# Specific Energy, Lorentz Factor & WIMP Annihilation

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**Abstract.** The time dilation formulas of both the Special Relativity and General Relativity could be studied using an expression dependent on specific energy. Should such factor be used to define the relativistic mass, the equation that arises is an approximation of the mass and energy relation. An entangled mathematical definition of mass that is finally compared to the equations that define Dark Matter Annihilation into charged states via loop-level processes.

**Keywords.** Specific energy — relativistic time — relativistic mass — dark matter

## 1. Introduction

The way time and mass is understood has been accurately predicting most of the research it has been carried out over the last century. But there are still many uncertainties in the universe for which it lacks sufficient understanding of these two variables, such like Dark Matter. For that reason, the present paper, an alternative mathematical perspective of the Lorentz Factor is proposed. The resulting expressions of time and mass are applied to three different cases to provide a brief comparison between the current proven knowledge from Special and General Relativity and the present theory.

Finally a fourth application is considered to benchmark the proposed formulation with the insights of photon production from WIMP annihilation into charged states via loop-level processes $(\chi\chi \rightarrow \gamma X)$ (Bertone 2010; Coogan, Profumo, Shepherd 2015).

## 2. Methodology

A particle of infinitesimal mass $(dm)$ can be identified in space-time with the three position coordinates $(x, y, z)$ and its time $(t)$. This particle also contains a differential energy $(dE)$, even though its mass is infinitely small. However, its specific energy $(\varepsilon)$ is much higher,

$$\varepsilon = \frac{dE}{dm} \tag{1}$$

Energy, position, time and even mass, need a reference. Kinetic energy, for example, requires a zero-speed reference. Similarly, gravitational energy is associated with its corresponding null potential. In fact, the same thing happens to the mass, its value depends on the observer.

Thus, the specific energy of particle of infinitesimal mass, $A$, can be redefined by taking another particle $B$ as a reference,

$$\varepsilon_{A_{r_B}} = \frac{dE_{A_{r_B}}}{dm_{A_{r_B}}} \tag{2}$$

Being $dm_{A_{r_B}}$ the mass, and $dE_{A_{r_B}}$ the energy of the particle $A$ having as reference $B$.

Se supone a continuación un conjunto de partículas $A$.

Let it be considered a set $S$ of particles $A$.

$$\int \varepsilon_{A_{r_B}} dm_{A_{r_B}} = \int dE_{A_{r_B}} \tag{3}$$

The total energy of the set $S$ $(E_{S_{r_B}})$ will be the integral of all the energetic contributions.

$$\int \varepsilon_{A_{r_B}} dm_{A_{r_B}} = E_{S_{r_B}} \tag{4}$$

The energy contribution made by each particle $A$ of the set can be the same regardless of the particle (condition (5)),

$$\varepsilon_{A_{r_B}} = \varepsilon_{S_{r_B}} \tag{5}$$

If so, $\varepsilon_{A_{r_B}}$ is constant throughout the mass of the set and therefore can be extracted from the integral.

$$\varepsilon_{S_{r_B}} \int dm_{A_{r_B}} = E_{Tot_{r_B}} \tag{6}$$

On the other hand, the mass of the set will be the sum of all the differential masses that compose it,

$$m_{S_{r_B}} = \int dm_{A_{r_B}} \tag{7}$$

Thus,

$$\varepsilon_{S_{r_B}} = \frac{E_{S_{r_B}}}{m_{S_{r_B}}} \tag{8}$$

If it is necessary to apply the formulas described below for a non-differential mass set, condition (5) has to be verified. Otherwise, portions of the subject matter where said condition is verified should be considered.

Let $A$ and $B$ be two particles of infinitesimal mass $dm_A$ and $dm_B$ respectively whose energy states are different. Consider also a light beam $D$ moving with a speed $c$.

