L A T E R A L   E A R T H   P R E S S U R E.

Thesis of

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FROM THE CODE OF LAWS OF **HAMMURABI** (2200 BC) KING OF BABYLONIA

A. If a builder build a house for a man and do not make its construction firm and the house which he has built collapse and cause the death of the owner of the house — that builder shall be put to death.

B. If it cause the death of the son of the owner of the house — they shall put to death a son of that builder.

C. If it cause the death of a slave of the owner of the house — he shall give to the owner of the house a slave of equal value.

D. If it destroy property, he shall restore whatever it destroyed, and because he did not make the house which he built firm and it collapsed, he shall rebuild the house which collapsed at his own expense.

E. If a builder build a house for a man and do not make its construction meet the requirements and a wall fall in, that builder shall strengthen the wall at his own expense.

Translated by R. F. Harper.
"Code of Hammurabi" p. 83-seq.

Jacob Feld 1922.

Contents.

Lateral Earth Pressure.

Synopsis.

In the numerous contributions which have appeared on this subject, the writer has been unable to find a single attempt to give a complete list of original work in developing the earth pressure theories. Todhunter in his "History of the Elasticity and Strength of Materials" mentions several of the more recent original articles, but unfortunately, does not go completely into the subject, because the "matter----lies outside our field." Finding that there exists a very confused notion of the oldest theories, the writer gives in detail the original contributions previous to 1773. Most of these are from original sources, the remainder are from books which appeared in the early part of the nineteenth century. In several instances, the writer makes mention where later authors have either incorrectly copied or have taken their material from incorrect translations. The most note-worthy instance is the fact, usually accepted and found in almost every article issued since 1800, that Coulomb assumes the tangent of the angle of repose as the coefficient of friction inside the mass.

The second part of this section deals with the development of the theory of the maximum wedge of pressure from Coulomb(1773) to Poncelet(1840). In spite of this advance in the theory, many articles appeared from 1773 to date, where the formulae of the earliest writers are accepted.

The third part briefly outlines a few of the most important of these. The general wedge theory, both analytical and graphical is developed in the fourth part, giving especial attention to the various methods of attack in finding the maximum earth pressure which can be derived from the theory of the wedge of rupture. Part 5 considers the recent addition of cohesion resistance in modifying the shape of the wedge.

It was considered advisable to entirely separate the stress theories from the wedge theories, tho this means a breaking up of the chronological order. Part 6 deals with the application of the Ellipse of Stress, both graphically and analytically, usually called the Rankine Theory. Part 7 is a short description of the use of Mohr's Circle of Stress to obtain similar results. The purely analytical work of Levy, n St. Venant and Boussinesq, based on the Theory of Elasticity is given in Part 8. The last part contains short accounts of some recent theories, notably those of Chaudy and Meem. In all cases, the source of the material is denoted by a number, referring to the bibliography. This list of references is believed to be the most complete bibliography on on the subject ever compiled.

The second section contains an account of all experimental work on Earth Pressure Determination and kindred researches. Up to 1840, the work was chiefly on lateral pressure and is contained in the first part. The second part consists of experimental determination of the lateral pressure of granular materials behind actual walls and behind test walls. The third part deals with foundations, especially the problem of the ratio of applied load to transmitted vertical pressure. The last two parts contain some of the more important work on the bin pressures and the lateral pressure of concrete mixes in forms.

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Many of the results are accompanied by a discussion and comparison with theories, the type of apparatus and method of procedure is given in each description.

The final section contains the report of the Cincinnati experiments, including a description of the apparatus, the method of procedure, detailed data and conclusions. The apparatus is the largest ever successfully installed. The final design was the result of several years study and includes suggestions made by numerous men who have made special study of the subject of Earth Pressure. Altho large ix size, the apparatus is quite easily handled, is very sensitive and consistent in results. A complete set of tests with sand have been completed. In general, the wedge theory which takes into account the wall friction is closely checked. Experiments were conducted with three types of walls; wood, glass and sheet-metal coated. Battered walls, both positive and negative, with the surface of the fill sloping at all possible angles, were included in this set of tests. The report also includes an investigation into the nature of the physical properties of a granular material as affected by changes of temperature, of humidity and of weather, as well as changes caused by static and dynamic loads.

Appreciation.

The problem of the experimental determination of Lateral Earth Pressure was assigned tomthe writer when he was appointed a Baldwin Fellow in Civil Engineering at the University of Cincinnati, in July 1919. The problem has been studied by the Civil Engineering Department since 1912, when G.M.Braune, Member A.S.C.E., then Professor in Civil Engineering at the University of Cincinnati, conceived of the idea of a large size apparatus to test the lateral pressure of soils. With the cooperation of Professor C.C.Meyers, of the Department of Industrial Engineering, a preliminary design was drawn up, based upon that of Mueller-Breslau. In 1916, H.Wolfkoetter, Fellow in Civil Engineering, developed details for this design. The writer, under the advisorship of Professor Braune, completed the present design. Suggestions were made by several men, as mentioned in the description of the apparatus. To Professor Braune is also due the credit for pushing the apparatus to completion, obtaining the necessary funds, etc. E.K.Ruth, Member A.S.C.E., checked the plans and computations of the design. Mr. Gerald Fitzgerald aided the writer in assembling the apparatus and in the preliminary tests. Thanks is due the Stacey Bros. Gas Construction Co., Elmwood, Ohio, for furnishing the measuring parts of the apparatus at a price far below the cost of the material. Mr. Harry Pockras read the first copy of this paper for improvement in expression.

The writer wishes to express his gratitude to Professor Braune, who, as advisor in 1919-1921, interested him in the subject and supervised the work of designing, constructing and erecting the apparatus. The writer is also indebted to his present advisor, Professor E.D.Gilman, Associate Member A.S.C.E., who has read the copy and made numerous suggestions which aided in bringing both the experimental and theoretical parts of this work to a successful end. Thanks are also due to Professor L.T.More, Dean of the Graduate School of the University of Cincinnati, Dr. D.B.Steinman, Member A.S.C.E., Mr. F.E.Schmitt, Member A.S.C.E., for advice and encouragement, without which the writer would probably not have completed the work.

The references, from which the material for the theoretical and historical discussion was obtained, were consulted at the Library of the City of Cincinnati and at the 42nd St. Branch of the New York Public Library. The former also, thru the Library Interchange System, borrowed a number of rare volumes from other cities. This aided greatly in collecting material from original sources, and in compiling the bibliography.

Lateral Earth Pressure.

INTRODUCTION.

Natural Repose. Any granular, semi-solid or semi-fluid, material, if unsupported, assumes a surface sloped at an angle to the horizontal; the maximum angle which a material can assume is called its natural slope, or angle of repose, and designated by the symbol ϕ. An attempt to pile the grains at a steeper slope results in the excess material rolling or sliding down. In liquids, the natural repose is horizontal; that is to say, liquids seek their level. In gra nular materials, the resistance that the grains or particles have to rolling over each other tends to neutralize the effect of gravity in bringing the material down to a level. Below the angle of repose, a grain is held by this resistance; above the angle of repose, the resistance is less than the gravitational attraction and the particle rolls down. Woltmann (1799) introduced the idea that the tangent of the angle of repose must be the coefficient of friction of the granular material on itself. Up to quite recently, this has been accepted. Since the angle of repose is a surface phenomenon, a plane of equilibrium on the surface, we are not justified in assuming the same amount of resistance inside the mass. Actual experiments have proven this contention - the angle of friction inside the mass is not the angle of repose.

Internal Resistance. The experimental method of determining the coefficient of friction is to measure the force required to move a known mass of material while resting on a plane of the same material. It is found that two coefficients may be determined: one corresponding to the force required to start the mass into motion, the second corresponding to the force required to keep the mass moving uniformly. In the first case the force required is equal to the total resistance exerted by the mass to shearing along the plane. This resistance consists of two parts - the resistance of the particles to rolling over each other, called friction, and the surface attraction between particles, called cohesion. This coefficient is called the "coefficient of internal resistance", and the corresponding angle is the angle of internal resistance. When motion occurs, the friction acts, but the cohesion does not, so that the second coefficient found, which is smaller than the first, is called the "coefficient of internal friction", and the corresponding angle is the angle of internal friction.

Internal Friction. It was pointed out by Boussinesq (1883) that the formulae for the phenomena in a granular mass should be functions of the angle of internal friction rather than the angle of repose. For in such phenomena the mass is originally in a state of rest and changes to a state of non-equilibrium, either motion or unbalanced stress, while the angle of repose is measured after the granular material has changed from a state of non-equilibrium to a state of rest. This prediction has been fully realized in recent experiments.

Laws of Friction and Cohesion. Coulomb (1781) formulated the laws of friction of solid bodies, and of the cohesion of bodies.

He assumed that the same laws hold for granular materials. With but few exceptions, all writers on the subject have held to this assumption.

1. The frictional resistance on any surface is equal to a constant, the coefficient of friction, times the total normal pressure on that surface.

2. The cohesion resistance is equal to a constant times the area of the surface.

Both forces act in the surface and in a direction opposite to the attempted motion. Whether the latter force is a cohesion, surface attraction between like particles, or an adhesion, surface attraction between unlike particles, is a doubtful point. It is probably an adhesion, because each grain is more or less coated by a water film; and in perfectly dry sand there is no cohesion force. The attraction is then between the grains of earth and water.

Active and Passive Pressures. If we assume a mass of earth to be divided into two sections by a plane, the forces exerted by one mass are just neutralized by the forces exerted by the other mass. Otherwise, the plane would not be in equilibrium. Either section may be removed and replaced by a single resultant force without changing the state of equilibrium. The minimum value of this resultant force is the required strength of a wall which is to hold the remaining section. This force is then called the "Active Earth Pressure." If it be increased, it will not only hold the earth in equilibrium as far as falling down is concerned, but will in addition tend to move it upward. Such motion is resisted by the friction and cohesion in the mass. When the force has been increased to a value which just overcomes these resistances, and the earth is on the point of moving upward, it is called the "Passive Earth Pressure". A better name for it would be the "Passive Resistance to Pressure", for it is equal to the latent resistance which the mass can exert to overcome forces tending to cause upward motion. The case of a retaining wall is an example of active pressure, usually called "Lateral Earth Pressure". The case of foundations is an example of passive pressure; where a heavy load causes the neighboring soil to heave, the passive resistance of the soil has been exceeded.

Determination of the Lateral Pressure. Three factors are necessary for the complete determination of the lateral pressure - magnitude, direction and point of application. The theories of lateral pressure cannot be based upon the laws of solid bodies, nor upon the laws of fluids. A granular material is neither a solid nor a liquid, but usually a combination of the two. The laws of granular materials have not been fully developed. All theories include some common assumptions:

1. That the mass is homogeneous.
2. The material consists of grains, which have the resistance to rolling over each other called friction.
3. The laws of friction hold.
4. The laws of cohesion hold. Cohesion is usually disregarded, since it cannot be counted upon to act at all times.

Wedge of Rupture. If a retaining wall were removed, some of the retained fill would immediately slip down. Not all the material above the plane of repose would fall at the same time. The first slip is called the wedge or prism of rupture. The surface that is left is fairly plane and usually called the plane of rupture. Changes due to weather and shock will soon cause the material to assume its natural slope. It is true that very often we see clay banks with overhanging tops, ditches with vertical sides, wtc., but these are merely cases where the cohesion is acting. Such banks and walls of ditches will soon slip and assume sloping sides unless retained.

Coulomb and Wedge Theories. The early wedge theories, derived by Coulomb, Mayniel and Prony assume that the pressure behind a wall is caused by the wedge of rupture exerting anormal force upon the wall. The resultant was assumed to act at the 1/3 point of the height of the wall. In 1840, Poncelet developed the general wedge theory, where the resultant pressure was assumed to act at the angle of friction between the fill and the wall from the normal to the wall. In all the wedge theories, an expression for the lateral pressure is derived in terms of the wedge, and a maximum value obtained by setting the first derivative of the expression equal to zero. Rebhann (1870) takes the area of the wedge as the variable with respect to which the differentiation is performed; all the other theories take the wedge angle, i.e. the angle between the wall and the plane of rupture, as the variable.

Rankine and Stress Theories. By assuming the mass indefinite in extent and incompressible, we can apply the theory of the elasticity of materials to a unit parallelopiped of volume in the retained mass of earth. In this way Rankine obtains his theory. Levy and Boussinesq generalized this stress theory by taking into consideration the effect of the wall surface. There are several other minor theories, all given below in the section on Earth Pressure Theories.

Notation. To facilitate comparison between the results of the various theories and the conclusions of the experimenters, a common notation is absolutely essential. In analogy to hydrostatic pressure, all resultant pressures are given in the standard form of a coefficient times half the product of the density of the fill and the square of the height of fill. No two authors agree entirely in their notation. Quite on the contrary - very often the same lett er is used by different authors for different things. For example, B usually denotes either the top or the bottom width of the wall; still, Paaswell uses B as the ratio of the height of the resultant to the total height of fill. Altho the symbol ϕ was very early recognized as the angle of natural slope, Arthur Jacob uses δ, Müller-Breslau uses ρ, and Slocum and Hancock use ω.

In the following standard notation, based upon that used most commonly in the literature of Earth Pressure, Roman letters of the lower case, denote distances, and ratios, capitals denote forces, Greek letters denote angles. The following symbols are in all the material given below put in place of the corresponding symbols from which the various theories and explanations are taken.

Distances.

a : top width of wedge of rupture.

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b : width of the wall, measured horizontally.

d : depth of the foundation.

e : eccentricity of the resultant on the base of wall.

h : height of the fill, measured vertically.

l : length of the wall, measured along the back.

s : height of the equivalent fill, in cases of surcharge.

x : height of the resultant above the base.

Densities.

w : weight of the wall per cu. ft.

y : weight of the fill per cu. ft.

Coefficients.

c : coefficient of cohesion, lbs. per sq. ft.

f : coefficient of friction corresponding to the natural slope.

f' : coefficient of friction of the fill on the wall.

f_1 : coefficient of internal resistance.

m : coefficient of friction on the foundation,

Forces.

E : total theoretical earth pressure.

G : gravitational force on the wedge of earth.

H : horizontal component of the lateral pressure.

N : normal component of the lateral pressure.

P : total resultant of the lateral pressure.

T : tangential component of the lateral pressure.

V : vertical component of the lateral pressure.

W : gravitational force on the wall.

S : gravitational force on the surcharge loads.

Angles.

Lateral Earth Pressure.

b : width of the wall, measured horizontally.

d : depth of the foundation.

e : eccentricity of the resultant on the base of wall.

h : height of the fill, measured vertically.

l : length of the wall, measured along the back.

s : height of the equivalent fill, in cases of surcharge.

x : height of the resultant above the base.

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T : tangential component of the lateral pressure.

V : vertical component of the lateral pressure.

W : gravitational force on the wall.

S : gravitational force on the surcharge loads.

Angles.

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α : between the vertical thru the toe and the back of the wall; negative if the top of the wall overhangs the toe.

δ : between the resultant earth pressure and the horizonta

ϕ : angle of natural slope or repose.

ϕ' : angle of friction between wall and fill.

ϕ_1 : angle of internal resistance.

ε : between the free earth surface and the horizontal; negative if the surface is below the horizontal.

ω : between the plane of rupture and the vertical.

Coefficients.

C : of $E = \frac{1}{2}yh^2C$, in the Poncelet theory.

$$C = \left[\frac{\cos(\phi-\alpha)}{\cos\alpha\ (1+n)}\right]^2 \frac{1}{\cos(\phi'+\alpha)}$$

$$n = \sqrt{\frac{\sin(\phi+\phi')\ \sin(\phi-\varepsilon)}{\cos(\phi+\alpha)\ \cos(\alpha-\varepsilon)}}\ ;$$

N : of $E = \frac{1}{2}yh^2N$, in the restricted wedge theory, (E normal).

$$N = \left[\frac{\cos(\phi-\alpha)}{\cos\ (1+n)}\right]^2 \frac{1}{\cos\alpha}\ ;$$

$$n = \sqrt{\frac{\sin\phi\sin(\phi-\varepsilon)}{\cos\alpha\cos(\alpha-\varepsilon)}}\ ;$$

R : of $E = \frac{1}{2}yh^2R$, in the Rankine theory.

$$R = \cos\varepsilon\ \frac{\cos\varepsilon-\sqrt{\cos^2\varepsilon-\cos^2\phi}}{\cos\varepsilon+\sqrt{\cos^2\varepsilon-\cos^2\phi}}\ .$$

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Compiled by Jacob Feld.

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Lateral Earth Pressure.

Earth Pressure Theories.

A. Theories previous to Coulomb. (1773).

1. Vauban (1687).
French Military Engineers.

The earliest attempt at scientific design of retaining walls is given by Vauban in "Traité de la defense des places" (1687). Altho no mathematical or theoretical discussion is given, Poncelet (556) (1840) tries to show that Vauban's empirical rules are based on advanced mechanical principles as well as experience and probably on some older rules. He says that Vauban's theory of the pressure of earth was based on the assumption of a wedge of rupture. For the case of a horizontal fill and a wall with a vertical back, the plane of rupture makes an angle ω with the back. For the usual filling materials ω has values between 0 and 45, but it is safe to always take $\omega = 45^0$. The center of pressure is at 1/3 of the height from the lowest point of the prism of rupture. The most economical form of wall is the triangular section. For this section, moments about the base, gives for the width of the base (see fig. 1)

$$\frac{1}{2} B h w \cdot \frac{2}{3} B = \frac{1}{2} \gamma h^2 \tan^2 \omega \cdot \frac{1}{3} h$$

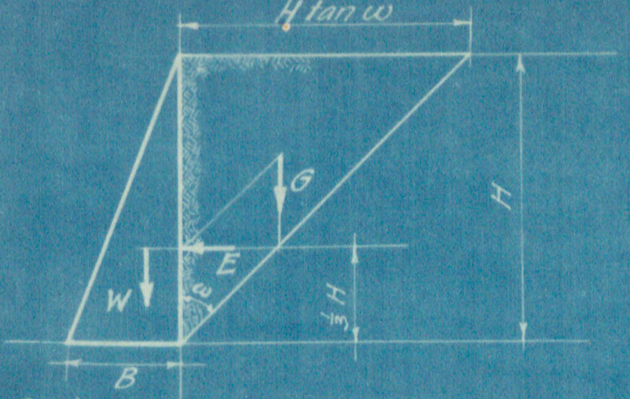
$$B = h \tan \omega \sqrt{\frac{\gamma}{2w}}$$

Vauban resolves the weight of the earth wedge into a horizontal earth pressure and a force along the plane of rupture.

His empirical rules were used in designing 150 fortifications of various heights, using 4000,000 cubic yards of masonry. (556)

1. For revetements $B = \left(0.2\frac{h'}{h} + 0.1\right) h + 1.225$ meters.

Fig. 1



$G = \frac{1}{2}\gamma H^2 \tan \omega$

$E = \frac{1}{2}\gamma H^2 \tan^2 \omega.$

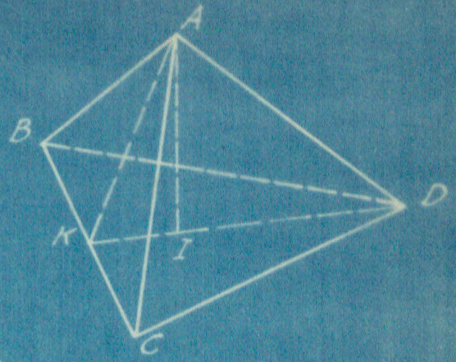
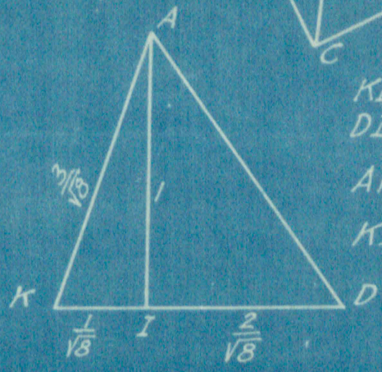


Fig. 2.



$KI = \frac{1}{3} KD = \frac{1}{3} KA$

$DI = \frac{2}{3} KA$

$AK = \frac{3}{\sqrt{8}} AI.$

$KI : AI : AK = 1 : \sqrt{8} : 3$

Diagrams for Earth Pressure Theories.

Fig. 3

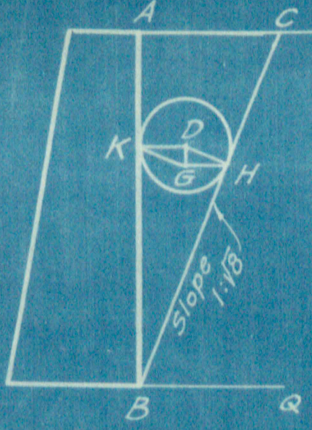


Fig. 4.

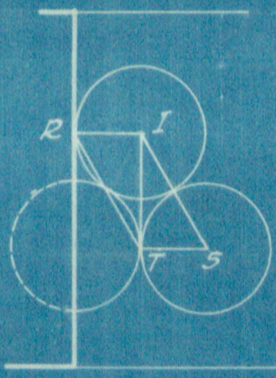
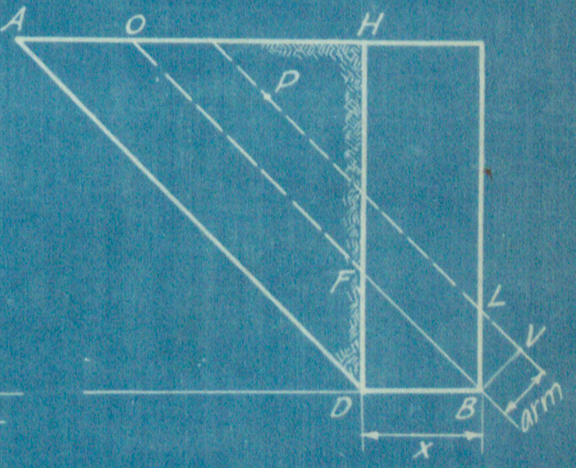
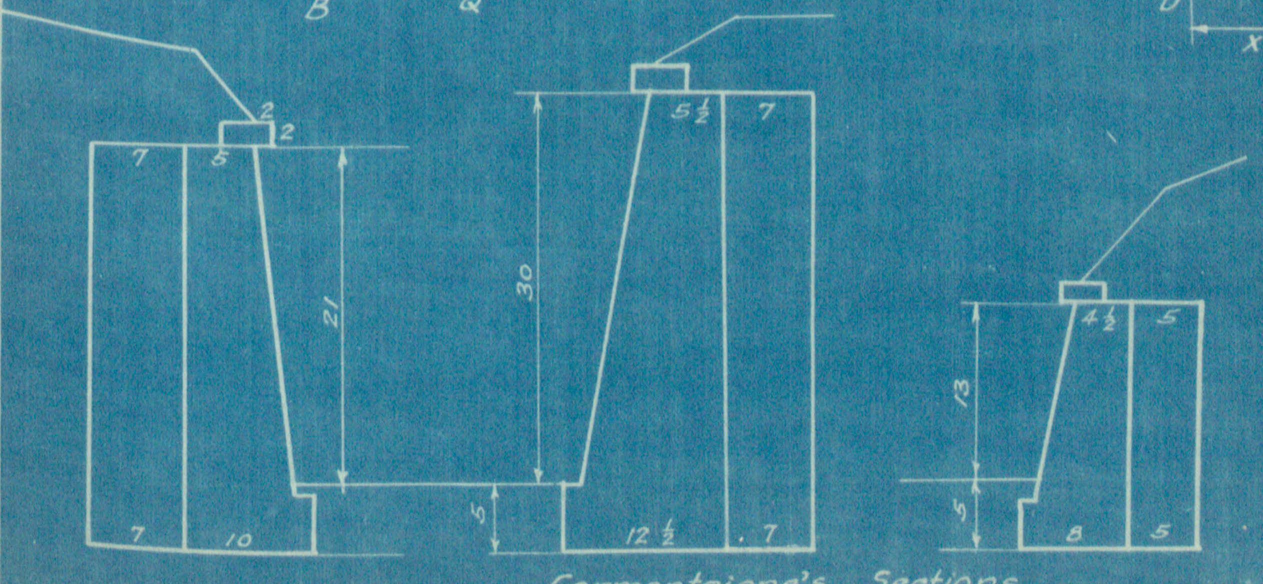


Fig. 5.





Cormontaigne's Sections.

Jacob Feld 1921.

Lateral Earth Pressure.

Counterforts to extend($0.2h+0.65$ meters) in back of the wall.

Thickness of counterfort at the wall: ($0.1h+0.65$ meter)

Thickness of counterfort at the end to be 2/3 of this. (556, 77)

2. For demi-revetements $B = 0.2h + 1.625$ meters.

Counterforts to extend [0.2 ($h+h'-2$ meters) + 0.65 meters] in back of the wall.

Thickness of counterfort at the wall:

[0.1 ($h+h'-2$ meters) + 0.65 meters].

Thickness of counterfort at the end to be 2/3 of this. (556)

3. Thickness of the top of revetement,

for new earth 1.625 meters (5 feet).

for old ditches 1 meter (3 feet). (556)

4. To transform a standard section into another section of equal stability, resolve the face line to the proper batter about a point on this line situated at 1/10 of the height from the base. If the batter is greater than 1 to 6, the error is 1 in 120; the usual error is about 1 in 70. For small batters, as 1 to 18, use a point at 1/9 of the height from the base. (125)

Vauban assumed that his walls would rotate as units about the toe, the counterforts acting as integral parts of the wall. His requirements for stability was that the resultant pressure must intersect the base at a point whose distance from the toe is 4/9 of the distance between the toe and a vertical thru the center of gravity of the wall. (457)

Altho often criticised, Vauban's rules have had a tremendous effect on the theory of earth pressure and the design of retaining walls. Couplet agrees with him in the action of counterforts; so does Poncelet. As late as 1877,

Lateral Earth Pressure.

Wheeler in his Manual of Civil Engineering for U.S. Military Cadets at West Point, advises the use of Vauban's formulae for military works of retaining nature. (645)

The following table gives some of the standard sections. (626)

h pi.	t pi.	b pi.	c pi.	h pi.	d pi.	e pi.pou.	V pi.pou.lig.poi. (cubic)
10	5	7	18	4	3	2- 0	2 -0 -11 - 1
20	5	9	18	6	4	2- 8	4 -5 - 0 - 5
30	5	11	18	8	5	3- 4	8 -3 - 3 - 1
40	5	13	18	10	6	4- 0	13 -2 - 6 - 2
50	5	15	18	12	7	4- 8	19 -3 - 8 -10
60	5	17	18	14	8	5- 4	27 -1 -10 - 2
70	5	19	18	16	9	6- 0	36 -3 - 9 - 4
80	5	21	18	18	10	6- 8	47 -4 - 5 - 4

h: Height in pieds t: Width at Top in pieds
b: Width at Base in pieds n: Length of counterforts in pieds
c: Distance between counterforts in pieds
Width of Counterforts - d: at wall, e: at end.
V: Volume of wall - including counterforts - per linear toise

Note: Old French System of Lengths. (Trautwine)

Point (poi.)	.0148in.	Ligne (lig.)	.0888in.
Pouce (pou.)	1.0658in.	Pied (pi.)	12.7892in.
Toise	6.3946ft.		

Vauban describes the action of counterforts as diminishing the total earth pressure acting on a wall by a decrease of the area of the wall, and also by the frictional resistance of the sides of the counterforts imbeded in the earth. (556) Audoy found that an ordinary wall must have a factor of stability for overturning of 3.80 and for sliding of 4.70, in order that its stability be equal to the Vauban section for the same height. He assumed that the counterforts hold to the wall in all cases, requiring the use of the 'best of hydraulic mortar'. Poncelet believes that these values are somewhat high, and compares the values given by Vauban with those from his formulae, computed both for sliding and overturning.

Lateral Earth Pressure.

2. Bullet (1691)

Royal Academy of Architecture.

Bullet in his "Traite' d'Architecture Pratique"(1691) gives the first earth pressure theory based upon mechanical principles. Altho his assumptions are many, the results are not very far from correct. (556,175) His formula

$$B = 0.35 H$$

is based entirely on theory; there is no record of experiments. He considers a wedge of rupture, acting as a solid body on a 45° inclined plane. The weight of the body is to the force acting parallel to the inclined plane as the length of the plane of rupture is to the height of the wall. If we assume the ratio of the diagonal of a square to the side to be 7/5, then 7/5 of the area of the wedge of rupture is equal to the area of the wall. (418)

$$\frac{7}{5}\,\frac{h^2}{2} = Bh \; ; \qquad B = 0.35 H.$$

Mayniel objects to this theory, because of the assumptions of a 45° plane of rupture, of a pressure acting parallel to the plane of rupture, and of the equality in density of wall and earth. Bullet's theory was used by Couplet(1727) to further develope the use of mechanics in determining the pressure of earth. The above is really the earliest known attempts to find a theoretical formula for designing retaining walls. (418)

Lateral Earth Pressure.

3. Buchotte (1716)

French military engineer.

Buchotte in his "De'pot des Fortifications" (1716) assumes a 45^{o} wedge of rupture, the density of the fill as two-thirds that of th e wall, and hense the weight of his wall is two thirds that of his prism of earth. This gives B = 0.33H. (556)

Some of his practical suggestions for the design and construction of walls are given below: (407)

1. It is inadvisable to follow Vauban in using batters greater than 1 to 5, because water getting into joints may cause cracks in the wall if freezing should occur.

2. Walls to resist cannon shock must contain a greater volume than those built to sustain earth; a width of 4.5 feet for a 15 foot wall is too small..

3. The height of counterforts is not increased in the same ratio as the height of the retaining wall. For a

H = 10 ft.; B = 4 ft.; width of counterfort is 1/3 H. For each 5 ft. increase in H, width of counterfort is increased 1 ft., giving for a 30 ft. wall, a counterfort thickness of 8 ft., or less than 1/3 H; and for a 60 ft. wall, a thickness of 14 ft., or less than 1/4 H. This last value is not suitable, it is too weak a design.

4. The width of the counterfort at the tail (back end) should be increased in proportion to the increase at the root (wall end). If the original width is 3 ft. at the root and 2 ft. at the tail, for each foot increase in length of counterfort, the increase in width at the root is 6 inches and at the tail 4 inches.

5. For the walls of 40 ft. and less in height, the counterforts should be spaced closer than 15 ft; for 50 ft. and

higher walls, they should be spaced further apart than 15 ft.
6. The theoretical design of a wall(see above) is
B = 1/3 H. The length of counterfort is 1/3 H; width of counterfort at the tail is 2 ft., and at the root greater than 2 ft., depending on the length. Location of counterforts to be 8 ft. on centers; Vauban's counterforts at 15 ft. intervals are in danger of being broken off.

Mayniel brings up objections to these suggestions of Buchotte. He cannot assume the wall to be stable if its weight equals that of the wedge, because the wall is rectangular and the wedge is triangular in section, and the wall rests on a horizontal base, while the base of the wedge is inclined. He forgets that the counterforts will aid stability much more if the tail is made wider than the root; this would also increase the frictional resistance to sliding.

4. Couplet (1726)

Academie Royale des Sciences.

Couplet in the "Histoire de L'Academie des Sciences de France" published three articles entitled
"De la Poussee des Terres contre Leurs Revetements et la Force des Revetements qu'n. Leur. Doit opposer,"
1726, vol. 28, pp. 106-64, 1727, vol. 29, pp. 139-85, and 1728, vol. 30, pp. 113-38. In these he developes Bullet's idea of the application of the laws of Mechanics, into two earth pressure theories.[136]

In his first article he assumes a frictionless wall, acted upon by a fill of an infinite number of small grains resting on each other. If equal sized spheres are piled on a triangu-

Lateral Earth Pressure.

lar base, they form a tetrahedron. The ratio of the height of a slope to the base is $\sqrt{8}$ is to 1.(see fig. 2.) This then is the slope of the plane of rupture. The effect of the prism of rupture is equal to the effect of a ball wedged in between the wall and the plane of rupture, its weight being equal to that of the prism, and touching the wall at a point 2/3 of the height of the wall above the base.(see fig. 3). This assumption was also made by Bullet, but Couplet definitely states as one of his theorems that the plane of rupture does not coincide with the plane of natural repose. Now arrange the spheres in tetrahedra acting against the wall (see fig. 4) Resolve the downward pressure of sphere I, into a force along the line of centers IS, and a force perpendicular to the wall, RI. The first is the lost or dissipated pressure, the second is the lateral earth pressure. The weight is IT. IS represents an edge of the tetrahedron, hence TS is 2/3 of the altitude of the base triangle, and $\frac{TS}{IT} = \frac{2}{\sqrt{8}}$.

or $\frac{\text{Earth Pressure}}{\text{weight of wedge}} = \frac{2}{\sqrt{8}}$

weight of wedge is $\frac{H^2}{2\sqrt{8}} y$ Earth pressure $= \frac{1}{8} y H^2$.

Its lever arm about the base is $\frac{2}{3}$ H; and assuming the density of wall as w.

$$BHw \cdot \frac{H}{2} = \frac{1}{8} y H^2 \cdot \frac{2}{3} H \quad \text{or} \quad B = H\sqrt{\frac{y}{6w}}$$

In this case, he has taken the earth pressure as acting normal to a perfectly smooth wall.

In his article of 1727, he assumes a rough wall, and takes the earth pressure not horizontal but perpendicular to the irregularities or grains of the back of the wall. He states six theorems as the basis of his theory.

Lateral Earth Pressure.

1. If the height of the tetrahedron and the altitude of the base are drawn, then the height, the shorter part of the base altitude and the slope or altitude of a face are to each other as $\sqrt{8}$ to 1 to 3.

2. The height, the longer segment of the base altitude and the edge of the tetrahedron are to each other as a to $\frac{a}{\sqrt{2}}$ to $\frac{a\sqrt{3}}{2}$. In this we have assumed one sphere piled on 3.

3. If we assume one sphere piled on 4, the ratio of the height, the slope and the projection of the slope on the base are to each other as $\sqrt{2}$ to $\sqrt{3}$ to 1.

4. In the case of 1 sphere on 3, the ratio of the weight of a sphere to the force that it exerts in the direction of the slope of a face is as $\sqrt{2}$ to 1.

5. In the case of 1 sphere on 3, the ratio of the weight of a sphere to the force that it exerts in the direction of the edge is as $\sqrt{6}$ to 1.

6. In the case of 1 sphere on 4, the ratio of the weight of a sphere to the force that it exerts in the direction of the slope of a face is as 1 to $\frac{\sqrt{3}}{2\sqrt{2}}$

In fig. 5, OFH is the wedge of rupture, OF, the plane of rupture passes thru the toe of the wall and has a slope depending on whether we assume one sphere to rest on three or on four other spheres. His reason for disregarding the earth OFDA, is that the triangle FBD may be left out and the earth OBDA would then be immovable.

Lateral Earth Pressure.

Let HD=h, ND=b, AD=c, BD=x; Area of HFO is $\frac{1}{2}OH \cdot HF$

$$= \frac{hb-hx}{b} \cdot \frac{b-x}{2} = \frac{hb^2-2hbx+hx^2}{2b}$$

for triangles BDF and HFO are similar, and $FD = hx/b$

$$HF = (hb - hx)/b \; ; \quad OH = b - x.$$

If E : total weight of earth wedge which gives,

F : pressure component, parallel to OF, causing shear along FB,

$$E : F = \frac{(hb^2-2hbx+hx^2)}{2b} : \frac{(hb^2-2hbx+hx^2)\,F}{2bE}$$

If the wall were to shear along FB, the lever arm of F will be BV at right angles to OB. If P is the center of gravity of the wedge OFH, and drawing PV parallel to OB, $\triangle BVL$ is similar to $\triangle$ DHA.

$$BV = BL \frac{AH}{AD} = 1/3\ HF \frac{AH}{AD} = 1/3\ h(b-x)(1/b)(b/c) = \frac{h(b-X)}{3c}$$

Moment of F causing rotation of wall about the toe is

$$F(BV) = \frac{yh\,(b-x)^2 F}{2bE} \cdot \frac{h\,(b-x)}{3c} = \frac{(b-x)^3}{2bE} \cdot \frac{Fyh^2}{3c}$$

The moment of stability of the wall is $\frac{1}{2}\frac{w}{y}hx^2$

The equation of equilibrium is

$$\frac{(b-x)^3}{2bE} \cdot \frac{Fh^2}{3c} = \frac{w}{y} \cdot \frac{hx^2}{2}$$

Assuming, as in the first theory, that one sphere rests on three others forming a tetrahedron, then (see fig. 2)

KI : AI : AK :: 1 : 8 : 3. Let the height, AI, represent the weight of the wedge and resolve it into two components, the earth pressure acting parallel to AK, the altitude of the face, and the frictional loss acting parallel to AD, the edge.

Since $KI = 1/3KD$; $KE = 1/3KA = \frac{2}{\sqrt{8}}$; letting $AI = 1$. Then

$AI : KE = 1 : 2/\sqrt{8} = \frac{\sqrt{2}}{1}$ weight of wedge to the force causing rotation,

or $\quad E/F = 1/\sqrt{2}$

Substituting above, for $F/E = \sqrt{2}$, $b = h/2\sqrt{2}$, $c = 3h/2\sqrt{2}$

$$\frac{(h/2\sqrt{2} - x)^3}{2\frac{h}{2\sqrt{2}}} \cdot \frac{2h^2}{3 \cdot \frac{3h}{2\sqrt{2}}} = \frac{w}{y}\frac{hx^2}{2}.$$

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$$\frac{\sqrt{2}\left(\frac{h}{2\sqrt{2}}-x\right)^3}{9/4}=\frac{w}{\gamma}\frac{hx^2}{2} \quad \text{or} \quad \left(\frac{h}{2\sqrt{2}}-x\right)^3-\frac{9}{8\sqrt{2}}\frac{w}{\gamma}hx^2=0.$$

The solution of this cubic equation is discussed in Appendix A. The following table computed by the writer, shows the values for ordinary cases.

w/γ:	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
x/h:	.124	.120	.1175	.115	.1125	.110	.108	.106	.104	.1025	.101

The results are far below the values ordinarily used.

Mayniel brings up the following criticisms of Couplet's theory. That it is not substantiated by experiment is evident from the assumptions of a natural slope of $\sqrt{8}$ to 1, and also of an earth pressure acting parallel to the plane of repose or the natural slope. Note that in his first theory he assumes a horizontal earth pressure. Gauthey tried to prove this theory experimentally, but discovered that the plane of rupture did not coincide with the plane of slope and that it was much harder to rupture the wall than to overturn it. These assumptions cause the very low values obtained for the base width.

Tho his hypothesis of one sphere resting on four is not developed in full, he cannot expect reliable results because he must assume that the upper spheres can only transmit stress to the lower ones either along ridges or along the lines bisecting the sides. This assumption is not reliable.

In assuming the wall to rupture, the theory takes the line of rupture to be the continuation of the plane of rupture of the fill, but Gauthey showed that in rupturing mortar walls, the line of break is very closely horizontal. This is to be expected, for a section of the wall parallel to the natural slope or parallel to the plane of rupture has a greater area than a horizontal section and will therefore offer greater resistance. Rupture, if it does occur, is across the weakest section.

Mayniel sums up his objections under three issues, the assumption that the triangle of pressure does not include the trapezoid pressing against the base of the wall; the assumption of rupture of wall along the plane of rupture and the disregard of actual conditions, such as the fact that walls are built of bricks or stones with horizontal and vertical sides.

5. Belidor (1729).

B. F. Be'lidor's first contribution appeared in his "Cours d' Architecture militaire, civil et hydraulique," 1720; but the following account of his theory is taken from his "Science des Ingenieurs," 1st ed. 1729.[40]

Experience, he says, shows that in the case of a horizontal fill retained by a vertical wall, the prism of rupture is a 45 degree right triangle. This tends to slide on the plane of slope, which is inclined to the horizontal at 45^0 and passes thru the heel of the wall. If we divide the wedge into as many parts as there are feet in the height of the wall, by planes drawn parallel to the natural slope, the areas of these parts times their lever arms represent the amount of pressure that they exert. If we take a wall 15 ft. high and divide the wedge into 15 parts, the area of the first segment, a tri-

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angle to be a and its lever arm 15, then the areas of the successive segments will be 3b, 5b, 7b,.....29b, and their respective arms will be 14, 13, 12,.....1, about the base of the wall, assuming the pressure to act horizontally. The summation of the products of the corresponding areas times their lever arms divided by the height of the wall gives the arm ratio; for the above case, he finds it to be 82.67. The arm ratio times the area of the wedge gives the total overturning moment due to the earth pressure. He assumes that half of this is lost in friction along the plane of rupture; this being the first case where friction is considered. By equating this overturning moment to the resisting moment of the wall, he gets his value for the base width. The center of moments is the toe of the wall.

Mayniel says that Belidor assumes his total pressure to act at the top of the wall. In editing a second editition of Belidor's "Science des Ingenieurs," 1813, Navier calls attention to this and shows the falsity of this statement. By his arm ratio method, Belidor really gets the pressure resultant to act at 1/3 the height of the wall from the base. Navier is perfectly right in his contention.(418)

Belidor's overturning moment is given as $\frac{rHh^2}{4}$ by Mayniel where r is his arm ratio; H the height of the wall and h the height of the unit of fill. Assuming $H = hn$; $M = \frac{1}{4} rh^3n$.

To find the value of r, Let x = vertical distance along the wall from the top to the trapezoid in question.

