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I hereby recommend that the thesis prepared under my
supervision by Frank Lerman
entitled Studies on Liquid Distribution
in Packed Towers

be accepted as fulfilling this part of the requirements for the
degree of Doctor of Philosophy

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STUDIES ON
LIQUID DISTRIBUTION
IN PACKED TOWERS

A dissertation submitted to the
Graduate School
of the University of Cincinnati
in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

1938

by

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To the Faculty of the Graduate School
University of Cincinnati
Cincinnati, Ohio

Gentlemen:

I herewith submit a thesis entitled "Studies on Liquid Distribution in Packed Towers" in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

The object of this investigation was to study, theoretically and experimentally, the distribution of liquid over tower packing. Such information would aid in the understanding of packed tower operation for absorption, rectification, and other gas-liquid interchange operations.

Distribution equations were mathematically derived from the Binomial and the Normal Laws of Frequency Distribution to express liquid spread over tower packing, with and without wall effects, for center and for area feed. Based on these equations and on experimental data from sand and liquid distribution set-ups, the effects of the variables of packed tower construction and operation were discussed. Recommendations were made for obtaining the best liquid distribution for packed tower operation.

Respectfully submitted,

Frank Lerman

Frank Lerman

Acknowledgement

The writer wishes to express his appreciation for the inspiring guidance given this work by his research advisor, Professor R. S. Tour, Head of the Department of Chemical Engineering.

Abstract

A set of equations, mathematically derived from the Normal and Binomial Laws of Frequency Distributions, are presented expressing liquid distribution with and without wall effects.

Experimental runs, using a specially designed sand distributing model and a 20" coke packed tower, six feet high, were used. Results from these runs served as experimental checks for the basic equations of unbounded distribution from center feed and gave insights into distribution from a uniform source and into the wall effects on distribution.

The effects of the various factors of packed tower construction and operation on liquid distribution are discussed in detail. Recommendations, as to operating conditions, type of packing, and on the use of redistributing devices, are made for obtaining the most effective liquid distribution.

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Introduction

The problem of getting economic transfer of material components between gaseous and liquid phases arises in several important unit operations such as absorption, rectification, stripping, and washing. To accomplish such an interchange of materials efficiently it is necessary to get intimate contact for a sufficient length of time between large areas of the interphase surface.

Plate columns and packed towers meet these requirements for interchange, and both have widespread application in industry. Plate columns produce an intimate contact between large areas of the interphase surfaces primarily through the use of bubble caps which break up the gas into small bubbles as they direct it into the liquid on each plate. Here the operation is a controlled one as both the gas and the liquid are directed by proper equipment design into paths best suited for the interchange.

On the other hand, packed towers produce the intimate contact and the large exposed surface by counter-current flow of the two phases through an enclosure filled with a large number of fragments or units of some inert solid, the packing. The interchange surface cannot be readily controlled since the liquid on flowing downward

over the packing picks out favored paths or, at best, moves randomly through the packing. Under such unregulated conditions, the surface that the liquid presents is not likely to be the most effective for interchange.

Many forms of patented packings for towers have been designed. Practically all are judged as to how well a single unit of packing, or contingent group of packing units, spreads the portion of the liquid meeting it rather than on how uniformly the packing as a whole spreads the liquid through the tower cross-section. In fact, it has been generally believed in all practical operations and in most research on packed towers that the liquid is distributed uniformly through the packing of the tower except for accidental irregularities of flow known as channelling and occurring only in specific cases. In all design equations for packed towers, it has been assumed, usually tacitly, that the liquid distribution is uniform.

Previous Investigations

The uniformity of liquid distribution in packed towers has been questioned by several investigators, and in general, conclusions have shown this assumption to be incorrect.

In 1893, Hurter concluded, from runs made on a large coke-packed tower with water distributed from a center feed, that:

"...the water spreads over a circular area of about 39 inches diameter in descending through a column of coke 14 feet high, and has nearly reached that distribution after a descent of 4 feet of coke. The distribution is not uniform over the whole area of the circle, the central portions receive rather more per square inch of water than the circumferential part." (9)

After this initial study and up to the last ten years, little research was done on this subject. A series of investigations starting in 1931 and carried out by Kirschbaum and Weimann (10) (18), showed that the liquid from a point feed spreads in a cone down the packing and toward the wall and results in high liquid flows at the wall and non-uniform distribution through the tower cross-section.

These investigators concluded that increased size of packing units increases both the liquid concentration on the wall and the tendency for channelling, that increased flow rates produce only proportionate changes

in the distribution, that increased tower diameters give a decreased difference between center and wall flows, and that after a given height the distribution remains practically constant. Weimann has indicated that a ratio of 25 to 1 for tower diameter to size of packing is desirable. On the basis of such studies, Kirschbaum has developed and patented (11) a redistributing device for correcting this non-uniformity of liquid spread.

After extensive research at the Du Pont experimental Station, T. H. Chilton and his co-workers have concluded that:

"....in sufficiently large packed towers, the tendency for liquor flow to concentrate near the walls, which is quite marked in small towers packed with relatively coarse material, is absent, and that uniform distribution, when once attained, will persist down to any reasonable depth of packing." (1)

Based on this and later research (4) they concluded that redistribution does not appear to be necessary but that adequate initial water distribution is essential to secure maximum absorption efficiency, that there is no definite effect of depth in the range of 8 to 16 tower diameters, and that there is no effect of the tower diameter on the absorption efficiency of the packing when the ratio of the tower diameter to packing diameter is 8 to 1 or greater.

Thormann (17) in his text on distillation and rectification stated that the distribution approximately followed the normal Probability Curve. With this in mind, Scott (16) approached liquid distribution from the probability concept and used a modification of Galton's apparatus for obtaining the distribution of shot falling downward over obstructions. The curves from this apparatus bear distinct resemblance to the complete set of liquid distribution curves that Scott obtained from various sizes of towers packed with $\frac{1}{2}$ " coke, $\frac{1}{4}$ " coke, or $\frac{1}{8}$ " Lessing rings. He concluded that the effect of increasing the height of packing to the tower diameter was to increase the spread of liquid to the walls, the rate of increase of the spread being greatest for the $\frac{1}{8}$ " ring, least for the $\frac{1}{4}$ " coke, and intermediate for the $\frac{1}{2}$ " coke. For all three packings the channelling was most marked.

In the early stages of our work on liquid distribution at the University of Cincinnati, experimental curves indicated that there was a regular trend in the liquid distribution in a packed tower which appeared to be in accordance with a natural law rather than haphazard or due to chance channelling. After a mathematical study, the distribution trend appeared to be a probability function. In the ensuing investigation, presented as a Master's thesis (12), mathematical equations expressing distribution where there is no wall effects were derived. The data obtained

by Hurter (9) was used for experimental verification of the equations. Unbounded distribution was then expressed as a probability function in terms of the radial displacement from the tower centerline, the height of the tower packing, and size of the packing.

Distribution runs were made at the various liquid flows in 5' and 10' lengths of a tower 10" in diameter, packed with 1" coke, and under both center and uniform feed. This bounded distribution appears to be expressible in terms of a probability function except for the distribution near the walls. At the walls the flow rate was high ranging from about 30% of the total feed rate of .5 cu. ft. per min. to about 10% of the total feed rate of about 4 cu. ft. per min. Center feed gave the lower wall flows as did the shorter lengths of tower.

A redistributing device was recommended for correcting the non-uniformity persisting in most cases of packed tower operation.

Later investigators at the University of Cincinnati have made additional distribution runs on towers 10" and 20" in diameter (5) (14).

The Mathematics of Unbounded Liquid Distribution

In the Master's thesis (12), it was shown that the spread of liquid from a point feed through unbounded packing was a frequency distribution expressible by the Normal Law of Distribution.

Any of the several methods used to derive the Normal Law may serve as a basis for the derivations of equations for liquid distribution, but for our investigations a modified form of Hagen's derivation (13) was found most suitable. This derivation is based on the assumptions that a final accidental deviation is the summation of a large number of ultimate, minute deviations, each equally likely of occurring in either of two opposing directions. The final deviation may be a plus or minus error in a measurement, the winnings in coin flipping, the horizontal displacement of a fired projectile to the left or right of the target center, or the final displacement of a liquid portion to the left or to the right of the center line through the feed point. On such assumptions the Binomial Law of Distribution is set up from which the Normal Law is derived mathematically as an approximate form.

Hagen's method and the analogies it permits was found more helpful than the other derivations in understanding the Normal Law when applied to liquid distribution and in extending the law to apply to the various conditions found in actual tower operation.

However, other derivations have been equally effective in applying the requirements of the Normal Law to actualities. Outstanding examples in Physics are Maxwell's equation for the distribution of velocities of molecules in any perfect gas and Einstein's equations for the displacement of a particle due to Brownian movement (6), (7).

The Norman Law as applied to liquid distribution is expressed by the equation

$$q_x = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \quad (\text{I})$$

Here q_x is the fractional part of the total feed from the center source that on distribution will be displaced a horizontal distance x from the centerline and is expressed as a fraction per unit increment of x ; h^2 is the constant of distribution for a given type and arrangement of packing units and for a given height of packing.

The distribution of liquid in the y direction, the horizontal direction perpendicular to the x direction, is expressed by

$$q_y = \frac{h}{\sqrt{\pi}} e^{-h^2 y^2} \quad (\text{II})$$

where the symbols are the same as in equation I except that all horizontal displacements are considered parallel to the y axis.

A requirement of the Normal Law is that the x component of a displacement does not in any way effect the y component of the displacement. This requirement permits the application of the multiplication theorem of probability for the simultaneous occurrence of independent events. Multiplying together equations I and II on the basis of this theorem gives an expression for the fraction of the total liquid feed that will have a final displacement of both x and y or the fraction per unit area that will fall at (x, y) ; that is

$$q_x \cdot q_y = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \cdot \frac{h}{\sqrt{\pi}} e^{-h^2 y^2} = \frac{h^2}{\pi} e^{-h^2(x^2+y^2)} \quad (\text{III})$$

Since $x^2 + y^2 = R^2$ where R is the horizontal radial displacement from the centerline, equation III becomes

$$q_R = q_x \cdot q_y = \frac{h^2}{\pi} e^{-h^2 R^2} \quad (\text{IV})$$

In the derivation of the Normal Law, it was shown that $h^2 = \frac{1}{2S\Delta^2}$ where S is the number of displacements the liquid portions undergo on encountering packing in the downward path and Δ is the average horizontal length of the displacement in either the $+x$ or $-x$ directions (or in the $+y$ or $-y$ directions). Here S and Δ are constant for a given packing set-up where the liquid feed rates are low enough so as not to fill the interpacking crevices appreciable. At these feed rates S is proportional to $\frac{L}{D}$ and

Δ is proportional to D , where L and D are the height of packing and the diameter of a packing unit, respectively. Then $h^2 = \frac{C}{LD}$ where C is a constant dependent only on the arrangement and the shape of the packing.

At high feed rates, because of unequal plugging effects of the liquid in the interpacking crevices at the various parts of the tower, h^2 becomes a function of the flow rates, of the displacement χ , and of the physical constants of the packing and of the liquid used.

In terms of the packing dimensions the equation for radial distribution (IV) applicable to low flow rates becomes

$$q_R = \frac{C}{\pi LD} e^{-\frac{C}{LD} R^2} \quad (V)$$

where q_R is the fraction of the total feed that will fall on a unit area, or the fractional concentration, at a distance L downward and a distance R radially and horizontally outward from the point feed.

The fractional concentration at R fed from a point source, displaced a horizontal radial distance r from the center line at the top of the packing, may be derived by use of the cosine law of trigonometry to give the equation

$$q_R = \frac{h^2}{\pi} e^{-h^2(R^2 + r^2 - 2Rr \cos \theta)} \quad (VI)$$

where θ is the horizontal angle between the radius of the displacement R and the projection of the radius r upon the horizontal plane through R .

The liquid concentration at R for distribution from an annular feed may be set up from equation VI in the form of an integral

$$Q_R^\odot = \int_{r_1}^{r_2} \int_{\theta=0}^{\theta=2\pi} \mu \frac{h^2}{\pi} e^{-h^2(R^2+r^2-2Rr\cos\theta)} r d\theta dr \quad (\text{VII})$$

where $Q_R^\odot$ is the actual quantity per unit area at R due to feed from an annular feed area bounded by concentric circles of radii r_2 and r_1 . Here μ is the liquid feed per unit area or the feed concentration at the annular source, and it may or may not be a function of r or θ .

In the following integration μ is assumed to be a constant throughout the annular area. Integration of equation VII could only be accomplished by expanding the factors of the integral into a product of two power series and integrating the product term by term. A complex but converging series is obtained, expressible in condensed mathematical form as

$$Q_R^\odot = \mu e^{-h^2 R^2} \left\{ -e^{-h^2 r_2^2} + e^{-h^2 r_1^2} + \sum_{n=1}^{\infty} \frac{(hR)^{2n}}{(n!)^2} \left[\sum_{k=1}^{\infty} \frac{h^{2(n+k)} (r_2^{2(n+k)} - r_1^{2(n+k)})}{(n-k)[(k-1)!]} \right] \right\} \quad (\text{VIII})$$

For circular feed, the special case of annular feed where

$r_2 = r$ and $r_1 = 0$, equation VIII becomes

$$Q_R^{\odot} = \mu e^{-h^2 R^2} \left\{ -e^{-h^2 r^2} + 1 - \sum_{n=1}^{\infty} \frac{(hR)^{2n}}{(n!)^2} \left[\sum_{K=1}^{\infty} \frac{(hr)^{2(n+K)}}{(n-K)!(K-1)!} \right] \right\} \quad (\text{IX})$$

For small values of h^2 and of the radii, the summation terms are negligible. Then

$$Q_R^{\odot} = \mu e^{-h^2 R^2} (-e^{-h^2 r_2^2} + e^{-h^2 r_1^2}) \quad (\text{X})$$

and

$$Q_R^{\odot} = \mu e^{-h^2 R^2} (-e^{-h^2 r^2} + 1) \quad (\text{XI})$$

In such case, the terms in the parantheses are constant when the area of the feed is fixed. Therefore, for uniform annular or circular feed,

$$Q_R = \mu K e^{-h^2 R^2} \quad (\text{XII})$$

where K is constant for a given feed area.

Thus the Normal Law of Distribution again appears in a modified form as an equation for liquid distribution from an area feed.

The Mathematics of Bounded Distribution

In the Master's thesis (12), the theory of bounded distribution was discussed only in a qualitative manner since mathematical expressions for such distribution had not been derived at that time. In that discussion, a tower wall was conceived as having two effects on liquid distribution, one effect collective and the other redistributive.

The irregular, curved surfaces of the boundary packing, resting against the smoother and less curved surfaces of the wall, form crevices or openings whose sizes and shapes depend on the size and the arrangement of the packing units and on the curvature of the wall. These boundary crevices permit ready flow down the wall. Liquid portions reaching the wall will tend to cling to it and to run down it at a more rapid rate than the portions would through the packing itself. This retention on the wall of liquid portions distributed to it is denoted as the collective effect of the wall.

Since the liquid quantity at any region on the packing depends on liquid feeds from all sides above and adjacent to the region, the decrease in feed from the wall side due to this collective effect will result in a decrease in that liquid quantity. The decrease in liquid quantities on the packing due to this wall effect becomes progressively less as one approaches the centerline of the tower and progres-

sively greater as one moves down the tower.

In the redistributive effect, liquid flowing down the wall is shunted back to the packing by the packing units adjacent to the wall. As the liquid collects on the wall, it begins to fill the boundary crevices and to increase the redistributive effect. This effect opposes the collective effect since it increases the liquid quantities on the packing, progressively less as one approaches the centerline of the tower and progressively greater as one moves down the tower.

At the upper part of the tower, the collective effect is most prominent, but as the liquid moves downward, the redistributive effect compensates for and soon overbalances the other effect. The closer the wall to the feed source and the larger the size of the packing units, the greater will be the collective effect. The smaller the packing size, the sooner will the redistributive effect come into play.

The special case of bounded distribution where there is no liquid return from the wall is first considered mathematically. The redistribution effect is zero, and the difference between the liquid concentration on the packing due to bounded distribution and that due to unbounded distribution will be a measure of the collective effect of the wall.

In the concluding parts of the Master's thesis, a Binomial equation for distribution from a feed a fixed distance from a straight boundary wall was developed through

analogy with the following coin problem:

The probability that a person, gambling with unlimited funds, will attain a given win (or loss) after a given number of flips of a coin is expressed by the following Binomial Law of Distribution:

$$p_x = C_m^S \left(\frac{1}{2}\right)^S \quad *$$

where S is the number of flips of the coin and m is the number of losses in S flips which result in the given win (or loss).

If his funds are limited, the decrease in his chances of attaining that given win (or loss) is expressed by the modified Binomial equation

$$p_w = C_{m+w}^S \left(\frac{1}{2}\right)^S \quad *$$

where w is the excess of losses over wins he has to attain to go broke in S flips or less (19).

Reworded for the conditions of liquid distribution, the analogy becomes complete:

The probability that a liquid portion, moving downward in an unbounded region, filled with packing, will attain a given horizontal displacement from its source after a certain number of horizontal shifts either to the left or right is expressed by the Binomial Law

$$p_x = C_m^S \left(\frac{1}{2}\right)^S$$

* C_m^S is identical with the factorial quantity $\frac{S!}{(S-m)! m!}$

C_{m+w}^S is identical with the factorial quantity $\frac{S!}{(S-m-w)! (m+w)!}$

where S is the number of horizontal shifts either to the left or right of the liquid portion in its path down the tower and m is the number of shifts to the left in S shifts which are required for the given final displacement.

If the distribution is bounded by a straight wall at the left, the decrease is the chances of the liquid portion attaining the given horizontal displacement is expressed by the modified Binomial equation

$$p_w = C_{m+w}^S \left(\frac{1}{2}\right)^S$$

where w is the excess of left shifts over right shifts that the liquid portion must receive in order to attain the wall in S shifts or less.

The probability that the liquid portion attains a given displacement is equal to the fraction of all the portions from the feed source (the fraction of the total feed) that will actually reach this displacement.

Then the Binomial equations are expressed as liquid fractions, distributed discontinuously and concentrated at points of 2Δ intervals in the direction of the displacement, where Δ is the horizontal displacement either to the left or right per shift. The final displacement, if only directions parallel to the X axes are considered, would be at $x = (S - 2m)\Delta$, and the wall in the bounded case would be at $x_w = W = w\Delta$.

A continuous Normal equation was obtained from the Binomial Law for unbounded distribution by means of a modification of Hagen's derivation (13). Only recently,

however, has an equivalent Normal equation been derived for bounded distribution (see Appendix I for this derivation). This Normal equation for liquid distribution bounded by one straight wall from which there is no liquid return is

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} - \frac{h}{\sqrt{\pi}} e^{-h^2 (2W-x)^2} \quad (\text{XIII})$$

where q is the fractional quantity per unit increment of x at the displacement x , and where W is the distance from the feed source to the straight wall perpendicular to the axis. The feed to obtain such a distribution may be from a point source at $x=0$ or from a horizontal line source at $x=0$ and parallel to the wall.

The fractional quantity q in equation XIII is expressed by two terms, the first, the fractional quantity at x for unbounded distribution, and the second, the decrease in the fractional quantity at displacement x due to the collective effect of the walls.

The more general approach to the mathematical expression for bounded distribution, as used in the derivations below, became obvious only after studying the significance of equation XIII.

Consider in Figure 1, the Normal distribution curve (q_x, q_x, q_x) set up by distribution from feed at $x=0$ and down through an unbounded region in accordance with the equation $q_x = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$.

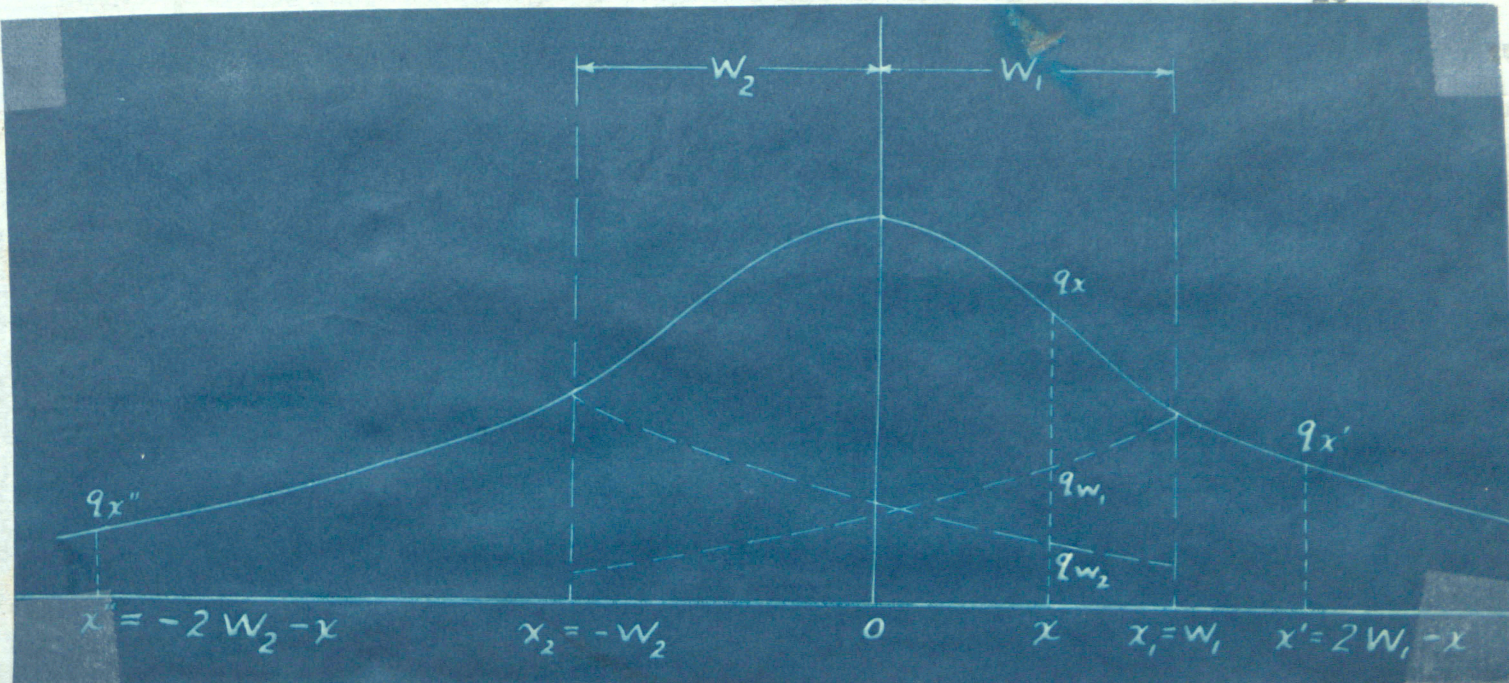


Figure 1. The Wall Effects on the Unbounded Distribution Curve

Consider, also, a liquid portion P which attains at a given instant during such distribution a given displacement, say $x_1 = W_1$. A certain fraction of that portion, say αP , will attain at the end of the distribution process a final displacement at some point $x' = 2W - x$ to the right of W .

Since a quantity distributes itself symmetrically to the left and to the right from any point at which it may be temporarily located, that same fraction, αP , of the quantity P at $x_1 = W_1$, will attain a final displacement $x' = W_1$ to the left of W_1 or at x . Of the totaled quantities attaining at any time during the process of distribution the displacement $x_1 = W_1$, a fraction q' will attain a displacement x' and an equal fraction q_{w_1} will attain a displacement x .

Let a barrier which collects without further distribution all quantities displaced to it be introduced

at W_1 . This barrier will collect the quantities q' and q_{W_1} , and will diminish the final quantity at x from q_x to $q_x - q_{W_1}$, or by the amount $q_{W_1} = q_x$. But q_x from the unbounded distribution curve is expressed by $\frac{h}{\sqrt{\pi}} e^{-h^2(x)^2}$. Therefore, with a barrier at W_1 , the fractional quantity at x per unit increment of x is

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} - \frac{h}{\sqrt{\pi}} e^{-h^2 (2W_1 - x)^2}$$

$$= \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left[1 - e^{-4h^2 W_1^2} e^{4h^2 W_1 x} \right]$$

which checks equation XIII.

In place of the barrier in the previous case, let there be introduced a barrier which immediately returns for redistribution a fraction f of all liquid portions attaining it. The barrier will collect the quantities q' and q_{W_1} , but will return the quantity $f(q' + q_{W_1})$. As a result, it will diminish the final quantity at x by q_{W_1} and increase it by $f(q' + q_{W_1})$, a net increase of $f(q' + q_{W_1}) - q_{W_1}$. Since $q' = q_{W_1}$, the net increase is $2fq' - q' = (2f - 1)q'$. Setting q' equal to $\frac{h}{\sqrt{\pi}} e^{-h^2 (2W - x)^2}$ gives for the quantity at x due to the barrier, the expression

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} + (2f - 1) \frac{h}{\sqrt{\pi}} e^{-h^2 (2W - x)^2}$$

$$= \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left[1 + (2f - 1) e^{-4h^2 W^2} e^{4h^2 W x} \right] \quad \text{(XIV)}$$

where the fraction f expresses the degree of redistribution from the wall.

For two walls, one at a distance W_1 , to the right, and the other at W_2 to the left of the feed,

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left\{ 1 + (2f-1) \left[e^{-4h^2 W_1^2} e^{4h^2 W_1 x} + e^{-4h^2 W_2^2} e^{-4h^2 W_2 x} \right] \right\} \quad (\text{XV})$$

When $W_1 = W_2 = W$, that is, when the feed is centered between the two parallel walls,

$$\begin{aligned} q &= \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left[1 + (2f-1) e^{-4h^2 W^2} (e^{4h^2 W x} + e^{-4h^2 W x}) \right] \\ &= \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left[1 + 2(2f-1) e^{-4h^2 W^2} (\cosh 4h^2 W x) \right] \quad (\text{XVI}) \end{aligned}$$

In these equations for two walls, wall reflections of the second order or greater are neglected, that is, no liquid portion redistributed from the wall is assumed to reach the other wall for further redistribution. If these higher order reflections be considered, the equation XVI becomes

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2} \left[1 + 2 \sum_{i=1}^{\infty} (2f-1)^i e^{-4h^2 i^2 W^2} \cosh (4h^2 i W x) \right] \quad (\text{XVII})$$

where i is the order of reflection.

For a feed displaced a distance X from the center-line between the two walls at W and $-W$, the fractional

quantity at X is

$$q = \frac{h}{\pi} e^{-h^2(x-X)^2} + 2 \sum_{i=1}^{\infty} (2f-1)^i e^{-4h^2 i^2 W^2} e^{-h^2 [(-1)^i X - X]^2} \cosh\{4iW[(-1)^i X - X]h^2\} \quad (\text{XVIII})$$

Feed from an area between any two limits X_1 and X_2 within the walls gives distribution to a displacement in accordance with the integral of equation XVIII

$$Q_X^{X_1, X_2} = \int_{X_1}^{X_2} \mu \frac{h}{\pi} \left[e^{-h^2(x-X)^2} + 2 \sum_{i=1}^{\infty} (2f-1)^i e^{-4h^2 i^2 W^2} e^{-h^2 [(-1)^i X - X]^2} \cosh\{4iW[(-1)^i X - X]h^2\} \right] dX \quad (\text{XIX})$$

where μ is the feed quantity at X per unit increment of X , and it may or may not be a function of X .

When $f = \frac{1}{2}$, that is for 50% redistribution from the wall, all reflection terms in the bounded distribution equations for both centered and area feed drops out since the factor $(2f-1)$ becomes zero. Under such redistribution, the quantities within the wall limits is the same as though the distribution were unbounded.

When $f = 1$, or at 100% wall return, distribution starting as an area feed between $-W$ and W and uniform with respect to X remains uniform and constant throughout the length of the packing. The quantities reflected from

the wall can be considered as though they come directly from feed points beyond the wall limits were the distribution unbounded. The summation of these reflected quantities is as though the feed comes from a uniform source of infinite limits in unbounded distribution; such feed produces uniform distribution at any packing depth. Therefore, for this case, equation XIX is a constant equal to μ for all values of x between the wall limits.

The redistribution fraction f has been assumed constant. This would not be the usual case in actual operation. In general, f would vary in some manner from zero to unity as the liquid moves down the tower and as the wall flow increases.

In all the bounded distribution equations, the wall flow can be expressed as the total feed quantity minus the integral of the distribution equation between the limits set by the boundaries. The wall flows, expressed in this manner, can be made a function of L , the distance down the tower, since h^2 in the distribution equations is inversely proportional to L for low flows. The variation of f with L is then a function of the wall flow and perhaps could be evaluated through a study of the channel or orifice flow of the liquid at the wall through the crevices between the wall and the boundary packing.

An Extension for Bounded Distribution

The derivation of equations applicable to distribution bounded by cylindrical walls presents difficulties which have not as yet been overcome. The approach likely to be most effective for such a derivation is through a Binomial equation. This equation has not yet been obtained, but the concepts for the solution are clear and will be stated below.

Under unbounded conditions, the equation $p_w = C_{m+w}^S (\frac{1}{2})^S$ expresses the probability that a liquid portion from a feed at $x=0$ will attain a displacement $x=X$ at least once during the distribution and then attain after the S shifts a displacement x , where $X=w\Delta$ and $x=(s-2m)\Delta$, the displacement being measured parallel to the horizontal x axis.

Likewise the equation $p_v = C_{n+v}^S (\frac{1}{2})^S$ expresses the probability that a liquid portion from a feed at $y=0$ will attain a displacement Y at least once during the distribution, and then after S shifts attain a displacement y , where $Y=v\Delta$ and $y=(s-2n)\Delta$, the displacements being measured parallel to the horizontal y axis which is perpendicular to the x axis.

The product $p_w p_v = C_{m+w}^S C_{n+v}^S (\frac{1}{2})^{2S}$ expresses the probability that the liquid portion from a point feed, at $x=0, y=0$, will attain a displacement $x=X, y=Y$ at least

once and a displacement $y = Y$ at least once during its distribution and then attain after S shifts the displacement to the point (x, y) .

The problem is to find a Binomial equation expressing the probability that a liquid portion from a point feed at $x=0, y=0$ will attain a displacement $x=X$ and a displacement $y=Y$ simultaneously, or the displacement (X, Y) , at least once during the distribution and then attain after the S shifts the displacement to the point (x, y) .

