Euros in units of $1 million); and Principal refers to the amount of outstanding GNMA principal being hedged.

Frequently, equation (38) produces fractional hedge ratios. To reduce the influence of rounding errors, a second computation is performed. In this computation, HR_E is assigned the nearest integer number of contracts based on the first solution to (38), and then, with this value substituted into (38), it is solved again for HR_T. In effect, the smaller denominated T-bond futures are used to fine tune the hedge durations.

2. Convexity Calculation For Strategy 3

Convexities for Strategy 3 are computed on an incremental basis, as they were for Strategies 1 and 2.

E. Strategy 4: Purchase Put Options On Treasury Bond Futures

1. Computing Duration

This study computes the duration of a long position in put options on T-bond futures with a procedure which is very similar to the one used to value the call component in the option-adjusted GNMA framework. The primary difference between the two methods arises because the GNMA calls are long term options whereas the futures options possess at most nine months to expiration. The result of the shorter lives of the futures options is that the Barone-Adesi and Whaley (1986) analytical approximation can be used. Specifically,

$$D_{FP} = d_p \, D_F \, FP \qquad (39)$$

where D_{FP} = the duration of the futures put;

d_p = the put delta; and

D_F = the duration of the underlying T-note future;

FP = the price of the underlying T-note future.

Computing D_{FP} from market data follows directly. D_F and FP are determined as they were for each of the first three strategies. d_p is computed as the change in the value of the put divided by the change in the value of the underlying T-bond future, given a one point decrease in FP. The value of the put is determined using the Barone-Adesi and Whaley formulas presented in the Appendix to Chapter II.

The procedure used to generate incremental changes in the value of the put for the purpose of calculating put deltas parallels the method used on the calls embedded in GNMAs. It is complicated only slightly by the recognition of the early exercise premium. As was the case for GNMAs, five parameters, including the current price of the underlyig security, are observed from market data:

P_{put} = the current market price of the put;

P_f = the price of the underlying T-note future;

X = the exercise price of the option, which varies depending on the level of cash and futures prices;

T = the time to option expiration; and

r = the risk-free rate, taken to be the yield on ninety day Treasury debt.

To this set of inputs is added the cost of carry, where b = the cost of carry, assumed to be zero for futures contracts.

Using these inputs, an iterative procedure is used to calculate the critical futures price, FPC.[21] This value, in combination with each of the above parameters, is substituted into the Barone-Adesi and Whaley expression. Another iterative procedure is employed to calculate the implied standard deviation of the underlying futures price.

This estimate of futures price volatility is retained and used to calculate the value of the put for a one basis point decrease in the price of the underlying future. The put delta is then computed as,

$$d_p = \frac{P_{put}(FP) - P_{put}(FP - .01)}{(FP - (FP - .01))} \qquad (40)$$

The duration of the T-bond futures put is then calculated from equation (36).

2. Convexity Calculation For Strategy 4

Once again, convexity is computed as the change in duration produced by a 100 basis point decline in interest rates. The calculation parallels the method described in Section III C 3 for computing GNMA option-adjusted convexities. V. HEDGE RATIOS

The motivation for the duration values developed for the cash and hedge positions in the preceding section is the calculation of hedge ratios for each strategy to be tested. As noted earlier, duration-based hedge ratios are computed for cash position i, strategy j, and period t from,

$$HR_{ijt} = \Delta P_{c,i,t} \,/\, \Delta P_{h,j,t}$$
$$= P_{c,i,t}\, D_{c,i,t} \,/\, P_{h,j,t}\, D_{h,j,t} \qquad (41)$$

In the present research, P_c and D_c refer to the seven GNMA portfolios defined as the cash positions to be hedged. Values for P_c are available in the data base, while values for D_c are calculated using each of the alternative methods described in Section III. Similarly, values of P_h are available in the data base, while values for D_h are determined according to Section IV.

V. SUMMARY

The most important aspect of any hedging strategy is the hedge ratio used to implement the strategy. Consequently, from a conceptual standpoint, this chapter constitutes the heart of the present research. It explains how Macaulay's duration can be modified and used to calculate hedge ratios for a variety of GNMA hedge strategies. In the process, several different views of the price behavior of GNMAs are noted and reflected in the numerator of the hedge ratios. The hedge ratios corresponding to these different views serve as the principal inputs to the hedge simulations discussed in the following chapter, Chapter V.

FOOTNOTES TO CHAPTER IV

[1] This ratio is of course valid only to the extent of the volatility estimates on which it depends.

[2] This expression applies to the full price (flat price plus accrued interest) of the security.

[3] Convexity refers to the curvature present in the bond price profile. The term is presently used loosely in market research publications. This study uses the term convexity to summarize that part of a bond's price performance profile not captured in the duration measure.

[4] See for example Jacob, Lord, and Tilley (1987) and Asay, Guillaume, and Mattu (1987).

[5] As noted in the literature reviewed in Chapter II, one approach to estimating GNMA prepayments is to assume no prepayments occur until the twelfth year, at which time the entire outstanding principal is repaid in full (a "bullet" loan). Calculating GNMA durations under this assumption is as straightforward as it is for government bonds. The modified Macaulay formula may be applied directly.

[6] Latane and Rendleman (1976) and Beckers (1981) have made noteworthy contributions in this area.

[7] The P.S.A. (Public Securities Association) model was developed in 1985. It combines the simplicity of the CPR method with information in the FHA survivorship tables to generate a table of "benchmark" CPR values for MBSs as a function of age. MBSs are then described in terms of "percent of PSA."

[8] The Financial Publishing Co. provided data specifying the total principal outstanding for each GNMA coupon group, by year of origination and original maturity, for January 1985 and January 1986. This data was augmented with new issue data for 1986 (provided by Shearson Lehman), which permitted the construction of a benchmark for January 1987. The age of a given coupon group in a given month was then determined through linear interpolation between the appropriate benchmarks.

[9] The underpinnings of the assumption of a twenty-five basis point FHLMC-GNMA spread are tenuous. Waldman and Guterman (1985) report on FHLMC-GNMA 8 1/2 % coupon spreads over the period 1977 - 1985. From their work, fifteen to thirty basis points is an appropriate spread to compensate for differential FHLMC-GNMA credit quality. Unfortunately, as WG pointed out, this spread has

exhibited much volatility over time, ranging from -90 to +90 bp. A great deal of this volatility is attributable to the increased value of discount FHLMC 8 1/2's which arises from expectations of faster prepayments relative to the GNMAs (FHA and VA loans are generally assumable, whereas the conventional mortgages in FHLMCs generally are not).

Alternative specifications of this discount yield were tested. Of particular interest in this regard was the definition of y as the yield on the long Treasury bond plus fifty basis pints. This assumption was suggested by recent empirical findings by Toevs and Jacob (1987). Their work indicates that arbitrage profits were available to investors on a fairly consistent basis to investors who purchased GNMAs with option adjusted yields more than fifty basis points above the Treasury curve. While the nature of the evidence provided by their study is not necessarily robust for all time periods, it seems preferable to alternative assumptions which are even more arbitrary. This specification was ultimately rejected in favor of the FHLMC-GNMA spread.

[10]The principle derivative mortgage securities are IO's (Interest Only securities), PO's (Principal Only securities), CMO's (Collateralized Mortgage Obligations), and REMICs (Real Estate Mortgage Investment Conduits).

[11]This convention assumes the securities in question possess positive convexity.

[12] The call delta is defined as the rate of change of the value of the call option with respect to changes in the value of the underlying security.

[13]Alternative specifications of the maturity and cash flow assumptions for the annuity were investigated. In particular, the annuity was treated as a simple non-callable annuity of maturity equal to the weighted average maturity of the parent GNMA. This treatment produced duration values which were less well-behaved than those produced by the twelve year bullet assumption.

To be more specific, the longer annuity approach produced a large proportion of negative values for the embedded call option. While incongruous at first glance, these results conveyed important information: the market at times expects irrational exercises of the embedded calls (i.e. prepayments when market rates exceeded the borrower's loan rate). From this view, the twelve year bullet assumption for the annuity component is a means to account for prepayments which are driven by factors other than interest rates.

Two final points should be noted. First, even with the twelve year bullet assumption, some implied option values were observed which could not be generated with plausible estimates for the implied variance. In these few cases (less than one per cent), the option components were ignored in the duration calculations. The result of this modification in every instance produced a duration value between those estimated for the prior and succeeding months, providing a measure of confidence that this procedure did not unduly skew results.

Second, Latane and Rendleman (1976) encountered similar difficulties (no implied variances) in their work with equity options. They attributed the problem to market imperfections.

[14]The selection of a ten basis point risk premium is based on discussions with market participants.

[15]The GNMA parity series created by Pinkus and Chandoha was defined as a series of price changes of GNMAs trading in the range between 99.5 and 100. Because this narrow range produced a limited number of observations, the original series was augmented with price changes for the GNMAs trading closest to par on a given day and adjusted with simple regressions.

[16]This value is taken from the Pinkus-Chandoha results.

[17]Source: Chicago Board of Trade publication, "10 Year Treasury Note Futures," 1986.

[18]See Hjerpe, E. (1987a).

[19]Strategies using T-bond futures make the same assumption regarding cheapest to deliver. Cheapest to deliver problems do not arise for Eurodollar futures: they are "cash settled."

[20]These portfolio weightings account for the differentials in contract denominations: Euro futures trade in units of $1,000,000, while T-bond contracts trade in units of $100,000.

[21]The critical futures price is the price at which early exercise of the put becomes optimal.

CHAPTER V: RESULTS

I. INTRODUCTION

The primary purpose of this chapter is to present the results of this research effort. In so doing, it integrates and builds from the results of previous chapters. To recount, Chapter II provided a conceptual foundation for this work by reviewing in detail theoretical and empirical literature germane to the topic of hedging GNMAs. Chapters III and IV drew heavily from this base. Chapter III introduced a new measure of hedge effectiveness, K, based on the observed limitations of another measure, Ederington's (1979) "E." Chapter IV explained the application of theory presented in Chapter II to the development of duration-based hedge ratios. Thus, the role of this chapter is to employ both these hedge ratios and K in tests of the research hypotheses.

By way of review, the three research hypotheses explored in the study are as follows:

1. THE INTEREST RATE RISK OF GNMA PASS-THROUGHS IS REDUCED BY THE USE OF DURATION-BASED HEDGING STRATEGIES WHICH EMPLOY FINANCIAL FUTURES AND OPTIONS ON FUTURES.
2. EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY INCORPORATE THE EMBEDDED CALL OPTION ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT INCORPORATE THIS COMPONENT.
3. STRATEGIES WHICH INCORPORATE DYNAMIC REBALANCING OF HEDGE POSITIONS OUTPERFORM STATIC APPROACHES.

As noted in Chapter III, each of these hypotheses represents a generalization which is widely accepted by financial managers and institutional investors participating in the secondary mortgage market. The primary purpose of this study is to provide empirical evidence documenting or disproving their reliability. The results reported in this chapter are significant because they show that each of these hypotheses can be rejected on the basis of simulation experiments performed with market data over the period April 1985 through March 1987.

The development of the chapter proceeds as follows. Section II describes the strengths and weaknesses of the various data inputs to the simulation process, while Section III discusses the simulation procedure itself.

Section IV is the heart of the chapter. It explains the statistical methods used to test the research hypotheses. The statistical results are then interpreted in terms of the parametric characteristics of the various cash and hedge positions. The generality of these results is then qualified carefully, based especially on limitations in the data. The chapter concludes with a summary in Section V.

II. DATA

Before proceeding to a presentation of the simulation method and a discussion of the results, it is important to consider the data used in this study. The principal inputs are daily closing prices for the GNMAs, financial futures, and futures options contracts used in simulating hedges.[1] The price series employed in this study spanned the time from January 2, 1985 to March 10, 1987, a period including 551 trading days. The GNMA price data represent TBA closing prices provided by a primary dealer which serves as a market maker for GNMAs.[2] The futures and futures option data were provided by the Commodity Futures Trading Commission, as were the futures delivery data.[3]

Other data inputs used in the empirical work include a collection of interest rates. Daily closing yields on a constant maturity ten year and thirty year Treasury bonds were provided by a primary dealer. FHLMC posted yields on thirty year mortgage commitments for delivery within thirty days were taken from The Wall Street Journal. This source was also used to acquire yields on the Treasury bonds assumed to be cheapest to deliver into the T-bond futures and to acquire LIBOR rates. The LIBOR rate applicable for a given day was taken to be the midpoint of the reported closing range.

The GNMA price series utilized in this study may at once constitute its greatest strength and its greatest weakness. On the positive side, GNMA price data are extremely expensive to collect and retain. The high cost involved often renders these data bases as proprietary and thus inaccessible to academic researchers. This problem more than any other has hampered research on this subject. It has often forced researchers to "construct" price series.[4] GNMA hedging studies based on "constructed" price series are obviously dubious. Thus, the present research is fortunate to be able to conduct empirical analysis based on actual GNMA market data.

On the other hand, a major limitation of this study concerns the relatively brief time period for which market data were acquired. A period covering a mere 551 trading days- almost all of which occur during a strong bull market in fixed income securities- is quite modest in comparison, for example, to the fifty-three years (1925-1978) spanned by the data used in the landmark duration study by Bierwag, Kaufman, and Toevs (1983).

The limited span of the data base greatly limits the generality of the results of this study and the strength of any probability statement which can be made regarding the hypotheses, as is noted later in this chapter. Nonetheless, this study is worthwhile because at the present time Finance literature lacks GNMA hedging studies based on actual market data. In addition, it is encouraging to recall that the original Black Scholes work was supported by results drawn from traders' diaries over a period of only 766 trading days.[5]

III. SIMULATION METHODOLOGY

As noted earlier in CHAPTER III, twenty-one hedge periods have been constructed for empirical testing. A detailed description of these periods is presented in Table 3-2, which is reproduced on the following page. The simulation model employed in this study assumes that an initial cash position of $25 million GNMAs of a particular coupon is held throughout the hedge period. On the first day of the period, a hedge is initiated at the market close.

Transaction costs are computed and the value of the portfolio is reduced accordingly. On succeeding days, the value of the portfolio is adjusted to reflect changes in the value of the GNMA position and changes in the value of the hedge position as indicated by closing prices. These adjustments are equivalent to marking to market each day.

This approach to measuring daily portfolio value is consistent with prevailing market conventions. GNMA mutual funds, for example, report net asset values daily on the basis of TBA price series similar to the one employed in this study. In addition, futures markets participants experience daily the marking to market of their positions. Thus, the simulation model used to test hedging strategies measures portfolio value in a manner which is in agreement with current market practices.

At the conclusion of a hedging period, the model computes the six measures of hedging effectiveness set forth in Chapter III: K, R, E, L_{min}, G_{max}, LV_0, and GV_0. These values are then retained for statistical analysis.

Simulation of the dynamic strategies referred to in Hypothesis III proceeds in parallel fashion. The only distinction occurs on hedge adjustment days, where the hedge position is modified to account for changes in the hedge ratio. Hedge adjustment days have been chosen as the eighth business day of the second and third months of the hedge period. Adjustments are made on these days with the assumption that any dramatic changes in market prices will have occurred in response to GNMA factor information released the preceding day.[6] Thus, the simulation model avoids adjusting hedges just prior to the release of new information likely to change price levels. At the same time, the choice of monthly intervals between adjustments preserves a measure of clarity in the performance of hedges which would likely be obscured in a more frequently adjusted scheme. It is also appealing as a realistic interval for real world portfolio managers who would shun day-to-day adjustments as churning. Thus, the dynamic adjustment procedure used in the simulation model reflects a reasonable approach to real world hedging applications.

For all its merits, this simulation model contains an important abstraction. Implicit in the procedure is the assumption that any principal payments received during the hedge period are reinvested in the present GNMA coupon at market prices. This assumption is used to isolate the effectiveness of hedge adjustments made solely in response to perceived changes in the interest sensitivity of the GNMAs in the cash position. In practice, principal payments alter the amount of principal to be hedged and hence necessitate adjustments in the size of the hedge position. This distinction is emphasized here to preclude confusion of adjustments made due to changes in interest sensitivity with those made to account for changes in underlying principal amounts. The value of the latter is straightforward and undisputed, whereas the value of the former is directly questioned in a test of Hypothesis III.

In a final note on the simulation method employed in this study, it is worthwhile to consider Pritsker's (1984) view of the simulation process itself. He identifies ten distinct stages:[7]

1. Problem Formulation : The definition of the problem to studied including a statement of the problem-solving objective.
2. Model Building: The abstraction of the system into mathematical-logical relationships in accordance with the problem formulation.
3. Data Acquisition: The identification, specification, and collection of data.
4. Model Translation: The preparing of the model for computer processing.
5. Verification: The process of establishing that the computer program executes as intended.
6. Validation: The process of establishing that a desired accuracy or correspondence exists between the simulation model and the real system.
7. Strategic and Tactical Planning: The process of

establishing the experimental conditions for using the model.

8. Experimentation: The execution of the simulation model to obtain output values.

9. Analysis of Results: The process of analyzing the simulation outputs to draw inferences and make recommendations for problem resolution.

10. Implementation and Documentation: The process of implementing decisions resulting from the simulation and documenting the model and its use.

For purposes of the current research, stages 5 and 6 are crucial. The empirical portion of this study utilizes a wide variety of computer models to characterize the various instruments involved. In particular, the models used to generate mortgage cash flows and option values are important. Each of these models was validated and verified. GNMA models were tested against published mortgage cash flow analyses while option models were tested against the results presented in the Barone Adesi and Whaley (1986) paper.[8] The simulation program itself, as well as the data inputs it employed, was audited extensively. Reasonableness tests were used as a final criterion for detecting possible errors.

IV. RESULTS

A. Introduction

The data inputs described in Section II were used in the simulation procedures presented in Section III to produce a value of K for each of the combinations of cash position, hedge strategy, hedge type, and GNMA duration model. In this section, the three research hypotheses are explored by means of an analysis of variance procedure applied to the simulation results. The results of these statistical tests are then interpreted in terms of the parametric characterisitics of each hedge and qualified as appropriate.

B. Statistical Methods and Hypotheses Tests

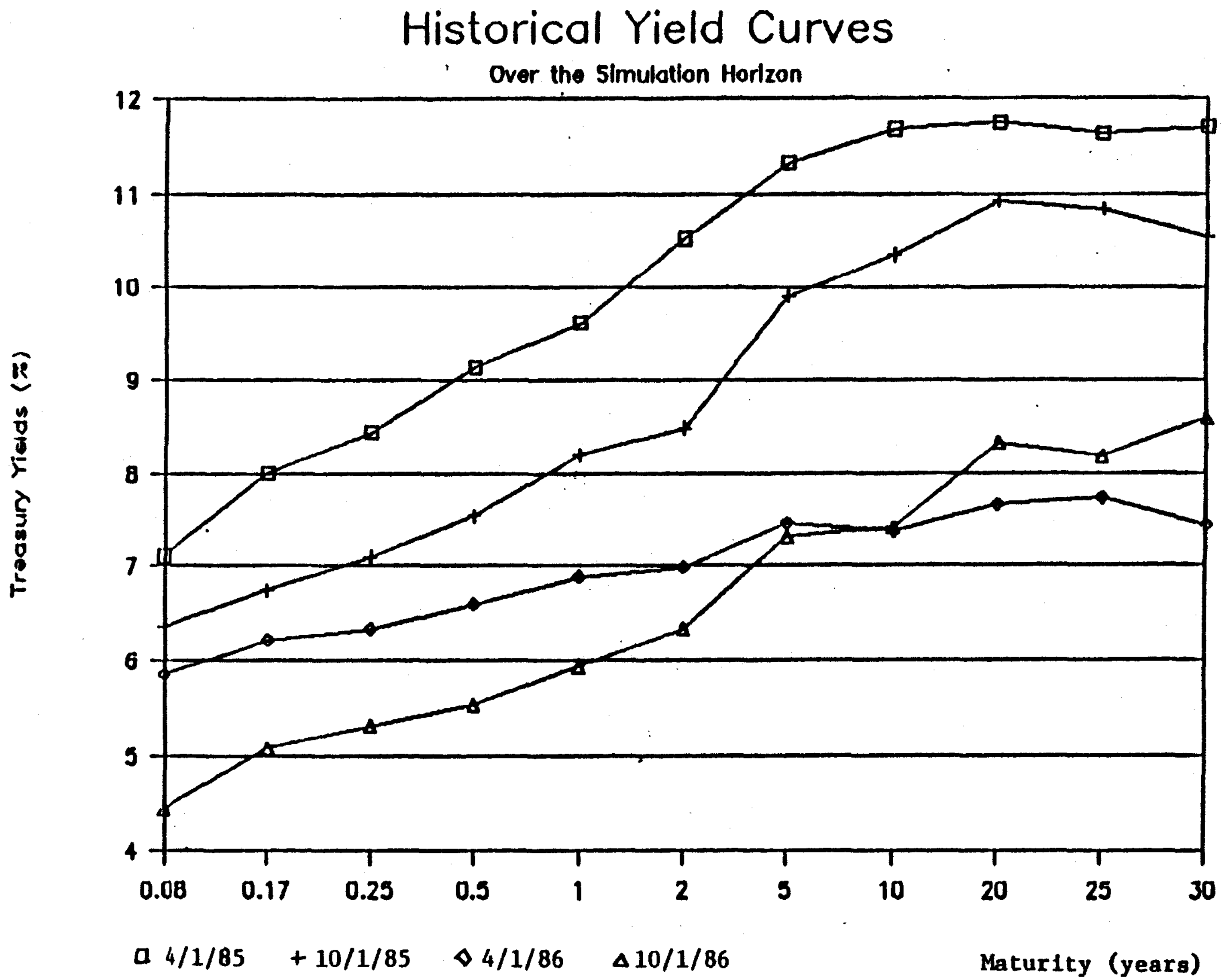
1. Preparation of Results Data Sets

From a statistical standpoint, a necessary precursor to testing the hypotheses of interest in this study concerns partitioning the computed K values. As noted earlier, much overlapping occurs in the hedge periods. K values computed from observations shared by overlapping periods are obviously not independent. Thus, three data sets are constructed for analysis: the first contains K's computed from every third period beginning with the first

(1,4,7,...19), the second contains K's from every third period beginning with the second (2,5,8,...20), and the third contains K's from every third period beginning with the third (3,6,9,...21). These three data sets are not independent, but testing each of them eliminates the possibility that conclusions are dependent on the choice of the first month in the data set.

Before proceeding, it should also be noted that despite the preceding effort to partition the K's into independent hedge periods, the data sets so constructed are not truly independent. The driving force in interest rate hedging is the behavior of the term structure of interest rates during the hedge period. Unforeseen quirks in the shape and level of the term structure can greatly distort hedge performance. As illustrated by the yield curves presented in Figure 5-1, the time period under study was characterized by a term structure the level and slope of which declined fairly steadily. Thus, even observations drawn from periods which are separate in time are not independent; they both hail from periods when interest rates were falling.

While troubling from a statistical standpoint, the interest rate behavior exhibited during the study period holds special appeal for research in hedging GNMAs. This appeal derives from the fact that a falling rate



Historical Yield Curves
Over the Simulation Horizon
Treasury Yields (%)
12
11
10
9
8
7
6
5
4
0.08
0.17
0.25
0.5
1
2
5
10
20
25
30
4/1/85
10/1/85
4/1/86
10/1/86
Maturity (years)

Figure 5 - 1

environment increases GNMA prices.[9] As prices rise, the influence of the call options embedded in GNMAs increases dramatically, and the hedging task becomes significantly more complex. Thus, hedging GNMAs is most difficult and therefore most interesting in periods of falling rates.

After construction of the three data sets, the distributions of the K's in each set were tested for symmetry. This testing was advisable in view of the fact that the maximum value of K is one (occurs when the variation of the value of the hedged portfolio about the initial value is zero) while the minimum K is unlimited (occurs when the standard daily deviation [SDD] for the unhedged portfolio is zero):

$$K = 1 - \frac{\text{SDD (Hedged portfolio)}}{\text{SDD (Unhedged portfolio)}}$$

The testing was accomplished by computing the normalized third moment ("skewness") of the K's collected for each combination of research factors. The results of these tests indicated the distributions of K were not asymmetric. This knowledge was important to the extent that it allowed the use of standard deviations in testing hypotheses.

