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DIFFERENCE FORMULA

be accepted as fulfilling this part of the requirements for the degree of DOCTOR OF PHILOSOPHY

Approved by:

Dr. Carl A. Ludeke

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requirements for the degree of

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1952

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ABSTRACT

When two colored samples are to be compared visually, the spatial proximity will in general affect the facility of matching. Considering chromaticity differences and reflectance differences as separately applicable features of a color difference, it was pointed out by Judd (1939)* that the perception of reflectance differences is reduced markedly by spatial separation while the same is not conspicuously true of chromaticity differences. The proximity effect is accounted for in the Judd formula for color difference where it appears as a weighting factor for the reflectance difference term, reducing the importance of the perceived lightness difference as sample separation is increased. The functional relationship between proximity factor and sample separation was not known, but a few tentative values were assigned by Judd. The present study finds an empirical expression for the proximity factor as a continuous function of angular width of dividing line. A divided visual field is used whose halves are variable separable. Calibrated pairs of colored papers provide the stimuli. Each pair differs in itself only slightly in the lightness attribute, with various pairs exhibiting different degrees of reflectance difference. Chromaticity differences are absent. The observer is asked to identify the darker member of each pair a large number of times for each value of dividing line width. The percentage of errors

*References are indicated by year of publication and listed in bibliography at end.

indicates the difficulty experienced by the observer in distinguishing the light and dark halves of the field as separation is varied. A scale relationship is then established between lightness differences and percent errors for zero separation by basing observations on a series of calibrated sample pairs with no line of demarcation. Lightness differences are expressed in NBS units using Dr. Judd's maximum value of 120 for the proximity factor k_1 . The scale relationship is extended to other combinations of reflectance difference and separation on the assumption that all such combinations which yield equal numbers of errors are equally perceptible and hence correspond to equal lightness differences. On the basis of the data, the Judd expression equating lightness difference to the product of k_1 and a function of reflectance difference is solved for numerical values of k_1 for various angular separations. The values of k_1 are plotted versus separation and a geometrical curve is fitted to the points. The corresponding analytic expression yields values of k_1 as a continuous function of angular separation. The effect of proximity on chromaticity discrimination is also examined, with no significant change reported. All data are based on one observer and specialized conditions of viewing.

I. INTRODUCTION

The attempt to introduce order into the apparently chaotic realm of color sensations involves the selection of a mode of representation and the formulation of rules whereby each color sensation is uniquely associated with some element of the representation. The gamut of visible colors lends itself conveniently to graphical representation in three dimensional space by virtue of the three dimensional nature of color. Each perceived color can vary independently in three attributes. A given color may appear lighter than a reference color while differing from it in no other way, or it may be of a different hue without varying otherwise, or it may show more or less saturation and yet have the same hue and lightness.

Starting, then, with a given color as a reference point, the entire range of visible colors may be described by manipulation of these three attributes alone. Consequently the use of physical space as a medium for the graphical representation of the color gamut suggests itself immediately.

Less obvious is the metric to be established for this representation for, aside from its tridimensional nature there is no characteristic inherent in color to suggest the geometrical properties which a color space should possess. Numerous color spaces have been devised since the time of Helmholtz, all differing in metric, and the number of color spaces which it is possible to devise

is limited only by the ingenuity and ambition of workers in the field.

The absence of a single, universally adopted color system throughout all of science, art, and industry indicates that no one system fulfills all the needs of these diverse branches of endeavor. The situation is not of great consequence however, since each color system possesses its own characteristics which render it useful for some one phase of color work.

In the problem of specifying color tolerances, the desirable features of a color space are homogeneity and isotropy with regard to visual distances between colors. That is to say, equidistant pairs of points in the color space should represent equal visual distances between the pairs of colors represented by the points. With a visually uniform color scale, color tolerances would be conveniently and unambiguously specifiable. The color of a given standard would be designated by the coordinates of a point in color space, and the permissible tolerance would be specified by the magnitude of the concentric sphere within which all permissible departures were to fall.

Because of the convenience afforded by plane diagrams over solid figures in graphical representations, the manifold of colors, with the lightness dimension suppressed, is often represented by a chromaticity diagram. Since the terms hue and saturation refer to

sensation as experienced subjectively, the physically measurable quantities dominant wavelength and purity are often to be preferred in the designation of color. Each point in the chromaticity diagram represents some combination of dominant wavelength (or complementary wavelength for purple colors) and purity, thereby designating by a pair of coordinates a complete set of colors which differ only in lightness from one another. A third specification, lightness, is necessary in order to single out a particular color from the set.

Lest the psychological import of the chromaticity diagram be overlooked by the reader, it is pointed out that such diagrams are constructed on the basis of color matching experiments and are obviously bound up with the color matching characteristics of the human eye. The possible geometrical arrangements of chromaticity diagrams are nearly as diverse as are those of color space.

A chromaticity scale possessing visual uniformity has not yet been satisfactorily achieved, although in the past two decades perhaps a dozen noteworthy strides have been made in this direction (Burnham, 1949). The general procedure is to modify the existing C.I.E. system (Commission Internationale de l'Eclairage, 1931; Judd, 1933) by a transformation of coordinates, amounting to a change of primaries, which seeks to reduce in the C.I.E. twofold the distortions which are present in any mapping of visual distances (MacAdam, 1944). The first successful color two-fold which was visually uniform

is just such a transformation and is due to Judd (1935).

More difficult of achievement is a uniform color three-space, in which the third dimension relates to lightness. Visually uniform color three-folds have been devised by Nickerson (1936), Balinkin (1939), Judd (1939), Adams (1940), Hunter (1942), and others. It is the color space due to Judd with which this research is concerned.

II. THEORY

The geometry of a color space is completely described by its metric, which is the expression for the square of the distance between two points in terms of the coordinates used. For a visually uniform color space, the metric should obviously be identified with "visual distance" in the manifold. The expression for the metric then becomes a formula for color difference.

It is well known that the visual distance between a given pair of samples depends upon a multitude of variables, both physical and psychological. Chief among these are:

1. The spectral reflectance characteristics of the samples.
2. The spectral energy distribution of the illuminant.
3. The illuminance level.
4. The spatial proximity of the samples.
5. The angular subtense of the samples at the eye of the observer.
6. The part of the retina affected.
7. The nature of the background against which the samples are viewed.
8. The properties and state of the visual mechanism.
9. The duration of exposure to the eye.

Of these, the first three are of the most immediate significance and are the parameters generally taken into account in the color difference equations extant. The fourth is a matter of some significance in the evaluation of lightness differences, as pointed out by Judd (1939), and has been incorporated into a color difference formula of wide applicability. This formula treats a color difference, (ΔE), as the sum of two mutually perpendicular vectors in Euclidean space, one vector representing a lightness difference, (ΔL), and the other a chromaticity difference, (ΔC):

$$(\Delta E)^2 = (\Delta L)^2 + (\Delta C)^2 \quad (1)$$

where chromaticity is a combination of the dominant wavelength and purity attributes of a colored surface under a given illuminant. The separation of a color difference into a lightness difference and a chromaticity difference is warranted by the evidence of several psychophysical experiments (Brown and MacAdam, 1939; Hecht, Peskin, and Fatt, 1938) and is a matter of common procedure in the treatment of color differences. The psychological aspects of this procedure are discussed at length by Bouma (1947, pp. 34-37).

The chromaticity difference, ΔC , is proportional to the separation of the given colors on the uniform-chromaticity-scale triangle derived by Judd (1935). The U.C.S. Triangle was derived so as to provide uniform spacing for equally perceptible differences between colors

of equal lightness and hence does not take lightness differences into account.

At this point a distinction should be made between lightness, which is a psychological phenomenon, and luminous directional reflectance, which is psychophysical (physically measured, psychologically significant). The luminous reflectance of a sample is the ratio of the amount of light reflected by it to the amount incident upon it when measured by an instrument whose spectral sensitivity is adjusted to that of the normal human eye. It is known experimentally that under ordinary viewing conditions and with samples against a light background, the psychological experience of lightness is correlated quite well with the square root of reflectance. Thus, a sample having four times the reflectance of a given standard will ordinarily appear twice as bright.

The addition of a third dimension to the U.C.S. Triangle, normal to its plane and accounting for colors of varying lightness, completed the color solid which is partially described by Eq. (1). To describe the solid completely it is necessary to establish the visual relationship between a given reflectance difference and a given chromaticity difference. That is, a unit of length along the lightness axis should correspond to a visual distance equal to that represented by a unit length on the chromaticity plane. The adjustment of units is based on the average estimates of a large number of observers as to what sizes of stimulus difference yield equal visual

lightness and chromaticity differences respectively. The ultimate basis for this comparison is of course psychological and has little appeal for the physicist. It is difficult to envision a lightness difference which can always be said to be equal in magnitude to a given chromaticity difference. The comparison has real meaning, however, in terms of sensation differences experienced by the observer. This is evidenced by the consistency shown in the results of large numbers of observers in such experiments. It was shown by Balinkin (1939) that very good agreement may be expected among observers in estimating the relative sizes of color differences, be they predominantly lightness differences or predominantly chromaticity differences.

Provision must be made, then, in the formula for ΔE , for weighting the ΔC and ΔL terms appropriately so as to ensure the proper visual relationship of one to the other. Several features of this relationship are noteworthy. The first is its dependence upon the average reflectance of the samples being compared. For dark samples, a given separation on the U.C.S. Triangle is not as significant visually as it is for light samples. The ΔC term, then, in addition to being proportional to the separation on the U.C.S. Triangle, varies with the reflectance also. Denoting a given length on the U.C.S. Triangle by ΔS , it is assumed by Judd that the visual importance of a given ΔS varies with the fourth root of the reflectance, which we denote by A .