On letting $X^2 + Y^2 = R^2$ and $x^2 + y^2 = r^2$, this last Binomial equation, if obtained, would then be integrated with respects to θ between the limits $\theta=0$ and $\theta=2\pi$, where θ is the horizontal central angle between the x axis and R . This integral form will give the probability that the liquid portion will attain a horizontal radial displacement R in any direction from its point feed at least once during the distribution and then attain the displacement to a given point a horizontal distance r from the centerline through the feed point.

After considering the equations as expressions for the fraction of all the liquid portions from a feed instead of for the probability of a single portion from that feed, this sought-for discontinuous Binomial equation for the liquid fraction, concentrated at points 2Δ intervals apart in both the x and the y directions, need be expres-

sed as a continuous Normal equation.

This Normal equation, could it be obtained, would express the collective effect of a cylindrical wall of radius R , that is, the decrease in unbounded liquid concentrations on the packing when the cylindrical wall, from which there is no liquid return, is introduced.

Once this equation for the collective effect is obtained, the effect of redistribution should follow readily, perhaps through reasoning similar to that for straight walls as applied to the set-up indicated by Figure I.

Distribution Studies using a Sand Distributing Model

(a) Shot Distributing Models

The usual demonstration and experimental models, used in obtaining frequency distribution curves for verifying the theoretically derived Normal and Binomial Laws of Distribution, are some modified forms of the original "Quincunx for the Illustration of the Law of Errors" designed by Francis Galton in the latter part of the eighteenth century.

Galton's Quincunx is an inclined board into which about 100 nails are driven forming a pattern known as the quincunx, where every nail is at the vertices of equilateral triangles marked off on and covering the total distribution area. Small spherical shot roll downward from a hopper at the top of the board and around the nails in a zigzag path, each shot being deviated either to the right or to the left for every encounter with a nail. The shot are collected in and distributed through a bank of compartments in accordance with the distribution laws.

In a research conducted by Gruber (8) with such a model, using two sizes of shot and two slopes of the board, the experimental curves obtained correspond well with Gaussian Error curves having the same mean divergences as those for experimental runs when the shot in the outer compartments near the boundary were neglected. Gruber also found that for larger shot size and larger slopes of the board

a larger average divergence arises.

A modification of Galton's model, designed by K. Pearson (15) was further changed by Scott (16) to permit a study of shot distribution under bounded conditions. Triangular wood obstructions fastened in horizontal rows on sliding strips of wood are held on a wooden easle and covered by a glass plate to serve as the distribution area. Below this section a glass panel containing a bank of compartments rests on the easle.

Scott introduced vertical walls bounding the distribution area and then made runs on this model to see if such distribution, believed to be in accordance with the laws of probability, were comparable to the distribution curves obtained from experiments on the spread of water in packed towers. His results showed a rough correspondence between the two types of distribution and hence indicated that liquid distribution in packed towers was of a probability nature. He concluded from a study of the shot distribution curves that the wall effect is less noticeable on decreasing the ratio of packing height to tower diameter and on increasing the number of obstructions between the walls. He stated that changes in slope of the distribution area, changes in the velocity of the shot fall, and collisions between the shot themselves did not appear to change the distribution.

F. Bernstein (2) (3) also used a shot distributing device for distribution studies and devised a new type of model for studying bounded distribution for application to tortion problems.

(b) The Sand Distributing Model

The sand distributing model built in the shops of the Department of Chemical Engineering at the University of Cincinnati avoids many of the inherent inaccuracies of the Galton type of model used for verifying experimentally the laws of frequency distribution.

In model I, used in this research, the distribution unit is a single layer of accurate $3/8$ " steel balls regularly arranged between two vertical plates of glass $1/4$ " thick. The distribution area between the glass plates is $20\frac{1}{4}$ " (or 54 ball diameters) wide and is bounded at the sides by metal strips separated from the glass by lengths of rubber. The lower layer of steel balls rests in a slot between two parallel strips of metal running across the bottom width of the distribution area. By putting 53 balls in the bottom row and 54 balls in the next row above and then alternating 53 and 54 balls per horizontal row, 107 rows of steel balls are arranged in the quincunx pattern and fill, almost to the top, the three-foot high distribution area.

Fine sand, carefully screened between 35 and 48

mesh, is stored in the hopper at the top of the model and flows into the sand chamber through four slots at the bottom of the hopper. There the flow is controlled by a sliding strip moved by a crank through a worm gear.

The sand chamber is a 20" by 1" compartment, 1 foot high, open at the top, and glass covered in front. The bottom is bounded by a fixed metal strip with 27 holes $\frac{1}{4}$ " in diameter drilled along its length at $\frac{3}{4}$ " intervals. Another identically drilled metal strip is constructed to slide horizontally and laterally across the bottom surface of the fixed strip. The openings formed by the overlapping of the holes on each strip can be closed instantly by moving the sliding strip, or the size of the holes can be adjusted by changing the extent of overlap of the holes by means of a thumbscrew. Nine metal tabs, one for each of three adjacent openings, can be slid across the openings to close any or all of the groups of three openings.

Sand flowing from the hopper builds up the desired head in the sand chamber and passes through the openings into the distribution area through the open end at the top of the plates of glass. A steel rod is placed on the top row of steel ball to direct the sand toward the front plate of glass. The sand flows downward in a zigzag path through the crevices bounded by the steel balls and the front glass plate. It leaves the distribution

area at the bottom through the slot between the parallel metal strips supporting the lower row of balls and fills the compartment bank below.

The compartment bank consists of strips of sheet metal with one edge inserted into vertical slots spaced $3/4$ inches apart in the wood backboard and with the other edge against the plate glass in front, thus forming an adjacent series of compartments $3/4$ inches wide, 1 inch deep, and 1 foot high.

One of each of these sheet metal strips, forming the compartment sides, is directly under the center of every other ball in the bottom row of the distribution area so that each compartment collects sand flowing from two openings between the balls in the lower row. The height of sand collecting in each of these 27 compartments is measured to determine the quantities distributed to each compartment displacement. A hinged trap door at the bottom of the compartment bank permits a rapid emptying of the compartments.

Since all parts of the model are painted black except the glass plates and the steel balls, the fine white sand against the black backgrounds become quite visible at the various parts of the model and particularly in the compartments. Such contrast makes the model ideal for demonstration purposes.

Steel balls of hardware grade (accuracy, $\pm .001$) were used in the first set-up and were found satisfactory



Figure 2 -- Model I Operating for a Center Feed Run.

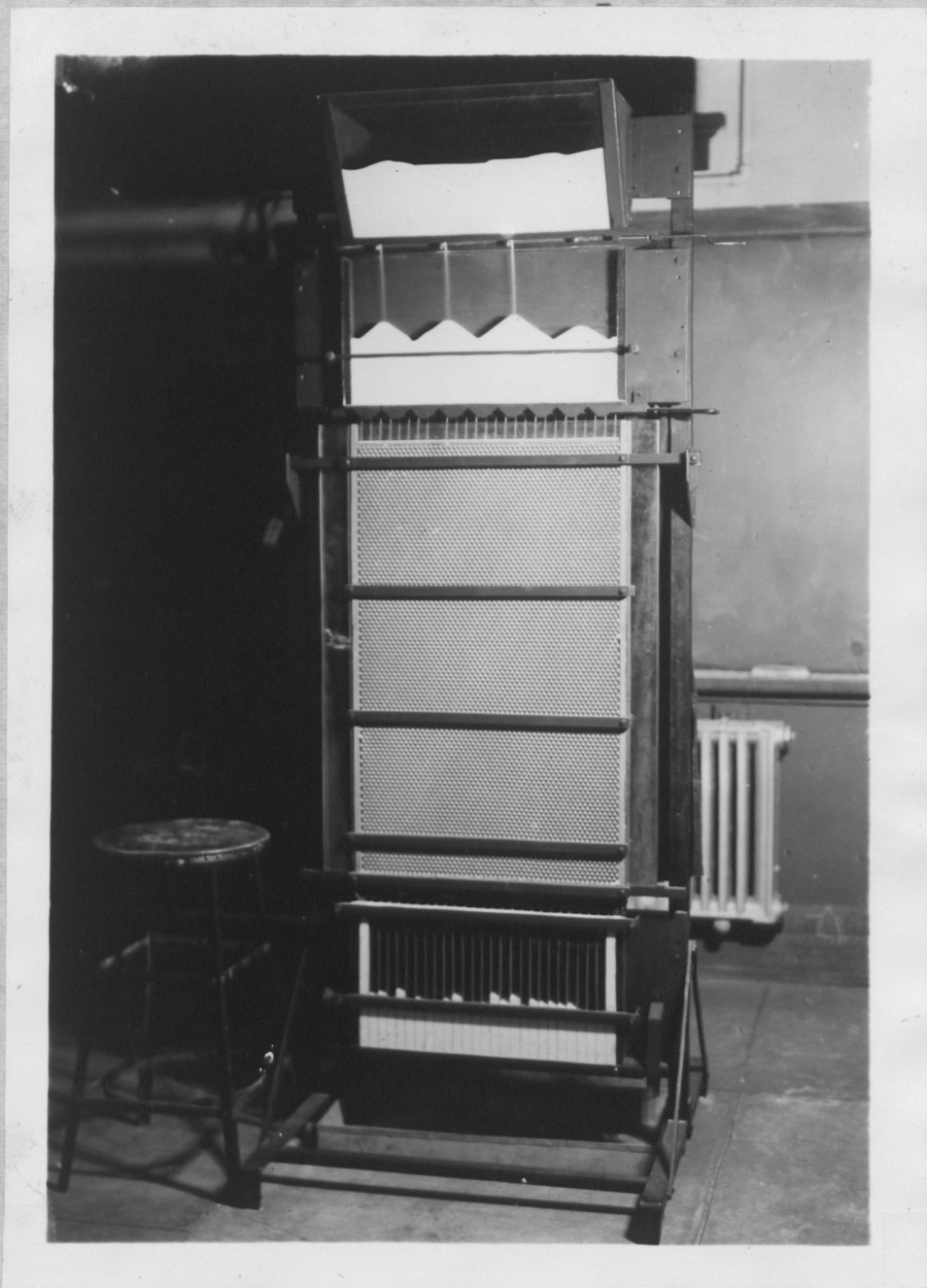


Figure 3 -- Model I Operating for a Uniform Feed Run.

for center feed but gave too irregular curves when uniform feed was used. Therefore steel balls of standard grade (accuracy $\pm .0001$) were used and gave more satisfactory curves. In this thesis, the first balls used were labeled No. 1 and the second, No. 2.

(c) Data from the Sand Distributing Model

Each compartment was calibrated in turn by adding 50, 100, and 150 cc. of sand and then measuring the height of sand in the compartment. It was found that 1 mm. height was equal to 0.568 cc. of sand. Also the following corrections to be added to the readings were found necessary: Compartment 6, add 5% of the reading; 7, 6%; 11, 3%; 12, 5%; 16, 13%; 17, 7%; and 23, 5%; where the compartments are numbered from 1 to 27 from left to right. In the compartments not mentioned, the corrections were not considered since they were within 3%.

To make a run, the hopper outlets and the chamber outlets are adjusted to the desired sand flow rate, keeping a constant head of sand in the chamber. For uniform flow, all of the metal tabs at the bottom of the sand chambers are pulled out, unblocking all openings. For center feed only the center tab is pulled out leaving the middle group of three openings unblocked. The outer two of these three openings is closed with a piece of tape leaving a single opening for true center feed.

The sand flow is continued for a while with the trap door at the bottom of the compartment bank left open until dynamic equilibrium is established. Then the trap door is closed at the same instant that a stop-watch is clicked to time the run. After a sufficient quantity of sand is collected in the compartments, the openings at the bottom of the sand chamber ~~are closed~~ and the time reading is recorded.

The height of sand in each compartment is measured in mm. and then tabulated. The reading for each compartment is calculated in cc. of sand and totaled for each run. This total divided by the time of flow gives the total rate per minute for each run.

The fractional quantity per unit compartment is calculated by dividing the cc. of sand per compartment by the total flow.

(d) Conclusions for Center Feed Sand Distribution

For center feed, if the sand distribution follows the Normal Law, equation I applies since the wall effects are negligible for this feed. On taking the natural logarithm of both sides of the equation, it becomes

$$\ln q_x = \ln \frac{h}{\sqrt{\pi}} - h^2 x^2$$

If the sand distribution follows this Normal Law, plotting the logarithm of the fractional quantity against the square of the displacement should rectify the curve giving a straight line having a slope h^2 and an intercept $\ln K$

Data for center feed runs on Model I, are tabulated and then plotted in the above manner to give the rectified curves (Appendix II). For low total flows (below 30 cc. per min.), the curves are straight lines whose slopes and whose intercepts are tabulated in Appendix II as h^2 and $\ln K$, respectively. It is shown that the ratio $\frac{K\sqrt{\pi}}{h^2}$ closely checks with unity, proving that K closely approximates $\frac{h}{\sqrt{\pi}}$, a beautiful verification of the theory with experiment.

At intermediate and high flows, the curves appear to be composed of two straight lines of different slopes connected by a curved line. The primary Normal curve extending from the centerline of the compartments, one-third of the distance to the outer boundary in each direction, is flatter than the Normal curve for low flows. A transition occurs within the next third of the distance from the center toward the wall. At the outer thirds of the distance from the center to the boundary walls, secondary Normal curves, steeper than the primary curves, exist.

In Figure 4, the primary and the secondary slopes of the rectified Normal curves are plotted against the total

Effect of the Feed on the Values of h^2 ,
the Slopes taken from the Rectified Curves
for Center Feed in Model I

Total Feed Rates in Gc. per Min.

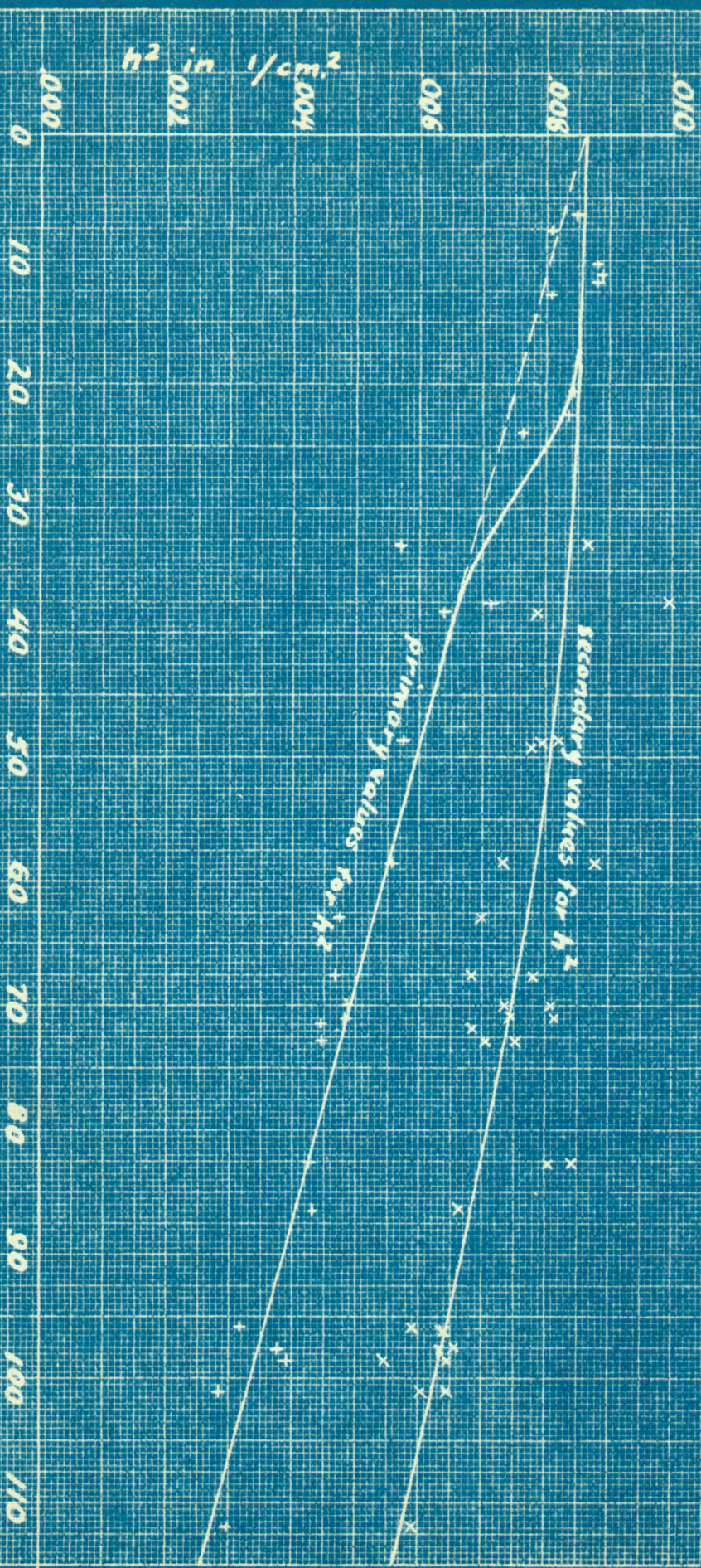


Fig. 4

flow rates, and in Figure 5, the values of K , the antilogs of the primary intercepts, are plotted against the total flow rates, the antilogs of the secondary intercepts being too irregular for plotting.

In each case, the primary values give a straight line except for the low flow values. The equation of these straight lines are

$$h^2 = .00860 - .0000525F$$

where h^2 is in $1/\text{cm}^2$, and F is the total flow rate in cc. per min., and

$$K = .0552 - .000180F$$

where K is in $1/\text{cm}$.

The empirical equation, applicable to intermediate and high flows for the given set-up in Model I becomes

$$q = (.0552 - .000180F) e^{-(.00860 - .0000525F)x^2} \quad (\text{XX})$$

where x^2 is in cm^2 ;

and for low flows the equation

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2},$$

*Effect of the Feed on Values of K
Taken from the Rectified Curves
for Center Feed in Model I*

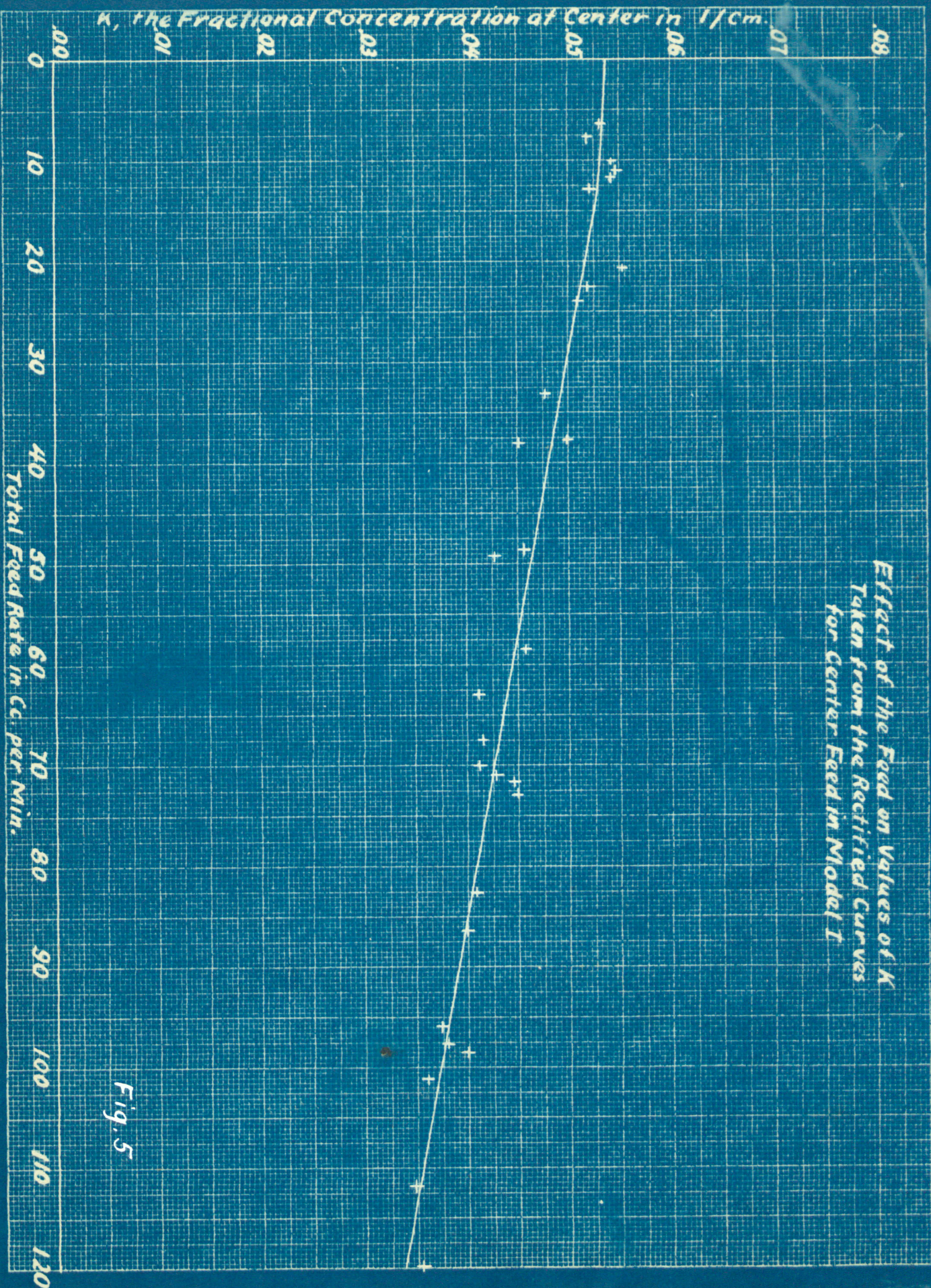


Fig. 5

where h^2 equals .00860, applies.

Let it be assumed that each sand particle, analogous to a liquid portion in liquid distribution, falls from the top of each ball that it strikes to the top of one of the two balls supporting it in the horizontal row below. Then the total number of shifts of a particle, S , equals the total number of horizontal rows it meets, 107 for the set-up used, and the horizontal displacement per shift, Δ , in the $+x$ or $-x$ direction equals the radius of the ball, which for the $3/8$ " ball used is $\frac{1}{2}(\frac{3}{8}) \cdot 2.54 = .476$ cm. Then from theory

$$h^2 = \frac{1}{2 S \Delta^2} = \frac{1}{2 (107) (.476)^2} = .0211$$

The much smaller value of h^2 from the experimental curves, indicate a greater spread of the sand particles and a longer actual length of particle displacement per row than would be expected from theory.

This larger displacement per shift is believed caused by the rebound of the sand particles from the balls and by the tendency for the sand to continue moving in the direction of motion. Both the elastic effect and the inertia effect of the sand particles were noticed in the model during the runs.

Since $h^2 = \frac{c}{L D}$ for low flows, c may be calculated from the model data taking the experimental value

$$h^2 = .00860 \frac{1}{cm^2}, L = 107 \frac{\sqrt{3}}{2} \cdot \frac{3}{8}'' = 34.8'' = 87.2 cm, \text{ and } D = \frac{3}{8}'' = .952 cm.$$

Substituting these values in the equation $c = h^2 L D$ gives $c = .718$.

Therefore, the general equation for unbounded sand distribution from a center feed at low flow rates and through a quincunx arrangement of balls is

$$q_x = \frac{\sqrt{.718}}{\sqrt{\pi L D}} e^{-\frac{.718}{L D} x^2}, \quad (\text{XXI})$$

where q_x is the fraction of the total flow per increment of x at displacement x , and L and D are the height of the distribution plane and the diameters of the balls, both in the same units as x .

If the theoretical assumptions as to number and length of shifts are made as previously,

$$\frac{c}{L D} = \frac{1}{2s\Delta^2} = \frac{1}{2\left(\frac{2}{\sqrt{3}}\frac{L}{D}\right)\left(\frac{D}{2}\right)^2} \text{ or } c = \sqrt{3} = 1.73.$$

a value 2.41 times the experimental value of C .

(e) Conclusions for Uniform Feed Sand Distribution

In Appendix III, data and graphs are presented for sand distribution by Model I at various uniform feed rates from 170 to 859 cc. per minute total feed.

The fraction per compartment of the total feed is plotted against the compartment displacement from the centerline of distribution. The fractions for pairs of compartments, equally distant from the centerline, one to

the left and the other to the right are plotted at the same compartment displacement.

The data is too irregular and the flow rates too high for accurate application of the derived mathematical equations for uniform flow and wall effects to these experimental curves. However, these curves are satisfactory for verifying and studying, in a qualitative way, the wall effects on distribution.

Since there are 27 compartments collecting sand from equal widths across the distribution plane, the fraction of the total feed that would be collected in each compartment, if the distribution remained uniform, is $\frac{1}{27}$ or .037. This average fraction is used as the basis of comparison for the curves, the fractions in the compartments being expressed as percent of the average.

The distribution as shown by the curves in Appendix III agrees well with the qualitative description of distribution with wall effects given previously. These experimental curves show an inversion of distribution from predominatingly collective at the wall to predominatingly redistributive from the wall as the flow rate passes through a range somewhere between 400 and 560 cc. per minute total feed.

Below this inversion range, almost all of the sand, reaching the boundary walls during the distribution, collects there with little return. Thus high wall flows

are rapidly built up, and there is a progressive decrease from uniformity as one moves outward from the centerline to the wall. At these total feed rates the fraction at the center compartment is little changed and remains at about 100% of the average fraction for uniformity. As one moves outward from centerline, the fractional quantities decrease progressively and uniformly from 100% until at the compartment next to the wall compartment the fraction is about 90% of average for a feed rate of 170 cc. per minute, and progressively smaller for higher feed rates up to 400 cc. per minute where the fraction is about 80%. The large fractions in the compartments at the wall increase rapidly with feed rate from about 160% of average at 170 cc. per minute to about 240% of average at 400 cc. per minute.

Above the inversion range, the fraction in the center compartment has dropped off sharply from the 100% at the lower feed rates and decreases as the feed rate is increased. As one moves from the center toward the wall the fractional quantity increases progressively because of the redistributive effect of the wall. This increase from the center compartment to the compartment next to the wall compartment ranges from about 80% to 94% at 560 cc. per minute total feed to about 60% to 130% at 859 cc. per minute.

The fractions in the wall compartments vary slowly with increased feed rates increasing from about 270% of the average for uniformity at 560 cc. per minute total feed to about 300% at 859 cc. per minute.

It has been shown in the study of center feed that the distribution constant, h^2 , decreases as the feed rate is increased. Also, it is known that the distribution constant decreases with the height, L , of the distributing section. Then, it may be assumed with some justification that a series of curves, obtained for a given value of L by increasing the feed rates, roughly approximates, at least qualitatively, a series of curves obtained for a given feed rate but by increasing the values of L .

Therefore, the series of curves in Appendix III may be considered, qualitatively, as distribution curves at various distances down a long distributing unit filled with the steel ball when the feed rate is constant. Through this approach, it is seen that on moving down the tower length the quantities of sand near the wall decrease progressively as the quantities at the wall increase progressively because of the collective effect at the walls. After passing an inversion range of length downward, the sand quantities at the wall increase more slowly but feed back to the steel ball enough sand to make the sand quantities progressively less as one moves toward the center line.

Water Distribution in Packed Towers

(a) Various Experimental Packed Towers for Distribution Studies.

The numerous set-ups for packed tower distribution studies, as used by the various investigators of this subject, vary greatly in tower dimensions, type of packing, method of feed, and kind of 'collecting' compartments.

Hurter (9) used a 14' tower of cross section 4' by 4' packed with coarse and with fine coke at various heights. Feed was from a point source and at low flows, and the water was collected in a bank of eight straight troughs each 6" wide. The data, obtained under such conditions has been found useful in studies of low flow, center feed, unbounded distribution as expressed in equation I.

Kirschbaum and Weimann (10) (18) used towers of 110 and 300 mm. (4.3" and 11.8") in cross section with packing heights of 4 meters and less, using 8 by 8 mm. and 15 by 15 mm. Rashig rings. Center feed was used and the water was collected in four annular compartments.

Chilton and his co-workers (1) (4) made runs using towers of diameters 3", 6", 11.3" and 24" packed at various heights up to 8 feet with various packings, particularly several small sizes of crushed stone and of clay spheres. Liquid feed was from one, four and more points

at the top of the tower at a total feed rate of one gallon per minute per sq. ft. of packing cross section. Water was collected in four concentric annular compartments of equal area. In some runs air or air-ammonia mixtures were introduced for counter-current gas flow.

Scott (16) used a 4" tower packed at various heights from 15" to 15' with $\frac{1}{4}$ " coke, with $\frac{1}{2}$ " coke, and with $\frac{1}{2}$ " Lessing rings. A center feed, varied from 10 to 50 pounds of water per minute per sq. ft. of cross section, was used. The water was collected in a bank of 30 octagonal compartments grouped in three zones.

In the research at the University of Cincinnati, the first set-up was a tower 10" in diameter packed with 1" coke (12) and with 1" Raschig rings (14) at several heights up to 10 feet. Both center and uniform feed were used and water was collected in five concentric annular compartments of equal area. The feed rates were varied from $\frac{1}{2}$ to 4 cu. ft. per minute by increases of $\frac{1}{2}$ cu. ft. per minute.

The second set-up was a tower 20 " in diameter and 6' high with 16 annular collecting compartments at the bottom, (Figure 6).

(b) Description of the Twenty Inch Tower

The irregularities of data obtained from the twenty inch tower when $\frac{1}{2}$ to 1" coke was used (5) made it appear advisable to use smaller packing. Therefore,

a 36 hour metallurgical coke was finely crushed and screened. The portions between $\frac{1}{4}$ " and $\frac{1}{2}$ " were retained. This coke was carefully poured in the tower keeping the increasing packing heights level to prevent any size segregation through rolling of the coke from high levels of the packing toward the lower levels. A three foot packing height was thus built up and was supported in the tower by a wire screen lying on the top edges of the walls of the collecting compartments. For later runs the tower was emptied and then repacked to a six feet packing height in the same manner as before.

Water is led through a 3" valve controlled pipe into the feed head, suspended just above the top of the packing by chains. The feed head is a cylindrical shell of sheet metal 15" high and of diameter to fit closely and yet freely against the inner walls of the tower at any level. Fitting tightly on a narrow, rubber-lined rim at the bottom edge of the shell is any one of a set of interchangeable discs of $\frac{1}{4}$ " bakelite, serving as the bottom of the cylindrical shell.

