Control of Linear Second-Order Systems by Fuzzy Logic-Based Algorithm

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Control of Linear Second-Order Systems by Fuzzy Logic-Based Algorithm

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The control of flexible structures employing the passivity approach has been extended to systems having noncollocated input/output pairs by introducing an observer that incorporates the nominal dynamical model of the plant. The passive observer-based control is applied to the American Control Conference benchmark problem, whereby, the control force emulates a dynamic vibration absorber attached to a virtual wall with passive control elements (spring, mass, and dashpot). The springs and mass elements of the controller are constant, whereas the damping coefficients are selected as time dependent in an attempt to choose continuously the most appropriate amount of damping in compliance with the design goals. A novel approach is introduced, whereby the passive observer-based control law is modified by varying the damping coefficient of the virtual dashpot by means of an adaptive fuzzy logic algorithm. This modified system exhibits quick settling times and desirable performance characteristics. Results from the statistical robustness analysis for the developed controller are compared to 10 other (linear) solutions of the benchmark problem. The comparison is based on robust stability, robust performance (settling time), and control effort. The results obtained by the adaptive fuzzy logic algorithm are superior to those obtained by all other methods, and, consequently, further application of the fuzzy algorithm is advocated.

I. Introduction

DURING the years 1990–1992, certain benchmark problems for robust control design were presented at the American Control Conference (ACC). One of these problems, referred to by Wie and Bernstein[1] as ACC benchmark problem 1, was concerned with vibration control of a two-mass system with an uncertain spring constant in view of a transient disturbance (Fig. 1). The simplicity of this problem provided a transparency that enabled it to be an interesting tool for comparison of a variety of robust control design methodologies. Nevertheless, this problem is nontrivial because it couples both rigid- and flexible-body modes with plant uncertainty and noncollocated sensor and actuator. In addition, sensor readings are contaminated by a high-frequency sensor noise.

The ACC benchmark problem may be classified as a flexible structure represented by a second-order dynamic system. For such a class of dynamic systems, Juang and Phan[2] presented a robust controller that is a passive design based on virtual second-order dynamic system comprising virtual mass, spring, and dashpot elements. The virtual mechanisms incorporated into the passive design serve only to transfer and dissipate the energy of the system, thereby maintaining the stability of the system. In addition, Juang and Phan[2] showed that overall closed-loop stability was guaranteed, independently of the system structural uncertainty and perturbations in the temporal plant dynamics.

These second-order controllers may also be termed collocated, and they consist of compatible pairs of actuators and sensors, which may be distributed throughout the structure.[3] However, in many real-life situations involving the control of flexible structures, because of physical placement and hardware limitations, absolute collocation may sometimes be impossible.[4] Furthermore, for the case of noncollocated actuator and sensor pairs, such as the ACC benchmark problem, strictly passive feedback no longer guarantees stability. To circumvent this problem, Hughes and Wu[5] presented an observer-based extension of the passive controller design for the described noncollocated case. This approach, based on the nominal dynamic model of the system, is a direct generalization of the dissipative controller, whereby the passive output is synthesized using an observer as opposed to the availability of physical measurement as required. This generalization, which includes application to the noncollocated case is, however, at the expense of two of the inherent characteristics of passive controllers, the sacrifice of model independence and the taking of stability robustness for granted.

In this paper, a second-order passive controller is applied to the ACC benchmark problem whereby the control force emulates a virtual dynamic vibration absorber attached to a virtual wall by means of a virtual spring (Fig. 2). The springs and mass elements of the controller are tuned to introduce two virtual low-frequency flexible modes instead of the single rigid-body mode. Furthermore, a control law, which is based on the principles of fuzzy logic control is introduced, to tune the damping parameter continuously of the earlier-described passive controller.

The main advantages of using a fuzzy approach are the relative ease and simplicity of implementation and the robustness characteristics. The parameters of the described absorber may be adapted to provide fairly fast control for large deviations, of the measured state of the plant from the desired state, and a minor amount of control for small deviations. The successful implementation of a fuzzy logic controller depends, among other design aspects, on the heuristic rule base from which control actions are derived. To obtain the required heuristic physically based insight, a single-degree-of-freedom (DOF) system based on optimal control theory will be examined analytically to observe the characteristics of a minimum time solution. Later, a fuzzy logic nonlinear mapping function, which has the potential of being a universal approximator,[6] is applied to emulate the minimum-time solution. The resulting rule base is the core of the control law that is applied herein to the ACC control problem.

Finally, we examine whether the closed-loop system should provide satisfactory stability and performance characteristics not only for the nominal plant but also over the range of values associated with parameter uncertainties. As mentioned by Wie and Bernstein,[1] the feedback controller should display reasonable performance/stability robustness. To this end, Stengel and Marrison[7] presented some evaluation criteria that concern the selection of appropriate measures of robustness of the ACC benchmark problem and that demonstrate that the described evaluation criteria may be satisfied by the application of stochastic robustness analysis (SRA).

SRA[7,8] involves determining the probability of unsatisfactory stability or performance resulting from expected parameter unstability. Furthermore, SRA was shown to provide a useful, unifying analytical framework that is intuitive and to have a direct, physical meaning. The definitions and principles of the SRA adhered to in

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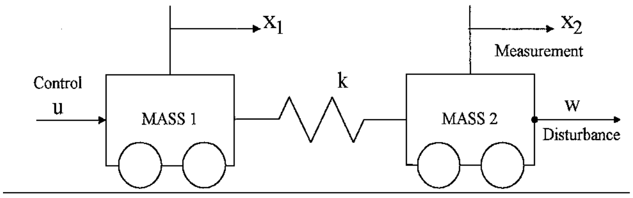

Fig. 1 ACC benchmark problem.[1]