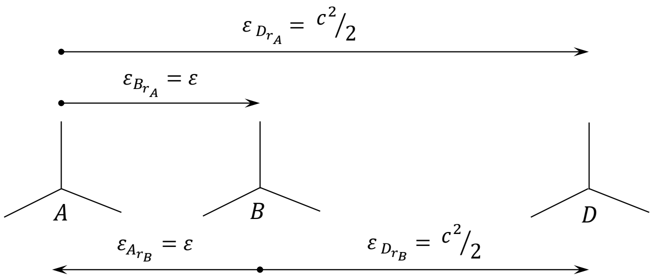


**Figure 1.** Specific energy differences of the particles

The specific energy of particle $A$ taking $B$ as a reference is,

$$\varepsilon_{A_{r_B}} = \varepsilon \tag{9}$$

By reciprocity, the specific energy of particle $B$ taking $A$ as a reference is,

$$\varepsilon_{B_{r_A}} = \varepsilon \tag{10}$$

Particle $D$ has only kinetic energy. This is defined as,

$$E_{kin} = \frac{1}{2}\, m\, v^2 \tag{11}$$

Being $v$ and $m$ velocity and mass of the particle respectively. Using the expression (1) the specific kinetic energy can be obtained,

$$\varepsilon_{kin} = \frac{v^2}{2} \tag{12}$$

Particle $D$ has no mass, but as seen in (12) its specific energy does not depend on it,
In the system described according to Fig. 1, $D$ travels at speed $c$. The speed of light is independent of the reference frame, therefore,

$$\varepsilon_{D_{r_A}} = \frac{c^2}{2} \tag{13}$$

$$\varepsilon_{D_{r_B}} = \frac{c^2}{2} \tag{14}$$

The particles of Fig. 1 can be infinitely close in the three-dimensional space. In fact, they both could be in the exact same point of the universe. Since the derivative is considered over the mass, not over the volume, they would still be different particles, even in such extreme condition.
However, despite their proximity, they are not the same particles, their specific energy differentiates them. Therefore, in order to distinguish the energy states of each particle, the $x, y, z$ position is not enough. In other words, the universe characterized by $x, y, z, t$ is not adequate to make the comparison of the present study.
Instead, an equivalent two-dimensional universe is used. This universe is defined by two variables $\Omega, t$. Where $\Omega$ is the equivalent spatial separation of the particles due to their specific energy.
Said spatial separation is defined below as the product of the equivalent velocity ($v_{eq}$) and the time advance of the particle ($t$).

$$\Omega = t\, v_{eq} \tag{15}$$

The equivalent speed is understood as that which would be necessary for all the specific energy of the particle to be specific kinetic energy.
Knowing that the expression of the specific kinetic energy is,

$$\varepsilon_{kin} = \frac{{v_{eq}}^2}{2} \tag{16}$$

The equivalent speed of $A$ (based on reference $B$) will be,

$$v_{eq_{A_{r_B}}} = \sqrt{2\, \varepsilon_{A_{r_B}}} \tag{17}$$

Por lo tanto, la separación equivalente entre las partículas $A$ y $B$ vista desde $B$ será,
Therefore, the equivalent separation between particles $A$ and $B$ seen from $B$ is,

$$\Omega_{A_{r_B}} = t_B\, \sqrt{2\, \varepsilon_{A_{r_B}}} \tag{18}$$

Cabe destacar que el avance temporal de las partículas no es el mismo. Puesto que $v_{eq_{A_{r_B}}}$ es la velocidad equivalente de la partícula $A$ observada desde $B$, es necesario utilizar el avance temporal del observador, que en este caso es $B$.
It should be noted that the time advance of the particles is not the same. Since $v_{eq_{A_{r_B}}}$ is the equivalent velocity of the particle $A$ observed from $B$, is necessary to use the temporal advance of the observer, which in this case is $B$.
This transformation is carried out for each specific energy of the Fig. 1, obtaining Fig. 2.