Then the vertical depth of any trapezoid is Δx, its altitude is $\frac{1}{2}\Delta x$ and its average base is $\frac{1}{2}x$. Its area is then $x\Delta x$,

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and its moment about the base $x \Delta x (H-x)$.

$$r = \frac{1}{H}\int_0^H x \Delta x (H-x) = \frac{1}{H}\left[H\frac{x^2}{2} - \frac{x^3}{3}\right]_0^H = \frac{H^2}{6}.$$

His $\underline{r}$ is based on $\underline{a}$ as a unit, hence we must use $r = \frac{H^2}{6a}$.

$$M = \frac{rHh^2}{4} = \frac{1}{2}\cdot\frac{H^2}{2a}\cdot\frac{H}{3}\cdot\frac{h^2}{2} = \frac{1}{2}\cdot\frac{A}{a}\cdot\frac{H}{3}\cdot a = \frac{1}{6}AH.$$

where A = total area of wedge;

H = height of wall

a = area of first segment of the wedge.

If we take, as Belidor advises, h = 1 ft. M = 1/6 AH, which is half of the theoretical overturning moment, the half factor being caused by friction along the plane of rupture.

Navier notes that Belidor's results are based on the assumption of a 45° plane of slope, the plane of rupture coinciding with the plane of slope, disregarding cohesion and that the earth pressure acts horizontally.[40]

If the back is sloped, the wedge of rupture is the mass between the back of the wall and a plane 45° to the horizontal passing thru the heel. If there is surcharge, the wedge of rupture is the mass between the back of the wall and two 45° lines passed thru the heel and the back of the top of the wall. Counterforts are of great advantage, and Vauban's sections are recommended.

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BELIDOR'S TABLE OF SECTIONS.

Height pi.	feet.	Earth Pressures. Retaining Wall. pi.po.li.	Revetment. pi.po.li.	Widths of Walls at Top. Ret.Wall: pi.po.li.	Revet. pi.po.li.	Retaining Walls. Base Width. pi.po.li. *	ft. *	Ratio b/h *
10	10.7	6- 5-0	15- 7- 0	1- 9- 1	3- 8- 4	3- 9- 1	4.0	.376
15	16.0	13- 9-4	27- 1- 4	2- 6- 2	4- 6- 8	5- 6- 2	5.9	.368
20	21.3	23-11-0	41- 5- 0	3- 3- 5	5- 4- 6	7- 3- 5	7.8	.364
25	26.6	36- 6-0	57- 6- 0	4- 0- 7	6- 1- 2	9- 0- 7	9.6	.362
30	32.0	52- 6-4	74- 4- 0	4- 9- 8	6- 9- 0	10- 9- 8	11.5	.360
35	37.3	71- 0-0	95- 3- 4	5- 6-11	7- 4- 8	12- 6-11	13.4	.359
40	42.6	92- 3-0	117- 8- 0	6- 3-10	8- 1- 2	14- 3-10	15.2	.358
45	48.0	116- 3-0	142- 7- 0	7- 1- 3	8- 7-11	16- 1- 3	17.1	.358
50	53.3	143- 1-0	170- 1- 0	7-10- 5	9- 3- 0	17-10- 5	19.0	.357
55	58.5	172- 8-0	200- 3- 0	8- 7- 6	9-11-10	19- 7- 6	20.9	.357
60	64.0	205- 0-4	233- 1- 0	9- 4- 9	10- 9- 1	21- 4- 9	22.8	.356
65	69.3	240- 2-0	271-10- 0	10- 2- 0	11- 4- 3	23- 2- 0	24.6	.356
70	74.5	278- 1-0	306- 9- 0	10-11- 0	12- 0- 8	24-11- 0	26.5	.356
75	79.9	318- 9-0	347-10- 0	11- 8- 3	12- 9- 1	26- 8- 3	28.4	.356
80	85.2	362- 3-0	391- 7- 6	12- 5- 4	13- 5- 6	28- 5- 4	30.3	.356
85	90.5	408- 6-0	438- 6- 0	13- 2- 7	14- 2- 1	30- 2- 7	32.2	.356
90	95.8	457- 6-0	487- 3- 8	13-11- 9	14-10- 9	31-11- 9	34.0	.355
95	101.0	526-10-6	556-10- 6	14- 8-10	15- 7- 5	33- 8-10	35.9	.355
100	106.5	563-11-0	594-10- 0	15- 6- 1	16- 4- 2	35- 6- 1	37.8	.355

Notes: For conversion table of old French system of linear measurements, see table under Vauban.

All walls have a front batter of 1 : 5.

Values in columns headed "Earth Pressures" are the equivalent forces acting horizontally at the top of the wall. Both types of wall are trapezoidal in section; the revetment or fortification wall has a parapet 3x4 pieds resting on the wall.

* denotes that the values in this column are not contained in the original article, but were computed by the writer.

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6 Lemaire(1737)

French Military Engineers.

Lemaire (1737) revised Vauban's dimensions for fortification walls, allowing a greater tenacity in earth. For a 30 ft. wall, he used a top width of 4 ft., in place of 6 ft., a batter of 1:8, therefore a base of 7 3/4 ft, giving a ratio of base to height of 0.26. He used counterforts 8 ft. deep, with a width of 4 ft. at the wall junction and 2 ft. 8 in. at the tail end.

To resist cannon shock, he recommends that a 30 ft. wall have a top width of 3½ ft, a batter of 1:8 and be backed with semi-cylindrical towers, radius of 9 ft., to form a solid back of 4 ft. thickness, and counterforts 6 ft. long and 2 feet thick. These cylinders are of a uniform thickness of 2 ft., and will aid the wall in withstanding the combined earth and cannon pressure. (418)

7 Querlonde (1743).

French Military Engineers.

Querlonde (1743) states that Buchotte's method, assuming that the natural slope was 45°, the coefficient of sliding was 1, and the ratio of density of earth to wall to be 2/3, applied only to a special case. Each type of material and wall must be separately considered. He takes up three cases - a wall retaining vegetation soil, stiff earth and sand. (556, 418)

From the principle of the inclined plane, we have the relation: the force which tends to cause sliding along the

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slope of the plane is to the part of the weight force which is lost because of the resistance of the plane as the height is to the base. Calling W the weight of the prism of rupture, which he assumed to slide on the plane of repose, he resolves it into F, a loss due to friction, and E the earth pressure, which is the cause of motion.

Hence E:F as the horizontal dimension of the wedge of rupture is to the vertical; and $E+F=W$

Case 1- Soil has a surface slope of 45°, or the wedge of rupture is isosceles. Therefore $E=F$ and $E=\frac{1}{2}W$.

Case 2- Stiff earth has a surface slope of 3/2. Then the Earth pressure is 3/5 of the weight of the wedge.

Case 3. Sand has a surface slope of $\frac{1}{2}$; then the earth pressure is 1/3 the weight of the wedge. In all of these, he assumes the pressure to act horizontally, and at 2/3 the height from the base of the wall, i.e. thru the center of gravity of the wedge.

To find the required width of wall, assume the ratio of densities of earth and wall to be as 2:3, also that because of friction and cohesion, only half the wedge of rupture will act.

In case 1; weight of wedge is $2 \cdot \frac{h^2}{2} = h^2$ (area x density)

Earth pressure moment is $\frac{1}{2}h^2 \cdot \frac{2}{3}h = \frac{1}{3}h^3$ and assuming that half is lost in friction and cohesion, the overturning moment is $\frac{1}{6}h^3$. The moment of the wall, about the toe, is $\frac{3}{2}hx^2$ where x is the width, and its density is 3. Equating these, we get $x=\frac{1}{3}h$.

for a natural slope of 1:1

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In Case 2; weight of wedge is $\cancel{\left[\frac{3}{5}\cdot\frac{2}{3}h^2\cdot\frac{2}{3}h=\frac{4}{15}h^3\right]}$ $2\times\frac{2}{3}h\times\frac{h}{2}=\frac{2}{3}h^2$.
Earth pressure moment is $\frac{3}{5}\cdot\frac{2}{3}h^2\cdot\frac{2}{3}h=\frac{4}{15}h^3$; and assuming that half is lost in friction and cohesion, the overturning moment is $\frac{2}{15}h^3$. The moment of the wall, about the toe, is $\frac{3}{2}hx^2$; and equating these we get

$x=\frac{2\sqrt{5}}{15}h$ for a natural slope of 2:3.

In case 3; weight of the wedge is $2\cdot\frac{1}{2}h\cdot 2h=2h^2$.
Earth pressure moment is (1/3) $2h^2\cdot\frac{2}{3}h=\frac{4}{9}h^3$; and assuming that half is lost in friction and cohesion, the overturning moment is $\frac{2}{9}h^3$. The moment of the wall, about the toe is $\frac{3hx^2}{2}$ and equating these we get.

$x=\frac{2\sqrt{3}}{9}h$ for a natural slope of 1:2.

In discussing this theory, Mayniel gives the following idea of earth pressure. There is a prism which will cause a pressure greater than that caused by a larger or by a smaller prism. Assumed the prism above the plane of rupture to be divided into an infinite number of small triangles with bases all of equal size on the top surface and their apices at the foot of the wall. These thin wedges will not fall down or break because of cohesion, but will follow each other successively. The first slipping gives the earth pressure, future slips do not concern the problem. Querlonde arbitrarily assumes that half of the possible pressure is lost thru friction and cohesion. He decomposes the acting weight of the wedge into two forces, that lost because of the resistance of the inclined plane, and the acting pressure. This is also an arbitrary assumption; it is more natural to assume the weight to be the resultant of the two other forces, i.e.

the vector sum; and not the algebraic sum.

This theory was generalized by Mayniel to hold for all kinds of earth and wall.

The area of the sliding wedge is $\frac{mh^2}{2}$, where **m** is the ratio of the horizontal length of the wedge to the vertical. The acting weight is half its weight or $\frac{wmh^2}{4}$, where **w** is the density of the fill. This is divided in the ratio of 1:m, and the acting lateral pressure is $\frac{wmh^2}{4(m+1)}$. Its lever arm is $\frac{2}{3}h$. the overturning moment $= M = \frac{wmh^3}{6(m+1)}$

Letting **x** be the width of the wall and **y** the density, the stability moment is $\frac{yhx^2}{2}$. Equating moments and solving for the base width:

$$\frac{wmh^3}{6(m+1)} = \frac{yhx^2}{2} \quad ; \quad x^2 = \frac{mwh^2}{3(m+1)y} \quad ; \quad m = \cot\phi .$$

$$x = \frac{h}{3}\sqrt{\frac{3wm}{y(m+1)}} = \frac{h}{3}\sqrt{\frac{3w}{y}}\sqrt{\frac{\cot\phi}{1+\cot\phi}} .$$

General equation $x = \frac{h}{3}\sqrt{\frac{3w}{y}}\sqrt{\frac{1}{1+\tan\phi}} = \frac{h}{3}\sqrt{\frac{3w}{y}}\sqrt{\frac{\cos\phi}{\sin\phi+\cos\phi}}$.

8. Gadroy (1745)

Gadroy (1745) is the first to base his theory upon experiments; or rather he tries to check Belidor's theory by experiment. He became interested in the subject of earth pressure when he noticed a slip and a bulge in a large wall at Place de Valenciennes. ~~(455)~~ (418).

His experimental work was with a box 3" x 3" x 10" long, having a free side 3" x 3". This free side could be placed in any position inside the box. He usually placed it 3 in. from the closed end, forming a 3" cube, which he filled with fine sand. The wall overturned when released. The wedge of rupture was 2 in. wide and 3" high. He repeated this

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with the depth of fill and amount of fill varied, and always found the ratio of the horizontal dimension to the vertical dimension of the wedge to be 2:3.

Mayniel objects to his drawing any but qualitative conclusions from the experiments. He notes that altho the natural slope was 1:1, the plane of rupture had a greater slope, 2:3. Gadroy errs in concluding that the greatest pressure occurs at the top of the wall because the side rotated and did not slide.

Gadroy's investigation into the condition of retaining walls leads him to make the following conclusions.

1. The most noticeable errosion is on the windward side.
2. The smaller the natural slope, the less the errosion.
3. Walls built of 4 ft. square blocks laid in sand are also affected by wind and exposure.
4. Such blocks are not like fire baked bricks, they crush while the brick cleaves.
5. If bricks are used, there should be both headers and spreaders. If a 9in. by 4½ in. by 2 in. brick is used, the spreaders course should be 4½ in. high. If failure occurs, the spreaders fall before the headers. He advises the use of a 12 in. by 4 in. by 2 in. brick, laid in slaked lime, slaked 24 hours before using, and a batter of 1:20 or less.

He claims that there are two errors in the existing earth

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pressure theory. That, first, the wall is usually built thicker at the base than at the top, even tho the force is less effective, in causing overturning, at the foot than at the top. Second, a wall supporting natural earth is usually made thicker than one supporting a filled earth.

He proposes a profile for a wall 50 ft. high. The thickness is 4 ft. thruout, and has semi-circular relieving arches 10 ft. on centers, which thickens the wall by 2 ft., and on each 10 ft. of elevation, he has a similar arch up to the ledge. His counterforts are then 10 ft. long and 5 ft. thick thruout.

Mayniel agrees with Gadroy's manner of laying brick, and summarizes the methods and fallacies of constructing walls:

1. A good masonry requires bricks made of good clay.

2. Also well prepared clay to make good bricks.

3. The bricks are not to be touched by flame during baking.

4. Ancient walls in Piedmont and Alexandria have larger bricks, than those used now, with better results.

5. Dr. Bryans - Hygins claims that the present masonry is weakened by the use of too rich a lime mixture. No mixture richer than a 1:7 lime-sand should be used.

9. Louis deCormontaigne (1749)

M -de Cormontaigne advises the use of Conplet's theory simplified to every day use. Assuming that the pressure is caused by a 45^{0} prism sliding on its plane of repose, he assumes that the value of the lateral pressure is one-half of the weight of this prism. From this he arbitrarily deducts 10, and also deducts for the batter of the wall. The remainder divided

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by the height gives the base width. (418)

For example, a 24 ft. wall with a batter of 1:6 area of pressure wedge is 24 by $\frac{24}{2}$ = 288.

$\frac{1}{2}$ of this is 144; and 144 − 10 = 134 is the pressure.

The batter causes a deduction of $\frac{1}{6} \cdot 24 \cdot \frac{24}{2} = 48$

Remaining pressure is $134 - 48 = 86$

Base width is then $\frac{86}{24}$ = 3 ft. 7 in.

In his book on Fortifications he gives the sections shown in figures (5 a,b,c,d.) The walls are built according to Vauban's standards, and retain surcharged fills. Above 12 ft. height he uses counterforts. Disregarding these we have the following dimensions, h being the total height.

h	b	Counterforts	b/h	B/H.
17	7	—	.412	.458
20	8	5×18	.400	.434
28	10	7×26	.358	.370
30	9	7×28	.333	.300
33	11	7×31	.333	.320
36	12.5	7×35	.348	.339

The wall is imbedded 5 ft. below the surface; $H = h - 5$ ft. the foundation is offset $1\frac{1}{2}$ ft., except in walls over 30 ft., where a 2 ft. offset is used. B is the width of the wall above the foundation.

9. Stahlswerd (1755)

See Forelaesningar uti reguliere Fortification - Stockholm 1755. (285)

Also Böhm's Magazine; IV Band; p. 145-62

10. Largna (1755).

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See Sterke der Walmuuren , Accademia delle Scienze de Siena.

Tom II p. 155-175

also Bohm's Magazine; IV Band, p. 119-144.

He assumes a 45^0 prism of rupture, and says that the earth pressure is to the weight of the prism as the sine of 45^0 is to 1; or $E = \frac{wh^2}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} wh^2$, acting parallel to the plane of rupture.

It may be resolved into a horizontal and a vertical component each of which is equal to $E \cdot \frac{\sqrt{2}}{2} = \frac{1}{4} wh^2$. (285)

11. Kinsky (1763)

See "DeDrukking der Aarde tegen Walmuuren" in the Magazien für Ingenieur und Artilleristen.

Also Bohm's Magazine, XII Band, p.127-160.

He assumes a 45^0 prism of rupture, and gets an overturning moment of $\frac{1}{12} wh^3$. (87)

12. N. Ypey (1765 ±)

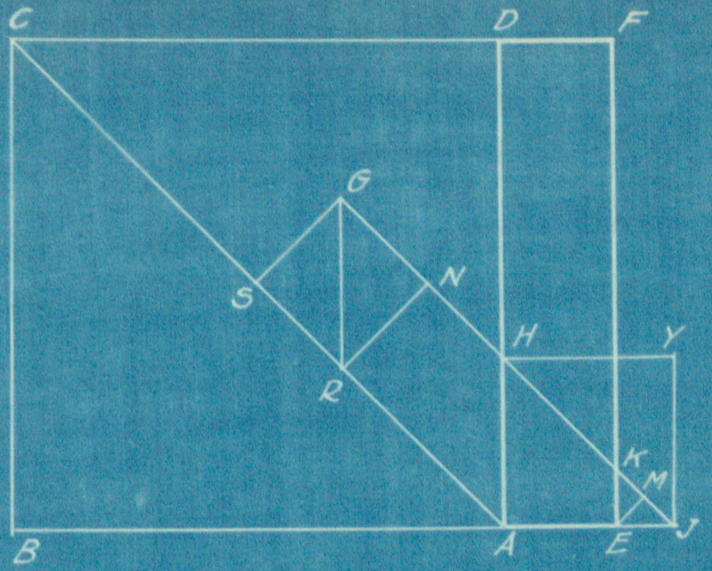
Ypey gives a theory based on the assumption of a 45^0 plane of rupture passing thru the toe of the wall. It is in the "Verhandelingen der Haarlemsche Moatschappij" VI deel p. 516-42. (87)

13. Sallonyer (1767)

French Military Engineer - Dept. of Fortifications.

Altho Sallonyer also assumed that the prism of rupture slides on the plane of repose, he resolves the weight of the prism into a different pair of components, parallel and perpendicular to the plane of rupture. The force causing sliding of the wedge is to the weight of the wedge as the height of the plane is to the oblique length. Since the mass is homogeneous, the pressure is parallel to the slope and acts thru the center of

Fig. 7



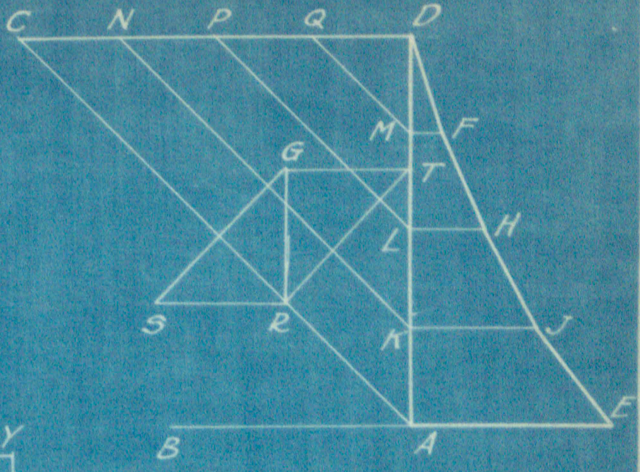


Fig. 9.

Diagrams for
Earth Pressure Theories.

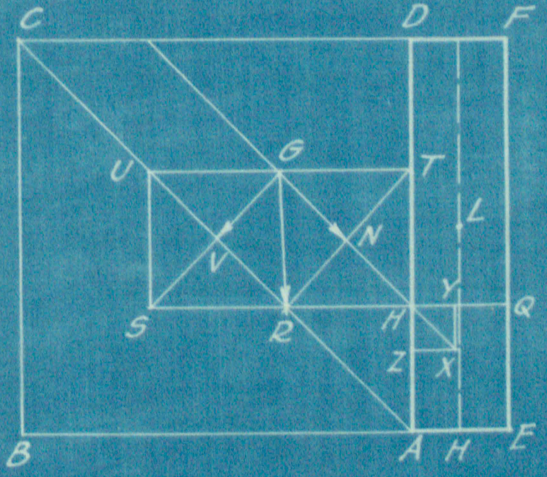


Fig. 8

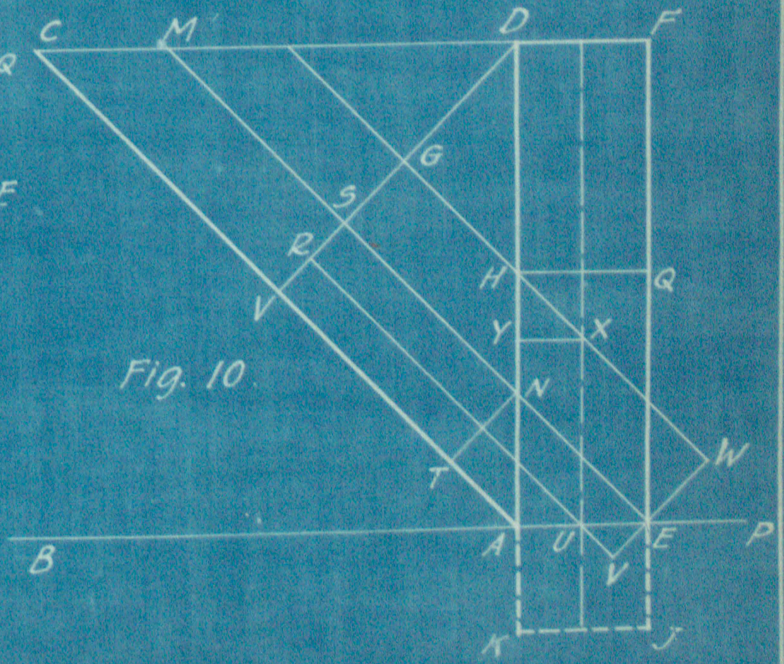


Fig. 10.

Jacob Feld 1921

gravity of the wedge. (see fig. 7) (418)

Let V be the force acting vertically, along AD.

Let E be the force acting horizontally, along HY.

Let Q be the force acting parallel to the plane of slope AC

Q is the weight of wedge ACD and acts thru its center of gravity G, parallel to AC; therefore $AH = \frac{1}{3} AD$.

At H, resolve Q into $HY = E$ and $HA = V$.

$V = Q \cdot \frac{AD}{AC}$; $\triangle HYJ \sim \triangle ACD$.

$$HJ : HY = AC : CD$$

If HJ represents V, then $E = HY = V \cdot \frac{CD}{AC} = Q \cdot \frac{AD \cdot CD}{\overline{AC}^2}$.

Force E has a lever arm $HA = \frac{1}{3} AD$.

Hence the moment of E about the base is $M_E = \frac{Q \cdot \overline{AD}^2 \cdot CD}{3 \overline{AC}^2}$

Since $Q = \frac{1}{2} CD \cdot AD$ (the wedge area).

$$M_E = \frac{1}{6} \frac{\overline{AD}^3 \cdot \overline{CD}^2}{\overline{AC}^2}.$$

$HA = JY$; $JY : HJ = AD : AC$.

Let $T = JY = \frac{AD}{AC} \cdot HJ = V \cdot \frac{AD}{AC} = Q \cdot \frac{\overline{AD}^2}{\overline{AC}^2} = \frac{1}{2} \frac{\overline{AD}^3 \cdot \overline{CD}}{\overline{AC}^2}$.

But the lever arm of $HA = x$, the width of the wall, $= AE$

$$M_T = \frac{1}{2} \frac{\overline{AD}^3 \cdot CD \cdot AE}{\overline{AC}^2}.$$

The real moment of overturning of the wedge is the difference of these two moments or $M_E - M_T =$

$$M = \frac{1}{6} \frac{\overline{AD}^3 \cdot \overline{CD}^2}{\overline{AC}^2} - \frac{1}{2} \frac{\overline{AD}^3 \cdot CD \cdot EA}{\overline{AC}^2} = \frac{1}{6} \frac{AD^3 \cdot CD}{\overline{AC}^2} (CD - 3\,AE).$$

The true pressure would then be $P = \frac{1}{2} \frac{\overline{AD}^2}{\overline{AC}^2} \cdot CD\,(CD - 3AE)$.

If the slope of AC is 45°; $DC = AD$.

$M = \frac{1}{6} \frac{\overline{AD}^4}{\overline{AC}^2} (AD - 3\,AE)$; But $\overline{AC}^2 = 2\overline{AD}^2$; $M = \frac{1}{12} AD^2 (AD - 3AE)$.

The moment of the wall is $m \cdot \frac{AD \times \overline{AE}^2}{2}$

where m is the ratio of the density of the wall to that of the fill.

Equating the moments, we find for $AE = x$, the value

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$$x = \frac{h}{3}\left[\frac{-3 \pm \sqrt{9+24m}}{4m}\right]$$

h = height of wall
x = base width
m = ratio of densities.

If m is	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
is	.229	.223	.219	.215	.210	.206	.203	.199	.195	.193	.190

which values show that the theory gives too low a value for the earth pressure. Yet Mayniel claims that Sallonyer errs on the other side, for he has not taken account of either friction or cohosion.

14. Blavean(1767)

French Military Engineer - Dep't of Fortifications.

Blavean formulated a very logical resolution of forces acting on a wall. He worked with Sallonyer, and this accounts for the similarity between the two theories.(see fig.7)
He assumes that the wedge ADC acts as a solid unit against an immoveable wall. He decomposes the weight of the wedge, GR, acting thru its center of gravity, into GS normal to the plane of rupture and GN parallel to the plane of rupture. (418)
Let GR = W, and IN = E. The wall must then resist the force E.

The triangle GRN and ADC are similar, therefore

$$\frac{GR}{AC} = \frac{GN}{AD} \quad \text{or} \quad \frac{E}{W} = \frac{GN}{GR} = \frac{AD}{AC}\,; \qquad E = W \cdot \frac{AD}{AC}.$$

Resolve E, acting at H, into two components; HY normal to the wall and HA tangential to the wall, or H and V respectively

$$H = E \cdot \frac{HY}{HJ} = E \cdot \frac{CD}{AC} = W \cdot \frac{AD \cdot CD}{\overline{AC}^2}.$$

But $W = \frac{1}{2} AD \cdot DC$ and $\overline{AC}^2 = \overline{AD}^2 + \overline{CD}^2$.

Call $CD = a$, $AD = h$, $AE = x$; $W = \frac{1}{2} y a h$; y = density of earth;

$$H = \frac{1}{2} y a h \cdot \frac{h a}{a^2+h^2} = \frac{y}{2}\,\frac{a^2h^2}{a^2+h^2}.$$

Since it acts at H, or 1/3 h above the base, its moment is $\frac{y}{6}\,\frac{a^2h^3}{(a^2+h^2)}$.

Note that if we assume a 45° degree slope, $M = 1/12h^3$, which agrees with Conplet's result.

If we assume a batter of 1:t on the wall, and call b the width at the top, the base width $x = b + ht$.

The cross section area of the wall is $hb + 1/2\,h^2t$.

The weight of the wall is $w(hb + 1/2\,h^2t)$

The moment of the wall about its toe is

$$w\,hb\,(1/2b + ht) + 1/2wh^2t \cdot 2/3ht = wh(1/2b^2 + bht + 1/3h^2t^2)$$

Equating moments and solving for b the top width:

$$\frac{y}{6}\,\frac{a^2h^3}{a^2+h^2} = wh(1/2b^2 + bht + 1/3h^2t^2); \quad \text{Let } m = w/y$$

$$b = -ht \pm \frac{h}{\sqrt{3}}\sqrt{t^2 + \frac{a^2}{m(a^2+h^2)}}$$

If we call ω the angle between the wall and the plane of rupture $a = h\tan\omega$; note that the total base is $(b + ht) = B$

$$\text{Total base } B = \pm\frac{h}{\sqrt{3}}\sqrt{t^2 + \frac{h^2\tan^2\omega}{mh^2(1+\tan^2\omega)}} = \pm\frac{h}{\sqrt{3}}\sqrt{t^2 + \frac{\sin^2\omega}{m}}.$$

$$\text{for } \frac{\tan^2\omega}{1+\tan^2\omega} = \frac{\tan^2\omega}{\sec^2\omega} = \sin^2\omega$$

If $t = 0$ the wall is vertical, $B = b = \frac{\pm h\sin\omega}{\sqrt{3m}}$

Assuming $\omega = 45°$ (to compare this theory with the previous)

$$b = \frac{\pm h}{\sqrt{6m}}$$ which gives the following table.

m :	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
b/h :	.408	.389	.373	.358	~~.333~~ .345	~~.323~~ .333	.323	.313	.304	.296	.289

These values are much higher than those given by Sallonyer, but they are closer to the present accepted values.

15. Rondelet(1767)

Jean Baptiste Rondelet's theory appeared in his "Traite theoretique et pratique de l'art de batir" a five volume book on construction which became widely recognized as a standard reference. Since he makes no mention of previous theories,

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except Belidor's, we can see why as late as 1818, Tredgold in the "Philosophical Magazine" says that Rondelet was the first to assume a pressure wedge. He repeats and developes Rondelet's theory, as is given under Tredgold.(616)

Rondelet bases his theory upon that of Belidor and also some experimental data which he collected. In order to determine the manner in which earth acted, he constructed a box, $16\frac{1}{2}$ in. long, 12 in. wide and $17\frac{1}{2}$ in. high with moveable sides, so that he could determine the natural slope at will. He concludes that there is a wedge of rupture which tends to slide on the plane of repose. Its effect or mement of rotation is equal to the product of its weight by the lever arm. This arm is the perpendicular distance from the point of rotation to a line drawn thru the center of gravity of the wedge and parallel to the plane of rupture. Soil and sand gave him different results. He decided upon 45 degrees as an average value.(418)

To determine the value of the earth pressure, which he takes as acting parallel to the natural slope, he resolves the weight of the wedge into components normal and tangential to the natural slope. The tangential component is the lateral pressure. Its lever arm is the perpendicular distance from the toe to GJ, or EM (See fig. 7) Let $AD = h$, $CD = a$

$$\frac{GN}{GR} = \frac{AD}{AC}$$

m = ratio of density of wall to density of fill.

Let GR = W, the weight of the wedge; x = base width of wall

GN = E, the lateral pressure.

$$E = W \cdot \frac{AD}{AC} = \frac{ah}{2} y \frac{h}{\sqrt{a^2+h^2}} = \frac{ah^2 y}{2\sqrt{a^2+h^2}}.$$

$$\frac{EM}{EJ} = \frac{AD}{AC} = \frac{h}{\sqrt{a^2+h^2}} \quad ; \quad EJ = AJ - AE = \tfrac{1}{3}a - x.$$

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for $\frac{AJ}{AH} = \frac{AJ}{1/3h} = \frac{CD}{AD} = \frac{a}{h}$

$$EM = EJ \cdot \frac{h}{\sqrt{a^2+h^2}} = (1/3a - x)\left(\frac{h}{\sqrt{a^2+h^2}}\right)$$

Moment of overturning is GN. EM =

$$\frac{ah^2 y}{2\sqrt{a^2+h^2}} \cdot \frac{h}{\sqrt{a^2+h^2}}(1/3a - x) = \frac{ah^3 y}{2(a^2+h^2)}(1/3a - x).$$

Moment of the wall is $1/2hx^2w$.

Equating moments $1/2\, hx^2w = \frac{ah^2y(1/3a - x)}{2(a^2+h^2)}$

$$hm(a^2+h^2)x^2 + a^2h^3x - 1/3a^2h^3 = 0; \quad m(a^2+h^2)x^2 + ah^2x - 1/3a^2h^2 = 0.$$

$$x = \frac{-ah^2 \pm \sqrt{a^2h^4 + 4/3ma^2h^2(a^2+h^2)}}{2m(a^2+h^2)}; \quad x = \frac{-ah^2}{2m(a^2+h^2)} \pm \frac{ah}{2m}\sqrt{\frac{h^2}{(a^2+h^2)^2} + \frac{4m}{3(a^2+h^2)}}.$$

For comparison with previous theories, assume an angle of slope of 45^0, a = h, then $x = h(-1 \pm \sqrt{1+8/3m})/(4m)$.

m	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
x/h	.229	.223	.219	.214	.211	.207	.203	.200	.196	.192	.190

In all cases, $a = h \cot\phi$, or $h \tan\omega$, ω = wedge angle.

$$x = \frac{h \sin\omega}{2m}\left(-\cos\omega \pm \sqrt{\cos^2 + 4/3m}\right)$$

Had he not disregarded his normal component, Rondelet's theory would have been much better and much closer to the present theory of the wedge of maximum pressure. For comparison with present theories, let m = 1.5 and note that

$$x = 1/3h \cos\phi\left(-\sin\phi \pm \sqrt{1/3(3\sin^2\phi + 6)}\right) = 1/3h \cos\phi\left(-\sin\phi \pm \sqrt{\sin^2\phi + 2}\right)$$

If ϕ=	30^0	35^0	40^0	45^0	50^0	55^0	60^0
x/h=	.285	.259	.232	.207	.180	.157	.132

16. Method of the Department of Fortifications.

(about 1767)

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as given in Maynial's "Traite de la Poussee des Terres."

Based upon the assumptions of a 45^o wedge (see fig. 8) ACD, sliding on its plane of slope against a rectangular wall ADFE, the weight of the wedge is decomposed into components, parallel and normal to the plane of slope

Let GR be the weight of the wedge ACD or W.

GN is the lateral pressure or E.

$$GN = GR\frac{\sqrt{2}}{2} \quad \text{or} \quad E = \frac{\sqrt{2}}{2} W = \frac{\sqrt{2}}{2} y \frac{h^2}{2} = \frac{\sqrt{2}}{4} h^2 y$$

where AD = DC = h; density of earth is y.

If f is the coefficient of friction along AC, the lateral pressure is reduced by f x GV; Since GV = GN,

Lateral pressure is $(1 - f)E = (1 - f)\frac{\sqrt{2}h^2y}{4}$, acting at 1/3 h against the wall at an angle of 45^o to the horizontal.

Resolve it into horizontal and vertical components HY and HZ;

$$HY = HZ = \frac{HX}{\sqrt{2}} = 1/4\ (1 - f)yh^2.$$

Moment of HY is $1/3\ h(1 - f)\frac{h^2y}{4} = \frac{1}{12} h^3y\ (1 - f)$

Moment of the walls is $1/2whx^2$; x is base width; w is density.

For equilibrium of overturning

$$\frac{1}{12} h^3y\ (1 - f) = 1/2\ whx^2. \qquad x = h\sqrt{\frac{1 - f}{6m}}; \qquad m = w/y.$$

If we assume the wall to slide, let f' be the coefficient of sliding on the foundation, then

$$1/4(1 - f)\ h^2y = whxf'. \qquad x = \frac{h}{4}\cdot\frac{1 - f}{mf'}$$

If we assume $f = f'$; $x_o = h\sqrt{\frac{1 - f}{6m}}$; $x_s = \frac{h}{4}\cdot\frac{1 - f}{mf}$

Solving for m in the last equation; $m = \frac{h}{4}\cdot\frac{1 - f}{xf}$

and substituting it in the previous value for x_o

we get $x = \frac{2}{3} fh$; the width of wall when the stability against sliding and overturning is the same.

Equating the values of x_o and x_s, we get an expression

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for $$f = -\frac{3}{16m} \pm \sqrt{\frac{3}{8m} + \frac{9}{256m^2}}\,,$$ for equating the values.

$$x = h\sqrt{\frac{1-f}{6m}} = \frac{h}{4}\cdot\frac{1-f}{mf} \quad ; \quad \sqrt{\frac{1-f}{6m}} = \frac{1}{4}\cdot\frac{1-f}{mf}$$

$$\frac{1-f}{6m} = \frac{(1-f)^2}{16m^2f^2} \; ; \qquad \frac{1}{3} = \frac{1-f}{8mf^2} \; ; \qquad 8mf^2 + 3f - 3 = 0$$

If we assume m = 1.5 (the usual ratio of densities)

$f = 0.39$, the coefficient of friction which will make the stabilities against rotation and translation equal.

$$x = h\sqrt{\frac{1-f}{6m}} = h\sqrt{\frac{1-0.39}{9}} = 0.26h$$

$$x = \frac{h}{4}\cdot\frac{1-f}{mf} = \frac{h}{4}\,\frac{(1-0.39)}{(1.5)(0.39)} = 0.26h.$$

The solution is, of course, not general; since it assumes a 45^0 plane of rupture, and disregards cohesion. The other doubtful assumptions are that the friction of the earth on earth is the same as the friction of the wall on earth; that the component of the earth pressure along the back of the wall has no effect, and that the same width of wall will give equal stability against overturning and against sliding.

17. Theory used by Various Engineers.

As given by Mayniel in his "Traite de la Poussee des Terres." (418)

The method used was to resolve the weight of the wedge of rupture into two components, one normal to the plane of rupture, the other horizontal. The horizontal component is taken as the lateral pressure and assumed to act at 1/3 h (see fig. 8).

Let ω = angle of the wedge, or angle CAD

$$CD = DA \cdot \tan\omega = h\tan\omega.$$

Weight of the wedge = $W = \frac{1}{2}\,AD \times CD \times Y = \frac{1}{2}h^2y\tan\omega = GR$

angle RTG = angle ω = angle CAD

$$TG = GR\tan(90^0 - \omega) = GR\cot\omega$$

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$= 1/2\ h^2y. =$ Lateral pressure.

Assuming it to act at 1/3 h, its moment is $1/6\ h^3y$.

If we assume a trapezoidal wall, vertical back

b = top width, (b + th)=base width, where t is the batter, the cross section area is $bt + 1/2\ th^2$

its weight, density being w, is $w(\ bh + 1/2 \cdot th^2\ )$

its moment of stability about the toe is

$$w(\ bh\ (\ th + 1/2b) + 1/2th^2 \cdot 2/3th\ ) = w(\ bth^2 + 1/2b^2h + 1/3t^2h^3)$$

Equating moments and letting $m = w/y$.

$$1/6h^3 = mh(\ bth + 1/2b^2 + 1/3t^2h^2\ );\ b = -th \pm h\sqrt{1/3t^2 + \frac{1}{3m}} = \text{top width.}$$

$$\text{Base width} = (b + th) = h\sqrt{\frac{mt^2+1}{3m}}$$

Mayniel notes that this theory disregards friction and cohesion and arbitrarily takes the point of application at 1/3 h. If the wall is vertical $t = 0$, $b = h\sqrt{\frac{1}{3m}}$.

m :	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
b/h:	.577	.550	.527	.506	.488	.471	.456	.443	.430	.419	.409

Very high values, at least 50 per cent over the usual values result from this method.

18. Trincanaux (1768)

Department of Fortifications.

Trincanaux goes back to the 45^0 wedge. However he assumes that friction and cohesion reduce the effect of this wedge against the wall. Assuming the wedge to slide on its plane of repose, he says that the lateral pressure is 1/4 of the weight of the wedge, and acts horizontally at 2/3 h from the base, i.e. thru the center of gravity of the wedge. He assumes that the ratio of the density of the wall to that of earth is 1.5 for all cases.

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Then the equation of equilibrium gives $x = 1/3\ h$.

for $E = 1/4\ W. = 1/4\ (1/2\ h^2y)$

$M = 1/4(1/2h^2y)(2/3h) = (1/4)(1/3h^3y) = 1/12h^3y.$

M of wall is $1/2hx^2w$.

$$1/2hx^2w = 1/12h^3y \quad ; \quad \text{But} \quad w/y = 3/2.$$

$$x^2 = 1/6h^2 \cdot \frac{2}{3} = 1/9\ h^2 \ ; \qquad x = 1/3h.$$

(418)

For a surcharged fill, he recommends a counterfort wall. His design for counterforts calls for a thickness of 5 ft., a depth of 12 ft., and a spacing of 20 ft. on centers. The counterforts are to be joined by arches, of 4 ft. radius and 18 in. thickness at the key or junction with the wall proper.

19. d'Antony (1768)

Papacini d'Antony performed some small scale experiments to determine the overturning moment of sand fills before developing a theory and a theoretical retaining wall section. His apparatus consisted of a wooden box in which he had a hinged gate. He tied a cord to the top of the gate and passing it over a pulley at the back of the box, hung a pan for weights at the end. The apparatus was crude and no satisfactory results were obtained.

As all the Italian engineers did, he assumed a 45° plane of rupture. (see fig. 9) Divide the height DA into four equal parts. The pressure due to ACD is to the pressure due to HKD as $\overline{AD}^2$ is to $\overline{KD}^2$. The pressure on a wall varies according to the square of the height, so assume a parabolic curve, DE for the outside of the wall. Let x represent the width at any point, and dz the element of height. The area of an

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element of height of the wall is $x\,dz$, and its moment about DA is $\frac{x^2}{2}\,dz$. The equation of the parabola is in general $z^2 = pz$ and the area is approximately $\frac{hx}{3}$, where h is the height of the wall.

The total moment is $\int_0^h \frac{x^2}{2}\,dz = \int_0^h \frac{z^4}{2\,p^2}\,dz$

He says that the moment over the whole area will be

$$\left(\int_0^h \frac{x^2}{2}\,dz\right)\text{(area of section)} = \left(\int_0^h \frac{z^4}{2p^2}\,dz\right)\left(1/3\ xz\right)_0^h =$$

$$= \int_0^h \frac{z^4 dz}{2p^2}\cdot\left[\frac{xz}{3}\right]_0^h = \left[\frac{z^5}{10p^2}\cdot\frac{z^3}{3p}\right]_0^h = \frac{z^8}{30\,p^3}.$$

The center of gravity from AD is $\frac{\text{Moment}}{(\text{Area})^2} = \frac{z^8}{30\,p^3} \div \frac{z^6}{9\,p^2} =$

$$= \frac{3\,z^2}{10p}\Big]_0^h = \frac{3\,x}{10}\Big]_0^h = \frac{3h}{10};\qquad x_h = \text{width of base.}$$

Assuming that the density of the wall equals w, the moment about the toe is $\frac{7x}{10}\cdot\frac{whx}{3} = \frac{7\,hx^2w}{30}$.

