

SPATIO-TEMPORAL ESTIMATION OF WILDFIRE GROWTH

Balaji R. Sharma

School of Dynamic Systems
College of Engineering
and Applied Science
University of Cincinnati
Cincinnati, Ohio, 45221
Email: balaji.rs@gmail.com

Manish Kumar*

Department of Mechanical,
Industrial and
Manufacturing Engineering
University of Toledo
Toledo, Ohio, 43606
Email: manish.kumar2@utoledo.edu

Kelly Cohen

School of Aerospace Systems
College of Engineering
and Applied Science
University of Cincinnati
Cincinnati, Ohio, 45221
Email: kelly.cohen@uc.edu

ABSTRACT

This work presents a methodology for real-time estimation of wildland fire growth, utilizing a fire growth model based on a set of partial differential equations for prediction, and harnessing concepts of space-time Kalman filtering and Proper Orthogonal Decomposition techniques towards low dimensional estimation of potentially large spatio-temporal states. The estimation framework is discussed in its criticality towards potential applications such as forest fire surveillance with unmanned systems equipped with onboard sensor suites. The effectiveness of the estimation process is evaluated numerically over fire growth data simulated using a well-established fire growth model described by coupled partial differential equations. The methodology is shown to be fairly accurate in estimating spatio-temporal process states through noise-ridden measurements for real-time deployability.

INTRODUCTION

Wildland fires across the globe pose a serious environmental and socio-economic threat with far-reaching consequences. Wildland firefighting is a challenging engineering endeavor and mandates co-ordinated allocation of resources to monitor and control fire spread. Conventional methods for monitoring forest fire growth, such as manned aerial missions or surveillance from lookout towers or the use of satellite imagery, have severe shortcomings in terms of the frequency of updates, constraints in terms of safety concerns and accuracy of estimates, and there is a strong need felt in the firefighting community to look beyond

existing methods, towards newer, state-of-the-art surveillance methods using unmanned systems.

There is a growing interest in the military and civilian domains for the deployment of unmanned aerial vehicles (UAVs) towards a range of applications. A particular area of deployment that has garnered a fair share of interest in this context is the generation of situational awareness for wildland fires using a fleet of UAVs working cooperatively. Enabling the deployment of such systems requires the development of effective methods for fusion and estimation of process variables based on data obtained from sensors onboard the UAVs. The generation of real-time situational awareness for spatio-temporal processes such as wildland fires, with the deployment of a multi-UAV system, presents a number of unique challenges in terms of fusion of data received from multiple distributed sources, often asynchronous, and additionally in terms of the computational complexity of processing large spatio-temporal data. The methods presented in this work form a precursor to the development of a broader framework for distributed monitoring of spatio-temporal events such as forest fires that can grow in space and time quite rapidly, requiring effective real-time monitoring and estimation.

Forest fire growths can be typically modeled using coupled Partial Differential Equations (PDEs), that can be solved using grid-based finite difference or finite element methods. Given the geographical spread of the environment over which such events occur and with the temporal aspect factored in, data from sensing systems for such processes can be very high dimensional in na-

*Address all correspondence to this author.

ture, and thus developing a filtering mechanism that incorporates sensor data for online and real-time estimation and prediction for applications can be very challenging. In the larger context, the use of a multi-agent system for the sensing process adds to the complexity of the problem by introducing the additional challenge of sparseness, both spatially and temporally, in the data received.

This work presents a real-time filtering method within the Bayesian framework that harnesses concepts from Proper Orthogonal Decomposition (POD) towards dimensionality reduction, and the use of a space-time Kalman filter for efficient, low-dimensional estimation of forest fire growth. Forest fire data is generated using a coupled PDE-based fire growth model with Gaussian noise added to the data in a grid-based framework. The efficiency of the space-time Kalman filter estimation, in conjunction with POD-based dimensionality reduction towards real-time implementation, is evaluated and presented in this work. The implementation is evaluated over simulated data, and its performance discussed in the context of feasibility towards the development of a broader framework for distributed spatio-temporal estimation for time-critical events such as forest fires.