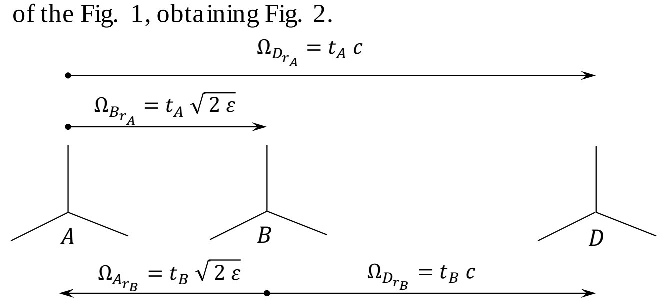


**Figure 2.** Equivalent distances between the particles

These equivalent distances are related to each other. However, they differ according to the reference from which they are observed.

$$dist(A, D)_{r_A} \neq dist(A, D)_{r_B} \tag{19}$$

$$dist(B, D)_{r_B} \neq dist(B, D)_{r_A} \tag{20}$$

To compare (19) and (20) the factor $k$ is used as seen in (21) and (22),

$$dist(A, D)_{r_A} = k\ (\, dist(A, D)_{r_B}\, ) \tag{21}$$

$$dist(B,D)_{r_B} = k\,(\,dist(B,D)_{r_A}\,) \tag{22}$$

Substituting the equivalent distances,

$$\Omega_{D_{r_A}} = k\,(\,\Omega_{D_{r_B}} + \Omega_{A_{r_B}}\,) \tag{23}$$

$$\Omega_{D_{r_B}} = k\,(\,\Omega_{D_{r_A}} - \Omega_{B_{r_A}}\,) \tag{24}$$

Then,

$$c\,t_A = k\,(\,t_B\,c + t_B\,\sqrt{2\,\varepsilon}\,) \tag{25}$$

$$c\,t_B = k\,(\,t_A\,c - t_A\,\sqrt{2\,\varepsilon}\,) \tag{26}$$

Considering the common factor,

$$c\,t_A = k\,t_B\,(\,c + \sqrt{2\,\varepsilon}\,) \tag{27}$$

$$c\,t_B = k\,t_A\,(\,c - \sqrt{2\,\varepsilon}\,) \tag{28}$$

Due to the symmetry of the problem, it is not possible to solve $k$ using the information from a single equation. It is necessary to incorporate the information of both to obtain the parameter $k$. Therefore (27) and (28) must be multiplied, obtaining (29).

$$c^2\,t_A t_B = k^2\,t_A\,t_B\,(\,c^2 - 2\,\varepsilon\,) \tag{29}$$

Simplifying,

$$k = \frac{1}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{30}$$

The factor obtained in (30) allows to relate the variables of two reference systems whose specific energies differ.

The last part of the development is analogous to the one made for the obtention of the Lorentz Factor (31) (Einstein 1905; Einstein 1915; Cenko *et al.* 2015). Indeed, the mathematical form of both factors (30) and (31) is very similar,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{31}$$

However, due to the initial transformations, the result is different. In the next section, (30) will be applied to different cases to observe the distinction between the current theories and the one developed in this study.

## 3. Applications

### *3.1. Velocity effect in time*

As it can be seen in (31), the Lorentz Factor depends on both the velocity ($v$) and the speed of light ($c$). This parameter ($\gamma$), defines the time dilation due to the velocity of a particle (Einstein 1916; Francis *et al.* 2013),

$$T' = T\,\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{32}$$

The relativistic time ($T'$) can be understood as the time advance of a certain particle $A$ whose velocity with respect to a reference $B$ is $v$.

Should the parameter suggested in (30) be used, the previous expression becomes,

$$T' = T\,\frac{1}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{33}$$

The specific energy of the particle $A$ with respect to particle $B$ is entirely kinetic. Thus,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{\frac{1}{2}\,m_A\left(v_{A_{r_B}}\right)^2}{m_A} = \frac{\frac{1}{2}\,m_A v^2}{m_A} = \frac{v^2}{2} \tag{34}$$

If this value (34) of the specific energy is substituted in (33) then it can be seen how the relation that arises is exactly the one defined by (32).