Thus:

$$\Delta C \propto A^{\frac{1}{2}} \Delta S \quad (2)$$

The lightness of a given sample is related to the reflectance through the square root relationship referred to above. Thus:

$$L \propto A^{\frac{1}{2}}$$

or the visual lightness difference in two samples varies as the difference of the square roots of the respective reflectances:

$$\Delta L \propto \Delta(A^{\frac{1}{2}}) \quad (3)$$

In addition, it is assumed by Judd that the perceptibility of the lightness component of a given color difference is affected significantly by the width of the line or area separating the samples and that separation has only a slight effect upon chromaticity discrimination.* It is the change in lightness discrimination as a function of separation which is the subject of the present research. The width of the dividing line will therefore change the relative importance of $A^{\frac{1}{2}} \Delta S$ and $\Delta(A^{\frac{1}{2}})$ in Eq. (1). The vectorial combination of the two need involve only one coefficient in order to adjust their relative importance in the formula. This coefficient is called the "proximity factor", k_1 , and with it at our disposal we are able to adjust the relative significance

*This distinction between the discrimination of chromaticities and of lightnesses is not generally recognized in the field and it is one object of this work to verify Judd's statement.

of the reflectance difference as dividing line width is varied. It is this factor for which a few tentative values have been assigned by Judd and for which a continuous function is here sought. We write Eq. (3) then as:

$$\Delta L = k_1 \Delta(A^{\frac{1}{2}}) \quad (4)$$

Having allowed for adjustment of the relative sizes of units of ΔL and ΔC we must make provision for adjustment of absolute sizes so that the units of ΔE into which they are combined will be of such size as to be useful in practical work. This is accomplished through a second coefficient which renders $A^{\frac{1}{4}}\Delta S$ capable of adjustment. Equation (2) is now written:

$$\Delta C = k_2 A^{\frac{1}{4}} \Delta S \quad (5)$$

whereupon the complete Judd color difference formula becomes:

$$[\Delta E]^2 = [k_1 \Delta(A^{\frac{1}{2}})]^2 + [k_2 A^{\frac{1}{4}} \Delta S]^2 \quad (6)$$

The ratio of the factors k_1 and k_2 is a function of the dividing line width, while their sum establishes the size of the unit of color difference. The numerical values of 120 for k_1 and 600 for k_2 have been assigned by Judd so that the unit of color difference is large enough to be perceived visually and yet small enough to be neglected in most commercial transactions. It is equivalent to one-fourth the chromaticity difference between the components of the sodium D line. The unit

is designated the "NBS Unit of Color Difference" or as suggested by Balinkin (1939) the "judd" as it will be referred to here.

The ratio 120:600 for the factors $k_1:k_2$ was established for the case of colors in close proximity, that is, separated by a very narrow or non-existent dividing line. As lightness difference becomes less conspicuous with increased dividing line width, the numerical value assigned to $k_1:k_2$ decreases to about 40:600 as found by Judd.

The exact behavior of this ratio as a function of dividing line width is not known and, as stated above, is the object of search in this investigation. On the basis of psychophysical data, a relationship will be determined between the proximity factor, k_1 , and the angular subtense of the dividing line so that color differences predicted by the Judd formula will accord with the average of visual estimates.

III. EXPERIMENTAL PROCEDURE

The experiment is a series of dichotomous tests in which the observer is asked to identify the darker of two colors under various conditions of separation. The 1 cm.² test colors are presented side by side against a dark background at an average viewing distance of six feet. Unrestricted binocular vision is used. At each observation the darker sample may be either on the right or the left and the observer indicates his decision by depressing one of a pair of push-button switches at his disposal. Vision is interrupted for a period of one second during which the observer's report is judged electrically and recorded as correct or incorrect, and the procedure is then repeated. Since the reflectance differences for the pairs of paper samples are liminal, the observer is expected to be in error a part of the time. After one hundred judgments have been made, the number of errors is noted and the procedure is repeated for a new value of separation. The variation of percent error with separation for each sample pair is the basis on which the proximity function is deduced.

The procedure employed is known by psychologists as the method of constant stimulus differences. The presentation of a stimulus difference, such as a reflectance difference, under a set of conditions which modify the perceived magnitude of that difference, such as at varying separations, is akin to the well known psychometric

experiment with lifted weights. In such an experiment a group of weights, varying in small steps about a standard, is compared one at a time with the standard, the observer indicating whether each weight feels heavier than, equal to, or lighter than the standard. Interpretation of the data leads to the difference limen for lifted weights (Guilford, 1936, p. 186). In the present experiment the equal-category is excluded and it is other information than the difference limen which is of interest.

The method of constant stimulus differences is to be distinguished from the method of minimal changes which is used frequently in psychometric work with liminal differences. In this method the set of constant comparison stimuli is replaced by a single variable stimulus which is adjusted until it appears to the observer as just equal to or just different from a given standard.

A. Testing Apparatus

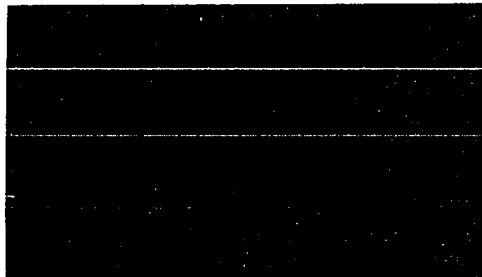
The colors to be used in the experiments consisted of small squares of colored papers mounted in pairs on cards of two by three inch heavy black paper. The colors of each pair differed essentially in lightness, and during the course of the experiments various means were developed for producing color pairs with the desired characteristics. It was essential that the chromaticity difference in any sample pair be much less than the reflectance difference, perceptually, since the latter alone was being studied. Reflectance

differences were to be liminal under the experimental conditions employed: reflectance differences which could be identified either positively or not at all under these conditions would yield no useful information in the experiment.

The first method consisted of selecting pairs of graded Munsell color chips which differed only in value, the Munsell equivalent of lightness. The value steps available, however, corresponded to supra-liminal differences. A second method employed the mixing of pigments which were then sprayed on smooth white paper. It was found that the addition of a small amount of black to a given pigment in order to darken it slightly resulted very often in a hue change, and the method yielded few acceptable sample pairs. Uniform rectangles of colored paper were then masked over half of their areas and subjected to a very fine spray of black poster paint without much improvement over the mixture method. An alternate method was the selection of sample pairs from a large number of prepared colored sample papers available from paint manufacturers. Selections were made visually and examined colorimetrically by means of a Hunter Multipurpose Reflectometer but again yielded few sample pairs which differed predominantly in lightness. Single rectangular paint samples were then masked and rubbed with cotton wadding which had been charged with pulverized pencil graphite but the results too often were non-uniformity of the darkened area and a change in the gloss characteristics.

When extensive psychometric data had shown no apparent dependence of the proximity function upon hue or saturation of samples, the decision was made to confine the study to achromatic samples. For this purpose, uniform samples of any desired degree of reflectance difference and no chromaticity difference were produced by exposing appropriately masked sheets of photographic paper to controlled illumination. The developed sheets were trimmed to size and calibrated on a Hunter Color and Color Difference Meter and provided most of the data upon which the present result is based.

Actual samples prepared by various methods are shown below. These are a few of the several hundred sample pairs which were prepared and rejected for various reasons including hue shift with darkening, non-uniformity of darkening, and too great or too small a lightness step.



For each series of tests, a sample pair was mounted in a 2 x 3 inch frame which was fixed centrally to the end of a rotor so that it could be made to rotate in its own plane. A small shaft mounted in ball bearings comprised the rotor. A solenoid was so disposed as to

provide an angular impulse to the rotor at a predetermined time. Thereafter the rotor spun relatively freely until coming to rest with the frame oriented randomly in one of two possible horizontal positions. The horizontal equilibrium position of the frame was achieved through the action of a permanent magnetic field on an elliptical soft iron plate mounted symmetrically on the rotor and perpendicular to its axis.

Various separations for the halves of the test field were achieved by a series of black cardboard masks, each mask with a pair of 1 x 1 cm. apertures. The series of seven masks provided separations from zero cm. to 2.60 cm. Provision was made for each mask at a time to be mounted directly in front of the sample pair under test, the dividing strips aligned with the dividing line between samples. During observation the mask was drawn up against the sample pair by solenoidal action but was pushed outward by spring action while the sample pair was spinning.

A pair of push-button switches was the only control at the disposal of the observer. Each judgment consisted of an identification of the darker patch as to "right" or "left" by pressing the corresponding push-button.

The electrical impulse arising from momentary depression of either push-button initiated a one second sequence of events controlled by a timing plate. The timing plate was driven by a 60 r.p.m. 1/200 hp. Telechron motor. The motor was continuously energized during the testing

procedure but rotation of the timing plate was impeded mechanically at the end of each cycle when a protuberance from its rim struck an electrically operated plunger. The cycle began with activation of the plunger which released the timing plate and motor.

A schematic circuit diagram is shown as Fig. I, p. 20. The observer's push-button switches are shown as a D.P.D.T. switch. A. Depression of either switch activates the motor release plunger, F, which frees the timing plate, E, allowing it to rotate under the action of the motor, D. Immediately thereafter a pair of solenoids, G, is de-activated allowing the cardboard mask to return to its original position. Within one-eighth of a second an evaluation is made of the observer's judgment as relay H closes a set of four contacts shown in routing circuit B. A short impulse to solenoid J then imparts the rotational motion to the rotor which spins three or four times and comes to rest by the end of the timing plate cycle. The observer having released switch A, the cycle is concluded when the timing plate is stopped mechanically and the mask solenoids G are activated. The timing plate is shown in the rest position in Fig. I.

