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Active control of flexible structures using a fuzzy logic algorithm

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Abstract
This study deals with the development and application of an active control law for the vibration suppression of beam-like flexible structures experiencing transient disturbances. Collocated pairs of sensors/actuators provide active control of the structure. A design methodology for the closed-loop control algorithm based on fuzzy logic is proposed. First, the behavior of the open-loop system is observed. Then, the number and locations of collocated actuator/sensor pairs are selected. The proposed control law, which is based on the principles of passivity, commands the actuator to emulate the behavior of a dynamic vibration absorber. The absorber is tuned to a targeted frequency, whereas the damping coefficient of the dashpot is varied in a closed loop using a fuzzy logic based algorithm. This approach not only ensures inherent stability associated with passive absorbers, but also circumvents the phenomenon of modal spillover. The developed controller is applied to the AFWAL/FIB 10 bar truss. Simulated results using MATLAB© show that the closed-loop system exhibits fairly quick settling times and desirable performance, as well as robustness characteristics. To demonstrate the robustness of the control system to changes in the temporal dynamics of the flexible structure, the transient response to a considerably perturbed plant is simulated. The modal frequencies of the 10 bar truss were raised as well as lowered substantially, thereby significantly perturbing the natural frequencies of vibration. For these cases, too, the developed control law provides adequate settling times and rates of vibrational energy dissipation.

1. Introduction and motivation

A major driver that affects the overall system performance of LFS (large flexible structures) involves their sizing for minimum mass, subject to both static strength and dynamic requirements. This need to increase structural efficiency in high performing systems has recently motivated a field of research referred to as *adaptive structures*, which involves the control of structural dynamics through interdisciplinary design of an integrated structure and control system. Adaptive structures may be introduced to influence the geometry, shape, apparent stiffness, damping, or inertia of the structural modes (Wada 1993). A research study on smart structures, prepared for the US Army Research Office (Parrish *et al* 1993), describes the promise of significant payoffs attainable in aerospace systems by introducing adaptive control surfaces, active acoustic coatings for signature suppression, vibration suppression and twist control, and active structural tuning and damping. The main motivation for using an active control system as opposed to passive means (e.g. dynamic vibration absorber (DVA)) is weight savings.

Aerospace facilities may generally comprise of repetitive latticed trusses, span large areas with a few intermediate supports, are light in weight and extremely flexible, and consequently are characterized by a large number of high-density low-frequency structural modes. These higher-order structural systems utilize feedback control laws that are based on system stimulus–response models, embedded sensors to sense their response to operational and environmental stimuli, and actuators to modify their response in such a way as to maintain or optimize structural performance.

In this study, a fuzzy logic controller is proposed and developed for the least settling time suppression of transient induced vibration of beam-like flexible structures. For such a

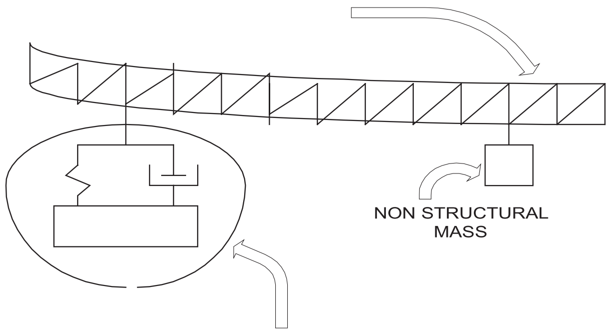

Figure 1. 'Passivity based' control for a LFS.

class of dynamic systems, Juang and Phan (1991) presented a robust passive design controller which is based on a virtual second-order dynamic system comprising of virtual mass, spring and dashpot elements. In addition, Juang and Phan (1991) showed that overall closed-loop stability was guaranteed independently of the system structural uncertainty and perturbations in the temporal plant of the system. These second-order controllers may also be termed 'dissipative', or 'collocated', and they consist of compatible pairs of actuators and sensors which may be distributed throughout the structure (Joshi 1989). The main reason for using the 'passivity' approach is its inherent robustness. Furthermore, the virtual mechanisms incorporated into the passive design serve only to transfer and dissipate the energy of the system, thereby maintaining the stability of the system.

The above passive design approach is also adopted in the present study, however, by treating the virtual system in a different manner than that suggested by Juang and Phan, i.e. in compliance with results yielded by non-linear time optimal control analysis. The proposed control law, which is based on the principles of passivity, commands the actuator to emulate the behavior of a DVA. The absorber is tuned to a targeted frequency, whereas the damping coefficient of the dashpot is varied in a closed loop using a fuzzy logic based algorithm. The purpose of the fuzzy logic based, variable damping strategy is to provide quicker settling times yielded by non-linear control action. In this paper, the proposed approach is applied only to transient disturbances. However, in a parallel effort, Cohen *et al* (2000) have shown that the developed methodology is effective for steady state excitations as well.

This paper represents part of an ongoing research effort conducted during the last six years at the Aircraft Structures Laboratory, Faculty of Aerospace Engineering, Technion, Israel Institute of Technology. In one of the laboratory's previous publications (Cohen *et al* 1995), a very detailed discussion on various controller mechanisms, including the usage of active struts, was presented. The proposed approach was preferred since it also lends itself to the application of actively shunted piezoelectrics for vibration damping and acoustic suppression. Furthermore, the developed strategy was also applied to a laboratory model of a flexible beam-like cantilever whose vibrations were actively controlled using piezoceramic sensors and actuators (Cohen *et al* 1997).

In this study, the 10 bar truss was selected for the numerical application since it is a well known benchmark, developed by AFWAL/FIB, for studies concerning vibration suppression of flexible space structures with several low natural frequencies and a high modal density. This benchmark serves as an ideal platform to demonstrate some of the important features of a control system design for a typical large space structure. Moreover, the main aim of the numerical exercise is pointing out the effectiveness of the fuzzy logic based controller in shortening the settling times. To this end, the results obtained are compared to those reached using LQG/LTR and H-infinity controllers, which serve as 'universal' baselines.

2. Actuator and sensor placement

Consider a large flexible beam-like structure illustrated in figure 1. The state space representation for such a large structure may be written in conventional form (neglecting noise and disturbances) as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $\dot{x}(t)$, $x(t)$, $u(t)$, A and B are the state vector, the state-rate vector, the control vector, a $n \times n$ state matrix and a $n \times m$ control matrix, respectively.

The corresponding measurement equation may be written as

$$y(t) = Cx(t) \tag{2}$$

where $y(t)$ is the output vector and C is a $l \times n$ measurement matrix.

Hughes and Skelton (1980) addressed the important issue of sensor and actuator positioning, for LFS, by proposing criteria for modal controllability (actuator placement) and modal observability (sensor placement). Another approach, based on passive control strategies for the vibration suppression of large beam-like truss structures, was introduced by Cohen and Weller (1994). Based on finite element analysis and using DVAs, Cohen and Weller (1994), showed the following:

- One DVA, tuned to a specific targeted mode, is sufficient for the vibration suppression of that particular mode.
- A DVA tuned to a certain frequency does not affect the characteristics of the modes above that frequency. Hence, after identifying the targeted modes, the highest mode is first tuned and then the structure is modified. In the next step, the second highest mode is tuned and so on (see details in the flowchart presented in figure 6 of Cohen and Weller (1994)).
- The energy absorbing property of the tuned DVA is most effective when placed on the maxima of the respective mode shape.