### *3.2. Gravitational effect in time*

Let it be studied the formula that defines time dilation due to the gravitational effect (Chou *et al.* 2010),

$$T' = T\,\frac{1}{\sqrt{1 - \frac{2GM}{R\,c^2}}} \tag{35}$$

The previous expression could also be rewritten as (33) where the particle considered is only submitted to the effect of gravity, and thus its specific energy is only gravitational,

$$\varepsilon = \varepsilon_{A_{r_B}} = \frac{E_{A_{r_B}}}{m_A} = \frac{m_A\,g\,R}{m_A} = g\,R = \frac{GM}{R^2}\,R = \frac{GM}{R} \tag{36}$$

It should be pointed out that it is curious that both equations (32) and (35) have (33) as a common ancestor.

### *3.3. Mass and energy relation*

In (37) is expressed the equation that relates the relativistic mass or total mass ($m_{Tot}$) with the rest mass ($m_0$) and the Lorentz Factor (Roche 2005),

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{37}$$

Equation (38) relates $m_{Tot}$ and $m_0$ if the proposed factor (30) is considered.

$$m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{38}$$

Equation (38) is an equation that relates the rest mass, the relativistic mass, and the total specific energy of a certain particle. But the famous equation (39) already relates those variables (Rainville, Thompson, Myers *et al.* 2005).

$$E = m_{extra}\ c^2 \tag{39}$$

Where the extra mass ($m_{extra}$) is the difference between the total mass ($m_{Tot}$) and the rest mass ($m_0$), that is,

$$m_{Tot} = m_0 + m_{extra} \tag{40}$$

Expressing (39) in a different form,

$$E = (\ m_{Tot} - m_0\ )\ c^2 \tag{41}$$

To make (41) comparable to (38) both equations should have the variable of specific energy $\varepsilon$. For such purpose, (41) can be modified as follows,

$$\varepsilon = \frac{E}{m_{Tot}} = \frac{m_{Tot} - m_0}{m_{Tot}}\ c^2 \tag{42}$$

$$\varepsilon = \left(1 - \frac{m_0}{m_{Tot}}\right) c^2 \tag{43}$$

Equations (43) and (39) are the same, but it would be truly remarkable if this last equation (43) is exactly equal to equation (38), which is proposed in this research.
Let equations (38) and (43) be compared. To do so, equation (38) will be rewritten as,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}} \tag{44}$$

Now (43) will be substituted in (44) and if both are the same, the resulting combined equation should lead to an identity,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{2\ \frac{m_0}{m_{Tot}} - 1}} \tag{45}$$

Calling $y$ the quotient of $\frac{m_0}{m_{Tot}}$,

$$\frac{1}{y} = \frac{1}{\sqrt{2\,y - 1}} \tag{46}$$

Simplifications end in two functions, one on the left of equality and one on the right,

$$f_1 = \frac{1}{y} \tag{47}$$

$$f_2 = \frac{1}{\sqrt{2\,y - 1}} \tag{48}$$

The functions defined by $f_1$ and $f_2$ are not the same. Thus, equations (38) and (43) are not the identical. To see how big the difference is, in Fig. 3 both are plotted together having $y$ as the independent variable.