Evaluation of the observer's judgment is performed by routing circuit B which consists of five button contacts set into a strip of plastic sheet and wired as shown. The rectangular plastic strip is mounted symmetrically and perpendicularly on the back end of the rotor and, as with the frame on the front end, comes to rest in one of two possible positions. The position shown in Fig. I corresponds

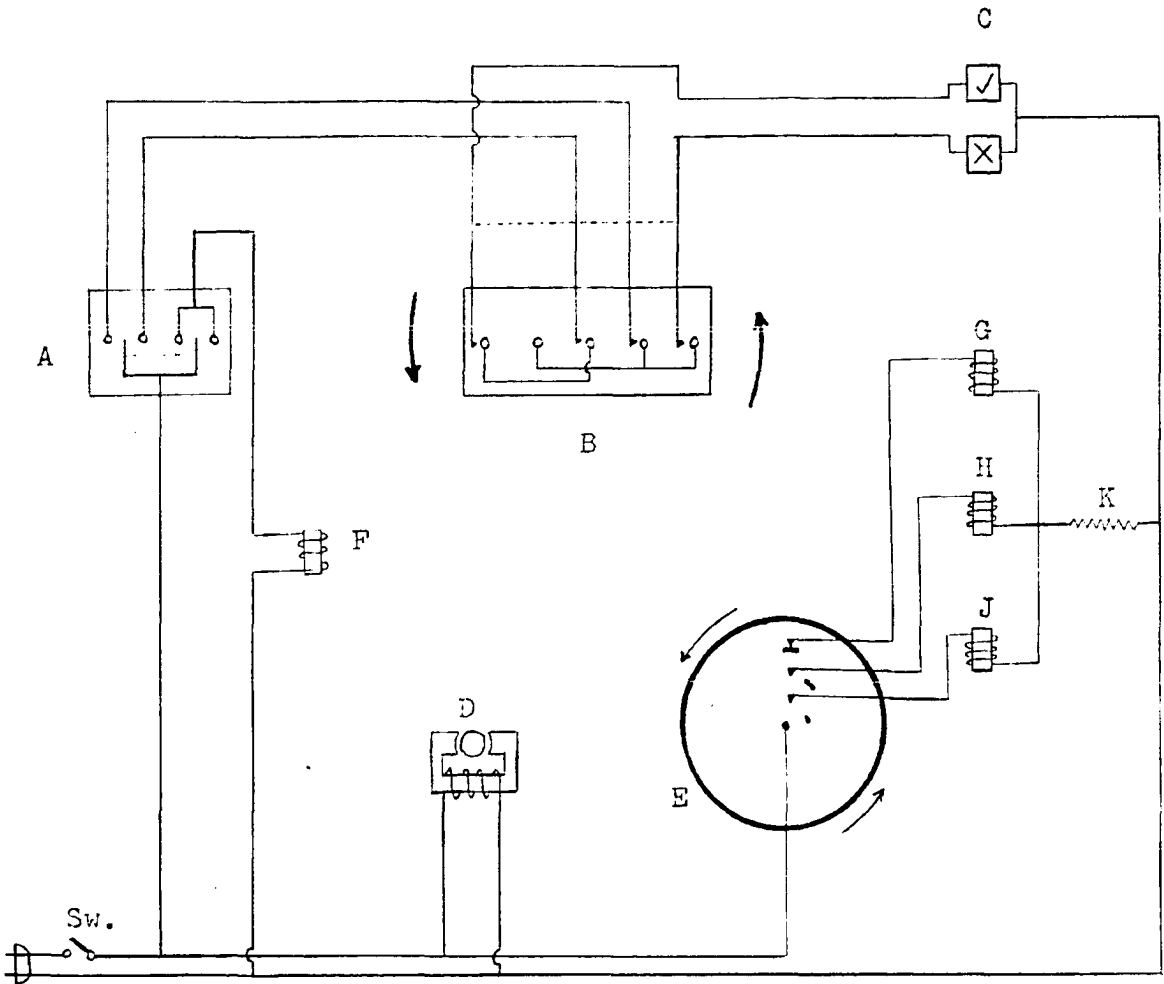


Fig. I. Schematic circuit diagram of apparatus showing

- | | |
|----------------------------|----------------------|
| A. Observer control switch | F. Motor release |
| B. Routing circuit | G. Mask solenoids |
| C. Impulse counters | H. Indicator relay |
| D. Synchronous motor | J. Spinner plunger |
| E. Timing plate | K. Dropping resistor |

to that position of the frame for which the darker sample is on the right. Momentary depression of either push-button, in addition to energizing plunger F, results in an impulse along one or the other of the parallel circuits to impulse counters C. Since the four contacts of routing circuit B are closed almost immediately, the impulse is directed to one of the two counters according as the position of the plastic strip is as shown or reversed. For each position of the strip, or darker sample, there is the possibility of either a correct or an incorrect judgment being indicated, depending upon which of the push-buttons is depressed. It is apparent that the card bearing the sample pair must be inserted in a particular position relative to the frame in order for correct judgments to be evaluated as correct.

The apparatus is enclosed in a cabinet of about ten inches square and six inches high and weighs about six pounds. A general view, with sample pair and mask in position, is shown in Plate 1. At the side of the cabinet are power switch, pilot light, and outlets for impulse counters and push-button switches. In Plate 2 the assembly is shown with the lid and mask removed. The mask, with a rectangular section removed from the lower edge, slides into slots at the top and sides of the rectangular plate seen just below the sample pair. Plate 3 shows the impulse counters and observer's control box. In practice the counters are not in the observer's view.

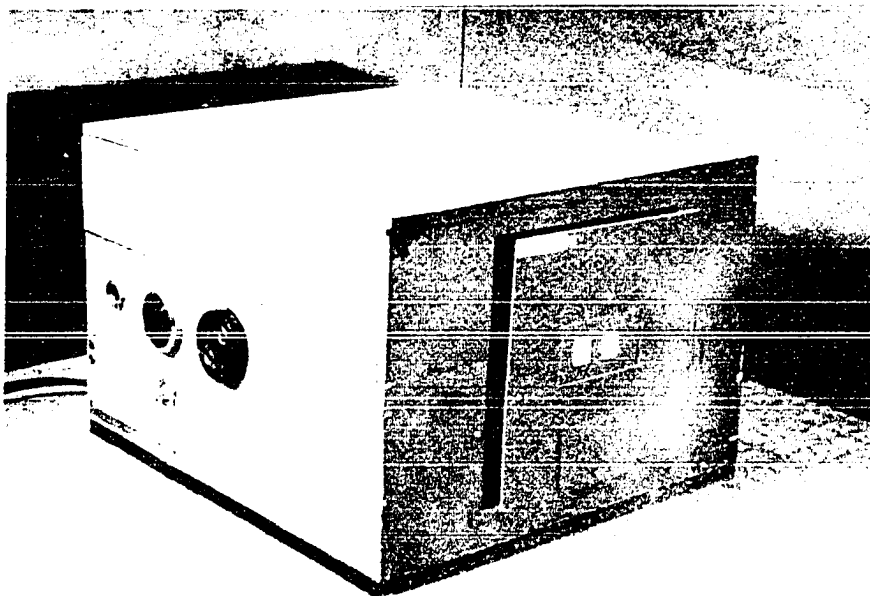


Plate 1. General view of apparatus with sample pair and mask in position.

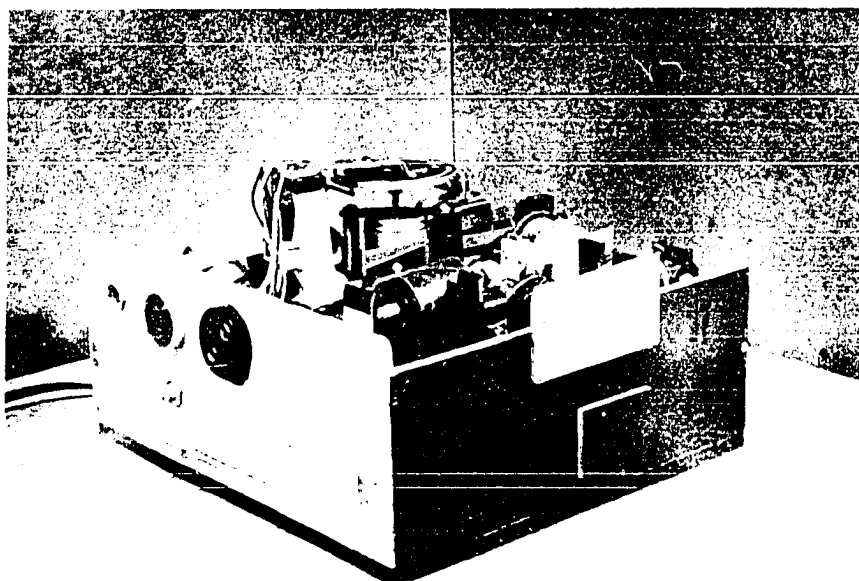


Plate 2. Apparatus with lid and mask removed.

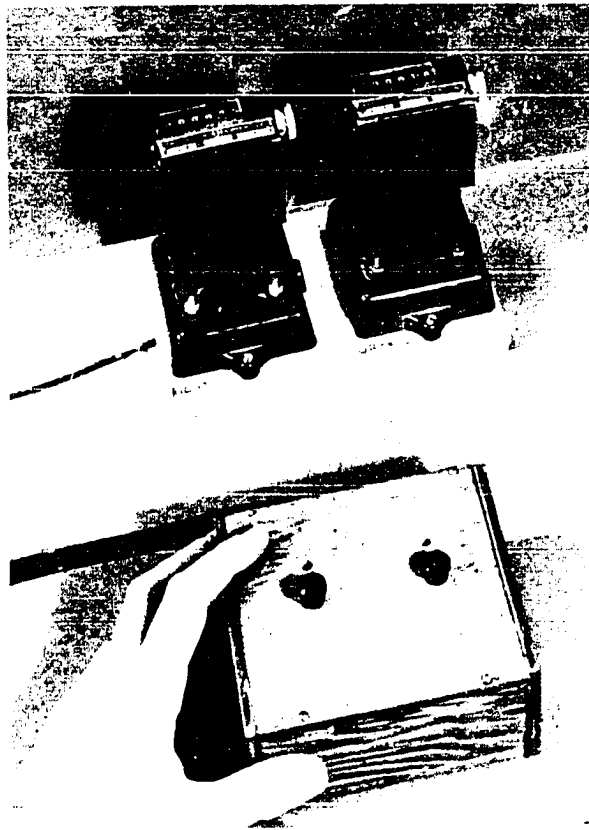


Plate 3. Impulse counters and control box.