The damping value of each of the DVAs may be obtained using fuzzy logic control based on the insight presented in Cohen *et al* (1996). The above-mentioned guidelines will make the task of selection and tuning of the actuator/sensor relatively simpler. Furthermore, Juang and Phan (1991) presented sufficient conditions on actuator and sensor that guaranteed overall closed-loop stability for robust controller designs for second-order dynamic systems.

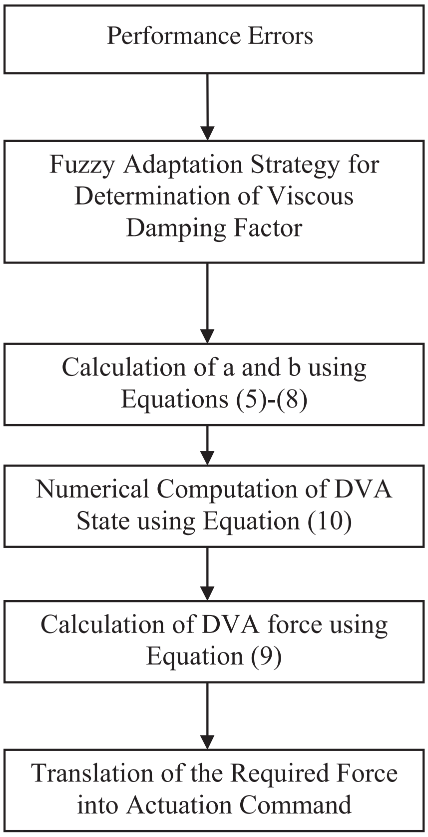

Figure 2. Calculation of actuation command.

3. Passivity based control

Consider a virtual DVA, attached at a distance 'm' from one of the ends of a large flexible beam-like structure having arbitrary boundary conditions as depicted in figure 1. For the sake of convenience, we denote the natural frequency of an undamped vibration of the absorber as '$\sqrt{a}$'. Based on the optimal tuning ratio of a 2-DOF system, for a given open-loop frequency of vibration ω, and a mass ratio of the absorber mass to the structure mass μ, 'a' is obtained by equating the first two peaks of the steady state response (Den Hartog 1956), and is given by

$$a = \left[\frac{\omega}{1+\mu}\right]^2. \tag{3}$$

Jacquot (1978) extended Den Hartog's 2-DOF theory to continuous systems. A DVA was applied at a distance 'm' from the fixed end of a cantilever Euler–Bernoulli beam, thereby transforming equation (3) to

$$a = \left[\frac{\omega}{1+(\phi_i(m))^2\mu}\right]^2 \tag{4}$$

where $\phi_i(m)$ is the value of the normalized mode shape of the natural mode 'i' at a distance 'm' from the fixed end. Following equation (4), the necessary plant information required to define 'a' is an estimate of the targeted frequency ω, which may be identified from the displacement–time sensor output of the open-loop system and $\phi_i(m)$. In certain applications, when there is a possible lack of information on mode shapes, we may introduce an empirical constant, μ^*, which is used in the tuning process to make up for the loss in performance. The value of μ^* is determined by a 'random walk' search during simulations. This practice has been found to be effective for beam-like structures (Cohen 1999). For such cases, the product of the mass ratio of the absorber, μ, and $\phi_i^2(m)$ may be written as a function of the damping factor, δ, and μ^* as follows:

$$\mu[\phi_i(m)]^2 = \mu^*[1/\delta + (1/\delta)^2]. \tag{5}$$

Equation (5) represents an empirical relation, whereby a *lightly* damped absorber corresponds to a *large* mass ratio and vice versa. Inserting the above equation into equation (4) gives

$$a = \left[\frac{\omega\delta^2}{\delta^2+\mu^*(1+\delta)}\right]^2. \tag{6}$$

Let the damping parameter, 'b', be defined as the ratio of the damping coefficient of the absorber to the mass of the absorber, for a typical second-order system, written as (Meirovitch 1986)

$$b = 2(\delta\sqrt{a}). \tag{7}$$

Inserting equation (6) into (7) yields

$$b = 2\delta\left[\frac{\omega\delta^2}{\delta^2+\mu^*(1+\delta)}\right]. \tag{8}$$

The resulting force (per unit mass) applied by the absorber may be written as

$$f = a[x_1 - y(m,t)] + b\left[\frac{\mathrm{d}x_1}{\mathrm{d}t} - \frac{\mathrm{d}y(m,t)}{\mathrm{d}t}\right] \tag{9}$$

where $y(m,t)$ and x_1 are the transverse displacements of the beam and virtual absorber, respectively. x_1 is obtained by solving the following 2-DOF equation of motion of the absorber:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a & -b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ a & b\end{bmatrix}\begin{bmatrix}y(m,t)\\ \dot{y}(m,t)\end{bmatrix} \tag{10}$$

where $\dot{x}_2$, x_2 and $y(m,t)$ are the transverse velocity of the absorber, the transverse acceleration of the absorber, and the transverse velocity of the beam, respectively.

The approach employed herein is based on MRAC (model-reference adaptive control (Friedland 1993)). This method may be used in the control of systems whose models possess significant uncertainties, and hence the application of a model-independent control law is a distinct advantage. As shown in figure 2, first the performance errors become the input to a tuner that adjusts the damping factor of the virtual DVA. The value of δ inferred from the measurements is reached by using a fuzzy control gain weighting adaptation strategy, which will be described in the following section. In the next step, the parameters that characterize the absorber, $a(\delta)$ and $b(\delta)$, are obtained using equations (5)–(8).

After obtaining the values of a and b, 2-DOF equations of motion of the absorber, given by equation (10), are numerically computed to give the displacement and the velocity of the virtual absorber. Then, the input control force, obtained from equation (9), is applied to the plant, which consists of the first n modes of a large, flexible, beam-like structure. Finally, the required force is translated into the actuation command.

For example, in the case of a piezoelectric actuator, the output force has to be converted into a term related to the required electric field (voltage), which in turn produces the desired mechanical deformation. Cohen *et al* (1997) present the equations concerning the respective conversion processes for piezoelectric actuators.

4. Fuzzy adaptation strategy

A fuzzy logic control system maps crisp inputs into crisp outputs based on the manipulation of linguistic variables. A linguistic variable is a variable whose value is defined in linguistic terms, i.e. words. For example, the following statement: 'the *frequency* is *low*', may be given the interpretation: the linguistic variable *frequency* has the linguistic value *low*, which is a fuzzy set on X. Generally speaking, a linguistic variable, with a universe of discourse X, may take on many linguistic values. Therefore, the variable *frequency* may have a set of values, referred to as its term set, which has values such as *high*, *medium*, *very low*, etc. The term set represents a fuzzy partitioning of X, where the membership functions of the linguistic values are made to overlap.

The fuzzy logic controller comprises of the following elements: fuzzifier, rule base, inference engine, and defuzzifier. We shall now examine the purpose of these principal components, based on Lee (1990), and the subsequent functional behavior in context of this effort.