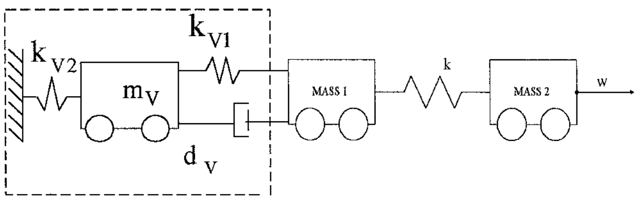

Fig. 2 ACC benchmark problem with virtual absorber and wall.

the present effort were based on the described approach, whereby a 1000-run Monte Carlo simulation was used to estimate the probabilities of stability/performance. The results obtained were compared with those presented in the comparative study conducted by Stengel and Marrison.[7]

II. Emulating Minimum-Time Control

Many dynamic systems in the field of aerospace/mechanical/electric engineering can be represented by a set of linear second-order ordinary differential equations. Shahruz et al.[9] showed that the optimal damping ratio for a single-DOF system (harmonic oscillator) that results in an open-loop minimum-time response to step inputs is of bang–bang type as follows.

1) Optimal damping ratio is zero from the initial state until an appropriate switch time (t_S).

2) At t_S, the damping ratio is switched to some maximum value.

3) Time t_S is a function of the maximum damping ratio and system natural frequency for a given set of initial conditions.

For certain applications, such as large flexible structures, the described open-loop control is impractical, especially if there are uncertainties in the plant temporal dynamics. Often, for such systems with uncertainties in their temporal plant dynamics, we seek a controller that incorporates (among other features) the following characteristics: 1) minimum-time responses, 2) robustness in face of uncertainties/variations plant temporal dynamics, and 3) closed-loop control that is insensitive to initial conditions.

We appreciate the implications of the result reached by Shahruz et al.[9] and furthermore suggest an alternative approach that leads to a more robust solution. Let us take a close look at the optimal control problem of a single-DOF system, with a variable damper, as described in Eq. (1), where ω and $u(t)$ are the natural frequency and the viscous damping factor, respectively:

$$x_1(t) = x_2(t)$$

$$x_2(t) = -2\omega u(t)x_2(t) - \omega^2 x_1(t) \tag{1}$$

with initial conditions

$$x_1(0) = x_1^0, \qquad x_2(0) = x_2^0 \tag{2}$$

and constraint on the viscous damping factor

$$0 \le u(t) \le 1 \tag{3}$$

For sake of convenience, we will consider the following numerical example: $\omega = 0.5$, with the initial conditions for sets 1 and 2, respectively, of

$$x_1^0 = 0.2, \qquad x_2^0 = 0, \qquad x_1^0 = 0, \qquad x_2^0 = 0.2 \tag{4}$$

The treatment is basically the same with other numerical values. The main objective is to drive the system in minimum time to the target set G, which represents a small neighborhood near the origin and is described as follows:

$$G = \{(x_1, x_2) : |x_1| \le 0.003, |x_2| \le 0.003\} \tag{5}$$

The Hamiltonian that describes the optimal control problem given by Eqs. (1–5) may be written as

$$H(x_1(t), x_2(t), \lambda_1(t), \lambda_2(t), t) = 1 + \lambda_1(t)x_2(t)$$

$$- \lambda_2(t)[0.25x_1(t) + u(t)x_2(t)] \tag{6}$$

The transversality condition requires that $H(t) = 0$ for all t along an extremal trajectory. The adjoint equations are obtained as follows:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = \frac{\lambda_2(t)}{4} \tag{7}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -\lambda_1 + u(t)\lambda_2(t) \tag{8}$$

The optimal control should satisfy the following (minimum principle):

$$\arg\min_u H[\lambda(t), x^*(t), u(t)] = H[\lambda(t), x^*(t), u^*(t)] \tag{9}$$

We obtain

$$u^* = \arg\min_u [-\lambda_2(t)x_2(t)u] \tag{10}$$

From Eq. (10) it is apparent that u is switched whenever the product $\lambda_2(t)x_2(t)$ crosses the zero axis going from negative to positive and vice versa. The resulting control law is as follows:

$$u = 0 \quad \text{for} \quad \lambda_2(t)x_2(t) < 0$$

$$u = 1 \quad \text{for} \quad \lambda_2(t)x_2(t) > 0 \tag{11}$$

Before proceeding, it is imperative to verify whether u can become singular, that is, $\lambda_2(t)x_2(t) = 0$ over a finite time interval. This implies that $\lambda_2(t) = 0$ for a singular solution because the state variable cannot lie permanently at the origin. From the adjoint equations, this would require $\lambda_1(t) = 0$. From Eq. (6), we obtain that $H = 1$. However, the transversality condition requires that $H(t) = 0$ for all t along an optimal trajectory. Therefore, u cannot be singular. Hence, the optimal solution represented by Eq. (11) is of a bang–bang nature.

After analytically examining the necessary conditions for a time-optimal control law for the variable damper, we examine the time-histories based on numerical studies conducted using MATLAB®. Equations (7) and (8) have a simple analytical solution, which has been used to verify optimality as shown by Cohen.[10] Figure 3 presents some numerical results in the phase plane (x_1–x_2). Three trajectories are shown: the one that resulted from set 2 of the initial conditions and two more, with higher initial velocities. The phase plane is partitioned by two switching curves S_1 and S_2. S_1 is the switching curve defined by $x_2 = 0$, whereas S_2 is the switching curve resulting from $\lambda_2(t) = 0$. For initial conditions that lie in the upper-right part of the plane, for example, sets 1 and 2, the control strategy is determined by the shown partitioning. Figure 4 presents time histories starting with set 2 as the set of initial conditions. The time-optimal settling time obtained, 7.45 s, compares very favorably to the 13.75 s obtained using a linear, time-invariant approach, that is, $u(t) = 1$. These results advocate the use of a variable damping system for vibration suppression in the presence of a transient disturbance. However, there are some limitations to the effective implementation of this minimum-time strategy. The presented optimal solution lacks robustness in view of uncertainties in plant model and external noise. The properties of the switch points (how many and when they occur) depend on the temporal plant model and the character of the transient disturbance,