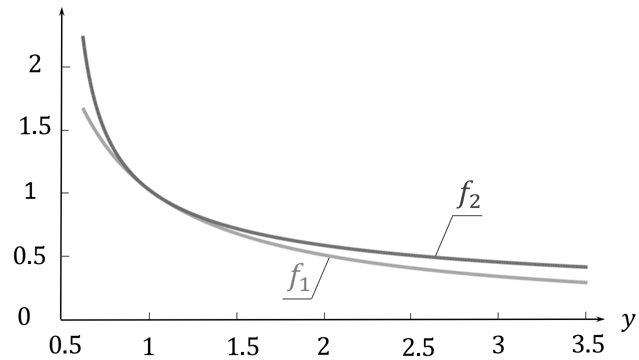


**Figure 3.** Tangency of functions $f_1$ y $f_2$

The resulting plot is of interest, since both functions are actually tangent at the point where $y$ is equal 1 (Fig 3.). It is thus observed that for those values close to 1, the error incurred is very small, while when being far from 1, the error can be very high.
In other words, when $m_{Tot}$ and $m_0$ are similar (usual in small particles, where $\varepsilon$ is often small), both formulas are applicable, (38) and (43). In fact, for the low-mass experiments carried out on Earth (small values of $m_{extra}$), if (43) was correct, no significant difference between (38) and (43) would be appreciated in the measurements.
Both formulas are very similar, and that is remarkable given the fact that their origins are completely different.
Each one understands the mass in its own way, and perhaps this is indeed the most interesting point to think about. Equation (39) says that mass is a linear property, that can be calculated adding up their parts. Simply with a sum. Indeed, (39) and (40) together they form a system of two equations. Formula (39) without (40) is meaningless. But equation (38) is suggesting that mass depends on its energy, and that the sum of the parts ($m_0 + m_{extra}$) is not the same as the whole ($m_{Tot}$).
To see more in detail the relation between these two versions of the theory, (38) and (43) will be compared using $E$ instead of $\varepsilon$. To do so, (44) will be transformed as follows,

$$\frac{m_{Tot}}{m_0} = \frac{1}{\sqrt{1 - \frac{E}{m_{Tot}\ c^2/2}}} \tag{49}$$

$$1 - \frac{E}{m_{Tot}\ c^2/2} = \left(\frac{m_0}{m_{Tot}}\right)^2 \tag{50}$$

$$E = m_{Tot}\ \frac{c^2}{2}\left(1 - \left(\frac{m_0}{m_{Tot}}\right)^2\right) \tag{51}$$

$$E = \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) \tag{52}$$

Equation (52) is exactly the same as (38), but allows the calculation of the bonding energy, being $m_0$ and

$m_{Tot}$ known.
Let it be studied the difference between (41) and (52) in a three-dimensional space being $m_0$ and $m_{Tot}$ the independent variables (Fig. 4 and Fig 5.).