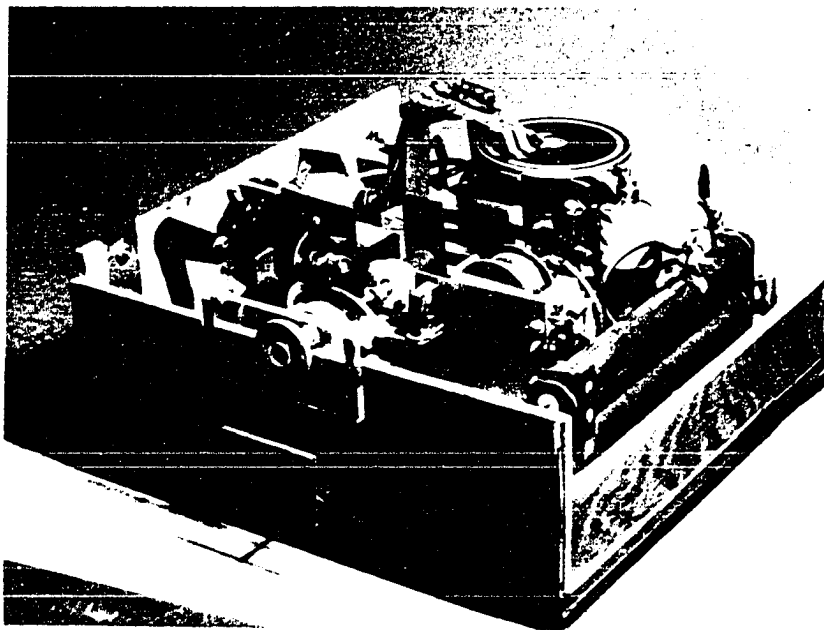


Plate 4. General view of apparatus.

The apparatus is seen from a different angle in Plate 4 with the sample pair removed. At the rear can be seen the timing plate in the rest position, with the small projection from the rim resting against the plunger which impedes it.

The timing plate and its three sliding contacts are seen in Plate 5. The outermost track on the face of the plate was originally used to control the timing plate motor, the contact being broken and the plate coming to rest with the contact arm touching the gap. A momentary impulse from the observer's controls would then start the motor for a new cycle. Overshooting of the gap and other complications resulted in the abandonment of this method of motor control.

Also visible in Plate 5 is the plastic strip at the end of the rotor with its five interconnected button contacts, as well as the four contact arms, the relay which closes them, and the strip of foam rubber fixed to all four arms in order to reduce contact bounce.

Plate 6 shows a top view of the rotor and allied parts, the slotted mask support and retracting arm, the rubber-tipped spinner plunger which strikes either end of a stud passing through the rotor, the short rod magnets which align the elliptical plate between them, and a tuft of cotton wadding against the elliptical plate which damps vibration of the rotor as it comes to equilibrium.

A side view of the apparatus is shown in Plate 7,



Plate 5. Detailed view of mechanism.

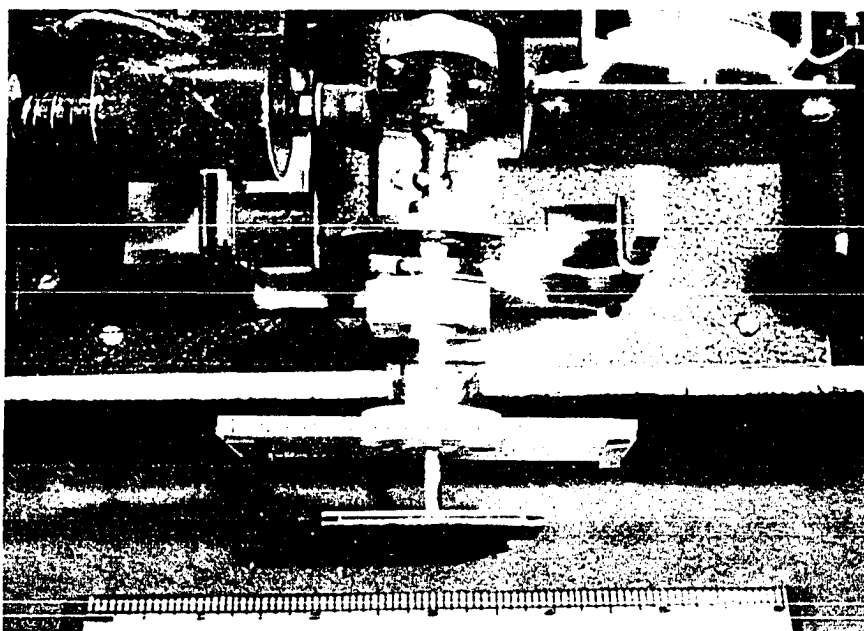


Plate 6. Forward part of apparatus, top view.

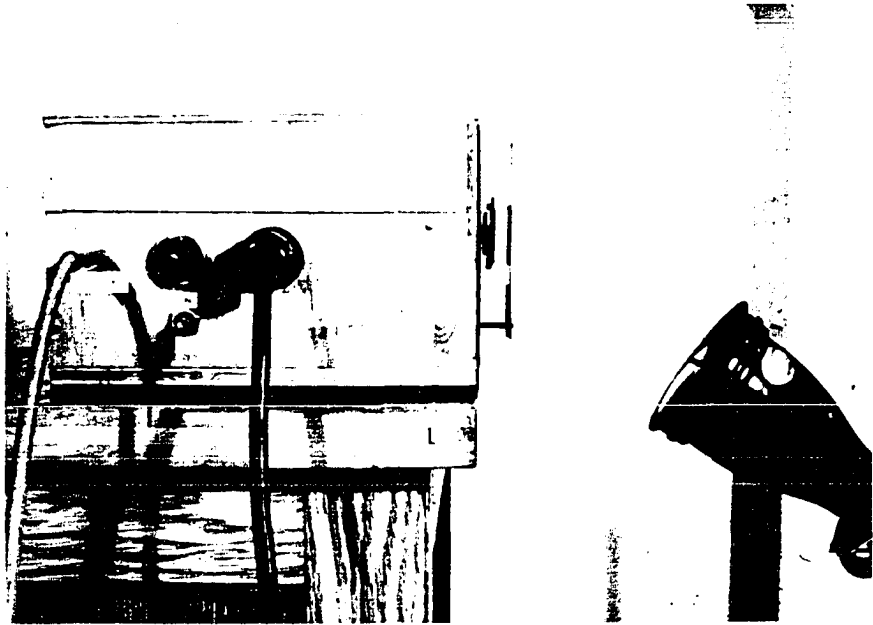


Plate 7. Side view of apparatus showing mask in open position.

with the mask in the released or open position. Illumination is provided by a Macbeth C.I.E. Illuminant C source, resembling average daylight, which is shown in one of the positions which were used. Forty-five degree illumination and normal viewing were the rule throughout. The lamp was positioned at various times for illuminations in the range from 10 to 35 foot candles.

B. Testing Conditions

It was originally intended to base results upon a large number of observers. The results of preliminary tests made on many observers, including the author, revealed the lightness discrimination characteristics of the latter to be fairly representative, that is, within the range of observer fluctuation encountered in the tests. Since the apparatus was completely automatic and required no attention during any one series of observations, it was convenient for the author to be the principle observer and the final results are based upon his observations alone.

Observation was made at viewing distances in the range from 4 to 10 feet, most of the present data being obtained at from 5 to 7 feet. It is known that discrimination is largely affected by angular field size, but the variation in the 4 to 10 foot range was not extreme in the case of the few data used here which were obtained from these limits. No significant variation is noticed in the 5 to 7 foot range, and the weighted average of 6 feet is used as the basis for

computing angular dividing line widths.

Various amounts of illumination were explored for comfort of seeing and acuity of discrimination. It was found that illuminations in the range from 10 to 35 foot candles were suitable, with most of the observations made at 15 to 20 foot candles. For very light or white sample pairs, the corresponding mean field brightness would be somewhat less than the 25 ml. lower limit found by Lowry (1931) for maximum photometric sensitivity. The data reported here are based on an illuminant of spectral composition similar to C.I.E. Illuminant C. No other light source was used during testing.

Angular field width at the average viewing distance was confined to the region immediately surrounding the fovea, where maximum retinal sensitivity to luminance differences is known to exist. The maximum separation of 28 mm. for the two 1 cm.² apertures at the minimum viewing distance of four feet corresponds to a total angular field width of 2.26 degrees.

A series of seven masks was used to provide separations of 0, 1, 3, 7, 12, 20, and 28 mm. The reflectance of the masks for forty-five degree illumination and normal viewing measured 4.5% on a Hunter Color and Color Difference Meter. The eight sample pairs on which the results are based range in reflectance from 33% to 80% as measured on the same instrument.

For each sample pair, 100 observations were made at each of the above separations. The succession of masks was

from maximum to minimum separation for each series. although random order was occasionally employed when repetitions of each series were undertaken as a check on the data.

For each set of 100 observations a point was plotted designating the number of errors versus separation. A smooth curve was drawn through the points for each sample pair when enough of the points were obtained to show regularity. During the course of the experiments, the error curves began to assume a characteristic form and the necessity of using all seven masks was obviated. Separations were then confined to 0, 3, 12, and 28 mm., with occasional checks using one or two other masks.

Since the equal category was excluded, a judgment of "equal" was relegated to one or the other of the push-buttons, resulting in an equal likelihood of a correct or an incorrect score. Since there is actually a greater possibility of correct than incorrect judgments when the observer is forced to guess (Boring, 1939), the observer attitude adopted was that one side of the test field was certainly darker than the other, whether visibly so or not, and that guessing would be repayed with a higher score of correct judgments. As a matter of fact it was found that after the observer had become accustomed to the liminal stimulus differences used, it was not difficult to envision a lightness difference, be it actual or imagined. The presence of imagined lightness differences where none

was actually seen was often verified by the large number of incorrect judgments which were scored on individual trials made for this purpose. The effect of imagination thereby largely reduced the need for an equal category, although this need was not entirely eliminated.

The observer was not limited as to length of observation for each judgment. It was found that a decision could be reached within a minimum period of about two seconds after the field was presented to the observer at the end of each cycle, this limit embracing the cases of real, imagined, or zero sensation differences. Observations were thus made at a fairly constant rate of about one every three seconds so that a set of 100 judgments could be made in about five minutes.

IV. RESULTS

Tests of either a preliminary or a final nature were made on more than sixty sample pairs, involving over 100,000 separate judgments. From the data, about thirty reasonably "good" error curves were available, the condition of "good" being fulfilled by those curves which showed a consistent increase in error with separation, fairly small spreads among percent errors at corresponding separations, and whose heights and shapes fell into some sort of logical order when curves for various $\Delta(A^{\frac{1}{2}})$ were examined.