If f' is the coefficient of friction along the base, the total friction is f'x, and its moment about the toe is $1/2\ f'x^2$.

The resistance of the wall is therefore $\frac{7}{30}hx^2w + \frac{f'x^2}{2}$.

Resolve the weight of the wedge into two components, one normal to the plane of rupture, the other horizontal.

GT = GR cot ω; ω being the wedge angle

GR is the weight of the wedge.

GT is the lateral pressure, acting at 2/3 h.

GR $\frac{1}{2}$ DA x DC $1/2yh^2 \tan\omega$.; let y be the density of earth. Assume that friction and cohesion permits only 1/4 of this to act.

Then $GT = E = 1/4(\ 1/2yh^2 \tan\omega \cot\omega\ ) = 1/8\ h^2y$.

Its lever arm is 2/3 h, then the overturning moment is $\frac{h^3y}{12}$

Then $\frac{7\ hx^2w}{30} + \frac{f'x^2}{2} = \frac{h^3y}{12}$.

$$x = \sqrt{\frac{1/12\ h^3\ y}{7/30\ hw + 1/2\ f'}} = h\sqrt{\frac{5}{2}\cdot\frac{yh}{7\ hw + 15f'}}.$$

Mayniel criticises his choice of a parabolic section as unpractical. He considers the friction along the base to aid in stability against overturning - this is a false idea.

If we assume f' = 1; we can practically disregard the 15 f', and approximately for $\frac{y}{w} = 2/3$; $x = .456h$. The very high value is due to the assumption that the pressure acts at 2/3h from the base.

B. The Theory of the Maximum Wedge of Pressure. (1773 - 1840).

August Charles Coulomb (1773).[133]

Up to Coulomb, the theorists had assumed the position of the plane of rupture, usually taking the plane of slope as the plane of rupture. This evidently gave results which were too high, so that arbitrary deductions for friction, cohesion, etc. were introduced. Coulomb, in his "Essai sur une application des regles de maximis et minimis a quelques problemes de Statique, relatifs a l'Architecture,"(1773), which is also reproduced in his "Theorie des Machines Simples"(1821 ed) assumes that the pressure on a vertical wall is caused by some wedge tending to rupture from the filling mass and slide down. He chooses a plane surface of rupture, because experiment has shown a plane of rupture. The pressure exerted by this wedge is due to its weight and is decreased by the friction and cohesion resistances along the plane of rupture. That wedge will tend to slide which exerts the maximum pressure. He gives no proof for this statement. In accordance with the laws of friction and cohesion, given in the Introduction, he obtains a mathematical formula for the lateral pressure. Since in this static case, he disregards friction along the wall, the pressure acts normal to the wall. Differentiating the expression for the pressure with respect to the width of the wedge along the top surface, and placing the first derivative equal to zero, he obtains a value for the width of the wedge which will exert the maximum pressure.

In discussing the results, he touches upon the case of passive pressure, or the force required to move the wedge upward. His value for the maximum pressure is independent of the cohesion, because cohesion always decreases the pressure. In the case of surcharged fills, he uses the total weight of the wedge plus surcharge over the wedge area in place of the weight of the wedge alone.

He next considers the possibility of friction along the wall. If motion occurs between the fill and the wall, the weight of the wedge is decreased by the value of the friction along the wall surface. The amount of this decrease equals the product of the lateral pressure(which is always normal to the wall) and the coefficient of friction along the wall.

The Essai ends with a discussion of the effect of cohesion. In earth where cohesion always exists the pressure should be calculated over the total height of the walls and from this should be subtracted the value of the pressure on the height of earth which is self-sustaining, due to friction and cohesion. Nowhere in this very complete discussion does he make mention of the angle of repose, but always uses the coefficient of friction. The mathematical discussion is a special case of the general wedge theory, which is given below.

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component, less 1750 ft. lbs. for the 9 inches of ballast held as part of the wall by the retaining action of the counterforts, or a resulting moment of 3830 ft. lbs. The changes in the shape of the wall tend to decrease the acting pressure. At a height of 10 ft., the overturning moment would be 1090 ft. lbs., while the moment of the wall is 2000 ft. lbs., so that the wall was stable, and stood. Altho Baker states that the binding of the gravel between the counterports increased the weight of the wall, he takes no account of this increase in stability.

The third wall, was 18 in. thick and was built with a batter of 1 to 5, as well as counterforts 3 ft. 9 in. thick, measured from the face of the wall, and spaced 10 ft. apart. This wall was carried to a height of 21 ft. 6 in. with no signs of weakness except a slight bulging about half way up of $2\frac{1}{2}$ in. at the panel and $1\frac{1}{4}$ in. at the counterfort. This can easily be explained from the method of construction--loose brick laid in wet sand. The wall was perfectly safe, due to both the batter and counterforts. Assuming perfect construction, the base width was 3 ft. 9 in., or a ratio of base to height of .174, a rather low value. Baker states that this wall was built with a base less than 1/10 of the height, and really had a factor of safety of 2. Assuming the fill to act with the wall, the stability moment was 100 lbs. x 3.75 ft. x 21.5 ft x $\left(\frac{3.75}{2}\text{ft} + \frac{21.5}{10}\text{ ft}\right) =$ 31,600 ft. lbs. The 1 to 5 batter causes the center of gravity of the

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Woltmann (651) (1790) in translating and discussing Coulomb's theory brought in the idea of the equality of the angle of internal friction and the angle of repose. Prony (418) (1797) simplified Coulomb's discussion by introducing trigonometric functions and showed that if the coefficient of friction equals the tangent of the angle of slope, the plane of rupture, for the case of a vertical wall, bisects the angle between the wall and the plane of slope. Since the coefficient of friction is a constant, the natural slope and the plane of rupture are constant. In place of the width of the wedge, Prony uses the angle of the wedge as the variable with respect to which he differentiates.

Mayniel(1808).(418)

Mayniel extended the theory to sloping walls assuming the lateral pressure normal to the wall and taking into account friction and cohesion along the plane of rupture and friction along the wall surface. He does not accept the assumption concerning the equality of the angles of internal friction and natural slope, tho he mentions the fact that Prony had made such an assumption. The mathematical work is very detailed and complicated, but as he says:"that is because of the general nature of the solution; we must take into account all the points of the problem."

In his discussion of the various wedge theories, he notes that researches have discovered two types of friction in soil:

1. Perfect friction, as in sand, is caused by the intercogging of particles, acted upon by a continuous pressure, and is to be distinguished from cohesion, which is a reunion of

masses, like a glueing together.

2. Imperfect friction is the rubbing or rolling of particles over each other, due entirely to their own individual weights. This is probably the earliest attempt at a scientific analysis of the granular actions in pulverulent materials.

Francois(1820)[256] repeats Mayniel's work, using the total angle between the plane of rupture and the back of the wall as the variable quanitity. He takes into account the cohesion and friction along the plane of rupture, and assumes the lateral pressure to be normal to the wall. The cohesion factor is eliminated in the process of differentiation and his value for the maximum pressure is independent of the cohesion. He accepts the assumption that the tangent of the angle of natural slope is the coefficient of friction:

Navier(1813-1826)[465] contributed the effect of surcharge on the lateral pressure against any slope of wall. He showed that the point of application of the resultant can no longer be assumed at the third of the height, but depended upon the amount of surcharge. He showed that the plane of rupture, in all cases, bisects the angle between the plane of natural slope and the back of the wall.

Audoy (1820)[24] does not accept the assumptions of uniform density, internal friction and cohesion. Assuming, however, that the coefficient of friction equals the tangent of the angle of natural slope, he derives an expression for the maximum pressure in general terms. The density and cohesion vary as the depth of fill, and the internal friction depends on the natural slope which also varies as the depth. Lack of experimental data, force him to accept the Coulomb hypotheses in order to obtain a useable formula. The result is that obtained by Francais, since Audoy has used the angle between the plane of rupture and the back of the wall as the variable.

C. Various Elementary Theories, 1773 - date.

In spite of the considerable advance in the theory of the maximum wedge of pressure, many authors have reverted to the older ideas, very often appearing quite ignorant of the fact that the older theories had been corrected.

Tersac de Montlong (1774) [418]

The Department of Fortification did not at first accept Coulomb's theory, but used the method developed by de Montlong. (see fig. 10). In this theory, the plane of rupture passes thru the toe, the material between the plane of rupture and a parallel plane passing thru the heel, namely MNAC, aids the wall in preventing rotation about J. He assumes the plane of rupture inclined at the angle of natural slope. Equating moments, he obtains the value for the base width:

$$x = h\left(\frac{-1 \pm \sqrt{1 + 8/3\ m}}{4m}\right),$$

where as before, h is the height of wall, and m is the ratio of the densities of wall to fill. The following values, which, as can be expected are quite low, are given by this formula:

m:	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
x/h:	.229	.224	.219	.215	.210	.207	.203	.199	.196	.192	.190

In making the assumption that the wall may fail by sliding, and using 45^0 as the angle of rupture, m as 3/2, he obtains the value b = 1/3 h. He does not notice, however, that by assuming part of the earth to resist rotation, he has practically eliminated the necessity of investigating that factor.

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Chauvelot(1783)

Chauvelot rejects the previous theories(he makes no mention of Coulomb's) because of the large number of assumptions; especially that the prism of rupture acts as a solid free body. The wedge CDA (see fig. 8) tends to slide, but in so doing it unites along the plane AC. The lateral pressure is GT - GU, where GT is the component of the weight of the wedge normal to AD, and GU represents the loss due to friction and cohesion. Since he makes no assumptions concerning the values of such losses, his formula is quite general. Mayniel notes that Chauvelot's experiments showed a natural slope for yellow sand of $39^{\circ}21'$. However, in his theory he recommends that 45° be used in all cases, and that the resultant be taken as acting at 2/3 h. It is noteworthy that when this theory was given before the Academy of Sciences(Jan. 22, 1783), this body announced that the problem of determining the lateral earth pressure was now susceptible of a rigorous solution for any special case, where experiment had determined the value of the cohesion, the friction and natural slope of the earth.

Gauthey (1785)

Gauthey was the first to base his theory upon a complete set of experiments. But unfortunately, he drew incorrect conclusions from his data, with consequent errors in his theory. Poncelet (1840) [656] brings attention to this, and shows how the experiments really point to the general wedge theory. Yet as late as 1865, Curioni and de Benedict advise the use of Gauthey's formulae.[524]

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Using a bin, 30 x 30 x 12 in., with a hinged gate(30 in high) he finds a value for the lateral pressure. $H = (0.123)\frac{1}{2}\gamma h^2$ The overturning moment on the wall was measured by means of weights hung on a string which passed thru the fill. Even disregarding the friction loss, due to the action of the sand on the string, the result is quite in close agreement with the wedge theory. By using a wall made up of horizontal strips and measuring the pressure on each, he showed that the pressure varied directly as the depth. Further experiments, where the slope of the wedge was controlled by inserting an inclined board, showed that the lateral pressure of wedges with angles of $67\frac{1}{2}^\circ$, 45° and $22\frac{1}{2}^\circ$ was the same, absolutely showing that the wedge of maximum pressure did not include all the material between the wall and the plane of slope.(418)

He uses all this experimental data to modify d'Antony's theory, by subtracting 1/3 the wedge weight to take account of friction loss. His arbitrary assumptions, not at all based on the results of his experiments, make his formulae of little value. Gauthey was a standard writer on engineering construction, see for example his book on bridges. This way explain why his earth pressure theory, really only a slight modification of d'Antony's, was so widely accepted.

Delanges (1788) made some careful observation of the effect of water content on the natural slope and density of soils, but used the older theories in applying his results.

Sengium(1792). edited the revised edition of Bullet's book "Traite d'architecture pratique." After giving Bullet's theory, he inserts a "newer" theory. The size of the wedge depends upon the nature of the fill. The lateral pressure is the component of the weight of the wedge which is parallel to the plane of rupture. The only condition for stability is that the resultant of the lateral pressure and the weight of the wall pass thru the toe.

Richard Woltmann(1799).

Woltmann's experiments are given in detail in the section on "Experimental determinations." In addition he translated the Coulomb theory into German and inserted the idea that the coefficient of internal friction was equal to the tangent of the natural slope. In his "Beitrage zur Baukunst schiffbarer Kanale" (1802) he goes into a mathematical investigation of the equilibrium of slopes.(see fig. 11)

Basing the equilibrium of an elementary length, ds, on the equality of moments, $y\,dx = a\,ds$, there results $y\frac{dx}{ds} = a$, a constant. Substituting and integrating, the equation of equilibrium of the sides becomes

$$x = a \log \frac{(y + \sqrt{y^2 - a^2})}{(b + \sqrt{b^2 - a^2})}$$

Woltmann is the earliest contributor to the theory of lateral pressure in bins(see fig. 12). The actual pressure on AB is the effect of ABDF less the effect of GDE. He calculates the pressure on a height AB due to a wedge bounded by the wall and the plane of slope, assuming a full plane of rupture. The lateral pressure is the horizontal component of the wedge less the friction loss along the plane of rupture. From this is subtracted the pressure of a similar wedge on the height DF and the frictional resistance along DF.

Goudriaan(1800)[87] assumes a 45° plane of rupture passing thru the heel (see fig. 13). He writes $\frac{1}{2}\,Uh^2$ as the lateral

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pressure, and then sets out to determine the form and value of u.

$$BA : (BA + BD) = \psi : \frac{uh^2}{2} \text{ (assumption).}$$

$$\psi = \frac{1}{2}uc^2\left(\frac{1}{1+\frac{BD}{BA}}\right) = \frac{1}{2}uc^2 \cdot \frac{1}{1+\cot\phi}.$$

This gives the value of the lateral pressure, but several more approximate assumptions must be made before a numerical result is obtained.

Brunings(1803)(87) developed a general formula for a horizontal pressure against a vertical wall. Substitution of values gives the same results as obtained from earlier theories. He does mention the fact that the surface of rupture may not be a plane, but uses an average plane to derive his formula.

Barlow(1837)(655) assumes that the material above the plane of slope acts as a solid sliding on a frictionless plane. The lateral pressure acts thru the center of gravity of this solid, and parallel to the plane of natural slope. The "general" equation of equilibrium is derived by assuming rotation about the toe and including cohesion along the base of the wall.

Woodbury (1845)(655) objects to Barlow's results, showing that the assumption amounts to a liquid sliding on the plane of repose. The actual thrust, he says, is due to a fluid of a density S, which has to be determined. The error in Barlow's result is much closer to the actual pressure, than the value given by Barlow's theory. In addition, he notes that Barlow has assumed the resultant pressure to act on the front face of the wall and therefore, in his equation for equilibrium, takes too small a lever arm.

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Moseley(1845)[457] takes up the pressure in bins and claims that the theory given above(under Woltmann), which he attributes to Hagen(1833) is wrong. The pressure against the side of a bin is the same as the pressure against an equal height in an indefinite mass, i. e. the back wall does not decrease the effect of the wedge, but reacts an amount equal to the pressure, which would be caused by the part of the wedge which is cut off.

He discusses in detail the equivalent fluid pressure theory, showing that in all cases the formula for the lateral pressure is of the hydrostatic form, so that by finding the value for the equivalent fluid, we can use the laws of hydrostatics.

Godfrey(1906)[277] is one of the many men who advise the use of the equivalent fluid theory. He disregards the vertical component of the pressure, as being variable and aiding the stability. The value of the equivalent fluid equals $y \tan^2 \frac{1}{2}(90^\circ - \phi)$, which makes this theory another form of the Coulomb formula. Other men, notably Baker(1881) use different values for the equivalent fluid.

Hoffman(1858)[315] disregards all previous theories and calculates an extensive set of tables based on the idea of a plane of rupture parallel to the plane of slope and passing thru the toe of the wall. The tables show a large amount of work, but from a practical point of view, they are perfectly useless.

Constable(1874)[125] accepts the idea of a plane of rupture thru the toe, because both experiment and experience showed that rupture of a wall occurs by an oblique shear thru the

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toe, at about 45° to the horizontal. However, he takes the plane of rupture at a greater angle from the horizontal than the plane of natural slope, beginning at the point of intersection of the back of the wall with a 45° line thru the toe. This method gives results only 8% less than the pressure caused by a wedge with a plane of rupture of the same slope passing thru the heel of the wall.

Gould(1877)(287) shows that the plane of rupture must bisect the angle between the wall and the plane of natural slope, because in the case of hydrostatic pressure, the correct value is obtained by assuming a 45° wedge. The lateral pressure equals the weight of the wedge times the tangent of the wedge angle.

Ludwig Debo(1901)(169) states that Coulomb's assumptions and results are "willkürlich ungebründet and entschieden unrichtig" (without foundation and entirely wrong). In all cases, no matter what the nature of the fill, shape of the fill or type of wall, all the material above the plane of slope slides and exerts a pressure against the wall parallel to the plane of slope. This very sweeping and revolutionary statement appears in his "Lehrbuch der Mauerwerks - Konstruktion" published in 1901.

Even more recently (1911)(16) the "Architect and Contract Reporter"(London) prints the statement:

" The earth pressure of loose earth is exactly the same as water pressure, being equal to the weight vertically above the area considered and the same in all directions." The pressure at the base of the wall is equal to the weight of a column of earth having the same height as the wall. The pressure on a sloped wall is calculated in the same manner, always perpendicular to the face. The pressure triangle is equal in area to the wedge of rupture(assuming a 45° plane of rupture). In this way, pressure of surcharged fills may be computed: by constructing a triangle of pressures equal in area to the wedge of rupture.

D. The Wedge Theory (1840 - date).

Returning to the development of the wedge theory, we note that between 1820 and 1840 but slight advances were made, by Huber-Bernard (250) (see section on experiments), Laurent(1831) (390) Hagen (1833) and Persy(1837) (563). The last named gave an analytical proof that the surface of rupture cannot be a plane. The outline of this proof is given below.

Poncelet(1840) (498, 500, 502).

Poncelet completed the development of the Coulomb theory, by deriving a general formula which took into account the friction on the wall and also gave a graphical construction for this formula. The general wedge theory should really be called the Poncelet theory, altho Poncelot advises that we disregard the wall friction for the sake of safety. The theory as given in the Memorial De l' Officier du Genie #13, with a modification required by the use of our notation, is reproduced below, as well as the usual graphical construction. Since the general formula is so much more easily obtained from the graphical construction, the details of the analytical method are omitted.

The assumptions made are that

1. The mass is homogeneous, in density, moisture content

and all physical properties, especially friction and cohesion.

2. The material is granular, i. e. it is not a liquid capable of transmitting stress undiminished in all directions, nor is it an elastic solid, which deforms at every application of force, coming back to its original shape upon the removal of the force. The particles of such a material possess a resistance to rolling over each other, called friction, and a surface attraction, calledcohesion.

3. The Laws of Coulomb on friction and cohesion(see Introduction) are assumed to hold. In the theory given below, cohesion is disregarded. The effect of cohesion is considered in the next part of this section.

4. A certain portion of the earth's mass, as abc(fig. 1) tends to slide down on a plane bc, which passes thru the heel of the wall, and is called the plane of rupture.

5. The wedge abc is acted upon by three forces, the weight of the wedge, the reaction of the wall, and the reaction of the material below the plane of rupture.

The weight will act thru the center of gravity of the wedge. The reaction along bc, see fig. 1, will act at some angle, say β, from the normal. Assuming that no cohesion acts, at the point when the wedge starts to slide down, R must make an angle ϕ below the normal, this being the case of active pressure. When the wedge starts to slide up, R must make an angle ϕ above the normal, this being the case of passive pressure. These angles cannot be exceeded, so it is assumed that the maximum pressure occurs when R is at the limiting inclination. Note that if we use for ϕ the angle of internal resistance(see Introduction)

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we are taking into account all resistances along the plane of rupture. The reaction of the wall is the lateral pressure. The oldest theories assumed it parallel to the plane of rupture. Coulomb assumed it normal to the wall, and Poncelet was the first to assume that it was inclined from the normal by an angle equal to the angle of friction on the wall surface, ϕ' . If ϕ' is greater than ϕ , then a thin layer of earth will stick to the wall, and the friction angle will be equal to ϕ . Coulomb himself did not fix the position of R in his theory; soon thereafter the angle of natural slope was brought into his derivation. Boussinesq(1883) objected to the use of the natural slope for ϕ because the maximum lateral pressure occurs when the material is changing from a state of rest to a state of motion, while the angle of natural slope is found after the material has changed from a state of motion to a state of rest. He advises the use of an experimentally determined angle of internal friction, and this has been accepted by most of the recent authors. In connection with the Cincinnati experiments, the writer found that the use of the angle of internal resistance (see introduction) gave a closer agreement between experiment and theory. The angle of internal friction is found during motion, while the angle of internal resistance is found under the same conditions as obtain in the fill, just before motion.

Drawing a force triangle for the three forces, these results

$$E = W \frac{\sin(90° - \omega - \phi)}{\sin(\phi + \omega + \alpha + \phi')} \quad \text{(see fig. 2).}$$

By substituting the area of the wedge times the density for the

W, we obtain an expression for the lateral pressure for one foot width of wall. The only unknown is the wedge angle ω. Differentiating with respect to ω and setting the first derivative equal to zero, we obtain a value for ω which makes E a maximum. Substituting this value in the original equation, we obtain the general formula for the lateral earth pressure.

$$E = \frac{1}{2}\gamma h^2 \frac{\cos^2(\phi-\alpha)}{\cos(\phi'+\alpha)\cos^2\alpha\,(1+n)^2} = C\frac{\gamma h^2}{2}.$$

$$\text{where } n = \sqrt{\frac{\sin(\phi'+\phi)\sin(\phi-\varepsilon)}{\cos(\phi'+\alpha)\cos(\alpha-\varepsilon)}}.$$

It can be shown, by setting the second derivative equal to zero, that this value is a maximum. By letting $\phi'=0$, we obtain the special theory, which assumes the resultant as normal to the wall. By proper substitutions, this formula gives all the special cases reducing to

$$E = \frac{1}{2}\gamma h^2 \tan^2\left(45^\circ - \frac{\phi}{2}\right) = \frac{1}{2}\gamma h^2 \frac{1-\sin\phi}{1+\sin\phi}, \quad \textit{for the case}$$

of horizontal fill and vertical wall, E acting horizontally. The notation is given in the introduction.

There is a fundomental error in the theory which has escaped most authors. Of the three forces acting on the wedge, we know one in magnitude, and all three in direction, leaving two unknowns. Since we have three conditions for equilibrium, we discard one and use the other two to find the unknowns. Usually the vanishing of the summation of moments is disregarded. Mohr(1871) (448) was the first to object to the theory on this account tho Rebhann (1870) and Weyrauch tried to bring this condition into the derivation. As can be seen from Fig. 3 only in special instances will the three forces be concurrent.

Poncelet's construction simplifies the use of the formula to a large extent(fig. 4) To show the method, assume that AC is the required plane of rupture and draw AD, the line of repose, and $\measuredangle ACI = (\phi + \phi' + \omega + \alpha)$.

$$E = W \frac{\sin(90° - \omega - \phi)}{\sin(\phi + \omega + \alpha + \phi')} = \frac{1}{2} y (AT)(BC) \frac{CI}{AI}.$$

Draw BG parallel to CI; then BC $= BD \frac{GI}{GD}$; $CI = ID \frac{BG}{GD}$.

$$E = \frac{1}{2} y (AT)(BD) \frac{GI}{GD} (ID) \frac{BG}{GD} \cdot \frac{1}{AI} = \frac{1}{2} y \frac{AT \cdot BD \cdot GB}{\overline{GD}^2} \cdot \frac{(x-b)(a-x)}{x}$$

where $x = AI$, $a = AD$, $b = AG$. The only variable is x, and setting $\frac{dE}{dx} = 0$, $x = \sqrt{ab}$ for the maximum value of E.

$E = \frac{1}{2} y \frac{AT \cdot BD \cdot GB}{\overline{DG}^2} \left(\frac{a - \sqrt{ab}}{a}\right)^2$, and since $AD = a$, $ID = a - x$,

$ID = a - \sqrt{ab}$; $BG : DG = CI : ID = CI : (a - x)$;

$$E = \frac{1}{2} y \frac{\overline{CI}^2}{(a - \sqrt{ab})^2} (a - \sqrt{ab})^2 \cos(\phi' + \alpha) = \frac{1}{2} y \overline{CI}^2 \cos(\phi' + \alpha).$$

which is the value of the earth pressure. The equality of this expression with the general analytical formula given above is easily shown and may be found in any book on Earth Pressure.

The construction of the plane of rupture follows from the above discussion. Draw BD making angle $(\phi' + \alpha)$ to BN. Construct the semi-circle on AD as diameter, and draw GM perpendicular to AD. Layoff AI equal to AM, draw IC parallel to BG, giving point C. AC is the plane of rupture. Since AI is equal to GM and is the mean proportional between AD and AG, the proof as given above applies.

Cousinery (1841)[137]

Cousinery developed a detailed graphical construction of the plane of rupture for the prism of maximum pressure. The method is a matter of trial and gives the value of the pressure. He also gives a construction for the theoretical shape of walls.

Kleitz(1844)[359] simplifies Cousinery's work and developed

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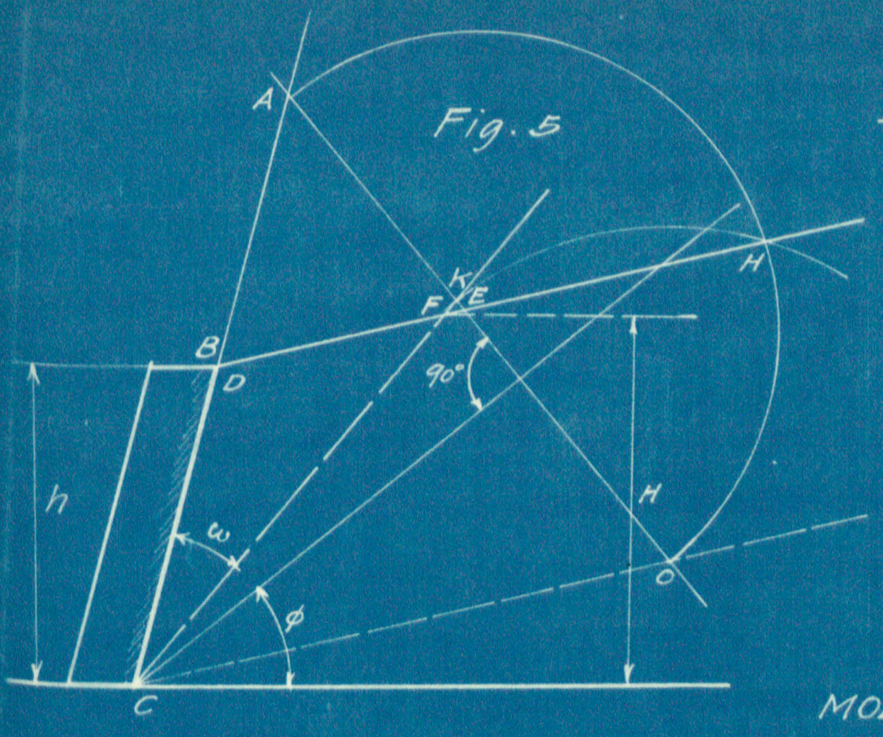
a construction for the plane of rupture based on the same idea as the Poncelet method. The mathematical work used in deriving the value of the pressure is identical with that given above, and, tho Kleitz does not mention the source of his material, it is probably not original.

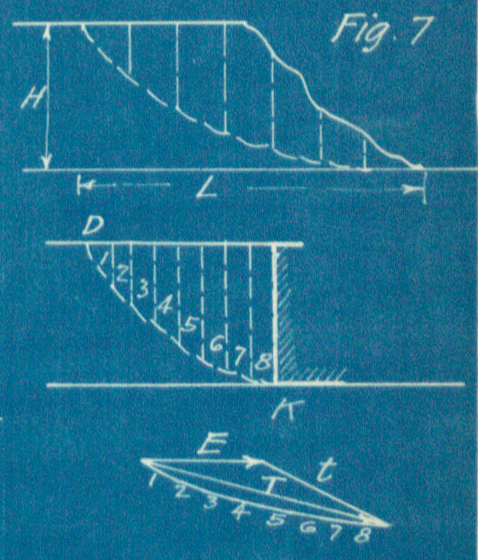
J. Neville(1845) (341, 467) gives the general formula for the lateral thrust as $P = \frac{1}{2} y H^2 \cos^2\phi (\tan\phi - \tan\omega)^2$. Since he assumes the pressure as horizontal, the formula is merely a special case of the wedge theory. When the maximum vertical height ~~is~~ of the wedge is greater than the height of the wall, that height is used in the formula. The construction is given in fig. 5. At any point in the line of natural slope CH, draw a perpendicular OA, lay off OK equal to OH; CH is the plane of rupture, and CDF is the wedge of rupture.

A.C.Scheffler(1851) (563, 564) showed that in order to keep the forces acting on the wedge in equilibrium, the lateral pressure must be assumed as horizontal. Otherwise the force will not be concurrent. With this requirement he derives the formula for the lateral pressure, taking into account cohesion; the results are identical with Coulomb's. He considered the possibility of obtaining a greater maximum pressure by assuming a curved surface of rupture. Investigating the stresses on an element of the curve and obtaining the value for the case of maximum pressure, there results an equation of a straight line as the line of rupture. He therefore concludes that the maximum wedge is bounded by a plane of rupture.

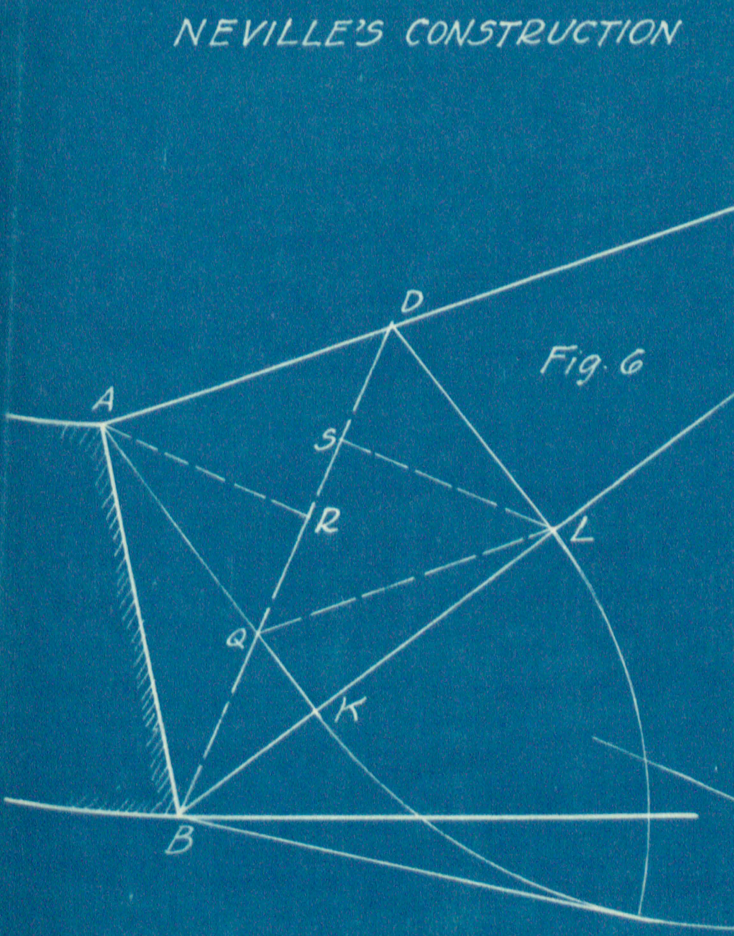
However, Persy (1837) (563) had used a very similar method to prove that the surface of rupture was not plane. Scheffler notes that Persy's method was in error because, having considered

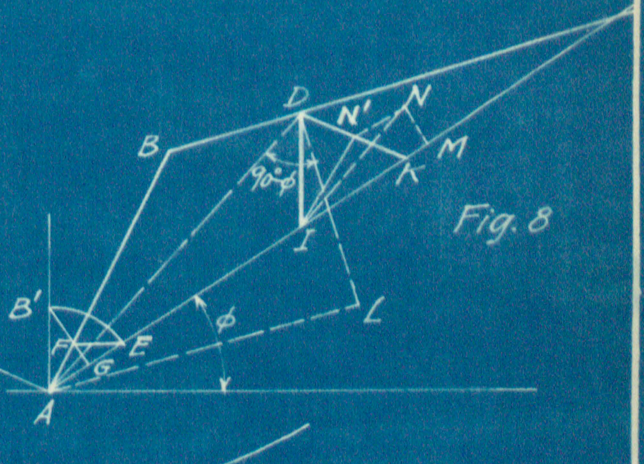


NEVILLE'S CONSTRUCTION



MOLITOR'S THEORY





ALLEN'S CONSTRUCTION
FOR REBHANN'S THEORY.

PRELINI'S DIAGRAM
EFFECT OF COHESION.

Diagrams for Wedge Theories.

Jacob Feld 1922.

a sloped wall he found the equation for the surface of rupture which would make the normal pressure a maximum. He should have considered the horizontal component only. In addition, Persy assumes that on an element of earth, the cohesion and friction acting on the face towards the wall equals the cohesion and friction between earth and the wall; on all other faces, he uses the cohesion and friction between earth and earth.

Rebhann(1870) (461, 504, 519) by assuming that the area of the wedge and not the wedge angle was the independent variable, showed that for all cases the plane of rupture which bounds the wedge of maximum pressure bisects the area between the wall, the plane of slope, the top surface and a perpendicular drawn from the end of the plane of rupture to the plane of slope. That is, in fig. 4, area AFPC equals ACH.

Winkler(1871) (649-652) pointed out the doubtful assumptions in the theory of the wedge of maximum pressure:

1. That the total pressure is a maximum when the horizontal component is a maximum.

2. That at the point of failure, the lateral pressure balances the friction and cohesion resistances along only one surface.

3. That altho the pressure varies with the position of the plane of rupture, there is but one position which gives a maximum.

He considered the possibility of curved surfaces of rupture but obtained no conclusive results.

J.R.Allen(1877) (8) gave a very simple construction for the Rebhann theory.(see fig. 6). Draw AK perpendicular to the line of slope, and lay off AL equal to BT, the tangent to the circle having KC as diameter. Draw DL parallel to AK, and BD is the plane of rupture. It is proven by showing the equality of triangles ABD and BDL. For AQ and DL are sides of a parallegram and therefore AR equals LS.

Weyrauch(1878)(641-3) objected to the existing wedge theory, because of the assumed direction of the resultant and because the forces acting on the wedge were not concurrent. Taking into account the requirement that the summation of the moments shall vanish, Rebhann had calculated the value of the resultant for the two special cases of the fill horizontal and sloping upward at the natural repose. Weyrauch generalized this analysis to all slopes and all inclinations of the wall forward.He does not claim that the formulae are accurate for a wall sloping backward. This first work of Weyrauch's was translated by Prof. Howe and is in the first edition of the latter's work.

Weyrauch's triangle of pressure is often used to evaluate the lateral pressure, tho the result is identical with that obtained from the Poncelet construction.(see fig. 4). The value of the lateral pressure equals the density of the fill times the area of triangle CKI, where K is the intersection of a vertical thru C and the plane of slope. A full discussion of this is in Prof. Cain's book on Earth Pressure.

Method of finding the Maximum by Trial wedges.

Ardant (1848)(17) computed the pressure caused by several trial wedges, and plotted the values on lines parallel to the directrix(a line at $(\phi+\phi')$ to the back of the wall), so that each line is bounded by the line of natural slope and corresponding line of rupture. The curve joining the points on the trial lines of rupture is a hyperbola. He draws a tangent parallel to the line of slope and the point of tangency is on the plane of rupture which gives a maximum pressure. Cousinery(1841) had used a very similar method.

Eddy (1878) (188) assumes the maximum obliquity of pressure on the wall to be at angle ϕ, takes several trial wedges and calculates the pressure due to each. After plotting the values, the maximum is obtained by inspection.

Prof. Cain (98, 101, 104) has developed some very good methods of determining the maximum pressure by this method. See especially Transactions of A.S.C.E. v. 72, p. 405, and Prof. Cain's book on Earth Pressure(1886 & 1916 eds.)

Cullman(1866) (153-4) used a method almost identical with that of Ardant; the resulting curve is often called "Cullmans E-line."

Mueller-Breslau(1906) (461) in his "Erddruck auf Stützmauern" gives some detailed computations to show that the assumption of a plane of rupture eliminates the obtaining of the maximum lateral pressure. He developes formulae for the pressure of wedges, where the curve of rupture consists of combinations of straight lines and curves, both circular and spiral. The increase in pressure so obtained is quite small, and it is doubtful whether several pages of computation are paid for by the additional accuracy so obtained in computing the lateral pressure, especially since so many assumptions as to the shape of the curve have to be made.

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E. Effect of Cohesion in the Wedge Theory.

Coulomb and many of the early investigators had included a cohesion factor in their formulae, but had not been able to apply their results. In cohesive earth, the rupture probably occurs along a curved surface and not along a plane. In the wedge theory, the reaction on the plane of rupture changes in amount and in direction. It is assumed that this reaction may be resolved into a normal component which is a constant, in as far as taking cohesion or not taking cohesion into account, and a tangential component which consists of two parts, the frictional resistance and the cohesion resistance. The maximum pressure is then obtained by an analytical method or else by stress conics, as is described below.

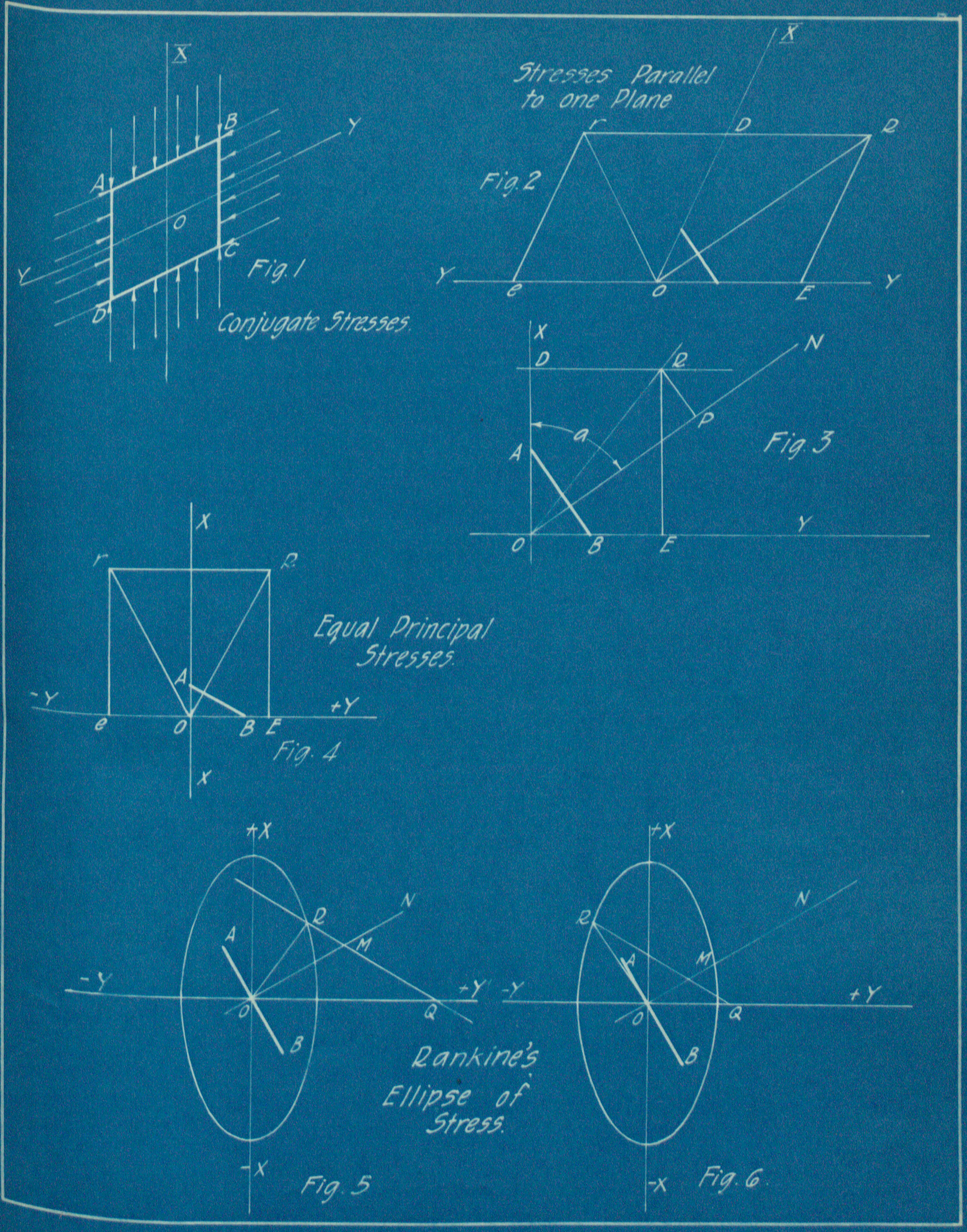
Another method is to determine the height of earth which is self supporting because of the cohesion and subtract from the calculated pressure the pressure on that height. The method employed in comparing the Cincinnati tests with theory, by determining a value for ϕ which would take into consideration both friction and cohesion, i.e. internal resistance as a whole, proved very simple and convenient, and gave good results.