2. Testing Hypothesis I

Four research factors were identified to facilitate hypothesis testing. Each of these factors represents a category across which hedge effectiveness was expected to vary. The four factors are: 1) hedge type (static or dynamic), 2) GNMA duration measurement technique (inferred, option adjusted, option-estimated, implied, 12 year bullet, and a special benchmark, "True" duration[10]), 3) hedge strategy (T-note futures, T-bond futures, T-bond and Eurodollar CD futures in combination, and puts on T-bond futures), and 4) GNMA price range. The motivation for evaluating the alternatives available in each of the first three of these factors has been discussed earlier. The fourth factor, GNMA price range, has not been discussed and bears amplification.

Table 5-1 presents output from a hedge simulation for GNMA 10s. Preliminary review of these results reveals that in some periods K is negative. This result means that in certain circumstances hedge strategies actually increased the variation of the portfolio value about its initial level. Further study of the K matrix suggests that hedge performance varied depending on the price level of the GNMAs in the cash position. For this reason, hedge results were assigned to one of six categories, depending on the price of the GNMAs in the cash position on the initial day of the hedge. The six categories correspond to the following price ranges: 75-90, 90-97.5, 97.5-100, 100-102.5, 102.5-105, and 105-110.[11]

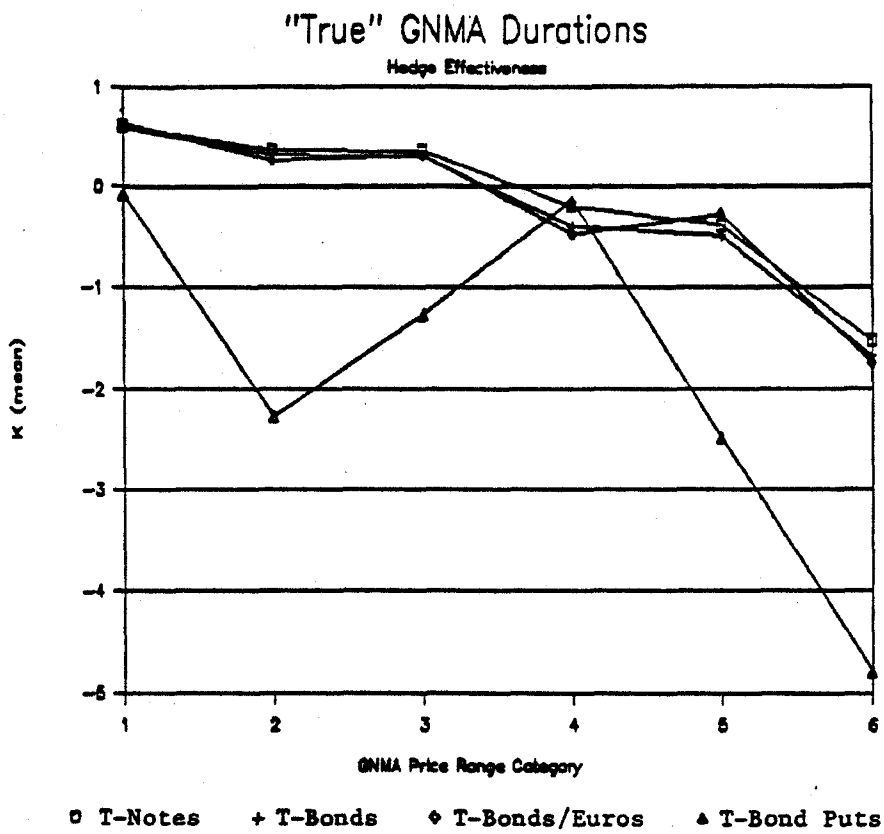
Two major considerations are reflected in the choice of price range boundaries. The first consideration is to provide a set of price ranges which are significantly different in terms of hedge effectiveness. The second concern is to define price categories which are large enough to accommodate observations sufficient in number to permit statistical testing. As will be noted later, these six price categories performed well on both counts.

Given the definition of these four research factors, t tests were performed on the K values grouped under each unique combination of factors.[12] These t tests were used to test Hypothesis I. K values significantly larger than zero indicate that the selected hedge reduced variation of portfolio value about the initial level; K's significantly less than zero indicate that the hedge actually increased variation about the initial level.

Summarizing the results of these tests is arduous because the number of unique experimental treatments is large. This problem notwithstanding, the general validity of Hypothesis I can be clearly rejected. The mean K for many treatments was significantly less than zero.

Figure 5-2 through 5-7 illustrate some trends in K values which occurred across GNMA duration measurement technique. Hedge effectiveness for each of these duration measures is presented in the following order: Figure 5-2: "True" GNMA Duration, Figure 5-3: Inferred GNMA Duration, Figure 5-4: Option-Adjusted GNMA duration, Figure 5-5: Option-estimated GNMA duration, Figure 5-6: Implied duration, and Figure 5-7: Twelve year bullet duration. The upper panel in each of these figures displays K values for all four strategies across all six GNMA price ranges. As shown, hedge effectiveness was extremely poor for GNMA cash positions the initial price for which fell into the super-premium 105-110 range. Additionally, hedge performance was very poor for strategy 4: T-bond futures puts.

Differences in the hedge effectiveness of strategies 1, 2, and 3 are more difficult to detect on the scale used in the upper panel of Figures 5-2 through 5-7. Accordingly, the lower panel of these figures is constructed to display K values for strategies 1, 2, and 3 across only the first five GNMA price ranges. The elimination of strategy 4: T-bond futures puts and the super-premium (105-110) GNMA range makes relative differences among these strategies more apparent. As shown in the lower panel Figures 5-2 through 5-7, strategy 1: T-note futures was consistently superior to strategy 2: T-bond futures and strategy 3:


"True" GNMA Durations
Hedge Effectiveness
1
0
-1
-2
-3
-4
-5
K (mean)
1
2
3
4
5
6
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros
T-Bond Puts

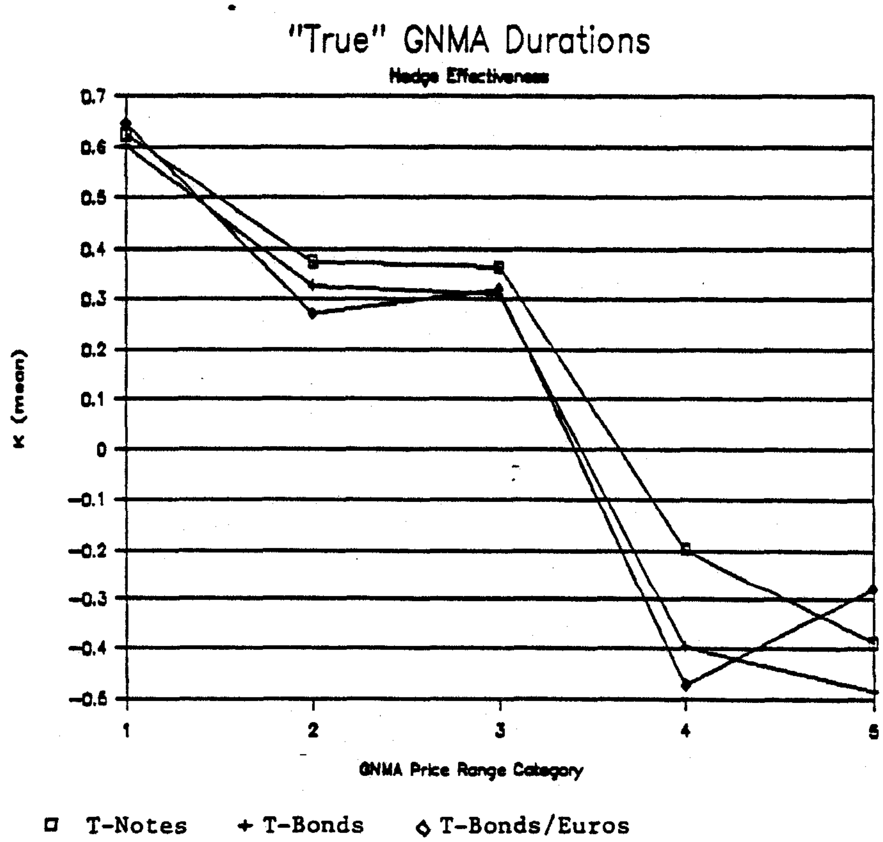
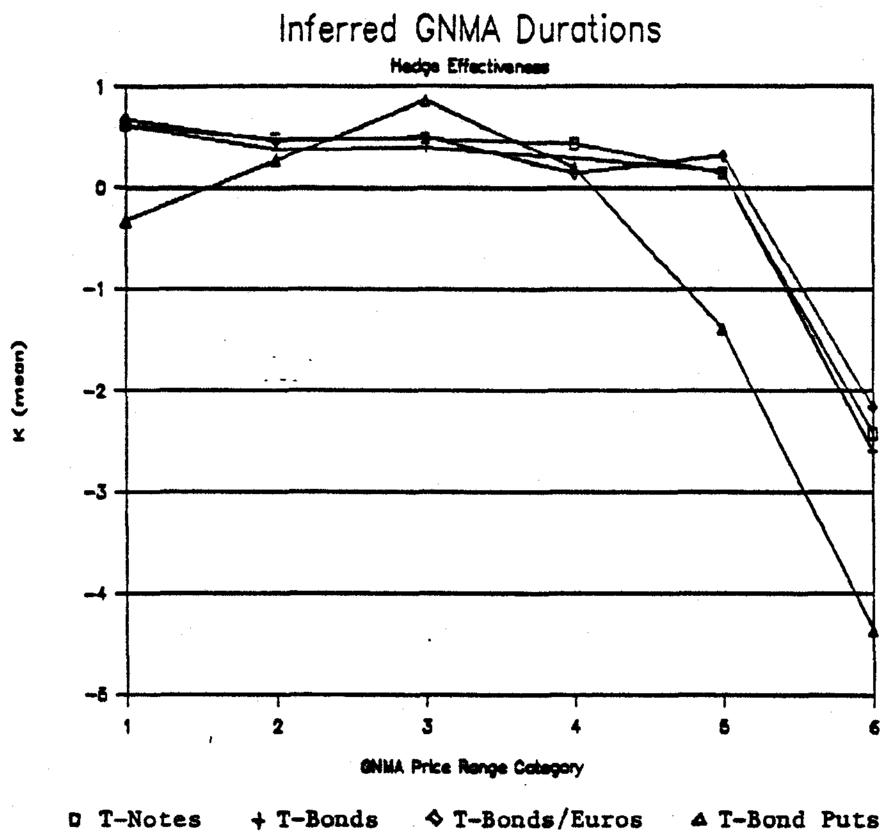
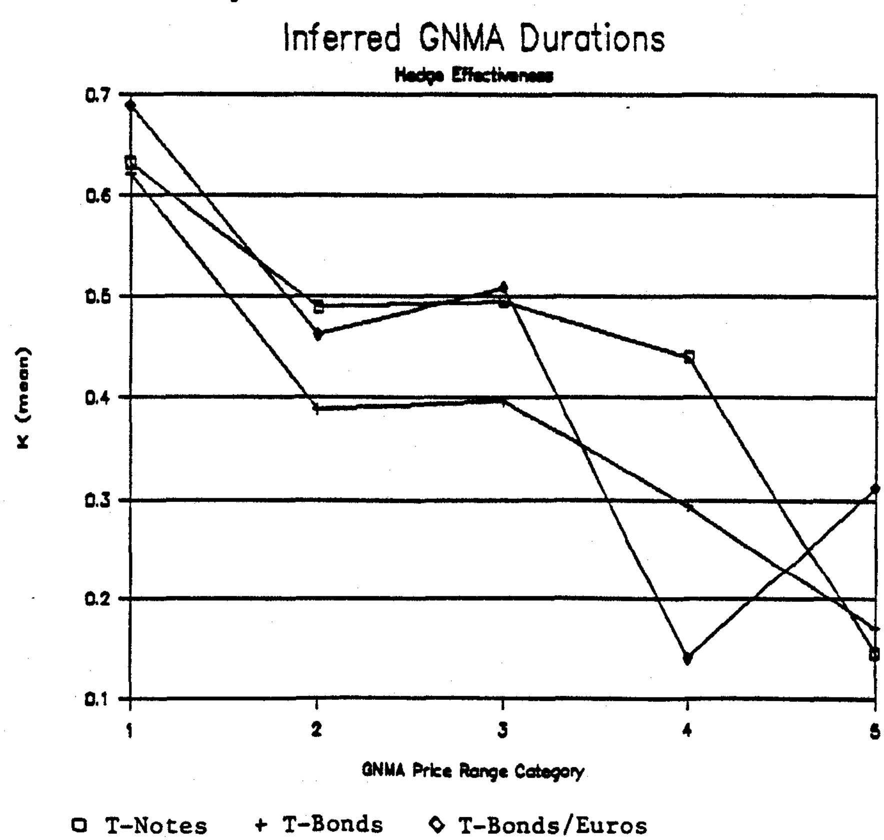

"True" GNMA Durations
Hedge Effectiveness
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
K (mean)
1
2
3
4
5
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros

Figure 5 - 2

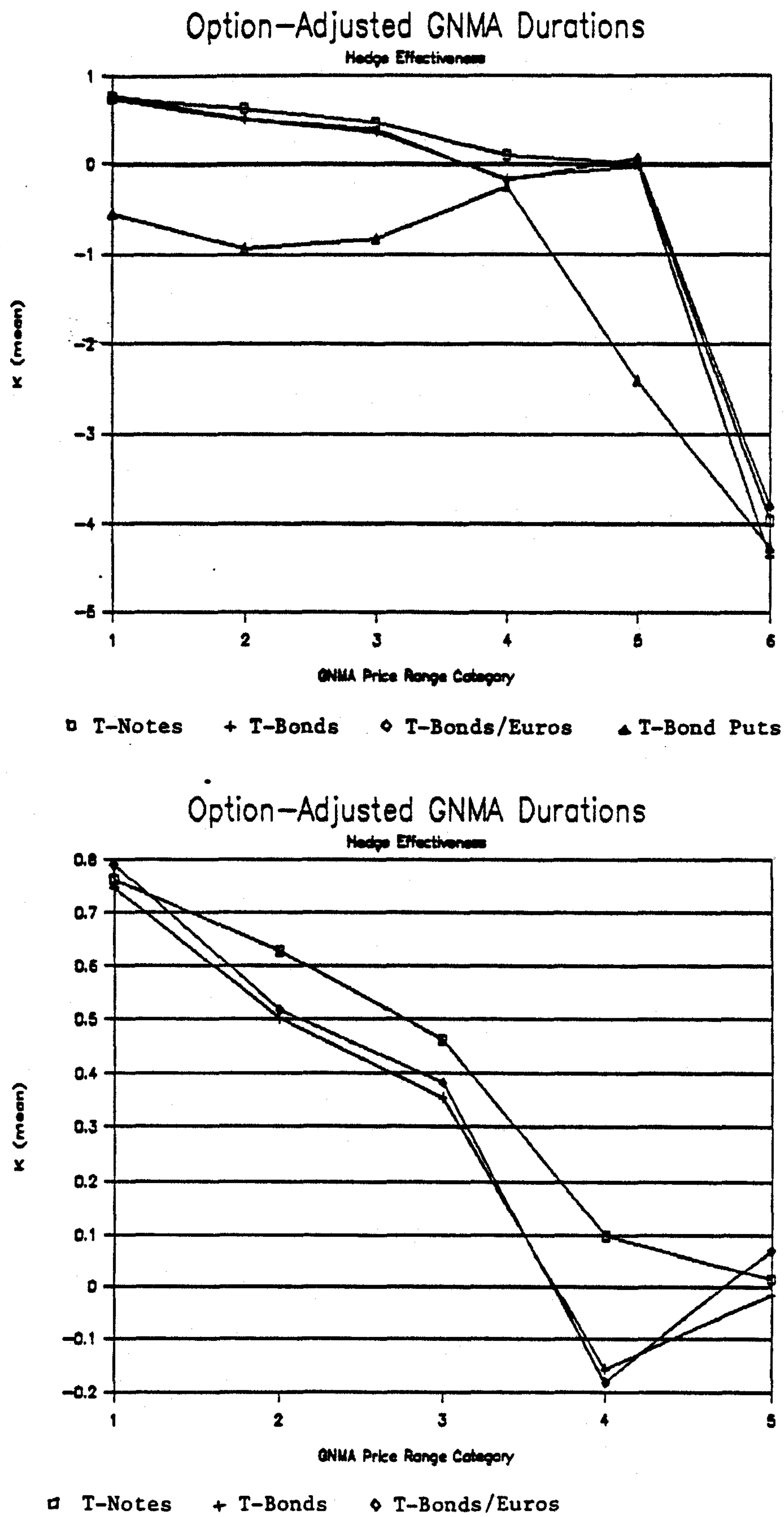


Inferred GNMA Durations
Hedge Effectiveness
1
0
-1
-2
-3
-4
-5
K (mean)
1
2
3
4
5
6
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros
T-Bond Puts



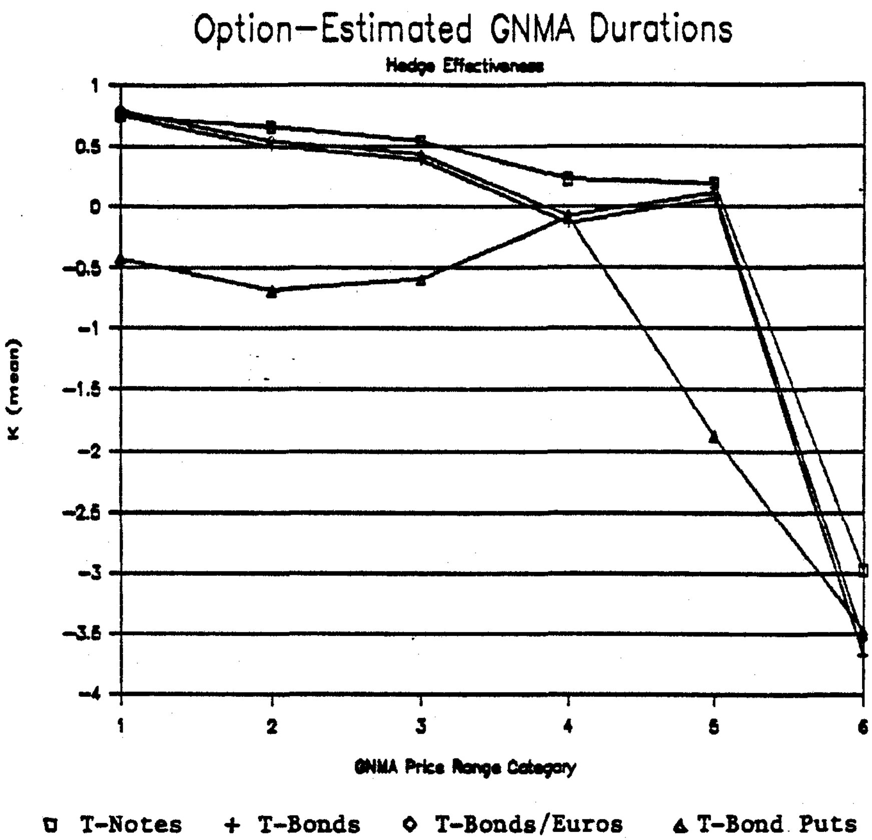
Inferred GNMA Durations
Hedge Effectiveness
0.7
0.6
0.5
0.4
0.3
0.2
0.1
K (mean)
1
2
3
4
5
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros

Figure 5 - 3



Option-Adjusted GNMA Durations
Hedge Effectiveness
K (mean)
GNMA Price Range Category
1
0
-1
-2
-3
-4
-5
1
2
3
4
5
6
T-Notes
T-Bonds
T-Bonds/Euros
T-Bond Puts
Option-Adjusted GNMA Durations
Hedge Effectiveness
K (mean)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1
2
3
4
5
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros

Figure 5 - 4


Option—Estimated GNMA Durations
Hedge Effectiveness
K (mean)
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros
T-Bond Puts

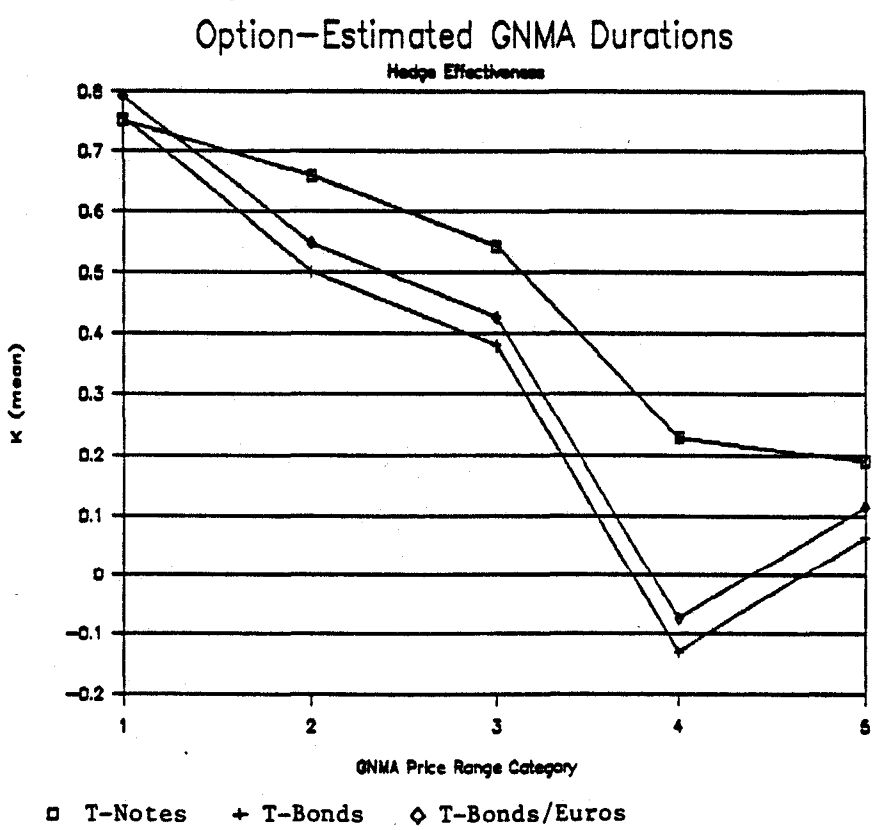
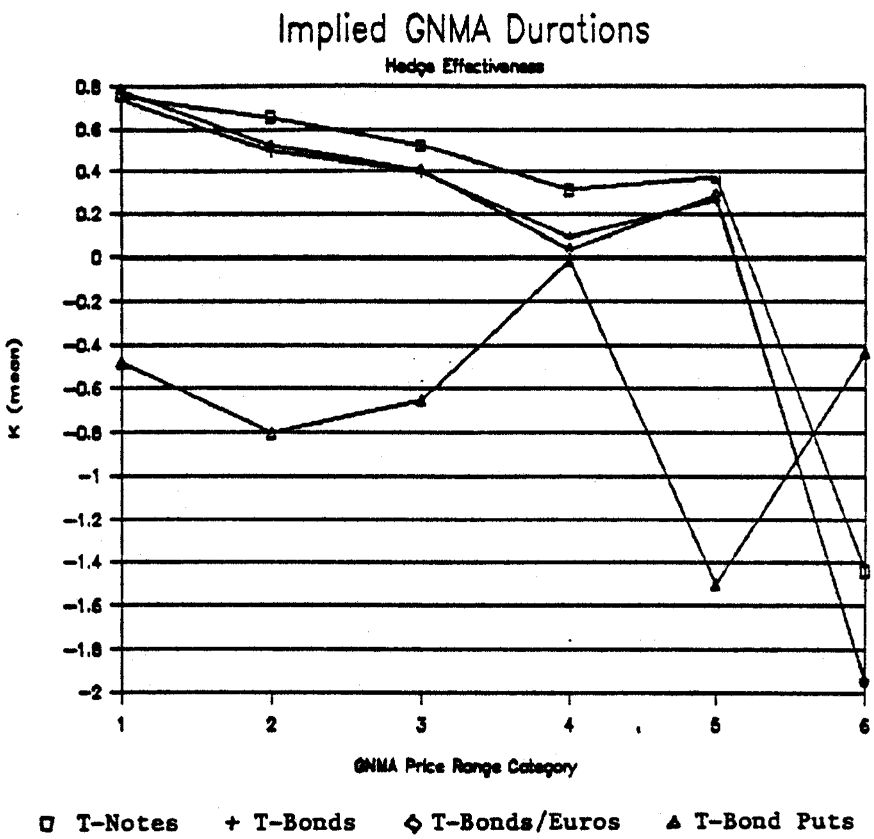