From these curves eight have been selected as representative of the group and are here designated by the letters A through H. Using A equal to unity for 100% reflectance, the values of $\Delta(A^{\frac{1}{2}})$ for the corresponding sample pairs are given below:

Pair:	A	B	C	D	E	F	G	H
$\Delta(A^{\frac{1}{2}})$:	0.010	.014	.016	.016	.019	.020	.034	.041

In these curves, percent error was first plotted versus linear separation in millimeters. The linear values were then converted to angular separation in degrees on the basis of an average six foot viewing distance. A typical set of data, arising from sample pair D. is shown in Fig. 2 with error plotted versus angular separation, θ .

A better graphical illustration of the change in lightness discrimination with θ is perhaps gained by replacing the concept of percent error with the concept of perceptibility in percent. This can be done approximately if it is assumed that subliminal stimulus differences are equally likely to result in correct and incorrect judgments. A stimulus difference which is seen $x\%$ of the time in 100 trials will result in at least x correct judgments. The remaining $(100 - x)$ observations will then be distributed equally, statistically speaking, among correct and incorrect judgments. The number of errors, y , will be $\frac{1}{2}(100 - x)$ or, solving for perceptibility in terms of error,

$$x = 100 - 2y$$

or perceptibility in percent is equal to 100% minus twice the percent error.

The perceptibility thus computed is not an actual measure of the percentage of trials in which a stimulus difference will be correctly perceived. It is known that when the observer is forced to guess, his answers are more often right than wrong and will fall proportionately rather than equally among correct and incorrect judgments. However, in the ensuing treatment of the data, it will become evident that the above transformation of error to perceptibility is superfluous, it having been performed as an intermediate step for graphical reasons.

The data for sample pair D have been plotted as

perceptibility versus θ in Fig. 3 and may be compared with Fig. 2.

The assumption is now made that visual equality exists between all stimulus differences which yield equal values of perceptibility. As applied to the data, all combinations of $\Delta(A^{\frac{1}{2}})$ and θ which result in equal numbers of errors, implying equal perceptibility, constitute stimulus differences which are perceptually equal. Thus if the eight error curves referred to are superimposed on a single graph and a horizontal line is drawn on the graph at an arbitrary height, the intersections of the line with the curves will yield pairs of values of $\Delta(A^{\frac{1}{2}})$ and θ which combine to yield equal lightness differences. The next question is, "What is the magnitude of the lightness difference (in judds) to which these combinations are equal?" More generally speaking, the problem is a correlation of the perceptibility scale with the lightness difference scale so that the lightness difference corresponding to any abscissa crossing the curves will be known. The question might be stated: "What is the value of lightness difference in judds which will be seen correctly 25% of the time? 50% of the time? 75% of the time?" etc. In the absence of an independent answer to these questions, data arising in these experiments were relied upon, as well as the assumption that $k_1 = 120$ for zero dividing line width was valid under the testing conditions used. The data were examined with special attention

given to the results obtained with zero separation. From the data of 36 sets of 100 trials each, the percent errors were noted which corresponded to the various values of $\Delta(A^{\frac{1}{2}})$. The percent errors were transformed to percent percentibility through the relationship given on Page 32 in order to accord with the form in which the eight curves, A through H, have been plotted. It is at this point that any error due to the non-rigorous definition of percentibility cancels out, for it is ultimately the values of percent error gained directly from experiment that are relied upon. The values of $\Delta(A^{\frac{1}{2}})$ referred to above were converted to judds by the multiplying factor 120. The thirty-six pairs of corresponding values are given below:

ΔL judds	% Percen- tibility	ΔL judds	% Percen- tibility
0.30	0	1.92	82
0.36	0	2.24	96
0.64	12	2.39	94
0.70	40	2.39	78
1.03	74	2.39	88
1.03	38	2.39	82
1.08	54	2.39	98
1.08	64	2.39	94
1.08	2	2.39	98
1.10	64	2.48	95
1.24	74	2.60	96
1.68	84	2.63	94
1.91	72	3.12	97
1.92	80	3.20	84
1.92	92	3.58	96
1.92	68	4.04	100
1.92	72	4.97	100
1.92	76	5.21	100

The first 3/4 value pairs are plotted in Fig. 1. The variability of the observer is readily noticeable. The curve which is drawn through the points is taken as the criterion for relating perceptibility values to lightness differences in judgs. On the basis of this curve, the perceptibility scale is replaced by the lightness difference scale. Using the latter, the curves representing the eight sets of data. A through H, have been plotted in Fig. 4. Each curve indicates the manner in which perceptibility falls off with increasing separation, and the series of curves taken together gives some idea of how perceptibility falls off with diminishing $\Delta(A^{\frac{1}{2}})$.

Each of the curves is assumed to satisfy

$$\Delta L = k_1 \Delta(A^{\frac{1}{2}}) \quad (4)$$

where $k_1 = k_1(\theta)$. The information relating ΔL and θ is taken from Fig. 4 at seven points on each curve. The value of $\Delta(A^{\frac{1}{2}})$ for each corresponding sample pair is known from photometric measurements and the eight values are tabulated on Page 31. Since ΔL , θ , and $\Delta(A^{\frac{1}{2}})$ are known for each point, eight values of $k_1(\theta)$ can be found for each value of θ by satisfying Eq. (4). It is to be noted that the intersections of the eight curves with the vertical axis have been adjusted to accord with the smooth curve of Fig. 1. Solution of Eq. (4) for $\theta = 0$ will therefore yield no information other than $k_1 = 120$.

A typical set of equations to be satisfied, arising

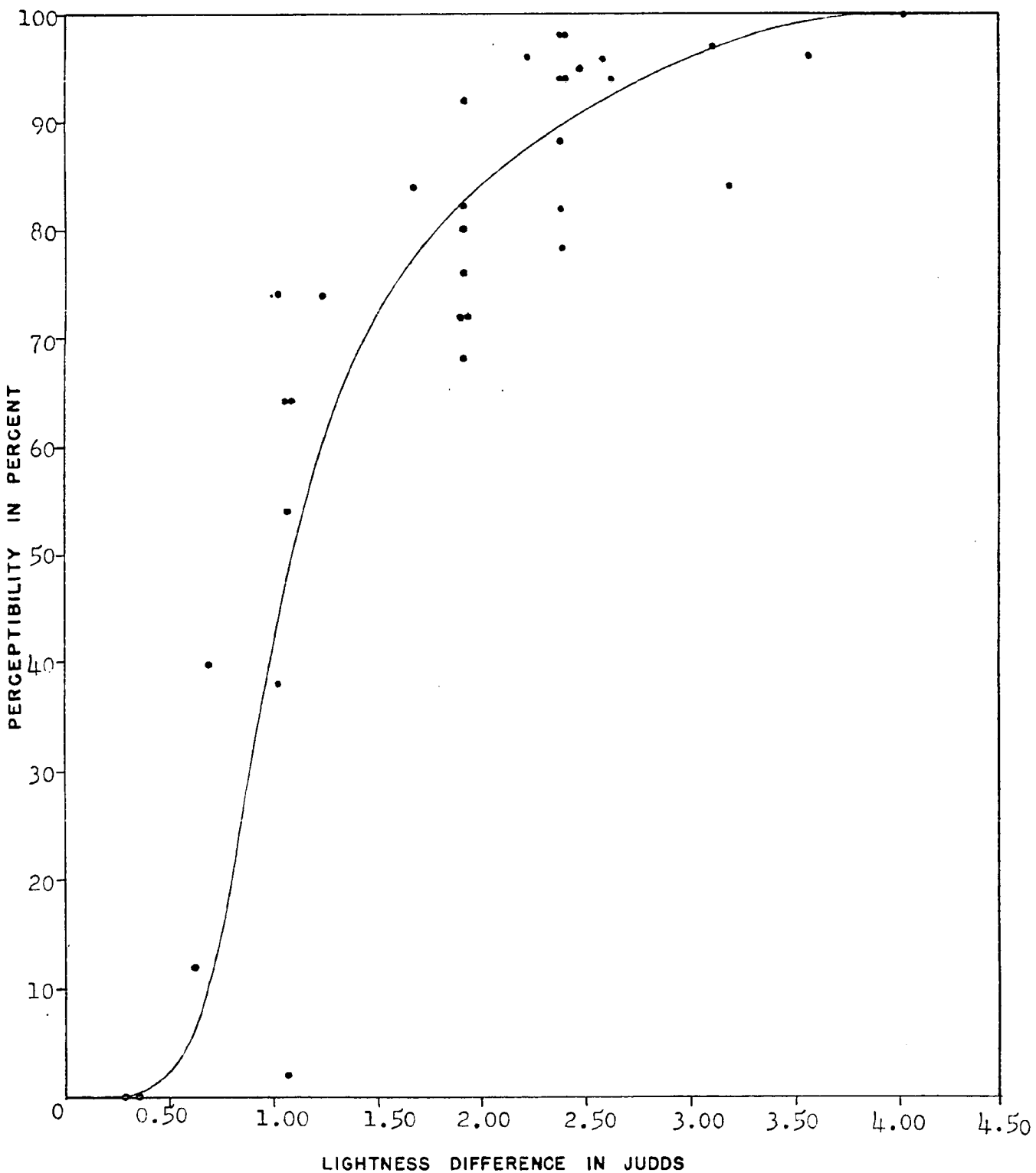


Fig. 1. Perceptibility vs. lightness difference at zero separation.