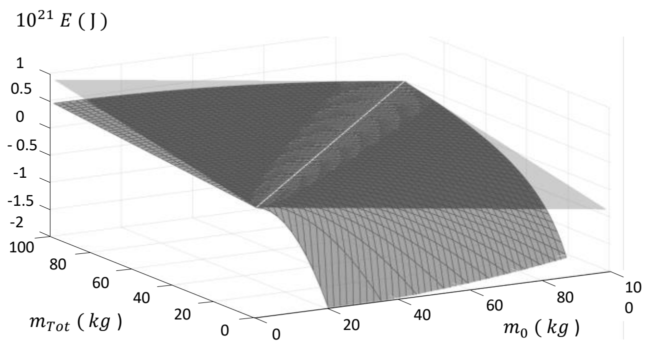


**Figure 4.** Differences between equations (41) and (52)

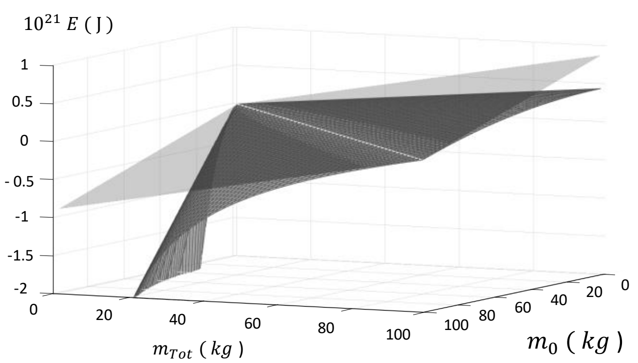


**Figure 5.** Differences between equations (41) and (52)

These representations (Fig. 4 and Fig 5.) are of vital importance. The plane represents equation (41) while the curved surface refers to equation (52).
Both equations provide the same value of the energy when $m_0$ and $m_{Tot}$ are equal, and therefore $E$ is zero. In the immediate vicinity of this line, (52) is a good approximation of (41). But when leaving the adjoining margin, the errors become much more noticeable, even reaching infinitely different values.
Interestingly, the plane defined by (41) is tangent to the curved surface (52). In fact, it is tangent along the entire line $E = 0$. An uncommon feature, between both three-dimensional functions.
Both surfaces predict a negative energy in case $m_0 > m_{Tot}$, positive when $m_0 < m_{Tot}$ and null in case the masses are equal, as explained.
The following development shows how the plane is tangent, across the straight line, to the curved surface.
The equation of the plane tangent to a given point $P$ of a function $F$ is given by,

$$F_x|_P(x - x_P) + F_y\big|_P(y - y_P) + F_z|_P(z - z_P) = 0 \tag{53}$$

Being $F$,

$$F = E - \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right) = 0 \tag{54}$$

Renaming with $x, y, z$,

$$F = z - \frac{c^2}{2}\left(y - \frac{x^2}{y}\right) = 0 \tag{55}$$

Being the point $P$ any point belonging to the line $m_0 = m_{Tot}$, thus $P(m, m, 0)$. Calculating the partial derivatives,

$$F_x = -\frac{c^2}{2}\left(-\frac{2\,x}{y}\right) \tag{56}$$

$$F_y = -\frac{c^2}{2}\left(1 + \frac{x^2}{y^2}\right) \tag{57}$$

$$F_x = 1 \tag{58}$$

Substituting $P$,

$$F_x|_P = -\frac{c^2}{2}\left(-\frac{2\,m}{m}\right) = c^2 \tag{59}$$

$$F_y\big|_P = -\frac{c^2}{2}\left(1 + \frac{m^2}{m^2}\right) = -c^2 \tag{60}$$

$$F_z|_P = 1 \tag{61}$$

Finally,

$$c^2\,(m_0 - m) - c^2\,(m_{Tot} - m) + (E - 0) = 0 \tag{62}$$

$$E = c^2\,(m_{Tot} - m) - c^2\,(m_0 - m) \tag{63}$$

$$E = (m_{Tot} - m_0)\,c^2 = m_{extra}\,c^2 \tag{64}$$

Checking in this way that the plane is indeed tangent to the curved surface.
But still there is some hidden relation between equations (41) and (52) that has not been covered. In order to see it, the variable $m_{Tot}$ will be extracted from equation (52), obtaining,

$${m_{Tot}}^2 - \frac{2\,E}{c^2} m_{Tot} - {m_0}^2 = 0 \tag{65}$$

$$m_{Tot} = \frac{\frac{2\,E}{c^2} \pm \sqrt{\frac{4\,E^2}{c^4} + 4{m_0}^2}}{2} \tag{66}$$

Considering only the positive value of the mass,

$$m_{Tot} = \frac{E}{c^2} + \sqrt{\frac{E^2}{c^4} + {m_0}^2} \tag{67}$$

Assuming that $\frac{E^2}{c^4}$ is negligible in comparison to ${m_0}^2$, then,

$$m_{Tot} = \frac{E}{c^2} + m_0 \tag{68}$$

Which in the end leads to,

$$E = (m_{Tot} - m_0)\, c^2 \qquad (69)$$

In such a way that the equation (41) could be understood as an approximation of (52), if $\frac{E^2}{c^4}$ is much smaller than ${m_0}^2$.
On the other hand, it should be remembered that $E = m\,c^2$ is actually a specific case in which the particle considered has no velocity according to the reference. If it has velocity, the expression becomes (Okun 2009),

$$E^2 = (m\,c^2)^2 + (pc)^2 \qquad (70)$$

Being $p$ the linear momentum of the study particle. However, this generalization is not necessary with the proposed equation (52), since it already considers all the specific energy according to the desired reference.
If the previous holds, if this proposed theory is indeed correct, it would imply that the energy cannot be converted into matter nor vice versa, but rather the energy affects the weight of matter.