Option—Estimated GNMA Durations
Hedge Effectiveness
K (mean)
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros

Figure 5 - 5


Implied GNMA Durations
Hedge Effectiveness
K (mean)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
1
2
3
4
5
6
GNMA Price Range Category
T-Notes
+ T-Bonds
T-Bonds/Euros
T-Bond Puts

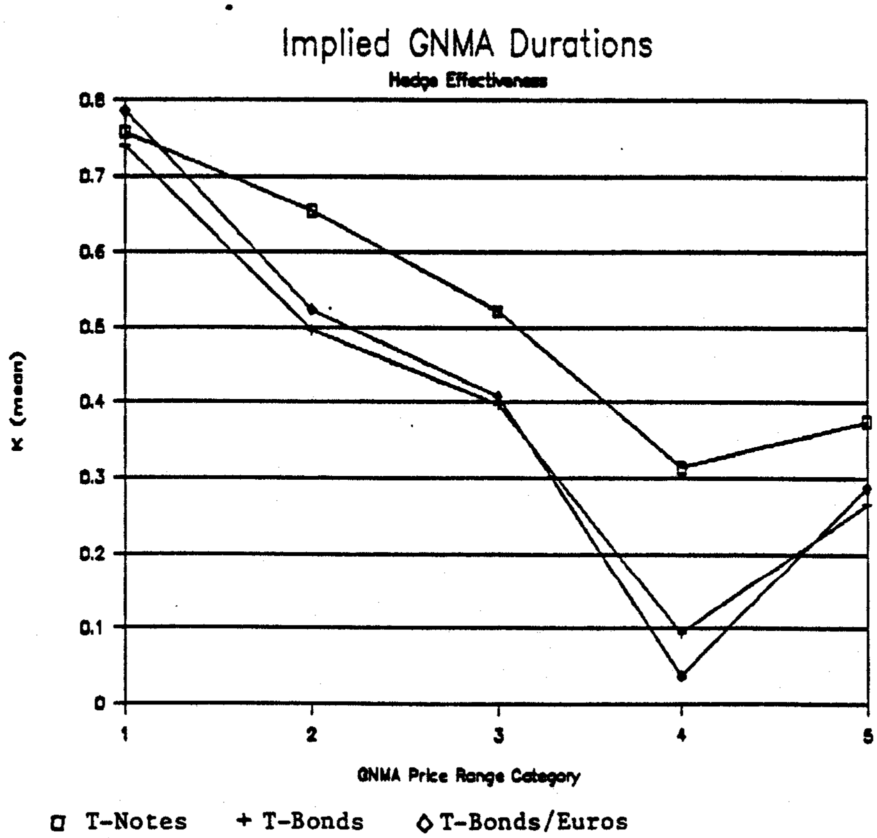
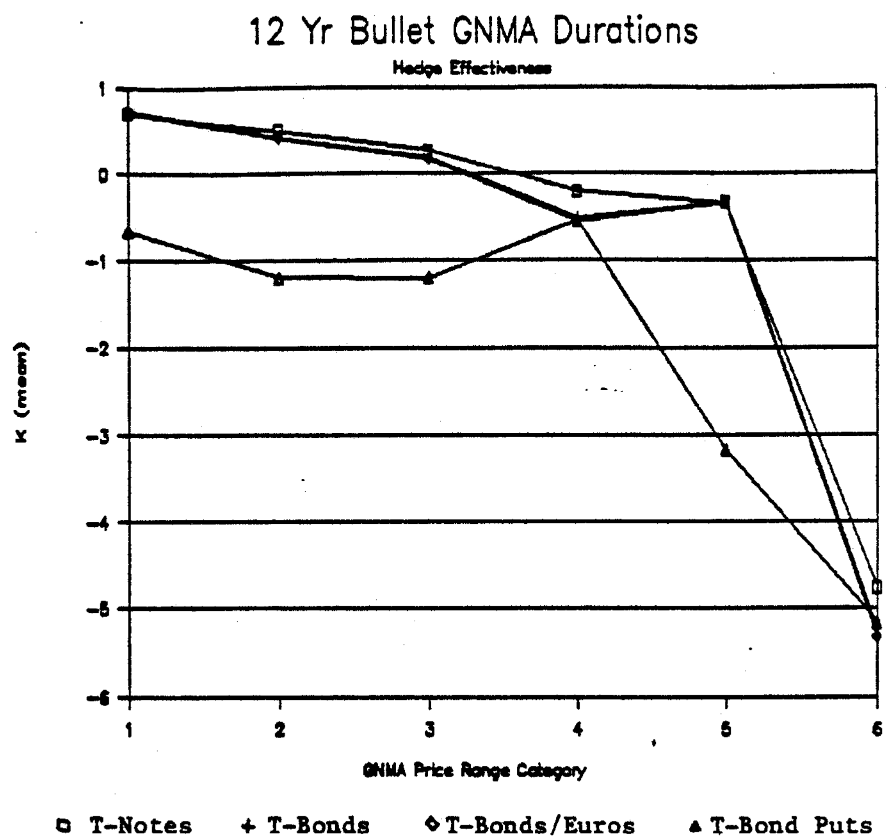
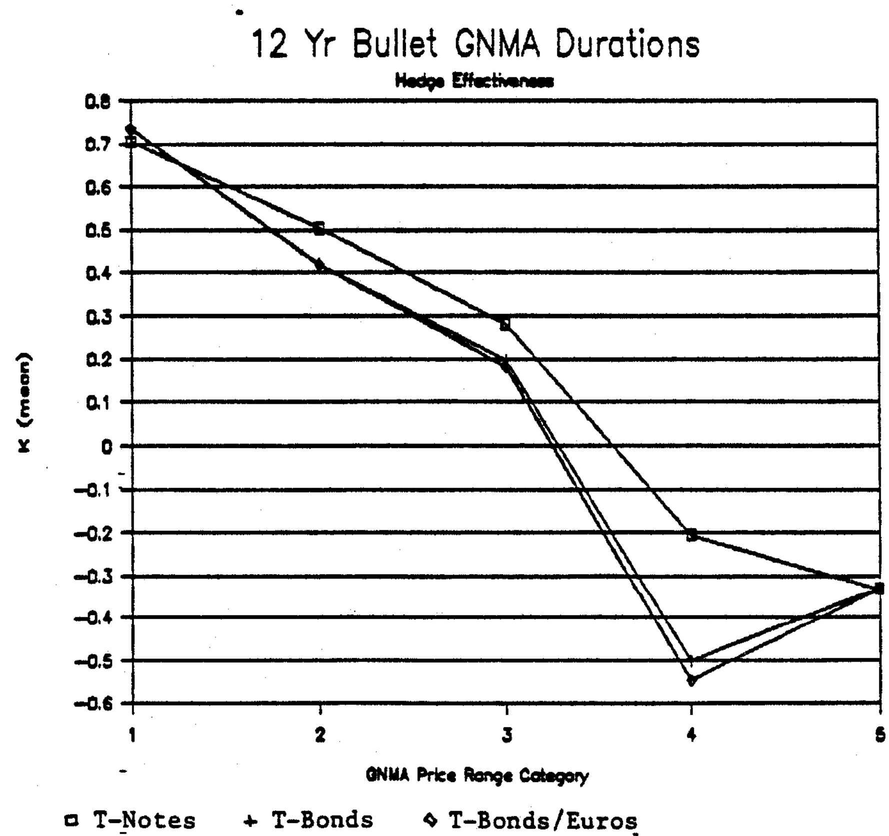

Implied GNMA Durations
Hedge Effectiveness
K (mean)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
GNMA Price Range Category
T-Notes
+ T-Bonds
T-Bonds/Euros

Figure 5 - 6



12 Yr Bullet GNMA Durations
Hedge Effectiveness
K (mean)
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros
T-Bond Puts



12 Yr Bullet GNMA Durations
Hedge Effectiveness
K (mean)
GNMA Price Range Category
T-Notes
T-Bonds
T-Bonds/Euros

Figure 5 - 7

T-bond and Eurodollar futures. The reasons for these trends are considered at length under the general discussion of hedge analysis which follows the present section on hypothesis testing.

3. Testing Hypotheses II and III

Testing Hypothesis I involved testing whether the mean K value for a given experimental treatment was significantly different from zero. Testing Hypotheses II and III, reiterated in the introduction to this chapter, differs in that it requires comparing K's from different experimental treatment. This process requires the pooling of variances among treatments. This study accomplishes this task by performing an analysis of variance (ANOVA) of a general linear regression equation encompassing four main effects and six two-way interactions. (The details of this methodology are presented in the Appendix.) The form of this regression equation is as follows:

$$K = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_1 x_2 + b_6 x_1 x_3 + b_7 x_1 x_4 + b_8 x_2 x_3 + b_9 x_2 x_4 + b_{10} x_3 x_4 \quad (1)$$

where b_0, b_{1-4} b_{5-8} are estimated regression coefficients and x_{1-4} are categorical variables representing the four research factors: GNMA price range, hedge strategy, hedge type (static or dynamic), and GNMA duration measure.

The analysis was repeated for each of the three data sets. Preliminary results indicated that two duration measures (option-adjusted and 12 year bullet), one strategy (T-bond futures puts), and one price range (super-premiums: 105-110) were responsible for most of the poor hedge results. A review of Figures 5-2 through 5-7 and the t tests in the previous section supported these results. The possibility that the somewhat extreme (and negative) K's observed for these treatments were adding a great deal of variance to the pooled variances, which in turn was obscuring significant distinctions among other treatments, recommended that the ANOVA be repeated for data sets devoid of these troublesome factor combinations.

The results of the second stage ANOVAs indicated that the vast majority of the interaction terms were not significant. This finding in turn led to reduced models which were tested with the ANOVA procedure.

Figure 5-8 summarizes the surprising results of these tests. For Hypothesis II to be accepted, GNMA duration measurement technique 3 (option-adjusted) or 4 (option-estimated) must produce significantly better

FIGURE 5-8: MEAN K VALUES

Duration Model	Period 1 Data Set	Period 2 Data Set	Period 3 Data Set
Implied	.518	.268	.396
Inferred	.456	.231	.352
Option-Estimated	.481	.193	.336
Hedge Type			
Static	.498	.252	.396
Dynamic	.471	.210	.327

Note: Scheffe's range tests support the ANOVA results. They indicate that not one of the three duration models is significantly different from the others. Similar analysis reveals no significant differences between static and dynamic hedge types.

hedging results than any of the other approaches. As noted earlier, the option adjusted technique was identified in the first ANOVA as significantly inferior to every technique except the 12 year bullet method. Thus, the prospects for accepting Hypothesis II hinge on the performance of the option-estimated approach. As shown in Figure 5-8, in not one of the three data sets was this method found to be significantly better (higher K) than the implied or inferred methods. In point of fact, the implied method was found uniformly to produce the best hedge results.[13,14] Thus, Hypothesis II is rejected.

The ANOVA results summarized in Figure 5-8 produce a similar result for Hypothesis III. As indicated, no significant difference was found for hedges which were adjusted on a dynamic basis when compared to hedges which were not adjusted. This result is surprising and justifies a rejection of the Hypothesis.

4. Other Observations: The Significance of Strategy and GNMA Price Range

The most significant differences in hedge effectiveness are those detected across GNMA price ranges. Even with the deletion of the super-premium range, significant differences were found across most price ranges in each of the three data sets. In general, hedge effectiveness was highest for the deep discount GNMAs found in range 1 (75-90) and next best for the discounts found in range 2 (90-97.5). Both of these ranges provided hedge effectiveness which was significantly better than that found in ranges 3-5 in each data set. The ordering of the remaining three ranges varied somewhat.

ANOVA results produced less significant results for hedge effectiveness evaluated across strategies 1-3. Although in each data set the mean K for strategy 1 (T-note futures) was greater than that for strategy 3 (T-bonds and Euros) which was greater than that for strategy 2 (T-bonds), in only one of the three data sets was there a significant difference among the three strategies.

Taken together, the results of the statistical procedures discussed in this section are somewhat surprising. Each of the three hypotheses introduced in earlier chapters as representative of conventional wisdom has been firmly rejected by the evidence at hand. It is left to the ensuing section to explain and interpret these findings.

C. Results: Analysis and Interpretation

1. GNMA Duration Measures

a. "True" Duration

The statistical analysis described in the preceding section produced the surprising result that efforts to hedge GNMAs can in some cases increase the variation of portfolio value about the initial level. To the extent that this variability is identified as interest rate risk, efforts to hedge GNMAs can actually be counterproductive. This finding is especially relevant when the GNMAs being hedged lie in the super-premium price range. Thus, Hypothesis I is not generally valid.

As was pointed out earlier, this conclusion is based on analysis of hedges simulated during a period of steadily falling interest rates. The failure of hedge strategies for the super-premium GNMAs can be attributed in part to the failure of the various GNMA duration measurement techniques to adequately describe the price performance of the super-premium GNMAs. Judging by the steady declines in mean hedge effectiveness apparent with increasing GNMA price in Figures 5-2 through 5-7, the usefulness of each of these measurement techniques seem to decline with increasing GNMA price.

This observation encourages a review of the GNMA duration values produced by each of the various techniques. A useful starting point in this regard is the set of values produced by the benchmark "true" duration. (Recall that this measure is computed on an ex post basis as the percentage change in the GNMA price divided by the observed change in rates. Thus, by construction, percentage price change equals the change in rates times "true" duration.) Figure 5-9 contains a plot of true durations against GNMA price.

Figure 5-9 illustrates some rather startling results. As GNMA price approaches par, true duration begins to gyrate wildly. Computed values range from fifteen years to negative thirty years.

This finding has several important implications. Perhaps the most important of these implications is that GNMA prices appear to be influenced by factors separate from interest rates. In particular, changing expectations for future prepayment rates and for future interest rates affect GNMA prices in a manner apart from changes in the level of interest rates. In addition, changes in the expected supply of and demand for GNMAs influence prices. Thus, GNMA prices are affected by many "technical factors" as well as by changes in interest rates.

The realization that GNMA prices are influenced by technical factors is important to the interpretation of

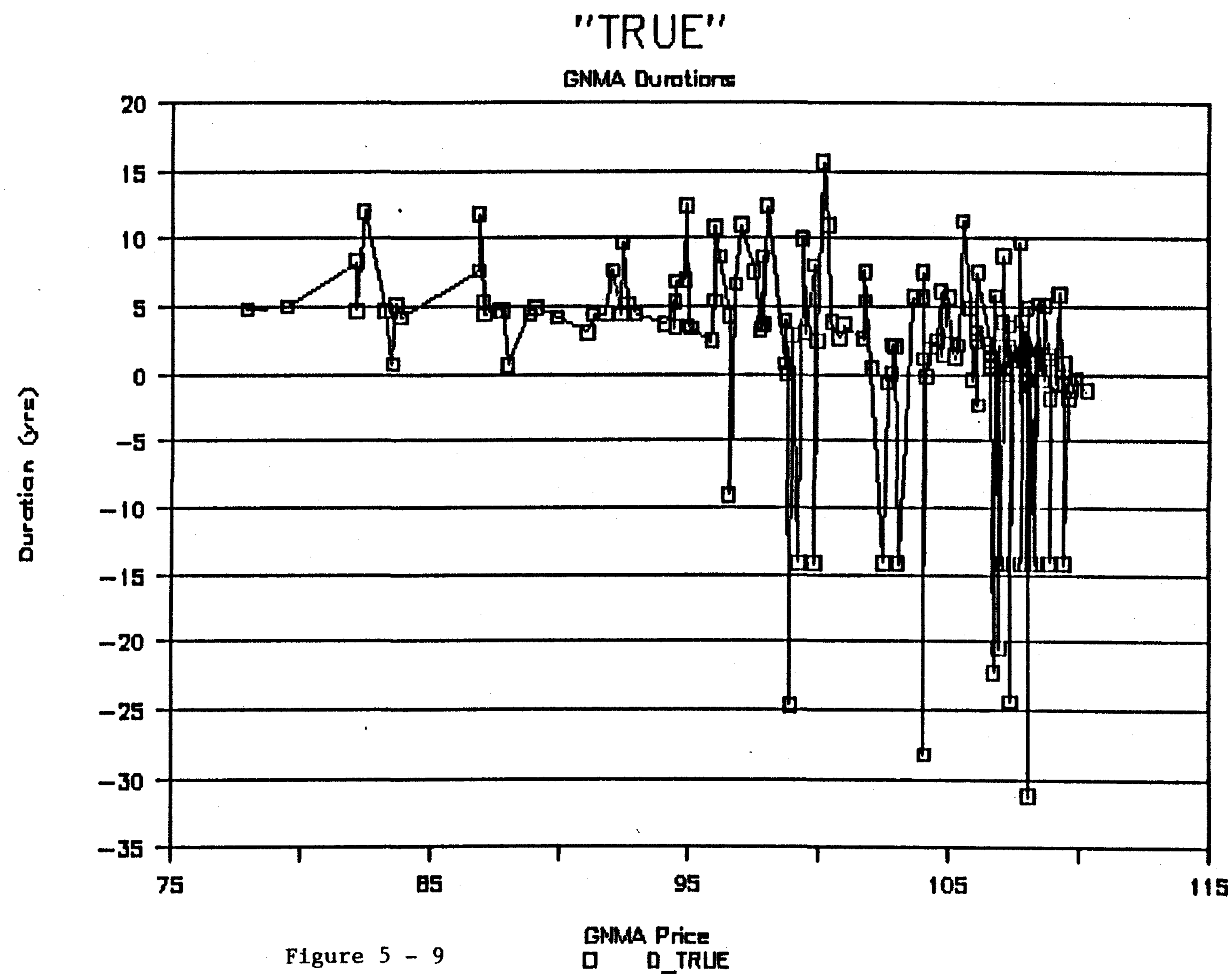

"TRUE"
GNMA Durations
Duration (yrs)
20
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
75
85
95
105
115
GNMA Price
D_TRUE

Figure 5 - 9

hedge results. This research has been designed as a study of interest rate hedging. Accordingly, hedge ratios have been developed on the basis of Macaulay duration. Implicit in this approach is the assumption that GNMA price changes are a result of changes in interest rates. To the extent that GNMA prices respond to technical factors apart from interest rates, hedge performance is likely to suffer; further, the deleterious influences of these technical factors is likely to be attributed to failures in the duration measurements used for the GNMAs. Thus, an important problem in interpreting hedge results concerns separating the influence of the technical and systematic (interest rate changes) factors. The large fluctuations in "true" duration are indicative of the dimensions of the problem.

Unfortunately, separating the influence of technical effects from those related to interest rate changes is extremely difficult. As noted previously, many of these technical effects are related to changes in expectations, which are unobservable. Thus, the interpretation of results proceeds with the understanding that the presence of technical effects may cloud distinctions in hedge effectiveness resulting from the various duration measurement techniques.

The wide range of true duration depicted in Figure 5-9 has other important implications. In general, it confirms the sentiments of many market participants to the effect that the GNMA market experienced great uncertainty during the period under study. Diller crystallized the views of many in explaining that the general valuation framework for mortgage securities - the prevailing paradigm - had collapsed during the period.[15] Market participants struggled to deal with the unprecedented volatility in this market by developing many new approaches to evaluating the risk return characteristics of GNMAs. (Indeed, the five duration measures tested in this study are evidence of this trend.) The result of this diversity of approaches was a market characterized by much disagreement about value.[16] This disagreement was translated into tremendous "technical" volatility, as prices responded to an extremely nonhomogeneous set of investor expectations. From the standpoint of interest rate hedging, these developments introduce a great deal of nonsystematic variation which is not reflected in duration based hedge ratios. Hedge effectiveness is expected to suffer as a result.

A third implication of the true durations shown in Figure 5-9 is that negative duration values may at times be appropriate for super-premium GNMAs. In this context, negative duration values accurately describe the behavior of GNMAs whose prices fall when rates fall. This phenomenon occasionally arises for premium GNMAs. A fall in rates often generates expectations for increases in prepayment rates. Increases in prepayment rates reduce the length of time over which premiums may be amortized, thus reducing rate of return. Price declines for GNMA premiums may therefore be observed during periods of falling rates. A related observation is that, unlike traditional fixed income securities, GNMAs exhibit negative convexity. Traditional fixed income securities, such as Treasury bonds, experience increases in duration as rates fall. This increase occurs because at lower rates the more distant payments assume greater present value weights in the duration formula, thereby increasing the overall weighted average, or duration. GNMAs, on the other hand, experience decreases in duration as rates fall. Once again, this effect develops because lower interest rates increase prepayment rates, shortening duration. These observations are useful in the discussion of strategies presented later in this section.

To summarize, the true duration values presented in Figure 5-9 are indicative of some important considerations:

1. Technical factors apart from interest rates influence GNMA prices and vitiate interest rate hedges.
2. The period under study was characterized by great volatility in the GNMA market, much of it related to technical factors such as expectations.

3. Super-premium GNMA securities can be expected to exhibit negative convexity and, to the extent that the market fails to anticipate increased prepayments and thus "over-prices" them, negative duration.

b. Clayton-Goldstein Inferred Duration

Figure 5-10 displays duration values computed with the Clayton-Goldstein inferred model discussed in Chapter IV. Duration values are fairly stable and gradually declining as GNMA price rises toward par. The range of values widens substantially beyond par.

At first glance, the range of these values is hard to explain. Recalling that this method relies on inferences made from market prices clarifies the matter somewhat. The large range in duration values shown for par and premium GNMAs is reasonable if the market distinguishes between premium GNMAs on the basis of the length of time they have spent above par. The fact that this method produced hedge results significantly better than those produced by the twelve year bullet benchmark (for which premium durations are all in the 5.5 to 6.5 year range) and not significantly different from the regression method (for which premium durations fall steadily from four years to two as price increases from 100 to 110) suggests that this may indeed be the case.[17] Thus, the inferred

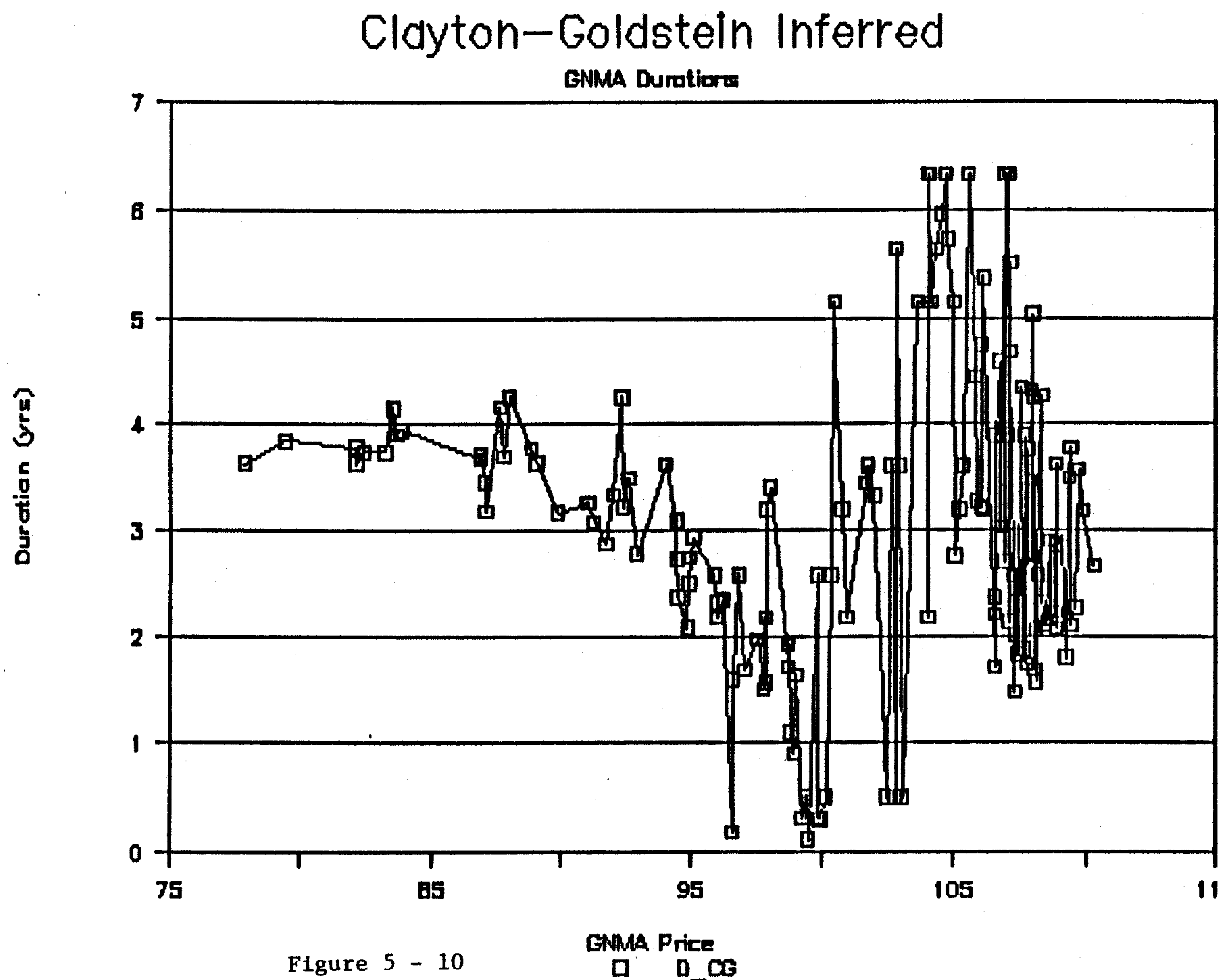

Clayton—Goldstein Inferred
GNMA Durations
Duration (yrs)
0
1
2
3
4
5
6
7
75
85
95
105
115
GNMA Price
D_CG

Figure 5 - 10

duration method introduced in this study shows promise and merits further testing.

c. Option-Adjusted GNMA Durations

As noted in Chapter IV, two different approaches were used to compute option-adjusted GNMA durations. The first method, referred to as the market implied or option-adjusted approach, calculated durations on the basis of call option values implied by the difference between the observable GNMA market price and the computed value of the annuity component. The second method, called the option-estimated technique, valued the call component based on the observed volatility of the ten year yield and calculated annuity values. The results of these two approaches are presented in Figure 5-11.