BACKGROUND

Spatio-temporal processes encompass a broad range of dynamic events that occur in nature and require effective methods for monitoring and estimation. Such processes include climate variations, pollution levels, wildland fires, oil spills and radiation spreads, among others. Many of these processes can often, directly or indirectly, have catastrophic influences on natural resources, affecting areas of human habitation and the ecosystem at large.

Wildfire monitoring

Of particular interest to this work is the development of effective methods for enabling efficient wildfire fighting. Wildland firefighting is a challenging engineering endeavor and mandates co-ordinated allocation of resources to monitor and control fire spread. Studies typically estimate monetary losses caused by wildland fires in tens of millions of dollars, with cumulative losses annually to the order of billions of dollars. In 2011 alone, 8,706,852 acres of forest lands were burned due to fire, with damages estimated in excess of one billion U.S. dollars [1]. The intangible impact in terms of short and long-term damages caused to the ecosystem cannot be overemphasized, reinforcing the need for more effective methods of firefighting.

Conventional methods of firefighting, typically consisting of lookout towers, manned aerial missions and satellite imagery, pose severe restrictions in terms of both accuracy and frequency of data gathering for effective situational awareness. Issues such as safety of firefighting personnel, high costs, and issues of noise and occlusion severely hinder the monitoring process. In recent

years, with the growth and development of low cost unmanned systems, there is a growing interest in the deployment of UAVs for forest fire monitoring. Among others, [2] and [3] discuss the issues and challenges involved in using vision-based monitoring of forest fires in a multi-UAV framework.

Events such as wildfires are very often large-scale events, inherently ridden with high dimensionality in the data received from sensing systems for their monitoring. In addition to the dimensionality of the problems, sensing systems for monitoring such processes are ridden with inherent limitations which result in reduced accuracy and effectiveness of the monitoring process. There is thus a need for an effective filtering mechanism that can handle such large amounts of data and integrate it with growth models for the event state variables for accurate real-time estimation of the event state values.

Spatio-temporal estimation methods

In recent years, there has been a growing interest in the research community towards the development of methods for spatio-temporal filtering under the Bayesian framework for estimation of large scale data. One of the early works in implementing Kalman filter-based estimation for a relatively simple dynamic space-time model was presented in [4], towards prediction of snow-water equivalence. Cane et al. [5] developed a state-space Kalman filter with a reduced order representation for dimensionality reduction of the error covariance matrices towards data assimilation for mapping tropical sea levels. A broad framework for space-time Kalman filtering with dimension reduction was presented in [6] and [7]. The process model is decomposed into an equivalent infinite series summation, and dimensional reduction is achieved by truncating the series while retaining maximum variability in the retained part. A space-time Kalman filter for estimation of ozone concentrations is implemented in [8], though the method proposed there still suffers from a relatively high computational burden. A space-time Kalman filter is developed in [9] towards estimation of soil redistribution attributes. Empirical orthogonal functions are used for dimensionality reduction in a space-time Kalman filter implementation in [10], where a process equation is capable of being described by an exact autoregressive model that can be parameterized by a relatively small number of parameters. This lends certain advantages in the estimation of processes that share similar models, but cannot be directly used for more complex process models involving coupled partial differential equations and a higher number of process parameters as is typical in fire growth models. More recently, Cressie et al. [11] build on the methods presented in [7] to develop an estimation model incorporating spatio-temporal random effects, allowing for computations in a space of a fixed dimension. The method is applied, over a temporal resolution of a day, for estimation of aerosol distributions across large datasets. These estimation methods are constantly being improved upon, with the objective of low-dimensional es-

timation of large datasets for a broad range of application areas.

Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (POD) is a powerful multi-variate statistical tool in data analysis towards the determination of lower-dimensional approximations of high-dimensional dynamic models or processes. The method was independently developed by several researchers that include Karhunen, Loeve, Kosambi and Pougachev [12]. Very often, the terms Principal Component Analysis (PCA) and the Karhunen-Loeve Decomposition (KLD) are used interchangeably with POD, while some researchers make a distinction between PCA, KLD and Singular Value Decomposition (SVD), and view them as the three components that make up the broader technique of POD [12]. PCA, for instance, is equivalent to the POD method applied to a finite-dimensional case and truncated after a few terms [13].