Fuzzifier. The first stage in building the fuzzy part of the controller is referred to as the fuzzification or interpretation interface of the input/output parameters. The function of this component may be described as follows:

Interpretation interface	(a) Measures the values of input variables. (b) Performs a scale mapping that transfers the range of values of input variables into corresponding universes of discourse. (c) Performs the function of fuzzification that converts input data into suitable linguistic values which may be viewed as lables of fuzzy sets.

Coming back to our concept of attaching a virtual DVA to a light flexible structure, the transverse displacement of the DVA, $y(m,t)$, and the corresponding transverse velocity, $\dot{y}(m,t)$, are normalized to the dimensionless variables y_N and $\dot{y}_N$, respectively, using the following relations:

$$y_N = N_y y(m,t) \qquad \dot{y}_N = N_{\dot{y}} \dot{y}(m,t) \tag{11}$$

where N_y and $N_{\dot{y}}$ function as tuning parameters for the membership functions. The use of these 'tuning knobs' substantially cuts the degrees of freedom involved in reaching the required membership functions. In addition, once N_y and $N_{\dot{y}}$ are found, the sensitivity of individual fuzzy sets to the closed-loop performance has to be re-examined. If no additional improvement is obtained, then the membership functions of y_N and $\dot{y}_N$ are frozen.

Fuzzy sets for the normalized transverse displacement, y_N, are characterized by membership functions μ_N, μ_{NS}, μ_Z, μ_{PS} and μ_P that map elements of the universe of discourse y_N, into the closed interval [0,1] as follows:

$$\mu_L = y_N \rightarrow [0, 1] \qquad \text{for } L = N, NS, Z, PS, P \tag{12}$$

where L stands for one of the linguistic terms used in this effort to categorize y_N, i.e. N (negative), NS (negative small), Z (zero), PS (positive small) and P (positive). The membership functions given in equation (12) express the degree to which y_N belongs to some category L. Similarly, fuzzy sets for the transverse velocity, $\dot{y}(m,t)$, are characterized by the membership functions μ_Q, for $Q = N, NS, Z, PS, P$. The fuzzy sets for the viscous damping factor are characterized by four membership functions μ_δ for $\delta = EL$ (extra large), L (large), M (medium), S (small).

Fuzzy rule base and inference. The second stage in constructing the fuzzy part of the controller involves the development of the fuzzy 'inference engine' that includes the *knowledge base* and the *decision-making logic*. The function of these components may be described as follows:

Knowledge base	(a) Provides necessary definitions, which are used to define linguistic control rules and fuzzy data manipulation in an FLC. (b) Characterizes the control goals and control policy of the domain experts by means of a set of linguistic control rules.
Decision-making logic	Has the capability of simulating human decision making based on fuzzy concepts and of inferring fuzzy control actions employing fuzzy implication and the rules of inference in fuzzy logic.

The conversion of inputs to outputs, using fuzzy logic, is based on rules of the form (Abihana 1993)

'IF *premise* THEN *consequence*'

where the premise is a set of conditions to be specified and the consequence is a set of actions to be taken. Both the premise and the consequence are fuzzy relations represented by linguistic variables and their linguistic values. For example, the controller of an inverted pendulum might include rules such as 'If the angle is PB (positive big) and the angle rate is NB (negative big), then set the motor input to a Z (zero) force'. Here, angle, angle rate, and motor input are linguistic variables, whereas, PB, NB, and Z are linguistic values in the term sets of each linguistic variable. The set of fuzzy rules may be provided by an expert or learned by an artificial neural network, as described by Kosko (1992).

Abihana (1993) defines inference as the process of applying the degree of membership, computed for a production rule premise, to the rule's conclusion to determine the action to be taken. The value assigned to the output may either be scaled (max–dot method) or clipped (max–min) to the degree of membership of the premise. Both of these methods provide similar results. The above inference methods are the most common methods used in fuzzy logic control. Nevertheless, several additional methods exist, as described by Wang (1994).

The fuzzy adaptation strategy, presented in this effort, is based on rules of the form 'if... then...' that convert inputs (normalized transverse displacement and velocity) to a single output (damping factor). Heuristic rules based on insight gained by approximating the time-optimal controller are coupled with fuzzy reasoning whereby *large* values of the inputs require a *lightly* damped absorber, which would provide quick rise times. However, when the plant state is in the vicinity of the desired state the damping factor is *large* to reduce the overshoot and steady state error. The resulting rule base that converts fuzzified inputs into a fuzzy output is presented in table 1.

For example, the rule described by the first row, first column, in table 1 reads as 'if y_N is negative AND $\dot{y}_N$ is positive, then the damping factor, δ, is *small*'. As observed in table 1, the rule base contains quite a few rules relating to the same output variable. Therefore, to obtain an overall output in the fuzzy state, an inference method is applied. First, the degree of fulfillment of each and every rule is found by applying the fuzzy 'AND' operation. Let us represent the individual elements of the rule-base 'matrix', presented in table 1, as

δ_{ij} ($i = 1, 5$; $j = 1, 5$), where

$$\delta_{ij} = \text{minimum}(\mu_Q, \mu_L) \tag{13}$$

for $Q = P, PS, Z, NS, N$, $L = N, NS, Z, PS, P$.

All the output values, obtained by clipping or scaling, are then brought together to form the final output membership function. After evaluation of the propositions, the output values represented are unified to produce a fuzzy set incorporating the solution variable. This unification of outputs of each rule, referred to as *aggregation*, occurs only once for each output variable. The aggregation process, always comprised of a commutative method, may be one of the methods as described by Jang and Gulley (1995): *max* (maximum), *probor* (probabilistic or), and *sum* (simply the sum of each rule's output set).

Applying the *sum* to the rule base given table 1, the union of the fuzzy sets for the same output variable is taken to reach the respective aggregation of the output as follows:

$$\begin{aligned}
\mu_S &= \delta_{11} + \delta_{15} + \delta_{21} + \delta_{31} + \delta_{35} + \delta_{45} + \delta_{51} + \delta_{55};\\
\mu_M &= \delta_{12} + \delta_{13} + \delta_{14} + \delta_{25} + \delta_{41} + \delta_{52} + \delta_{53} + \delta_{54};\\
\mu_L &= \delta_{22} + \delta_{23} + \delta_{24} + \delta_{32} + \delta_{34} + \delta_{42} + \delta_{43} + \delta_{44};\\
\mu_{EL} &= \delta_{33}.
\end{aligned} \tag{14}$$

The rule base is usually not made to be part of the tuning process. However, the sensitivity of the closed-loop performance to changes in the rule base is examined and minor changes are made in order to ensure as good an approximation as possible to the desired optimal solution.

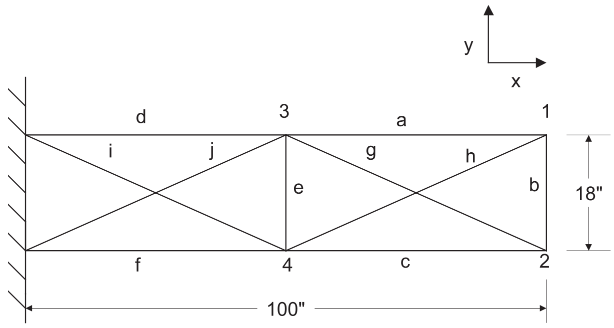

Figure 3. AFWAL/FIB 10 bar truss (Lynch and Banda 1988).