Despite the areas of overlap in this illustration, it may be observed that the option-estimated technique is more successful than the option-adjusted technique in capturing the negative convexity and lower duration values expected for premium GNMAs. The option-adjusted approach exhibits a higher level and a broader range of duration values in this price range. These results are undoubtedly a consequence of the turmoil prevalent in the mortgage markets in this period.

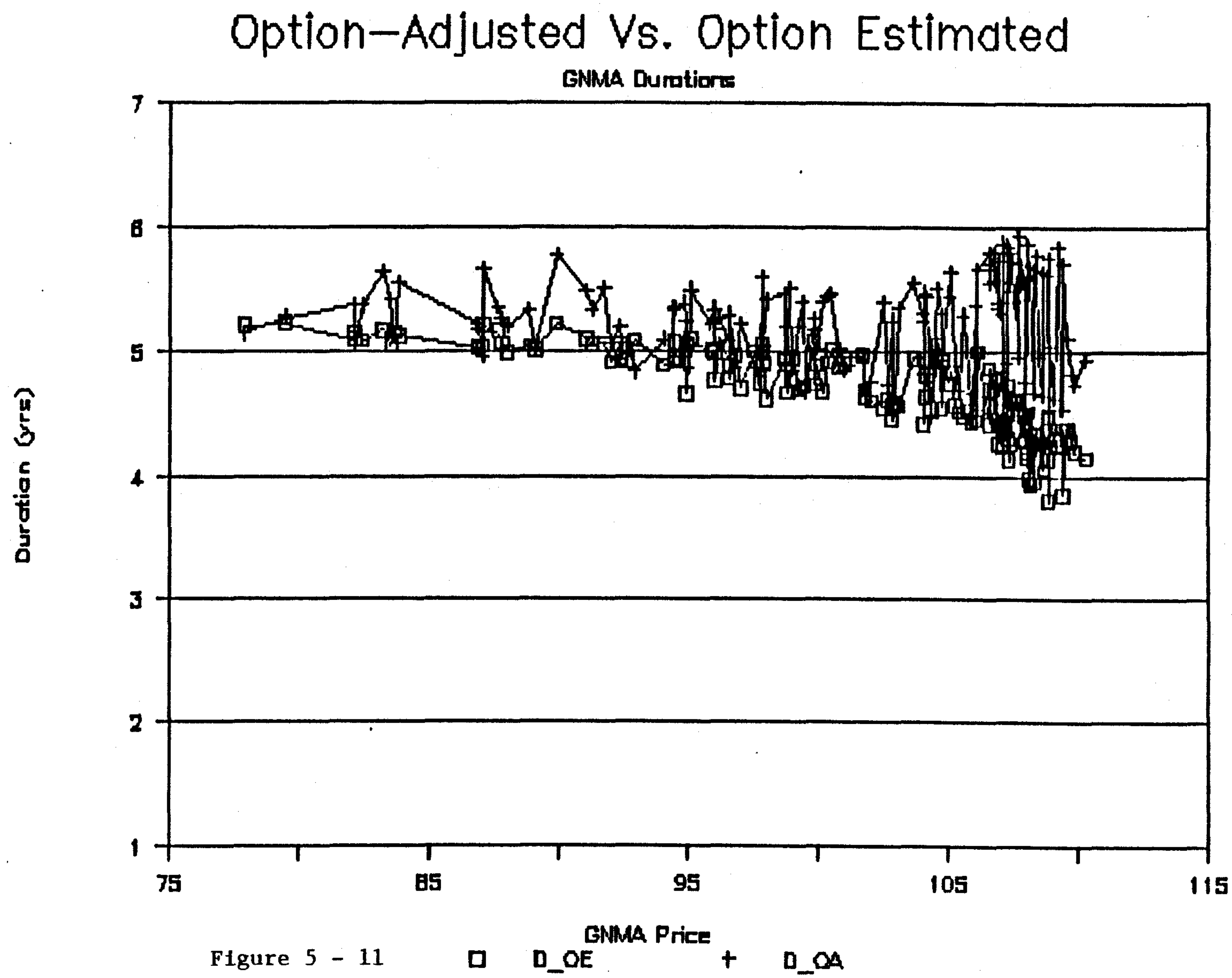

Option—Adjusted Vs. Option Estimated
GNMA Durations
Duration (yrs)
7
6
5
4
3
2
1
75
85
95
105
115
GNMA Price
D_OE
D_OA

Figure 5 - 11

Figure 5-12 compares the performance of the option-estimated method to the twelve year bullet approach. As shown, the option-estimated method produces significantly lower duration values, especially for the premium GNMAs.

The realization that the option-estimated method produces lower duration values than either the option-adjusted approach or the twelve year bullet technique assumes importance when considered in light of the statistical analysis which revealed that the option-estimated approach produced hedge ratios which were significantly more effective than those produced by these two other methods. In particular, this finding indicates that the option-estimated approach represents a significant advance over the twelve year bullet method in terms practical as well as theoretical.

d. Implied (or Regression) GNMA Durations

Figure 5-13 displays the implied duration values computed using the Pinkus-Chandoha method. It also displays the option-adjusted values discussed in the preceding section as a point of reference. As shown, the regression method produces lower duration values for premium GNMAs than any of the previous approaches. In addition, it attributes a great deal of negative convexity to premium GNMAs. The

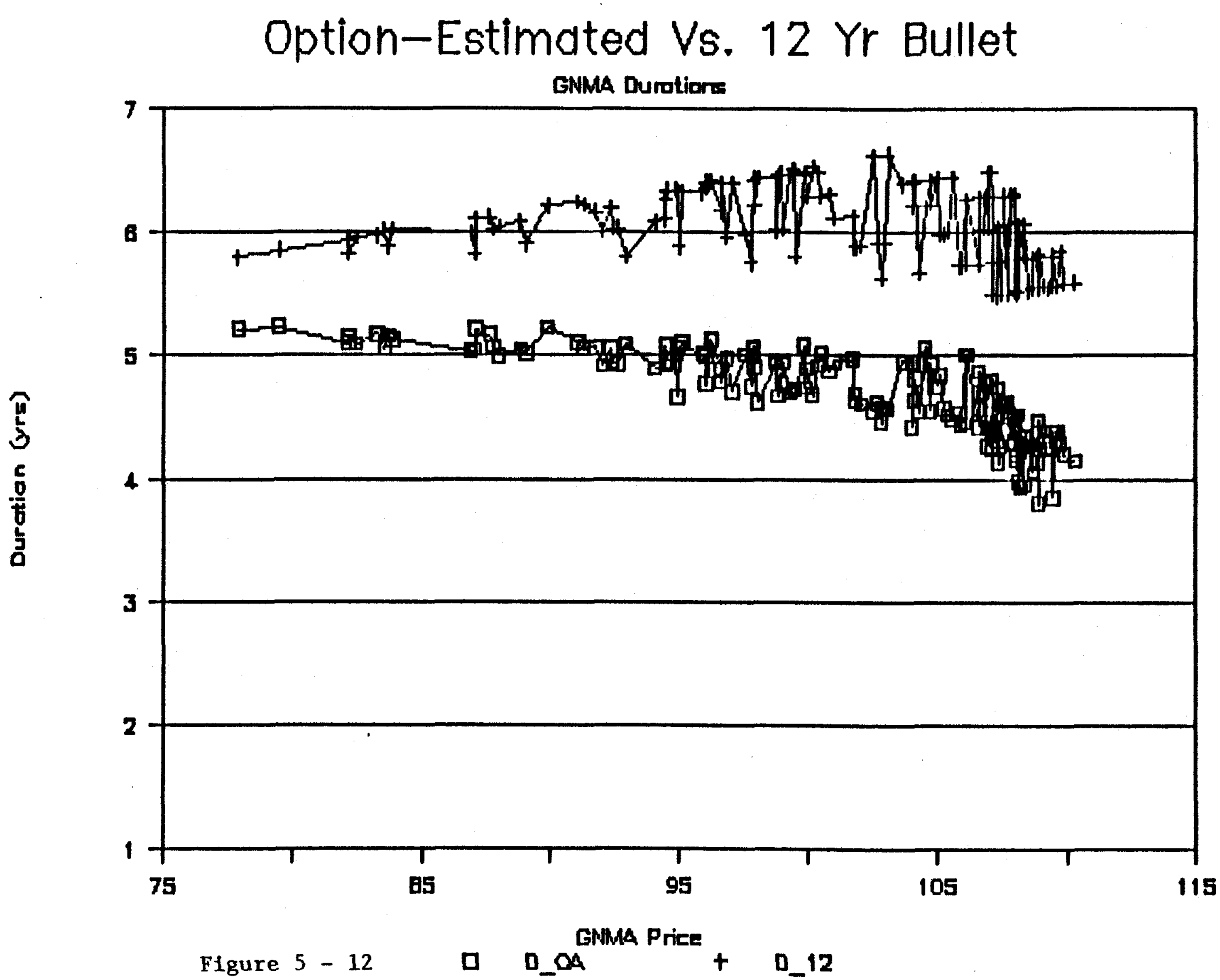

Option—Estimated Vs. 12 Yr Bullet
GNMA Durations
Duration (yrs)
7
6
5
4
3
2
1
75
85
95
105
115
GNMA Price
D_OA
D_12

Figure 5 - 12

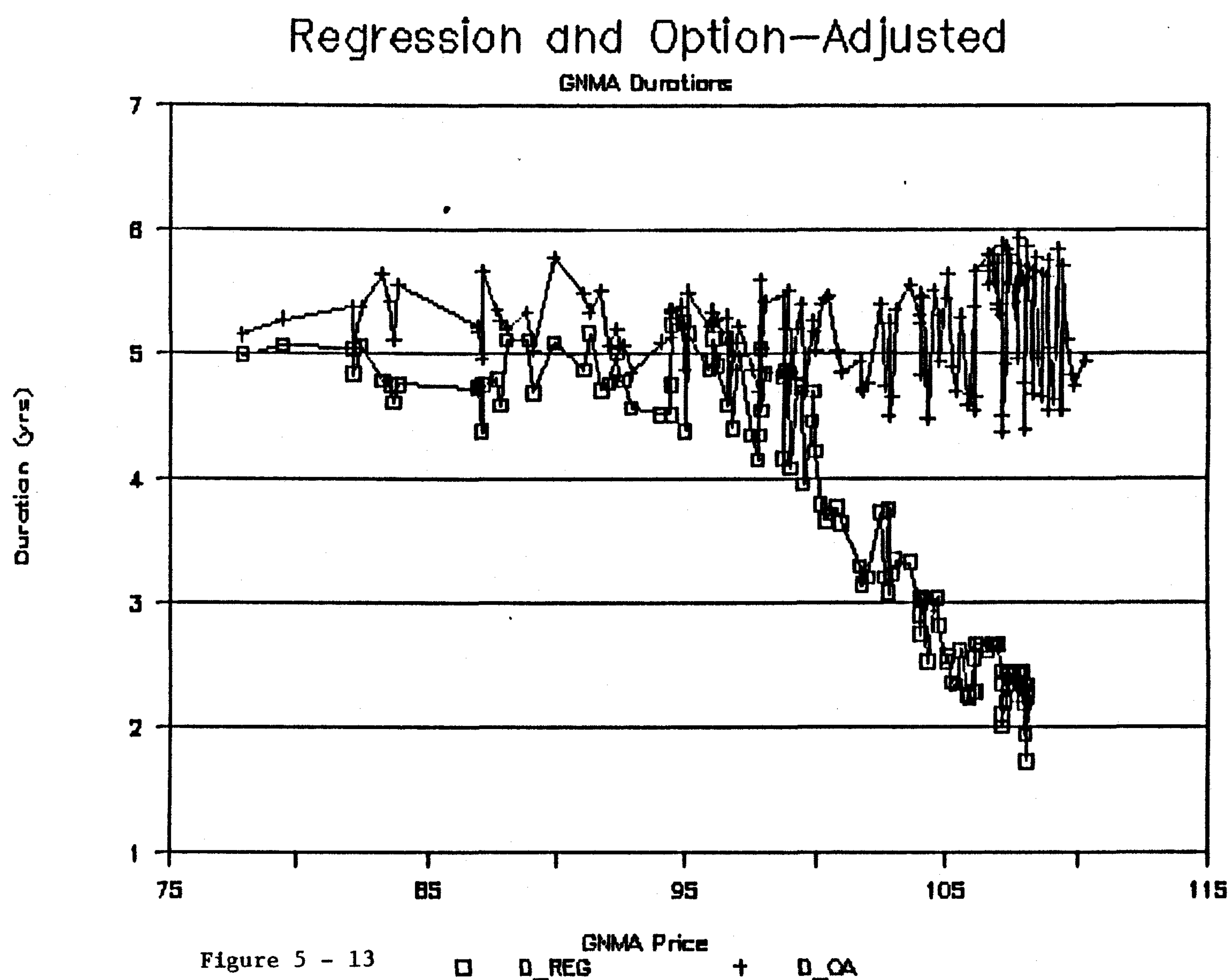

Regression and Option—Adjusted
GNMA Durations
Duration (yrs)
7
6
5
4
3
2
1
75
85
95
105
115
GNMA Price
D_REG
D_OA

Figure 5 - 13

superior hedge effectiveness of this method suggests these characteristics are aptly represented in duration values computed with this technique.

e. GNMA Convexities

It was pointed out in the section addressed to hypothesis testing that the regression, Clayton-Goldstein inferred, and option-estimated duration measurement techniques produced results which were significantly better than those produced by the option-adjusted and twelve year bullet approaches. It is interesting to note that these three approaches share two characteristics: each produces lower duration estimates than the two inferior measures, and each produces some negative convexity values, whereas the inferior measures do not. Figures 5-14 and 5-15 illustrate this finding for the Clayton-Goldstein inferred, option estimated, and twelve year bullet approaches.[18]

2. Analysis of Hedge Strategies

In the discussion of duration-based hedges in Chapter IV, it was noted that a "parametric matching" technique should be employed in constructing hedges. The essence of this technique is summarized in a concise hierarchy of rules:

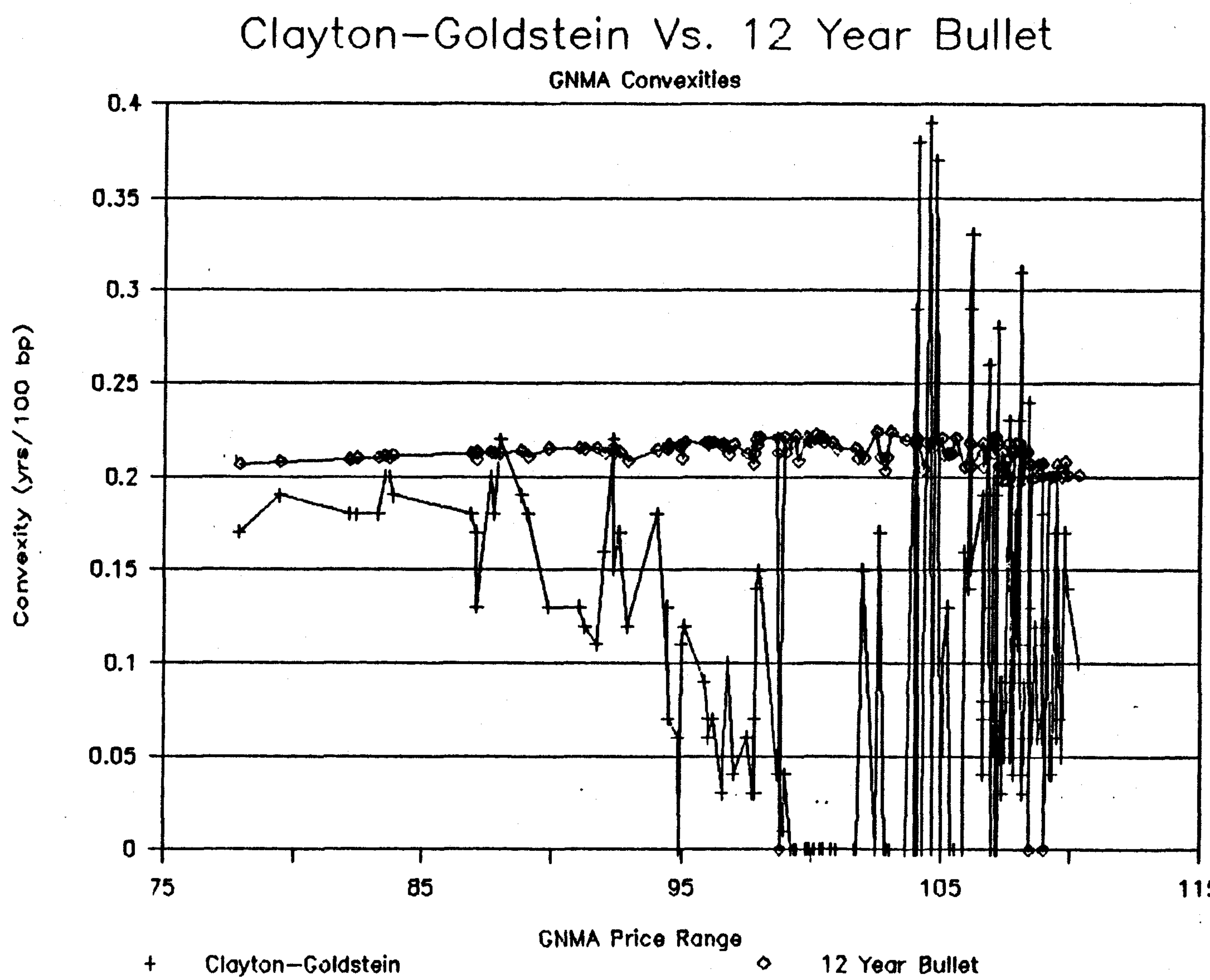

Clayton–Goldstein Vs. 12 Year Bullet
GNMA Convexities
Convexity (yrs/100 bp)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
75
85
95
105
115
GNMA Price Range
+ Clayton–Goldstein
◇ 12 Year Bullet

Figure 5 - 14

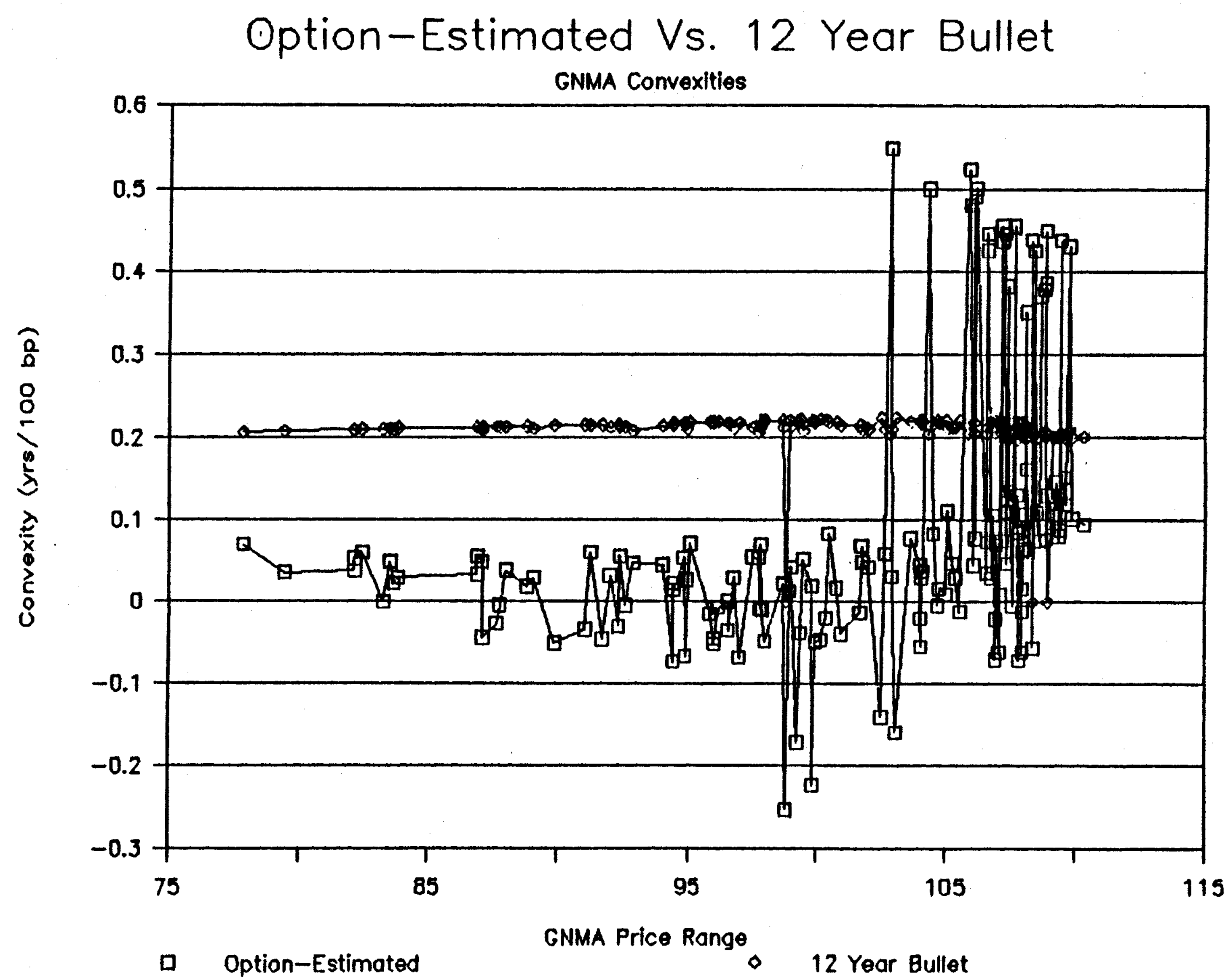

Option−Estimated Vs. 12 Year Bullet
GNMA Convexities
Convexity (yrs/100 bp)
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
75
85
95
105
115
GNMA Price Range
Option−Estimated
12 Year Bullet

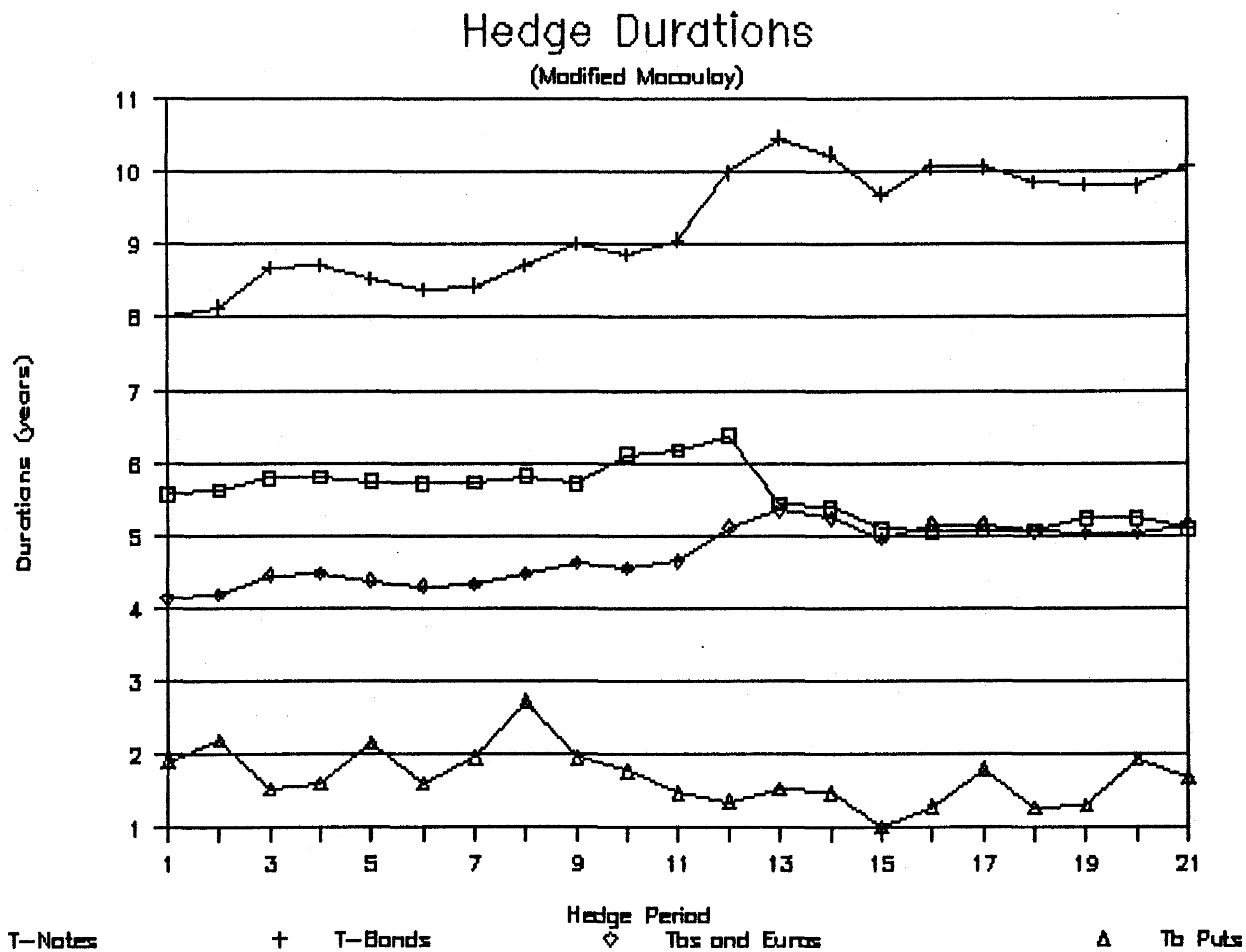
Figure 5 - 15

1. The first priority in designing a hedge is to match the modified duration of the cash instrument and the modified duration of the hedge instrument (presumed to be shorted). This is tantamount to matching the first order terms in the Taylor series expansions of price changes for both positions.