The reader is referred to [12–14] for a more detailed exposition on POD. POD essentially allows for a spatio-temporal process to be decomposed into a series summation of components - composed of spatial basis functions, or POD modes, and time coefficients. Dimensional reduction is achieved by a careful selection of modes that maximize the variance between them, allowing for a fairly accurate representation of the actual state variables with a reduced order model. The POD method can thus be harnessed for dimensional reduction in processing large spatio-temporal data, and has been applied to such problems as the spatio-temporal analysis of temperature patterns [14], in flow control problems [15–17] and in hydrodynamic evaluations of fluid beds [18], to mention a few examples.

Motivation for the present work

We stand at the verge of technological breakthroughs in the field of unmanned systems, with affordable unmanned systems equipped with powerful onboard sensor suites, are close to gaining entrance into the civilian domain. These systems can be deployed across a broad range of application areas, and one such area that stands to benefit from such a development is the field of wildfire monitoring. The advantages that a multi-UAV system can provide towards efficient data gathering for fire fighting applications are immense, reinforcing the emphasis on the need for developing real-time estimation algorithms that can make such deployments feasible and robust.

Much of the work on spatio-temporal estimation has been focused towards application areas in environmental modeling such as climate models, geological and meteorological estimations such as soil evaluations, aerosol distributions and so on, with temporal updates of the order of days, or longer. While many of these are large dimensional spatio-temporal problems and benefit from the reduced-dimensional estimation in space

and time, there is a strong need for development and evaluation of filtering methods for similar problems in time-critical scenarios - such as forest fires, where measurement information can be fairly noisy, spatially sparse and temporally intermittent, and potentially at a much higher acquisition frequency. Estimation methods that are robust to these constraints in data gathering and can allow for efficient filtering of noisy information are critical towards generation of improved situational awareness.

The current work uses a set of coupled Partial Differential Equations (PDEs) to describe the fire growth. Such models have been discussed and presented in works such as [19] and [20], among others. For this study, the model presented in [20] is utilized to generate simulated data for fire growth in a grid-based environment. Measurements are obtained from the process data with additive white Gaussian noise factored in. This would simulate noise introduced in the process of data gathering. The space-time filtering paradigm presented in [7] forms the basis for the filtering model implemented in this work. The emphasis of this work is on the estimation of fire growth patterns governed by coupled partial differential equations in the presence of process and measurement noise, with Proper Orthogonal Decomposition used for dimensionality reduction.

The mathematical details of the fire growth model, POD implementation and the overall filtering framework are presented in the section ahead.

DETAILS

Fire growth model

The fire growth model presented in [20] is used for the simulations in this study. The fire growth can be represented by the set of partial differential equations given by Eqn. 1-2.

$$\frac{dT}{dt} = \nabla(k\nabla T) - (\vec{v})\nabla T + A \left(S e^{\frac{-B}{T-T_a}} - C(T - T_a) \right) \quad (1)$$

The fuel consumption rate is described by:

$$\frac{dS}{dt} = -C_s S e^{\frac{-B}{T-T_a}}, \quad T > T_a, \quad (2)$$

with the initial values $S(t_{init}) = 1$ and $T(t_{init}) = T_{init}$.

$T(\mathbf{x}, t)$ and $S(\mathbf{x}, t)$ represent, respectively, the temperature of the fire layer and the fuel supply mass fraction (relative amount of fuel remaining - $S \in [0, 1]$). T_a represents the ambient temperature, and k represents thermal diffusivity. A is the rate of temperature rise at the maximum burning rate with full initial fuel load and in the absence of cooling. B and C are coefficients

corresponding to the modified Arrhenius law and the heat transfer to the environment respectively. The fuel disappearance rate is given by C_s . The wind speed is represented by \vec{v} .

The term $\nabla(k\nabla T)$ is the diffusion term, modeling short-range heat transfer by radiation in a semi-permeable medium. The influence of winds is described by the term $\vec{v}\nabla T$. The fuel consumption rate due to burning is described by $Se^{\frac{-B}{T-T_a}}$, and the term $AC(T-T_a)$ represents the loss of convective heat to the atmosphere. The reaction rate is given by the term $e^{\frac{-B}{T-T_a}}$, with an ambient temperature offset added to the Arrhenius Law to ensure zero reaction at ambient temperature.