For uniform feed five discs are available, each drilled across the face with about 250 holes of a given size equally spaced at the corners of 1" squares. In the low feed discs the holes, drilled with a No.55 twist drill, are each .0519" in diameter, and in the high feed discs, the holes drilled with No. 17 twist drill, are each .173" in diameter. The other discs for uniform feed have holes

of intermediate size. A three inch layer of 1" Rashig rings is built up on the uniform feed discs when in use to break the force of the water flow from the 3" pipe.

For center feed, a single 3" hole was drilled in the center of a similar $\frac{1}{4}$ " thick bakelite disc. A 6" ring was screwed on the disc concentric with the hole. Into the ring and above the 3" hole, any one of a set of interchangeable discs of $\frac{1}{8}$ " bakelite can be fitted tightly. Each disc contains a single centered orifice. The orifice sizes of the four discs prepared are approximatedly $\frac{3}{16}$, $\frac{3}{8}$, $\frac{3}{4}$, and $1\frac{1}{2}$ " in diameter. A splash plate, an 8" circular disc of sheet metal, is supported two inches above the bakelite disc to break the force of water flow from the centered 3" inlet water pipe.

The head of water in the feed head is controlled by a vacuum pump; a suspended 1" pipe is adjusted to the desired height of head and the excess water is constantly removed from the feed head through this pipe which is connected by a rubber hose to the vacuum pump.

The 16 annular collecting compartments are formed by rings of sheet metal strips about .015" thick and two inches high set on a brass plate. Each of the eight innermost compartments subtend an annular cross section area of tower of about 13 sq. in. while the eight outer compartments subtend about double that area. A discharge tube for



Figure 6 -- The Twenty-Inch Tower, Six Feet High, used for Distribution Runs.

each compartment passes through the brass plate, tapering from an annular segment at the bottom of the compartment to a short length of 1" pipe. The tubes are numbered from 1 to 16 consecutively from the center to the wall.

A rubber hose may be attached to any compartment outlet to lead the water into one of six 30 gallon collecting cans. Each can has a $1\frac{1}{2}$ " outlet pipe at the bottom which may be closed by a threaded plug or a rubber stopper. A full length gauge glass at the side of each can is calibrated for each of the 4 cubic feet contents of the can.

Plumb bobs are used to get the tower exactly vertical, an essential condition in obtaining distribution data.

(c) Determination and Plotting of Data

To make a run, the desired spray disc is placed in the spray head which is then set just above the top of the packing. The 1" pipe, controlling the water head by vacuum, is then adjusted to the proper height, and the vacuum pump is started. The inlet water valve is set to feed at a flow rate somewhat higher than the desired flow through the tower. When the water head is built up, the excess is continually removed by the vacuum pump, thus keeping a constant water head in the spray.

When dynamic equilibrium is attained in the tower, lengths of rubber hose are slipped over six of the water outlets at the bottom of the compartments. The free end of one of the hose lengths is placed in a collecting can as a stop watch^{is} clicked to time the water flow. The other free ends of the hose lengths are placed successively in their respective cans at 15 second intervals. The starting time for each can is recorded against the number of the outlet feeding into the can. Also recorded against the outlet number is the time when the water level in each can reaches each of the successive calibration marks on the gauge glass. Flows from the ten remaining compartment outlets are measured in a similar manner to complete the run.

The difference in seconds of the time for attaining the highest level reading (usually the 4 cu. ft. level) in the can and the starting time divided by the level reading gives the time of-flow in sec. per cu. ft. The time readings for the intermediate levels serve as a check on the constancy of flow. Sixty times the reciprocal of the flow time (in sec. per cu. ft.) gives the flow rate for the outlet in cu. ft. per minute. For a given run, the sum of the flow rates for the 16 compartments gives the total feed rate for that run. Each outlet flow rate divided by the known annular cross-section area of that compartment in square inches gives

the average concentration of water for that compartment in cu. ft. per min. per sq. in.

When the water flow is low for an outlet, the water issuing from it in a measured period of time is collected in a one-liter graduate calibrated to 10 cc. intervals. Several readings are made for each outlet so measured. From the time in seconds required to collect a given quantity of water in cc., the water concentration for the compartment is calculated in cu. ft. per min. per sq. in.

The concentrations at each total feed rate are tabulated against the compartment numbers and the mean compartment displacements, that is, the radial distances from the tower centerline to the circles dividing each of the compartments into equal areas.

Curves plotted from such data give fluctuations which conceal the general trend of distribution. An explanation for such fluctuations may be made based on the observed fact, noticed by several investigators of liquid distribution in packed towers, that the water flows through the packing in a definite number of streams. If the areas of the collecting compartments are small, the number of streams per compartment are few, and an accidental divergence of a single stream from a given compartment to an adjoining compartment may cause an appreciable fluctuation in the general

distribution trend as measured.

Such fluctuations are similar to those obtained in the limited measurements of a quantity when the frequency of the occurrence of an error of measurement in a very small interval of divergence is plotted against the divergence of the measurement from the true value. The frequency of an error occurring in a very small interval is low, when the number of measurements are limited, and the fluctuations in the plotted curve may conceal the trend of the curve which is in accordance with the Normal Law of Errors. Obviously, increasing the interval of divergence over which the frequency is counted decreases the localized fluctuations and gives a smoother, more continuous curve, consistent with the Normal Law.

Likewise, in the packed tower, increasing the compartment areas increases the number of streams collected and thus should decrease the localized fluctuations and give a smoother curve showing the true distribution trend.

Therefore the measured flow rates for adjacent pairs of compartments are added together and divided by the total area of the compartment pair to give combined data for water distribution as in Appendix IV. Such distribution data for 8 annular areas instead of the previous 16 gives the less fluctuating curves shown in Appendix IV. However, at the higher total feed rates for both center and uniform feeds, the

curves were not plotted as the distribution, although tending toward uniformity, becomes erratic as the plugging conditions in the packing are approached.

By plotting the water concentrations against the displacement squared on semi-log paper for all center feed runs except those at the high flow rates, rectified curves are obtained as shown in Appendix IV.

(d) Discussion of Results

The series of distribution curves for both packing heights at center feed appear to give Normal curves but with the center section somewhat flattened out as the feed rates are increased above 1 cu. ft. per min. Such lowered concentrations at the center are caused by the shunting outward of liquid quantities from the top center of the tower due to plugging and splashing as the feed from the centered orifice strikes the top of the packing. As the feed rate is increased this plugging and splashing effect increases until, at high flow rates, the feed approaches an equivalence in distribution to uniform feed. In fact, except for very low flow rates, the feed is as from an area source of radius increasing with the feed rate.

The rectified curves for feed rates at $\frac{1}{2}$ cu. ft. per minute and lower give a set of straight lines of constant slope. At the intermediate flows, the rectified curves move

away from centerline with increasing slopes until the slope approximating that of the straight lines for low flows is attained. These constant slopes persist as straight lines through the intermediate radial distances from the centerline. Near the wall there is a deviation from the straight lines in both low and intermediate flows, the deviation tending upward for the three feet packing heights and downward for the six feet heights.

The straight lines indicate that the curves follow the Normal Law of Distribution. Since the distribution is measured at radial displacements, equation III, $q_R = \frac{h^2}{\pi} e^{-h^2 R^2}$ should apply. Taking natural logarithms of this equations gives

$$\ln q_R = \ln \frac{h^2}{\pi} - h^2 R^2$$

where q_R is the fraction of the total feed that falls on a unit area at a radial displacement R . Plotting $\ln q_R$ against R^2 gives a straight line of slope $-h^2$ and intercept $\ln \frac{h^2}{\pi}$.

From the rectified curves, the values of h^2 for each straight line is obtained as the difference of the natural logarithms of the quantities at two points on the straight line divided by the difference of the squared displacements of the points. The intercept on the centerline is expressed as the fraction of the total feed per square inch of tower.

The slopes and the intercepts so obtained are tabulated in the data sheets at the end of Appendix III as h^2 and K^2 , respectively. There also, the values of $\frac{K\sqrt{\pi}}{h^2}$ are tabulated for all low flow runs. If Normal equation III applies strictly, $K^2 = \frac{h^2}{\pi}$ and $\frac{K\sqrt{\pi}}{h^2}$ equals unity.

The close check of the tabulated values with unity is another experimental verification that water distribution follows the Normal Law; The first verification for water distribution as an expression of the Normal Law was obtained in the Master's thesis using equation I applied to Hurter's data (9) for water distribution from a point feed and collected in a bank of straight troughs.

The average experimental values for the distribution factor h^2 for the low flows at the three feet packing height are:

From the slopes of the rectified curves — $h^2 = .0871$

From the intercepts of the rectified curves — $h^2 = K^2\pi = .0948$.

For six feet packing height:

From the slopes — $h^2 = .0356$

From the intercepts — $h^2 = K^2\pi = .0374$

The ratio of the factors h^2 for 3 feet and 6 feet packing is as 2.45 is to 1 or approximately as 5 is to 2 when data from slopes are used, or as 2.53 is to 1 or approximately as 5 is to 2 when data from intercepts are used. From theory h^2 is inversely proportional to the packing heights L or the

ratio above should be as 2 to 1 or as 6 is to 3. The difference between theory and experiment is believed caused by the lack of precision and the lack of sufficient low flow runs in the experiment for averaging values of h^2 with great accuracy.

From the four averaged values of h^2 tabulated above, the constant C which from theory equals $h^2/4D$ may be calculated on expressing L , the packing height, and D , the diameter of the packing units (an average value of $3/8$ " is taken), in inches of length.

The calculated values of C are:

	From 3 ft. packing	From 6 ft. packing
h^2 from slope	1.175	0.961
h^2 from intercept	1.285	1.010

The arithmetic average of these values gives $C = 1.107$, or approximately 1.1.

A general equation for distribution from center feed through coke on the basis of this value of C would be

$$q_R = \frac{1.1}{\pi L D} e^{-\frac{1.1}{4D} R^2} = \frac{.35}{L D} e^{-\frac{1.1}{4D} R^2} \quad (\text{XXI})$$

where q_R is the fraction of the total feed distributed per unit area to a displacement R after a packing height L for coke of size D , where the length units of L , and D are the same as of R . This equation should apply to all cases

for coke where the boundary effects on the distribution are negligible and where the feed rate is sufficiently low so that no plugging occurs to effect the distribution.

The uniform feed data for low and intermediate flows give an unusual set of curves in that maximum concentrations occur at radial displacement part way between the centerline and the wall. The water concentration remains fairly constant or increases slightly for about 4 inches outward from the centerline, then the concentration increases rapidly to a maximum at about a radial distance of 6 inches then drops sharply toward the wall (curves in Appendix 4). The wall flows are generally low, roughly half that at the center and one fourth that of maximum. Similar curves were obtained by Scott (16) for some set-ups using $\frac{1}{4}$ " coke.

In the runs on the tower using $\frac{1}{2}$ -1" coke made by Cuzner and Fischer (5) the distribution curves do not show this intermediate maximum but give the general curve for uniform feed at average packing heights, that is, a curve decreasing from a maximum at the center to a minimum at an intermediate point then increasing to high flow rates at the wall.

The intermediate maximum on the distribution curve for $\frac{1}{4}$ - $\frac{1}{2}$ " coke appears associated with the small size of the coke used and with the low wall flows. Apparently, due to the small crevices between the wall and

the boundary packing, the collective effect of the wall is low and the redistributive effect sets in rapidly to build up a maximum at the intermediate displacement. This explanation is not fully satisfying and is open to criticism. Accidental channeling is not believed an explanation, for after the three foot runs were made, the tower was emptied and carefully repacked for runs at the six foot height; yet the series of curves were very similar to those for the three foot runs.

General Conclusions

(a) Summary of Liquid Distribution Laws

The study of liquid distribution for packed towers has been approached through mathematical derivations from the Normal Laws of Distribution. The two basic laws extended for packed tower distribution from center feed where wall effects are negligible, are:

$$q_x = \sqrt{\frac{C}{\pi L D}} e^{-\frac{C}{L D} x^2}$$

and

$$q_R = \frac{C}{\pi L D} e^{-\frac{C}{L D} R^2}$$

, where q_x is

the fraction of the total feed that falls on a unit increment of x at displacement x , and q_R is the fraction of the total feed that falls per unit area at radial displacement R ; L and D are the packing height and the diameter of the single packing, respectively. When all lengths units in each equation are the same, C is a constant independent of the units used. It depends chiefly on the arrangement of the packing and to some extent on the type and shape of packing. For coke, the constant is 1.1.

These two basic equations have been extended mathematically to apply to area feed distribution and to special cases of bounded distribution from center and from area feeds. However, most of these equations need be simpli-

fied and modified for general use.

Sand distribution over steel balls, as carried out in Model I, described previously, acts analogously to liquid distribution over packing. The constant C for sand distribution down a quincunx arrangement of steel balls in a layer is equal to .718, giving the equation, applicable to center feed distribution at low flow rates,

$$q_x = \frac{\sqrt{.718}}{\pi L D} e^{-\frac{.718}{L D} x^2}$$

At higher flow rates the empirical equation for center feed distributing down 107 rows of 3/8" steel balls arranged in a quincunx layer, is

$$q_x = (.0552 - .000180 F) e^{-(.00868 - .0000525 F)x^2}$$

where F is the total feed rate in cc. per min. and x is the displacement in cm.

A study of the boundary effects on distribution was made using the concept that the wall has two effects on the distribution. The first effect is collective. The liquid, spread to the wall, tends to cling there as it flows downward at a rate believed more rapid than the flow down the packing itself. The second effect is redistributive. The liquid, after building up at the wall, tends to return in part to the packing for further distribution.

Mathematical equations developed from the Binomial and Normal Laws of Distribution using such a concept, involves a redistribution fraction, f , which is assumed constant for the derivation. Actually in most distribution runs, f varies as the liquid flows down the length of the wall from zero to unity by some functions probably dependent on the equations for channel and orifice flow of the packing down the wall.

(b) The Effects of Tower Variables on Distribution

Many factors in packed tower construction and operation effect the liquid spread through it. These factors include packing height, tower diameter, the size, shape, and arrangement of the packing, the type of feed, the feed rates, the effects of the wall, channeling effects, and the gas flow rates up through the tower.

Viewed from the Binomial and Normal Laws and their derivations and from the concept effects, conclusions may be drawn, from results of previous investigations and from the research presented here, which show the effect of these tower variables on liquid distribution.

An increased tower diameter increases the time required for the liquid to attain the wall and delays the effects of the wall on the distribution at cross-sections further down the tower. As the liquid moves down the tower the distribution of liquid from center feed becomes more

uniform and can be represented by increasingly flatter probability curves. These curves are modified by the wall effects first through a lowering near the wall because of the collective effect and then through an increase from the walls inward because of the redistributive effect. Figure 7 shows typical center feed distribution curves at various lengths down the tower under wall effects. The broken line curves show what the distribution would be were the liquid spread through unbounded packing.

The effect of the increased distance down the tower on uniform feed is first to cause progressively lowered concentrations toward the wall and then, farther down the tower, to cause progressively increased concentrations toward the wall. In the last case, inverted distribution curves, similar to those shown at the lower part of Figure 7 for center feed, are obtained but at levels higher up the tower.

Increased packing sizes cause more rapid liquid spread toward the wall. For center feed, the probability curves become more flattened at shorter lengths down the tower than when smaller packing is used. However, the larger packing sizes form large crevices next to the wall, resulting in high wall flows and lowered concentrations near the wall both for center and for area feed. The redistributive effect is delayed, not taking place until great lengths down the tower.

Small sizes of packing, lying closely against the

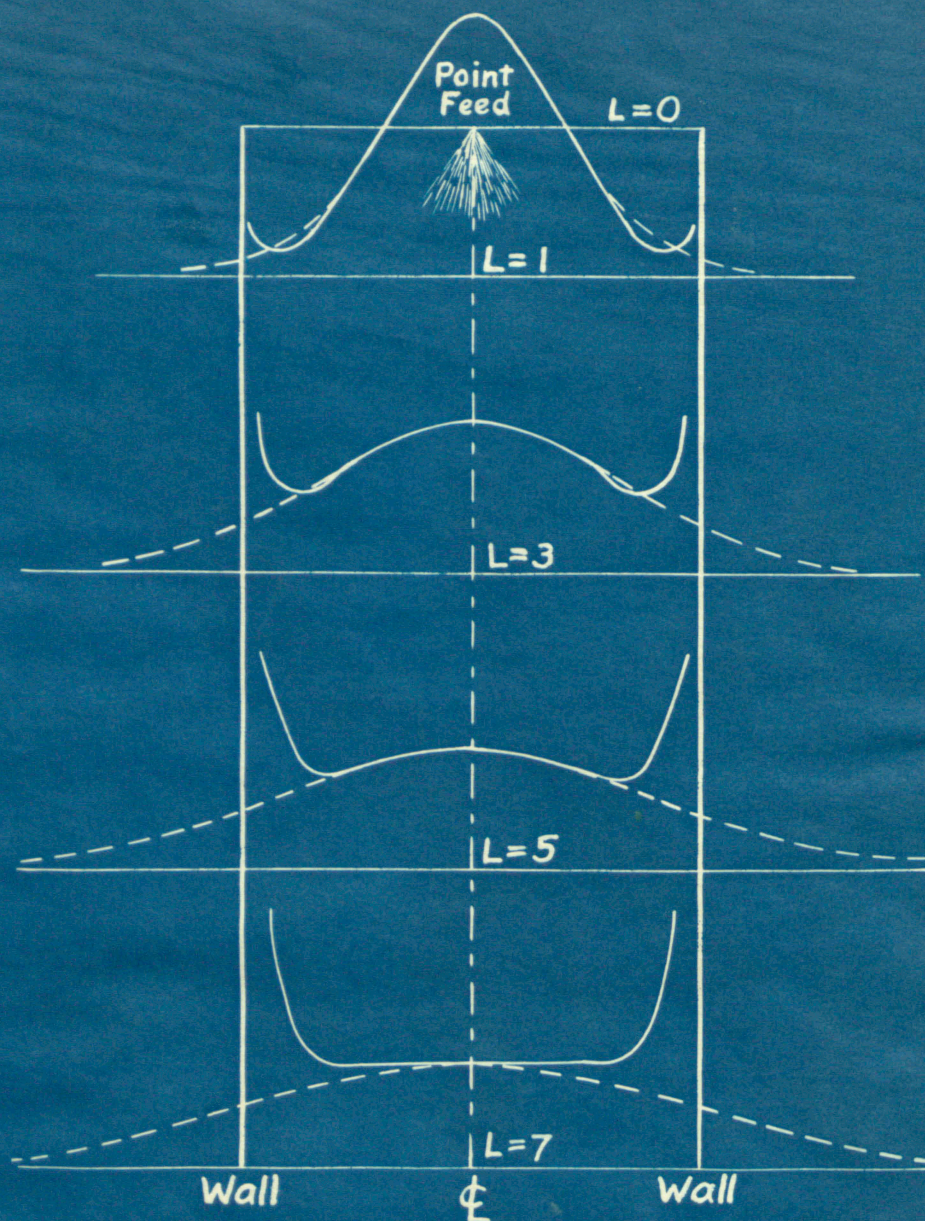


Figure 7 — Liquid Distribution, Bounded and Unbounded, at Various Lengths down the Packing.

wall form small wall crevices resulting in small wall flows, little collective effect, and a redistributive effect setting in quickly after liquid reaches the wall.

Center feed leaves unwetted packing surfaces at the top of the tower but delays the wall effects until some distance down the tower. In uniform feed, wall effects set in at the start. Increased flow rates (until plugging is approached) do not affect the distribution, as far as the packing spread is concerned, except in direct proportion. However larger actual quantities attain the wall and high wall flows set in sooner with the accompanying redistributive effects. Still higher wall flows load the packing with liquid, resulting in increased effective packing diameters with increased length of shift and greater spread toward the walls. When plugging sets in, a change from liquid flow over packing surfaces to channel and orifice flows occurs with large liquid quantities being shunted away from the "plugged" regions. The effects of channeling is not known but it is believed that distribution for the most part occurs through the downward flow of a number of streams of liquid taking erratic, random paths in accordance with the laws of probability.

The upward flow of gas is not believed to affect the liquid spread except at high gas velocities and near plugging conditions of liquid flow. In such cases, turbulence is set up at the interphase surface, and the tendency is for

the liquid to be spread into thinner layers over the packing, promoting uniformity.

(c) Recommendations for Actual Operation

The following criteria are accepted for judging best tower operation from the standpoint of liquid distribution:

(1) The best tower operation occurs when the distribution is uniform. The correct liquid concentration for maximum liquid-gas interchange will be constant throughout the cross-section, requiring uniformity of distribution.

(2) It is desirable to have large horizontal shifts of liquid portions per unit length of vertical drop. Increased horizontal movements of the liquid exposes more liquid surfaces for interchange and increases the length of time of the liquid passage through the tower. Large horizontal spreads also promote uniformity.

(3) Wall flows are undesirable. The liquid is believed to collect at the wall in relatively thick streams and to flow downward rapidly with little horizontal displacements. The time for contact with gas in the tower is thus lessened and the liquid surface exposes for interchange is decreased.

On the basis of these criteria, uniform feed is desirable at the top and large packing units are preferable that large horizontal shifts may be obtained. However,

both conditions set up the undesirable high wall flows rapidly, and large wall crevices formed with large packing sizes do not promote redistribution. The collective effect also causes decreases from uniformity at the regions nearest the wall.

Small packing will promote redistribution from the wall, the liquid return favoring the regions near the wall. The method used by some men in industry for packing towers with two different sizes of packing units, the smaller lining the walls and the larger units filling the bulk of the tower, has a sound theoretical justification.

An ideal packing for good liquid distribution is one composed of units of large diameter, specially designed to fit closely against the wall and whose shape and arrangement give low values of c . This distribution constant, C , for a packing may be determined by applying the modified basic Normal Laws of Distribution (given in part a of this section) to experimental data obtained for center feed unbounded distribution through the given packing.

A packing unit designed and arranged in the tower so that the horizontal units are large and the vertical dimensions are small will favor desirable distribution in that a large horizontal spread is obtained per unit vertical length of fall of the liquid.

Instead of specially designed packing, the use of any standard large size packing in conjunction with liquid redistributing devices at intervals down the tower would give good liquid distribution.

Kirschbaum has developed and patented (11) a redistributing device for leading water back from the tower wall to the inner portions of the tower. It consists of a conical shell of the same diameter as the tower to be placed point downward at intervals down the length of the tower. Perforations are made through the inner parts of the shell for passage of the liquid. Other perforations, ridged at the top to cause liquid to flow around not through them, permit the unobstructed flow of the gas phase.

Using such devices he obtained improved heat transfer coefficients in the condensation and reevaporation of vapor in a rectification process in a packed tower(10). A redistributing device recommended in the Master's thesis (12), consists of a narrow annular channel flush against the wall to collect the wall flows. Narrow bore tubes, extending from the bottom of the channel inward and downward, serve to lead the collected wall liquid to low concentration areas intermediately between the wall and the centerline of the tower.

These devices do not obstruct the natural liquid distribution of the packing, does not increase the pressure drop across the tower length, and leads liquid quan-

tities from the undesired position at the wall to the low concentration areas where it is needed.

Such devices were tested in a 10" tower 10 feet long used at the University of Cincinnati for research on absorption of CO_2 with water. The redistributing devices when placed at two foot intervals down the tower length resulted in markedly increased quantities of CO_2 absorbed as compared with quantities absorbed in the same set-up and under the same operating conditions but without the devices.

Summarized briefly, the requirements for good liquid distribution are uniform feed, large sizes of packing units, and redistributing devices, properly designed and properly placed at intervals down the tower length.

APPENDIX

- I Derivation of a Normal Equation Applicable to Bounded Distribution.
- II Data Sheets and Graphs for Sand Distribution from Center Feed.
- III Data Sheets and Graphs for Sand Distribution from Uniform Feed.
- IV Data Sheets and Graphs for Water Distribution in the Twenty Inch Tower.

Appendix I

Derivation of a Normal Equation
Applicable to Bounded Distribution

Appendix I

Derivation of a Normal Equation Applicable to Bounded Distribution

The Binomial equation

$$p = C_{m+w}^S \left(\frac{1}{2}\right)^S = \frac{S!}{(S-m-w)!(m+w)!} \left(\frac{1}{2}\right)^S$$

represents the fraction of the total feed, starting from a point source at $\chi=0$, which will attain at least once during the distribution the displacement X and finally attain the displacement χ . Here S is the total number of shifts, Δ is the horizontal displacement of the shift in either the $+\chi$ or $-\chi$ direction, and m is the number of the S shifts which are in the $-\chi$ direction that the displacement $\chi = (S-2m)\Delta$ may be attained; w is the excess of shifts in the $+\chi$ direction over the shifts in the $-\chi$ direction that the displacement $X = w\Delta$ may be attained.

As m is increased by unit values from $m=0$ to $m=S$, the equation gives a discontinuous curve with liquid fraction concentrated at 2Δ intervals along the axis of displacement.

To get a continuous curve representing liquid quantities per unit increment of χ for all value of χ , first plot the values of $\frac{p}{2\Delta}$ as ordinates at the concentration points along the χ axis. Then draw straight lines connecting the adjacent ordinates giving a distribution curve made up of segments of straight lines. Draw a continuous distribution curve tangent to the straight line

segments at their midpoints.

The slope of the straight lines segments and hence of the curve tangent to them is given by

$$\frac{d \frac{p}{2\Delta}}{dx} = \frac{dq}{dx} = \frac{p-p'}{2\Delta(x-x')}$$

where $p/2\Delta$ and $p'/2\Delta$ are adjacent ordinates and q is the ordinate of the tangent point. Since q bisects the straight line segment between the adjacent ordinates,

$$q = \frac{p/2\Delta + p'/2\Delta}{2} = \frac{p+p'}{4\Delta}$$

Then the function

$$\frac{dq}{q dx} = \frac{1}{\frac{p+p'}{4\Delta}} \cdot \frac{p-p'}{2\Delta(x-x')} = 2 \left(\frac{p-p'}{p+p'} \right) \cdot \frac{1}{(x-x')}$$

Let m be the number of negative shifts to attain displacement x , and $(m+1)$ be the number to attain x' .