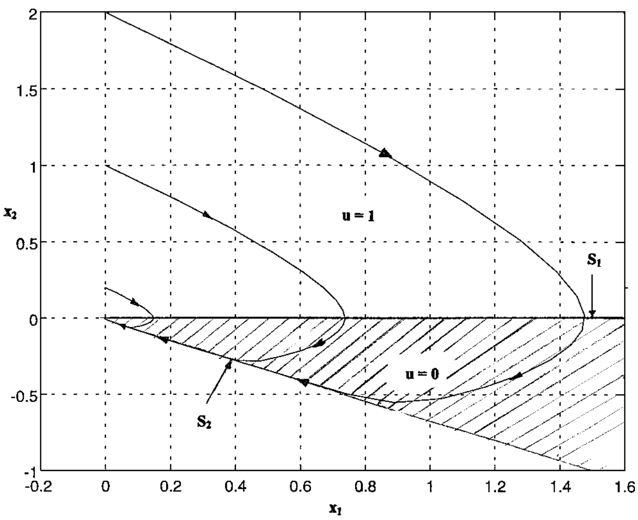

Fig. 3 Switching curves for time-optimal system: S_1, first switch at t = 2 s and X_2 = 0; S_2, second switch.

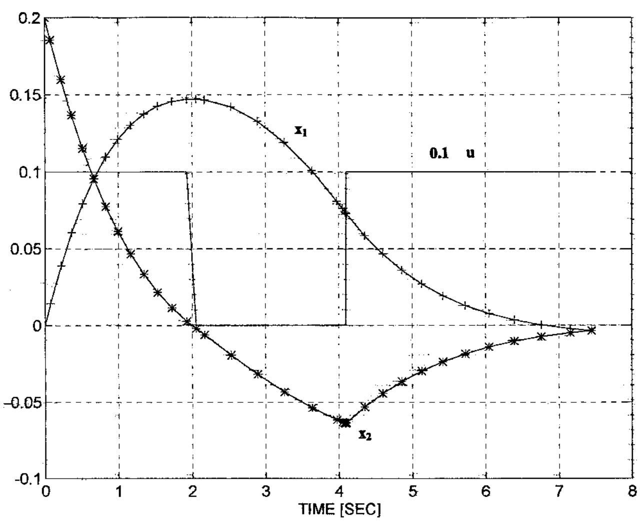

Fig. 4 Time histories for time-optimal system (set 2 initial conditions): settling time, 7.45 s; switch times, t_{S_1} = 2.0 s, t_{S_2} = 4.1 s.

Table 1 Comparison of settling times of one-DOF system

Initial condition	Critical damping $u(t) \equiv \xi = 1$, s	Time-optimal variable damping, s	Fuzzy logic-based approximator, s
Set 1: $x_1^0 = 0.2$, $x_2^0 = 0$	12.4	6.1	6.25
Set 2: $x_1^0 = 0$, $x_2^0 = 0.2$	13.75	7.45	9.25

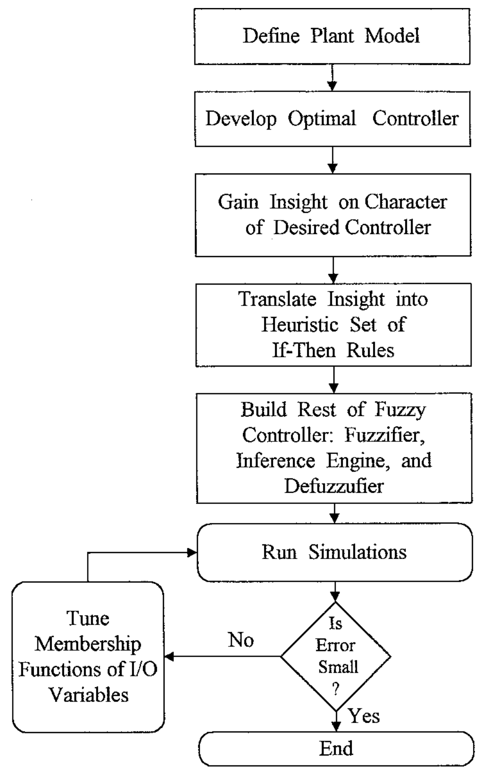

Fig. 5 Fuzzy approximation of time-optimal control.

that is, sensitivity to initial conditions. Furthermore, the implementation into a closed-loop system is cumbersome and practically ineffective.

Hence, incorporating optimal variable damping requires an approach that enables the integration of the control law into a large flexible structure with relative ease and simplicity, while providing the required robustness characteristics. An appropriate approach, based on fuzzy logic, is introduced to achieve the desired control. The expert knowledge required for the design of the fuzzy logic controller is obtained by observing the closed-loop system controlled by the minimum-time law. The insight gained with respect to the behavior of the desired closed-loop system is subsequently translated into a linguistic expression ("if...then..." rules) of the relationship between the input and output variables. In this case, the insight is indirectly based on the nominal model of the plant. For the class of systems such as flexible structures, it may be assumed that there always is some idealistic dynamic model. The uncertainties in the plant model and other sensitivities may be addressed by demanding robust control. Therefore, from this point of view, the described strategy differs from the model-independent approach usually associated with fuzzy logic control.

The entire process of developing a fuzzy logic controller for the single-DOF system, based on fuzzy approximation of time-optimal control, is shown in Fig. 5 and is detailed by Cohen.[10] The time history for initial condition, set 2, presented in Fig. 6, illustrates the fuzzy approximation of crisp bang–bang control. For the fuzzy approximator, we examine the time histories for the two sets of initial conditions given in Table 1. As shown in Table 1, the fuzzy approximator provides near time-optimal settling times. Furthermore, results obtained using fuzzy control can be compared very favorably to those obtained using a linear, time-invariant approach, that is, critical damping.