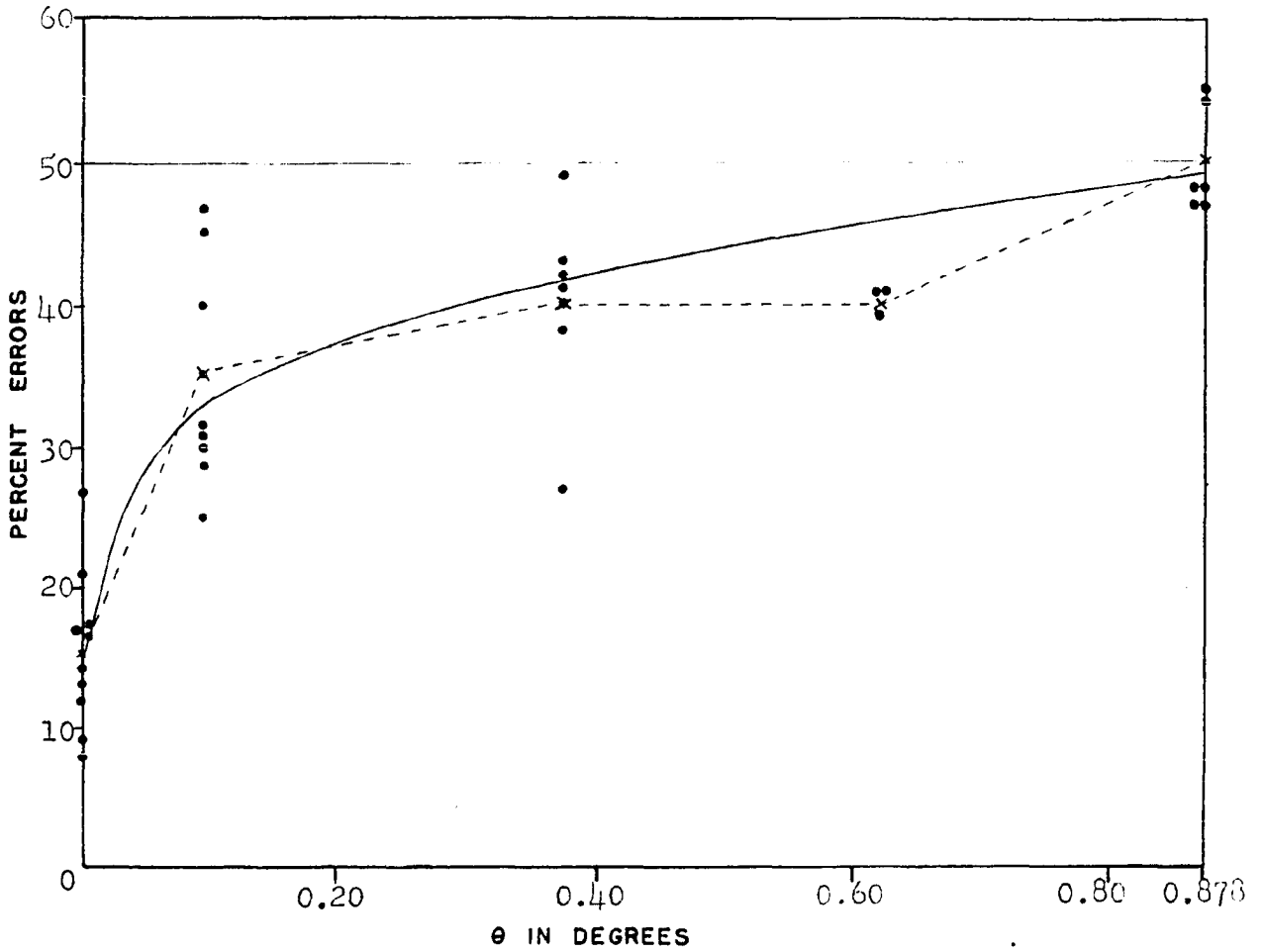


Fig. 2. Example of a well-behaved set of data, Set D. Percentage errors vs. angular separation θ . Each point represents 100 judgments. Crosses show averages at each value of separation.

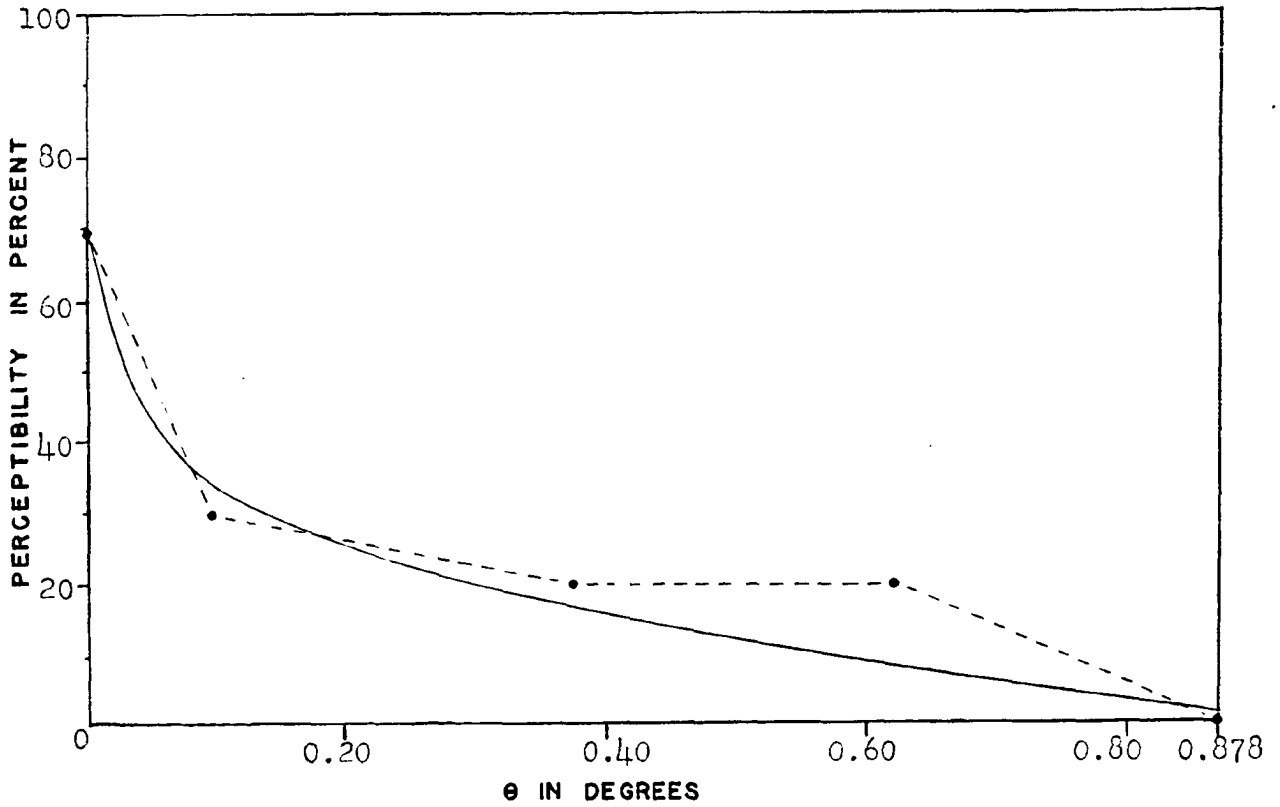


Fig. 3. Perceptibility vs. separation θ for Curve D, Fig. 2. Perceptibility is taken as $[100\% - 2(\text{percent error})]$.

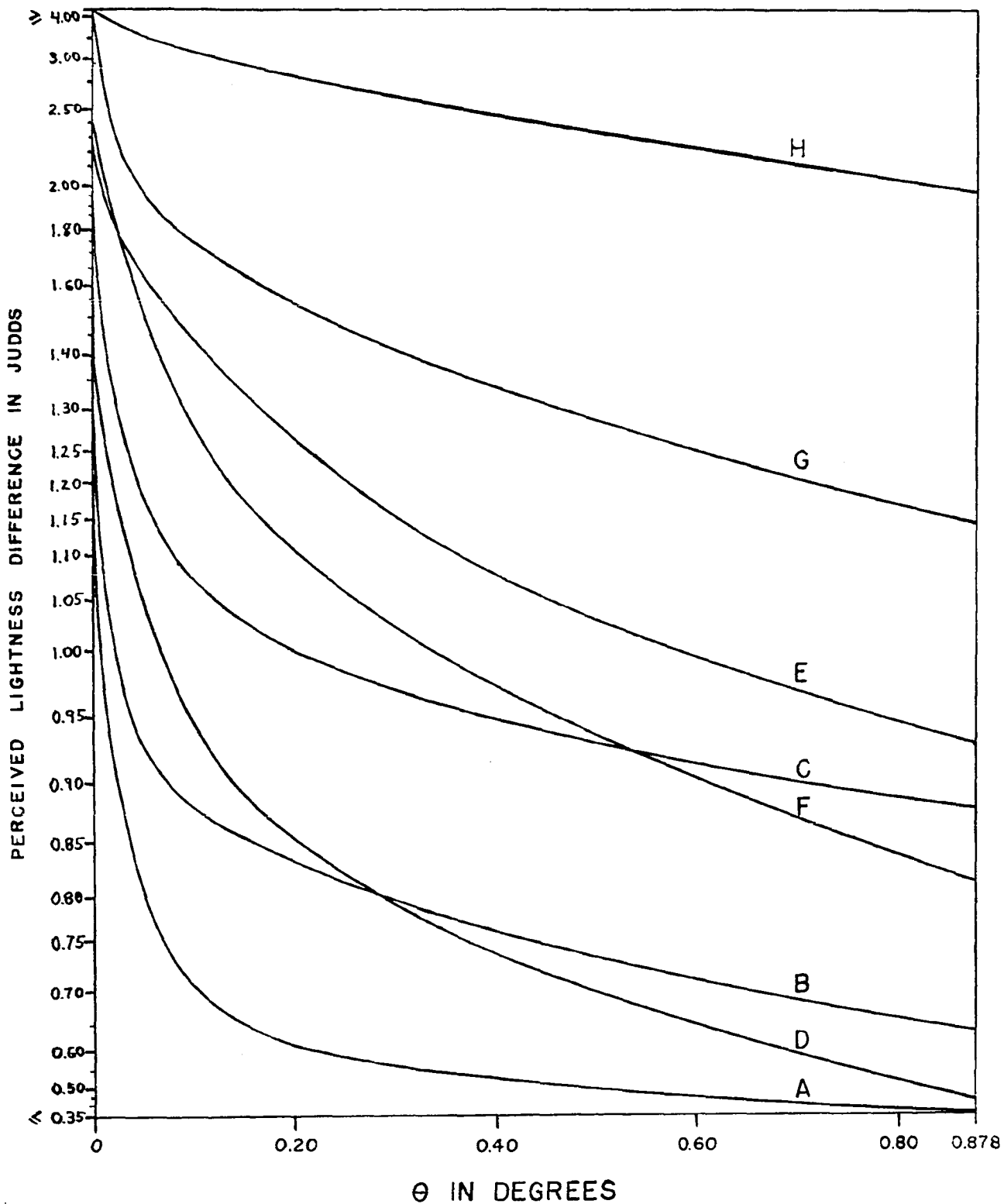


Fig. 4. Perceptibility vs. angular separation θ for eight sample pairs. Perceptibility is expressed in values of equivalent lightness difference as found from Fig. 1.