Then

$$p = \frac{s!}{(s-m-w)!(m+w)!} \left(\frac{1}{2}\right)^s \quad \text{and} \quad x = [s-2m]\Delta$$

Also

$$p' = \frac{s!}{(s-m-1-w)!(m+1+w)!} \left(\frac{1}{2}\right)^s \quad \text{and} \quad x' = [s-2(m+1)]\Delta$$

Then

$$x - x' = 2\Delta \quad \text{and}$$

$$\frac{p-p'}{p+p'} = \frac{\left[\frac{s!}{(s-m-w)!(m+w)!} - \frac{s!}{(s-m-1-w)!(m+1+w)!} \right] \left(\frac{1}{2}\right)^s}{\left[\frac{s!}{(s-m-w)!(m+w)!} + \frac{s!}{(s-m-1-w)!(m+1+w)!} \right] \left(\frac{1}{2}\right)^s}$$

or
$$\frac{p-p'}{p+p'} = \frac{\frac{1}{s-m-w} - \frac{1}{m+1+w}}{\frac{1}{s-m-w} + \frac{1}{m+1+w}} \quad \text{by factoring}$$

and cancellation. On multiplying, combined terms, and then neglecting the negligible unity values, is obtained

$$\frac{p-p'}{p+p'} = \frac{m+w+1-s+m+w}{m+w+1+s-m-w} = \frac{2m+2w-s+1}{s+1} = \frac{-(s-2m)+2w}{s}$$

Since $\chi = (s-2m)\Delta$ and $X = w\Delta$, substitution gives

$$\frac{p-p'}{p+p'} = \frac{-\frac{\chi}{\Delta} + 2w}{s} = \frac{-\chi + 2w\Delta}{s\Delta} = -\frac{\chi - 2X}{s\Delta}$$

Substitution in the function gives

$$\frac{dq}{q d\chi} = 2 \left(\frac{p-p'}{p+p'} \right) \frac{1}{(\chi - \chi')} = -\frac{2(\chi - 2X)}{s\Delta} \cdot \frac{1}{2\Delta} \quad \text{or}$$

$$dq/q = -\frac{(\chi - 2X) d\chi}{s\Delta^2}$$

Integration gives

$$\ln q = \frac{-(\chi - 2X)^2}{2s\Delta^2} + \ln K$$

Letting $h^2 = \frac{1}{2s\Delta^2}$ and putting the above equation into its exponential form gives

$$q = K e^{-h^2(\chi - 2X)^2}$$

To determine the constant K , consider the special case where $\chi = X$. In this case the liquid fraction attaining X at some time during the distribution and finally ending at χ is the same liquid fraction that finally ends at χ without other imposed restrictions, or expressed

mathematically,

$$k e^{-h^2(x-2X)^2} = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}.$$

Then $k e^{-h^2(x-2X)^2} = \frac{h}{\sqrt{\pi}} e^{-h^2 x^2}$ when $x = X$;

$$k e^{-h^2(-x)^2} = \frac{h}{\sqrt{\pi}} e^{-h^2(x)^2}, \text{ and}$$

$$k = \frac{h}{\sqrt{\pi}}.$$

giving

The equation for the general case is then

$$q = \frac{h}{\sqrt{\pi}} e^{-h^2(x-2X)^2}.$$

Appendix II

Data Sheets and Graphs for Sand Distribution from Center Feed

- (a) Data for Sand Distribution
- (b) Rectified Curves for Sand Distribution
- (c) Data Calculated from Rectified Curves

DATA FOR SAND DISTRIBUTION FROM CENTER FEED
Using Steel Ball No. 1. in Model I

Compartment	Displacement in cm.	Displacement Squared	Fractional Quantity per Compartment at Various Total Feed Rates in cc./min.									
			6.5	10.3	11.3	11.9	20.6	22.5	33.1	37.6	48.7	49.4
1	24.77	613.3	.00067	.00053		.00046	.00054	.00082	.00104		.00180	.00140
2	22.86	522.9	.00134	.00105		.00092	.00162	.00124	.00138		.00216	.00187
3	20.96	439.1	.00269	.00210	.00132	.00184	.00269	.00246	.00278	.00216	.00431	.00468
4	19.05	362.9	.00471	.00371	.00398	.00367	.00479	.00410	.00553	.00432	.00793	.00748
5	17.15	293.9	.00867	.00739	.00794	.00825	.0107	.00902	.0112	.00966	.0148	.0145
6	15.24	232.3	.0135	.0111	.01182	.0128	.0151	.0145	.0160	.0172	.0190	.0206
7	13.34	177.8	.0223	.0206	.0224	.0225	.0229	.0244	.0251	.0285	.0323	.0309
8	11.43	130.7	.0295	.0301	.0347	.0319	.0347	.0322	.0375	.0403	.0418	.0407
9	9.53	90.8	.0431	.0593	.0447	.0418	.0417	.0453	.0444	.0473	.0467	.0472
10	7.62	58.1	.0599	.0717	.0589	.0587	.0603	.0611	.0548	.0624	.0599	.0580
11	5.72	32.7	.0785	.0825	.0776	.0662	.0759	.0754	.0732	.0750	.0747	.0734
12	3.81	14.5	.0930	.0835	.0871	.0885	.0813	.0868	.0850	.0841	.0834	.0795
13	1.91	3.6	.0949	.0850	.0891	.0921	.0851	.0876	.0809	.0822	.0777	.0715
14	0.00	0.0	.0930	.0912	.0912	.0997	.0891	.0940	.0921	.0822	.0762	.0800
15	1.91	3.6	.1011	.0912	.1072	.1008	.1000	.0912	.0930	.0944	.0793	.0851
16	3.81	14.5	.0894	.0921	.1023	.0958	.0933	.0921	.0842	.0923	.0885	.0828
17	5.72	32.7	.0793	.0801	.0832	.0885	.0832	.0859	.0717	.0785	.0710	.0720
18	7.62	58.1	.0587	.0581	.0537	.0587	.0646	.0559	.0618	.0556	.0558	.0571
19	9.53	90.8	.0457	.0466	.0479	.0490	.0490	.0481	.0496	.0507	.0490	.0524
20	11.43	130.7	.0335	.0339	.0331	.0339	.0363	.0346	.0394	.0376	.0402	.0425
21	13.34	177.8	.0202	.0216	.0209	.0225	.0200	.0223	.0246	.0227	.0271	.0295
22	15.24	232.3	.0127	.0133	.0132	.0148	.0145	.0141	.0170	.0153	.0173	.0187
23	17.15	293.9	.00867	.00842	.00794	.00825	.00977	.0086	.0123	.00966	.0127	.0143
24	19.05	362.9	.00471	.00371	.00398	.00414	.00479	.00410	.00588	.00432	.00617	.00748
25	20.96	439.1	.00201	.00212	.00132	.00230	.00214	.00246	.00272	.00216	.00329	.00421
26	22.86	522.9	.00134	.00105		.00092	.00107	.00124	.00138		.00146	.00187
27	24.77	613.3	.00067	.00053		.00046	.00054	.00041	.00069		.00066	.00094

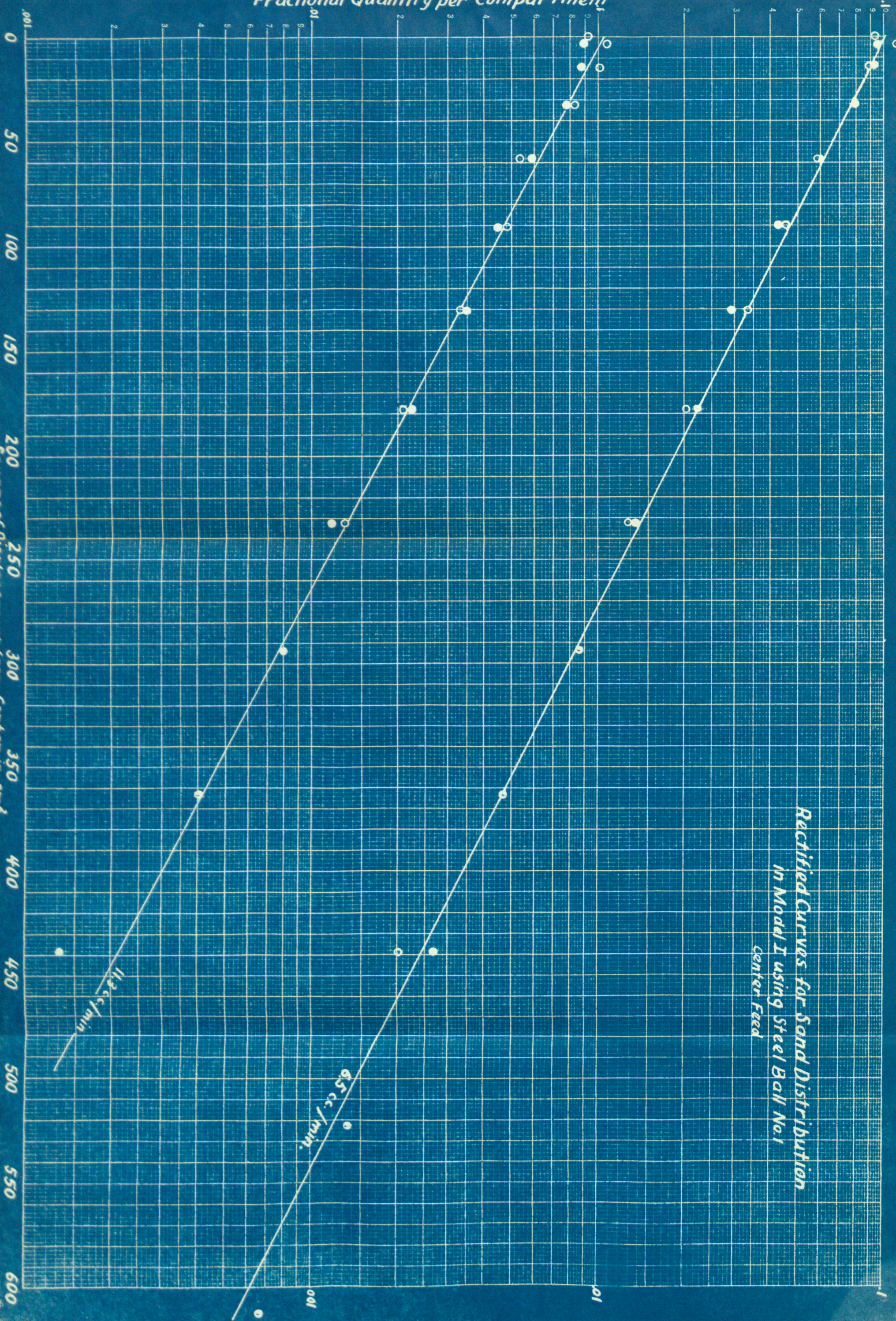
DATA FOR SAND DISTRIBUTION FROM CENTER FEED
Using Steel Ball No. 1. in Model I

Compartment	Displacement in cm.	Displacement Squared	Fractional Quantity per Compartment at Various Total Feed Rates in cc./min.									
			62.9	70.0	72.9	96.0	97.6	98.5	101.1	111.9	119.8	
1	24.77	613.3	.00234	.00184	.00187	.00439	.00232	.00267	.00324	.00476	.00289	
2	22.86	522.9	.00313	.00279	.00307	.00476	.00348	.00444	.00454	.00611	.00382	
3	20.96	439.1	.00623	.00556	.00614	.00921	.00696	.00789	.00876	.0109	.00801	
4	19.05	362.9	.0102	.00822	.00937	.0143	.0113	.0125	.0130	.0170	.0125	
5	17.15	293.9	.0172	.0168	.0196	.0246	.0212	.0222	.0237	.0269	.0220	
6	15.24	232.3	.0244	.0232	.0253	.0316	.0261	.0300	.0292	.0326	.0280	
7	13.34	177.8	.0374	.0351	.0364	.0414	.0354	.0361	.0399	.0431	.0371	
8	11.43	130.7	.0398	.0412	.0445	.0435	.0412	.0444	.0448	.0426	.0414	
9	9.53	90.8	.0466	.0473	.0487	.0466	.0464	.0454	.0467	.0495	.0435	
10	7.62	58.1	.0617	.0582	.0579	.0593	.0571	.0572	.0571	.0553	.0549	
11	5.72	32.7	.0710	.0684	.0686	.0617	.0638	.0671	.0678	.0611	.0605	
12	3.81	14.5	.0769	.0684	.0709	.0675	.0649	.0656	.0694	.0636	.0643	
13	1.91	3.6	.0696	.0684	.0694	.0630	.0606	.0641	.0613	.0564	.0553	
14	0.00	0.0	.0761	.0750	.0744	.0717	.0647	.0703	.0629	.0642	.0656	
15	1.91	3.6	.0793	.0804	.0809	.0724	.0707	.0736	.0701	.0674	.0689	
16	3.81	14.5	.0710	.0804	.0778	.0696	.0751	.0714	.0659	.0669	.0724	
17	5.72	32.7	.0731	.0750	.0717	.0732	.0664	.0736	.0678	.0662	.0717	
18	7.62	58.1	.0531	.0543	.0552	.0526	.0548	.0533	.0555	.0486	.0516	
19	9.53	90.8	.0537	.0556	.0512	.0547	.0571	.0585	.0561	.0526	.0599	
20	11.43	130.7	.0449	.0462	.0426	.0486	.0487	.0486	.0477	.0471	.0476	
21	13.34	177.8	.0301	.0299	.0311	.0342	.0391	.0321	.0367	.0386	.0406	
22	15.24	232.3	.0203	.0202	.0207	.0261	.0307	.0233	.0250	.0286	.0298	
23	17.15	293.9	.0150	.0157	.0157	.0196	.0232	.0177	.0195	.0259	.0271	
24	19.05	362.9	.00903	.00733	.00844	.0114	.0133	.00885	.0110	.0146	.0151	
25	20.96	439.1	.00466	.00367	.00422	.00662	.00783	.00533	.00649	.00817	.00894	
26	22.86	522.9	.00275	.00184	.00230	.00406	.00463	.00267	.00357	.00510	.00510	
27	24.77	613.3	.00157	.00092	.00087	.00295	.00319	.00088	.00227	.00375	.00382	

DATA FOR SAND DISTRIBUTION FROM CENTER FEED
Using Steel Ball No. 2 in Model I

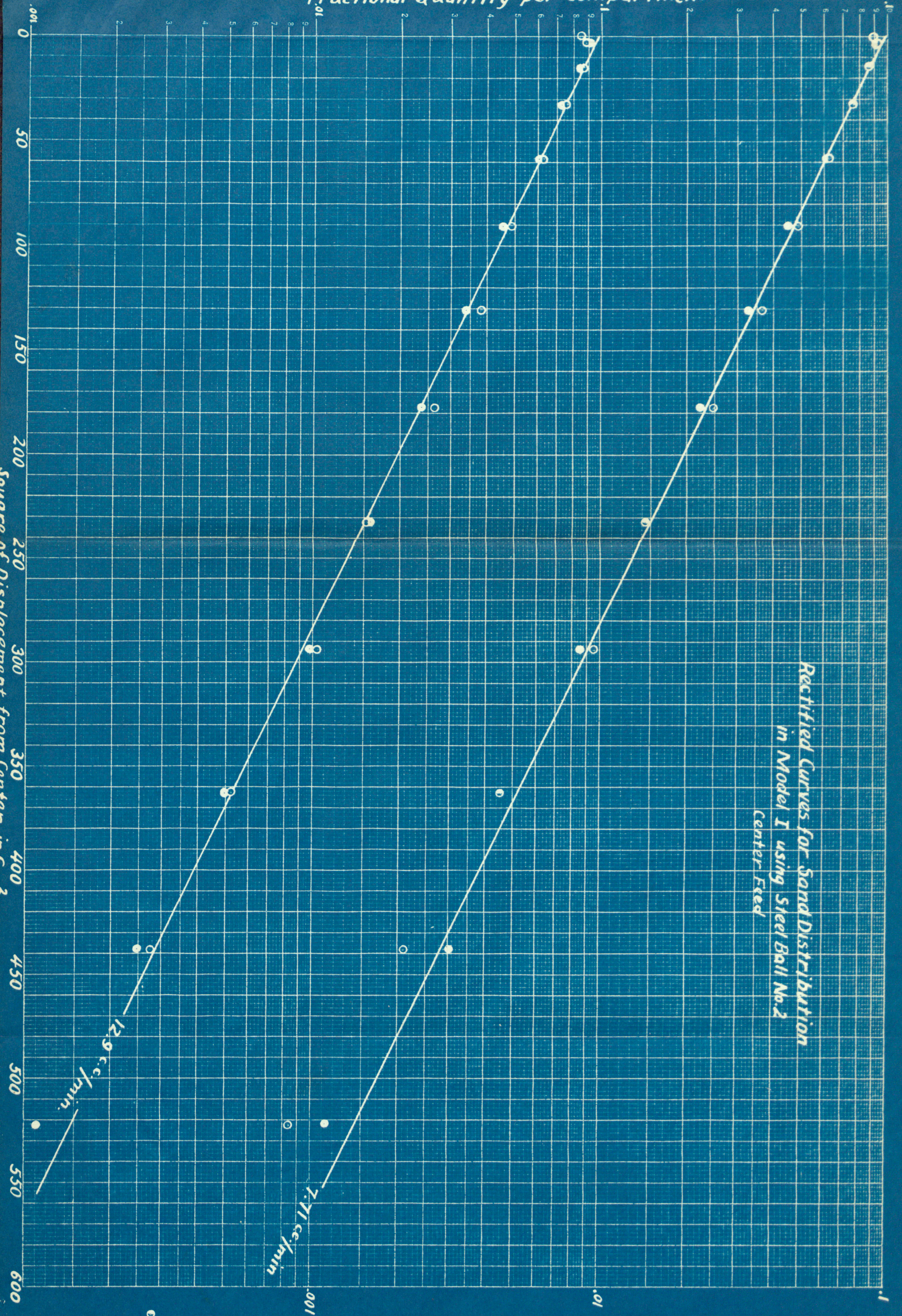
Compartment	Displacement in cm.	Displacement Squared	Fractional Quantity per Compartment at Various Total Feed Rates in cc./min									
			7.71	12.9	24.0	38.3	58.5	67.5	71.0	71.7	86.5	82.6
1	24.77	613.3	.00028	.00055	.00057	.00112	.0014	.0022	.0013	.00208	.00311	.0019
2	22.86	522.9	.00085	.00110	.00114	.00203	.0021	.0029	.0019	.00283	.00363	.0026
3	20.96	439.1	.00226	.00275	.00285	.00405	.0049	.0062	.0049	.00623	.00761	.0045
4	19.05	362.9	.00453	.00523	.00570	.00675	.0080	.0092	.0084	.00869	.01038	.0101
5	17.15	293.9	.00961	.01046	.01167	.0137	.0149	.0179	.0159	.01740	.0201	.0186
6	15.24	232.3	.01472	.01542	.0171	.0203	.0215	.0242	.0226	.0238	.0259	.0264
7	13.34	177.8	.0249	.0265	.0262	.0292	.0340	.0351	.0372	.0333	.0339	.0369
8	11.43	130.7	.0368	.0385	.0399	.0405	.0433	.0406	.0443	.0450	.0457	.0443
9	9.53	90.8	.0493	.0490	.0496	.0498	.0437	.0458	.0482	.0510	.0502	.0428
10	7.62	58.1	.0628	.0633	.0632	.0616	.0624	.0575	.0630	.0627	.0588	.0622
11	5.72	32.7	.0775	.0760	.0740	.0724	.0700	.0750	.0776	.0676	.0657	.0752
12	3.81	14.5	.0876	.0870	.0837	.0755	.0815	.0706	.0754	.0680	.0654	.0677
13	1.91	3.6	.0911	.0886	.0819	.0822	.0766	.0710	.0764	.0752	.0681	.0707
14	0.00	0.0	.0894	.0859	.0836	.0755	.0787	.0744	.0780	.0718	.0664	.0726
15	1.91	3.6	.0928	.0914	.0911	.0841	.0860	.0838	.0780	.0748	.0712	.0764
16	3.81	14.5	.0860	.0853	.0831	.0836	.0836	.0784	.0767	.0775	.0720	.0815
17	5.72	32.7	.0764	.0726	.0723	.0706	.0798	.0740	.0737	.0751	.0688	.0745
18	7.62	58.1	.0611	.0611	.0626	.0620	.0563	.0556	.0560	.0567	.0576	.0573
19	9.53	90.8	.0453	.0457	.0480	.0508	.0444	.0542	.0501	.0491	.0508	.0458
20	11.43	130.7	.0328	.0341	.0347	.0373	.0388	.0414	.0369	.0408	.0426	.0413
21	13.34	177.8	.0221	.0237	.0251	.0274	.0281	.0300	.0304	.0329	.0377	.0339
22	15.24	232.3	.0147	.0160	.01765	.0207	.0173	.0212	.0191	.0223	.0284	.0216
23	17.15	293.9	.00849	.00981	.01138	.01305	.0118	.0147	.0126	.0159	.0194	.0153
24	19.05	362.9	.00453	.00496	.00626	.0055	.0055	.0077	.0061	.00945	.0121	.0078
25	20.96	439.1	.00306	.00248	.00342	.00405	.0028	.0040	.0029	.00490	.0069	.0041
26	22.86	522.9	.00113	.00110	.00171	.00225	.0010	.0022	.0016	.00300	.00415	.0022
27	24.77	613.3	.00028	.00055	.00057	.00067	.0003	.0011	.0006	.00170	.00277	.0007

Fractional Quantity per Compartment



Rectified Curves for Sand Distribution
in Model I using Steel Ball No. 1
Center Feed

Fractional Quantity per Compartment

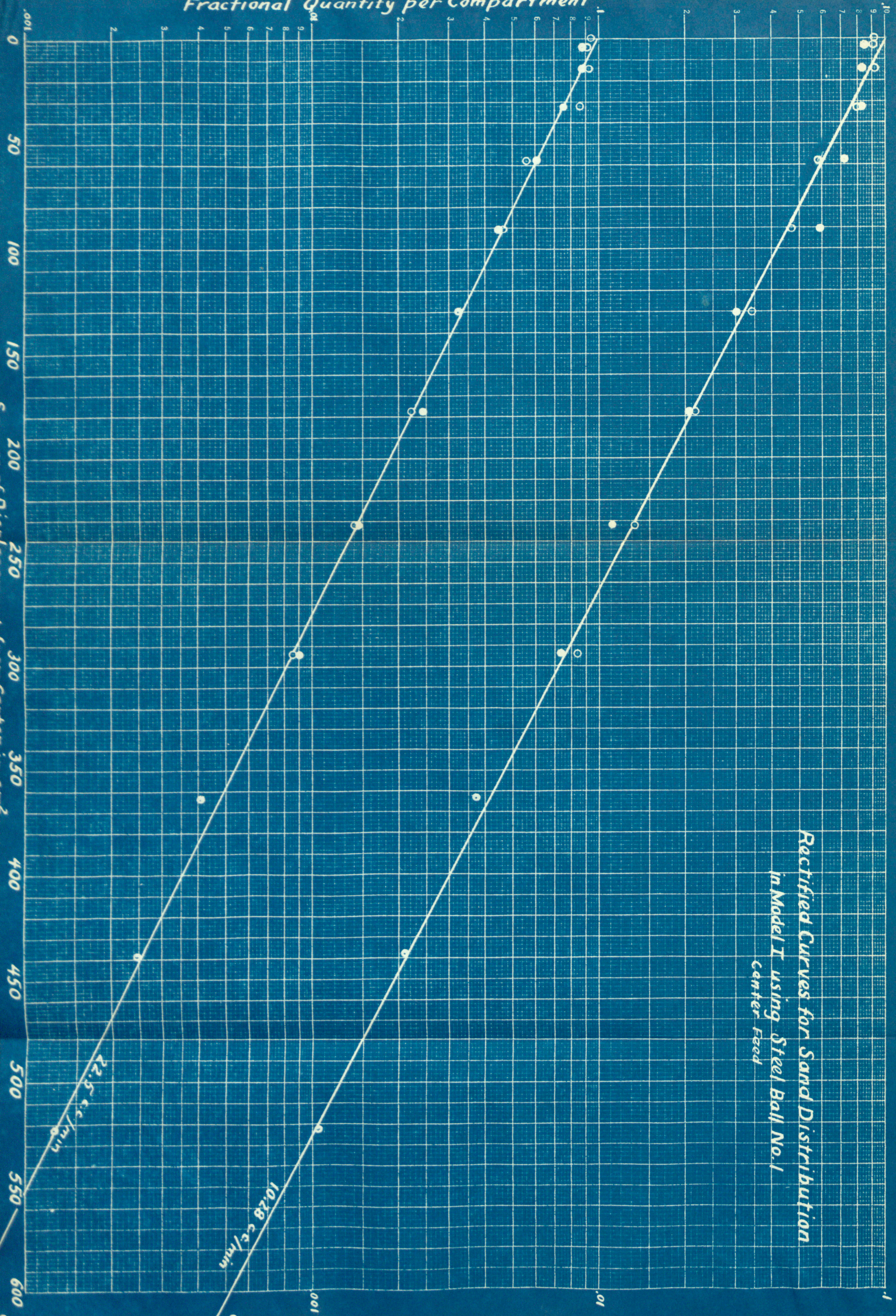


Rectified Curves for Sand Distribution
 in Model I using Steel Ball No. 2
 Center Feed

12.9 cc/min.

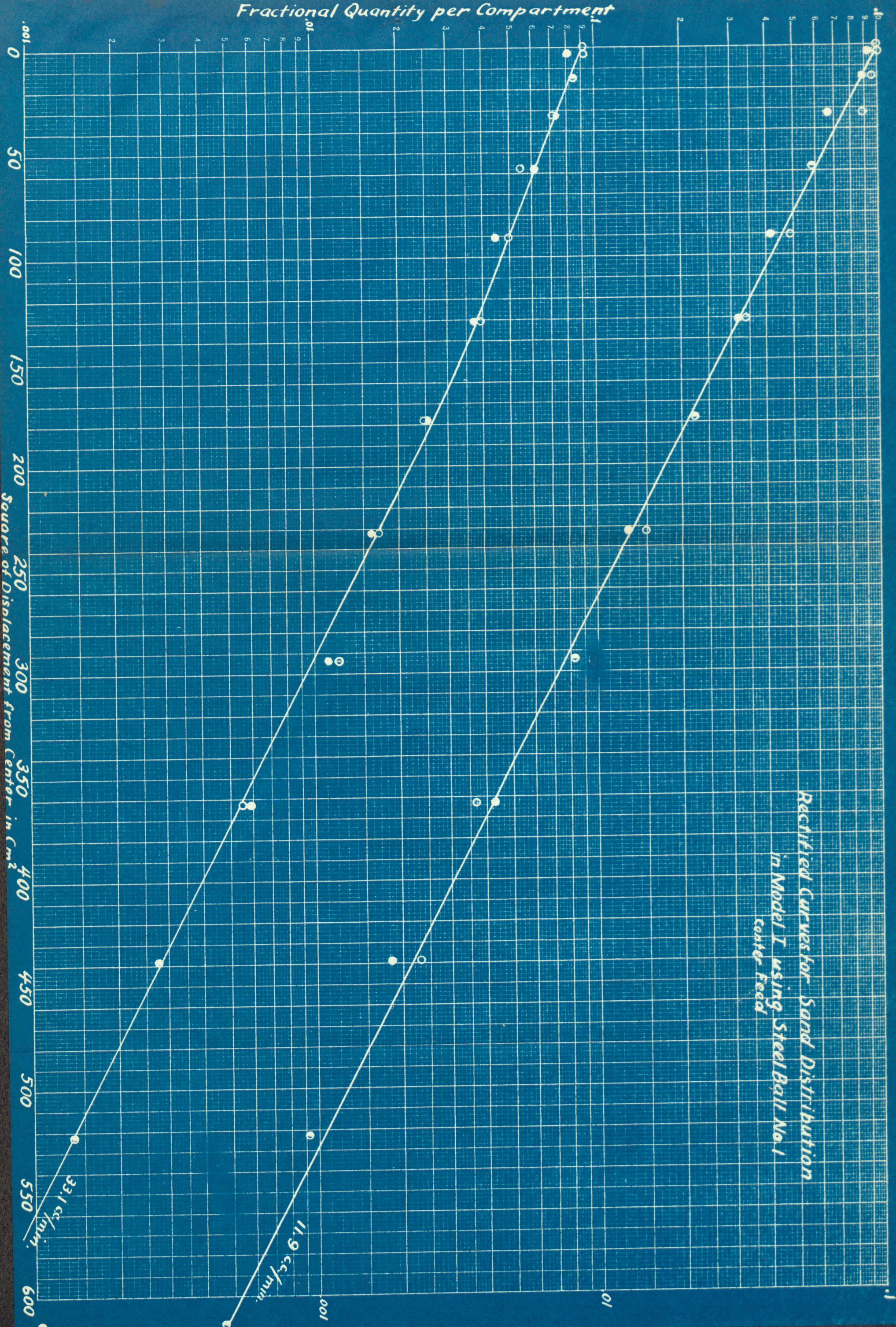
7.71 cc/min.

Fractional Quantity per Compartment

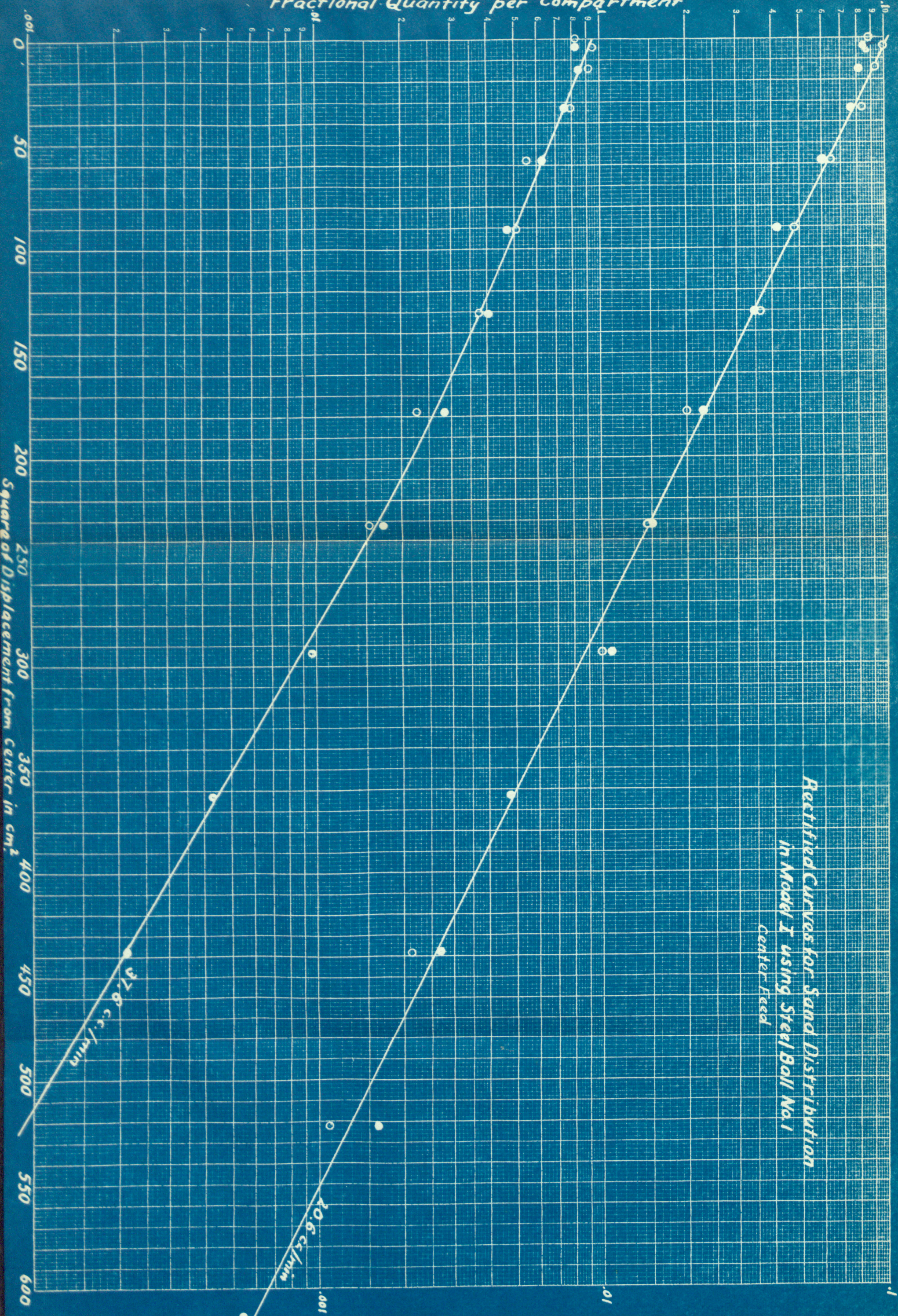


Rectified Curves for Sand Distribution
in Model I using Steel Ball No. 1
Center Feed

Fractional Quantity per Compartment

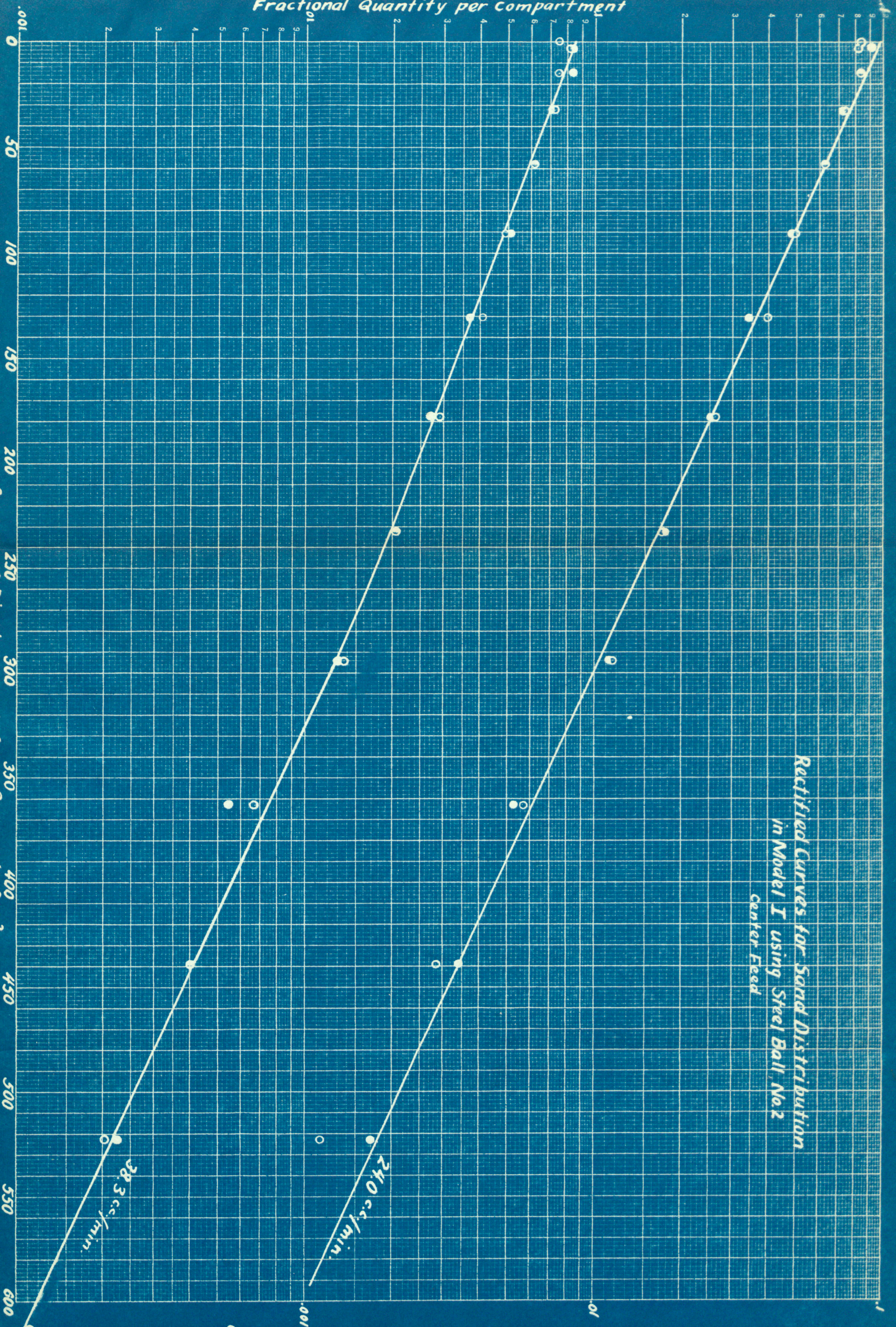


Fractional Quantity per Compartment



Rectified Curves for Sand Distribution
in Model I using Steel Ball No. 1
Center Feed