The model is solved using a grid-based finite-difference method to obtain the theoretical temperature distribution, henceforth represented by the term $u_o(x, y, t)$. Additive white Gaussian noise, $N(0, \sigma_p^2)$ and $N(0, \sigma_m^2)$, are successively added to the process to obtain the process and measurement data, respectively represented by $u_p(x, y, t)$ and $u_m(x, y, t)$.

Proper Orthogonal Decomposition

The POD method relies on decoupling the spatial and temporal components of the event state $u(\mathbf{x}, t)$ to allow for a linear approximation to be constructed in a form represented by Eqn. 3.

$$u(\mathbf{x}, t) = \sum_i a_i(t) \phi_i(\mathbf{x}) \quad (3)$$

where \mathbf{x} is the two-dimensional spatial information stacked into a one-dimensional spatial vector for mathematical convenience. $a_i(t)$ represent time-dependent amplitudes, referred to as time coefficients, and $\phi_i(\mathbf{x})$ are spatial basis functions, also referred to in some works as trial functions. The method relies on the determination of the functions $\phi_i(\mathbf{x})$ that have the largest amplitude $a_i(t) = \langle u(\mathbf{x}, t), \phi(\mathbf{x}) \rangle$ in a mean-square sense, with $\langle \dots \rangle$ representing the inner product on \mathbf{x} .

$$\max_{\phi_i(\mathbf{x})} \frac{\langle |u(\mathbf{x}, t), \phi_i(\mathbf{x})|^2 \rangle}{\|\phi_i(\mathbf{x})\|^2} \quad (4)$$

where $|\cdot|$ represents the modulus operator, and $\|\cdot\|$ represents the L^2 norm operator defined by $\|f\| = \langle f, f \rangle^{1/2}$. The POD method thus essentially comes down to the determination of dominant spatio-temporal structures $a_i(t)$ and $\phi_i(\mathbf{x})$ for the given spatio-temporal process. This term determines how well the function $\phi_i(\mathbf{x})$ captures the spatial behavior of the process state $u(\mathbf{x}, t)$ at an instant of time t . The eigenvalues $\lambda_i, i = 1, 2, \dots, \infty$ of the spatial autocorrelation operator

$R(\mathbf{x}, \mathbf{x}') = E[u(\mathbf{x}, t)u(\mathbf{x}', t)]$ are determined to be real and non-negative, since the autocorrelation operator is non-negative and self-adjoint. The corresponding eigenfunctions $\phi_i(\mathbf{x})$ thus form an orthonormal basis for functions on the domain. The eigenvalues are in turn normalized on the basis of the total energy of the system, and ordered in decreasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \dots \lambda_\infty$.

An approximation, and thus a reduction in the model order, is effected by a selection of the first N modes, i.e., modes with the N most dominant eigenvalues, and the basis is optimal for the reason that the aforementioned approximation captures more of the energy than any other N mode approximation. Thus, an infinite-dimensional spatio-temporal process is effectively reduced to a finite-dimensional model represented by the equation:

$$u(\mathbf{x}, t) = \sum_i^N a_i(t) \phi_i(\mathbf{x}) \quad (5)$$

The spatial domain is typically discretized into a grid, and can very often lead to a very large spatial autocorrelation matrix. Solving the eigenvalue problem for such high-dimensional data can be computationally very intensive. To circumvent this, the autocorrelation matrix for the eigenvalue problem can be constructed using what is referred to as the Method of Snapshots. Consider M snapshots of the event state $u(\mathbf{x}, t)$ taken at regularly spaced intervals Δt , with each snapshot defined by $u^i(\mathbf{x}) = u(\mathbf{x}, i\Delta t)$. The autocorrelation matrix is then approximated by Eqn. 6 and facilitates, subsequently, the POD process with reduced dimensionality.

$$R(\mathbf{x}, \mathbf{x}') \approx \frac{1}{M} \sum_{i=1}^M u^i(\mathbf{x}) u^i(\mathbf{x}') \quad (6)$$

The spatial basis functions can then be computed as the linear combinations of the data snapshots using the equation:

$$\phi_i(\mathbf{x}) \approx \sum_{k=1}^M A_k^{(i)} u^k(\mathbf{x}) \quad i = 1, 2, \dots, M. \quad (7)$$

where A_k represents the eigenvectors corresponding to the eigenvalues λ_k obtained from the solution of the eigenvalue problem discussed hitherto. The matrix of time coefficients, \mathbf{a} , can be computed as the solution of Eqn. 8.

$$\Phi \mathbf{a} = \mathbf{S} \quad (8)$$

where \mathbf{S} represents the $M \times M$ snapshot matrix and Φ represents the matrix of spatial basis functions computed using

Eqn. 7, i.e., $\Phi = [\phi_1, \phi_2, \dots, \phi_M]$.