For the control of dynamic systems, such as some flexible structures, an essential requirement from a controller constitutes robustness. Although the time-optimal controller performs well in an ideal situation (full knowledge of plant model and no measurement noise), such as the single-DOF problem described earlier, it is not a good choice at the real plant.[11] Modeling uncertainties and noise lead to inaccurate switching of the control u, and this causes settling times that are not only nonoptimal, but worse than those obtained with linear time-invariant (LTI) control (critical damping at $u = 1$).

One of the attributes of a fuzzy logic control system is the interpolation among rules, which combines the contributions of active rules, allowing the designer to bound the rate of change of control with respect to the state. This enables a gradual transition from one control action to the next as the plant traverses state-space regions.[12] This interpolation can implement the "if...then..." rule base, while avoiding the discontinuity of the minimum-time control. Furthermore, the attribute of interpolation among rules provides the additional

Table 2 Robustness comparison for one-DOF system

Natural frequency, rad/s	Control action	Settling time, s
0.25	Fuzzy control approximator	11.4
0.25	Time-optimal control	10.52
0.35	Fuzzy control approximator	8.5
0.35	Time-optimal control	8.0
0.75	Fuzzy control approximator	4.35
0.75	Time-optimal control	4.26
1.0	Fuzzy control approximator	4.8
1.1	Time-optimal control	3.3

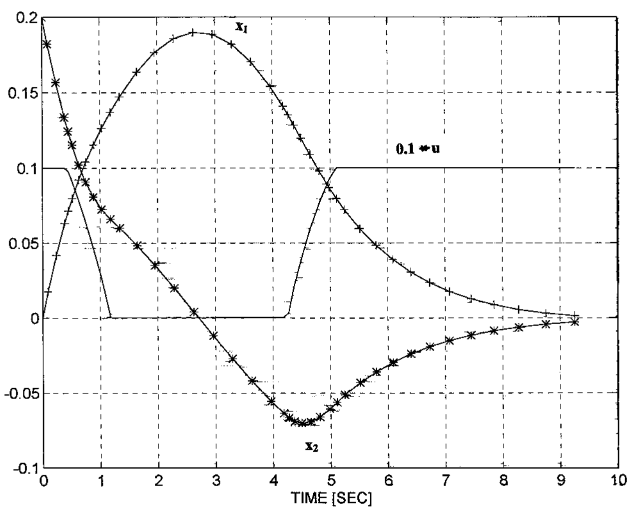

Fig. 6 Time histories for fuzzy near-optimal system (set 2 initial conditions)

benefit of robustness. Now, we shall put the inherent robustness of the fuzzy logic approximator to test with set 1 initial conditions. The frequency of the single-DOF system (nominal frequency 0.5 rad/s) will be perturbed for four separate cases, namely, 0.25, 0.35, 0.75, and 1.0 rad/s. For each of these cases the minimum-time control is obtained on an individual basis. On the other hand, we take the fuzzy logic approximator developed for the nominal frequency, and the frozen design is then applied to the perturbed cases.

As expected, the prescribed perturbations affect the switch points of the viscous damping factor u. The settling times, for the system to converge into the square determined by $|x_1| \leq 0.003$ and $|x_2| \leq 0.003$, for the time-optimal solution and the fuzzy logic approximator, are listed in Table 2. For example, as the frequency of the plant increases, the period of oscillation decreases, as does the settling times. Therefore, for the higher frequencies of $\omega = 1$ rad/s, the control needs to be switched earlier, with respect to the nominal controller, or else a slightly underdamped oscillation is experienced, thereby increasing the optimal settling time by a factor of 2.5–3. The closed-loop implementation of the fuzzy logic approximator inherently adjusts the switching points and provides near-optimal behavior for the perturbed range. The results presented in Table 2 emphatically demonstrate the capability of a fuzzy logic controller to approximate minimum-time control and to possess the quintessential robustness characteristics.

III. ACC Benchmark Problem

This section examines the strengths, weaknesses, and limitations of a constant-gain linear feedback passive observer-based (POB) controller. The main purpose is to examine the application of a POB controller to the ACC benchmark problem, presented in Fig. 1, that involves the vibration control of a two-mass–spring system in view of a transient disturbance. The ACC benchmark problem was chosen because it is characterized as having 1) uncertainties in the temporal plant (spring constant that varies within a very wide range), 2) a flexible mode as well as a rigid-body mode, 3) noncollocated sensor and actuator that introduce extra phase lag into the system (this makes the task of control more challenging), and 4) Sensor readings contaminated by a high-frequency sensor noise $[0.001\sin(100t)]$.

The two-mass–spring system shown in Fig. 1, which is a generic model of an uncertain dynamic system, contains a single rigid-body mode as well as one flexible mode. It is assumed that for the nominal system $m_1 = m_2 = 1$ and $k = 1$, with appropriate units, and that time is in seconds. A control force acts on body 1, and the position of body 2 is measured, resulting in a noncollocated actuator/sensor control problem. This system may be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/m_1 & k/m_1 & 0 & 0 \\ -k/m_2 & -k/m_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} (u + w_1) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} w_2 \tag{12}$$

$$y = x_2 + v \tag{13}$$

$$z = x_2 \tag{14}$$

where x_1 and x_2 are the positions of body 1 and body 2, respectively; x_3 and x_4 are the velocities of body 1 and body 2, respectively; u is the control input acting on body 1; y is the sensor output; w_1 and w_2 are the plant disturbances acting on body 1 and body 2, respectively; n is the sensor noise; and z is the output to be controlled, that is, the performance variable. The controller for the benchmark problem should have the following properties.

1) For a unit impulse disturbance exerted on body 1 and/or body 2, the controlled output z has a settling time of about 15 s for the nominal system with $m_1 = m_2 = k = 1$.

2) The closed-loop system is stable for $0.5 \leq k \leq 2.0$ and $m_1 = m_2 = 1$.

3) Reasonable performance/stability robustness are achieved with reasonable bandwidth.