Fractional Quantity per Compartment



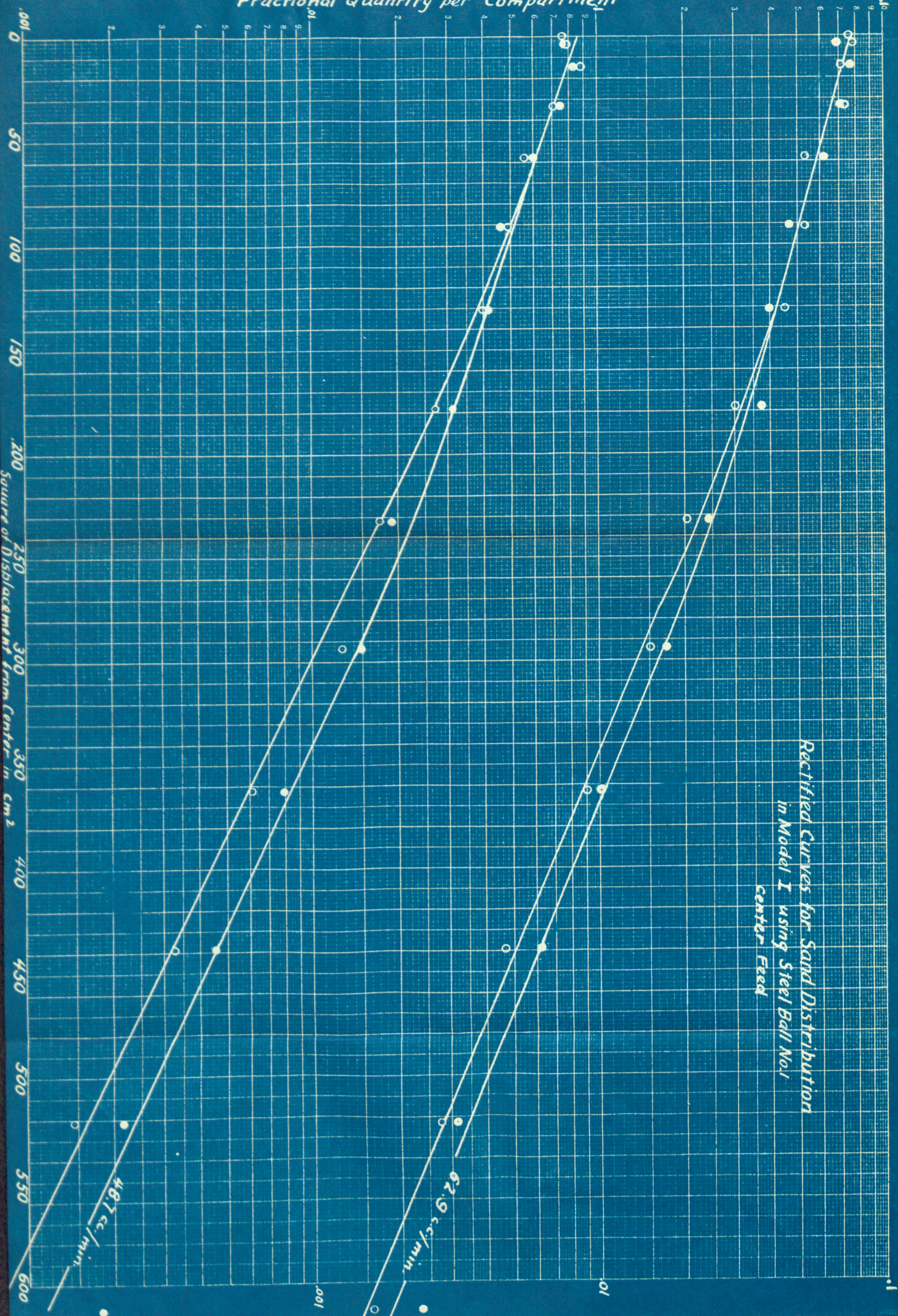
Rectified Curves for Sand Distribution
in Model I using Steel Ball No. 2
Center Feed

Center Feed

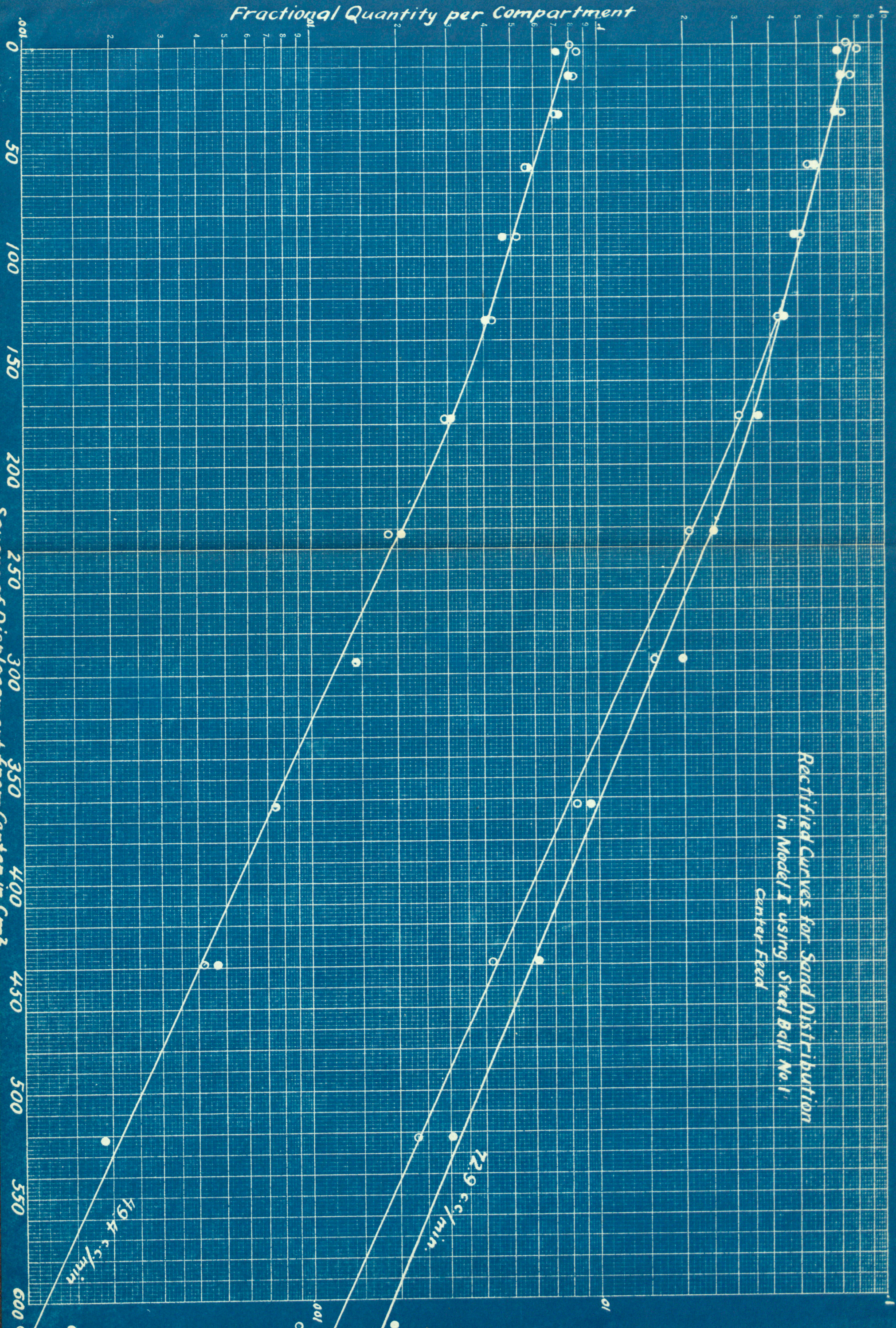
38.3 cc/min.

24.0 cc/min.

Fractional Quantity per Compartment



Fractional Quantity per Compartment

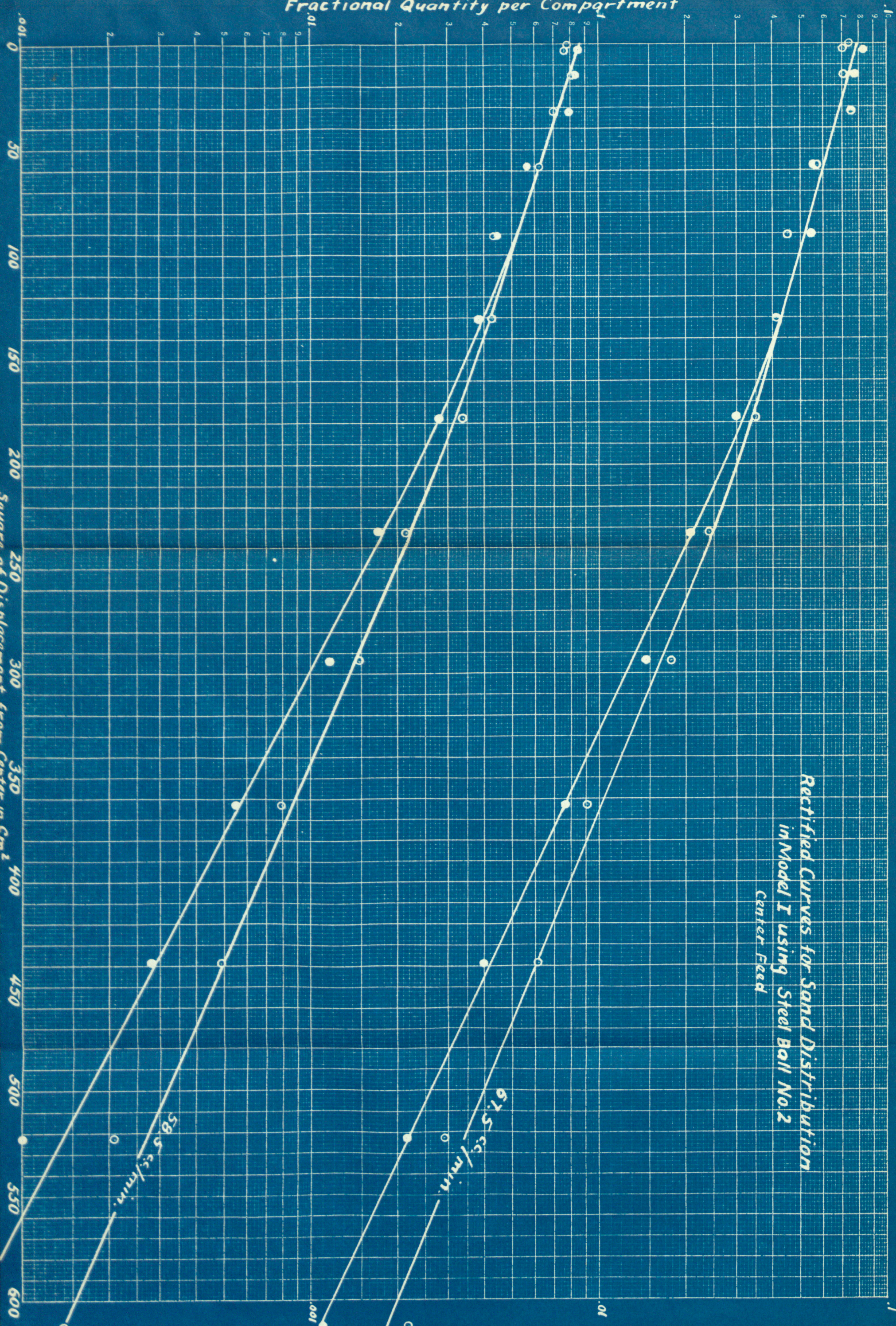


Rectified Curves for Sand Distribution
in Model I using Steel Ball No. 1
Center Feed

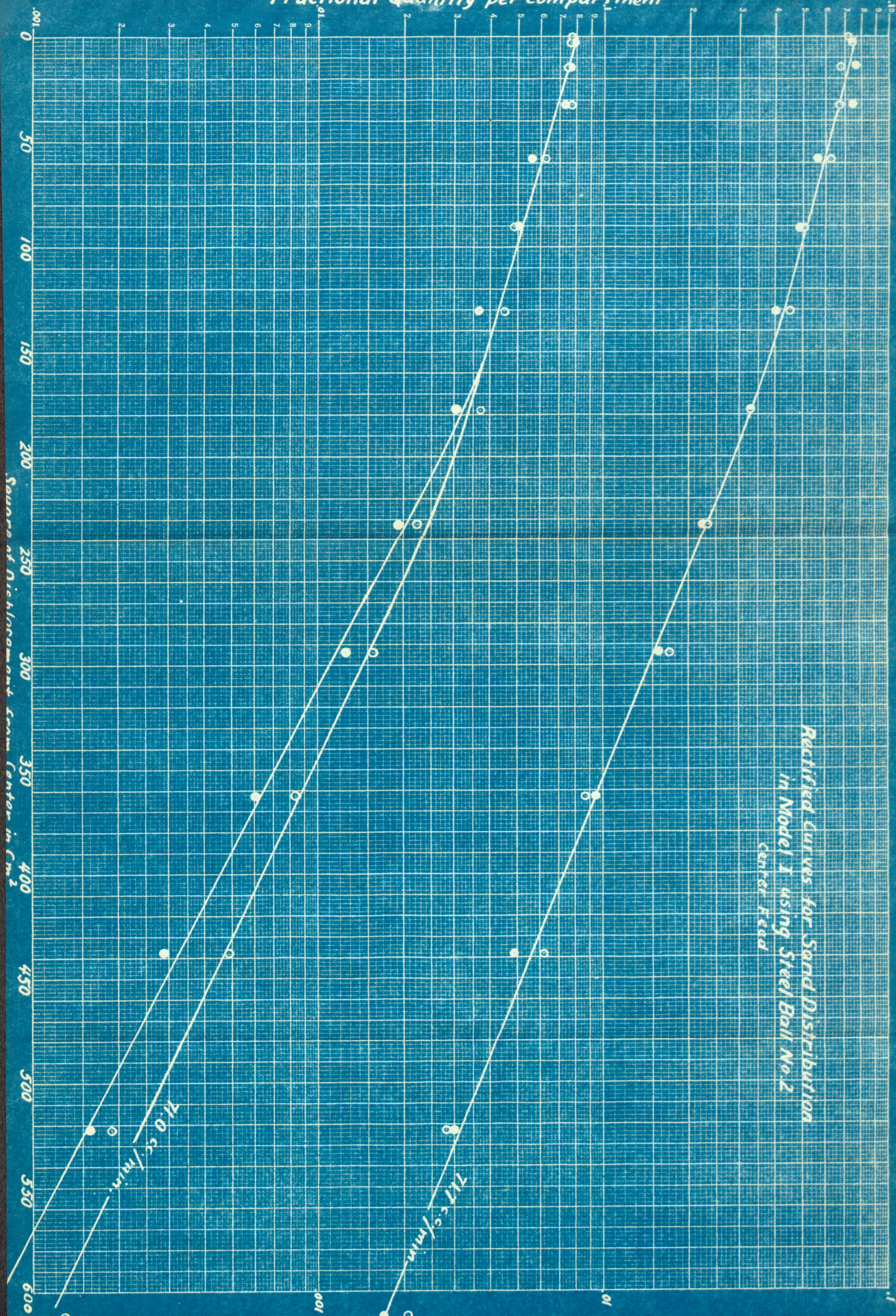
42.9 c.c./min.

49.4 c.c./min.