Defuzzifier. Finally, in order to reach a practical controller a control action comprising of a single numerical value is required. Therefore, the space of the fuzzy damping factor, obtained using the method described in the previous section, is mapped into a non-fuzzy space (crisp) in a process known as *defuzzification*. There are various strategies aimed at producing a crisp value. For example, a possible strategy involves the search for a scalar value that best represents the information contained in the aggregated output. Other commonly used strategies are the center of area (COA), the mean of maximum, and the max criterion. However, there is no accepted systematic methodology for selecting a defuzzification strategy (Lee 1990). The COA defuzzification method is the centroid calculation of the area under the curve. This strategy was found to yield better steady-state performance when compared to other defuzzification methods (Lee 1990). For this reason and for the relative ease in implementation, in this effort the COA scheme of defuzzification is adapted. The COA method projects the centroid of the output membership function, μ_R (for $R = S, M, L$ and EL), defined in equation (5.19), as the crisp value of the output viscous damping factor, δ:

$$\delta = \frac{\sum_R \mu_R A_R c_R}{\sum_R \mu_R A_R} \tag{15}$$

where A_R is the area under the 'R'th fuzzy set, c_R is the centroid of the area A_R, and R is the S, M, L and EL.

5. Application to a 10 bar truss

The 10 bar truss, described in figure 3, was introduced by AFWAL/FIB (Lynch and Banda 1988) to demonstrate some of the important features of a control system design, without the handling problems associated with typical higher-order large space structures. The mathematical model of the truss, modified by AFWAL/FIB from a similar structural model, is 100 inches in length and 18 inches high (Lynch and Banda 1988). The adaptive fuzzy passive control law, which was proposed and developed in this study, is applied to the problem of the 10 bar truss for two types of disturbances. The first type of disturbance consists of a 500 lbf pulse applied for a period of 10 s. The control objective for this work, as stated by Parlos and Jayasuriya (1990), is to reject the persistent disturbance and to damp out the resulting vibrations within the shortest possible time period. Other than closed-loop stability, the most important requirement that the control law must satisfy is the ability

Table 1. Rule base for computing the viscous damping factor (δ).

	y_N negative	y_N negative small	y_N zero	y_N positive small	y_N positive
$\dot{y}_N$ positive	Small	Medium	Medium	Medium	Small
$\dot{y}_N$ positive small	Small	Large	Large	Large	Medium
$\dot{y}_N$ zero	Small	Large	Extra large	Large	Small
$\dot{y}_N$ negative small	Medium	Large	Large	Large	Small
$\dot{y}_N$ negative	Small	Medium	Medium	Medium	Small

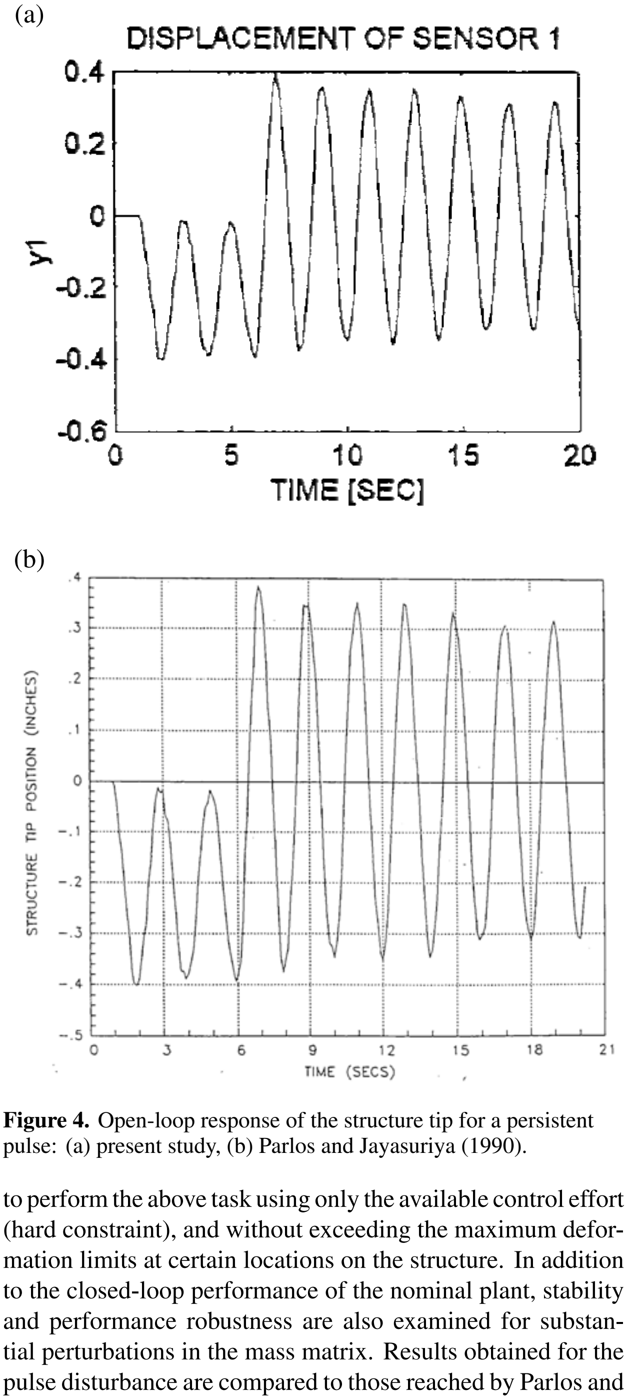

Figure 4. Open-loop response of the structure tip for a persistent pulse: (a) present study, (b) Parlos and Jayasuriya (1990).