Molitor(1900) (561-2) made careful study of the rupture and pressure of German clays. He concludes that the density varies with the depth, that the surface of slope is very close to a plane, but the surface of rupture is hyperbolic. The equation of the hyperbola is very closely $L = H\left(\frac{1}{3} + \frac{1}{\tan\phi}\right)$ for the materials he had; in general it is $y = \frac{H(L-x)}{L-5x}$ where H is the total height of fill above the base of the wall, and y is measured vertically; L is the total distance from the back of the wall to the furthest point of the wedge, x being measured in the same direction. To determine the lateral pressure, draw an equilibrium polygon made up of the weight and the resistance along the curve of rupture of the elements of the wedge, formed by several vertical sections(see fig. 7). The resultant acts at about 0.45 h and the direction is that given by the force diagram.

Prelini(1908)(504) gives a detailed discussion of coherent materials, both in free slopes and behind walls. He gives a method of extending the Weyrauch pressure triangle formula and construction to cohesion fills.(see fig. 8). Construct the pressure triangle IDK. Lay off AB' = AE, the height of earth which will stand vertically due to its cohesion. This height is equal to 4 k $\tan(45^\circ + \frac{\phi}{2})$, where k is the cohesion per unit area, c, divided by the density of the fill y. Draw FG $\perp$ AC, and EF horizontal. Then k = $\frac{1}{4}$ EF and c = $\frac{1}{4}$ y(EF). Let AD represent the cohesion along AD, and resolve it into two components AL and DL, where $\angle$ ADL is $90^\circ - \phi$, and $\angle$ BAL is $90^\circ - \phi'$. The lateral pressure is decreased by ($\frac{1}{2}$ y EF. AL) or by ($\frac{1}{2}$ y EF)($\frac{1}{2}$ AL). Lay off IM = $\frac{1}{2}$ AL, MN = EF, and $\perp$ AC, then the triangle IMN represents the decrease in pressure due to cohesion. If we make IKN' equivalent to IMN, the triangle IDN' is the new pressure triangle.

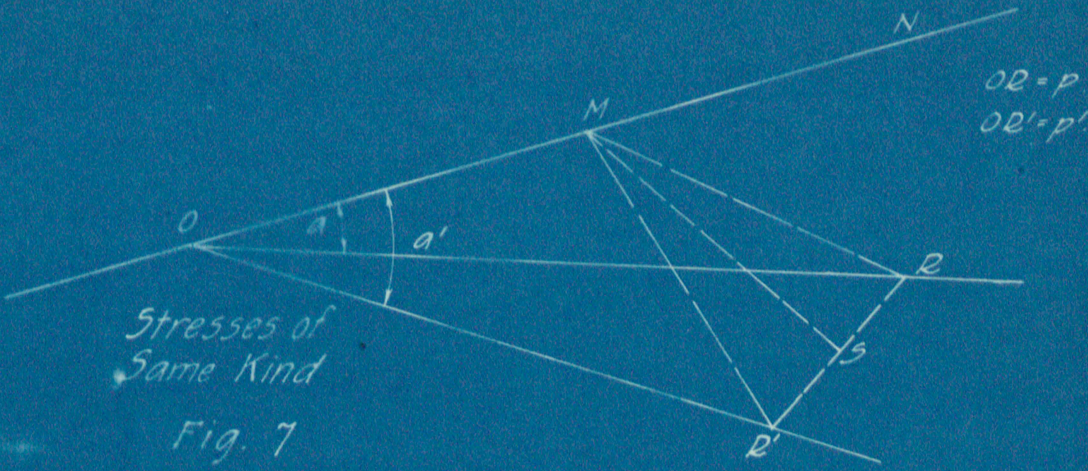
F. Stress Theories based on the Ellipse of Stress.

On June 19, 1856, Rankine reported a "Theory on the stability of Loose Earth, based on the Ellipse of Stress" before the



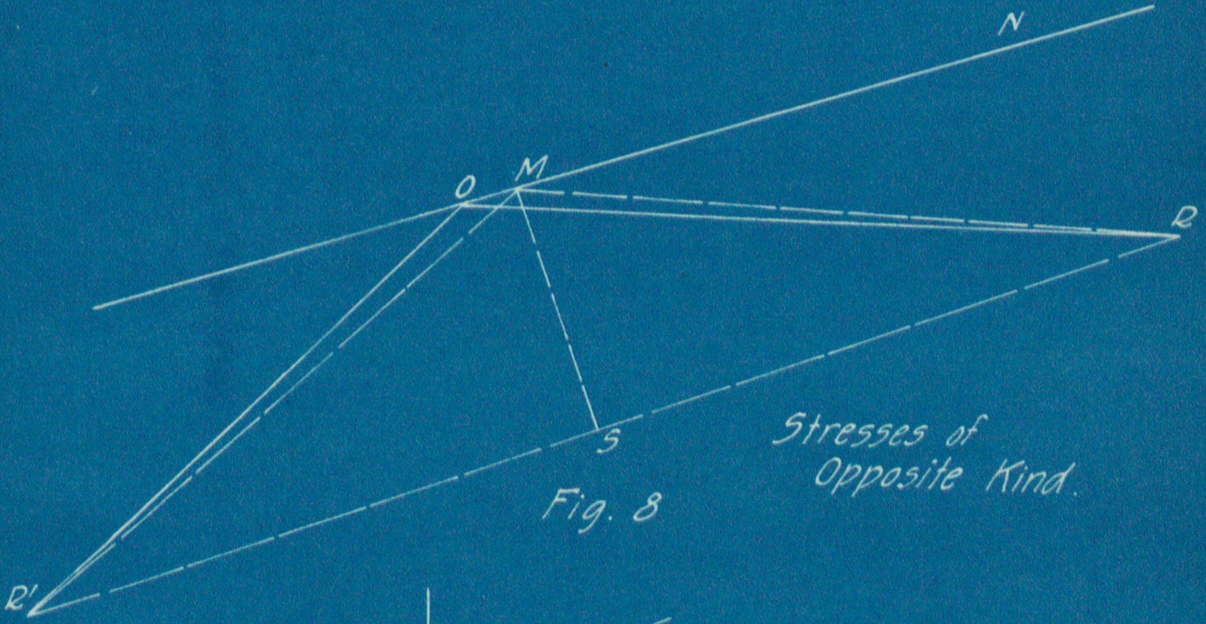
FIGURES FOR RANKINE'S THEORY.

Jacob Feld 1920



Stresses of Same Kind

Fig. 7



Stresses of Opposite Kind.

Fig. 8

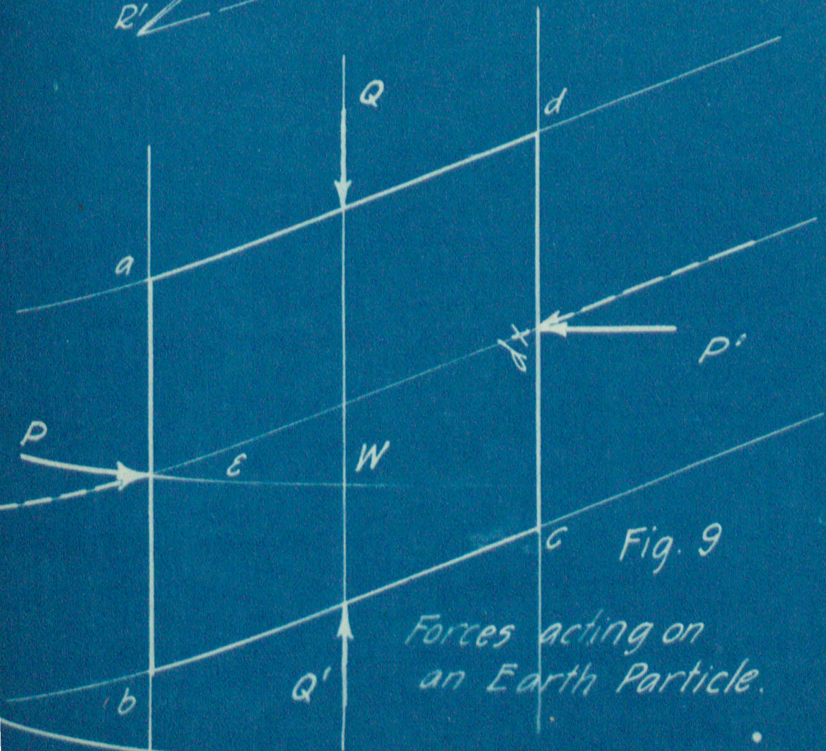


Fig. 9

Forces acting on an Earth Particle.

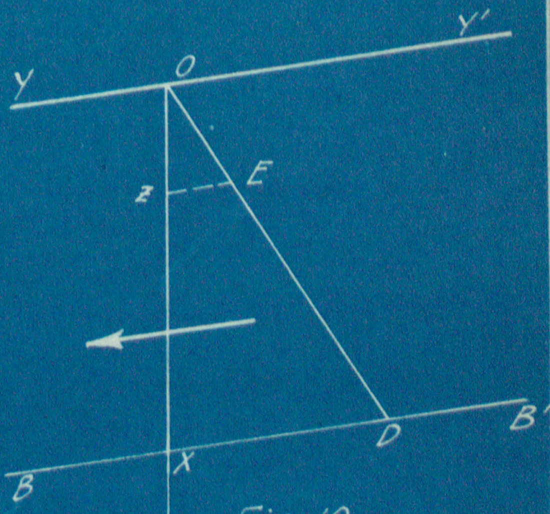


Fig 10.

Pressure on a Vertical Plane.

FIGURES FOR RANKINE'S THEORY.

Lateral Earth Pressure.

Royal Society in London, the theory appearing in the 1857 Proceedings. On Sept. 27, 1856, three months later, Carl Holtzmann's work "Ueber die Vertheilung des Drucks Im Inneren eines Körpers," Einladungeschrift der k. polytichnischen Schule in Stuttgart, derives the theory of lateral pressure for the special case of vertical wall and horizontal fill from the Cauchy and Lame' Ellipsoids of Stress. The methods are almost identical and probably derived independently. The theory is known as Rankine's theory.

The Rankine theory assumes a homogeneous, incompressible, granular mass of earth, of unlimited extent, the particles of which do not possess the property of mutual attraction called cohesion, but do possess the resistance to rolling over each other, called friction.

Consider an elementary volume, formed by planes at an infinitessimal distance apart, drawn parallel to the back of the wall and to the top surface (fig. 1). For equilibrium, the four forces acting on the four sides of the parallelopiped, must be concurrent, requiring, since the forces on the sloped sides are vertical, that the forces on the vertical sides be parallel to the sloped sides. Such forces are called "conjugate," i. e. If the force on a given plane acts in a given direction, stresses on planes parallel to that direction act parallel to the given plane. Of course this is true in all three dimensions, but we disregard all stresses acting in planes parallel to the back of the wall, since they have no effect on the wall itself.

In the case of earthworks, all the stresses to be considered are in a plane,(fig. 2 and 3), and if a pair of conjugate stresses OX and OY are given, the stress on any plane AB is the resultant OR in magnitude and direction. OD and OE represent the products of OA and OB and the unit stresses in the OX and OY directions, respectively. The conjugate stresses may be given in fig. 2 or only their components, as in fig. 3. The resulting stress may be resolved into a normal and a tangential component, OP and PR. It is evident from the figure 2, that on two planes, parallel and perpendicular to OR, there is no tangential stress. The stresses along these two planes are conjugate and are called the principal stresses. If the two principal stresses are equal and of the same kind, then every stress parallel to that plane is of the same intensity and same kind, and acts normal to its plane of action(fig. 4). If the given stresses are opposite in direction, all other stresses parallel to that plane have the same intensity but the angles which a stress makes with the normal to its plane are bisected by the axes of principal stress. In fig. 5 and 6, we have the solution of the direction and intensity of stresses acting on a plane perpendicular to the plane of the given pair of principal stresses of any intensities. The principal stresses are of the same kind in fig. 5, of opposite kind in fig. 6. The locus of R is an ellipse, whose semiaxes are the intensities of the principal stresses, and therefore the maximum and minimum stresses in the plane XOY are the principal stresses. The ellipse of stress can be constructed whenever two stresses whose planes of actions are perpendicular to the plane of their directions are given. In fig. 7 and 8, the given stresses are OR and OR! the normal to the plane is ON, and the principal stresses are

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OM+MR and OM − OR, the angles which the normals make with the axis of greater stress are ½ NMR and ½ NMR'.

In applying the above theorems to earthwork, (very little more than a mere statement of these has been given), we note that every stress is a thrust and the maximum obliquity of stress is ϕ, then the ratio of conjugate thrusts having a common obliquity is $\sin\phi = \dfrac{p_x - p_y}{p_x + p_y}$; px and py being the principal stresses. For stability, in a mass of earth, the direction of pressure on any plane must make an angle with the normal less than the angle of repose.

In addition, the ratio of the difference of the maximum and minimum pressures to their sum cannot exceed the sine of the angle of repose. In any indefinite homogeneous solid, bounded by a plane at any slope, the pressure on any plane parallel to that slope is vertical, and of uniform intensity equal to the weight of a vertical prism which stands on a unit area of the given plane, which equals the density of the earth times the vertical depth of the given plane times the cosine of the angle of slope of the plane. As can be seen from fig. 9, the stress on a vertical plane is parallel to the sloping surface and is conjugate to the stress on a plane parallel to that surface. The stress on a plane parallel to the top surface is uniform.

The upper and lower limits of the possible values of the pressure parallel to the top surface are

$$yx\cos\varepsilon\cdot\frac{1}{K} \leqq p \leqq yx\cos\varepsilon\cdot K,$$ where

E is the angle between the top surface and the horizontal.

p is the unit stress at a vertical depth x.

y is the density of the granular material.

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$$K = \frac{\cos\varepsilon - \sqrt{\cos^2\varepsilon - \cos^2\phi}}{\cos\varepsilon + \sqrt{\cos^2\varepsilon - \cos^2\phi}}, \qquad \text{where}$$

ϕ is the angle of natural slope, the maximum value of ε, Rankine must now resort to Moseley's Principle of Least Resistance, to determine the actual value of p from its limits, namely:

If there be a system of Pressure in equilibrium among which are a number of resistances, then is each of these a minimum, subject to the conditions imposed by the equilibrium of the whole. Moseley states the principle as follows: If the forces which balance each other in or upon a given body be distinguished into two systems, called respectively active and passive, which stand to each other in the relation of cause and effect, then will the passive forces be the least which are capable of balancing the active forces, consistent with the physical condition of the body. Assuming a uniform increase of pressure with depth, ~~the total pressure with depth~~, the total pressure on a depth h is $\frac{1}{2} yh^2 \cos\varepsilon \cdot K$ ~~$\cos \varepsilon K$~~, for the unit pressure at depth h is $yh \cos\varepsilon \cdot K$. This gives the amount and direction, and the uniform variation requires that the point of application be taken at 1/3 h from the base. (see fig. 10).

In this manner, we obtain the pressure on a vertical wall. If the wall is sloped, determine the pressure on a vertical plane thru the heel and add to it graphically the weight of the earth between the wall and this plane. The same theory using the maximum stress is used to determine passive pressure.

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This theory has been extended in many ways, including graphical constructions for the formulae. The greatest objection to this theory is the direction of the resultant, which is independent of the nature of the wall, and changes with the slope of the surface. If the slope is below the horizontal the theory requires an upward component, which is ridiculous. At the same time, all experiments have shown that the formulae of this theory give values far too large, especially for the cases of surcharges.

Altho Rankine makes no assumption of a wedge or surface of rupture, his formula is a special case of the general wedge formula if there is inserted in the latter the condition that the pressure acts at an angle ε to the horizontal, when the wall is vertical. In this case, the general wedge formula becomes

$$E = \frac{1}{2} y h^2 \left(\frac{\cos\phi}{1+n}\right)^2 \cdot \frac{1}{\cos\varepsilon} = \frac{1}{2} y h^2 \frac{\cos^2\phi \cos\varepsilon}{\left(\cos\varepsilon + \sqrt{\cos^2\varepsilon - \cos^2\phi}\right)^2}$$

$$= \frac{1}{2} y h^2 \frac{(\cos^2\varepsilon - \cos^2\varepsilon + \cos^2\phi)\cos\varepsilon}{\left(\cos\varepsilon + \sqrt{\cos^2\varepsilon - \cos^2\phi}\right)^2} = \frac{1}{2} y h^2 \cos\varepsilon \frac{\cos\varepsilon - \sqrt{\cos^2\varepsilon - \cos^2\phi}}{\cos\varepsilon + \sqrt{\cos^2\varepsilon - \cos^2\phi}}$$

for $n = \frac{1}{\cos\varepsilon}\sqrt{\sin(\phi+\varepsilon)\sin(\phi-\varepsilon)} = \frac{1}{\cos\varepsilon}\sqrt{\cos^2\varepsilon - \cos^2\phi}$.

This identity of formulae naturally requires that the assumptions made by Coulomb of a wedge of rupture, and that of Rankine's ellipse of stress are equivalent. The real difference between the theories is the assumption of the direction of the resultant. Lack of space prevents a more complete discussion of these stress theories, the outline of several of the more important is given below.

G. Stress Theories based on a Circle of Stress.

Cullman (1864).

In addition to his contributions to the general wedge theory,

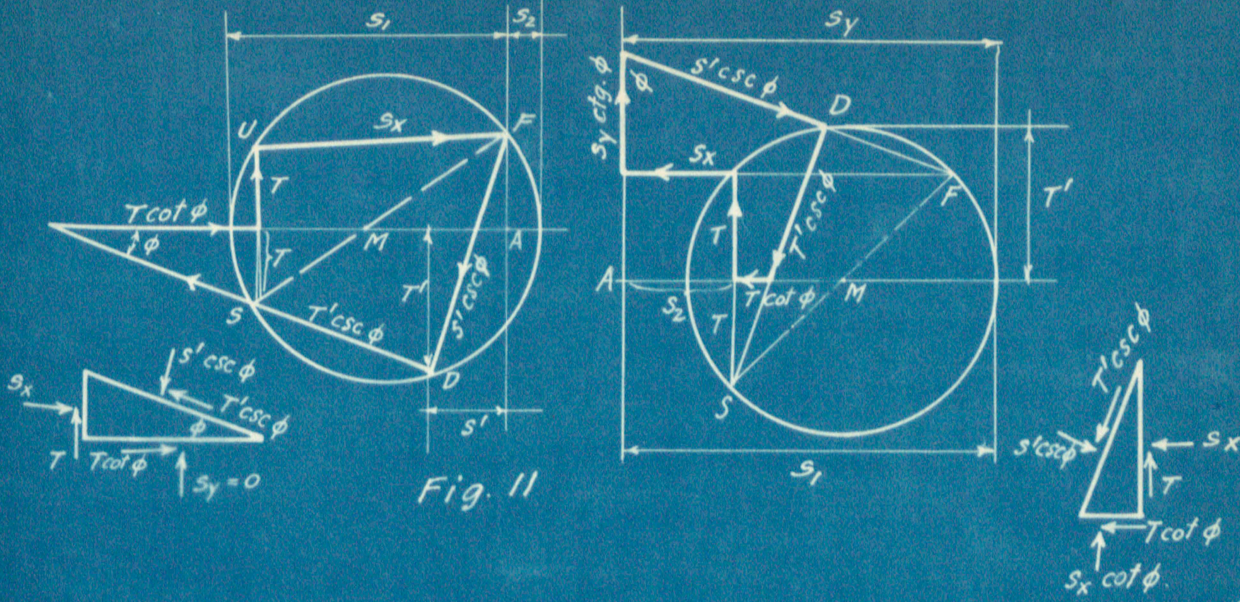

Fig. 11

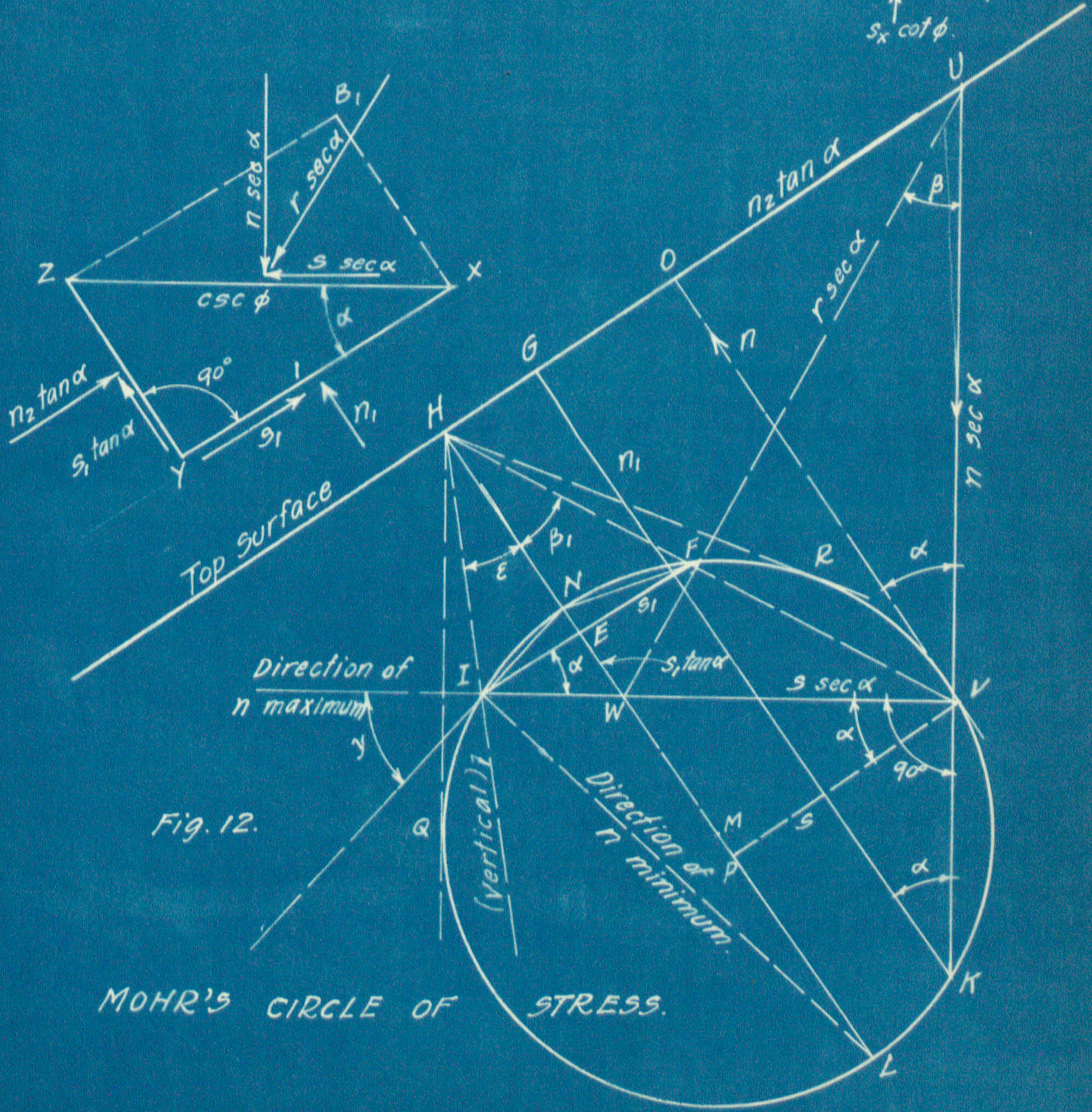

Fig. 12.

MOHR'S CIRCLE OF STRESS.

Jacob Feld 1922.

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Cullman developed a graphical analysis of the Ellipse of Stress and a method of determining the principal stresses by the use of a circle of stress.(see fig. 11.) He resolves the stresses on a triangular prism, vertical height being unity, into tangential and vertical components, and draws the force polygon. The Sy=0, for earth gives no upward push. He finds, that the points, S, D, F and U lie on a circle, while S is a fixed point, independent of the nature of the elementary prism. The figures show two cases. The principal stresses, S_1 and S_2, are much more easily obtained by this method than by the ellipse method. In all cases SD is parallel to the top surface of the fill.

Mohr(1871).

Mohr objected to the wedge theory, because the conditions for equilibrium did not hold inside the wedge. Winkler answered his objection by saying that the forces on the wedge did not have to act at the third points, and hence could be shown to intersect.

Mohr investigated the stresses acting on a triangular prism, with a unit hypotenuse drawn parallel to the top surface.(see fig.12) If β is angle between the resultant and the normal to the wall,

$$\sin \beta_{max.} = \frac{max.\,n - min.\,n}{max.\,n + min.\,n} = \frac{MQ}{MH}.$$

If θ = angle between the wall and the maximum n, then

$$\tan\beta = \frac{s}{n} = \frac{(max.\,n - min.\,n)\sin 2\theta}{(max.\,n + min.\,n) - (max.\,n - min.\,n)\cos 2\theta}$$

$$= \frac{\sin\phi \sin 2\theta}{1 - \sin\phi\cos 2\theta}, \text{ which is } \tan\phi, \text{ if } \theta = 45° - \frac{\phi}{2}.$$

The components of the stresses on the triangle XYZ form six forces in equilibrium, and hence must form a closed polygon, as EFGUVWE, where EF $= S_1$, FG $= n_1$, GU $= n_2 \tan\alpha$, UV $= n \sec\alpha$, VW $= S \sec\alpha$, WE $= S_1 \tan\alpha$. If these forces are drawn to scale we note (1) that E and G must be in the same vertical line, and

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EG is the intensity of pressure on XY or(yh), h being the vertical depth of XY. (2) UW and GF meet at K, so that GK = GU $\cot\alpha = n_2$
(3) EF and WV meet at I, so that EI = EW $\cot\alpha = s_1$.
That is, points, E, F, G, K, and I are independent of α, so that if we find these points for some plane XS, they are given for all other planes.
(4) Angles IFK and IVK are each 90°, so a circle may be drawn thru I, F and K, containing V; the diameter is IK, m is the center.
(5) If VP is drawn parallel to EF, to meet EW, VP = VW $\cos\alpha$ = S.
(6) In order to find the intensity of stress on any plane AB, draw IV parallel to AB and VO and VP are the normal and tangential intensities of stress on this plane. Draw HENPML, and HI, which is vertical, HV is the intensity of stress on AB, angle δ = PHV is the angle that the stress makes with the normal to AB. We can see from the circle, that the maximum value for the stress is HL on a plane parallel to IL, and the minimum is HN on a plane parallel to IN. On these planes there is no tangential stress, hence the stresses are the principal stresses and IL and IN are perpendiculars The maximum value for δ is on a plane IK, HR being a tangent, and also on a plane IQ, HQ being a tangent.

However, we are unable to draw the circle in the general case, but we can locate it within limits. We note that the limiting value of δ is ϕ, and circles drawn for such case represent the limiting states of equilibrium when the earth is just ready to slip. It is with these cases only that an engineer has to deal. If(in fig. 13) we draw HQ and HR at angles ϕ from the vertical, and set HI and HN equal to the principal stresses at the limiting value (from fig. 12), we obtain two circles of limiting stress, the smaller for active pressure, the larger for passive pressure. For the active earth pressure, the maximum pressure is HL_1 on a plane parallel to IL_1, and the minimum is HN_1 on a plane parallel to IH_1. The maximum obliquity occurs on planes parallel to IR_1, and IQ_1. Similar limits for the passive pressure are obtained from the larger circle. If the surface of the earth is horizontal, the two circles are tangent, and if it slopes upward at angle ϕ, the two circles coincide. In both figures IH is the intensity of pressure on a plane parallel to the top surface, and hence equals the depth of the plane considered times the density of the material, times the cosine of the angle of inclination of the surface.

Weyrauch(1880).(643)

Weyrauch resolves the stresses acting on a parallelopiped of infinitessimal dimensions, with faces parallel to the top surface and to the wall surface, into normal and tangential components. The stresses are conjugates, as was shown in the Rankine discussion. By a method very similar to Mohr's circle of stress, he obtains a circle of stress, from which may be obtained the Rankine formulae. Since the Rankine and Weyrauch theories are based on the same assumptions, they naturally give identical results. There were many more graphical stress methods, either modifications of those given above, or additions to their generality. Among others, Winkler, Engesser and Scheffler made original contributions.

H. Analytical Stress Theories.

J.J. Sylvester(1860)

Sylvester began a mathematical solution of the lateral pressure in "Mathematical", i.e. continuous and homogeneous, earth in

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1860. As is announced at the end of the article, the author intended to complete his investigation, but no other material on this subject is available. Sylvester objects to Rankine's use of the Principle of Least Resistance, because there is no proof that the resultant pressure in the state preceding motion is a minimum. The assumption upon which this theory rests is:

"If equilibrium can be preserved consistent with the imposed conditions, it will be preserved." Coulomb's plane of rupture is merely the surface of maximum frictional energy, which acts just before the phase of rupture can begin. However, it is dangerous to reason from conditions during equilibrium to those at incipient motion. Hence we must use the equations of stress, but Rankine did not use them correctly.

By considering the stresses acting on a particle, he finds that the curve of distribution of resultant thrust is an ellipse, and the law of variation of stress shows that the ratio between the maximum and minimum thrusts equal $\tan^2(45^\circ - \frac{\phi}{2})$. Having determined this, he constructs an ellipse of stress; ϕ being the angle of internal friction. The paper stops here, with the note that the solution requires the use of the Calculus of variations and ends with "To be continued." The writer has been unable to find any continuation.

Maurice Levy (1867)

Levy deduced a "rational" analytical theory of earth pressure from a study of the stresses on a unit parallelopiped. The result of the analysis, which makes no assumptions as to the magnitude or direction of the resultant, shows that the curves of

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sliding of particles are at constant inclination to the "lignes isostatiques," which are special cases of Lamé's "surfaces isostatiques." The equations are differential equations of the second degree, which Levy does not solve, but asks others to attempt rigid solutions. This paper appeared in 1867 and was reported to the Academy in 1869 by Saint Venant.

Saint Venant (1870)

Saint Venant, after reporting Levy's theory, attempts a solution of the equations by inserting end conditions, i.e. assuming the resultant stresses at the surface of the fill and along the wall surface. He uses Rankine's hypothesis, that the maximum resultant occurs when the inclination of pressure from the normal is ϕ. The solution however was not entirely general.

Boussinesq (1870 - 1884).

Boussinesq studied the stresses in a pulverulent mass by considering elastic equilibrium, during and before motion. At the free surface, the stress is zero. The atmosphere percolates thru the mass and since it presses equally in all directions, it has no resulting effect. Starting with a particle in the natural state, free from its own weight and a resultant pressure of zero, he studies the actions on it which cause it to reach the limit of elasticity. To obtain the constants in his equations, he takes the two limiting cases. At the top surface, the stress across the surface is zero. The other boundary is the wall, which may vary from a perfectly rough to a perfectly smooth case. In the extreme rough case, a particle in the natural state remains in the same position after the mass has become strained. But such cannot be the actual case. He constructs the ellipse of stress based on Cauchy's equations of mathematical theory of stress, and solves the problem independently for the extreme cases of wall surface. Then he deduces the values of the angles between stresses at the point when equilibrium breaks down and obtains the same results as are given in Rankine's theory.

In a latter paper (1884) Boussinesq gives the complete solution of Levy's equations, following the method outlined by Saint Venant.

Flamant computed several tables for this theory and simplified portions of it. The mathematics involved is too detailed and lengthy for a paper of this nature, and excellent summary of the methods employed is in Todhunter's:"History of the Elasticity of Materials," v.2, pt. 2.

Flamant gives a summary result of the Boussinesq theory: The actual retaining wall exerts a force against the earth which is not an arithmetical average between the limiting cases but equals the minimum value plus 9/22 the difference.

$$E = \frac{B}{\cos\phi} \cdot \frac{\gamma h^2}{2\cos\varepsilon}\ ; \qquad B = K' + \frac{9}{22}(K'' - K').$$

$$K' = \tan\left(45^\circ - \frac{\phi}{2}\right) \frac{\cos\psi \cos(\phi+\delta)\cos(\omega-\varepsilon)\cos^2\phi}{\cos(\phi-\delta)\cos(\omega+\psi)}.$$

$$\sin(\omega + 2\psi) = \frac{\sin\omega}{\sin\phi}\ ; \qquad \delta = \frac{\pi}{4} - \frac{\phi}{2} - \psi - \varepsilon.$$

K'' = same form as K', by replacing ϕ by Φ, where

$$\sin\Phi = \frac{\sin\Phi}{\cos\delta}.$$ Formulas do not hold, if $\varepsilon < \frac{\pi}{4} - \frac{\phi}{2} - \psi$.

δ = angle of rupture between wall and plane of rupture.

Recently (1917) J. Boussinesq has attempted a very rigid analytical discussion of Rankine's theory, taking into account the wall effects as end points of his theory.

Lateral Earth Pressure.

I. Various Theories recently advanced.

Chaudy's Pressure Theory (1895)

If a load P is placed on a fill in back of a wall, a thrust Q will be set up against the wall acting at angle δ from the horizontal. Q is equal to the component of P in the same direction, P sin δ, less the frictional resistance set up by the other component of P, P $\cos\delta$. Therefore $Q = P \sin\delta - f\, P\cos\delta$

$$= P \sin\delta \left(1 - \frac{f}{\cos\delta}\right).$$

P acts over a width dx, while Q acts over a width dx sin δ, therefore the ratio of intensities $\frac{q}{p} = 1 - \frac{f}{\tan\delta}$; where $f = \tan\phi$

In general, δ is greater than ϕ, and is determined by setting the derivative of the expression for Q equal to zero, thereby obtaining the maximum value of Q. A uniform load of p lb. per ft. on the fill, will give a total $Q = ph \frac{\cos(\delta - \alpha)}{\cos\alpha}\left(1 - \frac{f}{\tan\delta}\right)$; where α is the inclination of the wall. To determine the total earth pressure, summate this expression over a triangular area bounded by the wall, the surface and a plane at angle δ from the horizontal. The resultant earth pressure acts at an angle $(90^\circ \pm \alpha - \delta)$ with the wall, this angle being greater than ϕ'. The result is in the form of a cubic equations which is solved by approximation.

For $\phi = 45^\circ$, this theory gives a result 30% greater than the Rankine or Coulomb theory. For $\phi = 14^\circ$, the result is about 10% less than the older theories.

Meem's Arching Theory (1908)

For observations in timbered cuts, Meem concludes that the existing theories are far from correct. The maximum pessure occurs at the top of the cut, after a certain depth the earth gives no lateral pressure. There is a limit to the depth of cuts, for

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after a certain depth, bracing of the top is impossible, there is a sort of "topheaviness." The lateral pressure is caused by the arching of the earth between the wall and the plane of slope, causing a horizontal thrust which acts at 1/3h from the top of the wall.

Haine's Elastic Solid Theory(1908).

Earth is not a fluid, but an elastic solid. Pressure is caused by a semi-circular area of rupture, the surface of rupture being above the heel of the wall, probably because of the increase in density with depth. The lateral pressure is maximum at the top and equals the horizontal component of the rotating effect of this semi-circular wedge. The pressure at the base of and below the wedge is zero.

Ramisch's Work Theory(1910)

Ramisch derives an expression for the work done when a particle of a granular material moves along a plane. With this as a basis, he obtains the value of the work done when the wall rotates thru an infinitessimal angle. From which he obtains the value of the lateral pressure $0.215\ yh^2$, acting at 0.388 h. The passive pressure is $0.908\ yh^2$, acting at 0.408 h. If loads are placed on the fill, the height of the resultant is taken at $\frac{1}{2}$ h.

Dozal's Friction Theory(1918).

Starting with the ordinary laws of friction, Dozal derives the frictional resistance on the plane of rupture in terms of the weight of the wedge and the wedge angle. He decomposes the weight of the wedge into two components, normal and tangent to the plane of rupture. From the latter he subtracts the frictional resistance and obtains an"unbalanced force," L. L varies with the weight of the wedge, hence to obtain a maximum he differentiates with respect to the width of the wedge. Cubic equations result, which he solves by the Cardan formula. This he does for all possible cases of sloped surface of fill and sloped walls. He shows mathematically that the same result is obtained, whether we make the total pressure, the horizontal or the vertical component a maximum. The resultant acts parallel to the plane of rupture. The theory is but a slight advance over most of the pre-Coulomb theories.

Terzaghi's Stress Molecular Theory(1920).

The writer does not know whether Terzaghi bases his theory entirely upon experiment. For a description and comment upon the experimental work, see another part of this paper. Accofding to a letter received by the writer, Terzaghi has written a series of four articles for the "Engineering News - Record," but only the first has appeared up to the date of this writing.

Terzaghi makes a detailed study of the action of grains just before and during motion. He notes that slip is merely an after effect, after the equilibrium has been broken down, and that a true analysis must consider the elastic deformation of the grains. In addition we must take into account the statical resistance of the grains in position, the frictional resistance of the grains on their surfaces and the yield of the entire mass.

He considers the action in an earth mass during two phases, first, at rest, while equilibrium exists, second, while equilibrium is breaking down, but before motion occurs. During the second phase, there is an elastic yield, following by jerks as the volume of the backfill suddenly increases. These phases are not sharply defined. There is very little definite mate^r^ial in the first article, numerous new words with fine distinctions between them end in a maze of ideas, which he claims to have checked experimentally. For example: "One of the conditions for the occurrence of a slip is that the stress in the sand tangential to an ideal interface(an inclined plane passing thru the foot of the wall) exceeds a certain value," is merely Coulomb's assumption. Perhaps the reason for this rather unique use of English is that the author is at the Robert College in Turkey.

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The writer makes no attempt to derive any new or old theory. His experiments have shown some practical conclusions, which are given at the end of this paper. But the experimental work is not yet complete, there are numerous points which must be decided upon before the writer can scientifically develope a theory based on the experiments.

Lateral Earth Pressure.

History and Description of the Experiments in Earth Pressure and Related Phenomena.

A. Research to determine the lateral earth pressure behind test walls, both in models and in actual size.

1. Very little is said in the early literature on the subject concerning experiments, tho many of the writers on the subject probably performed some sort of tests. Mayniel (418) in his "Traite" states that Gadroy (1746) was the first to base theory upon the results of experiments. Yet, Belidor (40) in 1720 stated that experiment leads him to assume a 45° prism of rupture, so that we can safely assume that he had performed some sort of experiment. Gadroy's experiments were begun with the intention of checking Belidor's Theory. The apparatus was small, 3 in. high, and the only conclusions drawn were that there was a wedge of rupture. The experiments showed that the plane of rupture sloped at 2:3 while the natural slope was 1:1; the meaning of this was overlooked, and the plane of repose was assumed to also be the plane of rupture. A further account of these tests is in the section on "Theories previous to 1800." Rondelet (1767) (418) repeated these experiments upon a larger scale, 17½ in. high, and came to the same conclusions. In his theory, he therefore assumes that there is a wedge of rupture which tends to slide on the plane of repose, and that its effect is equal to its weight times the distance of the axis of rotation from a line drawn parallel to the natural repose thru the center of gravity of the wedge. Altho he

obtained different results for soil and for sand, he averages all his values and assumes a 45° plane of rupture for all materials.

2. D'Antony (1768)[(418)] was the first to construct an apparatus to measure the pressure against a test-wall. His box was small and had a hinged side. A cord was tied to the top of the gate and passed over a pulley at the back of the box. From the cord was hung a pan for weights. The crudeness of the apparatus prevented reliable results.

3. Gauthey (1785)[(418)] was the first to perform a complete set of earth pressure experiments. His apparatus was a box 30 in. long and 30 in. high, and 12 in. wide. He used a hinged gate, with a cord tied at 1/3 the height from the base, and passing thru the fill; the method used by D'Antony. The material used was sand, density 91 lbs. per cu. ft. and a force of 35 lbs. was required to keep the wall in equilibrium. This would require $\phi = 51°20'$. Since he does not take into account the resistance of the sand along the cord, which decreases the required force, his experiments were quite accurate.

Further experiments were in a smaller box, $7\frac{1}{2}$ in. long, $7\frac{1}{2}$ in. high, and 9 in. wide. A hollow hinged prism was used for a gate; and filled with lead shot or cast iron filings to give a wall of density equal to that of the fill. No results were obtained with this.

The wall was then changed into a gate made up of five parts, each 1½ in. high. Two cords, passing over pulleys in back of the fill were attached to each part. In this way, he could measure the pressure at each depth. To keep the wall from moving, using a sand fill, he finds that weights of 1½ oz., 4 oz., 6 oz., 8 oz., and 10 oz. are required for the various parts. From such a small model, accurate results cannot be expected, especially with the unknown resistances along the cords. But the variation of pressure with the depth is quite marked.

To determine the shape and size of the wedge of rupture, he inserted an inclined plane into the fill. By rotating this plane about the foot of the gate, he could control the lower surface of the fill. He found that when the plane was at 22½° and also at 45° to the horizontal, the pressures were the same as without the plane, showing that the material below the plane of repose has no effect on the lateral pressure. He then inserted the plane at 67½° to the horizontal, and the pressure still remained the same. Altho Gauthey drew no conclusions from this, he had proven that the wedge of maximum pressure does not include all the material above the plane of repose.