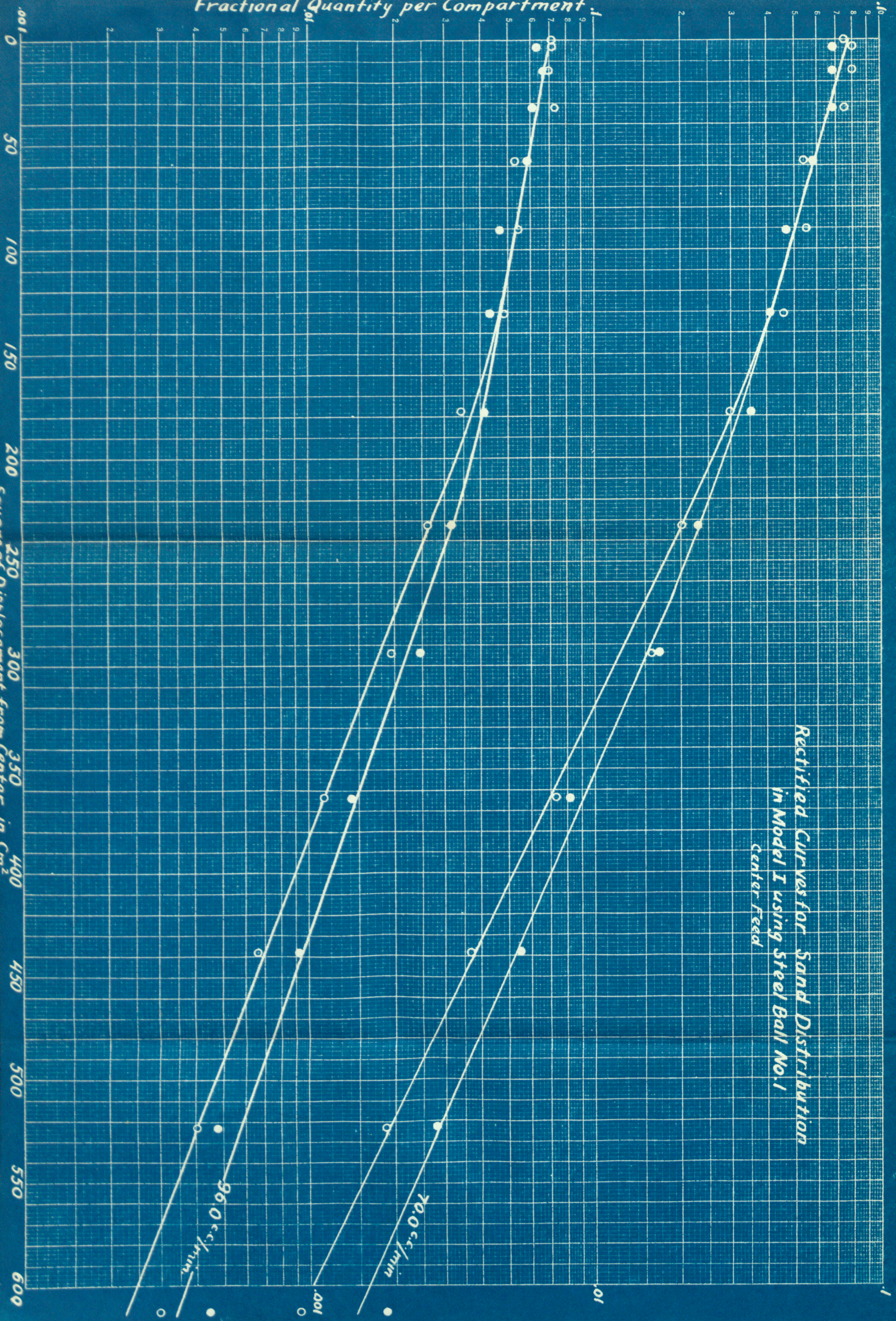
Fractional Quantity per Compartment



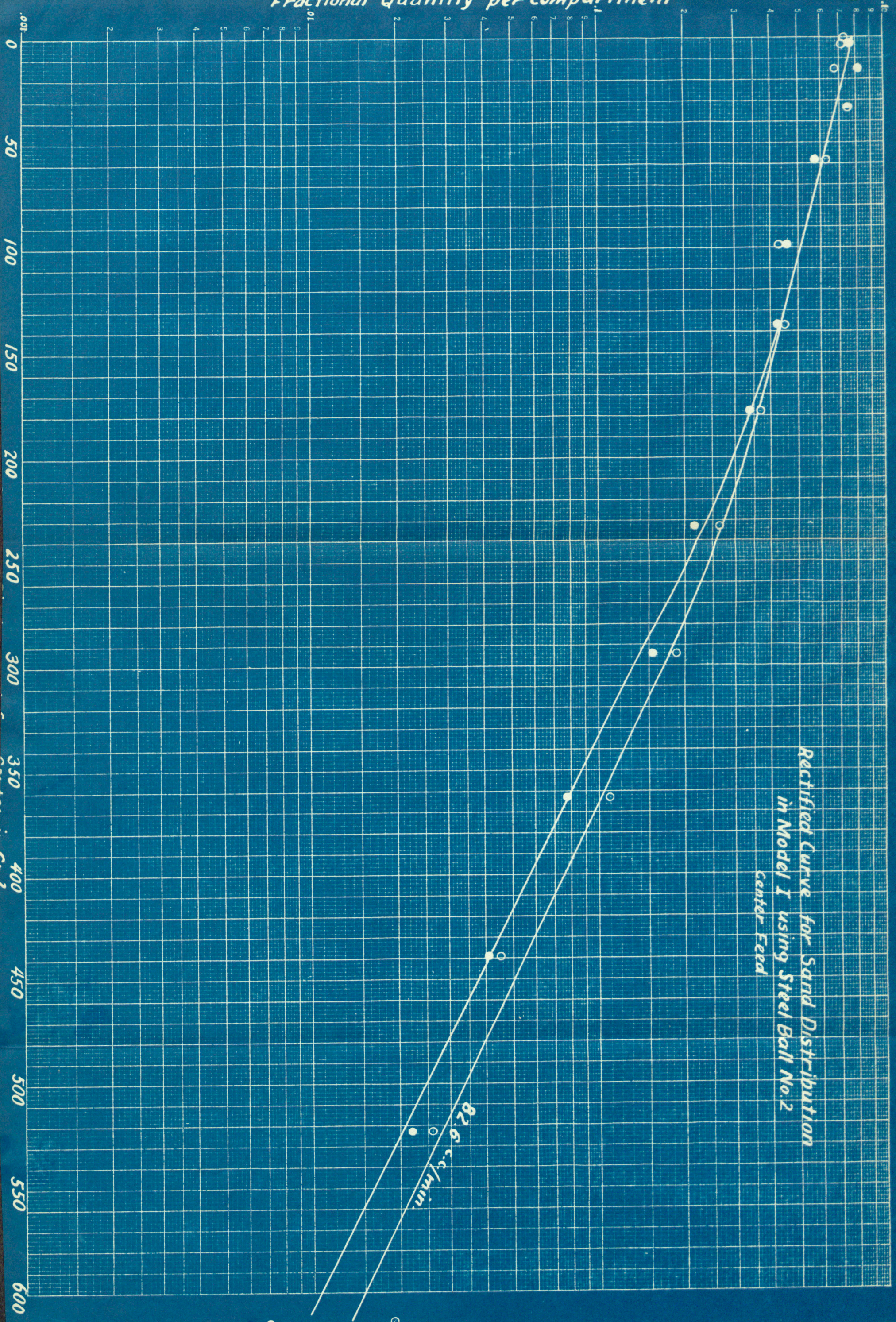
Fractional Quantity per Compartment



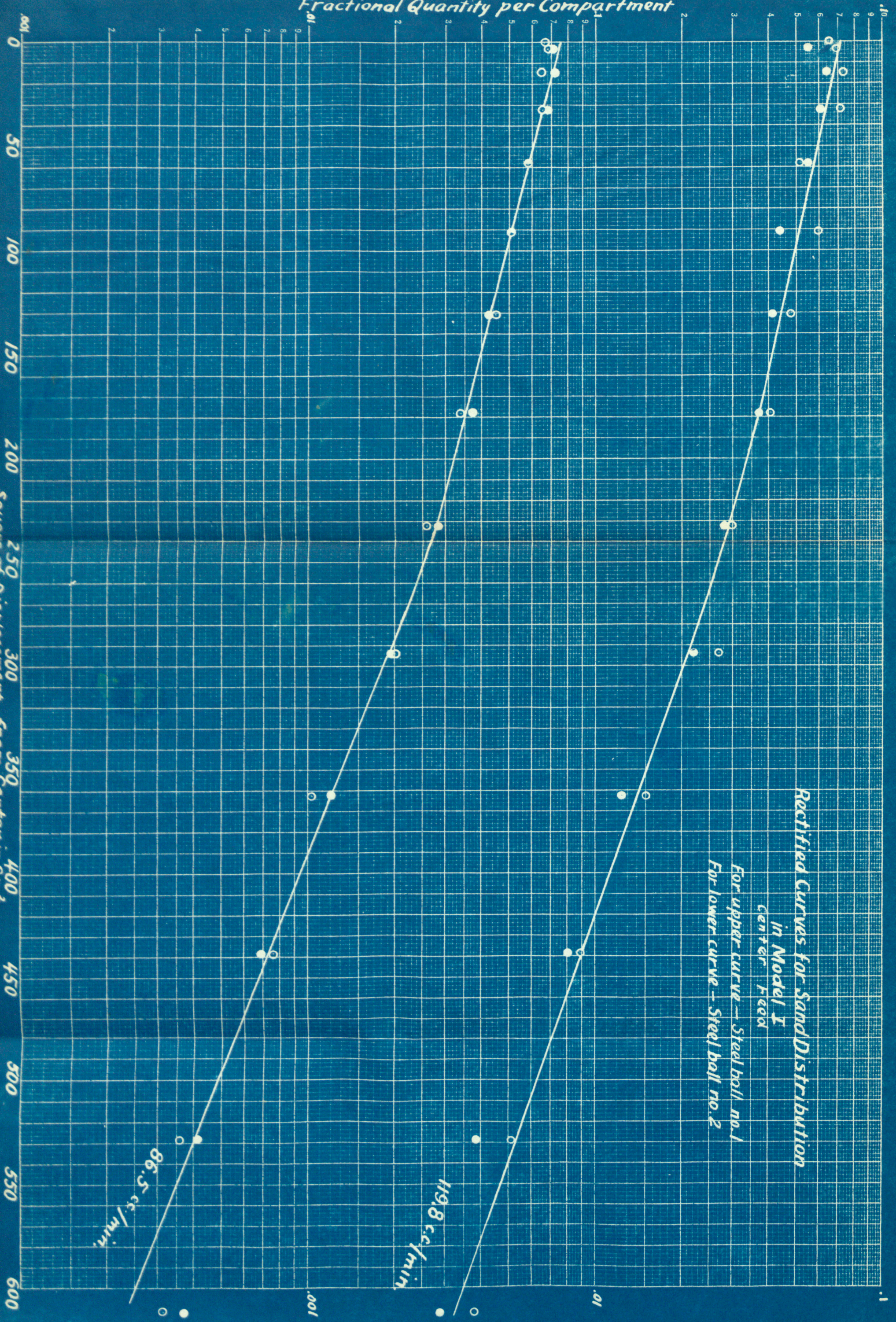
Fractional Quantity per Compartment



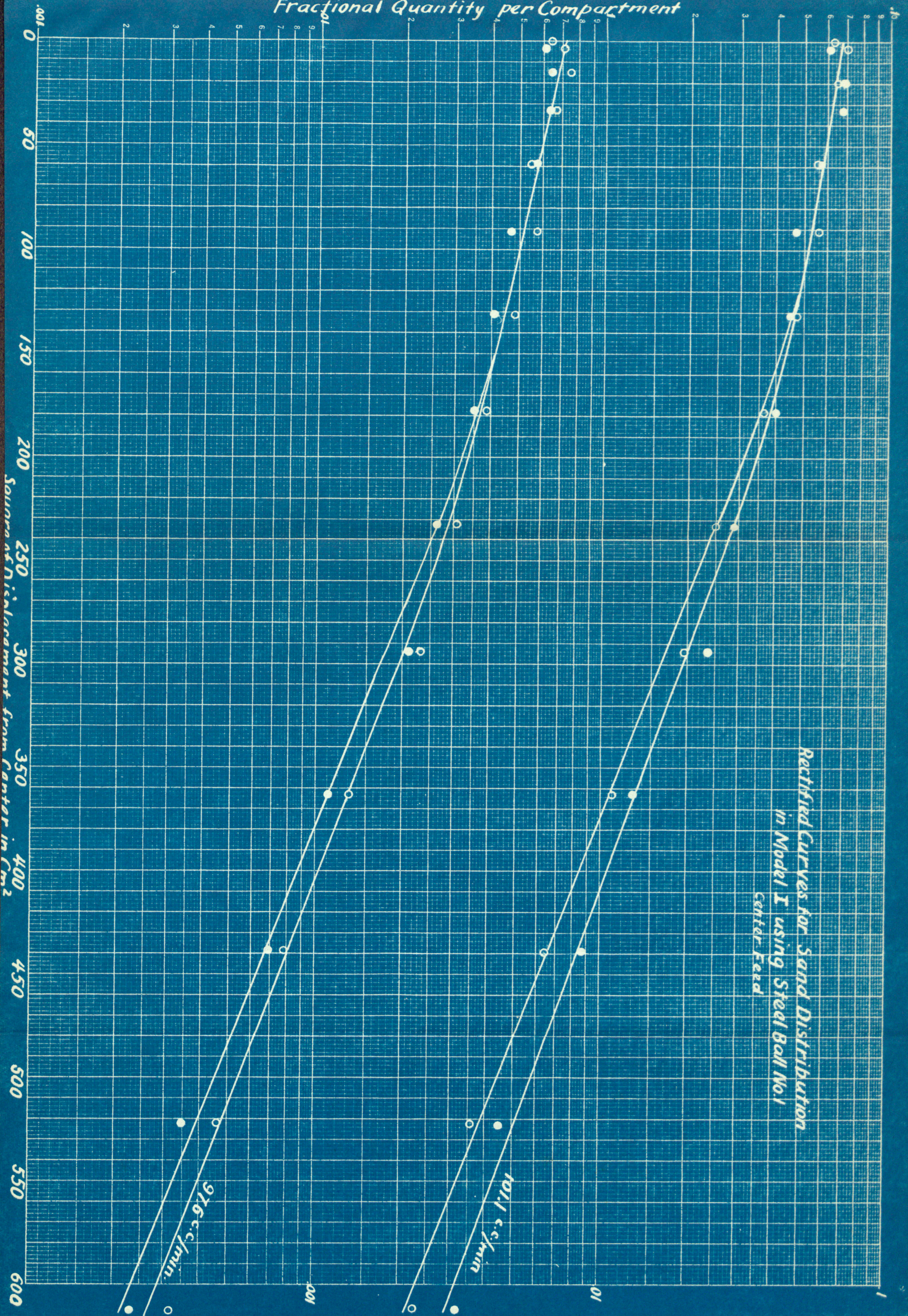
Fractional Quantity per Compartment



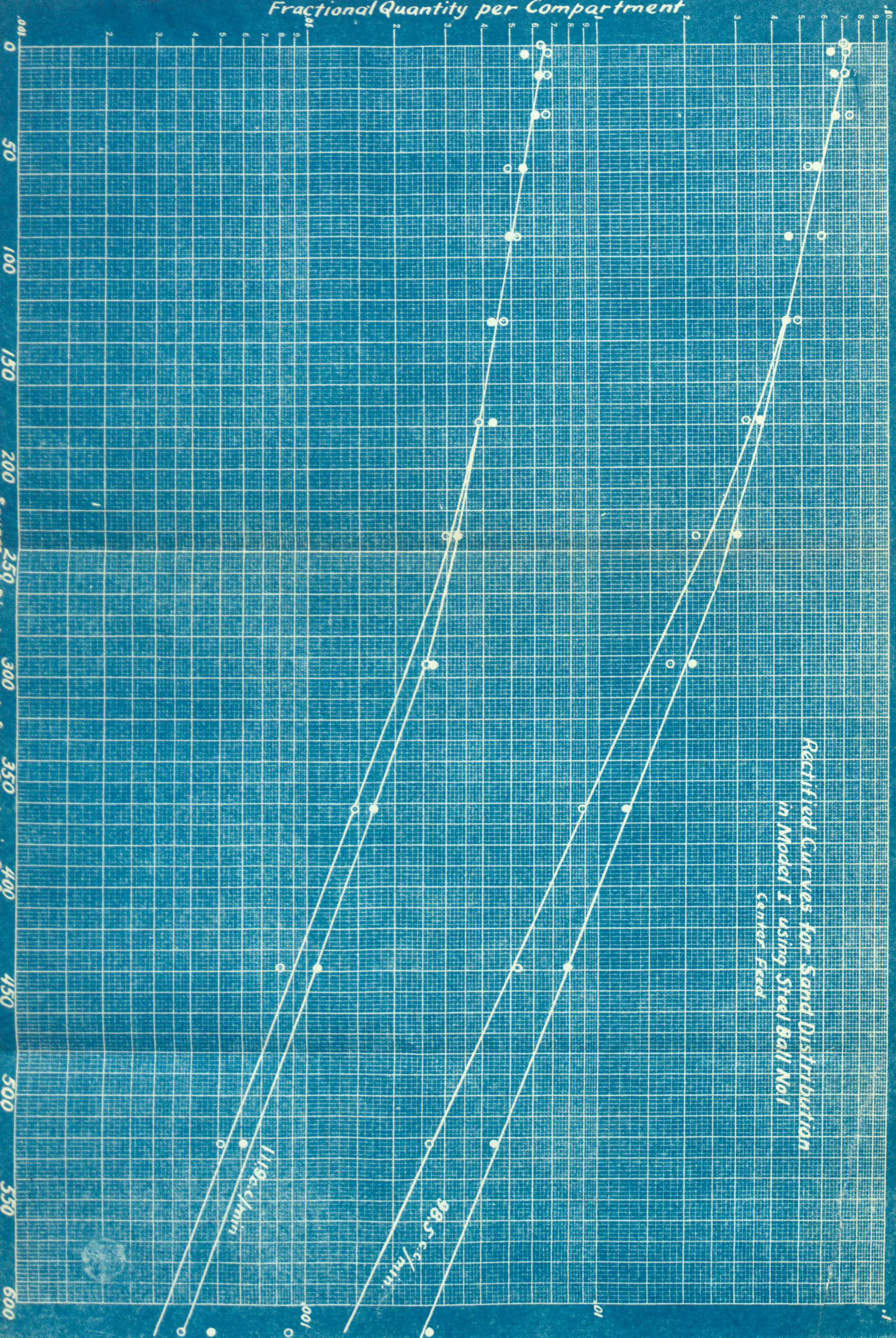
Fractional Quantity per Compartment



Fractional Quantity per Compartment



Fractional Quantity per Compartment



DATA CALCULATED FROM RECTIFIED CURVES
for Sand Distribution from Center Feed in Model I

(a) Low Flow Rates

Flow Rate in cc/min.	h^2 in l/cm^2	K in $l/cm.$	$\frac{K\sqrt{\pi}}{h}$
6.5	.00848	.0530	1.020
* 7.7	.00810	.0519	1.022
10.3	.00880	.0541	1.021
11.3	.00888	.0546	1.026
11.9	.00880	.0540	1.020
*12.9	.00806	.0520	1.027
20.6	.00844	.0552	1.063
22.5	.00833	.0519	1.006
*24.0	.00763	.0509	1.032

* Steel ball #2 used for this run, #1 for the others

DATA CALCULATED FROM RECTIFIED CURVES
for Sand Distribution from Center Feed in Model I

(b) Intermediate Flows

Flow Rate in cc/min.	h^2 in 1/cm. ²	K in 1/cm.	Secondary Slopes h_A^2 h_B^2		Secondary Intercepts K_A K_B	
33.1	.00568	.0478	.00867		.0656	
37.6	.00711	.0499	.00998		.0861	
38.3	.00634	.0452	.00784		.0656	
48.7	.00574	.0457	.00792	.00820	.0762	.0604
49.4	.00524	.0426	.00780	.	.0645	
*58.5	.00554	.0457	.00731	.00883	.0630	.0719
62.9	.00472	.0410	.00695	.00700	.0682	.0630
*67.5	.00467	.0415	.00687	.00783	.0656	.0693
70.0	.00484	.0410	.00734	.00811	.0709	.0735
71.0	.00489	.0426	.00814	.00743	.0840	.0813
*71.7	.00448	.0404	.00686	.	.0619	
72.9	.00449	.0409	.00706	.00755	.0645	.0671

* Steel ball #2 used for this run, #1 for the others

DATA CALCULATED FROM RECTIFIED CURVES
for Sand Distribution from Center Feed in Model I

(c) High Flow Rates

Flow Rate in cc/min.	k^2 in l/cm. ²	K in l/cm.	Secondary Slopes k_A^2 k_B^2		Secondary Intercepts K_A K_B	
*82.6	.00426	.0409	.00808	.00844	.0945	.0866
*86.5	.00431	.0399	.00664		.0693	
96.0	.00317	.0373	.00591	.00643	.0640	.0604
97.6	.00372	.0378	.00634	.00656	.0682	.0630
98.5	.00392	.0399	.00644	.00750	.0709	.0709
101.1	.00282	.0357	.00604	.00649	.0630	.0604
111.9	.00299	.00347	.00591	.00619	.0761	.0709
119.8	.00375	.0378	.00587		.0630	.

* Steel ball #2 used for this run, #1 for the others

Appendix III

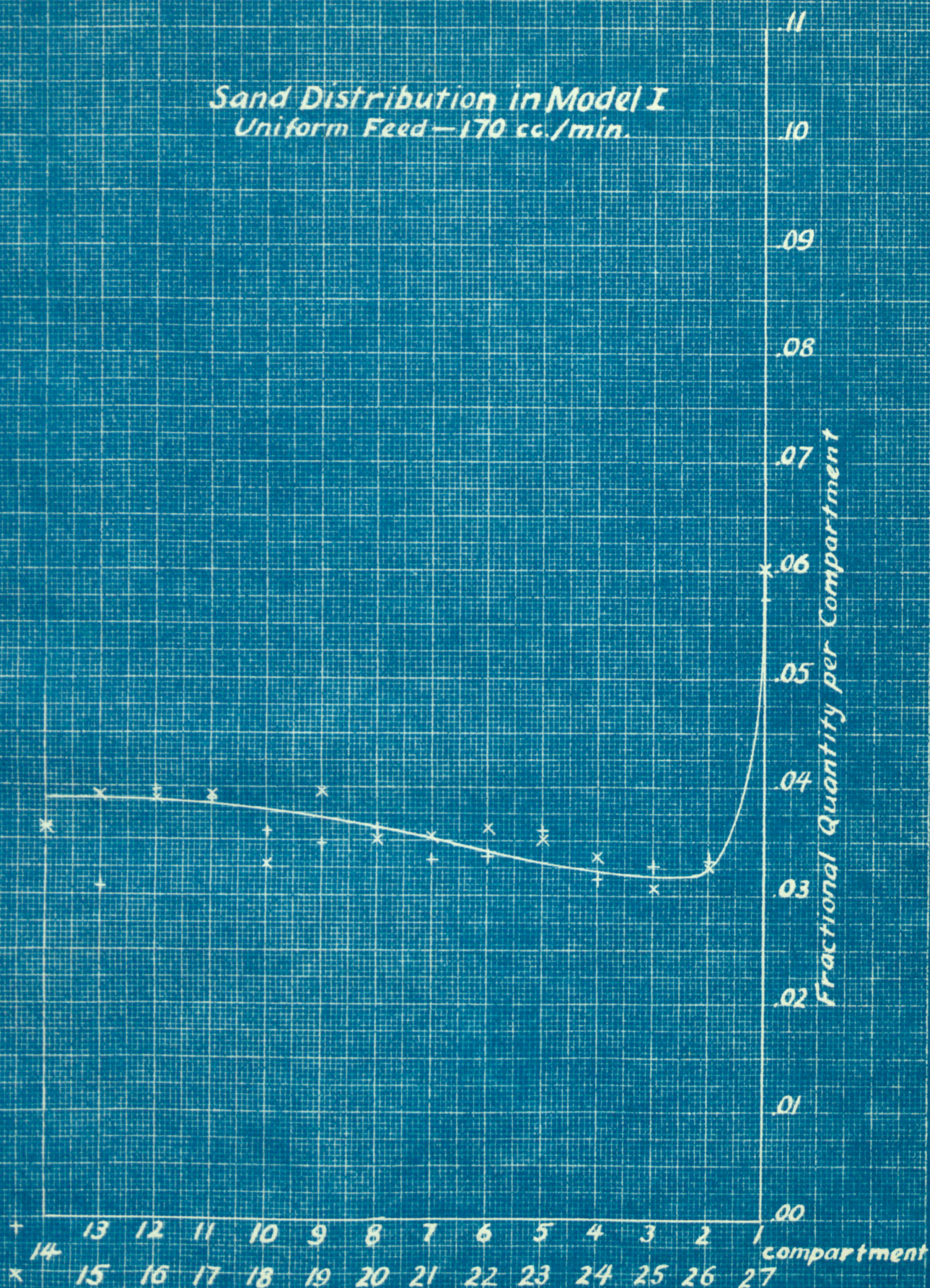
Data Sheet and Graphs for Sand Distribution from Uniform Feed

- (a) Data for Sand Distribution
- (b) Graphs for Sand Distribution

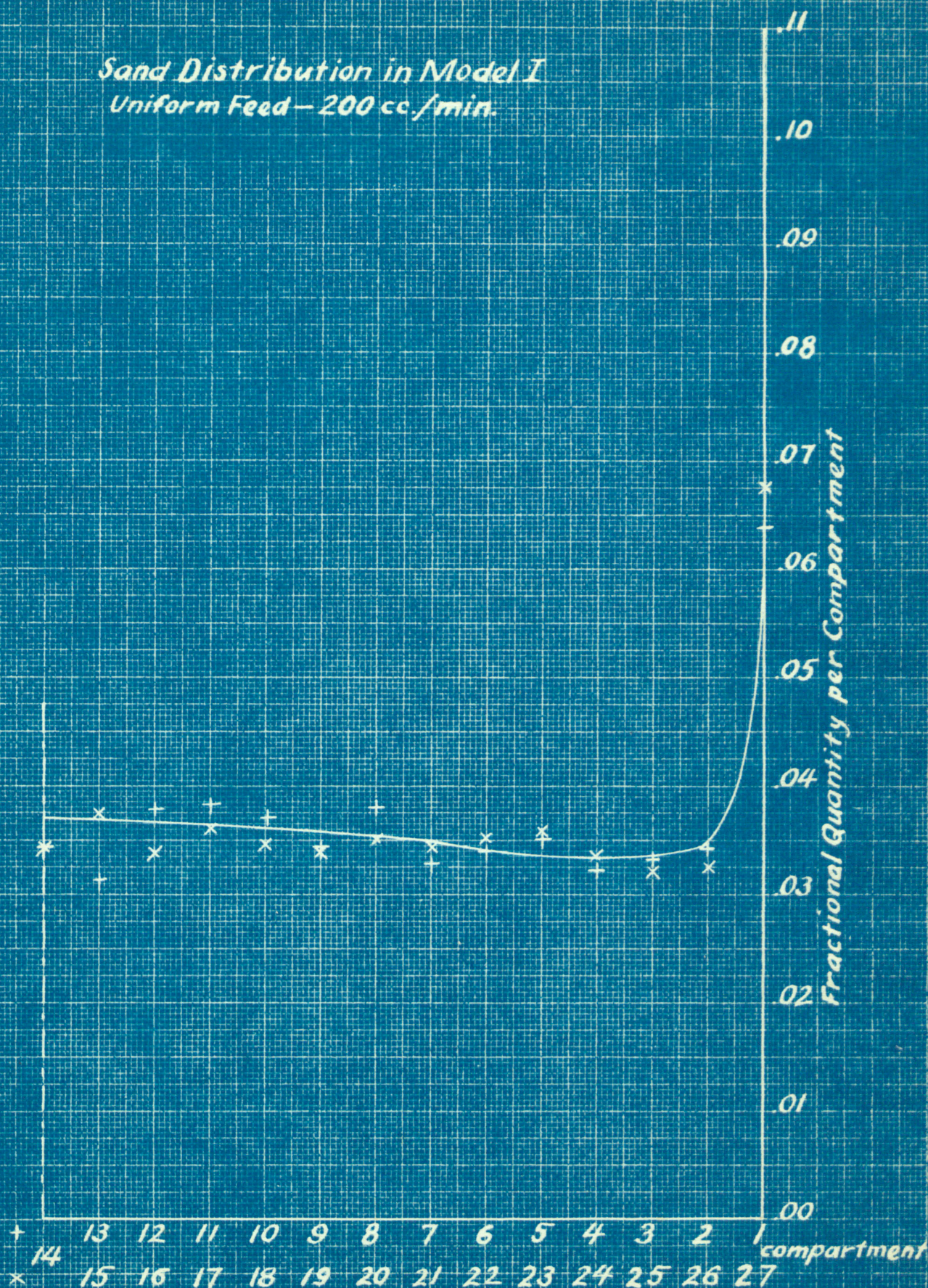
DATA FOR SAND DISTRIBUTION FROM UNIFORM FEED
Using Steel Ball No. 2 in Model I

Compartment	Displacement in cm.	Displacement Squared	Fractional Quantity per Compartment at Various Total Feed Rates in cc./min									
			170	200	245	247	379	400	561	594	858	859
1	24.77	613.3	.0573	.0639	.0605	.0646	.0917	.0840	.0907	.0938	.1215	.1326
2	22.86	522.9	.0332	.0342	.0324	.0319	.0322	.0315	.0312	.0333	.0675	.0678
3	20.96	439.1	.0329	.0333	.0329	.0322	.0318	.0305	.0304	.0318	.0544	.0542
4	19.05	362.9	.0315	.0322	.0314	.0296	.0303	.0284	.0312	.0303	.0516	.0524
5	17.15	293.9	.0360	.0352	.0342	.0322	.0341	.0337	.0336	.0353	.0503	.0467
6	15.24	232.3	.0335	.0340	.0335	.0319	.0314	.0308	.0300	.0310	.0343	.0330
7	13.34	177.8	.0332	.0328	.0317	.0317	.0299	.0299	.0292	.0303	.0292	.0291
8	11.43	130.7	.0362	.0380	.0371	.0373	.0337	.0356	.0332	.0345	.0306	.0291
9	9.53	90.8	.0346	.0344	.0327	.0340	.0311	.0331	.0272	.0287	.0228	.0229
10	7.62	58.1	.0357	.0372	.0358	.0363	.0337	.0350	.0296	.0307	.0251	.0234
11	5.72	32.7	.0388	.0383	.0376	.0378	.0367	.0356	.0296	.0318	.0256	.0251
12	3.81	14.5	.0396	.0377	.0394	.0393	.0359	.0371	.0304	.0326	.0251	.0238
13	1.91	3.6	.0307	.0312	.0288	.0294	.0284	.0284	.0251	.0257	.0187	.0185
14	0.00	0.0	.0363	.0342	.0327	.0350	.0326	.0331	.0284	.0287	.0215	.0203
15	1.91	3.6	.0393	.0374	.0376	.0383	.0337	.0362	.0296	.0291	.0233	.0229
16	3.81	14.5	.0388	.0339	.0360	.0363	.0359	.0356	.0296	.0299	.0233	.0220
17	5.72	32.7	.0391	.0361	.0371	.0373	.0371	.0343	.0300	.0299	.0224	.0212
18	7.62	58.1	.0329	.0347	.0363	.0365	.0292	.0346	.0296	.0283	.0206	.0198
19	9.53	90.8	.0396	.0339	.0350	.0375	.0381	.0356	.0300	.0287	.0196	.0225
20	11.43	130.7	.0351	.0350	.0337	.0350	.0326	.0315	.0308	.0307	.0192	.0242
21	13.34	177.8	.0354	.0344	.0324	.0345	.0322	.0337	.0320	.0299	.0224	.0264
22	15.24	232.3	.0363	.0353	.0371	.0363	.0333	.0356	.0345	.0353	.0238	.0291
23	17.15	293.9	.0351	.0358	.0368	.0360	.0337	.0315	.0389	.0420	.0302	.0308
24	19.05	362.9	.0337	.0336	.0345	.0352	.0337	.0315	.0376	.0391	.0297	.0322
25	20.96	439.1	.0309	.0320	.0327	.0333	.0307	.0284	.0442	.0345	.0329	.0308
26	22.86	522.9	.0326	.0325	.0329	.0368	.0307	.0299	.0361	.0398	.0354	.0326
27	24.77	613.3	.0603	.0678	.0724	.0636	.0946	.0940	.1171	.1065	.1160	.1040

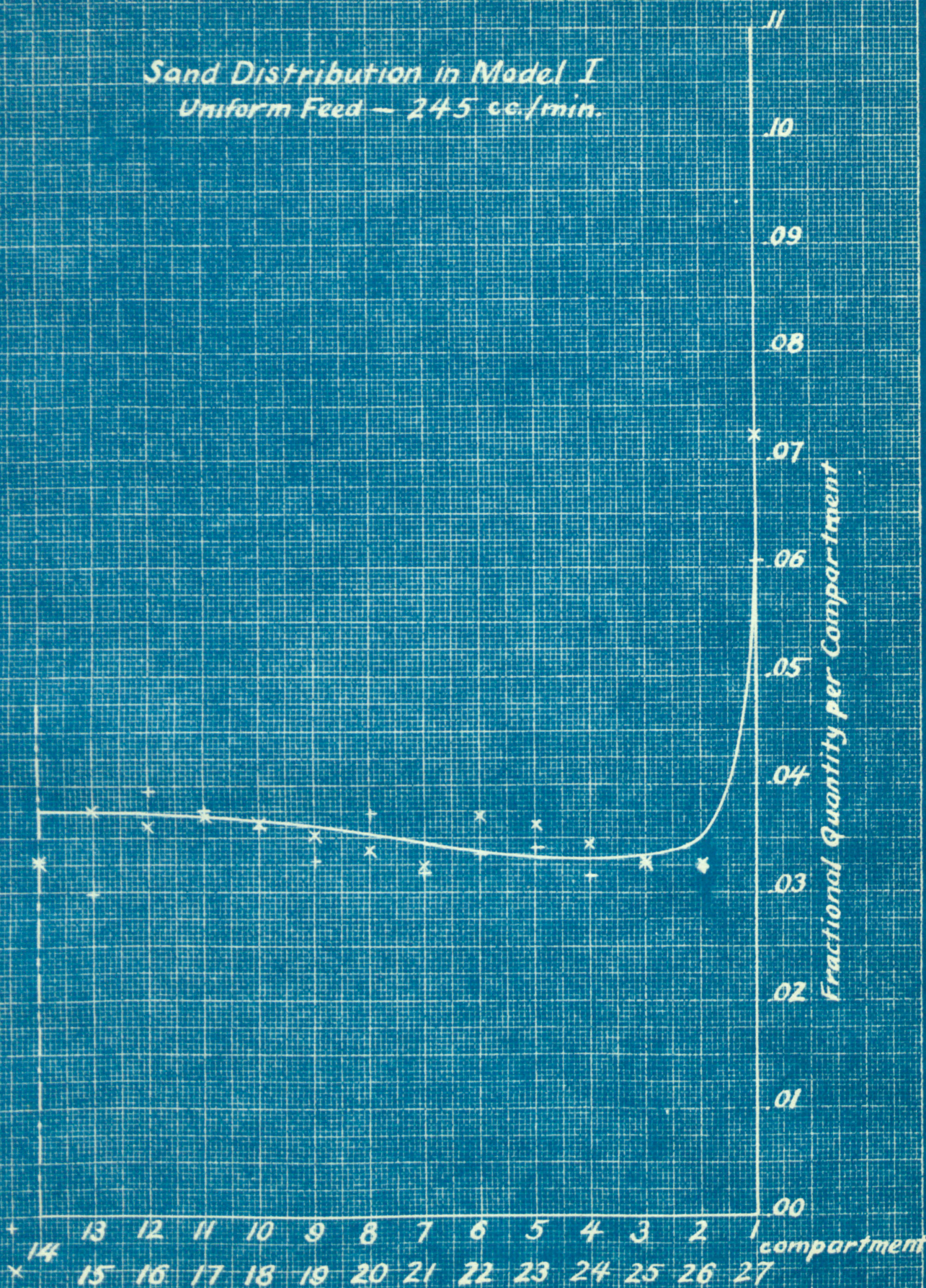
Sand Distribution in Model I *Uniform Feed—170 cc./min.*



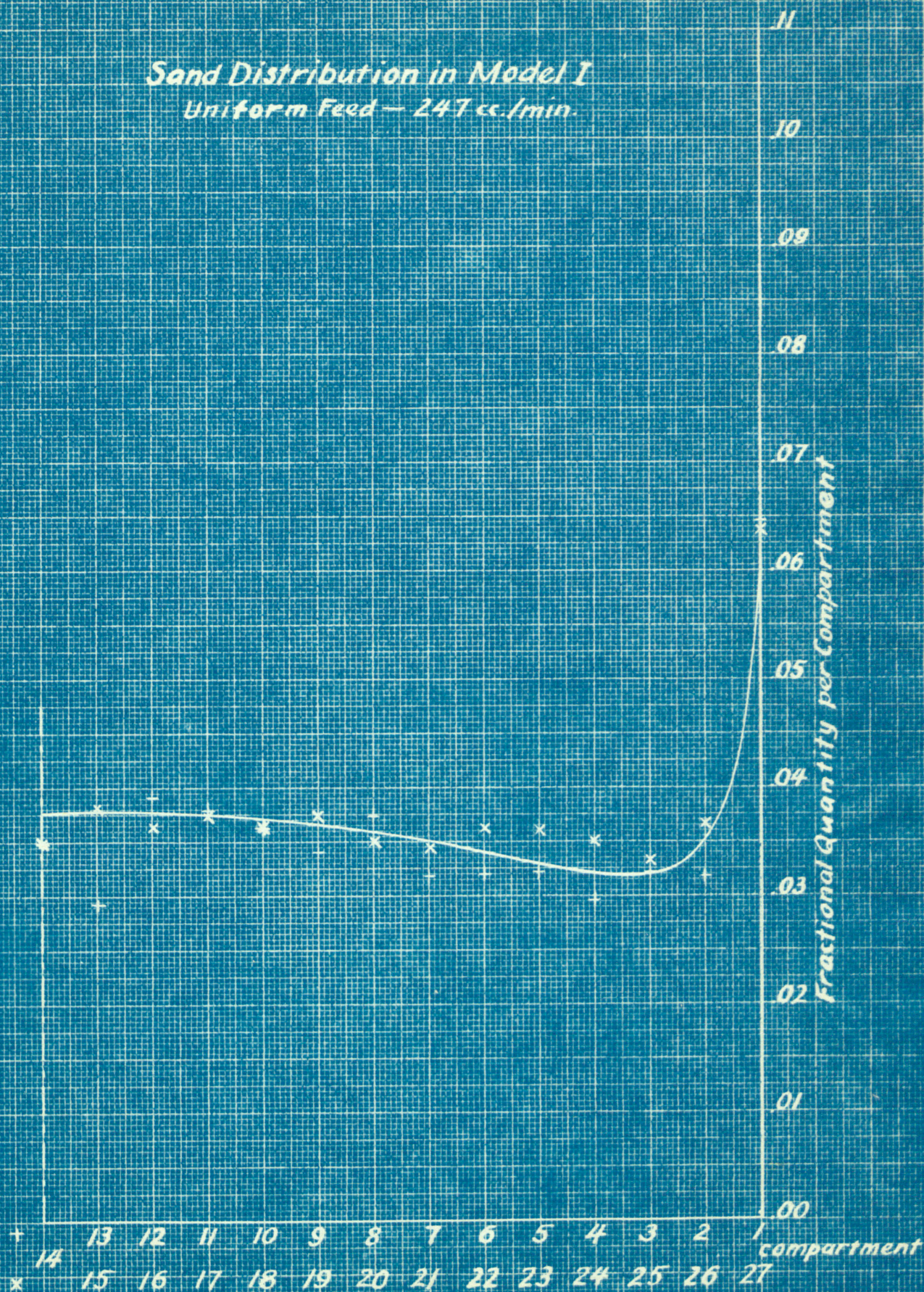
Sand Distribution in Model I
Uniform Feed - 200 cc/min.