2. The second priority is to match higher order terms, beginning with the second. Matching achieved in these higher order terms ensures that the initially matched duration values remain matched as yield levels change.

On this basis, it is advisable to analyze the parametric characteristics of the four alternative hedge strategies as a complement to the analysis of GNMA durations and convexities performed in the preceding section. Such an analysis provides insight into the differential effectiveness of the four strategies. It also sheds light on the value of considering convexity in the process of designing hedges.

Figure 5-16 presents modified Macaulay duration values for each of the four hedge strategies.[19] These values were computed as of the day prior to the first day in the hedge period for each of the twenty-one periods considered. As shown, Strategy 2: T-bond futures durations are highest,



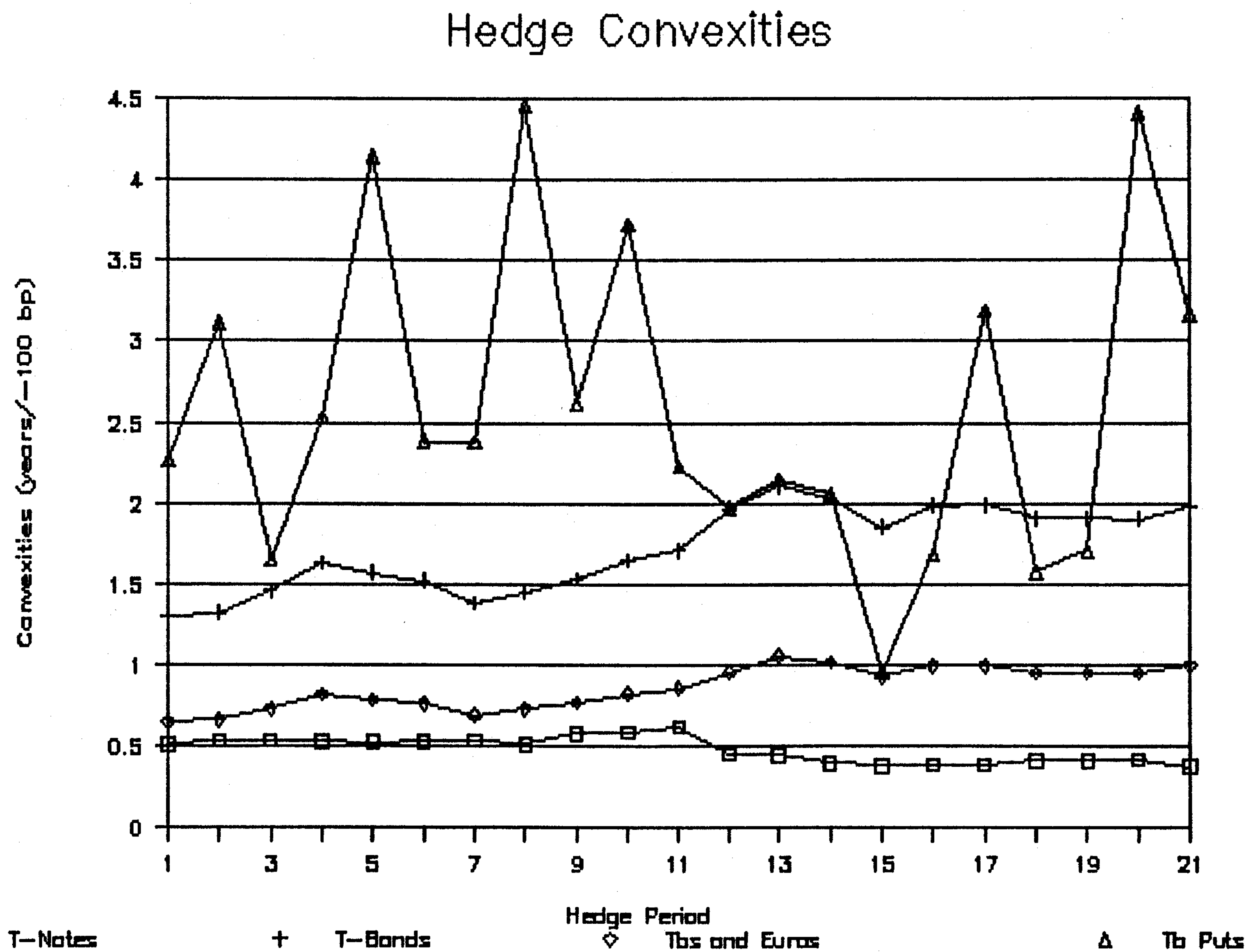
Hedge Durations
(Modified Macaulay)
Durations (years)
11
10
9
8
7
6
5
4
3
2
1
1
3
5
7
9
11
13
15
17
19
21
Hedge Period
T—Notes
T—Bonds
Tbs and Euros
Tb Puts

Figure 5 - 16

ranging from eight to ten years. The T-Note futures durations, on the other hand, appear in the 5.0 to 6.5 year range. Sudden changes in duration values are apparent in both of these curves at points where the assumed cheapest to deliver security (and its associated duration) changes. This effect is less prominent in the behavior of Strategy 3: T-Bonds and Eurodollar CD futures. The duration values for this strategy lie between 4.0 and 5.5 years. Finally, Strategy 4: T-Bond futures puts durations appear near the bottom of Figure 5-16, ranging between 1.5 and 2.5 years.

Figure 5-17 displays the convexity values computed for each hedge strategy. Strategy 4: T-bond futures puts exhibits the most convexity, while all the other strategies are ordered as they were in terms of duration with one notable exception. Strategy 3: T-bonds and Euros consistently exhibits more convexity than Strategy 1: T-note futures

The hedge durations and convexities presented in Figures 5-16 and 5-17 may be combined with the GNMA durations and convexities appearing earlier to gain insight into the effectiveness of the four hedge strategies. By way of review, statistical analysis of simulation results indicated that Strategies 1,2, and 3 were significantly more effective than Strategy 4: T-bond futures puts



Hedge Convexities
Convexities (years/—100 bp)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
3
5
7
9
11
13
15
17
19
21
Hedge Period
T—Notes
+ T—Bonds
◊ Tbs and Euros
Δ Tb Puts

Figure 5 - 17

across all three data sets. In addition, as shown in Table 5-2 below, hedge effectiveness for Strategy 1: T-note futures exceeded that for Strategy 3: T-bonds and Euros which in turn exceeded that for Strategy 2: T-bond futures in all periods.

Table 5-2: Differential Hedge Effectiveness
Strategy Comparison: Mean K Values

Data Set:	Period 1	Period 2	Period 3
Strategy 1:	.532	.346	.419
Strategy 3:	.485	.188	.339
Strategy 2:	.437	.158	.326

However, it was observed that differences among Strategies 1, 2, and 3 were generally not significant.[20]

To recall the preceding subsection, the GNMA duration measures which produced the best hedge results generally portrayed GNMA durations as decreasing from the 5.0 to 5.5 year range for deep discounts to the 2.0 to 4.0 year range for the super premiums. Similarly, the GNMAs seemed to exhibit decreasing and eventually negative convexity as price increased above par. In comparison, Strategy 1: T-note futures was characterized by duration values in the

5.0 to 6.5 year range and uniformly small but positive convexity values. Thus, it is not surprising to find Strategy 1 hedge effectiveness decreasing with increasing GNMA price, for it is in the upper price ranges where the duration and convexity mismatches become the largest.

The same trend was observed for Strategy 2: T-bond futures. However, Strategy 2 effectiveness was consistently lower than that for Strategy 1. This performance shortfall may be attributed to the higher convexity of Strategy 2 hedges. The higher convexity of Strategy 2 means that falling rates increase the duration of the T-Bond futures. At higher duration values, subsequent rate decreases generate larger futures price increases, which are translated as increased losses in the hedge position. These larger price increases would be expected to more than offset corresponding gains in the GNMA cash position. A discussion presented later in the present chapter notes that the simulated hedges generally produced losses in this period. This finding is expected and can be explained by the high level of convexity present in the Strategy 2 hedge.

This result is supported in a comparison of Strategies 1 and 3. Strategy 3, it should be remembered, is designed to match precisely the duration of the cash position by adjusting the weight of the T-Bond and Eurodollar futures in the hedge. This matching should be an advantage for Strategy 3 relative to Strategy 1. On the other hand, Strategy 3 has much more convexity than Strategy 1 (the margin of difference is over .5 years/100 basis points for the last eleven periods in the simulation) and may thus be expected to suffer as interest rates fall and duration values change. The finding that Strategy 1 produced slightly better hedge results is an indication that the advantage of duration matching may be quickly undone when convexity values differ and interest rates are volatile.

It was emphasized earlier that the presence of many technical factors makes it difficult to separate interest rate effects from non-interest rate effects in analyzing hedge performance. This consideration recommends cautious acceptance of the support found for the importance of convexity in studying Strategies 1, 2, and 3. It applies in double measure to the study of Strategy 4: T-bond futures puts. Figure 5-17 shows that Strategy 4 possesses much more convexity than any of the other strategies, by a wide margin. An easy but potentially misleading conclusion is that the very poor performance observed for this strategy is a result of its high convexity.

Upon further consideration, however, it is evident that T-Bond futures put prices are sensitive to factors other than interest rates as reflected in the price of the underlying futures.[21] In particular, decay of the time premium and changes in the volatility of futures prices have strong influence on put prices.[22] In the case of strategy 4, where liquidity considerations have in some cases required the use of relatively near-term options, the decay of the time premium is expected to have a substantial and adverse effect on hedge performance. Thus, although strategy 4 exhibits much convexity and would therefore be expected to provide imperfect hedge performance, it seems too great a leap to attribute all performance shortfalls to the convexity mismatch.

3. Results: Qualifications, Explanations, and Perspective

The present research has produced three principal results:

1. Hedging GNMAs with futures and options on futures does not necessarily reduce interest rate risk. In certain circumstances it may actually increase interest rate risk.
2. An option-adjusted GNMA duration model developed in the theoretical portion of this study (called option-estimated GNMA duration to distinguish it from another option-adjusted model developed here which is based on option values implied by market prices) performs as well as but not better than some other techniques.
3. The dynamic adjustment of hedge ratios to account for

changes in the interest sensitivities of the cash and futures positions does not significantly improve hedge effectiveness.

These findings become more meaningful in the context of certain qualifications.

With regard to the first result, it was generally apparent that hedge effectiveness declined with increasing GNMA price. Deep discounts were hedged in the .7 to .8 range of effectiveness, while effectiveness was often negative for super-premiums. The failure observed in these hedges may be attributed to inaccurate duration measurements and, broadly, to basis risk. Basis risk, in market parlance, refers to potential changes in the spread between GNMA prices and the price of the chosen hedge instrument. In the context of the present research, basis risk refers to the aggregate impact of technical factors at work in both the cash and futures markets. As noted earlier in different terms, basis risk is ignored in the construction of duration-based hedges. Its presence therefore diminishes hedge effectiveness.

The second result produced in this research refuted the hypothesis that option-adjusted GNMA duration models produce the best hedge results. This finding needs qualification. In point of fact, it is relevant only to the two specific models tested during the periods considered. Alternative option models may produce better results. What seems clear, however, is that an intuitively appealing method based on the Black Scholes model is an improvement over duration values based on the twelve year bullet prepayment assumption. At the same time, this approach does not appear to be significantly more effective than either the implied or inferred duration techniques which were tested.

Two aspects of the option-adjusted approach studied here seem attractive for refinement and further testing. The first of these concerns the early exercise feature which was ignored in the Black Scholes formulation. It may be that accounting for this feature in a binomial model would produce better hedge results. The second aspect of special interest concerns the maturity of the underlying annuity. In place of the twelve year bullet assumption used in this study, an improved method might estimate the maturity of the underlying annuity by means of a linear prepayment model which projects estimated prepayments as a function of non-interest rate factors. Taken in combination with a model which addresses the early exercise feature, this modification may produce GNMA durations which lead to more effective hedges. Given the extremely large influence of basis risk on hedge performance, however, it seems unlikely that such adjustments will provide quantum leaps in hedge effectiveness in volatile market environments. This consideration should appropriately temper expectations from and investments in more sophisticated models.

The third result produced by this research is that dynamic adjustment of hedge ratios to account for changes in the interest sensitivities of the cash and futures positions does not materially improve hedge effectiveness. This result is somewhat surprising, and, on closer inspection, somewhat deceptive. The distinguishing feature of K, the measure of hedge effectiveness developed for and used in this study, is that it is based on the sum of squared deviations of portfolio value from its initial level. During the periods included in the hedge simulation, cases where the current value of the portfolio was driven away from its initial level were observed frequently. In these cases, the dominant factor in the computation of K was the sum of the squared deviations from the initial level. Subsequent adjustments to the hedge ratio would be expected to stabilize this component of variation, but not reduce it. Thus, variation introduced by basis risk, which is the prime mover of portfolio value in the hedged position, makes it difficult to identify any significant improvements in hedging performance arising from dynamic adjustments to the hedge ratio.[23] Future research should consider testing this hypothesis in other periods of market history where basis risk may be less important.

A fair amount of emphasis has to this point been given to the limitations imposed on these research results by the brevity of the time period considered. It is equally important to recognize that the hypotheses were tested by analyzing K values produced by the simulations. Alternative measures of hedge effectiveness could conceivably produce different conclusions.

One alternative measure of special interest in this regard is Ederington's E. As noted in Chapters II and III, this measure is calculated on an ex post basis as the coefficient of determination for the linear regression of changes in the cash price as a function of changes in the futures price. As it turns out, this measure also corresponds to variance minimizing hedge ratios. Thus, an interesting question concerns how the use of a variance minimizing criterion such as E would alter the three principal conclusions based on the measure K.

This question was explored using the same ANOVA technique employed for testing hypotheses with K values. Briefly, no conclusions are changed. Statistical analysis of results using E is in fact unable to detect any significant differences across strategies 1, 2, and 3 or across the three best duration measures: implied, inferred, and option-estimated. Similarly, this analysis failed to distinguish the dynamic adjustment method as significantly better than the static approach. Although these findings are limited by the brevity of the simulation horizon in the same way as were those based on K, they do provide some support for the principal conclusions of this research effort.

In a related matter, it is interesting to compare the performance of K and E in terms of some simple, intuitive parameters describing hedge effectiveness. R_A and LV_O were introduced in Chapter III for this purpose. To recount, R_A represents the absolute value of the annualized return realized on a hedged portfolio during a given hedge period. An effective hedge would be expected to generate, on average, lower absolute returns than an ineffective hedge susceptible to much variation. Accordingly, K and E, in the ideal case, would exhibit negative correlation with R_A. Likewise, K and E would ideally exhibit negative correlation with LV_O, which represents the largest single day loss observed during a given hedge period.

Table 5-3 displays Pearson correlation coefficients for these four variables. As shown, the correlations for K exhibit the anticipated signs. E, on the other hand,

Table 5-3: Pearson Correlation Coefficients For K and E

	R_A	LVo	E
K	-.417	-.017	.278
E	-.429	.116	1.00

was slightly positively correlated with higher single day losses. Correlations are not strong for any of the four cases.

Further study of LVo is illuminating. A thorough review of hedge results reveals that daily losses to hedge positions reached levels as high as $500,000 (or two percent of initial value) on many occasions, even for hedges which registered positive K values. Thus, positions hedged "effectively" were still prone to loss in the period under review. This finding, in combination with the rather weak correlations cited above, serves as a reminder that GNMA hedges chosen to maximize effectiveness should not be expected to eliminate losses in every market environment.

V. SUMMARY

Statistical tests of the research hypotheses of interest in this study have produced some interesting results:

1. Efforts to hedge GNMAs with futures and options on futures do not necessarily reduce interest rate risk. They may in fact increase interest rate risk, especially if the GNMA cash position is priced in the super-premium range.

In terms of the four alternative strategies, simulation results for the period under study produced the following effectiveness rankings, beginning with the most effective approach:

1. Strategy 1: T-note futures
2. Strategy 3: T-bonds and Euros
3. Strategy 2: T-bond futures
4. Strategy 4: T-bond futures puts

This ranking was consistent across all three data sets, although differences among strategies 1, 2, and 3 were generally insignificant. Strategy 4, on the other hand, was significantly inferior in all cases.

2. The option-estimated GNMA duration model developed for use in this study performed significantly better than a duration model based on the twelve year bullet prepayment assumption. However, this model did not outperform either the implied or inferred GNMA duration models. From best to worst, these three measures were in every period ranked

as follows:

1. Implied (Regression) Duration
2. Inferred Clayton-Goldstein Duration
3. Option-Estimated Duration

This ranking should be viewed with caution, however, as differences among these models were for the most part not significant.

3. The dynamic adjustment of hedge ratios to account for changes in the interest sensitivities of the cash and futures positions does not significantly improve hedge effectiveness.

Several other contributions of interest emerged in the process of investigating these hypotheses:

1. Basis risk appears to be a very important factor in hedge effectiveness. Duration-based hedges, which ignore this factor, suffer accordingly.
2. Accounting for convexity in the hedge design process adds value during periods of volatile interest rates.
3. The use of a variance minimizing measure of hedge effectiveness does not alter the conclusions of the hypothesis tests.
4. K demonstrated some attractive statistical properties: it was not asymmetrically distributed, and it was negatively correlated with large daily losses and extreme

portfolio returns.

These findings have implications for future research and for practicing financial managers. These implications are explored in the following chapter, which concludes this dissertation.

FOOTNOTES, CHAPTER 5

[1] The futures and futures options prices used were settlement prices.

[2] Thirty year GNMAs are traded on "pool-specific" terms and "TBA." "TBA" trades refer to trades in which only the principal amount and coupon rate are specified in advance of settlement. Pool numbers for these trades are "To Be Announced" at settlement.

[3] Inquiry at the CFTC reveals that futures prices are carefully edited; option prices are less so.

[4] Gau and Goldberg (1984) and Gau and Markese (1986) are examples.

[5] See Black and Scholes (1972).

[6] See Chapter III for further discussion of the definition of the hedge periods.

[7] See Pritsker (1984), page 11.

[8] The Barone Adesi and Whaley paper presents computed commodity option values for several different levels of variance, cost of carry, time to maturity, and exercise price.

[9] During the period under study, the GNMA market underwent a significant transformation due to falling interest rates. In January 1985, the majority of outstanding GNMAs were trading at a discount. By June 1986, over three-quarters of GNMAs were premium (see Davidson, 1987).

[10] This measure is calculated as the ratio of the ex post change in the GNMA price over the hedge period to the product of the ex post change in the FHLMC posted yield and the initial GNMA price. In effect, this measure is the GNMA duration value which fully "predicts" the change in the GNMA price which occurs over the period based on the change in interest rates which occurs over the period.

[11] Observations falling on the boundaries of price ranges were assigned to the higher range.

[12] All tests were performed at the .05 level of significance.

[13] It is interesting to note that the implied method outperformed hedges constructed on the basis of the ex post, "true" duration measure.

[14] As shown in Figure 5-8, although it produced the largest

mean K in each of the three periods, the implied measure was not significantly better than the inferred method in two periods and not significantly better than the option estimated approach in the third.

[15]These remarks were shared at the Mortgage Backed Securities Conference at the New York Hilton on February 23, 1987.

[16]In a rather dramatic example of the magnitude of this general disagreement about value, a trader at one large Wall Street firm acknowledged missing a bid for mortgage securities during the period under study by six points. This six percent miss is extremely large in a market that normally trades in a few thirty-seconds of a percent. He was quick to point out that he knew of other traders who had also missed trades by several points.

[17]See Chapter IV for a discussion of the procedures used to assign GNMA durations for those cases in which no prepayment rate assumption satisfied the Clayton-Goldstein model.

[18]Ibid.

[19]The values for Strategy 3: T-bonds and Euros are computed assuming that the hedge is equally weighted in T-Bond and Eurodollar CD futures. In the simulations, these weights varied between .3 and .7. See the discussion in Chapter IV for details regarding the procedure for determining these weights for each hedge period and cash position.

[20]Significant differences occurred only in the period 2 data set, where Strategy 1 was significantly better than Strategy 2. Strategy 3, however, was not significantly better than Strategy 2 or significantly worse than Strategy 1 in this data set.

[21]Recall that liquidity considerations encouraged the use of at-the-money options. It is well known that at-the-money options are sensitive to changes in these variables.

[22]Kutner, McBain, and Sweeney (1986) provides a thorough analysis of the decay of early exercise premiums for alternative option models.

[23]The dominating effect of basis risk also swamps the influence of transaction costs which might otherwise be expected to influence the effectiveness of dynamic hedge strategies.

CHAPTER VI: CONCLUSION

I. SUMMARY

The primary purpose of this research has been to study the task of hedging GNMAs by exploring three hypotheses:

1. THE INTEREST RATE RISK OF GNMA PASS-THROUGHS IS REDUCED BY THE USE OF DURATION-BASED HEDGING STRATEGIES WHICH EMPLOY FINANCIAL FUTURES AND OPTIONS ON FUTURES.
2. EFFORTS TO HEDGE THE INTEREST RATE RISK OF GNMA PASS-THROUGHS WHICH EXPLICITLY INCORPORATE THE EMBEDDED CALL OPTION ARE MORE EFFECTIVE THAN STRATEGIES WHICH DO NOT INCORPORATE THIS COMPONENT.
3. STRATEGIES WHICH INCORPORATE DYNAMIC REBALANCING OF HEDGE POSITIONS OUTPERFORM STATIC APPROACHES.