4. Woltmann (1791-1799). (see fig.1)

The apparatus was a box 1.72 meters long, 1.15

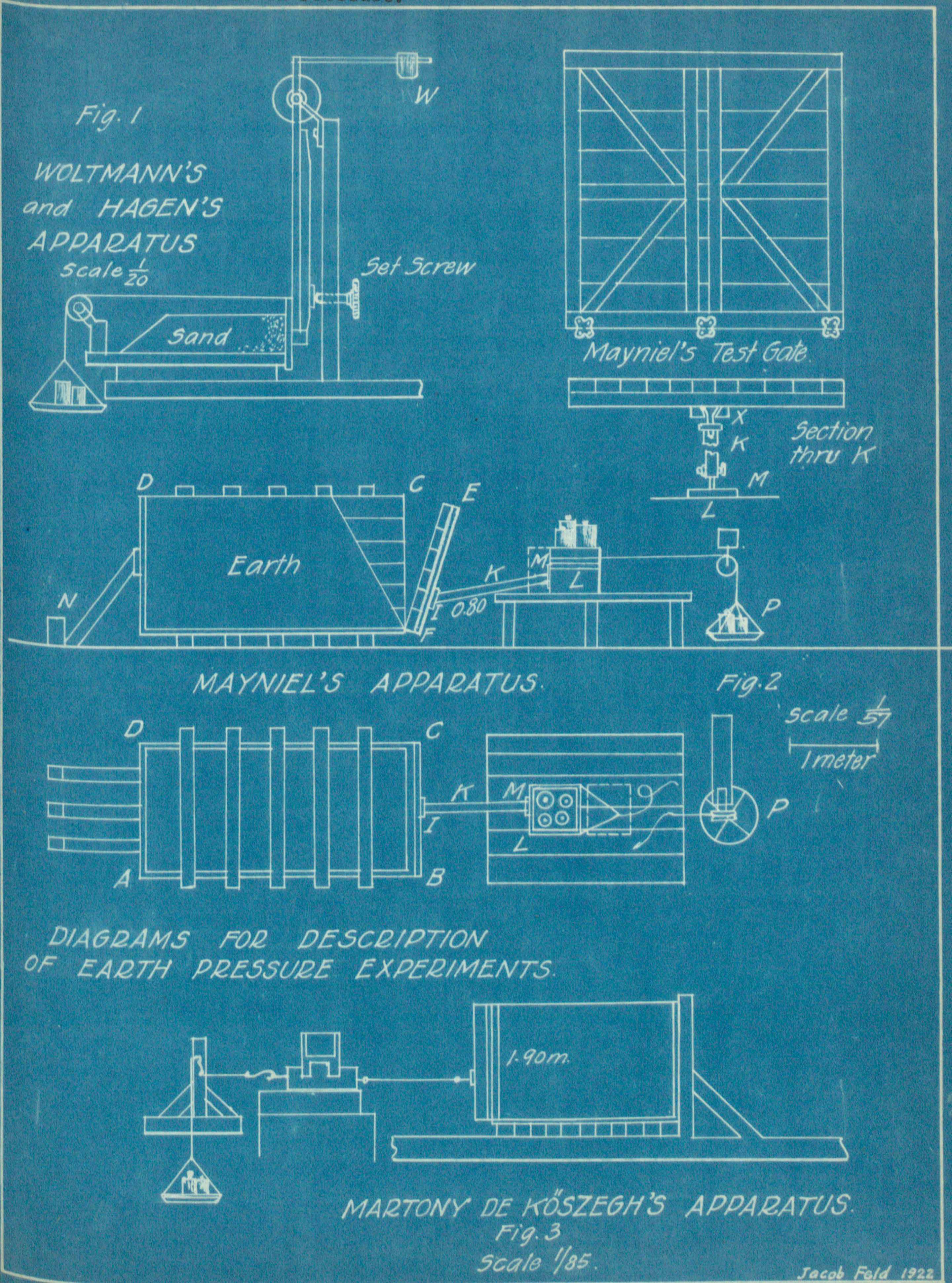


Fig. 1
WOLTMANN'S and HAGEN'S APPARATUS
Scale 1/20
W
Set Screw
Sand
Mayniel's Test Gate.
X
K
Section thru K
M
L
D
C
E
Earth
K
M
L
N
I 0.80
F
P
MAYNIEL'S APPARATUS.
Fig. 2
Scale 1/57
1 meter
D
C
K
M
I
L
P
A
B
DIAGRAMS FOR DESCRIPTION OF EARTH PRESSURE EXPERIMENTS.
1.90m.
MARTONY DE KÖSZEGH'S APPARATUS.
Fig. 3
Scale 1/85
Jacob Feld 1922

Lateral Earth Pressure.

m. high (4x4x6 ft.). The front wall was hinged on top and prevented from excessive rotation outwardly by an adjustable stop at about 1/3 the height. Two methods were used to measure the overturning movement on the wall. A string was tied to the back of the wall at its middle point and passed over a pulley in the back of the box. Weights were hung on this before filling the box. After the material was in place, weights were removed till the wall showed signs of moving. In this way, active pressures were always obtained. The second method was to counterbalance the moment in the wall by the beam and rider at the top. By getting the reading while moving the rider from an overbalanced to a balanced position, the active pressures were recorded. (303)

The following results are the averages of the tests:

The figure given is the value of C in the formula $E = \frac{1}{2} \gamma h^2 C$.

Material	Sand	Gravel	Soil	Rye	
Density	1.555	1.627	1.277	0.708	grs/cu.cm.
	97.2	101.5	79.8	44.3	lbs./cu.ft.
	0.142 - 0.164	6.078 - 0.104			
C (Experimental)	0.142-0.164 -0.164	0.078 - 0.104.	0.096 0.096 -0.120.	0.160 -0.212.	
C (Average)	0.153	0.091	0.108	0.186.	
C (Coulomb Theory)	0.308	0.260	0.172	0.406.	

Hagen suggests that Woltmann's results should be increased by 2/11 to take care of the loss in friction along the sides of the box. Even this correction will.

not take care of the difference from theory. If we note that Woltmann disregarded all tangential forces on the wall, and assumed the total pressure horizontal, and also assumed the pressure to act at 1/3 h, we can account for the difference. (651)

5. Mayniel (1808). (418)

The experiments conducted at Alexandrie (1805) Piedmont (1806) and at Juliers (1806-1807), started by Laulanier (Major of Engineers) and Derche (Lt. of Engineers) and completed by Mayniel are given in detail in the "Traite de la Poussee des Terres". The first apparatus built was a box 2 m. wide, $1\frac{1}{2}$ m. long and 1 m. high. When filled with sand, the front gate broke its hinges and fell with a "sharp detonation". However, the experimenters had curiosity enough to measure the surface of rupture and found it to be practically a plane at 64°42' to the horizontal. The Juliers apparatus is shown in Figure 2. The apparatus was made by Derche, probably also designed by him. The box ABCD is of matched boards with dovetail joints, 3 meters long, $1\frac{1}{2}$ meters wide, and $1\frac{1}{2}$ meters high (9.85x4.9x4.9 ft.) The box was stiffened by 5 ties over the top and a back strut N. The gate was also of wood, well stiffened by diagonal stays, and hinged at the bottom. At 1/3 the height, an iron strut K was connected to the front of the wall by a toggle joint. The rod was 80 cm. long and similarly attached to a tin box L, 0.495 m., by 0.495 m. by 0.585 m.

Lateral Earth Pressure.

high ($19\frac{1}{2}$x$19\frac{1}{2}$x23 in. high). In this way the rotation of the wall is changed into a push on this box. The location of I on the wall can be altered by changing the position of the box L. The earth pressure is measured by the weights in box L. To neutralize the friction, a pan P is connected to the box and enough weights are placed in the pan. During the filling process, the gate is clamped to the bin by means of four hooks.

Mayniel states that comparison between the results obtained and Coulomb's theory, show it to be the only true and simple theory. In general the earth pressure acts at 1/3 the height and equals $\frac{1}{4}$ to 1/3 the weight of the wedge of rupture when the fill is loose, and 1/7 to 1/20, when the fill is packed.

In the following discussion of the results and conclusions drawn, the formulas derived by Mayniel are used. They are based upon the theory of the maxiumum wedge of rupture.

Assuming the total pressure E, to act normal to the back of the wall, no matter how the wall slopes, taking into account cohesion as well as friction, $E = mh^2 - nch$, where m, n, and c depend upon the nature of the wall and of the fill. Assuming cohesion equal to zero, a coefficient of friction of 2, and a vertical wall, $E = 0.382 \frac{\gamma h}{2}$; or $C = 0.382$, in the general formula $E = \frac{1}{2}\gamma h^2 C$. The overturning moment is $1/6 \gamma h^3 C$ or $0.0637 h^3$. Equating this to the moment of the wall gives the required width. These theoretical results are checked by the experiments. Mayniel takes up numerous cases of wall design; they are given in the description of his theory. For sand his experiments give $E = 0.168\ (\frac{1}{2} yh^2)$. He accounts for the difference by assuming cohesive resistances in the fill.

6. Martony de Kőszegh (1828). (651)

De Koszegh used an apparatus very similar to that of Mayniel, but smaller in size. It was 2.85 meters long, 0.95 m. wide, and 1.90 m. high (9.35x3.10x6.25 ft). The pressure was measured by a friction dynamometer. A strut, DC, connected the wall with a box filled with shot. The box was fixed to a block of wood resting on a similar block of wood. (See fig. 3). The coefficient of friction was known. The bin, A, was filled, then some of the weight in the box, B, was removed till the wall began to give. The remaining weight in B times the coefficient of friction along DD gave the thrust of the earth, as transmitted thru CE. This is a very ingenuous method of measuring the horizontal force, but does not measure the vertical component.

For earth from a dam, density 1.369 ($85\frac{1}{3}$ lbs. per cu. ft.) and a natural slope of $58^{\circ}34'$, C was found to be 0.114 to 0.198. Theory for this case gives $C = \tan^2 \frac{1}{2}(90^{\circ} - 58^{\circ}34') = 0.080$. The ϕ was probably much less than $58^{\circ}34'$; the average C = 0.156 corresponds to $\phi = 47^{\circ}$, a perfectly possible value of the angle of internal resistance.

For dry sand, density 1.748 (109 lbs. per cu. ft.) and a natural slope of 37°, C was found to be 0.216 to 0.256. Theory for this case gives $C = \tan^2\frac{1}{2}(90° - 37°) = 0.249$. The average C = 0.236, corresponds to $\phi = 38°15'$, a close check.

7. Huber-Bernard (1829).
(Geneva)

Huber-Bernard investigated the flow of sand thru orifices and also the pressure of sand on the bottom of deep vessels. He concludes that sand is not a liquid because it does not seek its own level. Sand has a natural slope which is a constant under constant conditions. The flow of sand thru an orifice is independent of the size of the orifice. (66)

In the Bibliotheque Universelle for 1829, he reported on some experiments to determine the pressure on the bottom of a cylinder 65 cm. high and $3\frac{1}{2}$ cm. diameter ($25\frac{1}{2}$ x 1 3/8 in.). He connected a 1 cm. bore (0.4 in.) mercury tube with the bottom of this vessel and measured the pressures by the height of the mercury. He concludes that there is no pressure on the bottom. (326)

Prevost, in the discussion of this report, shows that the retaining effect, or frictional resistance, of the sides of the tube, prevented any readings on the bottom. This was the first test to determine the pressures in bins, or in limited masses of filled material.

8. Hagen (1833).

Hagen's investigations were to determine the direction of the total earth pressure. He repeated Woltmann's experiments, using practically the same apparatus, and draws three very important conclusions: (651)

1. The pressure is a function of the square of the height.
2. The total pressure acts at 1/3 the height.
3. The total pressure is inclined to the back of the wall, making a definite angle with the normal.

For sand, with $\phi =$ 37°30' natural slope, and 25°05' for the angle of internal friction, which he determined, he obtains C = 0.246 to 0.262. The average C = 0.254 corresponds to a $\phi =$ 36°30', a more probable value of the angle of internal resistance than 25°05', assuming the natural slope to be 37°30' as given.

He investigated the flow of sand thru orifices and gives the empirical formula $F = \pi r^2 \gamma h - 2\pi r \gamma l h^2$, where r is the radius of the orifice; h the head of sand above the orific; γ is the density of the sand; l is a frictional resistance along the edge of the orifice. (303)

In his paper on the experimental determination of the direction of the earth pressure, he calls attention to the fact that Coulomb had called the friction

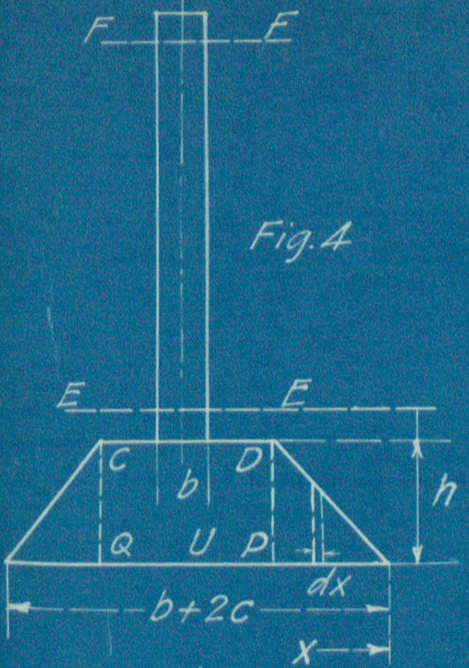


Fig. 4

HAGEN'S APPARATUS
Section thru Fill.

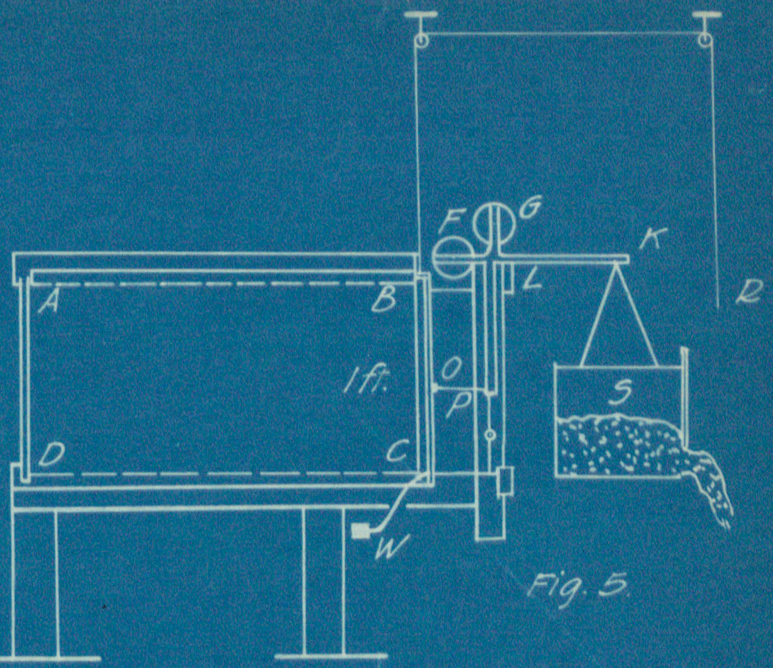


Fig. 5

LT. HOPE'S APPARATUS.
Scale 1 in. = 1 ft. (1/12)

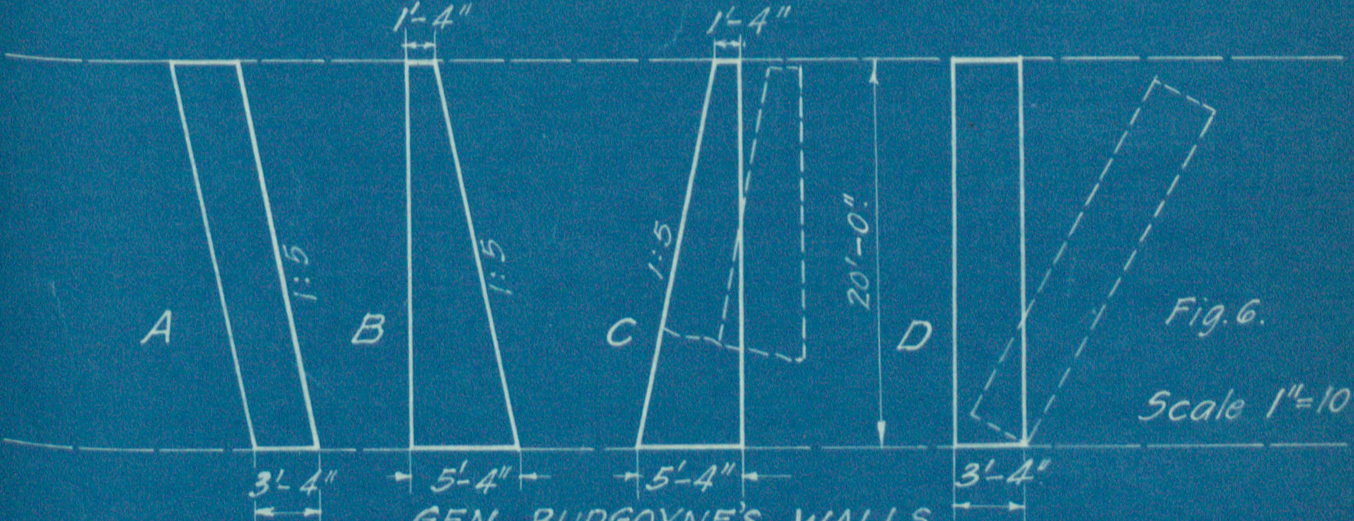
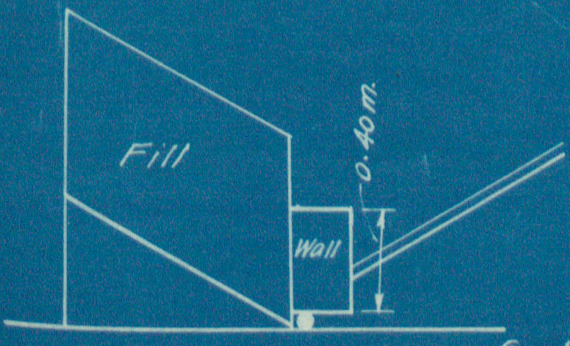


Fig. 6.
Scale 1" = 10'.

GEN. BURGOYNE'S WALLS.



Scale 1/40

Fig. 7

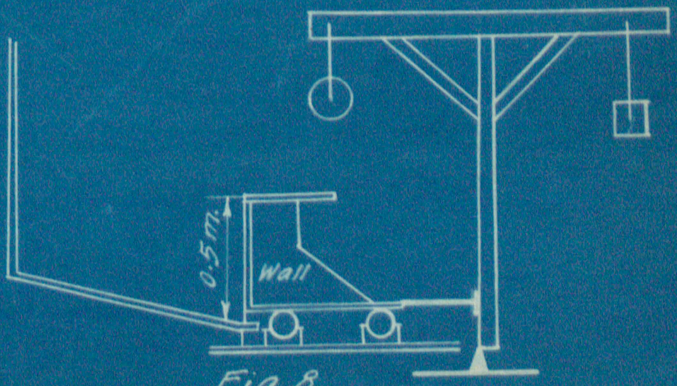


Fig. 8

AUDE'S APPARATUS.

Jacob Feld 1922.

Lateral Earth Pressure.

coefficient of sand 1/n, while it was Woltmann who had introduced the method of calling the friction coefficient the tangent of the natural slope. (304)

To determine the direction of the total pressure as well as the magnitude, he uses an apparatus similar to that in Fig. 1 but with no side walls. The section of his fill is then as is shown in fig. 4. The axis of the wall is FF, the cord is tied along EE. The height of the axis above the foot of the wall is l, the height of the cord is m.

The pressure as measured is not affected by side wall losses, and the longitudinal section of the fill is similar to the section in actual practice. The pressure on the area CDQR is $\frac{1}{2} y h^2 bC$, and its moment about FF is $\frac{1}{2} y h^2 b C(l-\frac{h}{3})$, assuming the point of application is at 1/3 h. This formula is the general form, y being the density, b the width, h the height of fill, and C the constant depending upon the material. The pressures on the areas ACQ and BDR is found by taking a width dx. The pressure on this elementary width is $\frac{1}{2} y \frac{h^2}{c^2} C x^2 dx$, and its moment about FF is $\frac{1}{2} y \frac{h^2}{c^2} C\, l(lx^2 - \frac{hx^3}{3c})$. The total M of ABCD is $\frac{1}{2}h^2 yC[b(l-1/3\,h)+2/3\,c\,(l-\frac{1}{4}h)]$.
If we call the height of the cord above the base n, then the stress in the cord is $E = \frac{1}{2}\frac{h^2 y C}{n}[b(l-\frac{1}{3}h)+\frac{2}{3}c(l-\frac{1}{4}h)]$.
In Coulomb's theory, the horizontal component is given by $H = \frac{1}{2} y b h^2 \tan\phi \sin\phi (\cos\phi - \frac{1}{n}\sin\phi)$.
For a maximum H, $\frac{dH}{d\phi} = 0$, or $\tan^3\phi + 3\tan\phi - 2n = 0$.

Lateral Earth Pressure.

The pressure on area CDQR $= \frac{1}{2} \gamma h^2 b C$; substituting, we obtain $\tan^2\phi = \frac{3C}{1-C}$; and $f = \tan\phi_1 - \frac{1}{2}(\tan^3\phi + 3\tan\phi)$ From this relation we can obtain the angle of friction from the surface slope.

The average of numerous tests, with dry sand, gave $\phi_1 = 55^\circ$.

For wet sand, density of 1.62 (101 lbs. per cu. ft) he finds $\phi_1 = 62^\circ$; natural slope being $43^\circ 50'$; $C = 0.235$

a =	1	2	3	4	5	zoll (in.).
b =	14.4	11.2	8.0	4.8	1.6	"
2c =	3.2	6.4	9.6	12.8	16.0	"
E =	6.0	20.5	38.45	55.0	65.4	kg.

$C = 0.235$ corresponds to an angle of internal resistance of $38^\circ 20'$; the value to be expected from a natural slope of $43^\circ 50'$! Hagen's values for the internal friction are too high. Experiments performed by the writer (see report) always showed that the angle of internal friction is less than the angle of natural repose.

9. Lieutenant Hope (1845). Royal Engineers.

Lt. Hope laid out a complete set of experiments to complete the determination of the lateral pressure of granular materials. Altho he did not complete his entire plan, his work gave many important results. He attacked the problem from two points of view - tests with a small apparatus and check determinations with full scale walls

Lateral Earth Pressure.

(320)

at Chatham.

The apparatus (see Fig. 5) consisted of a box, 2 ft. long, 1 ft. wide, and 1 ft. high. The test-wall was 1 ft. square. The wall, BC, was hung vertically by two cords passing thru pulleys and balanced by a box of sand, similar to box S, at R. The weight W was adjusted so that the wall did not touch the cords. The cords were tied to the bottom of the walls. The bent lever KLP was used to determine the horizontal component; the weight in box R measured the vertical component. A strut OP connected the lever to the wall; point O could be varied. The bin was filled after box S had been filled with sand or shot to overcome the possible horizontal pressure and after the wall had been counterbalanced vertically. Sand from S was then let out until motion occurred. The weight of the remaining sand measured the amount of the pressure.

To determine the existence and shape of the wedge of rupture, the bin had glass sides. His material in these tests was a sand, density 91 lbs. per cu. ft. when measured loose, and 99 lbs. per cu. ft. when packed. Its surface slope was 35°15'. For a height of 12" in all cases, he measured the distance of the plane of rupture from the top of the test-wall, i.e. the base of the wedge of rupture. By inserting a sloped board and filling above it only, he determines the pressures due to wedges smaller than the wedge of maximum pressure. The center of pressure in all cases was at 1/3 the height.

Shape of fill	Base of wedge	Horizontal pressure	
Bin full	$7\frac{3}{4}$ in.	9 lbs.	$0\frac{1}{2}$ oz.
Sloped board, 27°20' from vertical	6.2	9	4
18°25' " "	4.0	9	$0\frac{1}{2}$
14°00' " "	3.0	8	2
9°30' " "	2.0	8	2
4°45' " "	1.0	5	$5\frac{1}{2}$
2°25' " "	0.5	0	3

(320)

The wedge theory, for this case, gives an angle of rupture of $\frac{1}{2}(90° - 35°15') = 27°22\frac{1}{2}'$, a wedge base of 6.2 in., and a maximum pressure of $\frac{1}{2}$ (95) $\tan^2(27°22\frac{1}{2}') =$ 9 lbs. $2\frac{1}{2}$ oz., a result well checked by these tests. Lt. Hope states in his report that the actual plane of rupture was at $7\frac{3}{4}$ in. from the wall. It is curious to note, however, that the maximum reading obtained was when the wedge was of the shape required by the theory.

To determine the shape of the surface of rupture, he made numerous observations with walls sloping from 45° to 129° to the horizontal. In most of these the surface was not plane but somewhat hollowed, i.e., more inclined at the upper than at the lower end. In some instances, a well determined plane was found. He concludes that the surface of rupture does exist and may be assumed to be a plane bisecting the angle between the wall and the plane of slope.

Lateral Earth Pressure.

Sir Benj. Baker in his book on "Actual Lateral Pressure", states that these experiments did not agree with his theory - he was using the equivalent fluid theory. Lt. Hope had also measured the line of separation in fills 10 ft. high to determine the plane of rupture. In these tests, it was very difficult to obtain accurate data, since only the surface effect could be seen. In a gravel bank 10 ft. high, the wedge base varied from 3 ft. 8 in. to 5 ft. 8 in. or an average of 4 ft. 8 in. The natural slope of the material was 1 to 1 1/3 or 37°. The wedge theory would require a base of wedge of 5 ft. 0 in. The value 4 ft. 7 in. corresponds to a $\phi = 40^{\circ}$; again a faily close check. So Baker's objections to these tests are unfounded.(28)

Lt. Hope built several brick walls laid in wet sands, and tested them to destruction. The first wall was 20 ft. long and 1 ft. 11 in. thick. The filling material was ballast, density 95½ lbs., surface slope of 1 to 1 1/3 or 37°. When 8 ft. high, the top of the wall was inclined 1½ in. from the vertical, when 10 ft. high, the wall fell forward in one mass. At this height the wedge theory gives a pressure of 1190 lbs. per ft. length, horizontally acting at about 1/3 h, and, assuming an angle of friction of 37° between the fill and the wall. The overturning moment about the toe is then 2240 ft. lbs. The tipping of the wall probably decreased the pressure, giving a smaller moment than this. The moment of the wall

Lateral Earth Pressure.

was about 2000 lbs. x 11½ in. or 1920 ft. lbs., and the wall failed.

At the 8 ft. height, the overturning moment was 930 ft. lbs. and the moment of the wall was 1530 ft. lbs., so the wall did not fail. The bricks were laid in wet sand, so that a slight displacement may be expected. Baker, by disregarding the vertical component, concludes that the later-al pressure is only half of the value given by the theory.

The second wall had the same amount of material, but differently placed. It consisted of panels, 18 in. thick laid between counterforts 27 in. thick. After a heavy rain, the panel bulged 1½ in. when the fill was 8 ft. At a height of 12 ft. 10 in., the bulging was 4½ in. and there was an overhang at the top of 7½ in., and after several hours the wall fell. Baker computes the moment of this wall to be 2600 ft. lbs. per foot of length. Disregarding the counterforts, the wedge theory gives a horizontal pressure of 1960 lbs per ft., and a vertical pressure of 1480 lbs. acting on the back, 1.5 ft. from the toe. The overturning moment is then 1960 lbs x 4 ft.3 1/3 in. less 1480 lbs. x 1.5 ft = 6180 ft. lbs. per foot of length. Assuming that the counterforts have decreased the pressure by 10 per cent, and also aid stability by holding the earth between them as a part of the wall, the overturning moment is 7550 ft. lbs. for the horizontal component, less 2000 ft. lbs. for the vertical wall to move 2.15 ft. in back of the position when the wall is vertical. The overturning moment on a vertical height of 21.5 ft. (the wall has a back slope which really decreases this pressure, but we may disregard the amount of decrease) is 24,000 ft. lbs., assuming as before a vertical component. So the wall is theoretically safe, and stood. Baker's conclusions on these tests are therefore of little value. As stated above, his greatest error was in disregarding the vertical component on the back of the wall. Hope's experiments really give good experimental proof of the truth of the wedge theory.

10. Gen. Burgoyne (1834) - J. Owen (1845)

Gen. Burgoyne (28) built four walls of rubble masonry filled with loose mold at Kingstown. The report of these tests is given by J. Owen, (480) Commissioner of Public Works of Ireland, in 1845. Each wall was built 20 ft. high and 20 ft. long, but four different types were used. The average thickness in each was 3 ft. 4 in. or 1/6 of the height. The filling material weighed 87 lbs. per cu. ft. The tests were conducted during heavy rains. Baker claims that the rain increased the density to 112 lbs., but it is doubtful whether it could be over 100 lbs. per cu. ft. when saturated. The slope was not determined; but it was probably about 2 to 3 or 34°. We can assume that the angle of friction between the walls and the fill was also 34°. (See fig. 6.).

Lateral Earth Pressure.

Wall A was with parallel sides, sloping 2 2/5 in. per foot, or 1 to 5.

Wall B had a vertical back and the front was battered 1 to 5.

Wall C had a vertical front and the back was battered 1 to 5.

Wall D had both faces vertical.

Wall A was completed on Oct. 20, 1834, and the fill kept at the 20 ft. level till Dec. 1st, 40 days, showing no signs of weakness. Assuming the masonry to weigh 140 lbs. per cu. ft., the moment of stability of this wall was 20 ft. x 3.33 ft. x 140 lbs. x $(\frac{3.33}{2}+\frac{20}{10})$ft. = 34,200 ft. lbs. To find the earth pressure, use the general wedge formula for a horizontal fill with a negative batter on the back of the wall.

$$E = \frac{1}{2}\gamma h^2\left(\frac{\cos(\phi+\alpha)}{(n+1)\cos\alpha}\right)^2\frac{1}{\cos(\phi'-\alpha)}\ ; \quad n=\sqrt{\frac{\sin(\phi+\phi')\sin\phi}{\cos(\phi'-\alpha)\cos\alpha}}$$

where γ = 100 lbs. ϕ = 34° α = 11°20' E = 3620 lbs.

h = 20 ft. ϕ' = 34° n = 0.756

Horizontal component is E $\cos(\phi'-\alpha)$ = 3340 lbs. per foot of wall.

Vertical component is E $\sin(\phi'-\alpha)$ = 1390 lbs. per foot of wall.

The moment of the lateral pressure is $3340 \times \frac{20}{3} - 1390(3.33+\frac{20}{10}) = 22300 - 7400$ or 14,900 ft. lbs, and the wall is very stable with a factor of safety of over 2.

Wall B. after completion showed some slight fissures and the batter was decreased by 2½ in. This slight tipping probably relieved the pressures behind the wall, and the wall stood. This wall had an average width of 3 ft. 4 in., a top width of 1 ft. 4 in., and a base of 5 ft. 4 in. Its moment about the toe was 32,400 ft. lbs. The earth pressure on the vertical back was $\frac{1}{2} y h^2 \tan^2 (45° - \frac{\phi}{2})$ horizontally, and H tan ϕ' vertically.

$H = \frac{1}{2} \times 100 \times 400 \times (.532)^2 = 5660$ lbs., acts at $\frac{20}{3}$ ft. from the toe.

$V = H \tan \phi' = 5660 \times .675 = 3820$ lbs., acts at 5 ft. 4 in. from the toe.

The overturning moment is $5660 \times \frac{20}{3} - 3820 \times 5.33 = 17,350$ ft. lbs., and the wall has a factor of safety of 1.9, and stood.

Wall C failed when the fill had been raised to 17 ft. Failure was by overturning, the top overhung 10 in. just before failure. The wall burst at 5 ft. 6 in. from the base, about the third point of the fill, and the upper portion then fell vertically. The moment of this wall about the toe was 17,450 ft. lbs. For this type of wall, with positive batter,

$$E = \frac{1}{2} y h^2 \left(\frac{\cos(\phi - \alpha)}{(n+1) \cos \alpha} \right)^2 \frac{1}{\cos(\phi' + \alpha)} \; ; \quad n = \sqrt{\frac{\sin(\phi + \phi') \sin \phi}{\cos(\phi' + \alpha) \cos \alpha}}$$

$n = 0.867$; $E = 5250$ lbs. per foot of length

$H = E \cos(\phi' + \alpha) = 3680$; $V = E \sin(\phi' + \alpha) = 3730$.

The overturning moment is $3680 \times \frac{17}{3} - 3730 \left(1.33 + \frac{8}{3}\right) = 6000$ ft. lbs. and the wall is stable for overturning. However, along a plane at 1/3 h, the wall must overcome a shear of 3680 lbs. per foot width, hence the bulging and failure.

Wall D overturned as a unit when the fill was 17 ft. high. The overhang at the top just before failure was 18 in., then the wall "fell like a board." Its moment was 15,550 ft. lbs. The overturning moment was due to the pressure of 17 ft. of fill against a vertical wall, or 17,500 ft. lbs., and the wall failued by overturning. All of these walls acted consistently with the requirements of the wedge theory.

Baker in discussing these tests claims that these walls had a factor of safety of 2 even in the cases where failure occurred. There is no necessity for such assumptions, as is shown above, because the wedge theory explains the results in a very satisfactory fashion.

11. Gen. Pasley (circum. 1845).[28]

Gen. Pasley constructed model retaining walls, 26 in. high, and 3 ft. long, of various shapes, at Chatham. The filling material was shingle, density 89 lbs. per cu. ft., natural slopes of 1½ to 1. The walls weighed 84 lbs. per cu. ft. Each wall was pulled over before and after backfilling. The difference was taken as the measure of the overturning moment of the fill. The average of several hundred experiments for a vertical wall 8 in. thick was a difference of pull of 20 lbs. at the top of the wall. Assuming the resultant to act at the third point, this corresponds to a total pressure of 60 lbs., acting horizontally, or a moment of 20 lbs. x 26 in. = 43.5 ft. lbs.

Lateral Earth Pressure.

Taking $\phi = 37°30'$; $\gamma = 89$ lb.; $h = 26''$.
$\phi' = \frac{9}{10}\phi = 33°$; $w = 84$ lb.; $b = 8''$.

the wedge formula gives: $H = \frac{1}{2}\gamma h^2 \tan^2 \frac{1}{2}(90° - \phi)$

$$= \frac{1}{2} \cdot 89 \cdot \left(\frac{26}{12}\right)^2 \tan^2(26°15') = 51 \text{ lb.}$$

$$V = H \tan 33° = 33 \text{ lb.}$$

The moment of the earth pressure is $51 \times \frac{26}{12} \times \frac{1}{3} - 33 \times \frac{8}{12}$
$= 14.8$ ft. lbs. per foot of wall.

For 3 ft. of wall, the moment would be 44.4 ft. lbs., a good check of the theory. Baker gives the report of the above tests, and by disregarding the vertical component discovers that the theory gives about three times the actual pressure.

12. Gen. Cunningham (circum 1850) (28)

Gen. Cunningham tried to correct Pasley's results by using weighted model revetments 30 in. high, with a density of 129 lbs. per cu. ft. He also experimented with counterforted walls of larger heights. Baker gives very little data for these tests, and comparisons with theory is impossible, unless numerous assumptions are made.

13. Alex. Collin (1846) (92)

Collin made a study of the nature and extent of earth slips, occurring without outside influences. He concludes that rain has no effect. A wedge of rupture exists but the rupture of parts is not due to any pre-existing planes. That is, the mass will fall up to a definite surface, curved like a cycloid, but closely approximating a plane, called the plane of rupture.

14. Aude (1848).
French Engineers.

Aude[22] constructed an apparatus to measure the moment of the earth pressure. A miniature wall, 0.4 m. (15¾ in.) high, rested on rollers to prevent friction and bore against a bent lever to measure the moment of overturning (see figs. 7, 8). The length of fill was 1 meter. His tests included some inclined walls and also some inclined fills. He concludes that the total pressure acts at 1/3 h, for the average of all his readings is 0.34h. He proves the existence of a prism of rupture. For the case of a vertical wall and horizontal fill, his results are from 0.810 to 0.881 of the values given by Coulomb's theory. If he had taken into consideration the wall friction, he could have explained this low value of the experimental results. Domerque[174] developed a graphical construction of the wedge of maximum pressure from the results of Aude's experiments. Considere' (1870)[124] showed that Aude's results agreed very closely with the wedge theory, showing the resultant pressure to be inclined to the wall at an angle, equal to the angle of friction between the fill and the wall.

Lateral Earth Pressure.

15. Col. Michon (1863).(14)

Baker describes Michon's experiment with a special type of wall in considerable detail. The wall (see fig. 9) was 40 ft. high and consisted of a slab 1 ft. 8 in. thick, battered 1 to 20, with inverted buttresses in back, acting as counterforts. These were spaced 5 ft. apart on centers, were 1 ft. 8 in. thick, 2 ft. 4 in. deep at the base of the wall, and 9 ft. 2 in. deep at the top. The construction was very poor, of field stones set in lime. The work was during heavy rains. When the filling had been carried upto 29 ft., there was a slight bulge. When the filling was piled 3 to 4 ft. above the wall, failure occurred. The fall was preceded by a general dislocation of masonry at the base, a bulging at one third of the height and a slight movement of the top towards the bank. The lower portion of the wall fell outwards, the upper part dropped vertically. Baker attributes failure to the flexure of the thin wall at the center of pressure. Failure certainly was not due to overturning. The counterforts were so closely spaced that the earth between them can safely be regarded as part of the wall. We then have a wall 4 ft. wide at the base and 10 ft. 10 in. at the top. By this method, Michon showed that his experiment checked the wedge theory. Baker refuses to accept these assumptions and concludes that this test shows a lateral pressure of about 1/3 that given by theory.

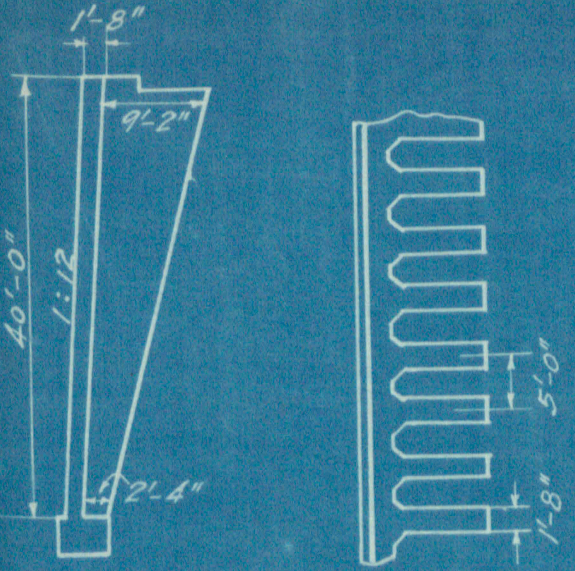


Fig. 9
COL. MICHON'S WALL.

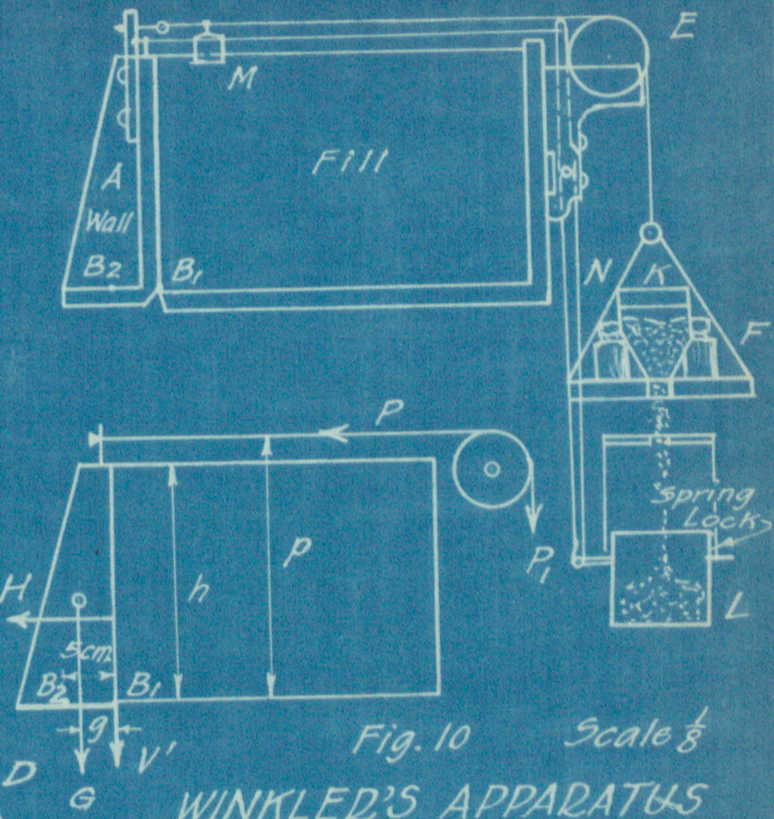


Fig. 10 Scale 1/8

WINKLER'S APPARATUS
(1863)

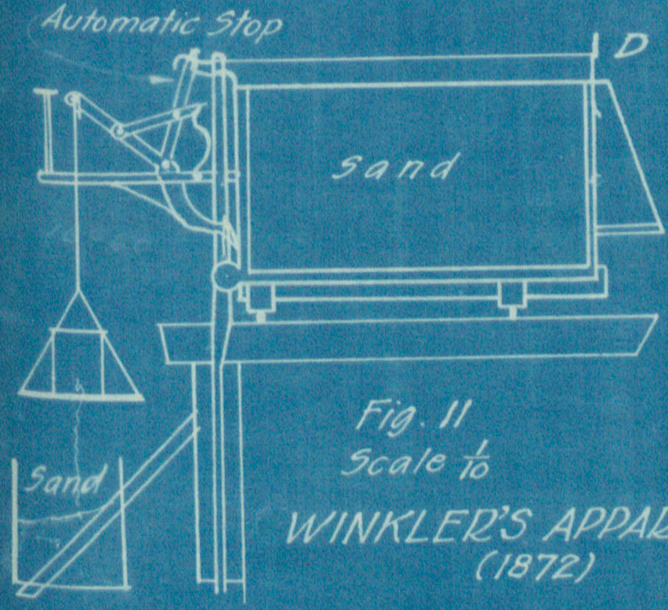


Fig. 11
Scale 1/10

WINKLER'S APPARATUS.
(1872)

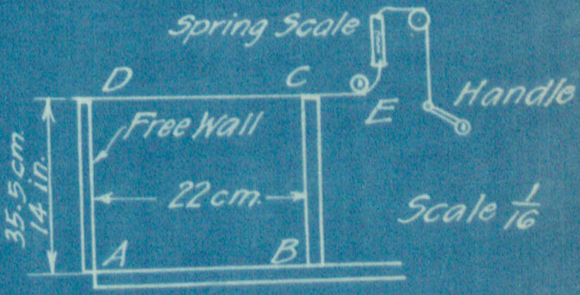


Fig. 12
DARWIN'S APPARATUS.