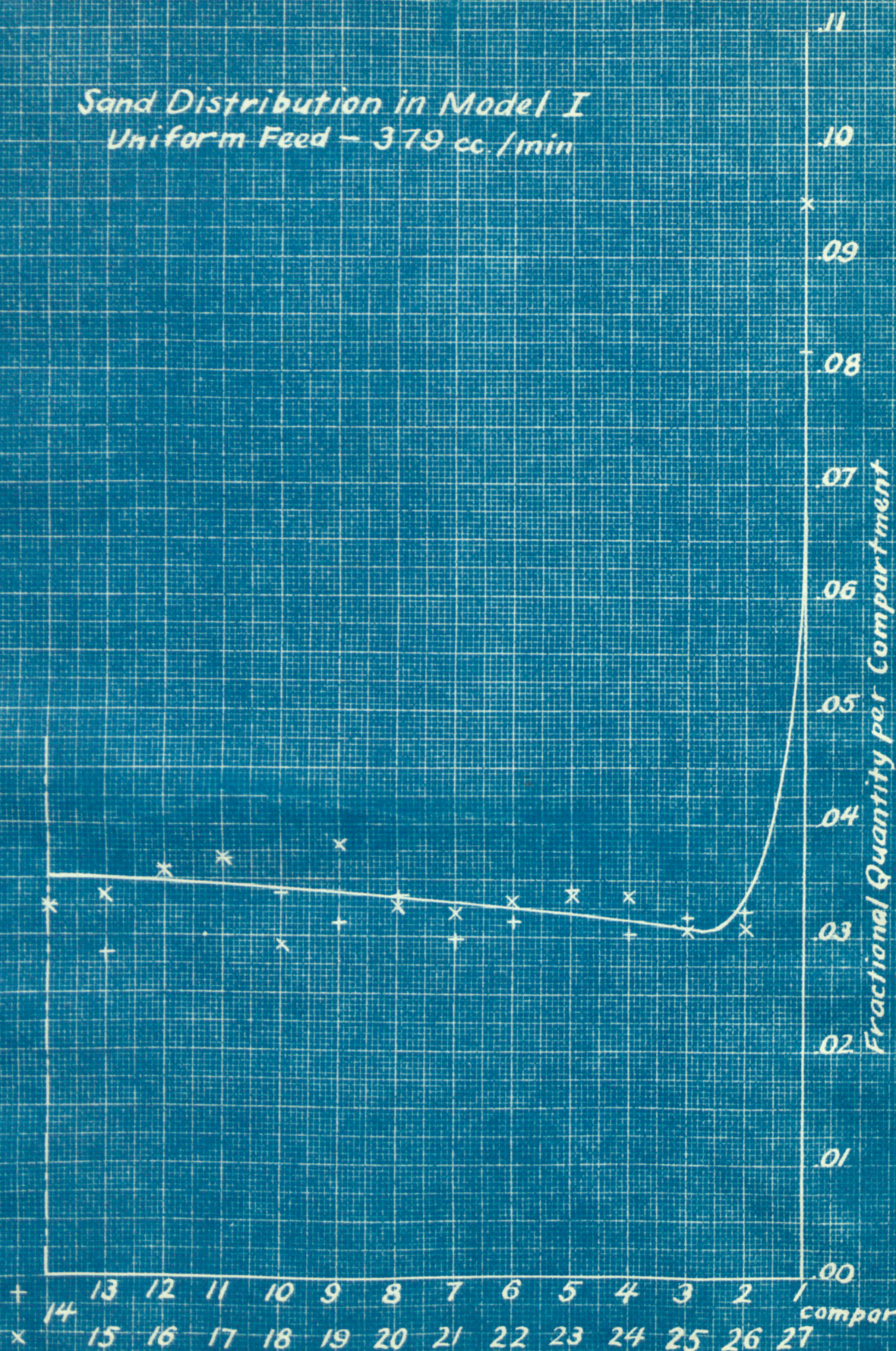
Sand Distribution in Model I *Uniform Feed - 245 cc./min.*



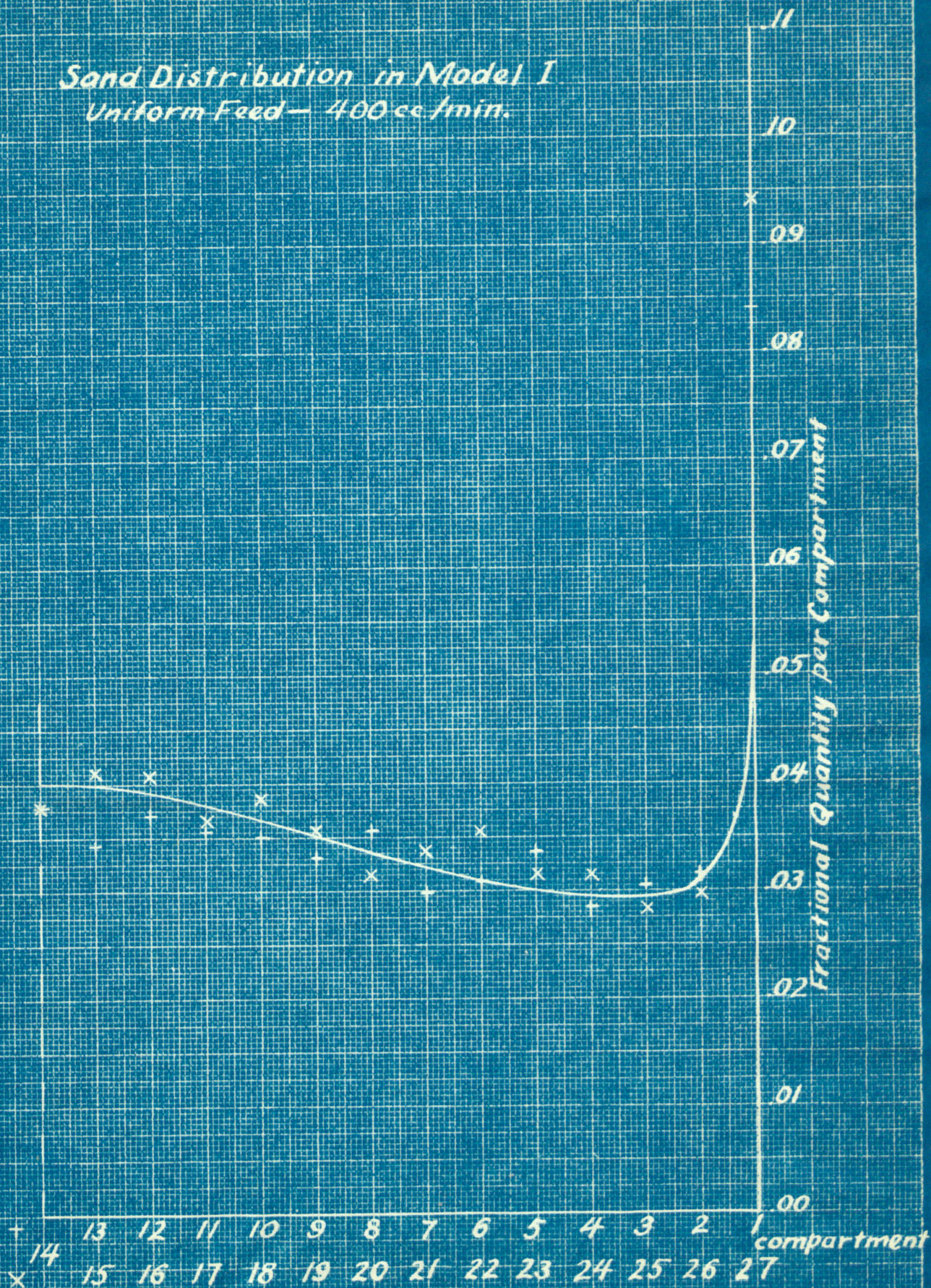
Sand Distribution in Model I *Uniform Feed - 247 cc./min.*



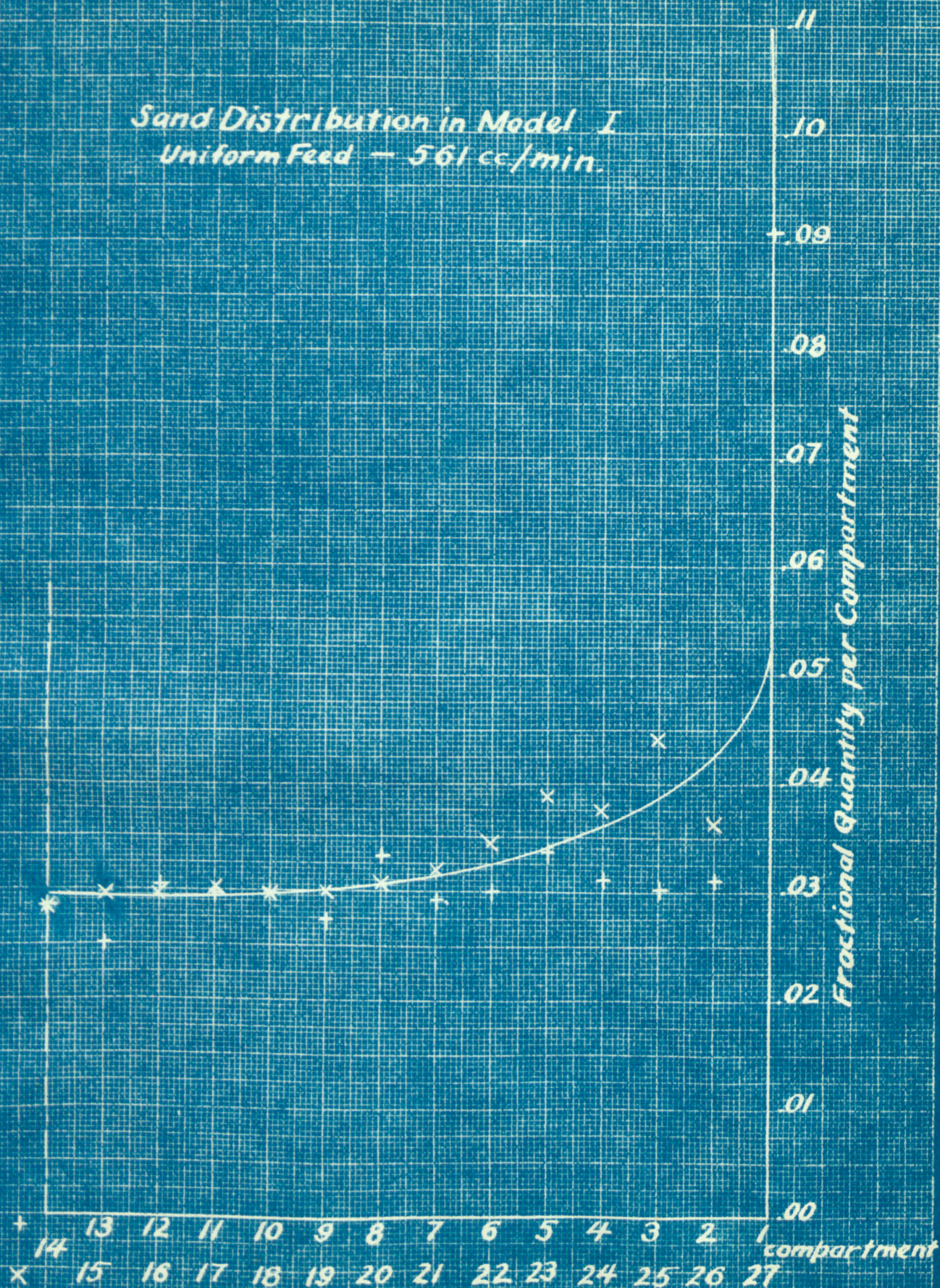
Sand Distribution in Model I
Uniform Feed - 379 cc./min



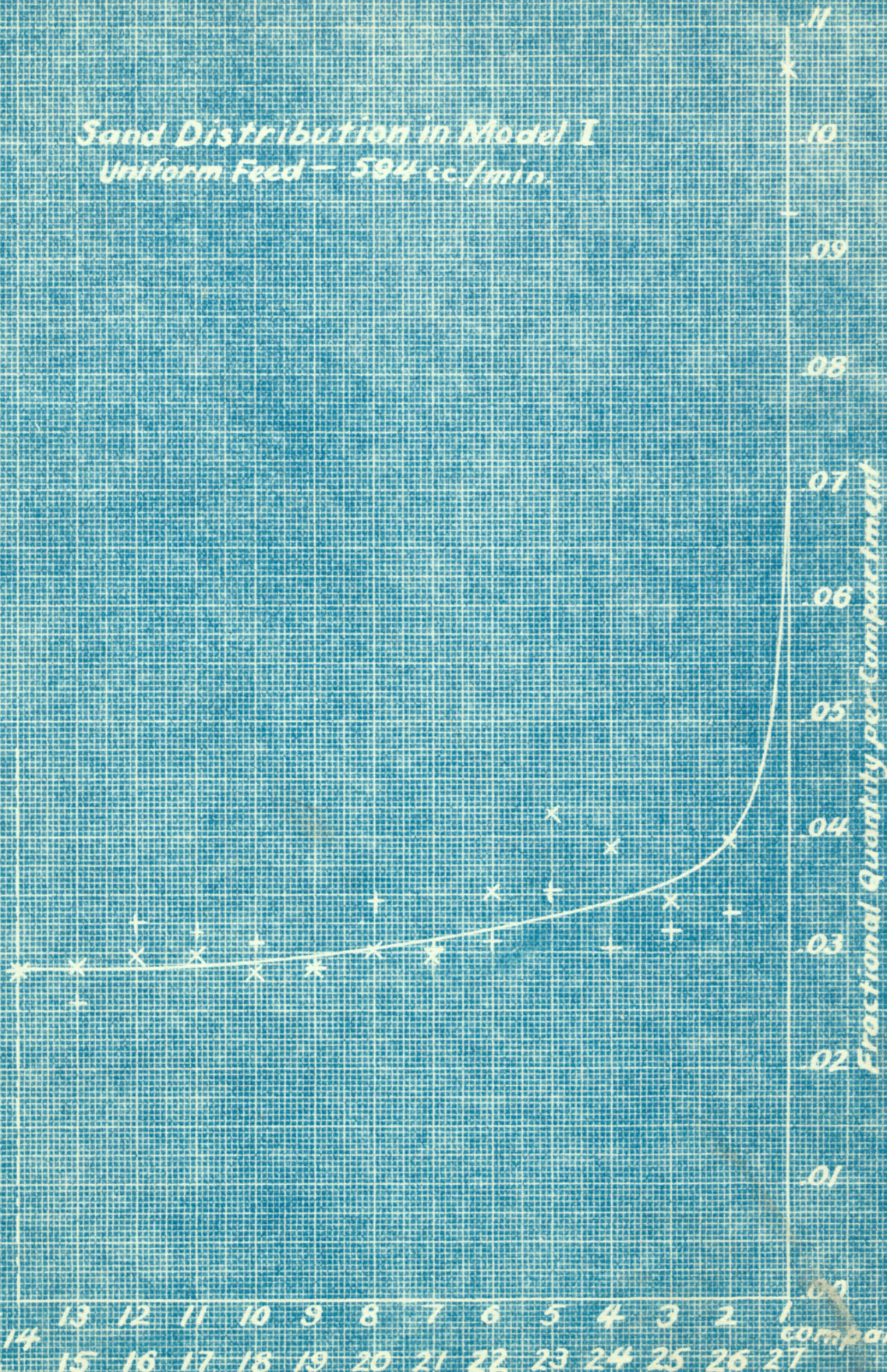
Sand Distribution in Model I *Uniform Feed - 400 cc/min.*



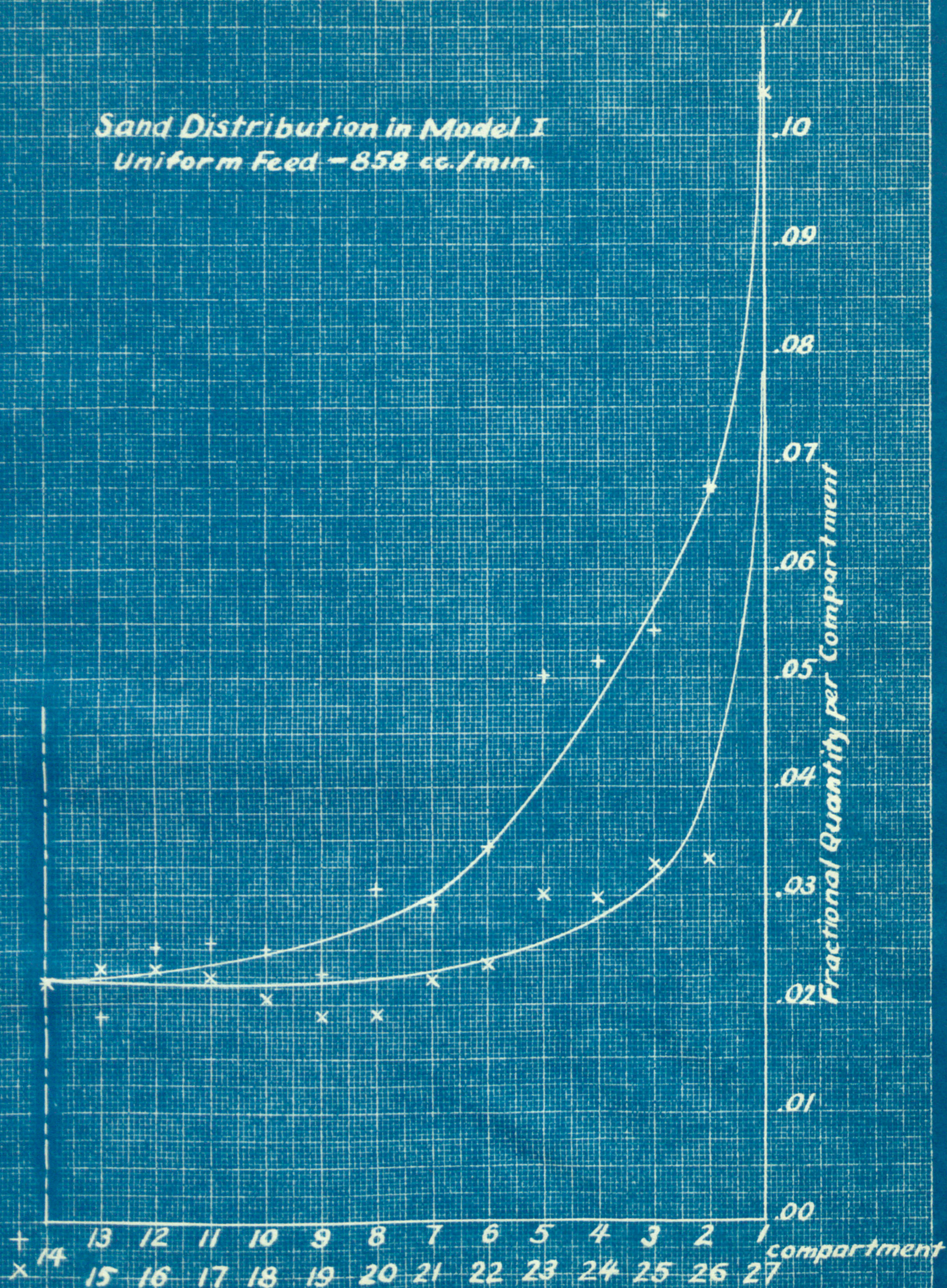
*Sand Distribution in Model I
Uniform Feed — 561 cc/min.*



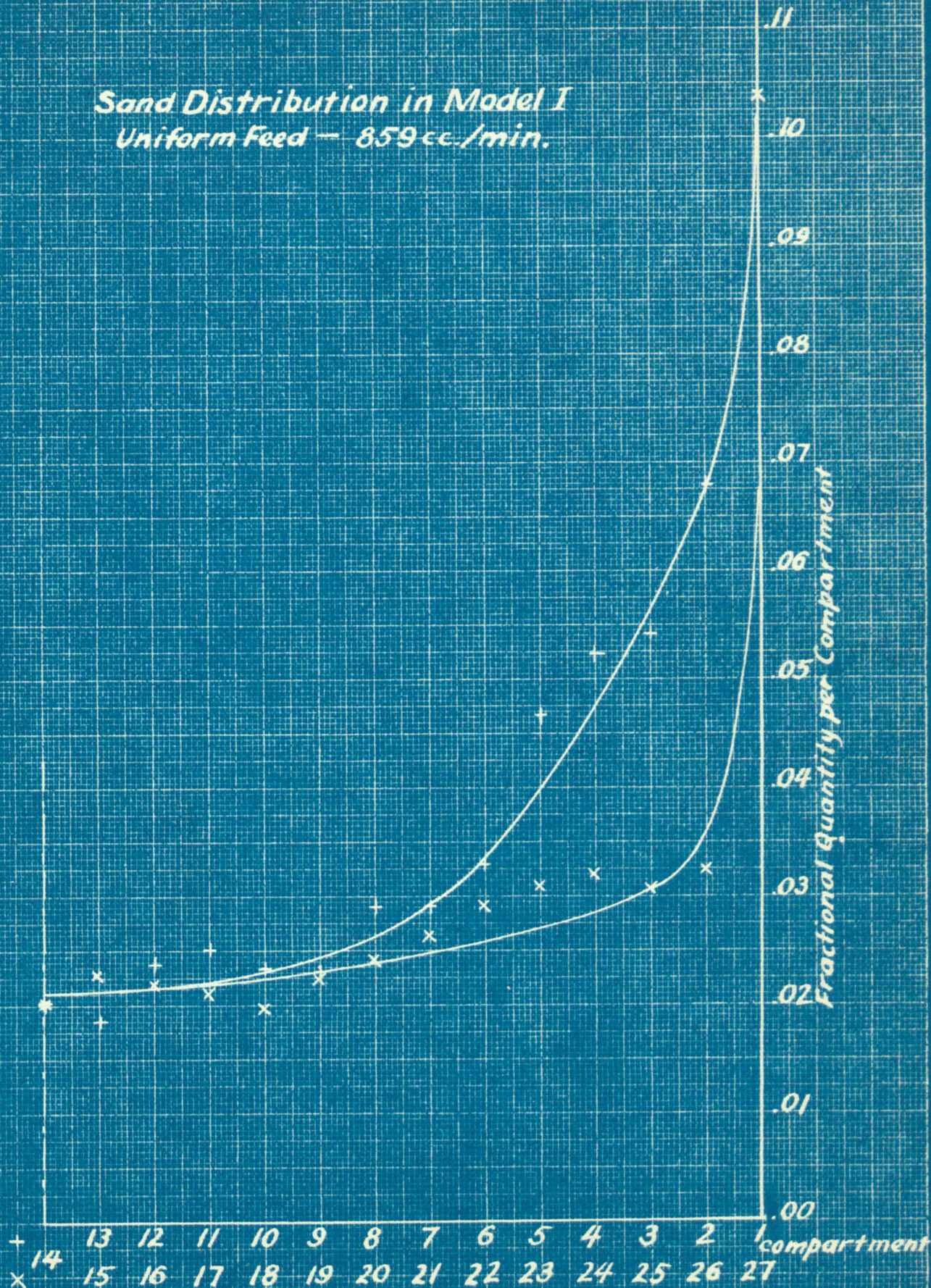
Sand Distribution in Model I *Uniform Feed — 594 cc/min*



Sand Distribution in Model I
Uniform Feed - 858 cc./min.



*Sand Distribution in Model I
Uniform Feed — 859 cc./min.*



Appendix IV

Data Sheets and Graphs for Water Distribution in the Twenty Inch Tower

1. Data for Water Distribution
2. Combined Data
3. Curves for Water Distribution
4. Rectified curves for Center Feed
5. Data from Rectified Curves

DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

I. Center Feed

(a) Three feet packing height

Compartment	Mean Radius	Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.									
		.139	.263	.564	1.05	1.08	1.65	3.24	6.57	12.5	
1	1.45	.00352	.00689	.0123	.0181	.0201	.0240	.0315	.0378	.0607	
2	2.49	.00157	.00323	.00786	.0132	.0120	.0160	.0191	.0251	.0396	
3	3.21	.00135	.00228	.00501	.00855	.00655	.00837	.0107	.0143	.0251	
4	3.80	.00139	.00273	.00594	.0113	.0112	.0154	.0199	.0258	.0420	
5	4.32	.000692	.00133	.00308	.00762	.00716	.0116	.0163	.0248	.0587	
6	4.79	.000640	.001311	.00372	.00849	.00881	.0173	.0302	.0378	.0651	
7	5.21	.000463	.000827	.00156	.00652	.00578	.0101	.01914	.0298	.0700	
8	5.58	.000332	.000635	.00136	.00325	.00442	.00916	.0222	.0325	.0509	
9	6.09	.000172	.000262	.000862	.00191	.00218	.00498	.0201	.0346	.0420	
10	6.74	.0000400	.0000558	.000147	.000520	.000467	.00148	.0104	.0355	.0339	
11	7.35	.0000544	.0000612	.000161	.000413	.000442	.000948	.00587	.0363	.0365	
12	7.89	.0000250	.0000238	.0000494	.000128	.000101	.000206	.00279	.0250	.0370	
13	8.37	.0000257	.0000191	.0000598	.0000625	.000133	.0000304	.000698	.0110	.0374	
14	8.89	.0000162	.0000197	.0000435	.0000855	.000105	.0000899	.000707	.0140	.0299	
15	9.34	.0000091	.0000035	.0000106	.0000223	.0000212	.0000533	.0000526	.0076	.0220	
16	9.75	.0000281	.0000389	.0000335	.0000537	.0000551	.000114	.0000447	.0131	.0347	

DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

I. Center Feed

(b) Six feet packing height

Compartment	Mean Radius in inches	Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.									
		.144	.259	.579	1.10	1.75	3.26	7.95	10.1	14.0	
1	1.45	.0012	.00321	.00660	.0112	.0162	.0299	.0580	.0523	.0720	
2	2.49	.00107	.00204	.00549	.0102	.0141	.0196	.0497	.0380	.0624	
3	3.21	.00103	.00183	.00422	.00843	.0125	.0179	.0406	.0388	.0584	
4	3.80	.000990	.00173	.00449	.00822	.0125	.0208	.0374	.0317	.0436	
5	4.32	.000944	.00164	.00424	.00827	.0123	.0186	.0334	.0443	.0680	
6	4.79	.000718	.00134	.00311	.00572	.00957	.0198	.0442	.0426	.0637	
7	5.21	.000615	.00108	.00244	.00480	.00740	.0140	.0343	.0342	.0466	
8	5.58	.000694	.00129	.00254	.00437	.00827	.0191	.0407	.0359	.0529	
9	6.09	.000414	.000642	.00160	.00334	.00576	.0122	.0314	.0298	.0412	
10	6.74	.000414	.000840	.00160	.00326	.00586	.0118	.0368	.0319	.0355	
11	7.35	.000253	.000376	.000709	.00164	.00331	.00728	.0181	.0342	.0315	
12	7.89	.000234	.000327	.000696	.00128	.00224	.00528	.0166	.0434	.0486	
13	8.37	.000252	.000370	.000513	.000998	.00199	.00453	.0161	.0293	.0403	
14	8.89	.000048	.000128	.000195	.000448	.00104	.00244	.0116	.0214	.0263	
15	9.34	.000029	.0000415	.000070	.000146	.00023	.00059	.0053	.0143	.0220	
16	9.75	.000055	.000131	.000224	.000369	.00368	.00128	.0104	.0253	.0576	

DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

II. Uniform Feed

(a) Three feet packing height

Compartment	Mean Radius in inches	1.225	1.823	2.67	4.36	6.52	7.73	9.95	10.39	11.05	13.81
Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.											
1	1.45	.00473	.00606	.00930	.01536	.0213	.0257	.0376	.0325	.0342	.0495
2	2.49	.00342	.00572	.00771	.01392	.0200	.0232	.0314	.0278	.0290	.0420
3	3.21	.00444	.00572	.00819	.01281	.01695	.01956	.0215	.0225	.0184	.0352
4	3.80	.00400	.00658	.00876	.01310	.0202	.0251	.0353	.0307	.0329	.0501
5	4.32	.00455	.00571	.00838	.01362	.0235	.0242	.0373	.0334	.0250	.0637
6	4.79	.00749	.00790	.01041	.01956	.0304	.0367	.0440	.0416	.0461	.0563
7	5.21	.00587	.00767	.01202	.01890	.0241	.0270	.0388	.0347	.0448	.0577
8	5.58	.01113	.01542	.01362	.0205	.0274	.0317	.0416	.0370	.0501	.0545
9	6.09	.00715	.01143	.01584	.01815	.0283	.0310	.0382	.0376	.0350	.0689
10	6.74	.00481	.00902	.01341	.0216	.0236	.0298	.0350	.0408	.0349	.0394
11	7.35	.00335	.00617	.01194	.0215	.0318	.0367	.0350	.0388	.0355	.0373
12	7.89	.001614	.00313	.00482	.00991	.0243	.0329	.0392	.0379	.0420	.0374
13	8.37	.00222	.00338	.00449	.00735	.01227	.01914	.01566	.0395	.0392	.0364
14	8.89	.00222	.00311	.00471	.00853	.01252	.01512	.0290	.0271	.0271	.0366
15	9.34	.001305	.001191	.00217	.00512	.00958	.00747	.0201	.01980	.0271	.0408
16	9.75	.00232	.00350	.00547	.00975	.01548	.01715	.0258	.0260	.0442	.0302

DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

II. Uniform Feed

(b) Six feet packing height

Compartment	Mean Radius in inches	Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.					
		.530	1.105	1.387	2.85	5.22	9.37
1	1.45	.001610	.00379	.00404	.00751	.01705	.01920
2	2.49	.001622	.00420	.00398	.00768	.01327	.01459
3	3.21	.00229	.00471	.00423	.00874	.01442	.0211
4	3.80	.00209	.00444	.00512	.00920	.01718	.0218
5	4.32	.00202	.00435	.00513	.01032	.0200	.0262
6	4.79	.001905	.00233	.00514	.01113	.0217	.0403
7	5.21	.001889	.00376	.00441	.00858	.01704	.0208
8	5.58	.00310	.00649	.00757	.01410	.0253	.0301
9	6.09	.00211	.00437	.00580	.01105	.0225	.0309
10	6.74	.00242	.00521	.00638	.01200	.0230	.0284
11	7.35	.001637	.00314	.00465	.01047	.01940	.0327
12	7.89	.001678	.00387	.00543	.01086	.01238	.0501
13	8.37	.001850	.00440	.00474	.01066	.01953	.0317
14	8.89	.001260	.00204	.00287	.00667	.01315	.0263
15	9.34	.000337	.000714	.000918	.00363	.00518	.0275
16	9.75	.000751	.001660	.00262	.00554	.01140	.0370

COMBINED DATA FOR
DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{3}{8}$ " Coke

I Center Feed

(a) Three feet packing height

Compartment	Mean Radius in inches	Square of Mean Radius	Water Concentration in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.									
			.139	.263	.564	1.05	1.08	1.65	3.24	6.57	12.5	
I*	2.03	4.1	.00255	.00507	.0101	.0157	.0154	.0200	.0252	.0314	.0501	
II	3.52	12.4	.00137	.00252	.00537	.00991	.00882	.0118	.0152	.0199	.0336	
III	4.56	20.7	.000669	.00132	.00340	.00804	.00776	.0143	.0228	.0311	.0619	
IV	5.39	29.0	.000399	.000734	.00146	.00375	.00512	.00947	.0206	.0311	.0607	
V	6.43	41.3	.000105	.000157	.000499	.00120	.00131	.00320	.0152	.0235	.0378	
VI	7.60	57.8	.0000406	.0000436	.000108	.000280	.000282	.000601	.00443	.0305	.0367	
VII	8.65	74.9	.0000206	.0000195	.0000511	.0000749	.000118	.0000642	.000703	.0126	.0355	
VIII	9.57	91.6	.0000194	.0000227	.0000230	.0000394	.0000394	.0000395	.0000866	.000484	.0105	.0289

* Compartment I represents both compartments
1 and 2 of tower, compartment II represents
3 and 4, etc.

COMBINED DATA FOR
DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

I Center Feed

(b) Six feet packing height

Compartment	Mean Radius in inches	Square of Mean Radius	Water Concentration in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.									
			.144	.259	.579	1.10	1.75	3.26	7.95	10.1	14.0	
I *	2.03	4.1	.00114	.00263	.00605	.0107	.0151	.0248	.0540	.0452	.0671	
II	3.52	12.4	.00101	.00166	.00436	.00834	.0125	.0193	.0391	.0353	.0512	
III	4.56	20.7	.000835	.00149	.00369	.00704	.0110	.0192	.0387	.0435	.0659	
IV	5.39	29.0	.000654	.00118	.00254	.00459	.00782	.0165	.0374	.0350	.0497	
V	6.43	41.3	.000515	.000742	.00160	.00330	.00581	.0120	.0290	.0308	.0382	
VI	7.60	57.8	.000244	.000353	.000704	.00149	.00281	.00629	.0174	.0385	.0395	
VII	8.65	74.9	.000143	.000241	.000343	.000703	.00148	.00341	.0135	.0250	.0328	
VIII	9.57	91.6	.0000453	.0000900	.000154	.000267	.000306	.000982	.00807	.0203	.0414	

* Compartment I represents both compartments
1 and 2 of tower, compartment II represents
3 and 4, etc.