## 5. WIMP annihilation via loop-level processes

Many are the challenges that, yet have not been solved with the current understanding of mass. Dark Matter is indeed one of those challenges (Evrard, Metzler, Navarro 1996; Merritt 2006; Navarro, Frenk, White 1997; de Blok *et al.* 2001; Wang *et al.* 2016). There are indisputable differences between predictions and observations related to certain gravitational effects in the universe, and in order to use current theories of gravitation, it would be necessary to have more mass than what it is observed. Indeed, that is the origin of Dark Matter, it should exist to reconcile both observations and theories. But its nature is so complex that its existence has never been proved.
The so-called weakly-interacting massive particles, or WIMPs are studied for being a potential candidate to explain the nature of Dark Matter (Sanders 1990; Borriello, Salucci 2001; Zaharijas, Hooper 2006; Gnedin *et al.* 2004). WIMP annihilation into charged states produces photons via loop-level processes. When two WIMP particles $\chi\chi$ annihilate each other at close to zero relative velocity into $\gamma X$, an energy of $E_\gamma$ is released (Abdo *et al.* 2010; Goodman Ibe, *et al.* 2010), as it is given by (71).

$$E_\gamma = m_\chi \left(1 - \frac{{m_X}^2}{4\,{m_\chi}^2}\right) \qquad (71)$$

Where $m_\chi$ refers to the mass of a single WIMP particle $\chi$, measured in energy units. And $m_X$ refers to the remaining mass after the annihilation, measured also in energy units. Thus, the total mass of Dark Matter before and after the annihilation can be expressed as $m_{Tot}$ and $m_0$ respectively ((72) and (73)), where $m_{Tot}$ is twice the $m_\chi$ (since there are two WIMP particles). Applying the transformation to get $m_{Tot}$ and $m_0$ in mass units instead of energy units, the resulting masses are,

$$m_{Tot} = \frac{2\, m_\chi}{c^2} \qquad (72)$$

$$m_0 = \frac{m_X}{c^2} \qquad (73)$$

Substituting $m_\chi$ and $m_X$ in (43),

$$E = \frac{c^2}{2}\, m_{Tot} \left(1 - \frac{{m_0}^2}{{m_{Tot}}^2}\right) \qquad (74)$$

Equation (74) is exactly the same equation as the one predicted in this theory (52).
Equation (52) has been obtained from a mathematical development that understands mass as a dependent variable of the specific energy. But equation (71) is a well-known relation developed from energy conservation laws, used to seek explanations for the observed data of Dark Matter presence in nearby halos. The fact that they are both the same it is not only an impressive result, but also a clarification of what the real nature of Dark Matter could be.

## 6. Conclusions

This development studies from a mathematical perspective the possible influence of the specific energy in the Lorentz Factor and its implication in the definition of time and mass. The major differences between the Theory of Relativity and the one proposed are shown in Table I. Which ultimately summarizes that both formulas in the right column are the same equation but ordered in two different ways, while the two on the left are different formulas. Going back to Fig. 4 and Fig. 5, it can be seen that there is a very important difference between both solutions. That although in the small world of the experiments carried out on Earth both theories predict similar results, in the vastness of outer space they lead to completely different predictions.

**Table I.** Differences between theories

| Theory of Relativity | Theory proposed |
|---|---|
| $m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ | $m_{Tot} = \frac{m_0}{\sqrt{1 - \frac{\varepsilon}{c^2/2}}}$ |
| $E = (m_{Total} - m_0)\, c^2$ | $E = \frac{c^2}{2}\left(m_{Tot} - \frac{{m_0}^2}{m_{Tot}}\right)$ |


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