Studying these hypotheses involves the application of research from a broad spectrum of subjects, including the term structure of interest rates, bond portfolio management, option pricing, financial futures, valuation of mortgage-backed securities, and hedging. Thus, following an introductory chapter, this dissertation began with a review of relevant theoretical and empirical literature in Chapter II. This review was necessary to establish firmly the theoretical underpinnings of the work to follow.

The literature reviewed in Chapter II held several important implications for this research effort:

DURATION: Empirical results, especially those of Lau (1983) and Bierwag, Kaufman, and Toevs (1983), recommended Macaulay duration as a measure of the sensitivity of bond prices to changes in interest rates.

OPTION PRICING: The quadratic approximation of American option values developed by Barone-Adesi and Whaley (1986) is well-suited to the task of estimating deltas for futures options.

The Black-Scholes European option model can be used to value the call option embedded in GNMAs. This model is recommended chiefly because the binomial model fails put-call parity for debt options and because there presently exists no persuasive empirical evidence to justify the expense of using finite difference methods to evaluate American formulas for long term debt options.

VALUATION OF MORTGAGE-BACKED SECURITIES: A modified version of the Clayton Goldstein (1986) model can be used to estimate GNMA price sensitivities without explicitly accounting for the option component.

The option explicit approach should rely on a combination of the Black-Scholes European model and an annuity valuation determined under the assumption of a flat term structure.

Neither of these approaches to estimating mortgage duration employ the stochastic model of the term structure advanced so successfully in the theoretical literature by Cox, Ingersoll, and Ross (1978, 1985b). This choice was warranted for two reasons. First, two extremely broad immunization studies -Lau (1983) and Bierwag, Kaufman, and Toevs (1983) - have recommended Macaulay duration in favor of the CIR measure. Second, there is a lack evidence demonstrating that the term structure is driven by a clearly identifiable, stationary stochastic process.

HEDGING: Empirical techniques which have dominated the literature on interest rate hedging are inconsistent with rational, arbitrage-free markets. Consequently, this research relied on a "fundamental" approach to hedge ratio computation based on Macaulay duration.

Following this literature review, Chapter III presented the specific research hypotheses to be examined in this study. In addition, it has described the characteristics of the institutional setting constructed for the purpose of specifying and evaluating hedging strategies. In particular, emphasis was given to the development of a new measure of hedging effectiveness, denoted by K, which evaluates hedging performance in terms of the average daily deviation of portfolio value from its initial value. This measure does a better job of measuring how well a given strategy performs in relation to the hedge objectives than variance based measures do. When supplemented with a few simple ratios describing the maximum observed single day deviations, K provides a great deal of meaningful information about hedging performance.

The penultimate section of Chapter III explained the rationale underlying each of the four hedge strategies chosen for testing:

1. Strategy 1: short T-note futures

2. Strategy 2: short T-bond futures
3. Strategy 3: short T-bonds and Euros
4. Strategy 4: long T-bond futures puts

Rules used to select futures contracts delivery months were explained in the context of liquidity constraints and the related needs of consistency and clarity in the structure of the research. In addition, this section presented the rationale for dynamic hedge adjustment and described the method used to simulate this process in the present research. And finally, a brief discussion identified other potential hedging strategies which were not selected for testing and explained the reasons for their exclusion.

Chapter IV was presented next. As noted there, the most important aspect of any hedging strategy is the hedge ratio used to implement the strategy. Consequently, from a conceptual standpoint, Chapter IV represents the core of this study. It explained how Macaulay's duration can be modified and used to calculate hedge ratios for a variety of GNMA hedge strategies. In the process, several different views of the price behavior of GNMAs are noted and reflected in the numerator of the hedge ratios. These alternative views produced four approaches to modeling GNMA duration:

1. Clayton-Goldstein inferred duration: In this model, it is assumed that expected prepayments are reflected in GNMA market prices. Iterative numerical search techniques can be used to "infer" the expected prepayments (adjusted for maturity considerations using the PSA schedule) which produces the observed market price for a given discount rate. Modified Macaulay duration can then be computed on the basis of the sum of anticipated prepayments and scheduled principal and interest.

2. Option-adjusted duration: This formulation conceptualizes GNMAs as the sum of two components: an annuity and a related call option representing the borrowers' rights to prepay the mortgages underlying GNMA securities. This dissertation employed two variations of this model in the hedge simulations:

a. Option-adjusted (market implied) method: In this case, the value of the embedded calls was determined as the difference between the observed market price for the GNMA and the computed value of the underlying annuity. Call values determined in this way were used to calculate implied volatilities; implied volatilities were in turn used to calculate the call deltas. GNMA duration was then computed as,

$$D_{GNMA} = D_A P_A * (1 - d_c) / P_{GNMA}$$

where D_A, P_A are the duration and price of the underlying annuity and d_c is the call delta.

b. Option-estimated method: This model computes the value of the embedded calls using the Black Scholes model. Volatility estimates for the price of the underlying annuity are made on the basis of observed interest rate volatility as captured in the daily yield on the ten year Treasury during the preceding sixty trading days.

3. Implied (regression) duration: This model is based on two assumptions: 1) GNMAs trading at par exhibit price volatility relative to the current ten year Treasury which is stable over time, and 2) GNMAs trading at price levels other than par exhibit price volatility relative to the par GNMA is stable over time. These two sets of relative price volatilities are estimated using regression models. The volatility factors which emerge from this process permit GNMA duration to be computed according to,

$$D_{GNMA} = B_1 * B_2 * D_{10\ yr}$$

where B_1 is the price volatility of the GNMA price range under study relative to the price volatility of the par GNMAs;

B_2 is the price volatility of the par GNMA relative to the price volatility of the ten year Treasury; and

$D_{10\ yr}$ is the duration of the ten year Treasury.

To these four models were added two benchmark duration models:

1. Twelve year bullet duration: This model computes modified Macaulay durations for GNMAs assuming only scheduled interest and principal payments are made for 143 months. The entire outstanding principal is assumed to be repaid in month 144.

2. "True" duration: this measure is an ex post computation:

$$D_{TRUE} = <>P/(P*<>y)$$

where <>P and <>y represent changes observed in the GNMA price and yield over the three month simulation period.

Each of these six measures of GNMA duration were used to calculate hedge ratios for use in the simulation experiments. The output of these simulation experiments was a collection of K values: hedge effectiveness was measured for every unique combination of GNMA duration measure, hedge strategy, hedge type, and GNMA coupon (ranging from 8% to 14%).

As explained in Chapter V, the output of these simulations was then used in statistical tests of the research hypotheses. The results of these tests are interesting:

1. Efforts to hedge GNMAs with futures and options on futures do not necessarily reduce interest rate risk. They may in fact increase interest rate risk, especially if the GNMA cash position is priced in the super-premium range.

In terms of the four alternative strategies, simulation results for the period under study produced the following effectiveness rankings, beginning with the most effective approach:

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This ranking should be viewed with caution, however, as differences among these models were for the most part not significant.

3. The dynamic adjustment of hedge ratios to account for changes in the interest sensitivities of the cash and futures positions does not significantly improve hedge effectiveness.

Several other contributions of interest emerged in the process of investigating these hypotheses:

1. Basis risk appears to be a very important factor in hedge effectiveness. Duration-based hedges, which

ignore this factor, suffer accordingly.

2. Accounting for convexity in the hedge design process adds value during periods of volatile interest rates.

3. The use of a variance minimizing measure of hedge effectiveness does not alter the conclusions of the hypothesis tests.

4. K demonstrated some attractive statistical properties: it was not asymmetrically distributed, and it was negatively correlated with large daily losses and extreme portfolio returns.

These findings have implications for future research and for practicing financial managers. These implications are explored in the following section, which concludes this dissertation.

II. IMPLICATIONS FOR FINANCIAL MANAGERS

Notwithstanding the qualifications cited in Chapter V, the results of this study have several implications for practicing financial managers. In particular, fixed income portfolio managers should be aware that efforts to hedge GNMAs with financial futures and futures options do not provide assurance that portfolio value will be protected from either large daily losses or extremely large negative returns. Evidence cited in this study indicates that basis risk and problems related to the assessment of GNMA duration may have serious, adverse effects on hedged portfolios during periods of falling interest rates.

For portfolio managers with a desire to hedge GNMAs, results of this research indicate that the use of short positions in T-note futures may be preferred to short positions in T-bond futures or in T-bond futures and Eurodollar CD futures. Furthermore, it is apparent that GNMAs priced as discounts or deep discounts are likely to be hedged more effectively than GNMAs in other price ranges, especially the super premium range. And finally, this study provides evidence that the hedging strategies suggested by Jones (1987) which is based on long positions in puts on T-bond futures are unlikely to be effective. The value of these puts varies as a function of many factors which are independent of interest rate changes, thereby reducing their attractiveness as hedge instruments. (The decay of the time premium component of the put price can be especially troublesome.)

In a related matter, portfolio managers reviewing research on the effectiveness of various proposed hedging strategies should be aware of two important considerations:

1. How does the measure of hedge effectiveness used in the research related to the hedge objectives they have identified? Specifically, the measure should monitor hedge performance over time (not just at hedge termination dates) and it should consider variation about the initial portfolio value, not about its period mean.

2. Does the time period over which hedge results were analyzed encompass periods of rising and falling rates and a variety of changes in the basis? If not, are the conclusions properly qualified?

An appreciation for the importance of these two questions is essential to the correct interpretation of any hedging study.

Aside from their interest in hedging, fixed income portfolio managers are in general concerned with measuring and managing the risk return characteristics of their holdings. The use of strategic frontier analysis, which is quite amenable to portfolios of traditional (option-free) fixed income securities, is complicated for managers of GNMA securities because of the difficulty associated with measuring the duration (and hence the systematic risk exposure) of GNMAs.[1] The results of this research suggest that the

Pinkus-Chandoha implied regression method is a simple and effective approach to measuring GNMA durations. It further suggests that GNMA prices are strongly influenced by factors other than interest rates, indicating that a two-dimensional strategic frontier does not describe fully the risk return characteristics of GNMAs. And finally, the results of this study suggest that portfolio managers may benefit by considering the convexity, as well as the duration, of their fixed-income holdings.

Effective management of interest rate risk and return has in the past decade become a primary concern to a broad class of financial managers whose responsibilities are entirely separate from the management of securities portfolios. Asset-liability management, as this task is commonly called, is a concept which came into being with the arrival of increased levels and volatilities of interest rates in the late 1970s. In general, it refers to the management of the maturity structure of assets and liabilities at financial institutions. The subject gained particular prominence in the thrift industry, where the traditional maturity mismatch of long term mortgage assets funded with short term deposit liabilities generated severe losses when interest rates rose in the late 1970s and early 1980s.

The losses experienced by the thrift industry as a result of the maturity mismatch inherent in their financial structure recommended a more active approach to managing interest rate risk. One regulatory response to this need called for the explicit accounting on a quarterly basis of the interest rate risk assumed at each institution on reports filed by members of the FHLB. The method used for this accounting was based on the "maturity gap" approach. While information provided by this approach represents a quantum leap forward in management information, it nonetheless has certain limitations. In particular, the gap method does not address the sensitivity of the market value of assets and liabilities to changes in interest rates.

Given this background, asset liability managers at institutions which hold large volumes of mortgage assets may be interested in the duration results produced in this research. The implied duration measure described for use in managing GNMAs has also been estimated for other mortgage-backed assets such as FHLMC PC's and FNMA MBS's.[2,3] Thus, asset-liability managers have at their disposal a tool which enables them to quickly compute values for the durations of those portions of their loan portfolios comprised of mortgage assets like those in the various mortgage pass-through securities.

As a final note of interest to asset liability managers, this work indicates that options extended to borrowers in the form of prepayment rights have great influence on the asset risk return characteristics. It is therefore essential that managers account for the value of these options when they extend financing.

III. SUGGESTIONS FOR FUTURE RESEARCH

This research effort has provoked many more questions than it has answered. Several of these questions are worthy of future research. Perhaps the most important of these questions concerns the reliability and generality of the conclusions. The hedge simulations on which the conclusions of this study are based should be repeated with a data base spanning the history of the GNMA market. In addition, other vehicles, such as strip hedges using Treasury futures, interest-only strips of mortgage securities, and over the counter GNMA options should be tested as hedge instruments.

Other potentially fruitful areas for research are related to the valuation of mortgage-backed securities.

The model developed by Kau, Keenan, Epperson, and Muller (1986), which is based on the Cox, Ingersoll, Ross (1985b) theory of asset pricing, is worthy of empirical testing. Similarly, extensive testing of available debt option models is needed to determine the nature of the biases in these models. One item of particular interest in this area concerns the nature and extent of the errors introduced by the use of binomial debt option models.

Finally, research is needed to investigate the problems created by basis risk. One interesting approach to this problem would be to reconsider the GNMA hedging problem as a "factor" hedge instead of an interest rate hedge, where the factors are those identified in the methods of the arbitrage pricing theory.[4] The modest success reported by Platt (1986) in composite hedging of low grade bonds, when viewed from the perspective of APT, provides limited encouragement in this direction.

In closing, it is important to note an important caveat which applies to all valuation-related research. Any valuation model, regardless of its theoretical appeal or empirical support, is only valid to the extent that it continues to describe the behavior of market participants. That is, it is only valid as long as market participants agree that it is valid and use it to assess value. When conditions in markets change and participants begin using different models, the validity of pre-existing models deteriorates. Diller referred to such a changing of models in the mortgage market as a "paradigm shift," and as we have seen, the diversity of opinion which develops during such a shift can create volatile, almost chaotic, markets.

A classic example of this phenomenon can be seen in the results reported by Long (1978). Theoretically, in the presence of taxes, investors should prefer the equity of firms which, instead of paying dividends, retain the cash and generate capital gains. Long examines this proposition in a study of the Citizens Utility Corporation. This corporation is unique because it issued two classes of stock which differ only in terms of their dividends and tax treatment: one class receives only stock dividends while the other receives only cash dividends. The unique situation created with these two classes of stock provides powerful controls in tests of dividend policy effects on financing and investment policy. Long finds that the stock which pays cash dividends is priced at a premium to that which pays stock dividends. Thus, an investor who, placing his faith and capital in the theoretically pleasing valuation model and transacting an arbitrage by purchasing the "stock-dividends-only" stock and selling short the "cash" stock soon after the stock was originally issued, would twenty years later still be waiting for the market to recognize its "error." This finding illustrates problems which can develop through the use of valuation models which are not used by the market.

With this final qualification firmly established, future research in hedging GNMAs should proceed with great promise for advancing the state of knowledge in finance.

CHAPTER VI: FOOTNOTES

[1]Strategic frontier analysis is discussed at length by Fong and Fabozzi (1985).

[2]FHLMC PC's (participation certificates) and FNMA MBS's (mortgage-backed securities are backed by conventional mortgage loans whereas GNMAs are backed by FHA (Federal Housing Administration) and VA (Veteran's Administration) loans. Because FHA and VA loans are assumable and conventional loans are not, FHLMC's and FNMA's tend to prepay faster. Thus, GNMA relative price volatility factors are likely to overstate the durations of mortgage assets backed by loans which are not FHA or VA loans.

[3]Pinkus and Chandoha (1987) provide some preliminary evidence that this approach is viable.

[4]APT (Ross, [1976]) does not specify the factors, or even the number of factors, which are theorized to drive asset prices. Roll, Ross, and Chen (1986) employ factor analysis to identify four factors.

APPENDIX

This Appendix is organized into three parts. The first part presents the quadratic approximation to the value of American commodity options as advanced by Barone-Adesi and Whaley (1986) and discussed in Chapter II. The second part presents the linear regression model developed by Peters, Pinkus, and Askin to predict prepayment rates for FHLMC mortgages. The last part of this Appendix explains the statistical methodology used to test the research hypotheses considered in Chapter V of this dissertation.

I. The Barone-Adesi and Whaley Quadratic Approximation to the Value of American Options

The Barone-Adesi and Whaley (1986) approximation of the value of an American put option is,

$$P(S,T) = p(S,T) + A_1(S/S^*)^{q_1} \quad \text{for } S > S^*$$
$$= X - S \quad \text{for } S <= S^*$$

where

$$A_1 = -(S^*/q_1)\{1-e^{(b-r)T}N[-d_1(S^*)]\}$$

$$p(S,T) = Xe^{-rT}N(-d_2 - Se^{(b-r)T}N(-d_1)$$

$$q_1 = \{-(N-1) - ((N-1)^2 + 4M/K)^{1/2}/2$$

$$N = 2b/o^2$$

$$M = 2r/o^2$$

$$K = 1 - e^{-rT}$$

$$d_1 = \{\ln(S/X) + (b + (.5)o^2)T\}/(o(T)^{1/2})$$

$$d_2 = d_1 - o(T)^{1/2}$$

N(.) = the cumulative univariate normal distribution

S = the current commodity price

X = the option exercise price

T = the term to expiration

r = the riskless rate of interest

b = the cost of carry

σ = the standard deviation of commodity price relatives

and S* is the critical commodity price which satisfies,

$$X - S = p(ST) - \{1 - e^{(b-r)T}N[-d_1(S^*)]\}S^*/q_1$$

Chapter II: Literature Review, Section III: Option Pricing Theory, Subsection D: Pricing Debt Options and Options on Futures in Continuous Time addresses this model and its implications for the present research in detail.

II. The Peters, Pinkus, and Askin Prepayment Model

The ordinary least squares regression model developed by Peters, Pinkus, and Askin (1984) for predicting prepayments on FHLMC mortgages is,

$$CPR_{jt} = -.07372\ AVAIL_{jt} - 1.79927\ POINTS_t - .53727\ RATECNG_j$$
$$+ .39199\ SMLADJ_j + .04092\ EARNS_j - .00146\ AGEBORR_j$$
$$+ .01803\ WEALTH_j + .00215\ MIGRN_t + .6148\ GNP_t$$
$$- .02618\ SE_j - .01827\ WEST_j$$
$$+ \sum_{k=1}^{8} d_k\ POLYR_k$$

where d_k = .01548, .05116, .09234, .12304, .12968, .12968, .12968, .12968

CPR_{jt} = the conditional prepayment rate in year t for cohort j;

$AVAIL_{jt}$ = the ratio of the mean principal balance or assumption value of the mortgages in the cohort to the mean borrowing potential in the market;

$POINTS_{jt}$ = buyer discount points in year t (in percent);

$RATECNG_j$ = for cohort j, the difference between the market mortgage rate and the contract rate on the mortgages expressed as a percentage of the contract rate;

$SMLADJ_j$ = zero if RATECNG is less than -5 percent, otherwise equals RATECNG;

POLYRk = a variable taking the value zero except in the kth year of the cohort's lifetime, when it takes the value one;

EARNSj = the mean monthly household earnings of the primary borrower (in thousands of dollars) for all borrowers in cohort j;

AGEBORRj = the mean age of all primary borrowers in cohort j;

WEALTHj = the ratio of the mean number of bedrooms in the house to mean number of dependents for all homeowners in cohort j;

MIGRNt = net migration per thousand of population in year t;

GNPt = the percentage change in the real gross national product (in 1972 dollars) in year t; and

SEj, WESTj = dummy variables equal to one if the region of origination for mortgages in cohort j is Southeast or West, respectively; zero otherwise.

Chapter II: Literature Review, Section IV: Mortgage-Backed Securities, Subsection C: Prepayment Estimation discusses this model along with other techniques for estimating prepayments.

III. Statistical Methods: Analysis of Variance

Statistical analysis of simulation output was performed using the General Linear Models (GLM) procedure in SAS.[1] This procedure performs analysis of variance,[2] which is a technique for analyzing experimental data. This technique is especially appropriate for research problems such as the present one where data observations do not fit a balanced design (i.e. different combinations of research factors have associated with them different numbers of observations). Briefly, in this procedure, a continuous response variable - in this case, hedge effectiveness, K - is measured under various unique experimental treatments identified by categorical variables. Variation in the observed response is partitioned between "explained" variation attributed to effects in the categorical variables and unexplained random error.

For each observation, the linear model, shown below, predicts the response with a sample mean. The difference between the actual and predicted value of the response is called the residual error. The algorithm underlying the GLM procedure in SAS fits the model parameters to minimize the sum of squares of residual errors. The variance of the random error is estimated as the mean square error.

$$K_i = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_1 x_2 + b_6 x_1 x_3 + b_7 x_1 x_4 + b_8 x_2 x_3 + b_9 x_2 x_4 + b_{10} x_3 x_4 \quad (5\text{-}1)$$

where

K_i is the ith observation of the response,
b_k are unknown parameters to be estimated, and
x_{ij} are design variables. (Design variables for analysis of variance are represent the various levels in the categorical research factors. SAS reparameterizes these variables as matrices of dummy variables [zero, one variables] to account for the various levels of the design variable.

As SAS proceeds through the task of fitting the linear model shown above, it calculates incremental improvements (reductions) in error sums of squares due to changes in the research factors. In addition, it computes the sums of squares of variation in K which is uniquely attributable to each combination of research factors. At the conclusion of the procedure, F tests are conducted on the ratio of the total variation (sums of squares due to the regression) explained by each term in the model to the mean square error. This ratio is F-distributed, hence large values of F indicate that a given research factor has a significant influence on hedge effectiveness.

In the case of the present research, the GLM procedure was first applied to the simulation results generated for every unique combination of the four research factors: 1) six GNMA duration models, 2) four hedge strategies, 3) two hedge types, 4) six GNMA price range categories. As noted in the text, a review of these results indicated that differences in hedge effectiveness across experimental treatments were in many cases difficult to detect. This difficulty arose because of the large amounts of variation introduced into the mean square error by Strategy 4: T-bond futures puts and GNMA price range 6: super-premiums (105-110). On this basis, the procedure was repeated with reduced data sets which did not include these troublesome factor combinations. The results generated with these methods are discussed in Chapter V.

In closing, many intermediate statistical methods textbooks consider the analysis of variance procedure. See for example Snedecor and Cochran (1980) or Kirk (1968). SAS manuals also provide helpful overviews.

FOOTNOTES TO THE APPENDIX

[1] SAS (Statistical Analysis Systems) is a system of computer software designed for data analysis. It is the product of the SAS Institute, Cary, North Carolina.

[2] Analysis of variance procedures were first advanced by R.A. Fisher (1925).

BIBLIOGRAPHY

For convenient reference, the entries in this bibliography are arranged in the following categories: I. Hedging; II. Measures of Interest Sensitivity , Theory of Bond Portfolio Immunization, and the Term Structure of Interest Rates; III. Option Pricing; IV. Mortgage-Backed Securities; V. Portfolio Management; and VI. Miscellaneous Sources.

I. Hedging

Ahmadi, H., Sharp, P. and Walther, C. "The Effectiveness of Futures and Options in Hedging Currency Risk." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticutt: JAI Press, 1986.

Anderson, R. and Danthine, J. "Cross-Hedging." Journal of Political Economy (December 1981): 1182-1196.

Anderson, R. and Danthine, J. "Time Pattern Hedging and the Volatility of Futures Prices." Review of Economic Studies (April 1983): 249-266.

Arak, M. Goodman, L. and Ross, Susan. "The Cheapest to Deliver Bond on the Treasury Bond Futures Contract." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticut: JAI Press 1986.

Arak, M. and Goodman, L. "How To Calculate Better T-Bond Hedge Ratios." Futures. (February 1986): 56-57.

Arnold, T. "The Current Status of Hedging Tools For Mortgage-Backed Securities." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Asay, M. and Edelsburg, C. "Can A Dynamic Strategy Replicate the Returns to an Option?" Journal of Futures Markets 6 (Spring 1986): 63-70.

Bear, R. "The Mechanics of Futures Trading." in Fabozzi, F. ed Handbook of Financial Markets, Securities, Options, and Futures. Homewood, Illinois: Dow Jones Irwin, 1986.

Belongia, M. and Santoni, G. "Interest Rate Risk, Market Value, and Hedging Financial Portfolios." Journal of Financial Research 10 (Spring 1987): 47-55.

Benninga, S. and Smirlock, M. "An Empirical Analysis of the Delivery Option, Marking to Market, and the Pricing of Treasury Bond Futures." Journal of Futures Markets 5 (Winter 1985): 361-374.