COMBINED DATA FOR
DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{3}{8}$ " Coke

II. Uniform Distribution

(a) Three feet packing height

Compartment	Mean Radius in inches	1.23	1.82	2.67	4.36	6.52	7.73	9.95	10.39	11.0	13.8
Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.											
I*	2.03	.00403	.00590	.00852	.0146	.0206	.0252	.0347	.0302	.0316	.0457
II	3.52	.00425	.00614	.00851	.0129	.0186	.0222	.0282	.0265	.0255	.0426
III	4.56	.00550	.00680	.00940	.0166	.0269	.0303	.0406	.0375	.0354	.0603
IV	5.39	.00796	.00999	.0128	.0197	.0257	.0293	.0402	.0358	.0474	.0561
V	6.43	.00597	.0102	.0147	.0199	.0259	.0303	.0366	.0398	.0370	.0539
VI	7.60	.00254	.00474	.00860	.0161	.0283	.0349	.0370	.0384	.0386	.0374
VII	8.65	.00223	.00324	.00455	.00800	.0125	.0170	.0227	.0329	.0327	.0366
VIII	9.57	.00185	.00244	.00395	.00762	.0128	.0127	.0231	.0158	.0362	.0350

* Compartment I represents both compartments
1 and 2 of tower, compartment II represents
3 and 4, etc.

COMBINED DATA FOR
DISTRIBUTION RUNS ON TWENTY INCH TOWER
Packed with $\frac{1}{4}$ " to $\frac{1}{2}$ " Coke

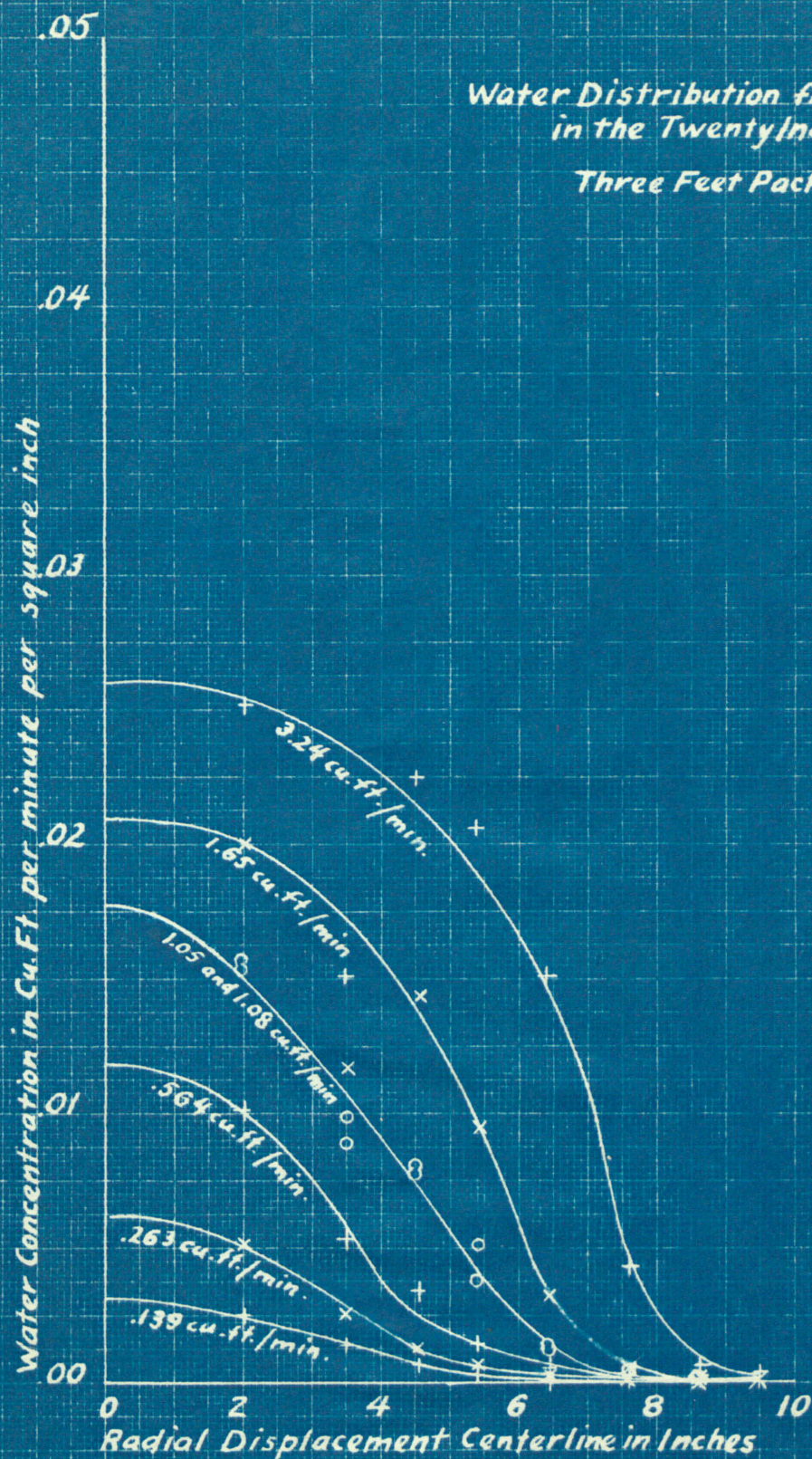
II. Uniform Distribution

(b) Six feet packing height

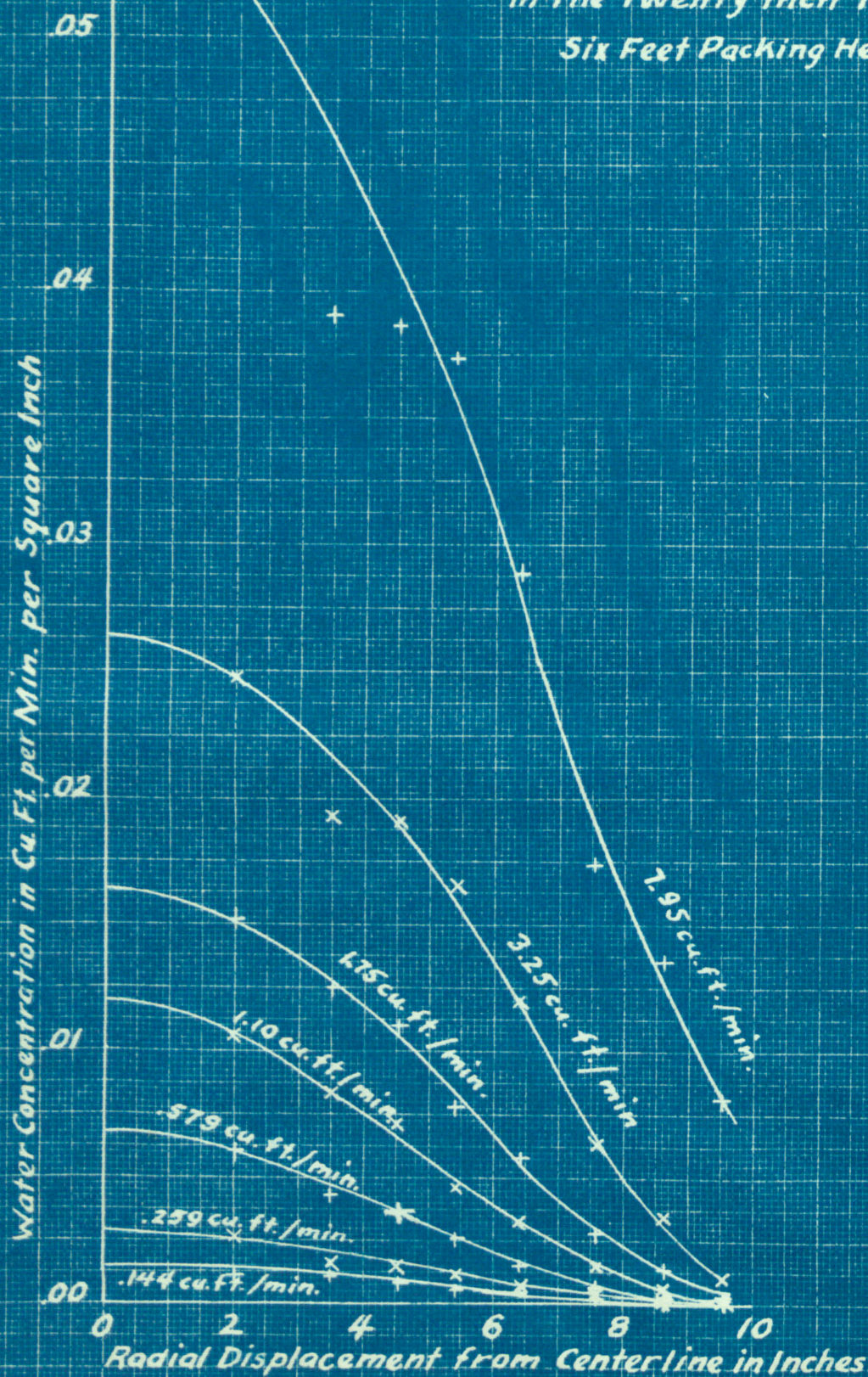
Compartment	Mean Radius in inches	Water Concentrations in cu. ft. per min. per in. ² at Various Total Feed Rates in cu. ft. per min.					
		.530	1.11	1.39	2.85	5.22	9.38
I *	2.03	.00162	.00399	.00401	.00759	.0152	.0169
II	3.52	.00219	.00457	.00465	.00887	.0158	.0173
III	4.56	.00196	.00337	.00514	.0107	.0208	.0331
IV	5.39	.00248	.00510	.00593	.0113	.0211	.0254
V	6.43	.00226	.00480	.00610	.0115	.0228	.0296
VI	7.60	.00165	.00349	.00502	.0107	.0161	.0408
VII	8.65	.00153	.00314	.00374	.00854	.0161	.0298
VIII	9.57	.000563	.00123	.00184	.00467	.0857	.0327

* Compartment I represents both compartments
1 and 2 of tower, compartment II represents
3 and 4, etc.

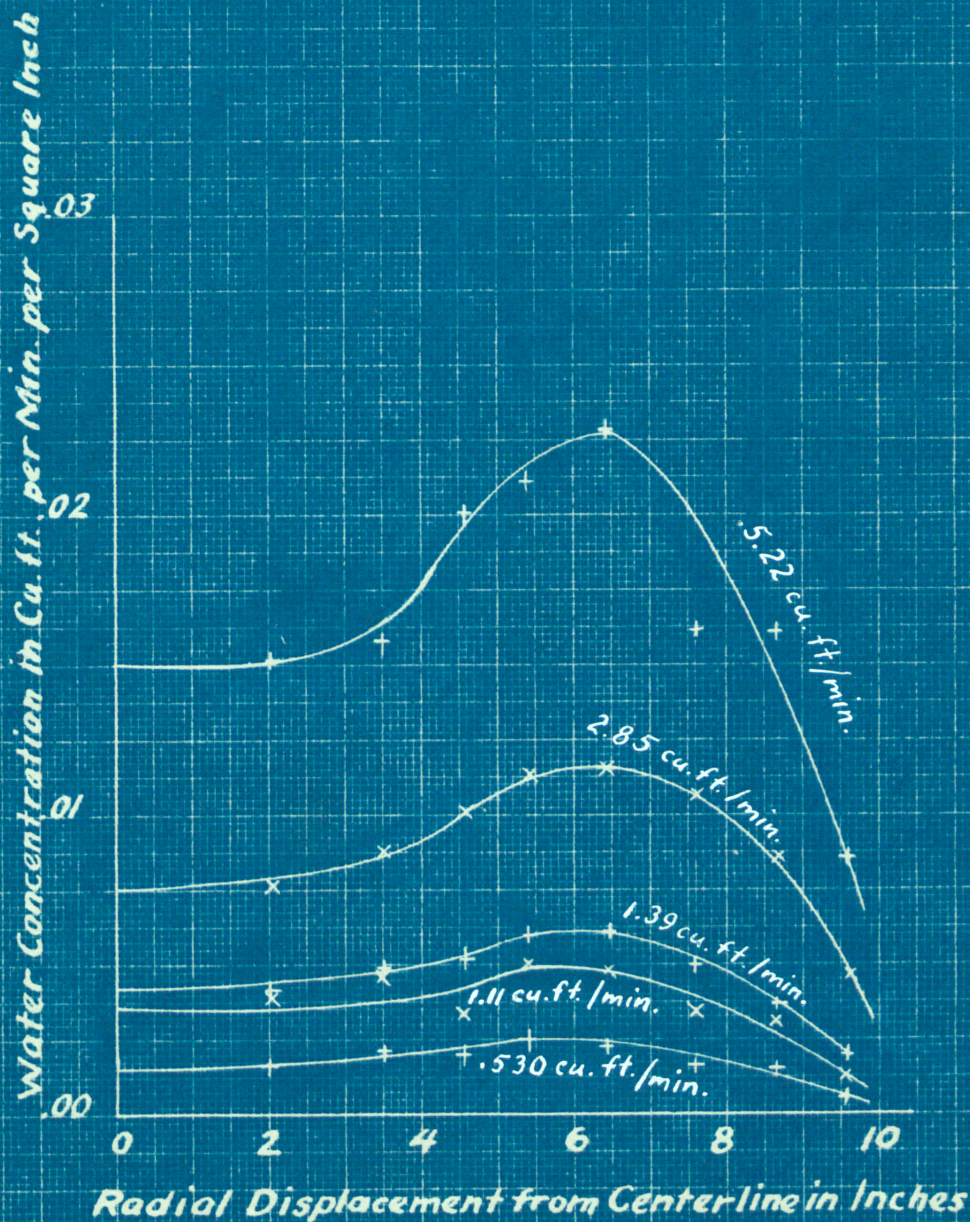
*Water Distribution from Center Feed
in the Twenty Inch Tower
Three Feet Packing Height*



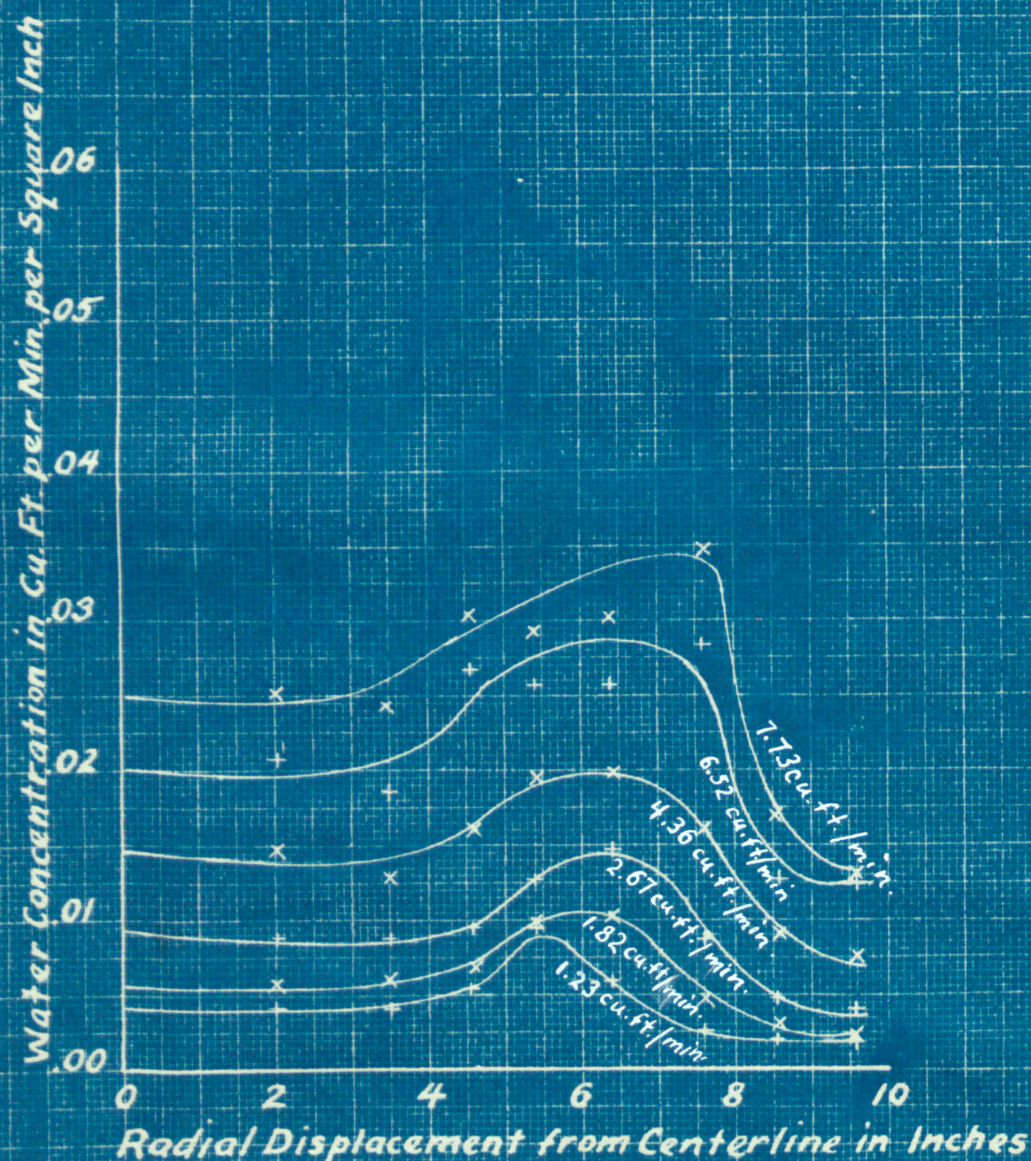
*Water Distribution from Center Feed
in the Twenty Inch Tower
Six Feet Packing Height*



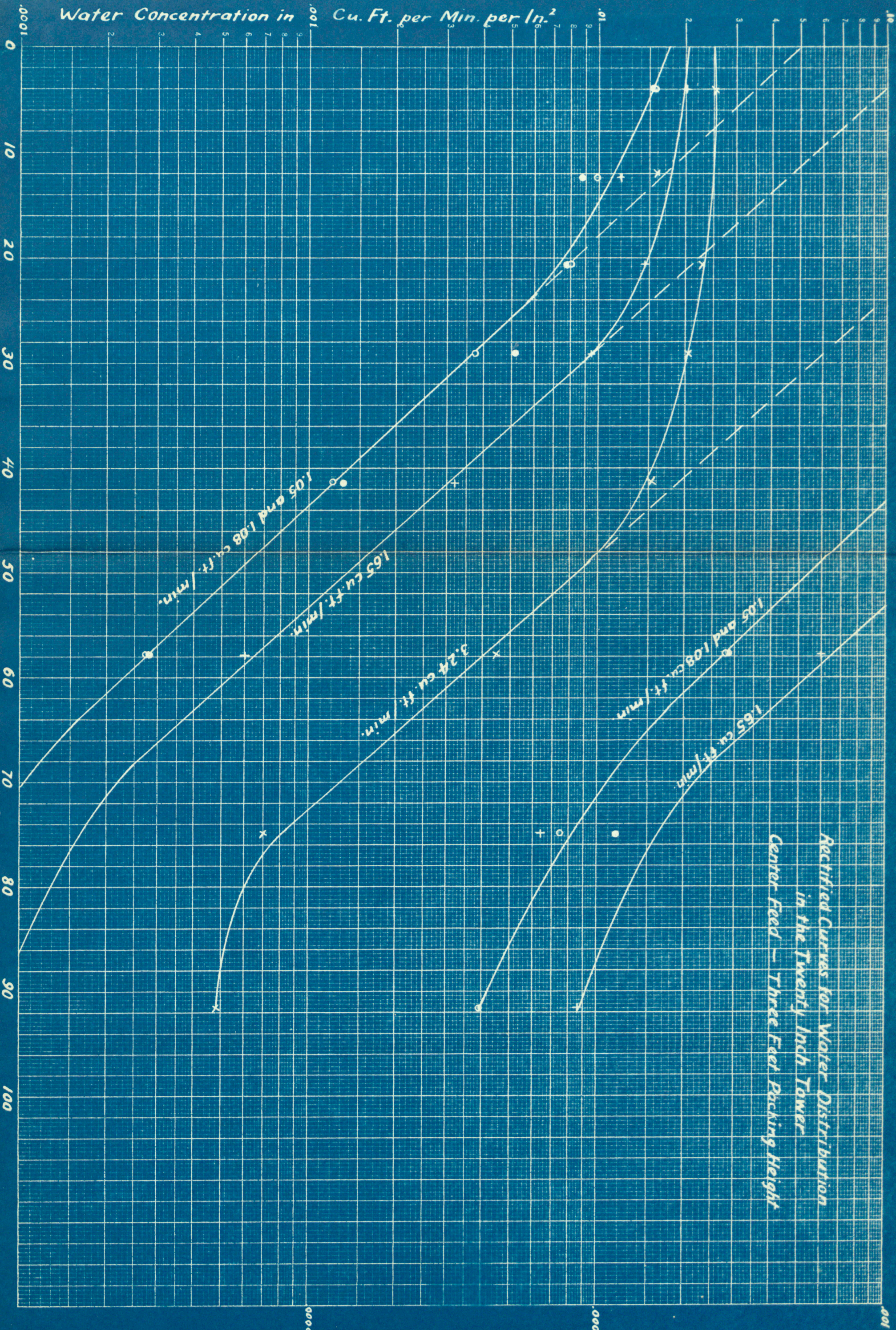
Water Distribution from Uniform Feed in the Twenty Inch Tower Six Feet Packing Height



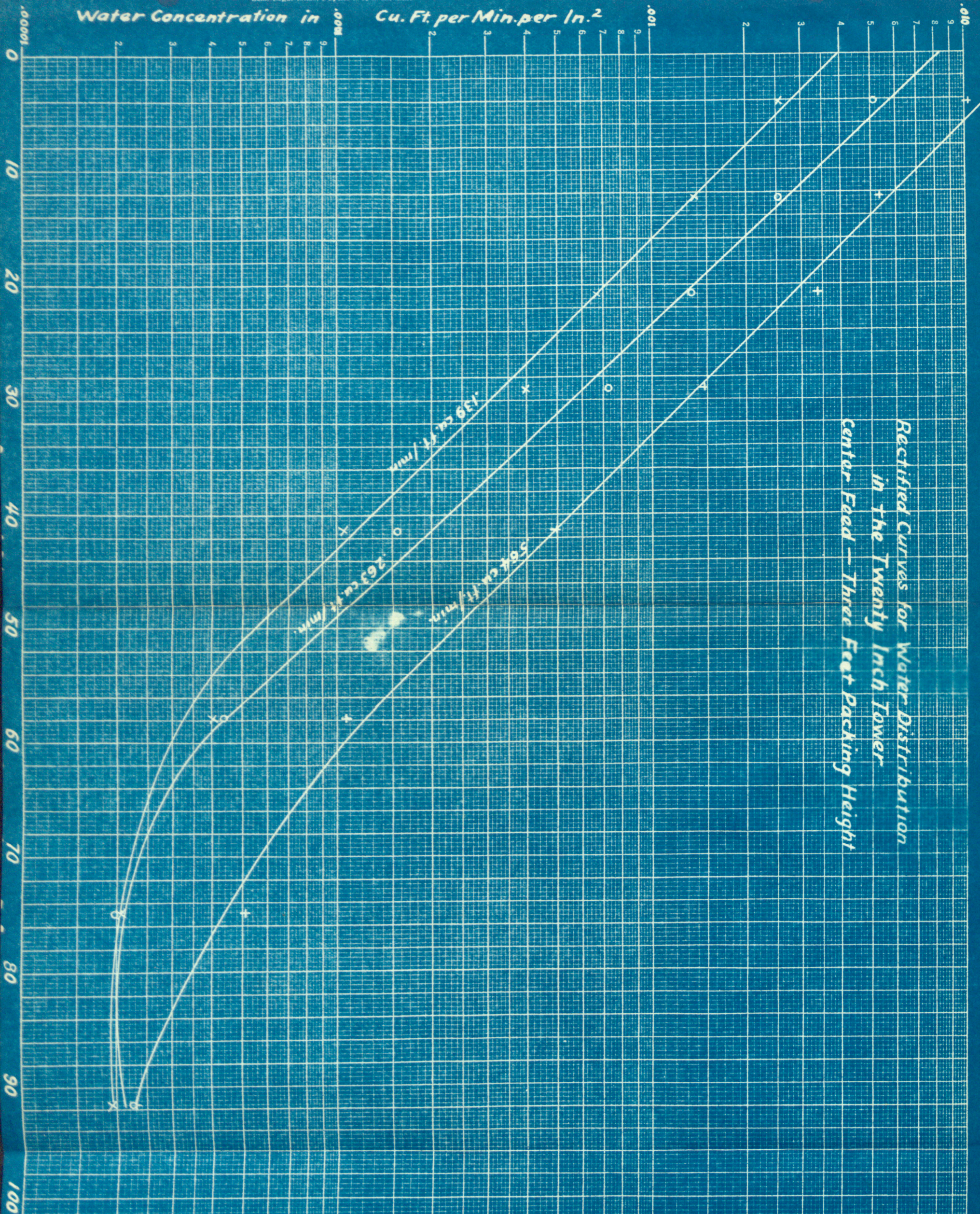
Water Distribution from Uniform Feed in the Twenty Inch Tower Three Feet Packing Height



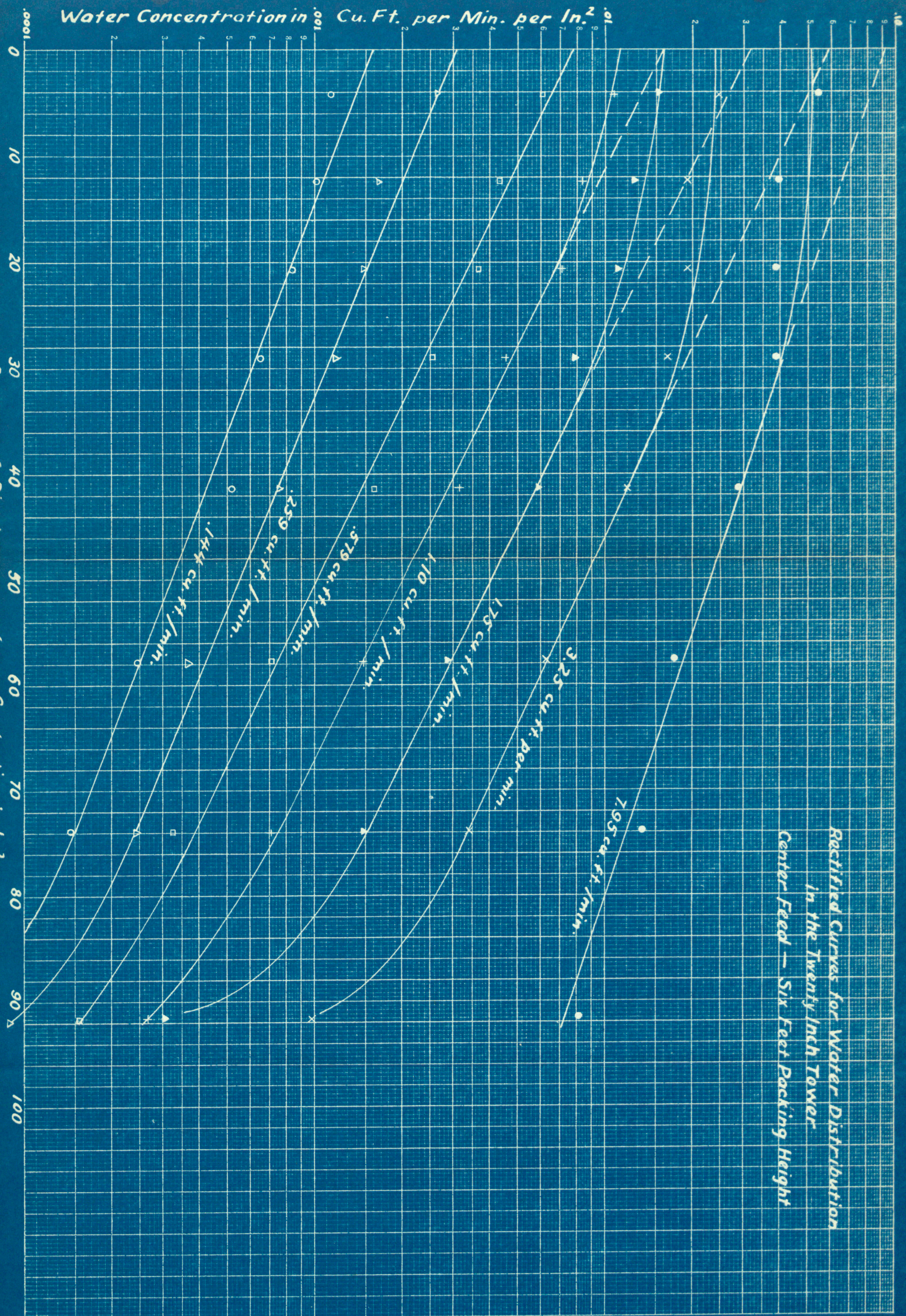
Water Concentration in Cu. Ft. per Min. per In.²



Water Concentration in Cu. Ft. per Min. per In.²



Water Concentration in $\cdot 001$ Cu. Ft. per Min. per In.²



DATA CALCULATED FROM RECTIFIED CURVES
for Water Distribution from Center Feed
in the Twenty Inch Tower

Three Feet Packing

(a) Low Flows

Total Flow Rate in cu.ft./min.	h^2 1/in. ²	K^2 1/in. ²	$\frac{K\sqrt{\pi}}{h}$	Flow at Centerline cu.ft/min.in. ²
.139	.0857	.0287	1.026	.0040
.263	.0904	.0319	1.052	.0084
.564	.0854	.0301	1.053	.0170

(b) Intermediate Flows

Total Flow Rate in cu.ft./min.	h^2 1/in. ²	K^2 1/in. ²	K^2 extrapo- lated	Flow at Centerline cu.ft/min.in. ²
1.05 1.08	.0900	.0164	.0468	.0175
1.65	.0935	.0124	.0876	.0205
3.24	.0945	.00801	.0293	.0260

DATA CALCULATED FROM RECTIFIED CURVES
for Water Distribution from Center Feed
in the Twenty Inch Tower

Six Feet Packing

(a) Low Flows

Total Flow Rate in cu.ft./min.	h^2 1/in. ²	K^2 1/in. ²	$\frac{K\sqrt{\pi}}{h}$	Flow at Centerline cu.ft/min.in. ²
.144	.0320	.0111	1.043	.0016
.259	.0346	.0112	1.042	.0031
.579	.0406	.0135	1.021	.0078

(b) Intermediate Flows

Total Flow Rate in cu.ft./min.	h^2 1/in. ²	K^2 1/in. ²	K^2 extrapo- lated	Flow at Centerline cu.ft/min.in. ²
1.10	.0414	.01026	.0146	.0113
1.75	.0413	.00914	.0183	.0160
3.25	.0386	.00706	.01815	.0240
7.95	.0277	.00641	.01155	.051

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