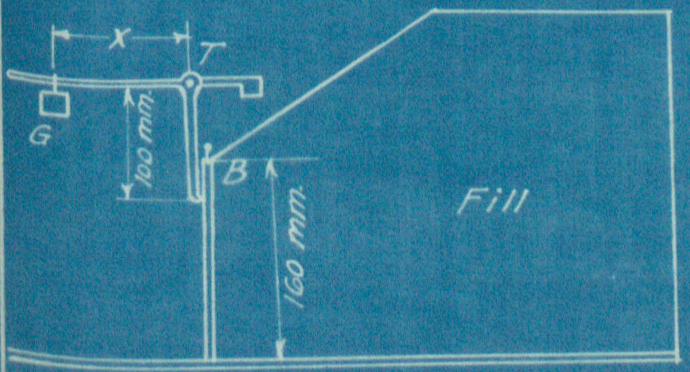


Fig. 13
STRUKEL'S APPARATUS.

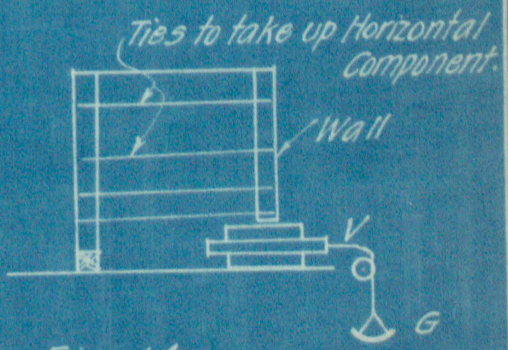


Fig. 14
ENGESSER'S APPARATUS.

Jacob Feld 1922

Lateral Earth Pressure.

16. E. Winkler (1863-1872). (649-652)
Vienna Polytechnicum.

The Vienna Polytechnicum, about 1825, had received royal aid to complete the earth pressure problem. The work done by de Koszegh in Vienna was due to this interest taken by the government. In 1863, Winkler constructed another apparatus at the Polytechnicum. His bin (see fig. 10) was 25 cm. high and 40 cm. long and wide (9.85 x 15.75 x 15.75 in.). The front wall was not attached to the bin, but could rotate about two axes B_1 and B_2. By allowing it to rotate about B_1, the horizontal component only was measured. By allowing it to rotate about B_2, both components were measured. The distance between the two axes was 5 cm. (1.97 in.). The overturning moment in each case was measured by the stress, P, in the string tied to the top of the wall and passing over a pulley in the back of the bin. A known amount of sand, enough to counterbalance the pressure on the wall was placed in the box K, which had a small orifice in the bottom. After the filling in the bin was complete, the sand in K was let out into a box L. As soon as motion of the wall started, a spring lock was automatically sprung, by the lever N being pulled by the wall, and the box L was closed. The remaining weight in K then measured the pressure exerted. In this manner, Winkler was sure to obtain the active pressure of the fill in the bin. The device was very ingenious and worked well. Outside of the objection to the size of the apparatus, which Winkler fully realized, there is also the fact that two tests had to be performed for the complete determination of the pressure. One, using axis B_2, gave the total effect; the second with axis B_1 gave the effect of the horizontal component alone. It might be expected that his determination of the vertical component would be in error, at least, not very accurate.

Winkler realized that his readings were being decreased by the resistance of the side walls. He repeated his tests with a thin partition wall inserted into the bin at the middle of the width. In this way he obtained readings which were affected by four side walls. The difference between the two sets he called the loss due to two side wall resistances. This method would be reliable if the width of the bin were so large that there would be no chance of arching action. With his center wall, the width was less than 20 cm. (7.85 in.) and there certainly was an extra loss due to arching of the fill between the walls. The loss in horizontal component per side wall was 0.065.

The result of several sets of tests showed results which checked the Coulomb theory quite well, for the case of horizontal fill and vertical wall. There was a vertical component equal to .45 of the horizontal, corresponding to an inclination of 24°30' of the resultant. The angle of friction on the wall was experimentally detemined as 29°40'. The loss is easily accounted for by taking into consideration the arching action and the double method in running tests. For a sand of specific gravity 1.405 (88 lbs. per cu. ft.) and a natural slope of 33°35', he obtains H =(114.8) (1.405 kg.) for a wall 1 meter high and 1 meter long. Changing the system of measurements to lbs. and ft. this corresponds to

$H = \frac{1}{2} \gamma h^2 (.292)$; The theory gives $H = \frac{1}{2} \gamma h^2 (.289)$

In 1872, Winkler built another apparatus (see fig. 11) of the same size and based on the same principle as the older apparatus. This newer device measured the horizontal component only. The same method was used to measure the pressure. He finds that the measured pressures must be multiplied by $(1 + 0.116 \frac{h}{b} \tan \phi')$ to give the actual values, corrected for the side wall resistances. <u>h</u> is the height of the test wall, <u>b</u> is the width, ϕ' is the angle of wall friction. The results are somewhat smaller than those given by the Coulomb theory, but afford a fair check.

17. <u>Casmir Constable</u> (1874).
American Society of Civil Eng.

Constable began his researches with a box 16 in. high, 24 in. long and 16 in. wide, with glass sides and a test wall made up of 1 in. by 1 in. by 2 in. pine blocks. The blocks were laid wet to prevent slipping. The fill was oats, having a natural slope of 30°. The test wall failed by bulging at about half the height. (125)

Lateral Earth Pressure.

It was found that the friction of the walls was insignificant, so the width of the box was reduced to 12½ in., the test wall being now 12 in. wide. Peas were used as a filling material. The natural slope was 28°. Constable decided on this material for fill, to "give the effect of a high wall". Just how he came to this conclusion is doubtful. The effect on the wall was the same as in the larger apparatus, but occurred more suddenly. He therefore concludes that the plane of repose of the fill should be drawn thru the toe and not thru the heel of the wall. He bases his theory upon this assumption. He computes two formulae, one assuming the prism of rupture to start at the heel, the second, assuming the prism of rupture to start at the intersection of the back of the wall with a plane of repose drawn thru the toe of the wall. If b = base width of wall, h = height of the wall, ϕ = angle of repose, γ = weight of the fill, and of the wall,

(I) Ordinary wedge theory: $b = 0.57 \cot \frac{90-\phi}{2}\sqrt{\frac{w}{\gamma}}$

(II) Constable theory : $b = 0.53 \cot \frac{90-\phi}{2}\sqrt{\frac{w}{\gamma}}$

Table of b for various values of h:

H	Experimental b	Wedge Theory	Constable's Theory
5 in.	2.1 in.	2.46 in.	2.28 in.
7 in.	3 "	3.44 "	3.15 "
10 in.	4	4.90 "	4.40 "
12½ in.	5	6.00	5.51 "

Baker's comment on those "toy" experiments is that in cases where a slight vibration or jarring can cause great variations in results, no conclusions can be drawn.

18. J. Curie (1873) (158,159)
French Academy of Science.

Curie's apparatus was a wall, 1 meter by 1 meter, hinged at the top and the bottom moving in an arc of a circle. The wall could be placed in any position. Bricks piled up in front of the wall showed the required thickness.

The material used was sand, density 1.555 (97½ lbs. per cu. ft.) with a natural slope of 33°30'. The angle of friction on the wall was also 33°30'. The wall weighed 106½ lbs. per cu. ft.

Number of tests.	Inclination of wall.	Nature of Sand.	Base Width. New theory.	Old theory.	Experiment.
1	55°	dry	0.565	0.423	0.556-.545
1	55°	humid	0.553	0.423	0.548
2	27°30'	dry	0.462	0.395	0.462-.450
2	4°	dry	0.350	0.350	0.450 } too
2	0	dry	0.324	0.335	0.450 } large.

The humid sand had a natural slope of 42°30', density 86.5 lbs. The new theory takes into account the wall friction. From these tests Curie concludes that neither theory is correct. The evidence for such a conclusion seems rather weak.

19. George Howard Darwin (1877).(130)
Institution of Civil Engineers, London.

During the summer of 1877, Darwin performed numerous experiments to completely solve the problem of earth pressures. His apparatus (see fig. 12) consisted of a box 22 cm. long and 35.5 cm. high and 30 cm. wide (8 2/3 x 14 x

Lateral Earth Pressure.

12 in.). The test wall was hinged at the base, and held by a cord at the top. This chord was connected to a spring scale, reading horizontally. To insure that the active pressure was determined, the spring scale was pulled as far as the wall would allow, and tightened. After the fill was in place, the scale was slowly released, and the reading taken just as the wall started to give.

The tension in the string, $T = \frac{Ex}{35.5}$, where E is the horizontal pressure, x is the height of the pressure, 35.5 cm. is the height of T. The material used was sand with a natural slope of 35°. After completing the tests a 1.3 cm. partition (½ in.) was inserted in the middle of the bin, and the tests repeated. The results showed an apparent loss in width of 2.5 cm. The clear width of the box without partition was 30 cm; with partition 28.5 cm. So that each side has an effect of decreasing the actual width by 0.5 cm.

There are several objections to this apparatus. The short box does not allow a full plane of repose, being $1\frac{1}{3}$ times as high as long; the use of a partition wall cuts the width of the bin into two parts, each less than 6 in. wide. The loss due to arching in such a small width is considerable. Darwin takes no account of this.

Sand in horizontal layers, horz. surf.,	$T = 0.0344\ h^3$ grs.
" " " " - shaken-" "	$T = 0.0285\ h^3$
" " layers sloping away from wall h.surf.	$T = 0.0315\ h^3$
" " " " towards w. - h surf	$T = 0.0360\ h^3$
" " " " away fr.w., final surface as far below the horiz. as possible. (35°)	$T = 0.028\ h^3$

Lateral Earth Pressure.

Sand in horizontal layers, final surface as far above the horizontal as possible (35°) $T = 0.055\ h^3$

The Coulomb and Rankine theories both give a coefficient of 0.270 for the cases of a horizontal fill. The above results correspond to

0.180 for the ordinary fill
0.132 for the shaken fill
0.165 for the fill sloping away from wall
0.189 for the fill sloping towards wall

But in computing these coefficients, Darwin has disregarded the losses due to arching action, and also the effect of the friction along the back of the wall in increasing stability, so no real comparison can be made.

Darwin made a noteworthy contribution to the solution of this problem in his statement: "Coefficient of internal friction is very different in different parts of the mass, and is a function of the pressure and of shaking."(282) He made no attempt to investigate this.

21. Philip Forchheimer (1882).(250)
Austrian Ingenieur und Architekt Verein.

Forchheimer's investigation was to determine the angle of the wedge of rupture. His box was very small, 84 mm. high, 13.3 mm. wide and 170 mm. long (3.3 x 5.25 x 6.7 in.). The pressure are was 46 mm. high and 12.6 mm. wide (1.81 in. x 4.95 in.). Let β be the angle between the horizontal and the plane of rupture. For Rheinsand (very fine sand), using smooth and rough boxes, he obtains $\beta = 60^\circ$, a little smaller than $\frac{1}{2}(180^\circ + \phi)$. For $\alpha = -70^\circ$ to 0, (negative sloped walls) and a horizontal fill, $\beta = 62^\circ$. In general he finds that

$$\cot\beta = \tan\tfrac{1}{2}(180^\circ + \phi) + \tan\alpha.$$

α = angle of wall from vertical. If ω is the wedge angle,

$$\tan\omega = \cot\frac{\phi}{2} + \tan\alpha.$$

22. M. A. Gobin (1883) Ponts et Chausees.

Gobin set out to experimentally show the accuracy of the Wedge Theory and the impossibility of the Rankine Theory. He conducted a complete set of tests with sand, using sloped and level fills against inclined and vertical walls. The front gate was free and the pressures measured by weights connected to the wall by strings. One string ran horizontally, the other over a pulley held the wall vertically. His fill was 0.50 meter (20 in.) high. He concluded that his experiments were no check on the theory because of the side wall effect. Some of his results are (276) as follows:

A horizontal fill of 0.50 meters against a vertical wall gave a horizontal component of 19.1 kg.; theory requires 25.74 kg.

A fill 0.35 meters high (13.8 in.) and with a surface slope of 14° above the horizontal exerted a horizontal force of 12.15 kg. (25.8 lbs.) on a vertical wall. The theory requires 14.5 kg.

A horizontal fill of 0.50 meters against a sloping wall showed 15.79 kg., where the theory required 25.90 kg.

In computing these theoretical values he used a value for the angle of internal friction less than the natural slope. This value was determined by measuring the coefficient of friction of the sand upon a sand coated cylinder which was imbedded and then pulled out.

23. M. Siegler (1887). Ponts et Chaussees.

Siegler's apparatus was the first to be made so that the motion of the wall could be controlled. Even tho it was on an extremely small scale, 9 in. high and 8 in. wide, he obtained some good results. He had four axes of rotation on his wall. He found that if the axis of rotation is at the top of the wall, the resultant is a little higher than the third point; if it is at the bottom of the wall, the resultant is a little below the third point. To measure the vertical component acting on the wall, he used several methods. An attempt to measure the variation in the resistance of coal placed underneath the wall proved to be inaccurate. He tried to measure the change in tension in strings tied to the top and bottom of the wall, but finally used a friction dynamometer. With this he proved the existence of a vertical component. The resultant pressure acts almost at the angle of repose from the normal to the wall. He also found that settling of the fill decreased the vertical component, as was found in the Cincinnati tests. This decrease in vertical component is probably caused by a drying out of the fill along the wall. (√74)

Lateral Earth Pressure.

20. Sir Benj. Baker. (1881). Institution of Civil Engineers, London.

Baker repeated Lt. Hope's experiments, using the same type of apparatus, a test wall 12 in. square, held by a string attached at one third the height. He tested several materials, obtaining the following results:

Material	Natural Slope	Lateral Force (horizontal)
Ballast- wet	1 1/2 to 1;(33°40')	7 lb.
Ballast and Sand	1 to 1	3 lb.
Ballast and Sand, with vibration.	1 1/4 to 1	4 lb.
Greasy coal	1 3/4 to 1	4.0 lb.
Schistose Rocks, ten experiments.	1 1/2 to 1	5.1 lb.
Large sized Coal	1 3/4 to 1	6.0 lb.

Baker's values are much lower than those found by Lt. Hope. The scale of the experiments is so small, that no conclusions can be drawn from them. Coarse material seems to give smaller pressures than fine material, but this may be caused by the small size of the bin.

Lateral Earth Pressure.

24. M. Strukel (1888)

Oesster. Ing. u. Arch. Verein.

Strukel constructed two types of apparatus, the first to measure the horizontal component, the second to measure the vertical component. In the former, see Fig. 13, he had a vertical wall 160 mm. (6.3 in.) high, hinged at the base and resting against one arm of a bent lever. A weight G travelled on the other arm, which was horizontal. The distance BT was 100 mm., the force at B was equal to 1/3 the total horizontal component, if we assume the resultant to act at 1/3 the height. If x is the distance between the rider G and the pivot T, the horizontal force $H = 3/100\ Gx$. He assumed the resultant to be inclined an angle ϕ from the normal, then the value of the resultant $P = H \sec\phi$.

All the tests were repeated with a thin partition wall inserted in the middle of the bin. The correction for each side wall was found to be 2/150 or $1\frac{1}{3}\%$. The objection to this procedure is given in the discussion of Winkler's work.

His results are summarized in the ratio:
$H_1 : H_2 : H_3 = 1:1.56:3.01$, where H_1, H_2 and H_3 are the horizontal components of the pressure of fills against a vertical wall when the top surfaces are respectively inclined at ϕ below the horizontal, level and inclined at ϕ above the horizontal. These values have been corrected for the side wall resistances.

The second apparatus was made later, and was in-

Lateral Earth Pressure.

tended to determine the direction of the resultant pressure. The box was cubical, a side being 0.3 meters (11.8 in.). One of the walls was not connected to the box, but held in place by four ropes horizontally and a friction dynamometer vertically. For a horizontal fill of 0.1, 0.2 and 0.3 meters, the vertical component was 0.3, 0.8 and 1.8 kg. respectively. He noted that the vertical component could be decreased by filling the bin in horizontal layers rather than throwing the fill into the bin in any manner.

Altho the tests are on a very small scale, note that the last two vertical components 0.8 and 1.8 kg are in the ratio of the squares of the corresponding heights of fill 0. 2 and 0.3 meters. This shows that the vertical component is a constant function of the horizontal component.

He later devised still another apparatus where the vertical and horizontal components were both measured by friction methods. For a height of 120 mm. (4.7 in.), and surfaces at ϕ below the horizontal, level and at ϕ above the horizontal, he obtains the values of the horizontal component as

H_1 : 1.77 - 1.92 kg;) Theoretical values if the
H_2 : 2.82 - 2.96 kg; (resultant is assumed at ϕ
H_3 : 5.33 - 5.55 kg;) and horizontal are

	$\phi' = \phi$	$\phi' = 0$
H_1:	1.45	2.09
H_2:	2.04	2.78
H_3:	3.74	4.66

The theoretical values give results which are not checked by the experiments. The variations are different.

Lateral Earth Pressure.

The experimental results are in the ratio - 1:1.56:2.94.
The theory where $\phi'=\phi$ requires the ratio - 1:1.40:2.58.
The theory where $\phi'=0$ requires the ratio - 1:1.33:1.68.

25. Wm. Cain (1888).(104)
Univ. of North Carolina; A.S.C.E.

Cain has done considerable work in comparing experimental results with the various theories, and in discussing the type of apparatus best fitted for the accurate determination of the earth pressure. He advises the use of a light test wall, at least 5 ft. high. In smaller models, side wall effects and cohesion play too great a part.

26. Wm. Bland (1890).(44)
London.

Two sets of tests were performed by Bland in order that he could base his recommendations for the construction of Piers and Buttresses upon experimental results. The first tests were with small wooden walls, to determine the stability of various sections. The second tests were to determine the lateral pressure of granular materials.

The walls were built up of wood bricks, 8 to the pound, each being 1 in. x 1 in. x 4 in. Weights were put on the top and a weight attached to the bricks over a pulley. The latter gave the resistance of the wall to horizontal pressure. These tests are discussed in detail in his book on Arches, Piers and Buttresses. Very little can be learned from them. Failure usually occurred because the units separated.

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To determine the lateral pressure of granular materials, he used a cubical box, 8 in. edge, and closed one side by building up walls out of his wooden units. His filling material was peas.

His first experiment is with a wall 1 in. thick. When 4 in. high, a fill of 3 in. overturns it. When 6 in. high and loaded with a 2 lb. weight, making a total weight of 2⅔ lbs., a fill of 3 2/3 in. was just held. When 8 in. high, and loaded with a 6 lb. weight, making a total of 7 lbs., a fill of 8 in. was just held. Another wall, 2 in. thick and 8 in. high required a load of 1 lb. on it, the total weight then being 3 lbs., to retain a fill of 8 in. So he concludes that by doubling the width of a wall, less than half the weight is sufficient to maintain the outward pressure.

By stepping the bricks, he obtained the effect of sloping walls. A wall, 1 in. thick, weighing 1 lb. and inclined 15° to the vertical held a fill of 8 in. When inclined 23° to the vertical it held the fill plus a 2 lb. weight placed on the fill. The size of these tests is so small, that no real conclusions may be drawn. The angle of natural slope of the peas is given as 36°.

27. N. E. Youkovsky (1890). (660-1)
Moscow Scientific Society.

Youkovsky was the first investigator to use a manometer or pressure gauge (seitenroehre) to measure the lateral pressure of granular materials. He was investigating the effect of the water content upon the lateral pressure of sand. He connected a thin tube to the side of a box. When the box was filled with water, the level in the tube and in the box were the same. In the case of sand, he measures the difference in level. For a 2 mm. dry sand, he obtains his maximum difference in level: 38 to 40 cm. As water is added, this difference decreases, showing that water increases the pressure.

28. Ad Donath (1891). (172)
Technische Hochschule in Berlin.

Donath conducted experiments on the determination of the lateral pressure of earth in 1889 at Charlottenberg. The apparatus consisted of a box 60 x 60 x 110 cm; this, as he says, being the maximum convenient height and size for experimental purposes. The sand used weighed about 1320 lb. The test-gate was hinged at the bottom, and the amount of overturning moment was obtained by measuring the pressure of a piston fixed to the gate against a cylinder of mercury. The cylinder was connected to a vertical mercury tube by means of which the pressure was read. The amount of rotation of the gate could be governed to 14 sec. of arc, the exact value being measured with a microscope. He notes that all the previous types of apparatus have had some amount of motion of the test-gate, and thereby concludes that his apparatus must also have motion in order that readings be obtained. By using two different axes of rotation, he obtains a value for the vertical component on the wall. To determine the effect of the side walls, the method of inserting a center partition was employed.

Lateral Earth Pressure.

The following table gives the values of the rotating moment for various amounts of rotation of the wall.

With Center Wall. Rotation	Moment	Without Center Wall. Rotation	Moment
0° 0' 14''	715	0° 0' 17''	886
28''	681	29''	851
42''	657	42''	811
1' 03''	632	1' 57''	786
1' 17''	610	1' 06''	738

Because of the continuous variation, he concludes that the actual moment for practically zero movement of the wall would be 762 and 960 for the cases of with center wall and without center wall respectively.

The average of six sets of tests gave values of 745 and 936 for the two cases. The sand used had a natural repose of 33°42', tan ϕ being 0.667. The coefficient of friction on the wall was 0.666, so that he assumes $\phi' = \phi$. The horizontal pressure E was 35.29 kg., using the average experimental value for the moment.

The formula $E = \frac{1}{2} \gamma h^2 \tan^2 \frac{1}{2}(90-\phi)$ gives 49.79 kg. Assuming that the resultant acts at an angle of ϕ' to the wall normal, E = 45.39 kg. and the horizontal component is 37.77 kg. These tests were for a height of 60 cm.

Likewise for a height of 48 cm., experiment gives H = 21.70 kg., the Coulomb-Rankine formula for this case (vertical wall and horizontal fill) gives E = H = 31.42 kg., and the Poncelet formula gives H = 23.86 kg. He found practically no vertical component.

He therefore concludes that the lateral pressure is horizontal, but its amount is much closer to the value of the horizontal component as given by the general wedge theory than the value given by any other theory.

29. Fr. Engesser (1882-1893). (201-2)
Karlsruhe.

The experiments performed in 1881 were an investigation into the pressure of earth on a tunnel. To give this effect he built a box 20 cm. square and 40 cm. high. In the bottom, there was left an opening 4 cm. wide and 20 cm. high, and closed by a board larger than the opening. By means of a lever and a can of water hung at the other end, this board was kept in place. Enough water was placed in the can to overcome the pressure on the board. The box was then filled with sand, and water taken away till the point of balance was reached.

Engesser very explicitly describes the procedure of these tests. He was careful to measure the pressure from a point of over-balance, thereby recording the active and not the passive pressure. His tests were conducted with the idea of deriving formulae for the design of tunnels. In order to eliminate the side wall effects, the opening in the bottom was made only 1/5 of the entire width. The sides of the box were of glass so that movements in the mass could be detected. He noted that the point of balance was reached quite suddenly, and as the plate under the opening sank a little, a thin stream of sand flowed out. There was very little movement in the sand mass itself.

The experiments reported in 1893 were on the investigation of the lateral earth pressure. Donath had announced that there was no vertical component. Engesser constructed an apparatus, (See fig. 14) to detect the presence of a vertical component if such existed. The ties N take up the horizontal forces on the wall while the force required to pull out the block V measures the vertical component. This method using a friction dynamometer was not new. He concludes that the wedge theory is experimentally checked, the direction of the resultant, as far as he could determine, was at an angle between 0 and ϕ from the normal. All his tests were with a vertical wall.

30. M. L. Leygue (1885).
Ponts et Chaussees.
(Eng., Travaux de l'Etat)

In a 220 page report in the "Annales des Ponts et Chaussees," M. Leygue announces the results of the most complete tests on the Lateral Pressure of Earth conducted up to the present day. The report is in three sections:-

Part 1 - Verification of hypotheses concerning cohesion, the deformation of the wall, the direction, point of application and magnitude of the lateral pressure, and a comparison between theory and practice.

Part 2 - Comparison of Empirical formulae with experiments. Semi-empirical and theoretical formulae. Boussinesq's formulae.

Part 3 - Application to retaining wall design.

Coulomb had attempted to prove the coexistence of friction and cohesion. Ardant had refused to admit this hy-

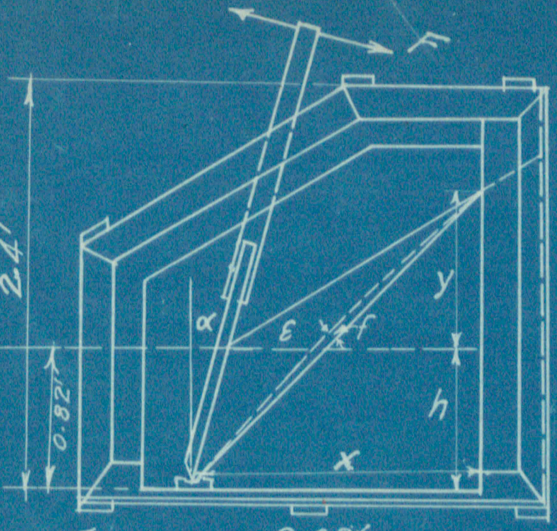

Fig. 15a

LEYGUE'S APPARATUS

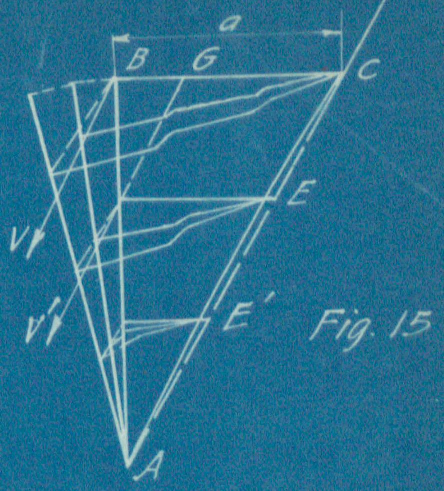

Fig. 15

Changes in Shape of Fill caused by Wall Rotation. (Leygue)

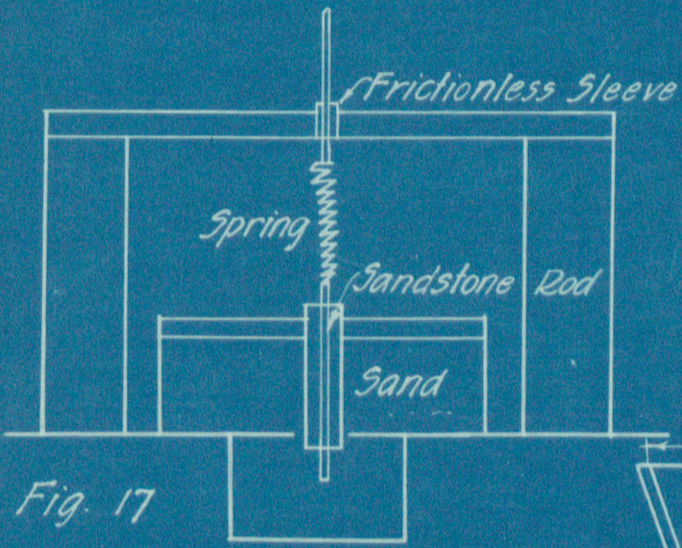

Fig. 17

ENGEL'S APPARATUS.

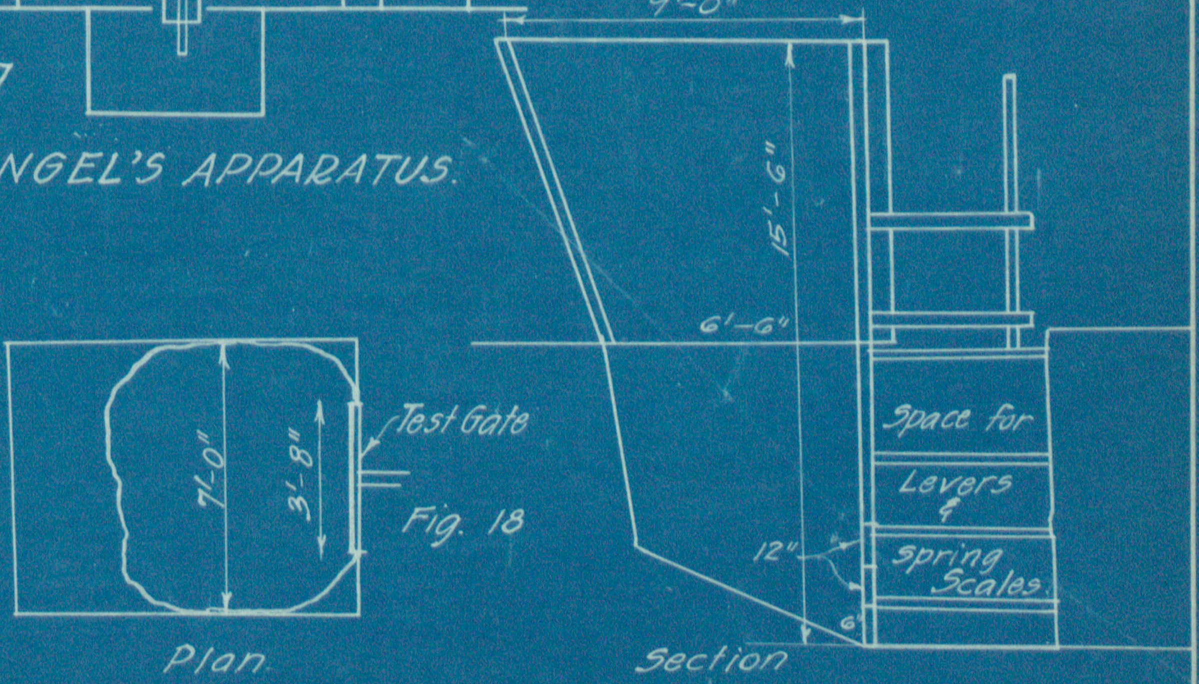

Fig. 18

A. A. STEEL'S APPARATUS.

Jacob Feld 1922.

Lateral Earth Pressure.

pothesis, because he maintained friction can only act when there is motion, and cohesion cannot act when there is motion. In the first part of this report, Leygue experimentally proves the correctness of Coulomb's idea.

The apparatus consisted of a box of earth, 144 mm. wide, 30 mm. high and 500 mm. long (5.65x1.18x19.6 in.) so hinged that it could be tipped at any angle from 0 to 45° to the horizontal. A smaller box 100 mm. square and 95 mm. high (3.94x3.94x3.75 in.) had frictionless wheels attached to the sides, so that it rolled on the sides of the larger box. The smaller box was bottomless. The test consisted of measuring the angle of inclination of the larger box when the smaller box just started to roll down. Various materials were filled into the boxes and the force measured was the resistance to rupture of the material in the box across an area equal to the section area of the smaller box.

The moving box, or "chariot", had a cross section of .01 sq. meter (15.5 sq. in.). Let

s = horizontal cross section area of the chariot.
h = height of earth in chariot; y = density of the earth
P = Weight of chariot without contained earth.

c = coefficient of cohesion for unit area
f = coefficient of internal friction.
K = coefficient found by experiment, the fraction of the earth weight which is lost by the friction of the chariot walls, and which therefore is transmitted to the walls of the box and does not act on the earth in the box.

β = angle at which the empty chariot moves uniformly
θ = angle at which the earth starts to break, i.e. the chariot starts to move; then if we admit Coulomb's hypothesis, the equation of equilibrium is

$$(Syh + P)\sin\theta = P\sin\beta + Sc + fKSyh\cos\theta$$

$$\text{or} \quad C = \frac{P}{S}(\sin\theta - \sin\beta) + yh(\sin\theta - Kf\cos\theta).$$

Lateral Earth Pressure.

According to Ardant's hypothesis, before rupture occurs, $f = 0$ and therefore $C = \frac{P}{S}(\sin\theta - \sin\beta) + yh\sin\theta.$

and after rupture $c = 0$, so $f = \frac{1}{K}\left[\tan\theta + \frac{P}{Syh}\left(\tan\theta - \frac{\sin\beta}{\cos\theta}\right)\right]$.

To find K, the chariot was lifted from the table, and weighed. The ratio of the weight of the earth which did not fall out to the weight of the earth put into the chariot was taken as K. The weight of the chariot P was 0.125 kg. (.255 lbs) when not loaded. It was found that the friction of the wheels reduced the effective weight of the chariot and the contained earth by 5 per cent. If the effective weight of the chariot is denoted by $F = P - 0.05\,(P + p)$ where p is the portion of the earth in the chariot which is held by the side walls,

$$K = \frac{S\,y\,H - F}{S\,y\,H}.$$

The following table gives the values for K:

h	Syh	P	F	K
.03 meter	0.433 kg.	0.100 kg.	0.088 Kg.	0.79
.06	0.866	0.250	0.230	0.73
.09	1.300	0.480	0.450	0.65

Four tests are reproduced below. A different material was used in each, and various loads were placed on the chariot. To compute the value of c from Coulomb's hypothesis, a value for f is assumed. The values of c and f from Ardant's hypothesis are also given.

Leygue's Cohesion and Friction Tests.

Sand, y:1.840, approx. f:0.70;
Load on chariot: 0

h(met.)	$\tan\theta$	C (Coulomb)	Ardant's Hyp. c	Ardant's Hyp. f
.03	.566	7.17	26.41	.958
.06	.524	7.16	43.40	.840
.09	.469	7.28	56.72	.801

Fine sand, y:1.440, f :0.85
Load on chariot:1 kg., P:1.143

h	$\tan\theta$	C (Coulomb)	Ardant's Hyp. c	Ardant's Hyp. f
.03	.578	40.40	72.7	1.911
.06	.610	40.00	102.7	1.391
.09	.660	40.20	132.6	1.226

tan / 0.016

Sand & Soil, y:1.640, f:1.70
Load on chariot: 2 kg. P:2.143

h	$\tan\theta$	C (Coulomb)	Ardant's Hyp. c	Ardant's Hyp. f
.03	.458	31.04	107.04	2.569
.06	.663	31.02	170.35	2.075
.09	.815	31.32	225.78	1.976

$\tan\beta = 0.012$

Humid Soil, y:1.320, f:1.65
Load on chariot:3.7kg. P:3.843

h	$\tan\theta$	C (Coulomb)	Ardant's Hyp. c	Ardant's Hyp. f
.03	.440	90.65	149.69	4.409
.06	.525	90.12	204.78	2.912
.09	.600	90.81	242.30	2.598

$\tan\beta = 0.029$

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These experiments conclusively prove the coexistence of frictional and cohesional resistances.

The next study was the shape of the prism of rupture, and the effect of various movements of the wall, shape of fill and slope of wall upon the prism. The apparatus consisted of a wooden bin 1.3 ft. long, 2.6 ft. wide and 2.4 ft. high, open in front and with glass sides. The test wall was pivoted at the base. To determine the movements in the fill, the sand was placed in thin layers, each layer being separated from the next by a very thin stratum of white plaster. The breaks in these strata as seen thru the glass walls gave an indication of the lines along which motion occurred. See fig. 14.

To show the theoretical values of the deformation of the earth mass due to movements of the wall, Leygue calls h = height of wall; $f = \tan \phi$ = friction coefficient earth on earth

b = width of wall; $f' = \tan \phi'$ = friction coefficient earth on wall.

Normal component of lateral pressure $N = n y h^2$.

Tangential component of lateral pressure $T = t y h^2$.

$$f' \left[b h w + n y h^2 (f \cos\alpha - \sin\alpha) \right] \geqq n y h^2 (\cos\alpha + f \sin\alpha).$$

$$\text{or } b \geqq n \frac{y}{w} h \left[\cos\alpha \left(\frac{1}{f'} - f \right) + \sin\alpha \left(\frac{f}{f'} - 1 \right) \right].$$

where α = angle of wall back from vertical

y = density of earth; w = density of wall.

If we assume $f = 2/3$ and $f' = 3/4$, $\frac{y}{w} = 0.80$ and determine the value of n experimentally we obtain a practical formula for the width of a retaining wall. To find the values for n

Lateral Earth Pressure.

he first determines the variation in the prism under various influences.

In the general cases of fill and wall slope, he concludes that the surface of rupture is not a plane, but a convex curve. If angle of surface slope, x and y are the coordinates of C, the point of intersection of the surface of rupture with the surface of the fill; DC is drawn at the natural slope thru C, cutting the back of the wall AB into two parts hz and $h(1-z)$, see fig. 15; then AOD equal OBC

$$z : 1-z :: \sqrt{\tan\phi - \tan\varepsilon} : \sqrt{\tan\phi}\text{, or } z = \frac{1}{1-\sqrt{\dfrac{\tan\phi}{\tan\phi - \tan\varepsilon}}}.$$

$$x = \frac{zh}{\tan\phi - \tan\varepsilon} = \frac{h}{\tan\phi - \tan\varepsilon + \sqrt{\tan\phi(\tan\phi - \tan\varepsilon)}}$$

$$y = x\left(\frac{\tan\phi}{2} + \sqrt{\tan\phi\left(\frac{\tan\phi}{4} + \frac{1}{x}\right)}\right) - 1.$$

$$\frac{dy}{dx} = \frac{\tan\phi}{2} + \frac{x\tan\phi + 2}{2x\sqrt{1 + 4 \div (x\tan\phi)}}$$ the formula for a vertical wall.

The following values are given:

$\tan\varepsilon$	z/h	x/h	y/h	dy/dx
0	0.50	0.75	0	0.889
1:3	0.41	1.23	0.41	0.889
1:2	0.33	2.00	1.00	0.75
7:12	0.26	3.12	1.82	0.75
2:3	0.00	Indeterminate		0.66

The experiments showed $\frac{dy}{dx} = 1$, in place of 0.889 to 0.66.

When y was 0, x was 0.662.

By a similar process when the wall is inclined at an angle α from the vertical, he obtains the formula

$$\frac{dy}{dx} = \frac{\tan\phi}{2} + \frac{\tan\phi(x - \tan\alpha) + 2}{1/2\sqrt{(x - \tan\alpha)^2 + 4/\tan\phi}}.$$

$$z = \frac{\sqrt{\tan\phi(\tan\phi - \tan\varepsilon)(1 - \tan\alpha\tan\varepsilon)} - (\tan\phi - \tan\varepsilon)}{\tan\varepsilon(1 - \tan\alpha\tan\varepsilon)}$$

$$x = \tan\alpha + \frac{\sqrt{\dfrac{\tan\phi(1 - \tan\alpha\tan\varepsilon)}{\tan\phi - \tan\varepsilon}} - 1}{\tan\varepsilon};\quad y = \frac{\tan\phi(x - \tan\alpha)}{2} + \sqrt{\frac{\tan^2\phi(x - \tan\alpha)^2}{4} + x\tan\phi} - 1.$$

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If $\tan \varepsilon = \tan \phi$, $z = 0$. $x = \infty$, $dy/dx = \tan \phi$

the following table of values is given:

$\tan\alpha$	$\tan\varepsilon$	x	y	dy/dx
-1:3	1:2	1.987	1.161	0.76
0	1:2	2.000	1.000	0.75
+1:3	1:2	1.987	.827	0.74
+1:2	1:2	1.964	.732	0.73
+2:3	1:2	1.932	.633	0.72
+3:2	1:2	1.500	.000	0.66

For a horizontal fill, any type of wall, let ab be the width of the wedge of rupture and bc be the distance from the rupture to the plane of repose intersection with the top surface; let $h = 1$ $\tan \varepsilon = 0$, $y = 0$,

$$x = \frac{1 + \tan\alpha \tan\phi}{2\tan\phi}, \qquad \frac{dy}{dx} = \frac{4}{3}\frac{\tan\phi}{\tan\alpha \tan\phi}.$$

$$\frac{ab}{ac} = \frac{x - \tan\alpha}{\cot\phi - \tan\alpha} = \frac{1 - \tan\alpha\tan\phi}{2(1 - \tan\alpha\tan\phi)} = \frac{1}{2}.$$

i.e. the width of the prism bounded by the wall and the plane of repose is bisected by the surface of rupture for the case of a horizontal fill. The following table shows how the experimental results agreed with the above formulae:

$\tan\alpha$	$\tan\varepsilon$	Value of x		dy/dx		Ratio ab:ac	
		Formula	Exper.	Formula	Exper.	Formula	Exper.
-1:3	0	0.583	0.552	0.960	1.00	0.50	0.48
0	0	0.750	0.662	0.889	1.00	0.50	0.44
+1:3	0	0.916	0.786	0.827	1.00	0.50	0.39
+1:2	0	1.000	0.856	0.800	1.000	0.50	0.35
+2:3	0	1.083	0.933	0.774	1.00	0.50	0.32
+3:2	0	1.500	1.500	0.666	1.00	0.50	0.50

For a horizontal fill, the above formula only held true for the extreme case of the wall sloping 3:2. The surface of rupture in each case had a slope of 1:1.

The following table gives the area of the wedges of rupture in sq. meters. for all cases, for a height of

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wall of 1 meter; $\phi = 33^{\circ}37'$.