Bennings, S. Eldor, R. and Zilcha, I. "The Optimal Hedge Ratio in Unbiased Futures Markets." Journal of Futures Markets 4 (Summer 1984): 155-159.

Bhattacharya, A. "The Joint Effect of Housing Starts and Inflation Announcements on GNMA Futures Prices." Journal of Futures Markets 6 (Winter 1986): 645-657.

Bond, G. and Thompson, S. "Optimal Commodity Hedging Within the Capital Asset Pricing Model." Journal of Futures Markets 6 (Fall 1986): 421-431.

Bookstaber, R. and Jacob, D. "The Composite Hedge: Controlling the Credit Risk of High Yield Bonds." in Platt, R. Controlling Interest Rate Risk. New York: John Wiley and Sons, 1986.

Breeden, D. and Giarla, M. "An Introduction to Hedging Interest Rate Risk With Futures, Options, and Swaps." Paper prepared for FHLBB, Washington D.C. (May 31, 1986).

Breeden, D., and Giarla, M. "An Introduction to Risk-Controlled Arbitrage." Paper prepared for FHLBB, Washington D.C. (August 1986).

Brinkman, W. "The Role and Operation of Clearing Houses." in Rothstein, N. ed. The Handbook of Financial Futures. New York: McGraw-Hill Book Co., 1984.

Cecchetti, P., Dale, C., and Vignola, A."Usefulness of Treasury Bill Futures as Hedging Instruments." Journal of Futures Markets 1 (Fall 1981): 319-387.

Cechetti, S. Cumby, R. and Figlewski, S. "Estimation of the Optimal Futures Hedge." Research Working Paper RWP 86-10, Research Division, Federal Reserve Bank of Kansas City, December 1986.

Chen, A. Park, H. and Wei, K. "Stochastic Duration and Dynamic Measure of Risk in Financial Futures." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticut: JAI Press (1986).

Cornell, B. "The Relationship Between Volume and Price Variability in Futures Markets." Journal of Futures Markets 1 (Fall 1981): 303-316.

Driver, D. and Lacey, J. "Trading Mechanics and Procedures." in Rothstein, N. ed. The Handbook of Financial Futures. New

York: The McGraw-Hill Book Co., 1984.

Eaker, M. and Grant, D. "Managing Foreign Exchange Risk With Cross-Hedging and Currency Swaps." in Handbook of Modern Finance 1986 Update. Logue, D. ed. New York: Warren, Gorham,and Lamont, 1986.

Easterwood,J. and Senchak, A. "Arbitrage Opportunities With T-Bill/T-Bond Futures Contracts." Journal of Futures Markets 6 (Fall 1986): 433-442.

Ederington, L. "The Hedging Performance of the New Futures Markets." Journal of Finance (March 1979): 157-170.

Fishe, R. and Goldberg, L. "The Effects of Margins on Trading in Options and Futures Markets. " Journal of Futures Markets 6 (Summer 1986): 261-271.

Gastineau, G. "Futures and Options on Fixed Income Securities: Their Role in Fixed Income Portfolio Management." in Fabozzi, F. The Handbook of Fixed Income Securities. Homewood, Illinois: Dow Jones Irwin Inc., 1985.

Gau, G. and Goldberg, M. "Further Tests of the Effectiveness of Cross-Hedging Among Mortgage and Futures Markets." ORER Working Paper No. 14, University of Illinois at Urbana Champaign, 1985.

Gay, G. and Kolb, R. "The Management of Interest Rate Risk." Journal of Porfolio Management 9 (Winter 1983): 65-70.

Gay, D. et al. "Interest Rate Hedging: An Empirical Test of Alternative Strategies." Journal of Financial Research (Fall 1973): 187-198.

Gay, D. and Manaster, S. "Implicit Delivery Options and Optimal Delivery Strategies For Financial Futures Contracts." Journal of Financial Economics 16 (May 1986): 41-72.

Geske, R. "An Options Approach To Controlling Interest Rate Risk." Working Paper, Federal Home Loan Bank Board, November 1986.

Goodman, L. "GNMA vs. Bond Futures." Mortgage Banking 45 (September 1985): 77-87.

Goodman, L. "Hedging With Debt Options." in Fabozzi, F. ed. Handbook of Financial Markets: Securities, Options, and Futures. Homewood, Illinois: Dow Jones Irwin, 1986.

Goodman, L. "Managing a Mortgage Pipeline: Instruments and Alternatives." in Fabozzi, F. Mortgage-Backed Securities.

Chicago, Illinois: Probus Publishing, 1987.

Grant, D. "Rolling the Hedge Forward: An Extension." Financial Management (Winter 1984): 26-28.

Hegde,S. "The Impact of Interest Rate Level and Volatility on the Performance of Interest Rate Hedges." Journal of Futures Markets 2 (Winter 1982): 341-356.

Hegde, S. and Nunn, K. "Interest Rate Volatility, Trading Volume, and the Hedging Performance of T-Bond and GNMA Futures - A Note." Journal of Futures Markets." 5 (Summer 1985): 273-296.

Hjerpe, E. "Calculating the 'Best' Hedge Ratio For Treasury Bonds." Working Paper, Office of Economic Policy and Research, March 13, 1987.

Hill, J., Liro, J., and Schneeweis, T. "Hedging Performance of GNMA Futures Under Rising and Falling Interest Rates." Journal of Finance 3 (4) (1983): 403-413.

Hilliard, J. "Hedging Interest Rate Risk With Futures Under Term Structure Effects." 39 Journal of Finance (December 1984): 1547-1570.

Hoeven, J. "The Relative Effectiveness of Stack (Roll) Vs. Strip Hedging Strategies." Paper presented at the 1987 annual meeting of the Midwest Finance Association, St. Louis, Missouri.

Howard, C. and D'Antonio, L. "A Risk Return Measure of Hedging Effectiveness." Journal of Financial and Quantitative Analysis (March 1984): 101-112.

Johnson, L. "The Theory of Hedging and Speculation in Commodity Futures." Review of Economic Studies 27 (1960): 139-160.

Jones, F. "The Yield Curve: Cash and Futures Markets." in Rothstein, N. ed. The Handbook of Financial Futures. New York: McGraw-Hill Book Co., 1984.

Jones, F. and Jain, A. "Hedging Mortgage Backed Securities." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Kamara, A. "The Behavior of Futures Prices: A Review of Theory and Evidence." Financial Analysts Journal (July-August 1984): 16-29.

Kidder, W. "The Implied Repo Rate." in Rothstein, N. The Handbook of Financial Futures. New York: The McGraw-Hill

Book Co., 1984.

Klemkosky, R. and Lasser, D. "An Efficiency Analysis of the T-Bond Futures Market." Journal of Futures Markets 5 (Winter 1985): 607-620.

Kolb, R. and Chiang, R. "Improving Hedging Performance Using Interest Rate Futures." Financial Management (Autumn 1981): 72-79.

Kuberek, R. and Pefley, N. "Hedging Corporate Debt With Treasury Bond Futures." Journal of Futures Markets 3 (Winter 1983): 345-353.

Labuszewski, J. and Kamradt, M. "Cash Factors That Make Treasury Futures Tick." Futures (August 1985): 80-83.

Labuszewski, J. and Meisner, J. "Gauging the Effectiveness of Treasury Bond Futures on GNMA." Futures: The Magazine of Commodities and Options 14 (June 1985): 76,78.

Lady, G. and Lady, J. "Cash/Futures Pricing and Hedge Ratios." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticut: JAI Press (1986).

Landes, W., Stoffels, J. and Seifert, J. "An Empirical Test of a Duration-Based Hedge: The Case of Corporate Bonds." Journal of Futures Markets 5 (Summer 1985): 173-183.

Lebeck, W. "The Origins of Futures Markets." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Leibowitz, M. "The Analysis of Value and Volatility in Financial Futures." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Livingston, M. "The Cheapest Deliverable Bond for the CBT Treasury Bond Futures Contract." Journal of Futures Markets 4 (Summer 1985): 161-172.

Little, J. "What Are Financial Futures." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Little, P. "Financial Futures and Immunization." Journal of Financial Research 9 (Spring 1986): 1-11.

Loosigian, A. "Financial Futures: Hedging Interest Rate Risk." in Fabozzi, F. The Handbook of Fixed Income Securities. Homewood, Illinois: Dow Jones Irwin Inc., 1985.

Losey, R. and Kelsey, S. "Interest Rate, Default, and Basis Risk in Hedging Fixed Rate Conventional Mortgages Via the Use of GNMA Futures." Working Paper, Federal Home Loan Bank Board, Washington, D.C., 1984.

Maness, T. "Optimal vs. Naive Buy-Hedging With T-Bill Futures." Journal of Futures Markets 1 (1) (1981): 393-403.

McCabe, G. and Frankle, C. "Effectiveness of Rolling the Hedge Forward in the Treasury Bill Futures Market." Financial Management 12 (Summer 1983): 21-29.

McEnally, R. and Rice, M. "Hedging Possibilities in the Flotation of Debt Securities." Financial Management (Winter 1979).

Meisner, J. and Labuszewski, J. "Treasury Bond Delivery Bias." Journal of Futures Markets (Winter 1984): 569-577.

Milonas, N. "Price Variability and the Maturity Effect in Futures Markets." Journal of Futures Markets 6 (Fall 1986): 443-460.

Milonas, N. "Liquidity and Price Variability in Futures Markets." Financial Review 21 (May 1986): 211-237.

Nelson, R. and Collins, R. "A Measure of Hedging's Performance." Journal of Futures Markets 5 (Spring 1985): 45-53.

Overdahl, J. and Starleaf, D. "The Hedging Performance of the CD Futures Market." Journal of Futures Markets 6 (Spring 1986): 71-81.

Parker, L."CMO's: Guaranteed Delivery." Mortgage Banking 46 (March 1986): 35-40.

Pitts, M. "Cross-hedging, Autocorrelated Errors, and the Tradeoff Between Risk and Return." Paper presented at the Financial Management Association Meetings, October 21, 1983.

Pitts, M. "OTC Options on Mortgage Securities." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Pitts, M. "Cross-Hedging, Hedge Effectiveness, and the Tradeoff Between Risk and Return." in Fabozzi, F ed. Advances in Futures and Options Research. Greenwich, Conn:

JAI Press (1986).

Powers, M. and Weissman, R. "Designing the Hedge Strategy." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Powers, M. Inside the Financial Futures Markets. New York: John Wiley and Sons, 1984.

Rebell, A. "Numerical Relationships of Cash and Futures: Keys To Analysis of Hedge and Speculative Strategies." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Rendleman, R. and Carabini, C. "The Efficiency of the Treasury Bill Futures Market." Journal of Finance 34 (1979): 859-914.

Rothstein, N. and Little, J. "Managing The Hedge Program." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Rothstein, N. and Little, J. "The Market Participants and Their Motivations." in Rothstein, N. ed. The Handbook of Financial Futures. New York: The McGraw-Hill Book Co., 1984.

Sandor, R. and Mains, N. "Financial Futures and Mortgage-Backed Securities." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Scholes, M. "The Economics of Hedging and Spreading in the Futures Markets." Journal of Futures Markets 1 (Summer 1981): 265-286.

Schneeweis, T. and Besant, L. "Finacial Futures Markets." in Handbook of Modern Finance 1986 Update. Logue, D. ed. New York: Warren, Gorham, and Lamont, 1986.

Senchak, A. and Easterwood, J. "Cross-Hedging CD's With T-Bill Futures." Journal of Futures Markets 3 (Winter 1983): 429-438.

Stein, J. "The Simultaneous Determination of Spot and Futures Prices." American Economic Review 51 (December 1961): 127-140.

Stulz, R. "Optimal Hedging Policies." Journal of Financial and Quantitative Analysis (June 1984) 127-140.

Thompson, T. "What's New At The Board of Trade." Mortgage

Banking 46 (March 1986): 18, 20-22.

Toevs, A. "Hedging the Interest Rate Risk of Mortgage-Backed Securities." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Toevs, A. and Jacob, D. "Futures and Alternative Hedge Ratio Methodologies." Journal of Portfolio Management 12 (Spring 1986): 60-71.

Waldrep, B. and Buck, J. "Efficacy of Hedging With Financial Futures: An Historical Perspective." Journal of Futures Markets 2 (Fall 1982): 243-254.

Yawitz, J. and Marshall, W. "The Use of Futures in Immunized Bond Portfolios." Journal of Portfolio Management 11 (Winter 1985): 51-58.

II. Measures of Interest Sensitivity, Theory of Bond Portfolio Immunization, and the Term Structure of Interest Rates:

Arak, M. Goodman, L. and Snailer, J. "Duration Equivalent Bond Swaps: A New Tool." Journal of Portfolio Management 12 (Summer 1986): 26-32.

Asay, M., Guillaume, F. and Mattu, R. "Duration and Convexity of Mortgage-Backed Securities: Some Hedging Implications from a Simple Prepayment-Linked Present-Value Model" in Fabozzi, F., Mortgage-Backed Securities. Chicago, Ill: Probus Publishing, 1987.

Bierwag, G. "Immunization and the Term Structure of Interest Rates." Journal of Financial and Quantitative Analysis (December 1977): 725-741.

Bierwag, G.O., and Kaufman, G. "Coping With the Risk of Interest Rate Fluctuations: A Note." Journal of Business (July 1977): 364-370.

Bierwag, G., Kaufman, G., and Khang, C. "Duration and Bond Portfolio Analysis: An Overview." Journal of Finance and Quantitative Analysis 13 (November 1978): 671-682.

Bierwag, G.O., Kaufman,G., and Toevs, A. "Duration: Its Development and Use in Bond Portfolio Management." Financial Analysts Journal (July/ August 1983): 15-35.

Bierwag, G.O., Kaufman,G. and Toevs, A. "Single Factor Duration Models in a General Equilibrium Framework."

Journal of Finance (May 1982): 325-338.

Bierwag, G.O., Kaufman, G. and Toevs, A. "Bond Portfolio Immunization and Stochastic Process Risk." Journal of Bank Research (Winter 1983): 282-291.

Bierwag, G.O., Kaufman, G. and Toevs, A. "Recent Developments in Bond Portfolio Immunization Strategies" in Bierwag, G.O., Kaufman, G., and Toevs, A., editors, Innovations in Bond Portfolio Management: Duration Analysis and Immunization. JAI Press, 1983.

Bierwag, G.O., Kaufman, G., and Toevs, A., editors, Innovations in Bond Portfolio Management: Duration Analysis and Immunization. Greenwich, Conn.: JAI Press, 1983.

Bierwag, G., Kaufman, G., Schweitzer, R. and Toevs, A. "The Art of Risk Managment in Bond Portfolios." Journal of Portfolio Management 7 (Spring 1981): 27-36.

Bierwag, G., Kaufman, G. Latta, C. and Roberts, G. "Duration: Response To Critics." Journal of Portfolio Management (Winter 1987): 48-52.

Brown, S. and Dybvig, P. "The Empirical Implications of the Cox, Ingersoll, Ross Theory of the Term Structure of Interest Rates." Journal of Finance 41 (June 1986): 617-630.

Caks, J. "The Coupon Effect On Yield To Maturity." Journal of Finance (March 1977): 377-385.

Carlson, S. "Accurately Measuring Mortgage Duration: A New Method." , New York: Shearson Lehman Bros. Mortgage Finance Marketplace (August 1986).

Carlson, S., "Assessing the Duration of Mortgage Securities: A New Approach." New York: Shearson Lehman Bros. Special Report. (October 1986).

Chambers, D. "The Management of Default-Free Bond Portfolios." Unpublished PhD. dissertation, University of North Carolina, Chapel Hill, North Carolina: 1981.

Chambers, D. Carleton, W. and Waldman, D. "A New Approach to the Estimation of the Term Structure of Interest Rates." Journal of Financial and Quantitative Analysis (September 1984): 233-252.

Coleman, T., Fisher, L. and Ibbotson, R. "Estimating Forward Interest Rates and Yield Curves from Government Bond Prices: Methodology and Selected Results." Working paper, Yale University, 1986.

Cox, J., Ingersoll, J. and Ross, S. "A Theory of the Term Structure of Interest Rates." Research Paper No. 468, Stanford University Graduate School of Business, August 1978.

Cox, J., Ingersoll,J. and Ross, S. "Duration and Measurement of Basis Risk." Journal of Business (January 1979): 57-61.

Cox, J., Ingersoll, J., and Ross, S. "An Intertemporal General Equilibrium Model of Asset Prices." Econometrica 53 (March 1985): 363-384.

Cox, J., Ingersoll, J., and Ross, S. "A General Theory of the Term Structure of Interest Rates." Econometrica 53 (March 1985): 385-407.

Davidson, A., "Overview of Alternative Duration Measures for Mortgage-Backed Securities." Merrill Lynch, in Fabozzi, F., Mortgage-Backed Securities. Chicago, Ill: Probus Publishing, 1987.

Diller, S. "Parametric Analysis of Fixed Income Securities." New York: Goldman Sachs (April, 1984).

Fabozzi, F., The Handbook of Fixed Income Securities, Homewood, Illinois: Dow Jones Irwin, 1983.

Fabozzi, F. Winning The Interest Rate Game. Chicago, Illinois: Probus Publishing Co., 1986.

Fisher, L, and Weil, R. "Coping With the Risk of Interest Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies." Journal of Business (October 1971): 408-431.

Fong, H. and Vasicek, O. "Term Structure Modeling." Journal of Finance 37 (1982): 339-348.

Gay, G. and Kolb, R. "The Management of Interest Rate Risk." Journal of Porfolio Management 9 (Winter 1983): 65-70.

Homer, S., and Leibowitz, M. Inside the Yield Book. Englewood Cliffs, N.J.: Prentice Hall Inc, 1972.

Hopewell, M. and Kaufman, G. "Bond Price Volatility and Term to Maturity: A Generalized Respecification." American Economic Review (September 1973): 749-753.

Ingersoll, J. "Is Immunization Feasible? Evidence from the

CRSP Data." in Bierwag, G.O., Kaufman, G., and Toevs, A., editors, Innovations in Bond Portfolio Management: Duration Analysis and Immunization. Greenwich, Conn.: JAI Press, 1983.

Ingersoll, J., Skelton, J., and Weil, R. "Duration Forty Years Later." Journal of Financial and Quantitative Analysis 13 (November 1978): 627-650.

Jacob, D., Lord, G. and Tilley, J. "Price, Duration, and the Convexity of Mortgage-Backed Securities." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Kolb,R. and Chiang, R. "Duration, Immunization, and Hedging With Interest Rate Futures." Journal of Finance 5 (Summer 1982): 161-170.

Lau, P. "An Empirical Examination of Alternative Interest Rate Risk Immunization Strategies" Unpublished Ph.D. dissertation, University of Wisconsin, Madison, 1983, cited in Haugen, R., Modern Investment Theory. Prentice Hall Inc., Englewood N.J., 1986.

Leibowitz, N. and Weinberger, A. "Contingent Immunization." Financial Analysts Journal (November/December 1982 and January/February 1983): 17-32 and 35-50, respectively.

Livingston, M. "Measuring Bond Price Volatility." Journal of Finance and Quantitative Analysis 14 (June 1979): 343-349.

Macaulay, F., The Movement of Interest Rates, Bonds, Yields, and Stock Prices in the United States Since 1865, New York: Columbia University Press, 1938, 38-53.

Maloney, K. and Yawitz, J. "Interest Rate Risk, Immunization, and Duration." Journal of Portfolio Management 12 (Summer 1986): 41-48.

Malkiel, B. "Expectations, Bond Prices, and the Term Structure of Interest Rates." Quarterly Journal of Economics." 6 (1962): 197-218.

Nelson, J. and Schaefer, S., "The Dynamics of the Term Structure and Alternative Portfolio Immunization Strategies," in Bierwag, G.O., Kaufman, G., and Toevs, A., editors, Innovations in Bond Portfolio Management: Duration Analysis and Immunization. Greenwich, Conn.: JAI Press, 1983.

Pinkus, S., and Chandoha, M., The Relative Price Volatility of Mortgage Securities," in Fabozzi, F., Mortgage-Backed Securities, Chicago, Ill.: Probus Publishing, 1987.

Platt, R. Ed. Controlling Interest Rate Risk. New York: John Wiley and Sons, 1986.

Redington, F. M. "Review of the Principles of Office Life Valuations." Journal of the Institute of Actuaries 78 (1952): 286-340.

Shea, G. "Pitfalls in Smoothing Interest Rate Term Structure Data: Equilibrium Models and Spline Approximations." Journal of Financial and Quantitative Analysis (September 1984): 253-270.

Toevs, A. "Interest Rate Risk and Uncertain Lives." Journal of Portfolio Management 11 (Spring 1985): 45-56.

Toevs, A. and Jacob, D. "Interest Rate Futures: A Comparison of Alternative Hedge Ratio Methodologies." New York: Morgan Stanley, June 1984.

Toevs, A. and Dyer, L. "The Term Structure of Interest Rates and Its Use In Bond Portfolio Management." (New York) Morgan Stanley, October, 1986.

Van Horne, J. Financial Markets, Rates, and Flows. 2nd ed. Englewood Cliffs, N.J: Prentice Hall, 1984.

Yawitz, J. "The Relative Importance of Duration and Yield Volatility on Bond Price Volatility." Journal of Money, Credit and Banking (February 1977): 97-103.

III. Option Pricing Theory

Barone-Adesi,G. and Whaley, R. "The American Put Option Valued Analytically." Working Paper Series 86-104, Fuqua School of Business, Duke University, October 1986.

Bartter, B.J and Rendleman, F.J. Jr."The Pricing of Options On Debt Securities." Journal of Finance and Quantitative Analysis (March 1980): 11-23.

Beckers, S. "Standard Deviations Implied in Options Prices as Predictors of Future Stock Price Variability." Journal of Banking and Finance 5 (September 1981): 363-382.

Black, F. and Scholes, M. "The Pricing of Options and Corporate Liabilities." Journal of Political Economy (May -June 1973): 637-654.

Black, F. "Fact and Fantasy in the Use of Options." Financial Analysts Journal (July August 1975): 36-41.

Black, F. "The Pricing of Commodity Contracts." Journal of Financial Economics 3 (January-March 1976): 167-179.

Black, F. and Scholes, M. "The Valuation of Option Contracts and a Test of Market Efficiency" Journal of Finance 5(1) (1977): 67-88.

Black, F. "The Trouble With Econometric Models." Financial Analysts Journal (March-April 1983): 29-37.

Bookstaber R., Jacob, D., and Langsam, J. "Pitfalls in Debt Option Models." New York: Morgan Stanley Fixed Income Research (April 1986).

Bookstaber, R. "Exposure Management and Valuation of Bonds With Attached Options." Handbook of Fixed Income Securities. 2nd Ed, Fabozzi, F. ed. Homewood, Ill.: Dow Jones (1986).

Bookstaber R. and Jacob, D. "The Composite Hedge: Controlling the Credit Risk of High Yield Bonds." New York: Morgan Stanley Fixed Income Research (1985).

Bookstaber R., Jacob, D., and Langsam, J. "Arbitrage-Fre Pricing of Options on Interest Sensitive Instruments." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich Connecticut: JAI Press (1986).

Bookstaber R. and McDonald, J. "A Generalized Option Valuation Model for the Pricing of Bond Options." Review of Research in Futures Markets 4 (March 1985): 60-73.

Boyle, P. "Options: A Monte Carlo Approach." Journal of Finance (May 1977): 323-338.

Brennan, M. and Schwartz, E.S. "Valuation of American Puts." Journal of Finance (May 1977): 449-472.

Brennan M. and Schwartz, E. "Savings Bonds, Retractable Bonds, and Callable Bonds." Journal of Financial Economics 5 (August 1977): 67-88.

Brennan, M. and Schwartz, E.S. "Alternative Methods for Valuing Debt Options." Finance 4 (1983): 119-129.

Buser, S., Hendershott, P., and Sanders, A. "Does the Nature of the Interest Rate Process Affect the Value of Call Options On Default-free Bonds?" Paper presented at the 1986 annual meeting of AREUEA

Burton, T. "Observations on the Theory of Option Pricing on Debt Instruments" Option Pricing. Brenner, M., ed. Lexington, Mass: D.C. Heath, 1983.

Carlson, S. "The Prepayment Call Option on GNMA Securities: Implied Market Valuation and Prepayment Speeds." New York: Shearson Lehman Bros. Mortgage Finance Marketplace (April 1986).