Further, a selection of the first N dominant modes with the maximum variance between them allows for approximating the process with a reduced dimensional model. An optimal choice of N would in turn yield a fairly accurate, yet lower dimensional, equivalence without significant error. The matrices for the spatial basis functions, Φ , and time coefficients, \mathbf{a} , are consequently truncated to retain only those corresponding to the first N modes used for reconstruction of the process.

Space-Time Kalman Filter Estimation

Assuming the process and measurement models satisfy linear and Gaussian assumptions, a space-time Kalman filter can be implemented in a manner described ahead. Consider the estimate of the process and measurement noise variances to be represented by σ_ε and σ_η . The temporal dynamics of the time coefficients are modeled using a first-order multivariate autoregressive model represented by Eqn. 9.

$$\mathbf{a}(t) = H\mathbf{a}(t-1) + J\eta(t) \quad (9)$$

The terms H and $J\eta(t)$ are obtained as parameters of the multivariate AR(1) model, and form the basis for the process equations. The space-time Kalman filter, in conjunction with the decoupled spatial basis functions and time coefficients, is described in Eqns. 10-14.

The prediction equations for the state and covariance matrix are given by:

$$\hat{\mathbf{a}}(t|t-1) = H\hat{\mathbf{a}}(t-1|t-1) \quad (10)$$

$$P(t|t-1) = HP(t-1|t-1)H' + JQJ' \quad (11)$$

where Q is the spatial error covariance matrix, and $J = (\Phi'\Phi)^{-1}\Phi'$, with Φ representing the matrix of spatial basis functions for the truncated set of modes, N .

The Kalman gain $G(t)$ is computed as in Eqn. 12.

$$G(t) = P(t|t-1)\Phi' [R + \Phi P(t|t-1)\Phi']^{-1} \quad (12)$$

The measurement updates for the state and error covariance matrices are presented ahead in Eqns. 13-14 respectively.

$$\hat{\mathbf{a}}(t|t) = \hat{\mathbf{a}}(t|t-1) + G(t)\Phi P(t|t-1) \quad (13)$$

$$P(t|t) = P(t|t-1) - G(t)\Phi P(t|t-1) \quad (14)$$

Once the filtered time-coefficients are obtained through the estimation process, the estimated process is determined by resynthesizing the decomposed model using the equation:

$$\hat{u}(\mathbf{x}, t) = \sum_i^N \hat{a}_i(t)\phi_i(\mathbf{x}) \quad (15)$$

Simulation details and results are presented in the sections ahead.

SIMULATIONS

For this study, the fire growth data is generated using the model described in Eqn. 1-2. The parameters for the fire growth model are listed below. The influence of ambient wind conditions is considered negligible.

$$\begin{aligned} k &= 0.2136m^2/sK^3. \\ A &= 187.93K/s. \\ B &= 558.49K. \\ C &= 4.8372e(-5)/K. \\ C_s &= 0.1625/s. \\ T_0 &= T_a = 300K. \end{aligned}$$

The finite-difference solution for this model is obtained over a spatial grid of dimensions 31x31 units with a resolution of 2 units, and for a temporal length of 101 timesteps with a resolution of 1 unit. The initial conditions of the fire are represented in Figure 1.