4) The closed-loop system is insensitive to high-frequency sensor noise [modeled as $0.001\sin(100t)$ by Chiang and Safonov[13]].

5) Reasonable control effort is used, and reasonable controller complexity is needed.

The constant-gain linear POB controller consists of a set of virtual springs, a damper, and a lumped mass applied to m_1, as shown in Fig. 2. After each contaminated sensor reading, the control force is computed by calculating the force applied by the set of virtual passive elements (lumped mass, springs, and dashpot). This in turn requires the establishment of an estimate of the displacement and velocity of mass m_1 as shown in the block diagram of the closed-loop system (Fig. 7). A Luenberger observer[14] is introduced to estimate the state of the plant. A necessary condition for the development of a Luenberger observer is that the system be observable for $0.5 \leq k \leq 2.0$. Because the observability matrix of the system represented by Eqs. (12) and (13) has a rank of 4 for the entire field of variation in the spring stiffness k, the system is observable. Equations (12) and (13) can be written in conventional form as (neglecting noise and disturbances)

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{15}$$

$$y(t) = Cx(t) \tag{16}$$

The output error may be written as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + M[y(t) - C\hat{x}(t)] \tag{17}$$

The error $\hat{x}(t) - x(t)$ will decay to zero if M is chosen so that all of the eigenvalues of the matrix $[A - MC]$ lie on the left half-plane. Subsequently, the poles of the observer state were placed as follows:

$$[P] = [-1 - j \quad -1 + j \quad -2 - 2j \quad 2 + 2j] \tag{18}$$

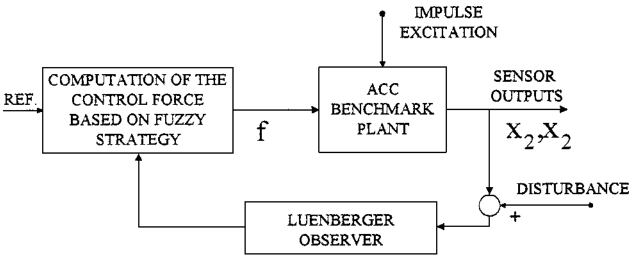

Fig. 7 Block diagram of closed-loop system.

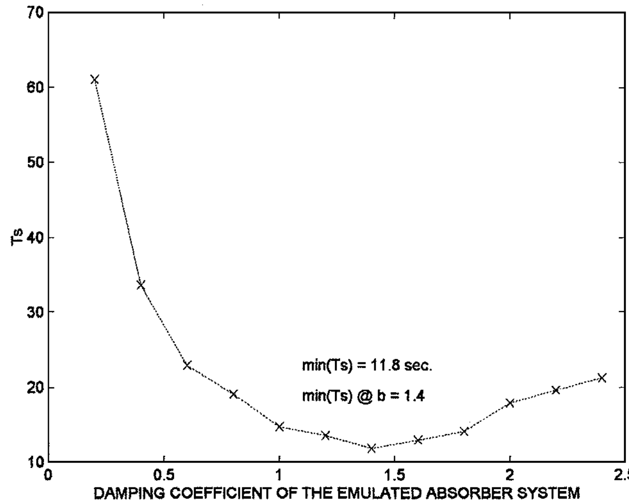

Fig. 8 Settling times for linear time-invariant controllers for various values of damping coefficient (k = 1).

An algorithm provided by MATLAB Control Toolbox (see Ref. 15) was used to obtain the value of matrix M:

$$[M]^T = [18.0 \quad 6.0 \quad 0.0 \quad 16.0] \tag{19}$$

The equations of motion of the system and the POB controller may be written in space-state form as

$$\overbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \\ x_1 \\ x_1 \\ x_v \\ x_v \\ x_2 \\ x_2 \end{bmatrix}}^{\cdot} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k & k & 0 & 0 & -k_{v1} & -d_v & k_{v1} & d_v & 0 & 0 \\ k & -k & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 18 & 0 & 0 & 0 & 1 & 0 & 0 & -18 & 0 \\ 0 & 0 & 0 & 0 & -(1+k_{v1}) & -d_v & k_{v1} & d_v & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{v1}/m_v & d_v/m_v & -(k_{v1}+k_{v2})/m_v & -d_v/m_v & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & -6 & 1 \\ 0 & 16 & 0 & 0 & 1 & 0 & 0 & 0 & -17 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \\ x_1 \\ x_1 \\ x_v \\ x_v \\ x_2 \\ x_2 \end{bmatrix} \tag{20}$$

The ACC benchmark problem suppressed by a POB controller, represented by Eq. (20), may be classified as LTI, thereby enabling the stability analysis of the closed-loop system using a conventional eigenvalue approach. The simulation of the closed-loop system is conducted using MATLAB. The size of the virtual lumped mass was arbitrarily chosen to be large, $m_v = 600$, whereas the stiffnesses of the virtual springs of the controller are, respectively, tuned to introduce two virtual low-frequency flexible modes instead of the single rigid-body mode. The values $k_{v1} = 0.35$ and $k_{v2} = 2.9$ were obtained after fine tuning the closed-loop system to provide quick settling times for the nominal system and satisfactory performance and stability robustness using reasonable control effort.

A wide spectrum of feasible damping coefficients is examined in an attempt to select the most appropriate amount of damping. First, the settling time (defined as the time for which the desired output z is captured within a 0.1-unit envelope about its zero steady-state value, given an initial unit disturbance impulse on m_2) and the maximum absolute peak value of the respective control effort, for the nominal plant ($k = 1$), are observed for a variety of damping coefficients.

The settling times T_S for 12 different controllers, each having a different damping coefficient, namely, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, and 2.4, are shown in Fig. 8. The quickest settling time reached is 11.8 s, and the corresponding damping coefficient $d_v = 1.4$. This settling time complies with the requirement of 15 s as prescribed by Wie and Bernstein.[1] The closed-loop time response of the nominal plant for $d_v = 1.4$, presented in Fig. 9, also shows that the maximum control effort required is 0.46 units, which is well below the constraint, $|u_{\max}| \leq 1$. At this point, the focus is on the robustness characteristics of the POB controller for $0.5 \leq k \leq 2.0$. For $d_v = 1.4$, two questions are asked. Is the closed-loop system stable for all k in the domain [0.5, 2.0]? How good is the closed-loop performance?