	Area of Wedge.			Ratio of Area to the Area when $\alpha = 0$.		
$\tan\alpha$	$\tan\beta = 0$,	1:2	3:2	$\tan\beta = 0$	1:2	3:2
-1:3	.394	.853	1.338	1.36	2.94	4.61
0	.290	.558	.818	1.00	1.93	2.82
+1:3	.201	.322	.454	.69	1.11	1.56
+1:2	.161	.236	.319	.56	.81	1.10
+2:3	.127	.172	.220	.44	.59	.76
+3:2	.000	.172	.220	.44	.59	.76

In all these tests, it was found the area of the prism did not depend on the nature of the wall movement, whether rotation or translation, nor on the amount, up to a limit of about 30° of rotation. The particles in the wedge moved in lines parallel to the surface of rupture. The influence of the shape of fill on the prism of maximum pressure is negligible.

To determine the influence of the wall friction on the shape of the prism of rupture, he ran two tests, identical in method, except that in the first he used a wooden wall with a coefficient f' of 0.810, and in the second, a glass wall, with a coefficient f' of 0.456. The table gives the areas of the prisms in sq. meters for a height of 1 meter.

		Area of Prism		Ratio of the two Areas.
$\tan\alpha$	$\tan\beta$	f':0.810	f':0.456.	
-1:3	0	0.394	0.366	1.08
	2:3	1.338	1.236	
0	0	.290	.271	1. 07
	2:3	.812	.756	
+1:3	0	.201	.186	1.06
	2:3	.454	.418	
+1:2	0	.161	.151	1.05
	2:3	.319	.295	
+2:3	0	.127	.119	1.04
	2:3	.220	.210	
+3:2	0	.220	.210	1.00
	2:3	.220	.210	

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He therefore concludes that the wall has but little effect on the shape and size of the prism of rupture.

To determine the influence of the nature of the fill upon the size and shape of the prism of rupture, he ran two tests with sand (density 1.430, slope 2:3) and millet seed (density 0.740, slope 1:2). The following table resulted. Areas are for h = 1 meter.

tan α	tan ε	Area of Wedge. Sand	Millet seed	Ratio.
-1:3	0	0.394	0.537	1.36
	1:2	.853	1.649	1.92
0	0	.290	.405	1.40
	1:2	.558	1.150	2.06
+1:3	0	.201	.275	1.36
	1:2	.322	.729	2.26

He ran these tests for several heights of fill, and found that the areas of the wedges were always in the ratio of the squares of the heights.

To show the similarity between wedges under different conditions of fill and wall, Loygue computed for a height of unity,

x, the distance of the highest point of the wedge from wall
y, the height of the highest point of thewedge above top of wall.

tan ε	tan α	y	x	r	tan ε	tan α	y	x	r
0	-1:3	0	0.552	.088	1:2	-1:3	0.885	1.434	.120
	0	0	.662	.053		0	.662	1.524	.077
	+1:3	0	.786	.031		+1:3	.453	1.237	.047
	+1:2	0	.858	.022		+1:2	.350	1.216	.033
	+2:3	0	.933	.015		+2:3	.267	1.200	.033

tan ε	tan α	y	x	r
2:3	-1:3	1.770	2.322	.150
	0	1.324	1.986	.096
	+1:3	1.161	1.692	.059
	+1:2	.906	1.574	.040
	+2:3	.716	1.437	.040

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The table also gives r, the maximum deviation of the observes surface of rupture from the line drawn between the heel of the wall and the highest point of the wedge.

This exhaustive study of the nature of the prism of rupture may be summarized as follows:

The surface of rupture is not a plane, but a slightly convex curve. It is independent of the height of fill, of the surface of the wall, of the inclination of the wall, and of the nature of the surface of the fill. It is dependent upon the material itself, the surface of rupture approximately bisecting a horizontal line drawn from the top of the wall to the plane of repose.

The shape taken by the fill as the wall was rotated is shown in diagram 15. This figure is merely one of a large number given in Leygue's report. A good summary of results obtained and discussion thereof is found in Professor Cain's book (1908 ed.).

Using the same apparatus as before, he then set out to investigate the magnitude of the lateral pressure. A string was tied at the top of the gate, 2.46 ft. from the toe, and the lateral pressure balanced by weights hung from the string. When the wall was inclined, the moment was balanced by the pull of a rope strung vertically. To determine the side wall friction, Leygue used the partition wall method. He notes that the fill often distorted the shape of this central wall, probably due to its arching action. Without the center wall, the force F required to hold the wall balances the lateral pressure across a width d, less

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two wall friction losses, 2x. With the center wall of thickness t, the measured force F$'$ balances the lateral pressure across a width d-t, less four friction losses, 4x. He does not consider the arching effect between walls.

$$F : F' = d - 2x : d - t - 4x .$$

His correction factor to take into consideration the wall friction depended upon the shape of the fill. This was to be expected, because of the different areas of wall in contact with the fill. The coefficient is given as 0.135(7-tanε). The following table gives the coefficients of (yh^3) to give the moment of the lateral pressure for sand. (y= 89 lbs., slope 2:3) and millet seed (y= 46, slope 1:2). The writer also inserts N, the coefficient of $(\frac{1}{2} yh^2)$ to give the normal component, assuming that the point of application was at 1/3 the height, and that the cohesive force was 1 lb. per sq. ft. These values are taken from Cain's discussion.

		Sand.			Millet seed.
tan α	tan ε	m	N(exper.)	N (Cain)	m
+3:2	0	0.535			
	1:2	0.970			
	2:3	~~0.1~~ 1.329			
+1:1	0	0.241			
	1:2	0.425			
	2:3	0.574			
+2:3	0	0.152			
	1:2	0.260			
	2:3	0.356			
+1:3	0	0.062	0.358	0.342	0.084
	1:2	0.103			
	2:3	0.136	0.774	0.534	0.154
0	0	0.030	0.180	0.186	0.057
	1:2	0.047	0.282	0.296	
	2:3	0.065	0.390	0.410	0.106
-1:3	0	0.015	0.086	0.084	0.037
	1:2	0.024	0.		
	2:3	0.032	0.182	0.168	0.063

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tan	tan	m (sand)
-2:3	0	.009
	1:2	.014
	2:3	.018
-3:2	0	.000
	1:2	.000
	2:3	.000

The apparatus was then changed by replacing the test wall by a double frame, consisting of two parallel walls separated by springs, whose compression gave the lateral pressure. Then pulling the wall in its own plane, he obtained the tangential component. He concluded that the lateral pressure acted at an angle ϕ to the normal. He also determined the height of the point of application of the resultant, with this apparatus. The apparatus was far from accurate, only two readings gave a value for the height of the resultant below 0.410 h, the average value being about 0.470 h. Cain tried to obtain a more reasonable value from these experiments by computing the earth pressure from the measured areas of the wedge of rupture. The results so obtained are somewhat low, but much closer to the reasonable values, about 1/3 h and below 5/8 h.

Altho Leygue's tests were with a very small model, the results obtained in his investigations are quite valuable. The magnitude of the lateral pressure being disregarded, he has shown the existence of a tangential component, and completely investigated the nature and variations of the prism of rupture.

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31. Hubert Engels (1896). (191-4)

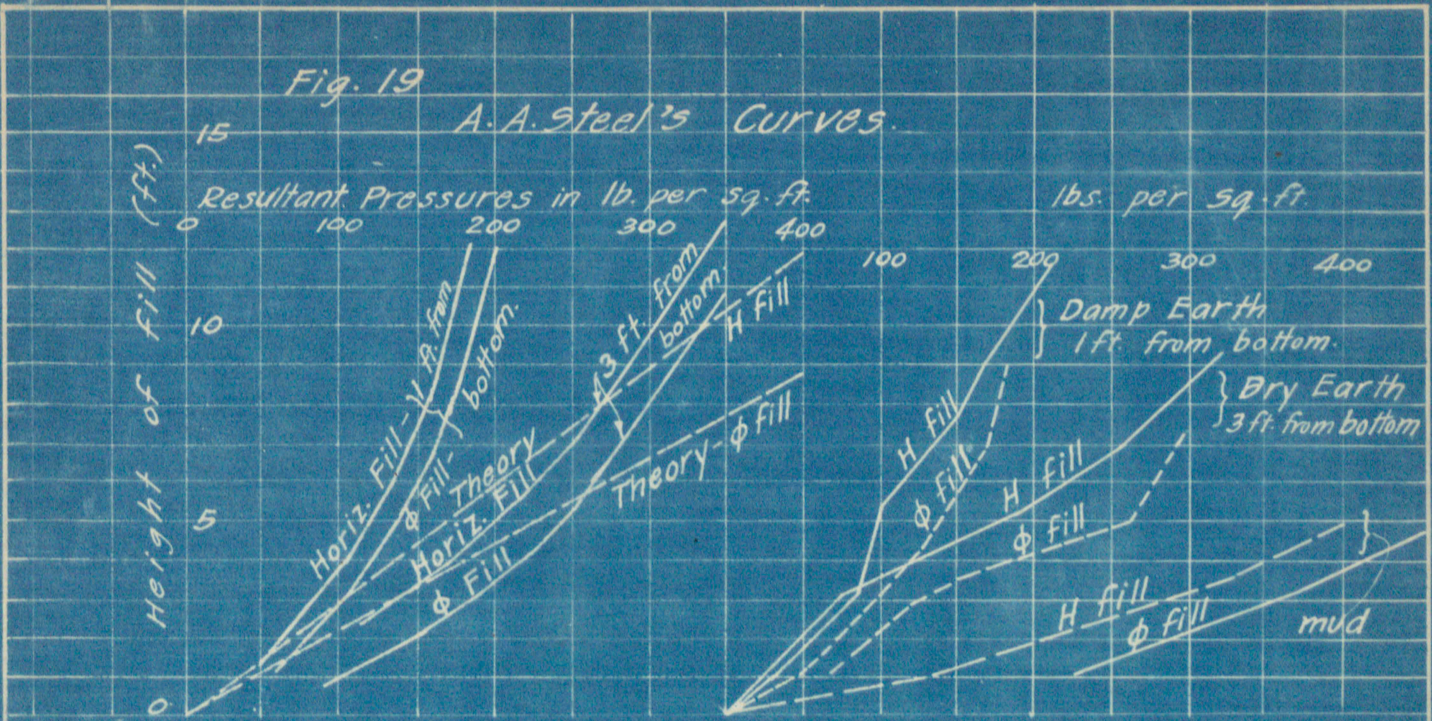
Engels set out to prove that the lateral pressure on a vertical wall was horizontal. He imbedded a circular rod of sandstone in a sand fill and measured the movements of the rod by detecting the deflection of the spring (See fig. 17). By means of an optical micrometer, he could detect a deflection of 0.001 mm. He concludes that when a wall is stable, the lateral pressure is horizontal. If the overturning moment exceeds the resisting moment, then there is motion and a frictional resistance is set up. But such resistance cannot be trusted and should be disregarded in designing a wall.

Tests were also run to investigate the variations in the values of ϕ. The results indicated that ϕ was not a constant but depended upon the height of fill and the method of filling. In tests with sand, he finds that dry sand has a natural slope of 31°, damp sand 40°, saturated sand 29°. To make experiment agree with theory, the value of ϕ in the formulae should be taken greater than the natural slope. His apparatus was of so small a scale that general conclusions cannot be drawn. Using a height of 50 cm. in a box he announces that he has checked Rankine's theory, forgetting that in the special case which he considered, both theories have the same formula. Two of his results are given as follows:

H	Experiment	Theory
49 cm	30.44	30.00
50 cm.	37.00	39.86

32. A.A. Steel (1899.) (598)
University of Nebraska.

The University of Nebraska in 1898 started the ex-



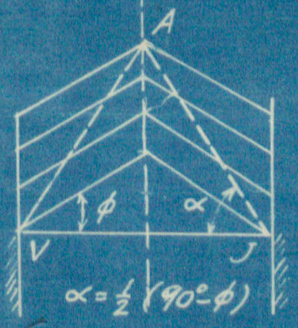
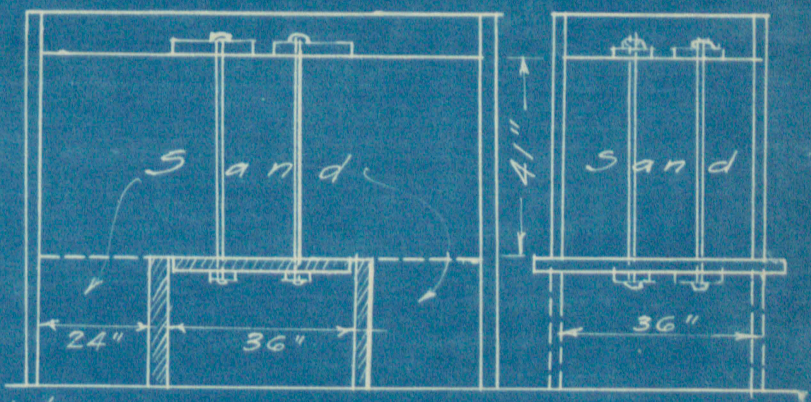
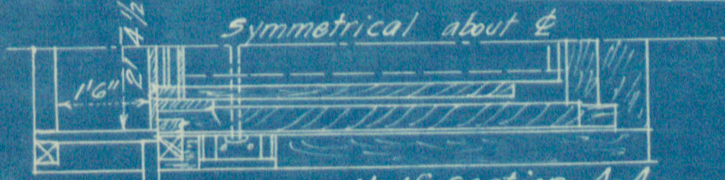


Fig. 23



MEEM'S APPARATUS.
Fig. 22.



Half Section A-A
J.H. SMITH'S APPARATUS.

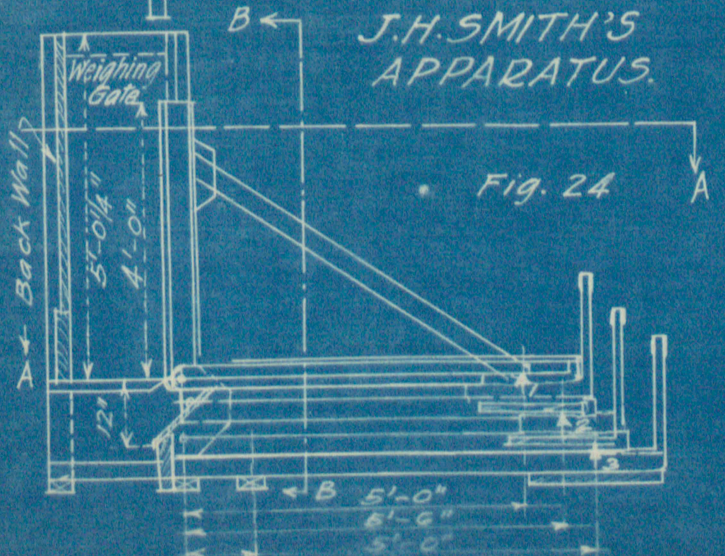
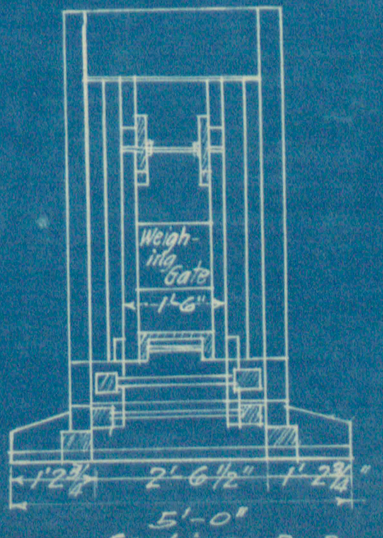


Fig. 24



Section B-B.

Jacob Feld 1922.

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perimental determination of the lateral pressure on a very large scale. The apparatus (see fig. 18) consisted of a pit 7 ft. wide and about 7 ft. deep. A test wall of wood was built to a total height of 14½ ft. There were two openings in the wall, 12 in. high and 12 1/8 in. wide, the lower edges being 6 and 30 in. above the base of the fill. Boards were inserted in these openings, making a close fit, but not so tight as to cause friction. Each piece was held horizontally by a strut which pressed against a lever. To obtain the vertical component, each piece was connected by wires to levers above. All the levers transmitted the forces to spring scales.

Readings were taken for every 2 ft. increase of fill It was intended to use sifted earth, but the amount of labor required soon caused the abandonment of the idea. The first material used had a surface slope of 38°22'. The lower board gave readings which increased slower than the readings of the upper board. This could not be explained, so the test was repeated with dry earth, which had less cohesion. The same result was obtained. In this test, the vertical pressure on the upper board fell 60 lbs. overnight; this did not happen again. The dry earth had a surface slope of 35°29'; 25 per cent of water made a fluid mud of it.

The curves shown in diagrams 19 give the intensity of total pressure on the two test areas under various conditions. The first set of curves show the relation between the pressure of dry earth for horizontal fill and for a fill at the angle of repose as measured on each test area with the theoretical values. The resultant is taken as acting at

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$28^{\circ}10'$ from the normal, this being the average value found in the tests. The theoretical formula used is from the general wedge theory, taking into account wall friction. The oblique surface readings at each height were obtained by filling in the extra material after the horizontal surface reading had been taken. The fill was then re-arranged for the next height with a horizontal surface. This method gave two sets of readings with one filling, a great saving of labor, but not a very uniform method of running a pressure test.

The fill was just wide enough to allow a full plane of rupture. No attempt was made to loosen the earth in back of the bin; the effect ofthe solid material in decreasing the pressure was not considered. Such binding action is the probable cause of the decreased readings on the lower test area. The width of the wall was 3 ft. 8 in; the pit was 7 ft., so that there was no side wall resistance.

In the Cincinnati tests, the sudden drop in vertical component over night was also found. It was carefully investigated, and is discussed in the report of the experiments. Steel notes that it was probably an accident, and ran the rest of his tests continuously, thereby preventing such changes.

The second set of curves show the relation between the pressures recorded for damp earth, dry earth and mud. Steel's conclusions are:

1. There is a large vertical component; the angle of inclination of the resultant was $28^{\circ}10'$.
2. The pressure is not a straight line function of the depth.

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3. The presence of solid earth considerably reduces the pressure. The second conclusion is not warranted, especially from the results in the second set of curves, which average fairly well to straight lines.

Assume that the total pressure is given by formula

$E = \frac{1}{2}\gamma h^2 \tan^2 \frac{1}{2}(90° - \phi_1) \sec\phi'$ (horizontal fill)

Assuming a straight line variation, the rate of increase of pressure, or slope of the curve, is $\gamma \tan^2 \frac{1}{2}(90° - \phi_1)\sec\phi'$.
The following table includes the experimental values for the density, natural slope and wall friction, as given in the report, the slope of the curve as taken approximately from the curves, and the required ϕ_1, to satisfy the above equation.

Material	Natural Slope	Density	Slope of curve	Req'd ϕ_1
Damp Earth	38°22'	83.5 lbs.	20 lbs./ft	40°40'
Dry earth	35°29'	78	33	27°20'
Mud	- -	75 (assumed)	80	0°50'
		80 "	80	1°50'

It is unfortunate that more care was not taken in conducting these tests. Had the pit been made larger, especially at the base, better results would have been obtained because the binding power would have been eliminated.

33. Heinrich Mueller-Breslau (1906).(461) Charlottenberg.

In a very detailed and comprehensive chapter in his book on "Erddruck", Mueller-Breslau describes the theoretical foundation on which he designed his earth pressure apparatus. He shows that for a general "Free" body, such as the test-wall should be, there are six equations for equili-

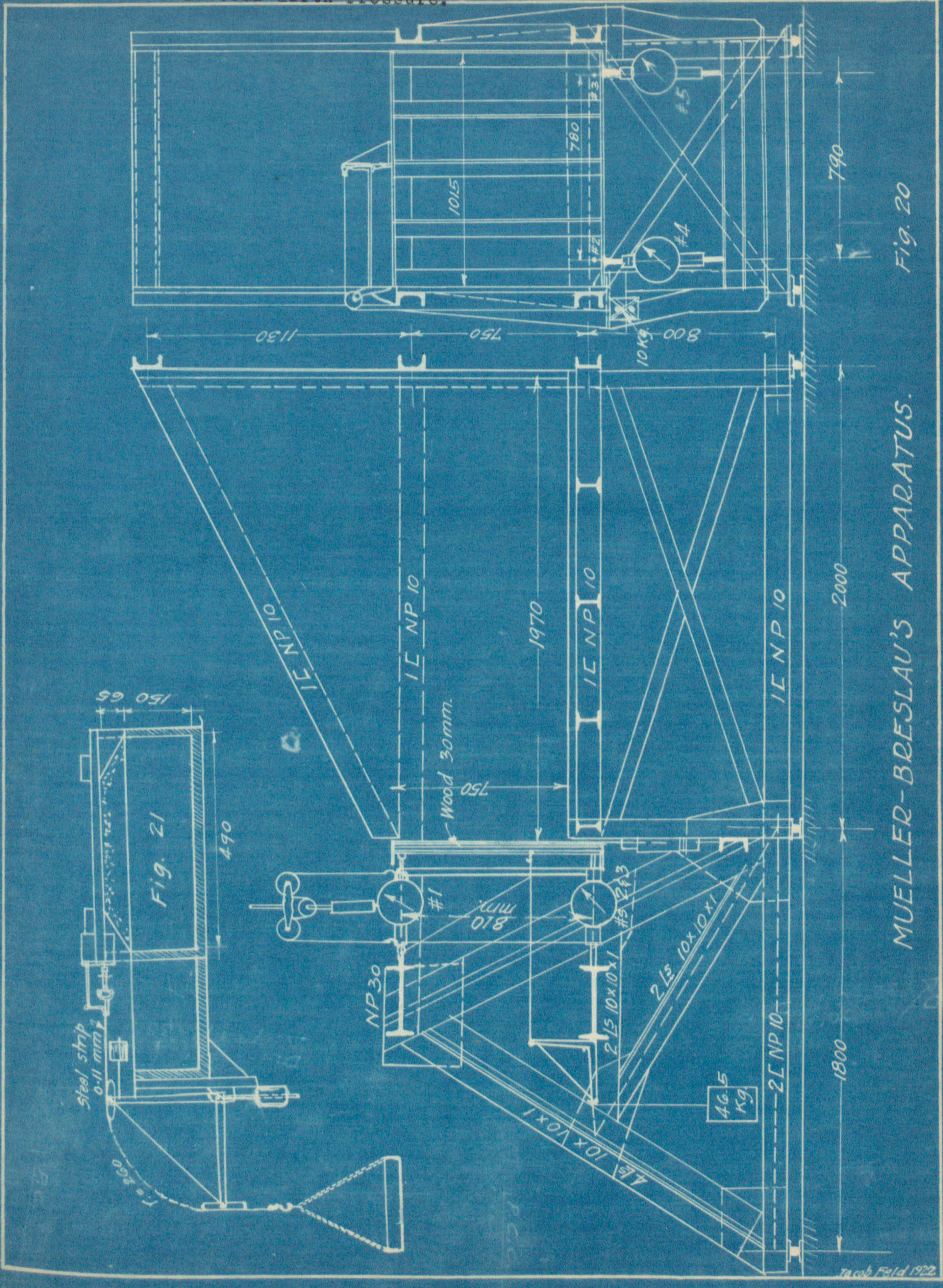

Fig. 20

MUELLER-BRESLAU'S APPARATUS.

brium, the summations of the forces along the three rectilinear axes and the summations of the moments about these three axes. In such a case the wall must have six supports and these must so be spaced that there will be but one solution to each equation. The general resultant of the earth pressure may be resolved into three components, two in the plane of the wall and perpendicular to each other, one vertical and one horizontal, and the third component normal to the wall. By assuming that the horizontal component in the plane of the wall ~~is nil,~~ which amounts to the assumption that the fill is homogeneous, the number of necessary supports is cut down to three. For convenience, two of the three required supports are made double, i.e. two parallel reactions to take the component in a certain direction. The method of support used in the Cincinnati apparatus is based directly upon this discussion.

The description of the apparatus and plan of work is: 'An attempt has been made to reproduce the actual action of the retaining wall under earth pressure. A wall resting on elastic supports is slowly back-filled with earth. The initial stresses, due to the weight of the wall, gradually increase due to the earth pressure. The amount of this pressure is measured on stress meters (Spannungsmessern), which record information regarding the earth pressure existing at the time of taking readings. This method enables a continuous observation of the existing earth pressure.'

He shows that the apparatus may be used to determine the influence of surcharge, changes caused by moving loads, by water content, shock, etc., while the changes are occurring.

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The work up to 1906 had been chiefly preliminary, many years are necessary for this work, 'for the subject is one dealing with time-robbing continuous experiments, which must be conducted on as large a scale as possible, if they are to give reliable data concerning the earth pressure occurring in practice.'

Each contact is provided with a measuring device, consisting of two slightly bent leaf-springs of cast crucible steel, 16 mm. wide and 5 mm. thick. The dimensions are so chosen that a compression of s in the length of the instrument (385 mm.) corresponds to a lateral expansion of the double spring of 12.5. This expansion is transmitted to a calibrated indicator which gives the pressure in kilograms. The apparatus is a very delicate and detailed affair. It was equipped with stops in all directions and various adjustments. The horizontal measuring units were counterbalanced at all times. Careful calibration was necessary. The spring struts were all tested for a strain-stress equation and found to average an axial deformation of 0.064 mm. for a load of 100 kg.

As a receptacle or bin for the sand whose lateral pressure is to be measured a box, open in the top and front, 1015 mm. wide and 1970 mm. long was used (40 in. by 77.5 in.) The sides of the box were covered with sheet steel rubbed with graphite to diminish friction. The sides increased in height at an angle of about 30° towards the back of the box. (See fig. 20). The test wall is a wooden slab 30 mm. thick, 740 mm. high (29 in.), reinforced by angle irons, and provided with steel bearing plates and lined with emery cloth. For tests with a smooth surface, a piece of plate glass 6 mm. thick was fastened to the rough surface by means of corner pieces. The sand in the bin was handled by a crane of 3/4 cu. meter capacity. The report also mentions a larger apparatus which was being constructed but no details are given.

The material used was a fine sand, which was dried for several months. It's density was 1.58, on the average. When poured it was 1.56 and when rammed it rose to 1.61. The natural slope was quite constant, 32°; this was to be expected from the uniformity of the sand. To determine the coefficient of friction between the sand and the wall, a special apparatus (see fig. 21) was constructed. A weighted board faced the same way as the test-wall is drawn over a level mass of sand by means of shot placed in a pan. The average value of the coefficient of friction is 31°08'. There was considerable trouble in obtaining consistent values in these tests. The coefficient was determined by taking the force required to start motion. Since the test wall was backed with emery cloth, this is practically the coefficient of internal resistance between sand and sand.

It was very difficult to determine the side wall resistance. It was not considered feasible to use the center partition method because of the difficulty in obtaining a uniform and even fill in each half of the bin. Mueller-Breslau states that the side walls not only influence the amount, but also the height and direction of the resultant. The coefficient of friction on the side walls was found to be 0.25, or an angle of friction of 14°. It is interesting to note that this was less than

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the coefficient of friction on the plate glass wall, where the angle of friction was found to be 22°. By means of a theoretical discussion, he obtains a value for the retarding effect of each side wall, but decides not to use the result, because the actual values are probably not so large.

Assuming that the horizontal component of the pressure of a horizontal fill on a vertical wall is given by the formula $E = \frac{1}{2} y h^2 \tan^2 \frac{1}{2}(90° - \phi)$, the effect of each side wall on the lateral pressure, $\Delta E = R \cos(90° - \omega - \phi') = R \sin(\omega + \phi')$, where ω angle of the wedge of rupture, ϕ' is the inclination of the resultant earth pressure from the horizontal and R, the frictional resistance on each wall, is $(11m + 0.028\ sm)$, m being the distance of the highest point of the wedge from the wall, and s being the load per sq. ft. on the surface of the fill. R acts in the direction of the plane of rupture. The correction is a function of the shape of the fill, because of the different areas in contact with the wall.

The tables of results and observations are omitted because of their great length. A translation of the entire report can be found in the Master's Thesis submitted by the writer to the University of Cincinnati (1920).

Mueller-Breslau gives the following conclusions; a discussion of the points by the writer is given with those requiring discussion.

1. "The measured earth pressures were always greater than those calculated by means of Coulomb's theory, when assuming a plane surface of sliding. This result ought not be surprising, for in the theoretical section (see Mueller-Breslau's theory)

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we proved that there exist non-planar surfaces of sliding which require greater wall resistances than plane section." The average results for dry sand, vertical and and sand coated wall were:

Density of fill: 1.58 Height of fill = 29 in. (750 mm.)

Natural slope: 32°; $\phi' = 31^\circ$

Experimental Results:

ε	Total E	Angle of Inclination.	Height of E	Theoretical Value of E (General wedge theory)
-32°	91 kg.	26°	0.31 h	89 kg.
-16°	113	26°	.33	98
0°	134	27°	.36	124
$+16^\circ$	195	28°	.375	155

If we assume the formula $E = 1/2\ \gamma h^2 \sec\phi' \tan^2 \frac{1}{2}(90 - \phi)$, for the case $\varepsilon = 0$, we obtain E = 153 kg, the horizontal component is 136 kg. If in place of $\phi = 32^\circ$, the natural slope, we had used the value for the angle of internal resistance (not determined by the experimenter), the theoretical values would have been larger and closer to the experimental. For example if we assume $\phi_1 = 30^\circ$, a very probable value of the angle of internal resistance, the general wedge theory gives 95, 108, 132 and 171 kg. respectively, fairly close to the experimental results.

2.' The earth pressure makes an angle of about $\frac{3}{4}\phi$, but in the cases of surcharge, the value often fell to $\frac{1}{2}\phi$. The surface slope has no effect on the inclination of the resultant."

With the emery cloth-backed wall, the coefficient of friction gave an angle of 31°, while the average ratio of vertical to horizontal components gave an angle of 27°. With the glass-backed wall, the values were 22° and 21° respectively.

Mueller-Breslau disregards the evident conclusion, that the inclination of the resultant is the angle of friction on the wall. The amount of the pressure (horizontal component) is not affected appreciably by the nature of the wall.

3. "Alternate loading and unloading causes the elevation of the resultant to rise." This phenomenon was also noticed in the Cincinnati experiments, where a maximum value was also obtained.

4. "A single load of 735 kg. placed across the entire width of the bin and placed just outside the theoretical plane of rupture doubled the pressure of a level fill, decreasing the ratio of the vertical to horizontal components from 0.46 to 0.28. The maximum pressure occurred directly after placing the load on the fill, and decreased about 10 per cent in 24 hours, slowly falling about 2 per cent in the next six days. Therefore loads outside the wedge do affect the lateral pressure." Exactly the same conclusions were drawn in the Cincinnati tests.

5. The effect of temperature on the pressure of a fill is shown by the following readings, taken in successive days:

Temperature (Centigrade)	16°	$9^{\circ}.3$	9°	$17^{\circ}.6$
Total Pressure	198	199	195	197

The number of readings is too few to allow any conclusions, but the general results are in agreement with the Cincinnati tests on this point.

6. Some preliminary tests on vibration show that there is no important effect on the amount or direction of the earth pressure caused by either horizontal or vertical vibration of the test wall. The blows were with a 5 lb. hammer, continu-

ously for 5 minutes, using about a 2 cm. (less than one inch) swing. The resultant dropped in value, the horizontal component much more than the vertical. The vibrations were therefore beneficial.

The experimenter also conducted some photographic work with a glass bin, 600 by 1000 by 500 mm. A wooden wall, coated with emery cloth closed one side; the wall was 395 mm. high. This wall could be displaced either by rotating about the toe or moving bodily thru a distance of 6 mm. The displacement was controlled by a hand operated screw. "It was found that the amount of movement of the wall, between the limits of 1 and 6 mm. had a remarkably small effect on the size and boundaries of the moving portions of sand." The author draws no conclusions from the results obtained so far. He merely calls attention to the fact that the border, between the stationery and the moving sand, which is of course somewhat influenced by the friction against the glass wall, is in the neighborhood of Coulomb's line of rupture, and that in both kinds of tests (rotation and translation displacement of the test-wall), the lines of rupture are curved down at the lower end. In the case where a heavy surcharge load was on the fill, the lower end almost coincides with the straight line determined by the foot of the wall and the back end point of the load; the upper portion is curved."

So far, Mueller-Breslau's later work has not been reported on. The writer has been unable to find any further data or information of the work, if any, performed after 1906. In general, the results of these investigations agree closely with the results of the Cincinnati tests, which were more or less modelled upon Mueller-Breslau's plan for a complete set of earth pressure tests. The report ends with the words, "Perhaps these reports will stimulate other Technical Hoch Schule to make similar efforts to get a firm basis for one of the most important, yet least investigated branches of engineering knowledge."

34. Jacquinot and Frontard (1910) (531)

Resal in the appendix of the second volume gives a short account of experimental work performed by Jacquinot and Frontard on the physical properties of the earth in the Charmes Reservoir Dam. The dam was built in 1902 to 1906, and gave no trouble till 1909, when large amounts of the earth started to slip. The material was 60% clay, 32% sandy earth, and 8 % sand. The grains varied in size from 0.5 to 10 mm. diameter, averaging about 3 mm. Assuming Coulomb's law of friction and cohesion, $F = P \tan\phi + C$, where ϕ is the angle of internal friction, F is the force required to start the motion of a mass of earth weighing P, while resting on a plane of similar material, C is the cohesion per unit area. The angle of internal friction was assumed as 8°, corresponding to $\tan\phi = 0.14$. The tests gave a remarkably close check on the Coulomb law:

P (kg)	F (kg)	P tan ϕ	C
3.380	2.400	0.470	1.930
14.550	4.090	2.040	2.050
27.660	5.840	3.870	1.970
34.930	7.080	4.890	2.190

All loads are in kilograms. With greater loads, the material seemed to compact and the value of C increased. The experimenters decided that the percentage of water influenced the coefficient of friction to a considerable extent. For a very wet earth, $\tan\phi$ was 0.14, but inside the mass $\tan\phi$ was 0.17. The dangerous height, when slipping may occur, is

$$H = \frac{c}{y} \frac{\sin \varepsilon \cdot \cos\phi}{\sin^2 \frac{\varepsilon-\phi}{2}} ; \quad \text{in this case} \quad \varepsilon = 33^{\circ}40' \quad \phi = 8^{\circ} \quad y = 1.800$$

The height was found to be 17 m. hence C was 2.750. The critical width is $\beta = \frac{c}{y}\frac{\cos\phi}{\sin(\varepsilon-\phi)}$; the width was found to be 7.5 m. and therefore C was 5.880. In general, only a few of the results were consistent.

35. Professor Charmock (1911)
Bradford Technical School, England.

J.H. Everest in the "Canadian Engineer" for 1911, mentions some small sized experiments performed by Professor Charmock on a 7 in. wall. The resultant pressure was assumed to act at an angle ϕ' to the wall, only the horizontal component was measured. The average of a number of experiments gave 2.72 lbs. for the horizontal component, corresponding to 3.25 lbs. resultant pressure, with $\phi' = 33^{\circ}$. The theoretical value for this case is 4.7 lbs. Very little can be concluded from such small-sized tests.

36. R. J. Mann (1913)
Cornell University

Professor A. P. Mills of Cornell designed an apparatus to determine the lateral earth pressure. It consisted of a 3/8 in. steel plate, 40 in. wide and 60 in. high; the pressures being determined by measuring the deflections. The work was started by Hoffert and Kimber, but was soon abandoned, because the results were in no wise consistent.

Further work was then taken up by Mann and Gons, using a pressure cell. The first report announces that Goodrich's results are checked, and that the work is to be continued. There is no later report. The cell consisted of a spring diaphragm pressing against water, the pressure being recorded outside. The experimenters were considerably troubled by air bubbles and by changes in temperature, both of which brought in large errors in the readings.

37. J. C. Meem (1908-1920) (419-25)
A. S. C. E.

Following the theory which he outlined in his paper in 1908, Meem conducted several experiments to give an experimental backing to his ideas. The first experiment was to determine the value of the arching action in sand fills. Two wooden abutments, 3 ft. wide, 3 ft. apart and 1 ft. high, were built and filled solidly with sand. Two wooden walls, 3 ft. apart and 4 ft. high, were built crossing the abutments and solidly cleated and braced frames placed across the ends about 2 ft. back of each abutment. A false bottom, made to slide easily between abutments and projecting slightly beyond the walls on each side, was blocked up snugly to the bottom edges of the sides, getting a box 3 x 4 x 7 ft. (see fig. 22). Bolts were run thru the bottom and thru 6 x 15 x 2 in. pine washers. The box was then filled with sand and compacted. The bolts were tightened and the blocks knocked from under the bottom.

The result was that the unsupported sand sank 2 in.; it did not fall because of the initial compacting of the sand

and the arching stress. Three hours later, 600 lbs. load was placed on the arch of sand, causing another settlement of 2 in. About an hour later, failure was caused by a slight vibration.

Meem concludes that the arch of sand is not only self-sustaining, but can also hold loads, if the point A falls inside the mass (see fig. 23). The natural slope was about 45°. If l is the span, VJ and is the weight of the material per cu. ft., W is the weight per lin. ft. dead weight on the plane VJ (not necessary for the arch),

$$W = 2\frac{l}{2}\cdot\frac{\tan\alpha}{2}y = \frac{1}{2}yl\tan\{\tfrac{1}{2}(90^\circ-\phi)+\phi\}$$
$$= \frac{1}{2}yl\tan(45^\circ+\frac{\phi}{2}).$$

The second test was to study the nature of submerged sand. The sand in the water did not act as a fluid. To show that water had no influence on the arching action, the first experiment was repeated on a small scale, using saturated sand. The results were the same. The next test showed that the pressure of water is against and not thru a saturated fill. To determine the difference in pressure on a body immersed in water and one immersed in saturated sand, the force required to raise a piston while in water and while in sand 8 in. deep was measured. A force of $8\frac{1}{2}$ lbs. lifted the weight immersed in water— the pressure being applied at the water surface. A force of 22 lb. was required to lift the weight while in the sand, but as soon as the weight cleared the sand surface, a pressure of $8\frac{1}{2}$ lbs. was required to lift it out of the water. The ratio is 40%, which is the percentage of voids in the sand. Hence the conclusion that sand only transmits hydrostatic pressure thru the voids. Meem states that fluid masses do not give hydrostatic pressure, that liquid concrete in forms does not give a thrust of a liquid of equal density. The work of Shunk(573) and E.B. Smith(378) in experimentally determining the lateral pressure in forms filled with concrete mixes shows that Meem's statement is absolutely incorrect. A report of the experimental work on this subject is found in a latter part of this section.

38. J. H. Smith (1915).(589)
University of Pittsburg.

Basing the design upon the principles which are discussed under Mueller-Breslau's work, in connection with the number of unknowns in the determination of the lateral earth pressure, an apparatus was constructed to solve the problem. Moments determined about three different axes gives the values of the three unknowns, the horizontal and vertical components, H and V, and the height of the resultant, x. (see fig. 24). From the equations of moments about axes 1, 2 and 3, he obtains $H = (R_2d_2 - R_1d_1)/n$; $V = (R_2d_2 - R_3d_3)/a$; $x = R_1d_1n/H$
When the values for a, d and n are substituted, he obtains $H = 5(1.1\ R_2 - R_1)$; $V = 5(1.1\ R_2 - R_3)$; $x = 5R_1/H + V/4H$
The additional term V/4H, in the last equation is introduced because the vertical weighing surface is 3 in. back of the vertical plane thru axis 1. The scales gave values for R_1, R_2 and R_3. The unknows were computed from the above equations.

The weighing gate of the apparatus, or test wall, is 18 in. wide and 4 ft. high. It consists of two walls, the inner wall forming the retaining surface, while the outer wall is intended to catch any of the filling material which may run over the inner wall, and return it to the space underneath the

the bin. The height of the inner wall is adjustable by means of removable sections. The weighing gate occupies only a part of the side of the bin, in this way eliminating the side wall effect of reducing the lateral pressure. The clearance space between the gate and side ledges and bottom is covered with loosely fitting strips of muslin, glued to the inner surface of the bin.

The bin has vertical side and back walls, is 28½ in. wide inside, and 5 ft. high above its floor. The back-wall is made adjustable, so that the thickness of fill between back wall and weighing gate may be varied. This ranges from 6 to 18 in. The greatest objection to this apparatus is the bin, for it gives no data of any value. There is no room for a full surface of rupture, it is even too narrow to be called a grain bin. The effect of the back wall, both in friction and arching action was not considered. Woltmann, in 1799, gave a theoretical discussion of pressures in such containers. For this see the section containing Woltmann's theory.

The following table gives the results for heights of 4 ft. of fill:

Material						Thickness of Fill	E	H	V	X	R	ϕ
Gravel -	moist & loose;		115	lbs/cuft.		6"	0	157	80	1.70	175	
"	"	"	113	"	"	10"	0	247	82	2.27	260	
"	wet	"	118	"	"	6"	40°	167	106	1.89	198	40°
"	dry	"	113	"	"	10"	0	186	90	1.60	206	
"	"	packed	113	"	"	6"	0	229	79	2.38	241	
"	"	loose	113	"	"	14"	0	183	88	1.61	202	
"	"	"	113	"	"	18"	0	194	98	1.60	219	
Sand	"	"	90	"	"	6"	0	128	99	1.69	162	
"	"	"	90	"	"	10"	0	148	116	1.88	189	
"	moist & loose		94	"	"	10"	0	162	92	1.55	183	
"	"	packed	94	"	"	6"	0	183	59	2.56	188	

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39. Max Miller (1916). (459)
Public Service Commission, N.Y.