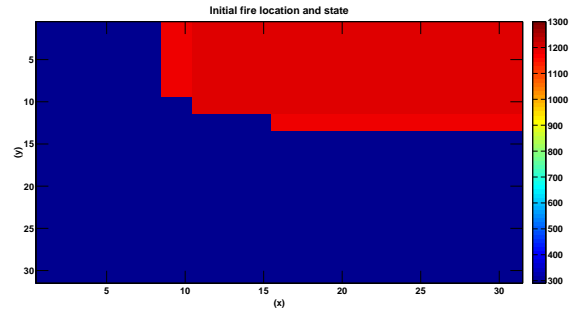


Figure 1: INITIAL FIRE LOCATION (T (in K))

For evaluating the estimation process, process and measurement noises with variances $\sigma_p^2 = 2.25$, $\sigma_m^2 = 16$ are added to the fire data obtained from the PDE model. The estimated process and measurement noise variances for the filter are set at $\sigma_\eta^2 = 1$ and $\sigma_\varepsilon^2 = 6.25$, and the corresponding covariance matrices for estimation are $\sigma_\eta^2 I(n^2, n^2)$, where n represents the length of the stacked one-dimensional spatial vector and $I(n^2, n^2)$ represents

an identity matrix of dimensions (n^2, n^2) . A total of 101 measurement snapshots are obtained for the POD process, yielding the same number of spatial modes and time coefficients. Figure 2 shows the errors in the POD reconstruction process as a function of the number of modes used for reconstruction, corresponding to two spatial gridpoints. It can be observed that the errors largely stabilize at a relatively low magnitude after the first 6 modes, and with a small tolerance, the first 8 modes are considered for the POD process to represent the overall process dynamics with marginal error, which allows for a significant reduction in computational costs.

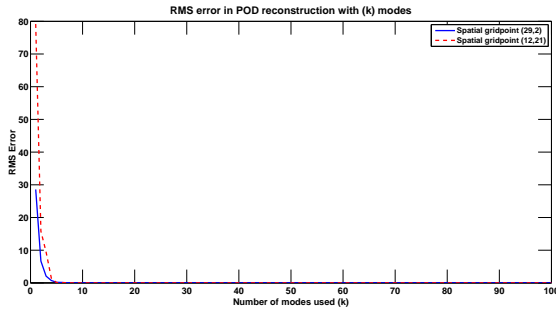


Figure 2: ERRORS RELATIVE TO NUMBER OF MODES USED FOR POD-BASED RECONSTRUCTION.

The estimation process is implemented as discussed hitherto, and the results are presented ahead. For the multivariate autoregressive model fitting, the MATLAB package described in [21] is utilized. All computations are performed in the MATLAB R2011b environment on a 64-bit Windows-based PC with a dual core processor with a clock speed of 3.10 GHz and 8 GB of RAM.

Figure 3 shows the effectiveness of the estimation process at a specific spatial grid location over time. Comparative plots of the different states are presented, and it can be observed that the filtering process is able to accurately track the actual state variable. Comparisons in space are presented in Figure 4 at a specific instant of time.

Similar comparisons are shown for a second set of spatial and temporal points in Fig. 5 and Fig. 6, and the effectiveness of the filter in seeing through noisy measurements can be observed through these plots.

Plots of the RMS errors between the estimated and actual processes, and of those between the measurements and actual process data, are compared in Fig. 7 to demonstrate the effectiveness of the estimation method in filtering out noise in space and time. The reduction in noise in the state estimates can be observed in these plots and indicates the effectiveness of the pro-

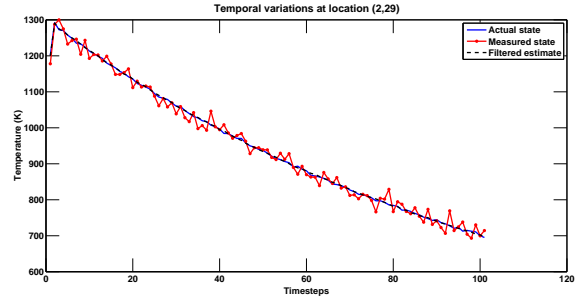


Figure 3: TEMPORAL COMPARISON OF ESTIMATES WITH PROCESS AND MEASUREMENT DATA AT GRID-POINT (29,2).