Examination of the poles that result from Eq. (20), for $d_v = 1.4$, indicates that the closed-loop system is stable in the interval $0.61 \leq k \leq 2.7$. However, this does not meet the requirement set by Wie and Bernstein,[1] namely, that the closed-loop system be stable for $0.5 \leq k \leq 2.0$. Therefore, the described approach to selecting d_v, based on the search for a corresponding minimum value of the settling time (Fig. 8), falls short of design demands.

The search may be redirected to find a value for d_v that stabilizes the closed-loop system for $0.5 \leq k \leq 2.0$. A reexamination of the resulting poles for $d_v = 0.75$ indicates that the closed-loop system is stable in the interval $0.50 \leq k \leq 2.04$, thereby satisfying the specified level of stability robustness. However, the settling time of the nominal plant for $d_v = 0.75$ is an unacceptable 20 s (Fig. 8). There seems to be a conflict between the demand for nominal plant performance and stability robustness. This conflict does not enable the selection of a single value for d_v that meets both requirements simultaneously. Because the quest for an appropriate value for d_v is not as straightforward as one might expect, given the demands of the ACC benchmark problem, an attempt will now be made to increase our understanding of the effect that the parameter d_v has on the stability of the closed-loop system for a wide variety of spring stiffness k. As seen in Fig. 10, the regions of stability, with respect to d_v and k, fall within the enclosure ABCD. In addition, the regions of interest are bounded by $d_v \geq 0$ (passive systems have positive semidefinite damping coefficients) and $d_v \leq 2.85$ (heavily damped systems provide poor nominal plant performance as shown in Fig. 8). Figure 10 clearly shows the limited range of stability for $d_v = 1.4$ and the unfulfilled promise for $d_v = 0.75$ (segment EF). Can the overall performance of the constant gain POB controller be augmented? If so, how?

Examining Fig. 8 in the vicinity around $k = 0.5$, we observe that the closed-loop system is stable for small values of d_v (when damping is light) and unstable for large values of d_v (when damping is

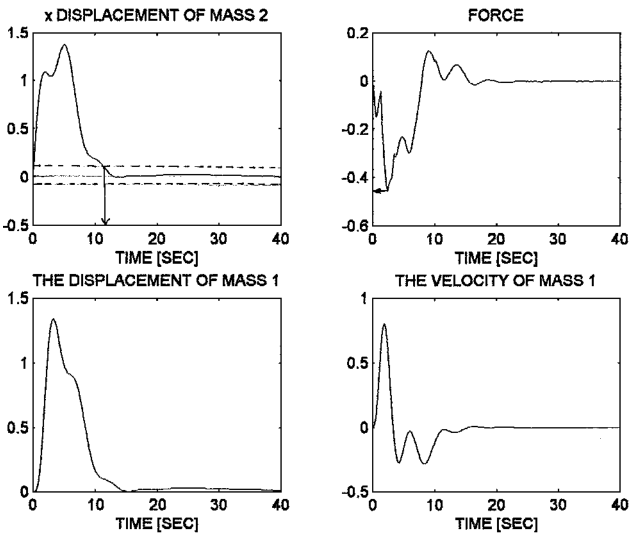

Fig. 9 Performance of the constant-gain, linear POB controller for the nominal system (d_v = 1.4, T_s = 11.8 s, $|u_{max}|$ = 0.46).

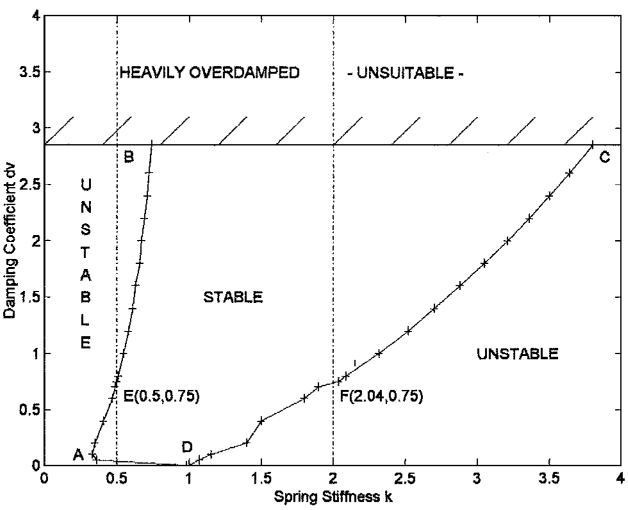

Fig. 10 Regions of stability of a POB controller.

heavy). Now, we introduce some heuristic rules based on insight obtained from research on control of flexible structures, including numerical simulations[16,17] and experimental studies.[17,18] Let the value of d_v vary along the line $k = 0.5$ in accordance with the following.

1) If the error (deviation from the desired state) is large then damping is light.

2) If the error is small then damping is heavy.

The application of these heuristic rules for $k \cong 0.5$ is shown schematically in Fig. 11a. Region A within the circle represents small errors that is, heavy damping, which in turn corresponds to large values of d_v. As seen in Fig. 10, the region to the left of segment BE is unstable. Hence, within A, arrows point outward signifying instability of the closed-loop system. On the other hand, region B outside the circle represents large errors, that is, light damping, which in turn corresponds to small values of d_v. In this case, it is seen from Fig. 10 that the region to the right of segment AE is stable and, accordingly, in region B the arrows point inward.