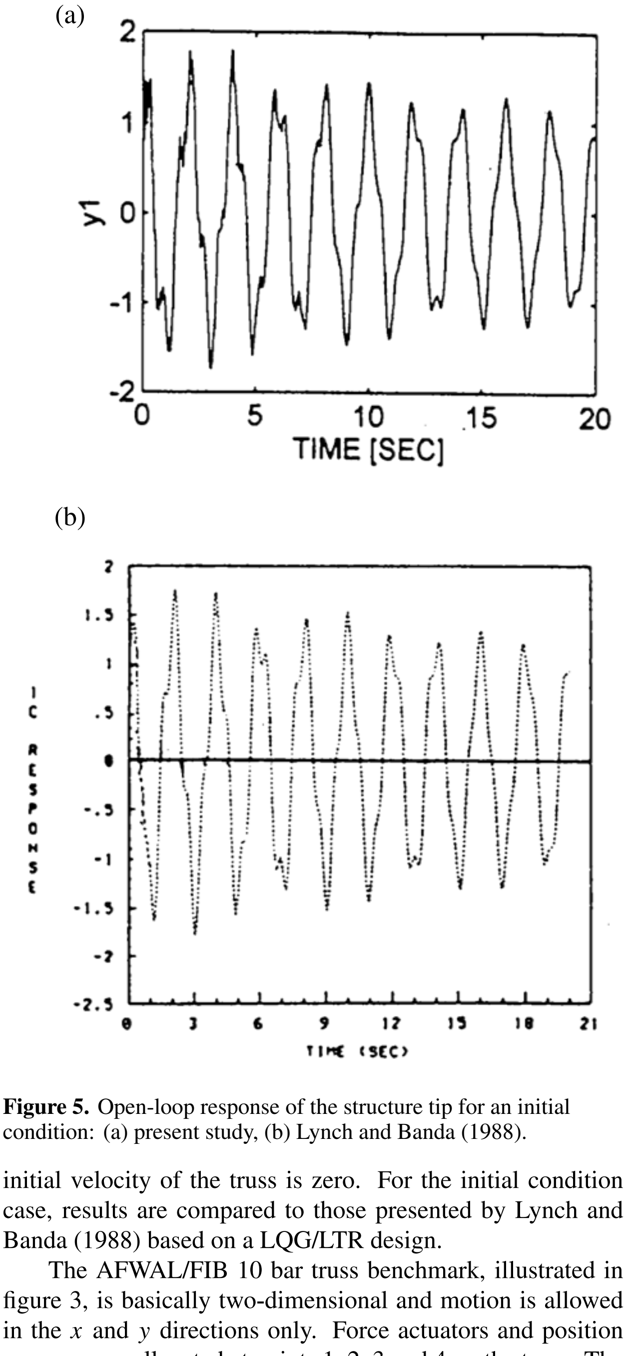

Figure 5. Open-loop response of the structure tip for an initial condition: (a) present study, (b) Lynch and Banda (1988).

to perform the above task using only the available control effort (hard constraint), and without exceeding the maximum deformation limits at certain locations on the structure. In addition to the closed-loop performance of the nominal plant, stability and performance robustness are also examined for substantial perturbations in the mass matrix. Results obtained for the pulse disturbance are compared to those reached by Parlos and Jayasuriya (1990) using a H_∞-optimal controller.

Finally, the evaluation of the developed control system will be based upon the system response to an initial condition. The initial condition vector corresponds to a tip displacement of $\approx$1 in and a mid-station displacement of nearly 2 in. The initial velocity of the truss is zero. For the initial condition case, results are compared to those presented by Lynch and Banda (1988) based on a LQG/LTR design.

The AFWAL/FIB 10 bar truss benchmark, illustrated in figure 3, is basically two-dimensional and motion is allowed in the x and y directions only. Force actuators and position sensors are collocated at points 1, 2, 3 and 4 on the truss. The actuators act along the y axis only whereas the sensors measure physical displacements in the y direction at the above four locations. The structure is constructed from a material with a modulus $E = 10^7$ psi, a weight density of $\rho = 0.1$ lbf in^{-3} and cross-sectional area of the structural members shown in table 2.

Table 2. Structural members cross-sectional areas.

Member	Area (in^2)	Member	Area (in^2)
a	0.003 21	f	0.010 49
b	0.001 00	g	0.003 28
c	0.003 21	h	0.003 28
d	0.010 49	i	0.004 39
e	0.001 00	j	0.004 39

Table 3. Non-structural mass.

Location	Mass [(lbfs^2) in^{-1}]
1	1.294
2	1.294
3	1.294
4	1.294

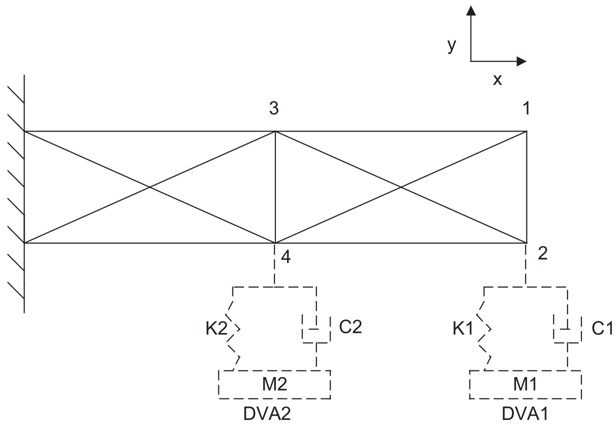

Figure 6. Application of two virtual DVAs to the 10 bar truss.

Non-structural masses are located at positions 1, 2, 3 and 4. Table 3 indicates the mass at each location. These masses can be associated with the additional mass from the actuators at the four locations. The non-structural mass is large relative to the structural mass to achieve the low-frequency structural modes typical of a large space structure.

Lynch and Banda (1988) assumed that uniform damping existed throughout the structure. This was achieved by selecting a passive damping level of $\delta = 0.005$. The modal frequencies for this uniformly damped eight-mode model are shown in table 4. The addition of the non-structural mass results in the lowering of the natural frequencies and a high modal density.

As already indicated, evaluation of the control system will be based upon the system response to two different types of disturbances. The first type of disturbance, as applied by Parlos and Jayasuriya (1990), constitutes of a 500 lbf pulse applied for a period of 10 s. The plant is subject to the persistent disturbance starting at $t = 1$ s and is ultimately freed at $t = 11$ s. The control problem of the above structure is determined by the following design requirements:

- It is desired that, after the disappearance of the disturbance, the controlled vertical displacement of the truss be within ± 0.01 in at points 1, 2, 3, and 4 in the shortest possible time period.
- The above is to be accomplished using only the available control power of a maximum 100 lbf, at the available control rate of a maximum 1000 lbf s^{-1}.

The second type of disturbance involves an initial condition as formulated by Lynch and Banda (1988). The above initial condition vector given in physical coordinates excites the first, second, fifth, and seventh modes of the structure. This vector corresponds to a tip displacement of ≈ 1 in and a mid-station displacement of nearly 2 in. The initial velocity of the truss is zero.

The different tasks involved in application of the developed controller to the 10 bar truss benchmark were as follows:

(a) The open-loop models were built on a MATLAB platform as follows: two-mode reduced model for control design and an eight-mode truth model for testing the performance of the system. Subsequently, the open-loop models were validated for persistent pulse disturbance by comparison with Parlos and Jayasuriya (1990). Furthermore, the open-loop models experiencing the above-mentioned initial condition were compared with Lynch and Banda (1988).

(b) The adaptive fuzzy passive control was applied, based on the strategy developed in the previous section, for both types of disturbances.

(c) The closed-loop performance for the nominal two-mode design model were compared with those using a H_∞-optimal controller (Parlos and Jayasuriya 1990) and a LQG/LTR controller (Lynch and Banda 1988).

(d) The closed-loop performance for the nominal eight-mode truth model were evaluated for both types of disturbances and results obtained were compared with those using a H_∞-optimal controller (Parlos and Jayasuriya 1990) and a LQG/LTR controller (Lynch and Banda 1988).

(e) The stability and performance robustness for substantial perturbations in the plant were examined for persistent pulse disturbance and results obtained were compared with those using a H_∞ controller (Parlos and Jayasuriya 1990).