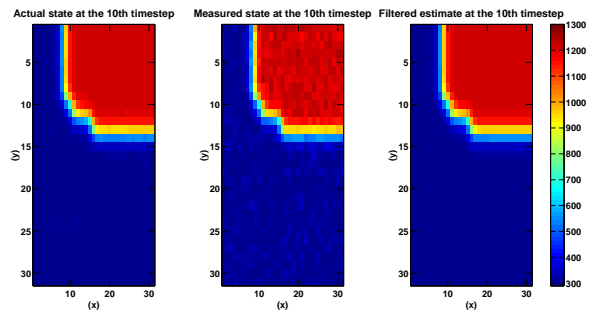


Figure 4: SPATIAL COMPARISON OF ESTIMATES WITH PROCESS AND MEASUREMENT DATA AT THE 10TH TIMESTEP (K).

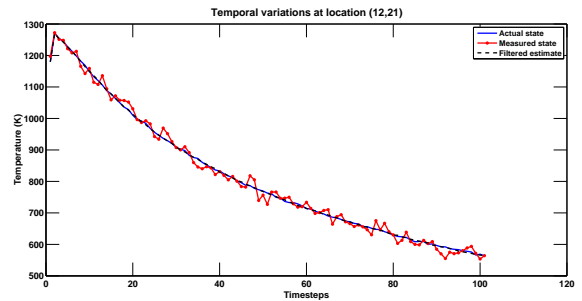


Figure 5: TEMPORAL COMPARISON OF ESTIMATES WITH PROCESS AND MEASUREMENT DATA AT GRID-POINT (21,12).

posed methodology towards accurate spatio-temporal estimation.

CONCLUSIONS

In this work, a spatio-temporal estimation model has been proposed for real-time monitoring of wildland fires. Considering the high dimensionality of such processes evolving rapidly in space and time, the need for prediction and estimation methods

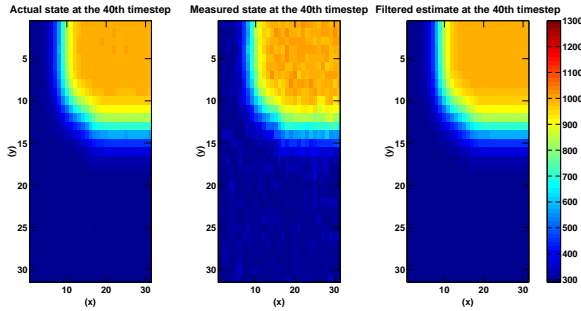


Figure 6: SPATIAL COMPARISON OF ESTIMATES WITH PROCESS AND MEASUREMENT DATA AT THE 40TH TIMESTEP (in K).

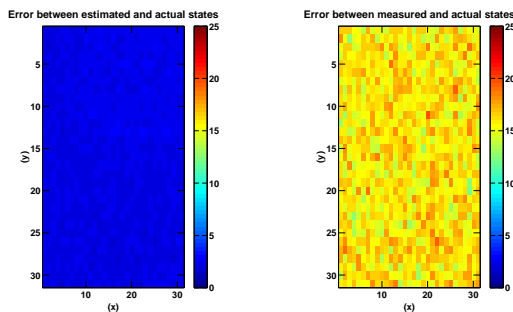


Figure 7: COMPARISON OF RMS ERRORS WITH RESPECT TO ACTUAL PROCESS STATES: (L) FILTERED ESTIMATES (R) MEASUREMENT DATA.

for efficient tracking of forest fires is critical. The shortcomings of current methods for monitoring wildland fires mandate the need for newer technologies towards such applications, and the use of unmanned multi-agent systems with onboard sensors has, in recent years, been regarded as a viable alternative in the near future, the feasibility of which largely depends on estimation algorithms such as presented in this paper, and their efficiency in terms of both accuracy and computational costs towards real-time application.

The novelty of this work lies in the development and implementation of an estimation framework incorporating a theoretical fire growth model based on coupled partial differential equations into a space-time Kalman filter, for state estimation in the presence of noisy measurement information. Proper Orthogonal Decomposition methods are incorporated for use with the spatio-temporal filter for achieving reduced dimensionality in computations, for real-time deployability. The estimation model is tested over numerically simulated forest fire data in the presence of Gaussian noise, and the performance of the reduced dimensional estimation methodology in estimation of hidden states in the presence of noisy measurements is demonstrated.

The present work would be pursued further in a distributed sensing framework in a multi-agent system, and issues with spatial and temporal gaps in data gathering, errors in process modeling and measurements, and cumulative influence of these agent-level issues on developing global situational awareness would be studied.

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