Curve C is the borderline between regions A and B, that is, between the region of stability and instability. About curve C, unstable small errors would move from region A to B only to transform into stabilized large errors, which flow back into region A and so on and so forth. Therefore, for $k \cong 0.5$, one may predict that a typical transient response of a variable damping POB will produce a mechanism of a limit cycle. Because this limiting cycle is bounded, the closed-loop system may be regarded as being stable in the sense of Lyapunov.

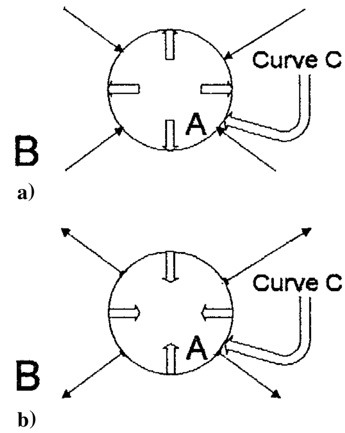

Fig. 11 Schematic of stable regions for k = 0.5 and k = 2.

The next step is to examine the effect of the heuristic rules, associated with an adaptive POB (d_v is varied), on the nominal plant $k \cong 1.0$. The damping coefficient is adapted to provide fairly fast control for large errors and a minor amount of control for small errors. Thus, control actions, corresponding to a lightly damped POB controller send the plant state hurtling toward the desired state. On the other hand, in the vicinity of this desired state, the system is heavily damped. Application of the mentioned heuristic rules to the vibration suppression of large flexible beamlike structures[16] produced fairly quick settling times for the nominal plant. A similar result may be predicted for the case of the ACC benchmark problem.

Finally, the heuristic rules are applied to vary d_v for $k \cong 2.0$ (see schematic in Fig. 11b). Once again, region A within the circle represents small errors, that is, heavy damping, which in turn corresponds to large values of d_v. As seen in Fig. 10, the region to the left of segment CF is stable. Hence, within A, arrows pointing inward signify stability of the closed-loop system. On the other hand, region B outside the circle represents large errors, that is, light damping, which in turn corresponds to small values of d_v. In this case, it is seen from Fig. 10 that the region to the right of segment DF is unstable, and accordingly, in region B the arrows point outward. Curve C is the borderline between regions A and B, that is, between the region of stability and instability. All errors within region A converge to the origin, thereby leading to asymptotic stability. Accordingly, once the magnitude of the disturbance is beyond a defined threshold, which in turn causes the error to belong to region B, the system is unstable Curve C, therefore, represents marginal stability. For the values of $k \cong 2.0$, the adaptive POB controller no longer provides a mechanism of a limiting cycle, and stability depends on the intensity of the disturbance. Although this situation may not seem to be a very comfortable one, it is acceptable in several instances concerning the control of aerospace systems.

The heuristic rules described and the qualitative analysis that followed have provided a fairly good picture of what to expect from an adaptive POB controller. However, an appropriate algorithm that provides the adaptive feature, incorporating the heuristic rules, needs to be introduced into the POB. At this point, the damping is varied based on the fuzzy logic controller developed for the one-DOF system. The fuzzy adaptation strategy selects the most appropriate value for the damper based on the estimated state. The various modules of the fuzzy logic controller, that is, the fuzzifier, inference engine, and defuzzifier are built according to the principles laid down by Cohen et al.[16] Data clusters mapping the sensor outputs and the input control force are examined to tune the membership functions. The determination of the damper is followed by the calculation of the actuation force, by solving the second-order differential equations concerning the equations of motion of the emulated passive system, similar to the approach used in the one-DOF system.

Results showing the performance of the nominal system are presented in Fig. 12. The settling time obtained is 8.8 s for a 0.1 unit x_2 response envelope, for a unit impulse applied at m_2. The peak control input reached is 0.53 units, which is well within the hard constraint imposed.[7] In comparison, the best settling time for a fixed-gain POB

Table 3 Comparison with other controllers (based on Stengel and Marrison[7])

Controller description	Design	Nominal T_S [a]	Nominal $u_{\max}$ [a]	P_I	P_{TS}	P_U
Fixed-order compensators achieving approximate loop-transfer recovery	A	21.0	0.514	0.160	0.971	0.160
Same basic design as A	B	19.5	0.469	0.023	1.000	0.023
Same basic design as A	C	19.7	0.468	0.021	1.000	0.021
H_∞	D	9.9	297.8	0.000	0.000	1.000
Nonlinear constrained optimization	E	18.2	0.884	0.000	1.000	0.000
Structured covariance terms added to linear quadratic Gaussian equations	F	13.7	2.397	0.000	0.633	1.000
Game theoretic controller based on linear exponential Gaussian and H_∞ concepts	G	31.3	1.458	0.000	1.000	1.000
H_∞ using the internal model principle	H	14.9	0.574	0.000	0.742	0.000
Same basic design as H	I	17.8	0.416	0.000	0.756	0.000
Same basic design as H	J	43.2	1.047	0.039	1.000	0.857
Adaptive fuzzy POB controller	K	8.8	0.53	0.000	0.468	0.042

[a] Values represent nominal characteristics (i.e., for $k = 1$).

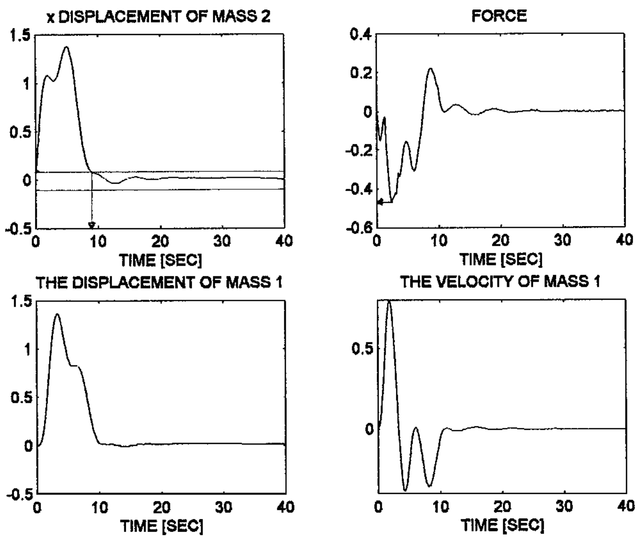

Fig. 12 Performance of the adaptive fuzzy POB controller ($k = 1.0, T_s = 8.8$ s, $|u_{\max}| = 0.47$).

controller having $d_v = 1.4$ (Fig. 9) represents a degradation of about 34% as opposed to the performance presented by an adaptive fuzzy POB controller. In addition to nominal plant performance, there is concern about the stability/performance robustness characteristics of the adaptive fuzzy POB controller. This all-important issue is dealt with using statistical robustness analysis (SRA), based on Monte Carlo simulations, and is applied to determine the probability of unsatisfactory stability or performance resulting from expected parameter variations.