The state-space representation of the eight-mode (16-order) model is presented in Parlos and Jayasuriya (1990). The eight-mode model is reduced for design purposes. Following Lynch and Banda's (1988) examination of the system's second-order modes, the system was found to be most controllable and observable with respect to the first two modes. The resulting design model is as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gd(t) \tag{16}$$

$$y(t) = Cx(t) + Du(t) \tag{17}$$

where $x(t)$, $y(t)$, $u(t)$ and $d(t)$ denote the state vector, the measurement vector, the control forces and a scalar external persistent disturbance, respectively.

With

$$A = \begin{bmatrix} -0.0314 & 0 & -9.8694 & 0 \\ 0 & -0.1039 & 0 & -107.86 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3142 & 0.3142 & 0.1161 & 0.1161 \\ -0.1040 & -0.1040 & 0.3337 & 0.3337 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

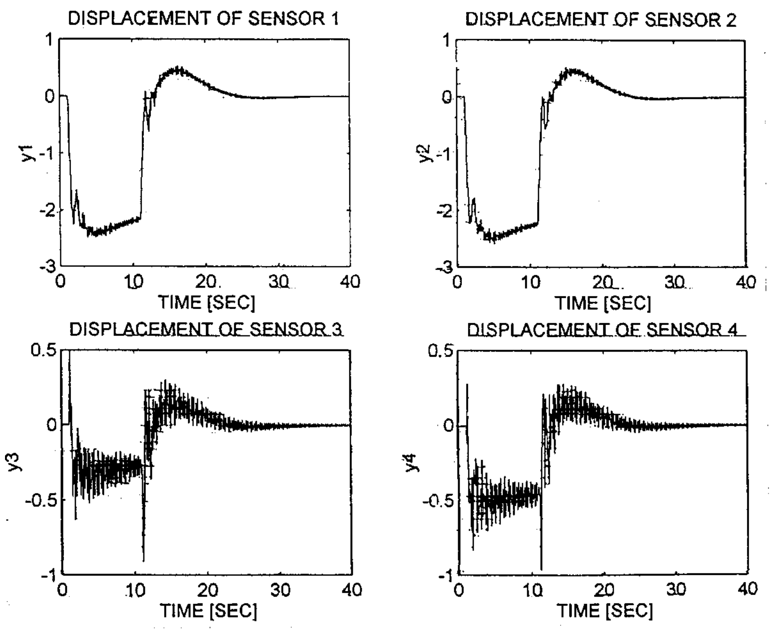

Figure 7. Transient response to persistent disturbance using adaptive fuzzy controller for the four sensors.

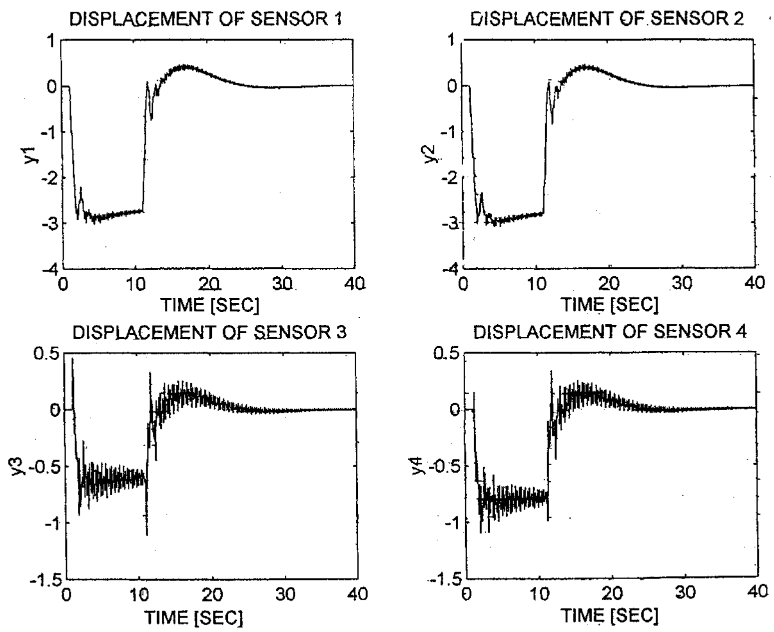

Figure 8. Robustness test using adaptive fuzzy controller—case 1 for the four sensors.

$$C = \begin{bmatrix} 0 & 0 & 1 & -0.3117 \\ 0 & 0 & 1 & -0.3117 \\ 0 & 0 & 0.3402 & 1 \\ 0 & 0 & 0.3402 & 1 \end{bmatrix}$$

$$D = 10^{-3} \times \begin{bmatrix} 0.4480 & -0.4230 & -0.0263 & 0.0010 \\ -0.4230 & 0.4480 & 0.0010 & -0.0263 \\ -0.0263 & 0.0010 & 0.4239 & -0.3970 \\ 0.0010 & -0.0263 & -0.3970 & 0.4239 \end{bmatrix}$$

$$G = \begin{bmatrix} -0.0393 \\ 0.0822 \\ 0.0000 \\ 0.0000 \end{bmatrix}.$$

The maximum singular values of the truth model and the reduced-order design model, as shown in Lynch and Banda (1988), are nearly identical at low frequencies. In addition, the open-loop initial condition responses indicate that these two models compare well. The open-loop responses obtained for model validation are as follows:

- Open-loop response of the structure tip position (point 1 in figure 3) obtained when applying a 500 lbf persistent pulse for 5 s to the eight-mode model is given in figure 4(a). This response compares well with the open-loop response, given in figure 4(b), presented by Parlos and Jayasuriya (1990) under similar conditions.

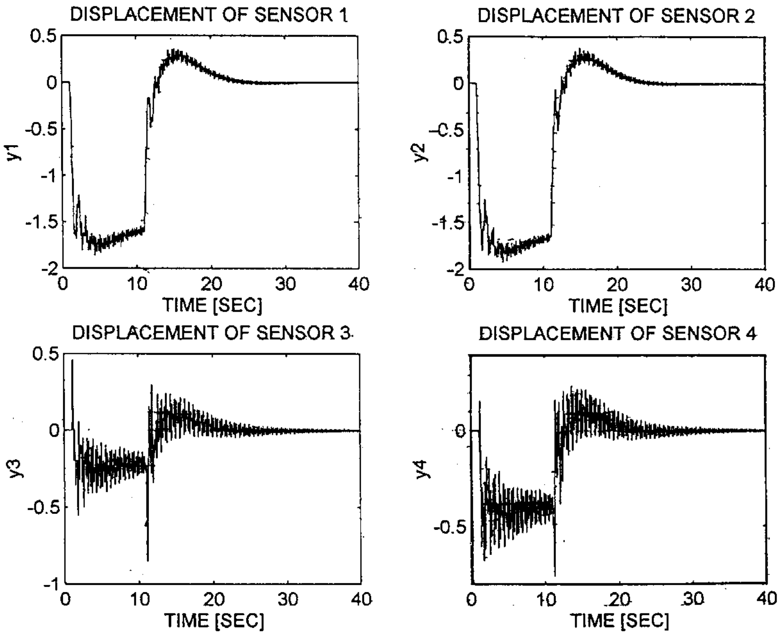

Figure 9. Robustness test using adaptive fuzzy controller—case 2 for the four sensors.

Table 4. Natural frequencies of cantilevered two-bay truss.