Copeland, T. and Weston, F. Financial Theory and Corporate Policy 2nd ed. Reading, Massachusetts: Addison Wesley Publishing Co, 230-270.

Courtadon, G. "The Pricing of Options On Default-Free Bonds" Journal of Financial and Quantitative Analysis 17 (March 1982): 75-100.

Cox, J., Ross, S., and Rubenstein, M. "Option Pricing: A Simplified Approach" Journal of Financial Economics (September 1979): 229-263.

Cox, J. and Ross, S. "The Valuation of Options Under Alternative Stochastic Processes" Journal of Financial Economics (January-March 1976): 145-166.

Cox, J., and Rubinstein, M. Options Markets. Englewood Cliffs, N.J.: Prentice Hall Inc. 1985.

Geske, R. and Johnson, H. "The American Put Option Valued Analytically." Journal of Finance 39 (December 1984): 1511-1524.

Geske, R. and Roll, R. "On Valuing American Call Options With Black-Scholes European Formula." Journal of Finance 39 (June 1984): 443-456.

Hall, A. "Valuing the Mortgage Borrower's Prepayment Option." AREUEA 3 (Fall 1985): 229-247.

Hicks, J. Value and Capital. Oxford: Clarendon Press, 1939.

Ingersoll, J. "An Examination of Corporate Call Policies on Convertible Securities." Journal of Finance (May 1977): 463-478.

Jarrow, R. and Rudd, A. Option Pricing. Homewood, Illinois: Dow Jones Irwin, 1983.

Jordan, J. and Seale, W. "Transaction Data Tests of Minimum Prices and Put-Call Parity For Treasury Bond Futures Options." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticut: JAI Press (1986).

Kau, J., Keenan, D. Muller, W., and Epperson, J. "Option Theory and Fixed Rate Mortgages." Working Paper, University of Georgia, 1986.

Koop, D. "Pricing Options: Variations on the Black Scholes Model." Review of Research in Futures Markets 5 (1) (1986): 81-89.

Kopprasch, R. "Contingent Takedown Options On Fixed Income Securities." in Fabozzi, F. The Handbook of Fixed Income Securities. Homewood, Illinois: Dow Jones Irwin Inc., 1985.

Kopprasch,R. "Early Redemption (Put) Options on Fixed Income Securities." in Fabozzi, F. The Handbook of Fixed Income Securities Homewood, Illinois: Dow Jones Irwin Inc., 1985.

Kutner, G., McBain, M. and Sweeney, R. "An Examination of Time Decay in the Black Scholes, the Absolute, and the Cox Call Option Valuation Models." Working Paper 87-111, College of Business Administration, Marquette University. Milwaukee, Wisconsin. 1987.

Luft, C. and Fielitz, B. "An Empirical Test of the Commodity Options Pricing Model Using GNMA Call Options." Journal of Financial Research (Summer 1986): 137-151.

Leland, H. and Rubinstein, M. "Replicating Options With Positions in Stock and Cash." Financial Analysts Journal (July-August 1981) 63-71.

MacBeth, J. and Merville, L. "An Empirical Examination of the Black-Scholes Call Option Pricing Model." Journal of Finance (December 1979): 1173-1186.

MacBeth, J. and Merville, L. "Tests of the Black-Scholes and Cox Option Pricing Models." Journal of Finance (May 1980): 285-300.

MacMillan, L. "Analytic Approximations for the American Put Option." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich, Connecticut: JAI Press (1986).

Merton, R. "Theory of Rational Option Pricing." Bell Journal of Economics and Management Science 4 (Spring 1973): 141-183.

Merton, R. "Option Prices When Underlying Stock Returns Are Discontinuous." Journal of Financial Economics 3 (January-March 1976): 125-144.

Merville, L. amd Overdahl, J. "An Empirical Examination of the T-Bond Futures (Call) Options Markets Under Conditions of Constant and Changing Variance Rates." in Fabozzi, F. ed. Advances in Futures and Options Research. Greenwich Connecticut: JAI Press (1986).

Pitts, M. "The Pricing of Options on Debt Securities." Journal of Portfolio Management 11 (Winter 1985): 41-50.

Redington, F. "Review of the Principle of Life-Office Valuations." Journal of the Institute of Actuaries 78 (1952): 286-340.

Rendleman, R. and Bartter, B. "Two-State Option Pricing." Journal of Finance 34 (June 1979): 1093-1110.

Roll, R. "An Analytic Valuation Formula For Unprotected American Call Options On Stocks With Known Dividends." Journal of Financial Economics 5 (November 1977): 251-259.

Samuelson, P. "The Effects of Interest Rate Increases on the Banking System." American Economic Review (March 1945): 16-27.

Sharfman, B. "Two-State Option Pricing Model For Options On Debt Securities." Technical Bulletin No. 4, Financial Planning Division, Office of Finance, Federal Home Loan Bank, December 1986.

Shastri, K. and Tandon, K. "On the Use of European Models to Price American Options on Foreign Currency." Journal of Futures Markets 6 (Spring 1986): 93-108.

Shastri, K. and Tandon, K. "Options on Futures Contracts: A Comparison of European and American Pricing Models." Journal of Futures Markets 6 (Winter 1986): 593-618.

Smith, C. "Option Pricing: A Review." Journal of Financial Economics 3 (January-March 1976): 3-51.

Stoll, H. and Whaley, R. "New Option Instruments: Arbitrageable Linakages and Valuation." in Fabozzi, F. ed. Advances in Futures and Options Research Greenwich, Connecticut: JAI Press (1986).

Thode, S. and Voas, C. "Optimal Refinancing Strategies For Corporate Debt." Working Paper, Lehigh University, 1986.

Whaley, R. "On the Valuation of American Call Options on Stocks with Known Dividends." Journal of Financial Economics 10 (March 1982): 29-58.

Whaley, R. "Valuation of American Call Options On Dividend Paying Stocks: Empirical Tests." Journal of Financial Economics 10 (March 1982): 29-58.

Wolf, A. "Fundamentals of Commodity Options on Futures." Journal of Futures Markets 2 (Winter 1982): 391-408.

Wolf, A. "Options of Futures: Pricing and the Effect of Anticipated Price Change." Journal of Futures Markets 4 (Winter 1984): 491-512.

IV. Mortgage-Backed Securities

Askin, D. "Forecasting Prepayment Rates For Mortgage Securities." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Askin, D. "Dollar Rolls: A Low Cost Financing Technique." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Askin, D., Hoffman, W. and Meyer, S. "The Drexel Burnham Lambert Mortgage Pricing Model: The Complete Evaluation of the Option Component of Mortgage Securities." New York: Drexel Burnham Lambert (January 1987).

Brennan, M. and Schwartz, E. "Determinants of GNMA Mortgage Prices." AREUEA 3 (Fall 1985): 209-228.

Buser, S. and Hendershott, P. "Pricing Default-free Fixed Rate Mortgages." Housing Finance Review 3 (1984).

Carlson, S. and Sears, T. "The Prepayment Call Option in GNMA Securities: Implied Market Valuations and Prepayment Speeds." New York: Shearson Lehman Bros. Mortgage Finance Marketplace (April 1986).

Carlson, S. and Sears, T. "Stripped Mortgage-Backed Securities: A New Product." New York: Shearson Lehman Bros. Mortgage Finance Marketplace (April 1986).

Carlson, S. and Sears, T. "A Guide to Mortgage Prepayment Analysis." New York: Shearson Lehman Bros. Mortgage Finance Marketplace (June 1985).

Clayton, R. and Goldstein, S. "Prepayment Implications of Mortgage-Backed Securities Prices." Working Paper, University of Tennessee, January 1986.

Curley, A. and Guttentag, J. "The Yield On Insured Residential Mortgages." Explorations in Economic Research (Summer 1974): 114-161.

Davidson, A. "Understanding Premium Mortgage-Backed Securities: Observations and Analysis." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Diller, S. and Beaulac, T. "Paydown Consistency of Individual Passthroughs." New York: Goldman Sachs Financial Strategies Group (April 1984).

Dunn, K. and McConnell, J. "A Comparison of Alternative Models For Pricing GNMA Mortgage-Backed Securities." Journal of Finance 36 (June 1981): 471-483.

Dunn, K. and McConnell, J. "Valuation of GNMA Mortgage-Backed Securities." Journal of Finance 36 (June 1981): 599-617.

Dunn K., and Singleton, K. "An Empirical Analysis of the Pricing of Mortgage-Backed Securities." Journal of Finance (May 1983): 613-623.

Foster, C. and Van Order, R. "FHA Terminations: A Prelude To Rational Mortgage Pricing." AREUEA 3 (Fall 1985): 229-247.

Goodman, L. and Rones, A. Using GNMA-Zero Combinations To Enhance Portfolio Returns." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Green and Shoven. "The Effects of Interest Rates on Mortgage Prepayments." NBER Working Paper No. 1246, 1983.

Hjerpe, E. "Risk-Controlled Arbitrage: Description and Associated Risks." Working Paper, Federal Home Loan Bank Board, Washington D.C., January 1987.

Hjerpe, E. "Stripped Mortgage-Backed Securities: An Economic Analysis and Valuation Simulation." Research Working Paper #130, Office of Economic Policy and Research, Federal Home Loan Bank Board, Washington D.C. June 1987.

Hu, J. "Evaluating Mortgage Pass-Through Securities." New York: Shearson Lehman Brothers Special Report (August 1986).

Jacob, D., Latainer, G. and Toevs, A. "Value and Performance of Mortgage-Backed Securities." (New York) Morgan Stanley, February, 1987.

Jacob, D. and Toevs, A. "An Analysis of the New Valuation, Duration, and Convexity Models For Mortgage-Backed Securities." New York: Morgan Stanley, January 1987.

McManus, B. "REMICS: The Legislation and Its Implications." New York: E.F. Hutton, November 20, 1986.

McManus, B. "An Introduction to Collateralized Mortgage Obligations." New York: E.F. Hutton, September 1986.

Navratil, F. "The Estimation of Mortgage Prepayment Rates." Journal of Financial Research (Summer 1985): 107-117.

Peters, H., Pinkus, S. and Askin, D. "Figuring the Odds: A Model of Prepayments." Secondary Mortgage Markets (May 1984) 18-23.

Peterson, B. and Stein, J. "Mortgage Hedging." New York: Goldman Sachs, February 1987.

Roberts, B. "The Consequences of the Pure Aging Effect on the Yields of Mortgage-Backed Securities." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Senft, D. "Pass-Through Securities." in Fabozzi, F., ed. The Handbook of Fixed Income Securities. Homewood, Illinois: Dow Jones Irwin, 1983.

Sullivan, K., Collins, B., and Smilow, D. "Mortgage Pass-through Securities." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Sullivan, K. "The Mortgage Market." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Waldman, M. and Guterman, S. "The Historical Performance of Mortgage Securities: 1972-1984." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Waldman, M. " Constructive Use of Fixed Rate Mortgages: Arbitrage Opportunities for Financial Institutions." in Fabozzi, F. Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1987.

Waldman, M. and Gordon, M. "Evaluating the Option Features of Mortgage Securities: The Salomon Brothers Mortgage Pricing Model." New York: Salomon Bros., January 1987.

V. Portfolio Management

Bradley, S. and Crane, D. "Simulation of Bond Portfolio Strategies: Laddered Vs. Barbell Maturity Structure." Journal of Bank Research 6 (Summer 1975): 122-134.

Bradley, S. and Crane, D. "Management of Commercial Bank Government Security Portfolios: An Optimization Approach Under Uncertainty." Journal of Bank Research 4 (Spring 1973): 18-30.

Dunford, D. and Kopprasch, R. "Futures and Options: Return/Risk Modification Properties and Use in Portfolio Management Strategies." in Maginn, J. and Tuttle, D. Managing Investment Portfolios: 1985-1986 Update. New York: Warren, Gorham and Lamont, 1985.

Fong, H. "Portfolio Construction: Fixed Income." in Maginn,J. and Tuttle, D. Managing Investment Portfolios. New York: Warren, Gorham and Lamont, 1983.

Fong, H. and Fabozzi. F. Fixed Income Portfolio Management. Homewood, Illinois: Dow Jones Irwin, 1985.

Fong, H. and Fabozzi, F. "Interest Rate Anticipation Strategies." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Francis, J. Investments. New York: McGraw Hill, 1986.

Haugen, R. Modern Investment Theory. Englewood Cliffs, N.J: Prentice Hall, 1986.

Maginn, J. and Anderson, M. "The Fixed Income Analysis Process: Types and Characteristics of Fixed Income Securities and Markets." in Maginn,J. and Tuttle, D. Managing Investment Portfolios: 1985-1986 Update. New York: Warren, Gorham and Lamont, 1985.

Maginn, J. and Anderson, M. "The Fixed Income Analysis Process: Return and Risk Analysis." in Maginn,J. and Tuttle, D. Managing Investment Portfolios: 1985-1986 Update. New York: Warren, Gorham and Lamont, 1985.

Reilly, F. Investment Analysis and Portfolio Management. 2nd ed. Chicago, Illinois: Dryden Press, 1985.

Ryan, R. "Return Enhancement: GNMA Time Swap Using Floaters." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Sega, R. "Mortgage-Backed Securities from the Portfolio Manager's Perspective." in Fabozzi, F. The Handbook of Mortgage-Backed Securities. Chicago, Illinois: Probus Publishing, 1985.

Tuttle, D. "Fixed Income Portfolio Management Strategies." in Maginn,J. and Tuttle, D. Managing Investment Portfolios: 1985-1986 Update. New York: Warren, Gorham and Lamont, 1985.

VI. Miscellaneous Sources

Chen, N., Roll, R. and Ross, S. "Economic Forces and the Stock Market." Journal of Business 59 (1986): 383-403.

Fisher, R. Statistical Methods For Research Workers. Edinburgh: Oliver and Boyd, 1925.

Kirk, J. Experimental Design: Procedures For The Behavioral Sciences. Monterey, California: Brooks/Cole, 1968.

Long, J. "The Market Valuation of Cash Dividends: A Case To Consider." Journal of Financial Economics 6 (1978): 235-264.

Lorrain, P. and Corson, D. Electromagnetic Fields and Waves. San Francisco: W.H. Freeman and Co., 1970.

Roll, R. and Ross, S. "An Empirical Investigation of the APT." Journal of Finance (December 1980): 1073-1103.

Roll, R. and Ross, S. "The Arbitrage Pricing Theory Approach To Strategic Portfolio Planning." Financia; Analysts Journal (May-June 1984): 14-26.

Ross, S. "The Arbitrage Theory of Capital Asset Pricing." Journal of Economic Theory (December 1976): 341-360.

Snedecor, G. and Cochran, W. Statistical Methods, Seventh Edition. Ames, Iowa: Iowa University Press, 1980.

GLOSSARY

American options: options which may be exercised prior to expiration.

Arbitrage: the simultaneous purchase and sale of similar financial instruments in order to benefit from an anticipated change in their price relationship.

Basis: the difference between the cash and the futures prices.

Basis risk: refers to potential variability in the basis.

Cash position: the portfolio to be hedged.

CFTC: The Commodity Futures Trading Commission is the independent federal agency created by Congress in 1974 to regulate futures trading.

Contract: a unit of trading for financial or commodity futures; also, an agreement between the buyer (seller) and the clearinghouse specifying the obligations of the respective parties.

Convexity: refers to the curvature present in the bond price profile. This study uses the term convexity to summarize that part of a bond's price performance profile not captured in the duration measure. It is defined as the change in duration corresponding to a 100 basis point decline in interest rates.

Cost of carry: the difference between the yield on a security and the cost to finance its purchase. In a normal, upward-sloping yield curve environment, the "cost" of carry for longer maturity bonds is not a cost, but a source of revenue: long term yields exceed short term borrowing costs.

CPR (Conditional prepayment rate): the proportion of loans prepaid in a given month expressed as a percentage of the principal balance outstanding at the close of the previous month.

Critical futures price: the price at which early exercise of a futures option becomes optimal.

Cross-hedging: a term used to describe hedging methods in which the securities underlying the financial futures and options on futures are not the same as the security being hedged. In this study, the securities underlying the futures and options on futures are U.S. Treasury notes and bonds and a ninety day Eurodollar Certificate of Deposit index. They are not GNMAs. For purposes of expositional convenience, this dissertation uses the term hedges generally, whereas in reality the hedges considered are cross hedges.

Delivery month: the calendar month during which a futures contract matures.

Delta: Option deltas are defined as the rate of change of the value of the option with respect to changes in the value of the underlying security.

Derivative mortgage securities: The principal mortgage securities are IO's (Interest Only securities), PO's (Principal Only securities), CMO's (Collateralized Mortgage Obligations), and REMICs (Real Estate Mortgage Investment Conduits).

Duration: the present value weighted average time to receipt of cash flows by a security owner.

Early exercise premium: value associated with the right of the holder of an American option to exercise prior to expiration.

Embedded options: rights provided to or by owners of financial instruments which are not explicitly accounted for in the pricing process.

Eurodollar Certificate of Deposit: a negotiable instrument evidencing a dollar-denominated deposit (usually in units of $1 million) made with a foreign bank (usually in London).

Eurodollar Futures: futures contracts traded on the International Monetary Market at the Chicago Mercantile Exchange. These contracts are "cash settled" (no physical delivery takes place) on the basis of prices determined by LIBOR (London Inter Bank Offered Rates).

European options: options which may be exercised only at maturity.

GNMA: the Government National Mortgage Association was created in 1968 by Congress to support the FHA and VA loan markets.

GNMAs: refers to thirty year Government National Mortgage Association pass-throughs. A pass-through security is created when one or more mortgage holders sells shares or participations in a "pool" or collection of mortgage loans. The security entitles the holder to a pro rata portion of the principal and interest payments made against the mortgages in the pool. In effect, the cash flow from the mortgages is "passed through" to the security holder. Pools consist of as few as one or as many as several thousand loans. Generally, each loan is serviced by its originator; a trustee is assigned to hold the titles of all mortgages in the pool and to ensure that all mortgages and properties are in acceptable form and all payments are properly made.

GNMA duration models:

1. Clayton-Goldstein inferred duration: In this model, it is assumed that expected prepayments are reflected in GNMA market prices. Iterative numerical search techniques can be used to "infer" the expected prepayment rates (adjusted for maturity considerations using the PSA schedule) which produce the observed market price for a given discount rate. Modified Macaulay duration can then be computed on the basis of the sum of anticipated prepayments and scheduled principal and interest.

2. Option-adjusted duration: This formulation conceptualizes GNMAs as the sum of two components: an annuity and a related call option representing the borrowers' rights to prepay the mortgages underlying GNMA securities. This dissertation employs two variations of this model in the hedge simulations:

a. Option-adjusted (market implied) method: In this case, the value of the embedded calls is determined as the difference between the observed market price for the GNMA and the computed value of the underlying annuity. Call values determined in this way are used to calculate implied volatilities; implied volatilities are in turn used to calculate the call deltas. GNMA duration is then computed as,

$$D_{GNMA} = D_A P_A * (1 - d_c) / P_{GNMA}$$

where D_A, P_A are the duration and price of the underlying annuity and d_c is the call delta.

b. Option-estimated method: This model computes the value of the embedded calls using the Black Scholes model. Volatility estimates for the price of the underlying annuity are made on the basis of observed interest rate volatility as captured in the daily yield on the ten year Treasury during the preceding sixty trading days.

3. Implied (regression) duration: This model is based

on two assumptions: 1) GNMAs trading at par exhibit price volatility relative to the current ten year Treasury which is stable over time, and 2) GNMAs trading at price levels other than par exhibit price volatility relative to the par GNMA which is stable over time. These two sets of relative price volatilities are estimated using regression models. The volatility factors which emerge from this process permit GNMA duration to be computed according to,

$$D_{GNMA} = B_1 * B_2 * D_{10\ yr}$$

where B_1 is the price volatility of the GNMA price range under study relative to the price volatility of the par GNMAs;
B_2 is the price volatility of the par GNMA relative to the price volatility of the ten year Treasury; and
$D_{10\ yr}$ is the duration of the ten year Treasury.

GNMA benchmark duration models:

1. Twelve year "bullet" duration: This model computes modified Macaulay durations for GNMAs assuming only scheduled interest and principal payments are made for 143 months. The entire outstanding principal is assumed to be repaid in month 144.
2. "True" duration: this measure is an ex post computation:

$$D_{TRUE} = <>P/(P*<>y)$$

where $<>P$ and $<>y$ represent changes observed in the GNMA price and yield over the three month simulation period.

GNMA factors: ten digit decimals released by GNMA on a monthly basis. The product of the factor for a given pool and the original balance of the pool is the outstanding principal balance for the pool. The release of factor information affects GNMA prices. In the context of a dynamic approach to hedging where adjustments are made on a monthly basis, the intuitive choice for adjusting hedge ratios is the first opportunity following the release of factor information, or at closing prices on the seventh business day of each month. To facilitate comparison of static and dynamic approaches, this study has defined the eighth business day of each month as the start of a new hedge period and as the "adjustment day" for dynamic strategies.

GNMA parity series: The GNMA parity series created by Pinkus and Chandoha was defined as a series of price changes of GNMAs trading in the range between 99.5 and 100. Because this narrow range produced a limited number of observations, the original series was augmented with price changes for the GNMAs trading closest to par on a given day and adjusted with simple regressions. Hedge ratio: the ratio of the value sensitivity of the cash position to the value sensitivity of the hedge position. Interest rate risk: refers to variability in holding period returns to a security due to changes in interest rates.

MBSs: mortgage-backed securities, including FHLMC PC's (Federal Home Loan Mortgage Corporation participation certificates, backed by conventional home loans), FNMA MBS's (Federal National Mortgage Association mortgage-backed securities, backed by conventional mortgage loans), GNMAs (backed by FHA (Federal Housing Administration) and VA (Veteran's Administration) loans, and mortgage derivative securities.

Mortgage for pass-through swaps: a phrase which refers to the process through which mortgage holders convert loans into securities. Investment bankers perform investment analysis to illustrate to lenders the returns to be earned in securitizing and swapping loans. They often follow up by helping mortgage holders select loans for inclusion in pools and by finding purchasers for the resultant security.

Pass-through securities: securities created to enable lenders to convert existing loans to "re-lendable cash," hence increasing the availability and decreasing the cost of mortgage credit. See Senft (1983) for a lucid introduction.

PSA schedule: a schedule of prepayment rates as a function of mortgage age (PSA stands for Public Securities Association). It utilizes information in the FHA survivorship tables to generate benchmark CPR (conditional prepayment rate) values for MBSs (mortgage-backed securities) as a function of age. MBSs are then described in terms of "percent of PSA." See Chapter IV for further discussion.

Stack hedge (vs. strip hedge): a hedge where all futures contracts are concentrated in a single delivery month.

Strategic frontier analysis: An analytical technique used in the management of fixed income portfolios. The technique is designed to help the manager assess tradeoffs between expected returns and worst-case returns. It is a framework for making portfolio decisions by comparing expected returns accruing to different interest rate scenarios.

Strip hedge (vs. stack hedge): a hedge where futures contracts are evenly dispersed across several delivery months.

TBA prices: Thirty year GNMAs are traded on "pool-specific" terms and "TBA." "TBA" trades refer to trades in which only the principal amount and coupon rate are specified in advance of settlement. Pool numbers for these trades are "To Be Announced" at settlement.

Transaction costs: costs incurred in trading securities which are apart from the value of the securities exchanged. Transaction costs include commissions paid to brokers and the bid-ask spread (the difference between the prices at which market makers buy and sell securities).

Treasury bonds: securities issued by the U.S. Treasury with maturities of not less than ten years. The timely payment of principal and interest is guaranteed by the U.S. government.

T-bond futures: futures contracts traded at the Chicago Board of Trade, the deliverable securities for which include Treasury bonds with at least fifteen years to maturity and call. These contracts are denominated in units of $100,000. Treasury notes: coupon bearing securities issued by the U.S. Treasury with maturities of not less than one year and not more than ten years. The timely payment of principal and interest is guaranteed by the U.S. government.

Treasury notes: coupon bearing securities issued by the U.S. Treasury with maturities of not less than one and not more than ten years. The timely payment of principal and interest is guaranteed by the U.S. government.

T-note futures: futures contracts traded at the Chicago Board of Trade, the deliverable securities for which include Treasury notes with at least six and one-half and no more than ten years to maturity. These contracts are denominated in units of $100,000.