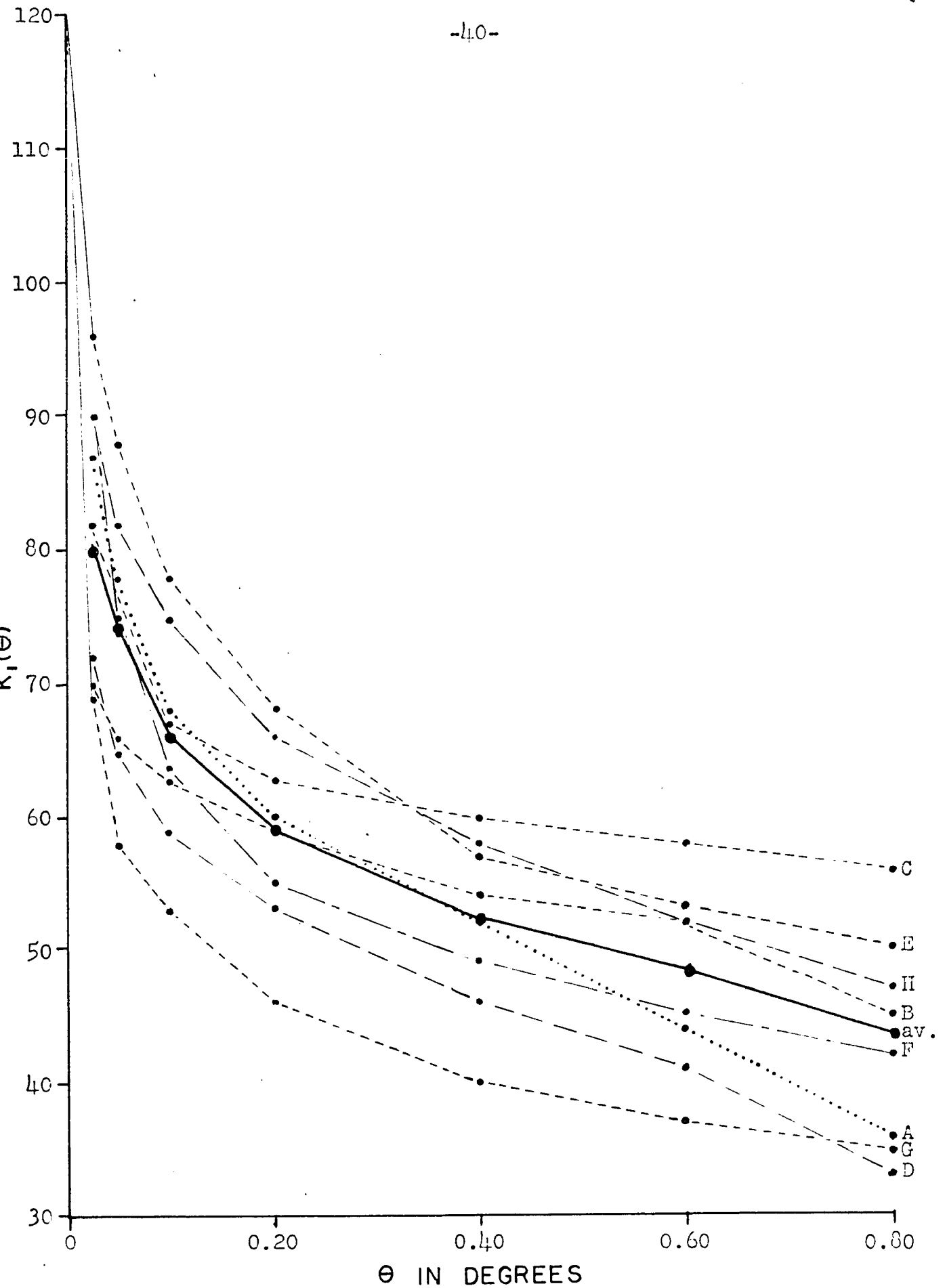


Fig. 5. Proximity factor $k_1(\theta)$ vs. angular separation θ assuming $k_1(0^\circ) = 120$. Shown for eight curves of Fig. 4. Solid line indicates numerical average of all curves.

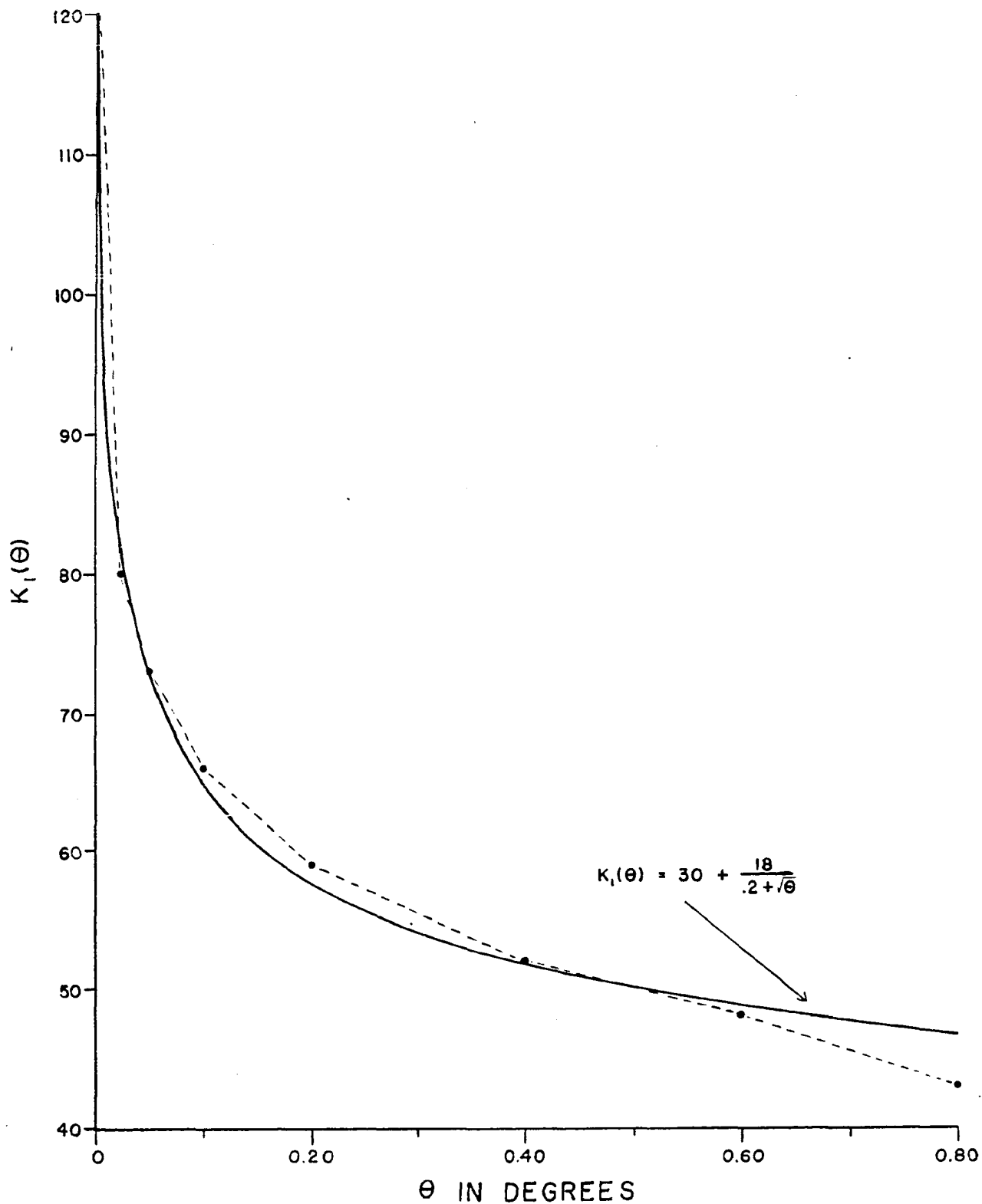


Fig. 6. Experimental (dashed) and theoretical (solid) curves for $k_1(\theta)$ vs. θ .

from Curve A for which $\Delta(A^{\frac{1}{2}}) = 0.0103$, is given below as well as the values of $k_1(\theta)$ vs. θ which result therefrom:

$k_1(.025^\circ)(.0103)$	= 0.90	judd	$k_1(.025^\circ)$	= 87
$k_1(.050^\circ)(.0103)$	= 0.80	"	$k_1(.05^\circ)$	= 77.6
$k_1(.100^\circ)(.0103)$	= 0.70	"	$k_1(.10^\circ)$	= 68
$k_1(.200^\circ)(.0103)$	= 0.63	"	$k_1(.20^\circ)$	= 60
$k_1(.400^\circ)(.0103)$	= 0.53	"	$k_1(.40^\circ)$	= 51.5
$k_1(.600^\circ)(.0103)$	= 0.45	"	$k_1(.60^\circ)$	= 44
$k_1(.800^\circ)(.0103)$	= 0.37	"	$k_1(.80^\circ)$	= 36

The proximity function is embodied in the second set of equations above. On the basis of Curves B through H, seven more sets of proximity data are found. The data from each curve are plotted and labelled in Fig. 5, the average of all data being indicated by the heavy line. The average data are shown again in Fig. 6 as dashed lines. The smooth curve is the locus of the analytic expression:

$$k_1(\theta) = 30 + \frac{18}{.2 + \sqrt{\theta}} \quad (7)$$

where θ is expressed in degrees.

V. DISCUSSION

A. Method

The method of constant stimulus differences employed here is a departure from the more customary method of minimal changes which is likewise used in investigations of liminal visual differences. In the latter case, one or more variable stimuli in combination are adjusted by the observer until the combination appears just (or just not) noticeably different from a given standard. The data are then interpreted according to the usual methods of statistical probability.