Mode	1	2	3	4	5	6	7	8
Freq (Hz)	0.5000	1.6529	3.6134	4.7020	4.9640	5.2315	8.8844	9.3551
Freq (r s^{-1})	3.1416	10.386	22.704	29.544	31.189	32.870	55.822	58.779

- Open-loop response of the structure tip position (point 1 in figure 3) obtained when applying the initial condition to the eight-mode model is given in figure 5(a). This response compares well with the open-loop response reached by Lynch and Banda (1988) for a similar study as shown in figure 5(b).

The sensors of the 10 bar truss, described in figure 3, sense displacements in the y direction only and the actuators act along the y axis alone. Therefore, the control effort at points 1 and 3 may be identical to those at points 2 and 4, respectively. This method is simpler and is also adapted by Lynch and Banda (1988) as well as by Parlos and Jayasuriya (1990). Based on the strategy developed in the previous section, two virtual DVAs were introduced at points 2 (DVA 1) and 4 (DVA 2), as shown in figure 6. For both the DVAs, the damping coefficient is varied in accordance to the rule base described by table 1.

After calculating the force applied by DVA 1, this value was divided by 2 to obtain the force applied at points 1 and 2. Furthermore, DVA 1 was tuned to the fundamental frequency (0.5 Hz) and DVA 2 was tuned to the second natural frequency (1.6529 Hz). In addition, the damping coefficients of the two virtual absorbers were varied using the adaptive fuzzy approach based on the rule base of the fuzzy logic controller given in table 1. On the other hand, the membership functions of inputs (displacement and rate at points 2 and 4) and outputs (damping of DVA 1 and DVA 2) required some further fine-tuning. If the velocities at positions 1, 2, 3, and 4 (required for calculating the force applied by the virtual absorbers) are obtained by calculating the change of displacement with time, then the signal may consist of a lot of high-frequency harmonics. These harmonics of the signal can affect the closed-loop performance. The transfer function of the estimator may be written as

$$G(s) = \frac{s}{(\tau_d s + 1)}. \tag{18}$$

In equation (18), when τ_d is selected as zero, the estimator performs as a real velocity but produces a lot of noise. However, when τ_d is increased, the estimation error of the estimator is increased but the noise is reduced. The estimation error is proportional to the parameter τ_d and, when t approaches infinite, the estimation error of velocity approaches zero. Generally speaking, τ_d is selected as a small value to allow the estimated velocity to approach the real velocity in a short time. In this effort, after several tuning attempts, the appropriate value for τ_d was found to be 0.4 s.

This disturbance (Parlos and Jayasuriya 1990) consists of a 500 lbf pulse for a period of 10 s. The plant is subject to the persistent disturbance starting at $t = 1$ s and is ultimately freed at $t = 11$ s. Figure 7 shows the decay of the truss displacement based on the eight-mode model, when subjected to the above disturbance. For the nominal plant, the peak value of the truss vertical displacement is 2.4 in (see figure 7) as opposed to 2.7 in reached using a H_∞ controller (Parlos and Jayasuriya 1990). For both controllers, the plant settles down after about 20 s.

It is interesting to examine the robustness characteristics of the two controllers. Consequently, the developed controller was applied to three perturbed plants as indicated in table 5. The perturbations consist of varying the frequencies of the

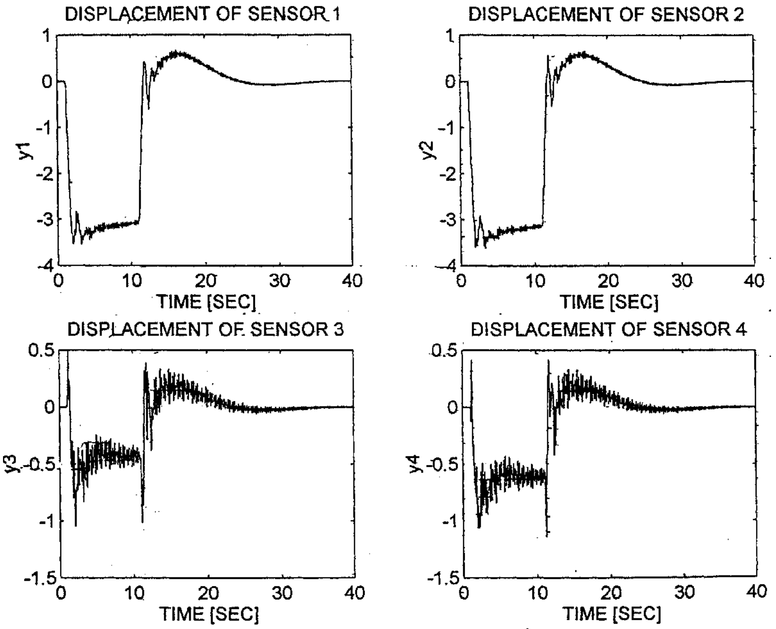

Figure 10. Robustness test using adaptive fuzzy controller—case 3 for the four sensors.

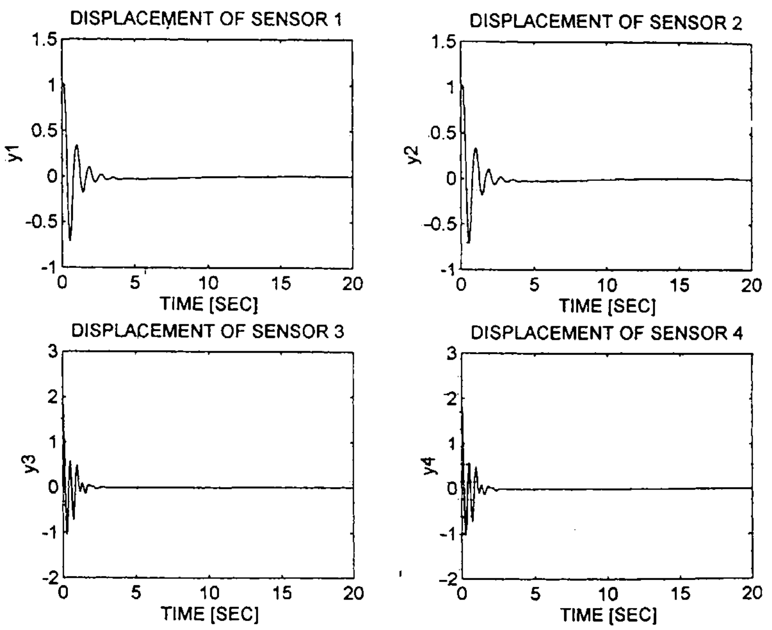

Figure 11. Transient response to initial condition using LQG/LTR control for the four sensors.

Table 5. Description of the perturbation models used for the robustness tests.

Case	Mode 1	Mode 2	Mode 5	Mode 7	Closed loop response
1	Down 15%	Up 25%	Up 40%	Up 40%	Figure 8
2	Up 15%	Up 25%	Up 40%	Up 40%	Figure 9
3	Down 15%	Down 15%	Down 15%	Down 15%	Figure 10

first, second, fifth, and seventh modes. The corresponding responses, given in figures 8–10, display very little sensitivity to the perturbations and negligible degradation in performance compared to that obtained for the nominal plant.