IV. Analysis of a Fuzzy POB Controller

The definitions and principles of the SRA adhered to in this effort are based on the approach proposed by Stengel and Marrison,[7] whereby Monte Carlo evaluations are used to estimate the probabilities of stability/performance. The performance metrics are defined as follows.

1) Probability of instability, P_I, shows the likelihood that the variations in the uncertain plant parameter will force at least one closed-loop root into the right half-plane.

2) Probability of settling-time exceedance, P_{TS}, shows the likelihood that the actual response of the targeted state variable will fall outside an arbitrarily chosen envelope. For the ACC problem, P_{TS} derived from the time-history of x_2 obtained using a unit-impulse input shows the likelihood that x_2 will fall outside a ± 0.1 unit envelope after 15 s.

3) Probability of control-limit exceedance, P_U, shows the likelihood that the peak actuator displacement will exceed a prescribed saturation limit. In the ACC benchmark, P_U corresponds to the requirement that calls for the restriction of controller effort to one unit in response to a unit disturbance impulse.[1]

The Monte Carlo analysis comprises three steps, namely, generation of random spring stiffness, solution of the deterministic problem for a large number of realizations, and statistical analysis of the results as described in detail by Cohen.[10] The number of Monte Carlo runs, m, was selected arbitrarily at 1000 and was found to be adequate.[10] The results obtained using a fuzzy POB are compared to other strategies as presented by Stengel and Marrison.[7] Table 3 presents the results of a comparison between the adaptive fuzzy POB controller and 10 other controllers. Table 3 indicates the substantial benefits obtained using an adaptive fuzzy POB controller. However, to conduct a fair comparison on the basis of the results presented in Table 3, the confidence intervals of the probabilities also have to be stated. This is also essential for the justification of the Monte Carlo simulation, and the subsequent calculation offers the following results for a confidence level of 95%.

1) The probability for instability for the adaptive fuzzy POB controller lies within the confidence interval [0, 0.0037].

2) The probability for settling-time exceedance for the adaptive fuzzy POB controller lies within the confidence interval [0.437, 0.499].

3) The probability for control-limit exceedance for the adaptive fuzzy POB controller lies within the confidence interval [0.030, 0.056].

In the preceding section, mention was made of reaching a limit cycle for small values of k. This phenomenon was closely examined, beyond the Monte Carlo analysis, to ensure no collapse of the solution in this region. The limit cycle was arbitrarily defined to occur when, for $t > 50$ s, the absolute response exceeded a value of 0.1 units. For this definition, a limit cycle was noticed for spring values of $0.5 < k < 0.66$, and it occurred 105 times during the Monte Carlo run. For a confidence level of 95%, the probability for limit-cycle occurrence is within the confidence interval [0.09, 0.13]. Several runs were made to ensure that the amplitude of the disturbance (between 1 and 10 units) did not affect the amplitude of the limit-cycle envelope in this region.

Revisiting the results provided by the Table 3 indicates that, on the one hand, the adaptive fuzzy POB has the best results as far as the stability and settling times are concerned, whereas, on the other hand, the control effort required is within the required limit for most cases. For linear controllers, the improvement of settling times is directly associated with the payment of a huge penalty for the control effort (see design D). The quality of the results obtained by the adaptive fuzzy POB controller together with the confidence intervals presented earlier justify using a 1000 run. By contrast, Stengel and Marrison[7] conducted 20,000 evaluations.

V. Conclusions

A novel approach is presented for the control of linear second-order systems using a fuzzy logic-based algorithm. The development of the fuzzy logic controller begins with a close examination of the minimum-time solution for a variably damped one-DOF system. Then, the input–output mapping of the optimal controller is expressed in linguistic terms. Finally, for a center-of-gravity defuzzification method, the membership functions of the inputs–outputs are tuned accordingly to provide a fairly good approximation (there is a trade off between the accuracy of the approximation and the complexity of the fuzzy logic controller).

The fuzzy approximation of the one-DOF system is generalized and applied to the ACC benchmark problem that represents a noncollocated system. This extension, a direct generalization of the dissipative controller, is based on the Luenberger observer. The main principle involves synthesizing the passive output using an observer as opposed to the availability of physical measurements as required. It was demonstrated that the developed approach yielded performance and stability robustness in view of plant uncertainties, sensor noise, and sensitivity to actuator/sensor noncollocation. The provision of fast and effective system responses demonstrated during the numerical investigations is primarily due to the capability of a fuzzy logic controller to emulate time-optimal bang–bang control.

The central idea that drives the developed control law implies that, for large values of system error, the damping effect of the error derivative control is blocked because full control authority is used to quickly drive the system to zero. On the other hand, as the system error tends to zero, a progressively greater damping effect is introduced. The most interesting result of this effort is the finding that the developed control strategy leads to robust near time-optimal control while requiring a relatively small amount of control effort. Comparative settling times, for a linear controller based on H_∞, require a maximum control force of about two orders of magnitude.

Studies should be pursued to develop further and to test the controller presented herein for the vibration suppression of structures, such as beams, plates, shells, and those possessing very high modal densities at the lower frequencies. Future research should include further comparisons with other linear and nonlinear control laws.

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