While the method of minimal changes is adaptable to investigations of aperture colors, which can be made continuously variable, the method used here is more suited to the study of surface colors, to which the Judd formula applies strictly. The test colors are presented in a mode of appearance similar to that for which the formula is intended. This mode of appearance precludes any provision for continuous variation of the stimulus at the will of the observer. Considering the stimulus difference provided by a given sample pair at a given separation, then as the stimulus difference decreases to subliminal values with increasing separation, no provision is made for the observer to increase the reflectance difference until the stimulus difference is again noticeable. Rather, he must indicate its relative magnitude as is, in terms of errors of identification.

The tendency of test procedures in recent years, in the investigation of psychophysical phenomena, has been toward a closer approach to actual conditions encountered in practice. An attitude shared by many investigators is summarized by Dimmick (1943) in speaking of color vision tests: ". . . practice usually demands that the test setting bear an obvious relation to the practical problem to which the test results are to be applied. This factor militates against laboratory techniques and apparatus that would otherwise make excellent tests of color vision."

The test method adopted here is a compromise between the generality of actual conditions and some of the specialization from which laboratory conditions cannot escape if the test conditions are to be at all reproducible. The use of surface colors rather than aperture colors is a sine qua non in this case, even though the specialized conditions of 45 degree illumination, perpendicular viewing, and dark surround may detract from the strictly surface appearance of the samples. Some restriction of generality is introduced by the small field sizes and the limitations of viewing distance. Actual conditions would allow the observer greater freedom of examination. However, the observer's freedom of vision was not otherwise hampered, as would be the case with the head or chin supports, monocular vision, or artificial pupils which are often characteristic of eyepiece instruments. The abrupt presentation and interruption of the stimuli, peculiar to this experiment, interspersed with one second waiting periods,

are certainly a deviation from actual conditions. It is presumed that these factors, as well as prolonged fixation, contribute to the effect of fatigue toward the end of a long session. Successive stimulation, on the other hand, is known and was observed here to increase the acuity of the observer for the first half hour or so of each session. The effects of fatigue and practice, however, are averaged out with the well known observer fluctuations due to unknown causes (Woodworth, 1938, pp. 394-5). The difference threshold is a transitory phenomenon, varying from moment to moment, and is influenced variously by fatigue, practice, systemic and metabolic conditions, and indeterminate causes. The most minute stimulus difference will sometimes elicit a response if it is presented at a moment when the observer happens to be favorably disposed toward perceiving it. It is only through the averaging of large numbers of data that the relationship between psychophysical stimulus and sensation has any meaning.

B. The Proximity Function

The analytic expression which fits the data,

$$k_1(\theta) = 30 + \frac{18}{.2 + \sqrt{\theta}} \quad (7)$$

is seen to yield values of $k_1(\theta)$ ranging from 120 to 30. The upper limit is predetermined by Dr. Judd's maximum value of 120 for k_1 and the present experiment does not allow an independent determination of this quantity.

The lower limit is based on the present data and is thus determined independently. In practice the value of $k_1(\infty) = 30$ has no meaning, but a lower limit of 40 is reached before θ increases to 2.6 degrees. The magnitude of the lower limit accords with Judd's estimate of $k_1 = 30$ or 40 for samples separated by a broad patterned area. The agreement may be accidental since there are several differences between the present experimental conditions and actual conditions for which the Judd formula

$$[\Delta E]^2 = [k_1 \Delta(A^{\frac{1}{2}})]^2 + [k_2 A^{\frac{1}{4}} \Delta S]^2 \quad (6)$$

is intended.

The perceptibility of a lightness difference is known to be influenced by background contrast. Given a certain reflectance difference, any specification of the lightness difference which will be experienced by an observer must include reference to the lightness of the background. Such reference is included in the Judd assumption that lightness varies with $A^{\frac{1}{2}}$, for it is stated explicitly that the relationship is most accurate for white and light grey backgrounds. As background reflectance diminishes, the lightness difference between a pair of light samples diminishes also. Lightness differences for the present samples were computed on the basis of $L \propto A^{\frac{1}{2}}$. An empirical formula given by Godlove (1933) predicts that for middle grey backgrounds of 19% reflectance, L varies as $(1.47A - 0.47A^2)^{\frac{1}{2}}$.

The backgrounds used in the present investigation showed a 4.5% reflectance. Five of the samples on which results are based showed reflectances exceeding 70%. The calculated lightness differences, assuming a light background, may be assumed to be somewhat larger than those actually perceived against a dark background.

At large separations, lightness perception is influenced by the uniformity of the background as well as its lightness. A conspicuous patterned area between samples will reduce lightness differences more than a uniform background of the same size. This consideration suggests that the value of $k_1(2.6^\circ) = 40$, as predicted by Eq. (7) for a uniform background, will be somewhat reduced by substitution of a patterned area of the same width.

The limitation of field width to two inches at six feet is a departure from the usual comparison of larger samples at the normal viewing distance, for which Judd's formula is intended.

Another limitation imposed by experiment is the small range of $\Delta(A^{\frac{1}{2}})$ which was adaptable. While Fig. 5 shows no orderly change in the proximity function as $\Delta(A^{\frac{1}{2}})$ increases from 0.01 for curve A to 0.04 for curve H, there is no assurance that the derivative of k_1 with respect to $\Delta(A^{\frac{1}{2}})$ will remain zero for much larger lightness differences.

The data reported here as well as all previous data showed no orderly variation in lightness discrimination

as a function of dominant wavelength or purity of samples. On this basis, Eq. (7) is equally applicable to samples of various chromaticities.

C. Chromaticity Discrimination

The error curves obtained in the experiments indicate definitely that the perception of lightness differences depends upon spatial proximity. Exploratory experiments were conducted with samples differing slightly in chromaticity alone to determine how chromaticity discrimination was affected by proximity. The conditions of experiment were identical to those of the lightness discrimination studies, although less data were taken. Four sample pairs were investigated under various conditions of illumination and viewing distance, and nine sets of error data were obtained. Three of these sets, which are representative, are shown in Fig. 7 where they are plotted as percent error versus angular separation. Within the limits of observer fluctuation, no consistent change in chromaticity discriminability as a function of separation is indicated. Comparison of Fig. 7 with Fig. 2, which is typical of the lightness difference curves, shows a striking dissimilarity between the perception of lightness differences and the perception of chromaticity differences as separation varies.

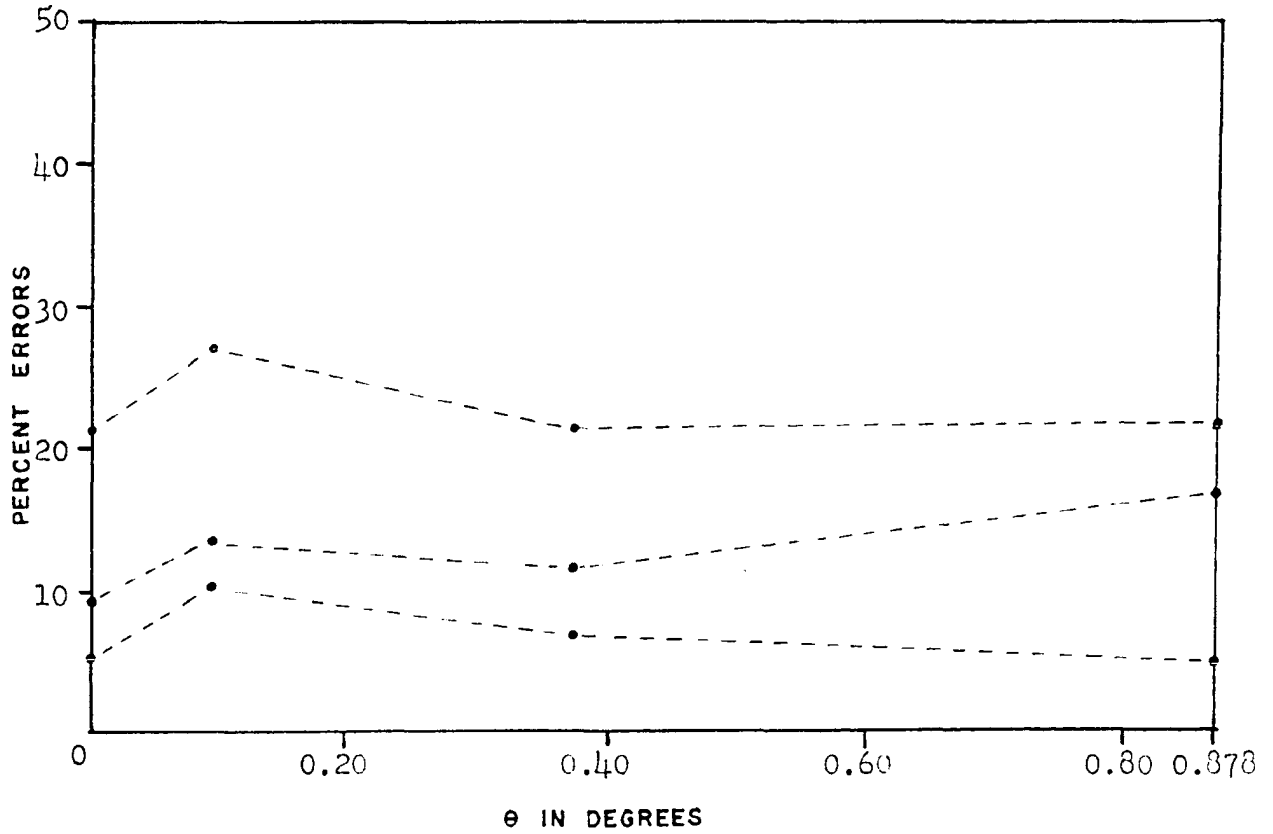


Fig. 7. Typical discrimination data for chromaticity differences alone. Percentage of errors vs. angular separation θ .

In the latter case, it was expected that some change, however small, would be noted (Judd, 1930). It is possible that the expected change has been obscured by observer fluctuation or other conditions of experiment.

A singular feature of the data for chromaticity discrimination is a tendency for the percent error values to reach a peak at the first non-zero value of separation used, that of 3 mm. which corresponds to 0.094 degree at six feet.

It is to be noted that any influence of separation on chromaticity discrimination would have to be accounted for by the factor k_2 of Eq. (6) and would not affect the behavior of the proximity factor for lightness, k_1 .

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