The above insensitivity compares well with the results obtained by Parlos and Jayasuriya (1990). The H_∞ controller performs relatively well for the case when the mass of the truss is increased by 80% (see Parlos and Jayasuriya 1990). However, an only 1.8% or more reduction of the truss mass results in unacceptable system performance (see Parlos and Jayasuriya 1990). In general, the above result is not acceptable in terms of a robust design. It is, therefore, surprising that,

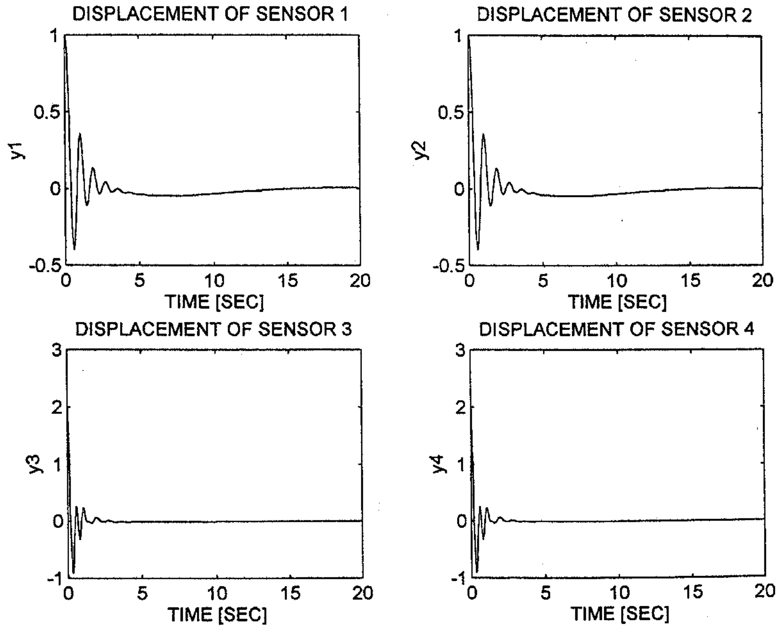

Figure 12. Robustness test for initial condition using adaptive fuzzy controller—case 1 for the four sensors.

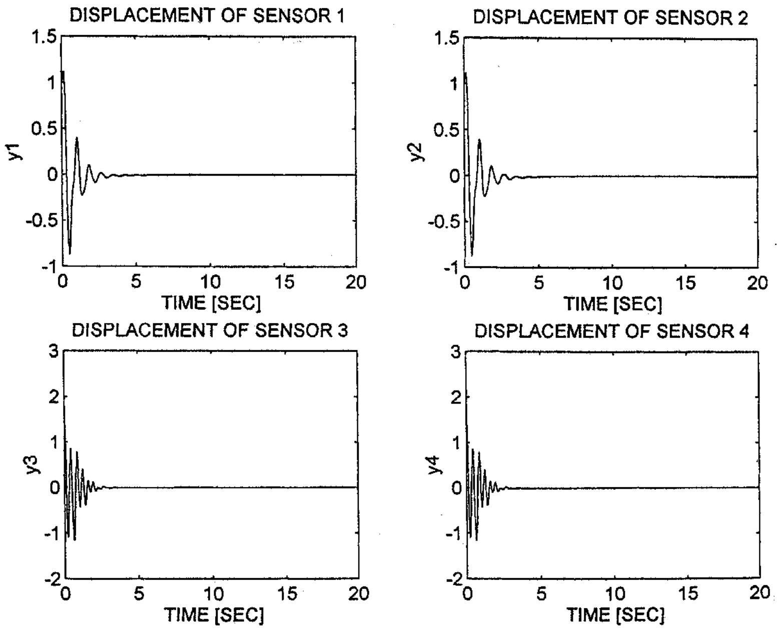

Figure 13. Robustness test for initial condition using adaptive fuzzy controller—case 2 for the four sensors.

on the basis of the above results, Parlos and Jayasuriya (1990) conclude that their approach 'exhibits good stability robustness to structural parameter variations'.

Lynch and Banda (1988) developed a LQG/LTR controller based on the two-mode design model. For the LQG/LTR controller, the initial vibrations are damped to within 0.1% of the initial amplitude in $\approx$12 s. In comparison, for the adaptive fuzzy controller, the initial vibrations are damped to within 0.1% of the initial amplitude in $<$6 s (see figure 11). This remarkable improvement in the settling times is obtained without exceeding the specified control power limits. The robustness characteristics of the developed controller are examined for the three perturbed plants described in table 5. For all three cases, the adaptive fuzzy controller yielded satisfactory results as depicted in figures 12–14.

6. Conclusions and recommendations

The present effort describes the development and application of a fuzzy based controller that emulates the functioning of an adaptive DVA tuned to the targeted frequencies. The central idea, which drives the developed control law, implies that, for large values of system error, the damping effect of the error derivative control is blocked as full control authority is used

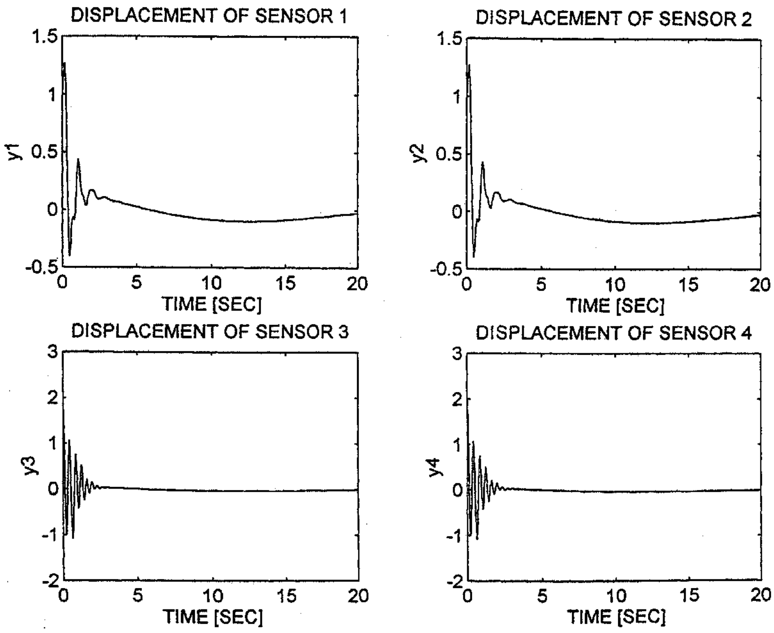

Figure 14. Robustness test for initial condition using adaptive fuzzy controller—case 3 for the four sensors.

to quickly drive the system to zero. On the other hand, as the system error tends to zero a progressively greater damping effect is introduced.

The controller is applied to a beam-like 10 bar truss that is subjected to two different types of transient disturbances. Four collocated pairs of sensors/actuators were used to suppress the transient vibrations. MATLAB© simulations of the closed-loop transient response, for the nominal and other perturbed plants, demonstrate quick settling times, a high rate of vibrational energy dissipation, and no control spillover to the higher modes. The results obtained demonstrate that the fuzzy based controller has superior settling times and is more robust when compared to designs based on LQG/LTR and H_∞. Future work will be aimed at providing additional comparisons with other controllers and may include non-linear designs.

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