In one of the deep subway cuts made during the rapid transit construction in New York City, along Eastern Parkway, Brooklyn, Miller determined the lateral pressure by measuring the deflection of rangers over an area 22 ft. horizontal and 55 ft. vertical. The soil was a coarse sand containing 20 to 30 per cent of clay. The results show that the maximum pressure occurs at a depth of 25 ft., remains constant to a depth of 48 ft., and below that depth, the pressure decreases. The material had a natural slope of $36^{o}53'$ and weighed about 80 lbs. per cu. ft.

The comparison between this experimental and theoretical values is:

Depth (ft).	15	45	55
Coulomb's theory	300	900	1100
Experimental	900	900	300

The Public Service Commission of New York designs the subsurface structures to withstand a pressure of 100/3 h, h being the depth in feet, and the pressure being in lbs. per sq. ft.

40. Ponsonby Moore Crosthwaite. (1916-1922). (147-152)
Institution of Civil Engineers, London.

The report of 1916 is on experimental work conducted to investigate the accuracy of Rankine's theory for penetration of loads resting on granular material:

$$d = \frac{P}{y}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)^2 ;$$

d = depth of penetration in feet
y = density of material in lbs/cuft

Loads were placed on a plunger with a bearing area of 3.25 sq. in. resting on a granular material in a bucket. The penetration was measured. Substituting in the above formula the values found, the value of ϕ can be computed. Comparison is then made between this ϕ and the natural slope. Sand, garden earth, cinders, ashes and clay were used. In general, it was found that in materials loosely deposited, the angle of friction equals the angle of repose. Upon consolidation of the fill, this did not hold. The results obtained are the combined effects of cohesion and friction, there is no necessity to consider these phenomena separately. Altho Crosthwaite draws definite conclusions we must note that in most of his experimental work, results were widely scattered, readings being often 50 per cent from the mean. He considers this due to the small-sized apparatus and says, "Model experiments are best . . . . of the experiments made by previous investigators to investigate the lateral pressure of earth, those in which model walls were used are of the greatest value, but if the models are of any size, the experimental difficulties are almost insuperable." The writer agrees with this last phrase, both in spirit and to the letter.

Experiments with sand ranged from dry sand to saturated sand (35% moisture), with a density range of 108 to 121 lbs. per cu. ft. The computed value of $\frac{1-\sin\phi}{1+\sin\phi}$ was 0.0500 to .0601, corresponding to values of $\phi = 64^\circ 27'$ to $62^\circ 27'$. The sand acted like a solid, the plunger came to rest immediately. The observed natural slope was $52^\circ 28'$.

Garden earth with a natural slope of $46^\circ 12'$, gave experimental values for ϕ from $43^\circ 08'$ to $60^\circ 51'$ with an average of $58^\circ 15'$. Ashes and cinders behaved like viscous solids, taking a long time to come to rest. The following table summarizes the results obtained.

Lateral Earth Pressure.	Angle of repose	Max. exp. ϕ
Materials: Leighton Buzzard sand, damp	52°28'	67°48'
" " " dry	43°04'	64°47'
Bournemouth sand	47°21'	65°32'
Garden earth	46°12'	60°51'
Ashes & Cinders thru 1/3 in. sieve	53°48'	72°59'
" " 2" 30x30 mesh	47°30'	68°08'

Experiments with clay were made with a bearing area of 1 sq. ft., loading being up to 12 tons per sq. ft. With mud,, the area was 16 sq. ft., with loads upto 1.25 tons per sq. ft. These tests were carried on in the natural undisturbed ground and showed that the internal coefficient of friction falls off rapidly with an increase in pressure. There were very great variations in the results.

Crosthwaite's conclusions are that the Rankine formula holds, but ϕ must be taken as the angle of internal friction and not the surface slope. For clay the penetration varies as the square of the load. Had the experimenter taken the trouble to determine the actual coefficient of friction in the material, he would have found that the angle of internal friction is always less than the natural slope, while in these tests, the average angle of friction is 68°21', with a corresponding natural slope of 48°24', almost 20° less! Without doubt, the friction effect along the sides of the bucket had considerable influence upon these results.

In his 1920 report, the author gives just the opposite conclusion for the same type of experiments. "The angles of internal friction from these experiments was much <u>less</u> than the angle of repose, and was not constant for any one material, but depended upon its state of aggregation, whether it was put loosely together, shaken or consolidated by tamping."

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Tests with a small vertical wall, hinged at the base and the overturning moment caused by sand fills being measured by the tension in a string tied to the top of the wall, cause Crosthwaite to conclude that the pressures as calculated from the Coulomb and the Rankine formulae are too high, especially for surcharges. Walls calculated by the Rankine theory, using the value of the angle of repose for ϕ would have a factor of safety of from $2\frac{1}{2}$ to 4.

To test the wedge theory, a false bottmm coated with glued-on sand was inserted, and could be set at any angle with the back of the wall, so that the pressure produced by any wedge could be measured. He found that the wedge theories which take into account the friction between the back of the wall and its backing, give correct results for the wedge of maximum thrust so long as the wall is not surcharged, but that the calculated pressures are 30 per cent too great for the cases of surcharged walls. These results, he states, made him think that something was wrong with the theory and that the friction between the wall and backing should not be taken into account, and that ϕ was really the angle of friction. In view of the above stated result that the theoretical and experimental results did agree, it is hard to see how the author obtained his conclusions. He calculated the value of ϕ required to satisfy the values obtained for the lateral pressure of a horizontal fill against a vertical wall. This value, $42^{\circ}20'$, he called the angle of internal resistance. The natural slope of this material was 35°, <u>less</u> than the angle of friction. With this new value for and disregarding the friction on the wall, "the maximum calculated pressures were practically identical with the observed."

The next experiment was intended to definitely test if wall friction did actually affect the horizontal pressures or not. A long box was cut into two compartments by a vertical saw cut transverse to its length. One half was fixed to solid supports, the other being on live rollers. When the box is full of sand the two compartments are urged asunder by the pressure of the sand on the plane passing thru the saw cuts. This is the pressure on a vertical plane in an indefinite mass of sand and it was determined by measuring the tension on a string that prevented the rolling half of the box from being pushed forward. The rolling box was then turned end for end, so that the sand in the fixed compartment was held in position by the solid end of the box, the face of which was coated with glued-on sand. "This is the same as if it had been retained by a rough wall, and if the friction between the wall and backing affects the amount of the horizontal thrust the amount of tension now required to keep the rolling box in position should have been less than before." A large number of experiments showed that there was practically no difference, and so, he concludes that "it is not correct to assumed that the friction between the wall and backing can affect the amount of the horizontal thrust."

Disregarding the truth of the conclusion, the writer doubts very much the right to draw such conclusions from the tests. What was really shown is that the thrust of a mass of sand upon a theoretical vertical plane bounding the mass is the same as the thrust upon a vertical wall replacing the material on the other side of the theoretical plane, if the surface of the wall is of the same material as the filling material. Had the experimenter used a smooth wall and obtained the same results, the conclusions might have an experimental foundation.

The final conclusions drawn by the author from all his work are:

1. That the plane of rupture may be a convenient mathematical friction, but has no existence in the granular material dealt with --at least, he was unable to trace any evidence of it in his experiments.

2. That the angle of repose is a physical constant that relates only to the surface, and is represented in the interior of the mass of sand by the angle of internal friction.

3. That the angle of internal friction is not a physical characteristic constant for any one material, but varies with the state of its aggregation.

4. That friction between the back of a wall and its backing does not affect the amount of the resultant thrust. The writer believes that "horizontal" component of the thrust is meant, because the apparatus used gives no measurement of the vertical component.

5. That the wedge theory which takes into account the wall friction and the angle of repose, tho giving correct results when applied to a wall without surcharge, or with negative surcharge, breaks down completely when applied to a surcharged wall.

The writer desires to bring attention to the statements concerning the relative value of the angles of repose and internal friction. Altho both the articles in "Engineering and Contracting" for March 31, 1920 and "Engineering News -Record " for Jan. 19, 1922 start with a statement that "the angle of internal friction from these experiments was much less than the angle of repose," all the data given in those and all the other articles by the author show just the opposite. The other conclusions are remarkably close to those drawn from the Cincinnati experiments, tho the experimenter's right to give them from the tests which he performed are very doubtful.

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Angus Robertson Fulton (1920).(265)
Institution of Civil Engs., London.

Using a rotating test-wall apparatus in a bin 7 ft. high, Fulton measures the overturning moment of various types of fill. To date (Feb. 1, 1922) it has been impossible to obtain a copy of the complete report. The magnitude of the resulting moment was measured by observing the extension of a bar fitted with a Ewing extensometer. The abstract in the "Engineering and Contracting" for March 31, 1920, gives no further details concerning the apparatus or method.

The results were compared with the various theories, and it was found that, (using the conclusions given in the abstract),

1. The overturning moment on retaining walls due to the filling, as calculated from a general formula based on the wedge theory, is approximately the same as ordinarily exists when the filling is within the limits of these experiments, and when the inclination of the inner face of the wall either outward or inward is not greater than usually obtains in practice. In calculating theoretical values, it was assumed that the resultant acts at a point on the inner face distant one-third from the bottom of the wall and that it makes an angle equal to the angle of repose with the normal to the wall.

2. That in the case of walls with a filling whose surcharge is not continuous and unlimited, the general formula requires modification. This can be done by reducing the sliding prism of irregular section to one equivalent triangular section from which the overturning moment can be deduced

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either by direct calculation or by a convenient graphical method. The position of the resultant pressure may under these conditions range from 0.33 to 0.364 of the inner face of the wall measured from its base.

For the case of a vertical wall with horizontal fill, the Rankine theory gave results for the overturning moment greatly in excess of the observed values, while the wedge theory approximated fairly closely to experiment. For surcharged fills and a vertical wall, the Rankine theory gave values 50% in excess and the wedge theory 20% in excess of the observed values. Altho three materials were used, clean river sand, gravel and garden soil, it seems that most of the tests were with sand. In order to keep a full plane of rupture inside the fill, surcharged fills were made smaller than the height of the bin, 7 ft., hence we may conclude that the bin was rather narrow. With sloped walls the theoretical values were from 40% to 50% in excess of the experimental values.

H. G. Moulton (1920). (459)
Amer. Inst. of Mining and Met.Eng.

Moulton divides all materials in the class between liquids and solids, as

A - solid, from moist sand to rock; dealing with walls
B - granular, from dry sand to wheat, dealing with bins
C - semi-liquid, from wet clay to quicksand.

He deals only with the first. From numerous field observations he concludes that the surface of rupture and surface of repose are not plane; that fracture in these materials is conchoidal in nature. He notes that in one of the subway tunnels, the floor being 68 ft. below street level, a break in the surface occurred at a distance of 50 ft. from the center line; the average line of rupture being about 1:2. In the placer mine gravel banks at Dawson, Yukon Co., the natural slopes were always concave, with an average slope of about 1:2, for heights up to 100 ft. He therefore concludes that the wedge behind a wall must exert pressure in accordance with Meem's theory.

Dr. Charles Terzaghi (1920). (610)
Robert College, Constantinople.

Terzaghi rejects all the existing theories and starts the investigation of the stresses in granular fills following the method of Couplet (1727) but including the frictional forces between grains. The lateral pressure per unit area due to liquids is given by the well known hydrostatic formula $p = wh$. The same law holds for a mass of perfectly smooth spheres of equal size. For many other assemblage of smooth spheres, the lateral pressure may be greater or less and is denoted by $kwh = p'$. The cause of the difference between p and p' is called the statical resistance, because it exists independently of any frictional resistances acting at the surface of the spheres. The statical resistance is merely the effect of the weight of individual grains transmitted by their neighbors to the wall of the confining vessels.

Should the spheres become rough, the value of p' is unchanged as long as the wall does not yield. After a given amount of yield, the frictional resistance will be set up due to the roughness of the grains requiring that work be done in deforming the shape of the body. During the failure of a wall, we may distinguish between the first phase, the interval in which the frictional resistance is increasing but has not yet reached a maximum and in which the statical resistance remains constant, and the second phase, during which the mutual positions of the grains change. In a detailed study, the elastic properties of the grains must also be considered.

The experimental work, described in the "Engineering News-Record" of Sept. 30th, 1920, consisted of several sets of tests, the object of each of which was to investigate one single point in the theory. Altho several diagrams are given, nowhere is there a scale or dimension. The writer, at the time this article appeared, was working on his apparatus and therefore quite interested in finding out the type of apparatus and method employed by Dr. Terzaghi. The "Engineering News-Record" could furnish no information further than what was published, but gave the author's address. In reply to a letter, the author states in part:

"The preliminary investigations made for my own experiments have shown, that the intensity of the earth pressure is strongly influenced by the side of the bin, unless the width of the bin is at least two and a half times greater than the height of the backfilling. As shown by Donath (1891) the use of scales to measure the vertical component is utterly unreliable because of the inevitable vertical displacement of the wall. That is the reason why I adopted an indirect method for measuring the vertical component (several comments are here inserted on the design of the Cincinnati apparatus).

"

Lateral Earth Pressure.

"My own experiments were made on a very small scale, the height of the retaining wall being 10 cm. (4 inches) only. I was not interested in the quantities but in the principle and in this case the size of the box makes no difference. The goal of my investigations was the determination of the elastic constants of the sand."

Remembering then the size of the appartus, the investigation covered the following problems:

1. Earth pressure at rest - the ratio of lateral to vertical pressure was determined. The vertical pressure was applied by a testing machine upon a mass of sand in a square frame (no dimensions given) one side of which was closed by a steel tape. Pressure on the sand caused this tape to press against a similar tape. A third tape was passed between these two and the frictional force on this third tape was measured by pulling it out. No account is taken of the resistances of the sides of the frame, nor the pressure lost in deflecting the inner tape before it began to bind on the measuring tape, altho the distance between the tapes could be made very small.

2. Horizontal Component - the test wall (10 cm. high) rested on frictionless rollers, and was tied to a weighing scale by means of a cord passing thru the fill. The connection to the wall was probably at the third point. Displacement of the wall was horizontally (sliding) and occurred by reducing the weights on the scale so that the active pressure was obtained. Coarse sand (2 - 3 mm.) was used, because it is easier to produce a fairly homogeneous backfilling with a coarse sand than with a fine sand. To obtain a picture of the size

of the apparatus, note that 40 grains of sand in a line will cover the entire height of the wall.

3. Vertical Component - By using a single roller under the wall, and using an indirect method, the author claims to measure the vertical component with only a horizontal displacement. The bottom of the front end of the wall is provided with a steel button resting on a ground glass surface. The filling exerts a vertical component on the wall causing this steel button (placed very close to the bin) to bind on the glass. Weights are placed on the further end of the base of the wall till the wall is balanced over the roller, which is at about the mid-point of the base. The computation of the vertical component requires the use of the horizontal component which was independently obtained. The author states "no pull is exerted in a vertical direction by the measuring device on the retaining wall". He forgets the process of counterbalancing which causes the face of the wall to move up a trifle of a distance, it is true, yet a vertical displacement does occur. The writer wishes to note that in all our present day methods, stress is determined by strain in the direction of the stress; in other words it is impossible to measure a vertical force unless there is a vertical displacement.

4. Passive pressures were obtained in the same way as the active components, but measrements were made while the wall was being forced against the fill. As shown in the Cincinnati experiments, the amount of such movement must be considerable, to correspond to the amount of yield in the "second

phase" (as termed by the author) before the passive pressure will occur.

5. Observing inter-granular movements - sheets of aluminum were cut in the shape of sand grain sections and a study made of the manner in which these sheets assembled. Various other tests, measuring the lateral pressure at rest and during the second phase are given. As far as the direction of the resultant is concerned, during the first phase, the value of the angle of friction on the wall being 34°, the resultant acted at angles from 5 to 20° for various kinds of fill. The measured angle of inclination increased with the total pressure. During the second phase the lateral pressure, for all cases, acts normal to the back of the wall.

In discussing the action of the sand during passive pressure, he also notes two phases. During the first phase, the resistance of the backfilling increases in direct proportion to the distance of compression until the frictional resistance which acts between the surface of contact of the sand grains assumes its maximum value. The second phase starts with inter-granular movements whose intensity increases more rapidly than the earth pressure decreases. These movements are confined to a definite wedge of the mass resting against the wall, which statement can be obtained by a study of the squilibrium of a mass of spheres. This "slip" occurs only if the angle between the forces acting on some surface (surface of rupture) and the normal exceed a certain value, which is the "angle of slip".

"The earth pressure against a perfectly rigid wall

is fairly independent of the density of the backfilling."
For sand its value is $\frac{1}{2}(0.42)\ wh^2$, and the minimum value due to any backfilling can be caused by a yielding of the wall corresponds to coefficients ranging from 0.15 to 0.05. (Note that all of these formulas include the density, in spite of the first statement.)

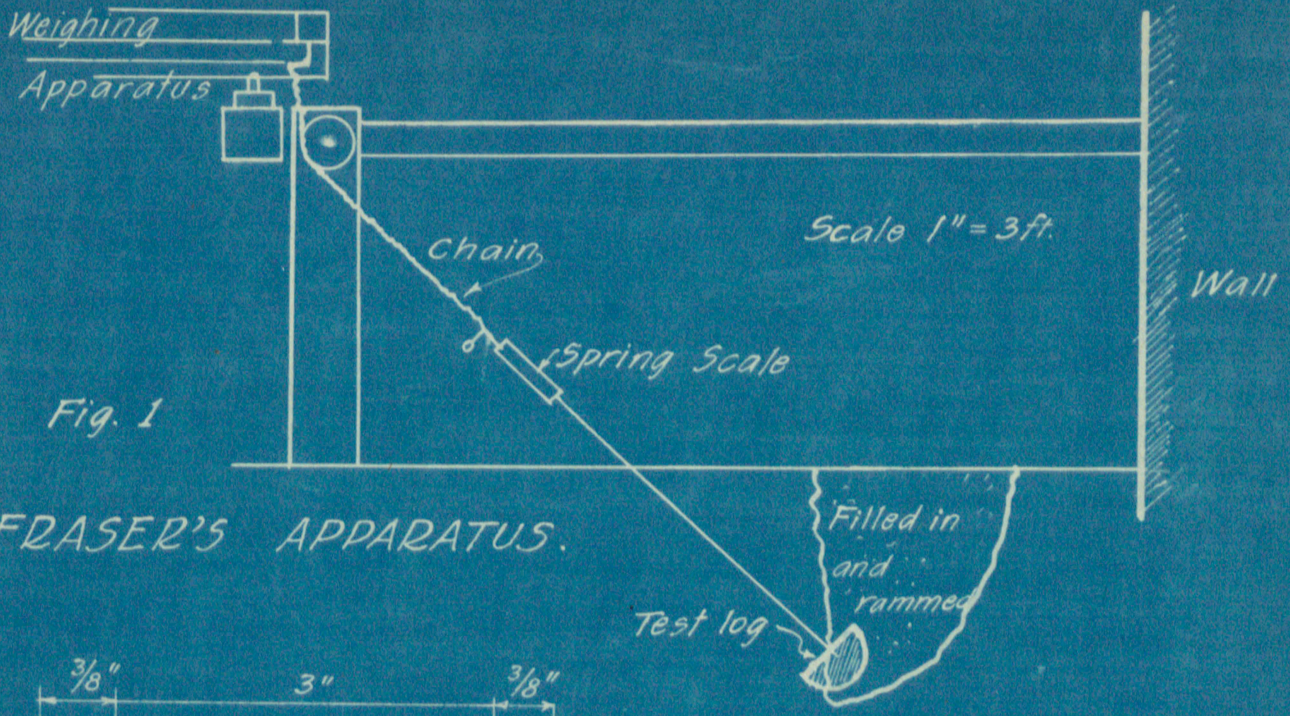
"The conclusions are given for the sake of completeness, not because of their value.)

Experiments on Foundations and the Holding Power of Earth.

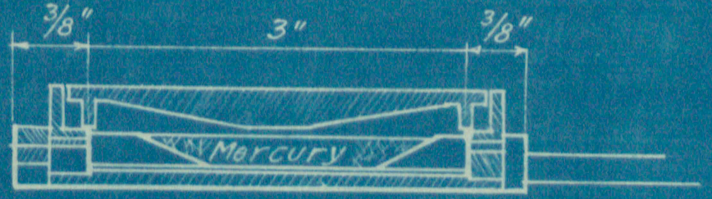
Some of the early experimenters did some work on this part of the subject, and such work is included in the description of the researches on Earth Pressures. The following descriptions deal with investigations into the holding power of earth, the ratio of vertical to lateral pressures, the safe bearing value of soils, etc.

T. Fraser. (1874)(264)
Royal Engs. of England.

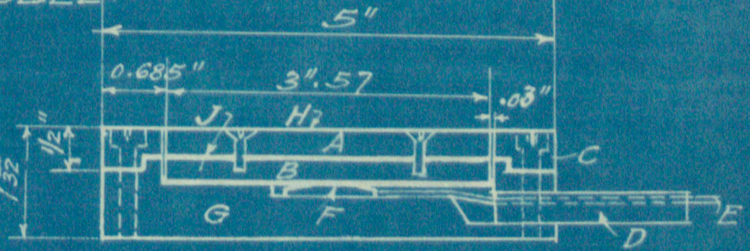
Fraser investigated the amount of pull required to dislodge logs of various sections and lengths imbedded in different soils (see fig. 1). His main conclusion is that anchors with the same cross section area normal to the direction of the pulling force have the same resistance. His tests were with square, circular, and semi-circular logs. Some of the more important tables of results are reproduced below.



FRASER'S APPARATUS.



WILSON'S PRESSURE CELL
Fig. 2



GOLDBECK'S CELL.
Fig. 6

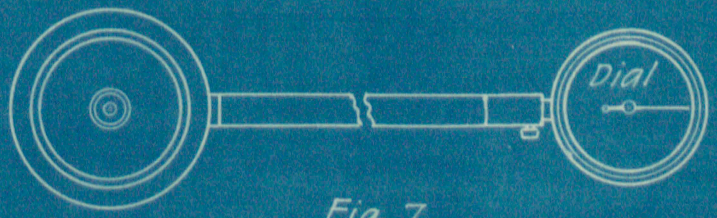
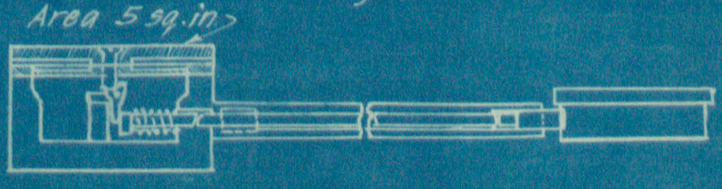


Fig. 7



ENGER'S PRESSURE CAPSULE.

Jacob Feld 1922

Lateral Earth Pressure.

I. Relative Holding Power of Earths. (VerticalPull).

lb. per sq. in. of exposed surface.

Size and shape of Anchor.		Mean depth of face.	Soil.		Density & slope.	Ultimate Resistance.	
Square.	13"x13"x6ft.	2ft.0in.	Dry loam.		82 lb.	900+)	Earth
"	"	"	Wet	"	2:3	900-)	pulled out.
"	9"x9"x4ft,	1ft.0in.	Dry	"	"	1167	"
"	"	2ft.0in.	" "	"	"	1700	
Semicircular.	12"x 6"x3ft,	3ft.	"	"	"	3024	"
(flat side up)	"	2ft,	"	"	"	2150	"
"	"	4ft.	"	"	"	5470	)Surface
"	"	1ft.	"	"	"	450	)disturbed. 4ft.around
Circular.	10"x 3ft.6"	3ft.	"	"	"	3000	"
Square.	9"x9"x4ft.	1ft.	Gravel.		118 lb.	680	Gr. drew out
"	"	2ft.	Damp	"	4:5	1430)	Surface 6x3½'
"	"	3ft.	"	"	"	3300)	disturbed. " "
"	"	4ft.	"	"	"	4550	" 7x5'.
Circular.	9"x4ft.	2ft.4½in.	"		"	1190	Failed suddenly
"	" "	3ft.0in.	"		"	2418	
Square.	9"x9"x4ft.	3ft.	Damp sand.		114 lb.	1587	Loose sand.
"	"	2ft.	"	"	2:3	942	" "
"	"	1ft.6in.	Loam.		82 lb.	2350	Earth bulged.

II. Relative Holding Power (Inclined Pull).

lb. per sq. in. of exposed surface.

Direction of Pull.	Shape of Anchor.	Size of Anchor.	Depth of Face.	Nature of Fill.			Ultimate Resistance.
1:3	Square.	9"x 9"x4'	3 ft.	Wet	loam.		4494
1:1	Semicircular.	6"x12"x3'	2	Dry	"	(82 lb.)	2632
1:1	"	6"x12"x1'	2	"	"	@ 2:3)	3516
1:1	"	6"x12"x3'	2	"	"		3822
1:1	"	"	2	"	"		2755
1:3	"	"	2	"	"		4032
1:2	"	"	2	"	"		3882
1:4	"	"	2	"	"		4368
1:1	"	"	1	"	"		933
1:1	"	"	3	"	"		4400
1 :1	"	6"x12"x1'6"	4	"	"		9400
1:1	"	6"x12"x1'	5	"	"		27552
1:1	"	6"x12"x8"	5	"	"		28560
1:2.5	"	6"x12"x1'	2	"	"		7837
Vertical	"	"	5	"	"		14112
1:1	"	6"x12"x3'	2ft,4½"	"	"		3660
Vertical	"	6"x12"x2'	2	Wet	river	clay	952
1:1	"	"	2	"	"	"	1344

Latetal Earth Pressure.

III. Relative Holding Power of Earth.

Compact Loam-	slope: 2:3,	density: 82 lb.,	dry,	1.0
Hard Compact Gravel-	" 4:5	" 118	"	0.9
River Clay-	1:4	" 83	wet	0.5
Loose non-coherent Sand-	2:3	" 114		0.5

IV. Influence of the Depth on the Resistance.

Values of the Resistance per sq.ft. of Exposed Surface.

Direction of Pull	Vertical	1:1	1:2	1:3	1:4
Relative Value:	1	1.5	2.0	2.1	2.3
Mean Depth: 1 ft.	808	933	1244	1300	1430
1 1/2 "	1040	1458	2100	2180	2360
2 "	1925	2700	3880	4032	4370
3 "	3024	4400	5860	6160	6750
4 "	5470	8000	10660	11200	12260
5 "	14112	22000	29330	30800	33730

The tests are most complete and the results are very easily seen from the tables.

Lateral Earth Pressure.

Fr. Kick (1883). (355)

Kick investigated the proportionality of the bearing area on soils and the unit pressures. He concluded that the unit pressures vary inversely as the area of the cross section, and that the bearing value, therefore, varied directly with the area of the cross section.

Geo. Wilson (1902) (282, 648).

Wilson started the investigation of the ratio of horizontal to vertical pressure in large masses of granular material, by the use of pressure cells. His preliminary tests consisted of determinations of the constants of the sand.

When poured on a flat surface, he found that the natural slope of sand varied from $29°32'$ to $32°53'$. When allowed to stand, the variation was from $30°00'$ to $30°36'$, averaging about $1°$ less than the initial readings.

The coefficient of friction on a metal wall was found by measuring the force required to pull a sheet metal plate vertically out of a sand fill. The force was a function of the height, the empirical formula covering the tests was

$$P = 0.434\, h^2 + 29.18\, h^{1.38}.$$

Wilson notes that since the sand grains are covered with water, the surface tension must be considered. The writer has discussed this point in the Introduction, where the coefficient of internal resistance is defined. This coefficient includes friction and all other resistances in the mass. The value obtained by Wilson corresponds to the internal kinetic friction,

Lateral Earth Pressure.

To measure the pressure, he devised a guage, consisting of a flexible disk, 3 in. diameter, bearing against a vessel of mercury (see fig. 2). The pressure guage was carefully calibrated. No attempt was made to consider the effect on the recorded pressure due to the deflection of the pressure area. The following values for the ratio of horizontal to vertical pressures were found; for sand with various moisture contents:

Percent Moisture	Hor. Press.	Vert Press.	Ratio
Dry Sand	2.857	8.97	0.319
6%	1.82	8.21	0.221
12%	2.08	9.79	0.212
17% (saturated)	2.266	8/096	0.280

Using different materials, and calculating the ratio from Rankine's formula, he gives the comparison with theory:

Experiment:	0.336	0.305	0.319
Theory	0.339	0.296	0.320

The most important conclusion to be drawn from these tests is the fact that the ratio of lateral to vertical pressure for sand is a minimum when the moisture content is about 9 percent, and increases as the percent of water goes above or below this value.

E. P. Goodrich (1904).(282)
A. S. C. E.

Most of Goodrich's experiments were aimed at the determination of the ratio of vertical to lateral pressure, so that the description of all his work is placed in this section.

The first set of experiments described are with a box

Lateral Earth Pressure.

3 ft. by 3 ft. by 6 ft. deep, made of matched boards. By means of a bent lever, the pressure on one of these boards was measured by running a weight along the horizontal arm of the lever and counteracting the deflection of the board. But he did not notice that this method gives the passive resistance to pressure and not the active pressure; for the sand had changed its shape in bending the board and the lever forced it back to its original shape. The results gave the formula H = V/15 + 15 lbs., where H and V are the horizontal and vertical pressures, the H being also the horizontal component of the lateral (in this case, passive) pressure. The results are of little value, because of the small-sized bin, the variation in readings, which amounted to 10 percent, and the 15 lb. correction quantity in the formula.

The next tests were with model retaining walls made up of double lines of sheeting in the shape of a box. The deflections of the yellow pine boards (placed vertically) was measure. One of the sheeting planks was then loaded with a uniformly varying load from zero to a maximum at the other end and the deflections measured. Using this as a basis, the pressures were computed. Using fine beach sand, he finds that the horizontal pressure was over half the total vertical pressure. The large deflections in the walls of the boxes make the results very uncertain.

To do away with the arching action and deformation of the earth, tests were then conducted in a cast iron cylinder, 5 in. high, and 6 in. diameter. The cylinder was accurately fitted with a plunger and a plug fitted to a 1 in. hole bored thru the side. The movement of the plug could be measured to 1/5000 in. One arm of a right-angled lever bore against the plug, the other arm was horizontal and graduated. The weight of the arm was counterbalanced by a spring at the end.

By means of a testing machine, loads were impressed upon the plunger, and the resulting lateral pressures on the plug were meassured on the horizontal beam. The curves of the results, plotting impressed load against lateral pressure were straight lines as long as dry materials were used. With wet materials, the curves start as straight lines, but a permanent set seems to occur and slight stratification of the fill has large effects. The 30-50 sand gives a maximum ratio of Horizontal to Vertical pressure; finer and coarser sands give smaller ratios. Repeated application of loads diminishes the lateral pressure.

The effect of moisture is to decrease the ratio from the value obtained for dry sand, then the ratio increases with a greater percentage of water, till it reaches a greater value than the original. More water causes a decrease to a lower minimum, then comes an increase to an average value, at the point of saturation. In moist earth, the first application of load causes a set which reduces the lateral pressure. The variation of the lateral pressure with moisture content gives a curve of the fourth degree. The one maximum and two minima values obtained between zero and 100 percent saturation can be explained by consideration of the adhesion forces between water and earth, the capillary action of the water, and the

Lateral Earth Pressure.

friction between grains with water as a lubricant. In the case of clay, the material does not flow, but crawls and relieves the pressure by extrusion.

The coefficient of friction between earth and earth was found by measuring the force required to move a 12 in. cube of the material while resting on a similar cube. Two boxes of the proper inside dimensions are filled with earth and placed with the open sides together. If the proper angle of friction is used, Goodrich checks Rankine's theory of conjugate pressures. For depths upto 6 ft., the angle of friction equals the angle of natural slope. Below 6 ft., the angle of internal friction falls off, at first rapidly, then slower.

The following table gives the ratio of horizontal to vertical pressures, the internal friction and the surface slope of various materials.

Ratio H/V	Material	Observer	tan Angle Int.Fric.	tan Angle of repose	Observer
0.10	Coal, shingle, ballast	Baker	1.423	1.11-0.70	Rankine
	bank sand	Goodrich	1.423	1.45-0.60	Goodrich
				1.00-0.67	Trautwine
				0.75-0.38	Rankine
0.15	rip-rap	Goodrich	1.097	1.00	Goodrich
	earth	Baker	1.097	0.66	Trautwine
0.20	quicksand 100-up	Goodrich	0.895	-----	----
----	clay	Baker	0.895	1.00	Trautwine
0.25	quicksand 50-100	Goodrich	0.750	----	----
	earth	Steel	0.750	0.66	Trautwine
	bank sand	Wilson	0.750	0.58	Steel
				0.58-o.62	Wilson
0.30	--- ---	-----	0.622	---	---
0.35	Sand 50-100	Goodrich	0.549	0.85	Goodribh
	bank sand	"	0.549	0.75 -0.38	Rankine
0.40					

Lateral Earth Pressure.

Ratio H/V	Material	Observor	tan Angle Int.Fric.	tan Angle of repose	Observer
0.40	cinders	Goodrich	0.474	0.86	Goodrich
	clay	"	0.474	1.00	Rankine
	½ in. gravel	"	0.474	0.66	Trautwin
0.50	¼ in. gravel	"	0.350	0.85	Goodrich
	bank sand	"	0.350	0.75-0.38	Rankine
0.60	sand 30-50	"	0.258	0.66	Goodrich
0.70	sand 20-30	"	0.179	0.75-.38	Rankine

R. B. Fehr and C. R. Thomas (1913-1914). (234-6)
Pennsylvania State College.

A very complete set of tests on the determination of the laws of the distribution of vertical pressures. The first report, issued by Fehr and Thomas in 1913 deals entirely with sand tests; the work on soils was continued by the former and the second report appeared in 1914.

The sand used was a clean dry river sand of medium sharpness. It weighed 88 lbs. per cu. ft. loose and 93.5 lbs. when packed; the angle of repose was 33°20', percent of voids was 36.6, the amount of silt was less than 1/4% by volume. The first apparatus used was a box about 8 ft. square and 1½ ft. deep, raised 3 ft. off the floor. A hole 24 by 36 in. was cut in the center of the bottom, and in it were placed six 6 by 24 in. slats supported at one end on a piece of 1-in. pipe. The weight on the slat was measured by a spring balance connected to the outer end of the lever, which was supported so as to hold the slat in a horizontal position level with the other slats and the bottom of the box.

At first short pieces of rope were used to take up the extension in the balances when weighing. The stretch

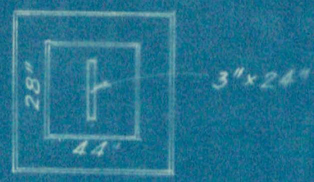
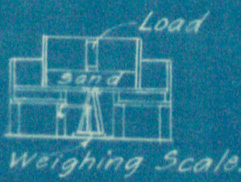


Fig. 3, 1st Apparatus.



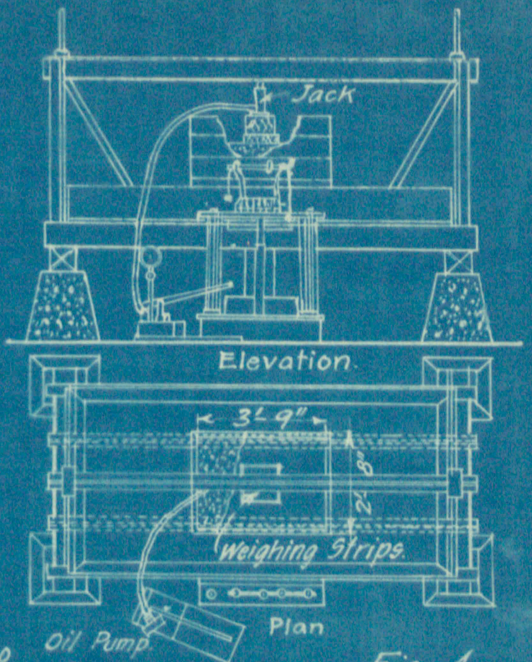
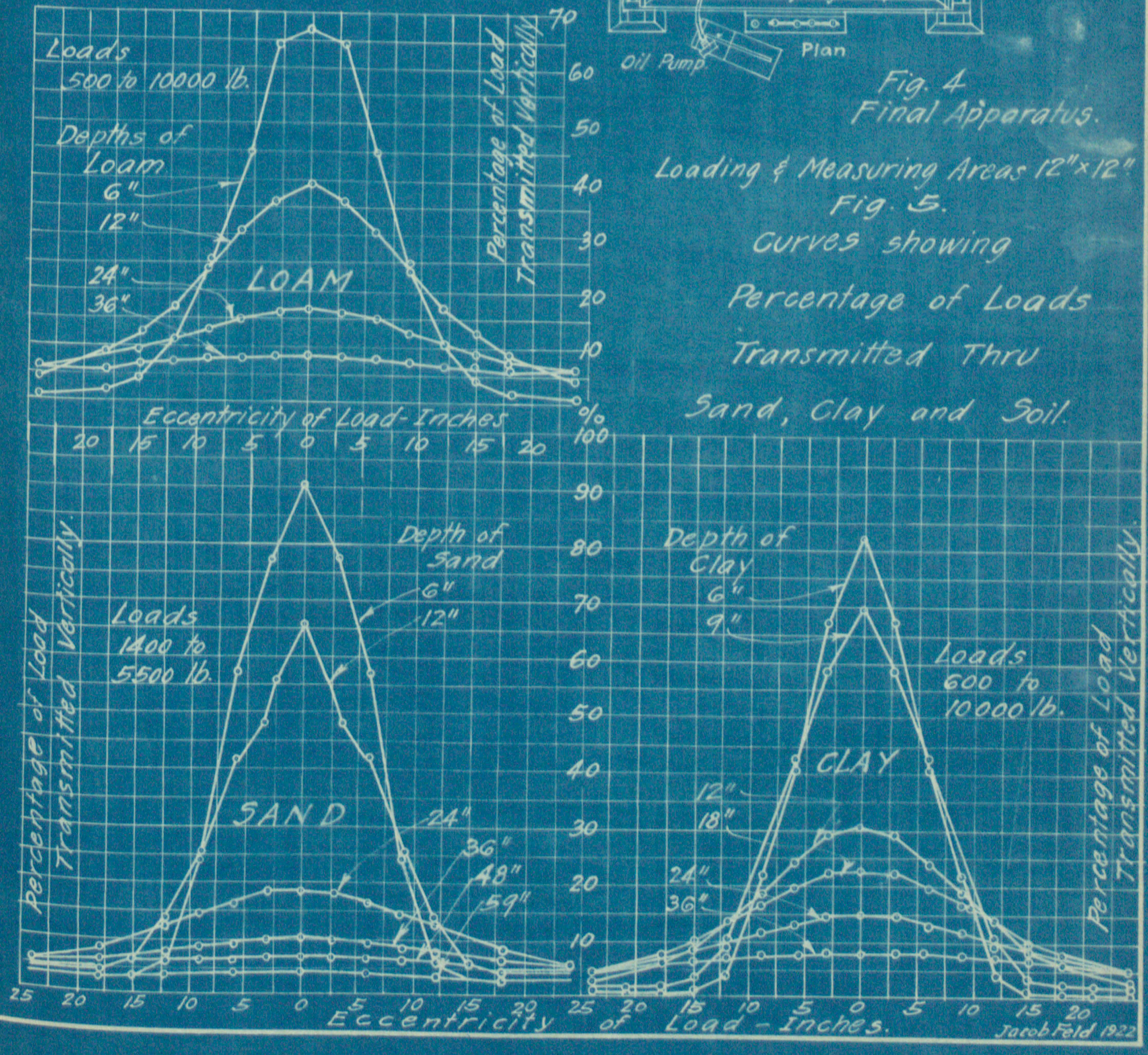


Fig. 4
Final Apparatus.

Loading & Measuring Areas 12"x12"

Fig. 5.
Curves showing
Percentage of Loads
Transmitted Thru
Sand, Clay and Soil.

FEHR & THOMAS TESTS.



in the ropes made this impracticable, and they were soon replaced by small threaded eye bolts run down thru a steel angle. A piece of cloth was spread lightly over the slat to prevent the sand which was to be piled on top of them from running thru. The sand was struck off to a certain depth and the weight on each slat was obtained by pulling down on the spring balance until the pressure against the supporting strip was relieved. Readings were then taken on the balances. A load of brick was placed on a loading slat 6 x 24 x $\frac{3}{4}$ in. over one of the movable slats and the spring balances pulled down. Readings were again taken on the balance. The difference between the two consecutive readings on the same balance multiplied by 2 gave the effect of the internal load on that slat.

The results showed in a general way that the pressure due to a contentrated load on the sand was distributed over a large area. The results, however, were not very consistent, in some cases the pressure transmitted on the slat directly below the load was less than on other slats. If we note that readings were obtained by pressing against the sand, thereby bringing into play the passive resistance of the sand, the variations and inconsistencies are accounted for.

The apparatus was then improved by introducing 14 slats, each 24 x $2\frac{1}{2}$ in., supported on knife edges. The amount of movement was also cut down to 0.005 in. The procedure was the same, and again proved unsatisfactory. The results fluctuated greatly and bore no relation to each other.

It was finally decided to cut the movement of the weighing strip to a minimum, by using platform scales. The bottom of the new box had an opening 24 x 36 in., in which the weighing strip was placed (see fig. 3). The weighing strip was wholly and directly supported by the platform of a small scale, with a scale ratio of 50 to 1. The maximum movement of the arm was restricted to 1/16 in., corresponding to an 1/800 in. movement of the weighing strip. Since there was but one weighing strip, the distribution of the pressure had to be obtained by varying the position of the load. To determine the percentage of load transmitted by the sand, two readings had to be taken, before and after the load was applied.

In these tests, the loads varied from 60 to 300 lbs. and consisted of bricks piled upon an area equal to the weighing strip. The position of the load was varied from the center directly over the weighing strip to a position where the eccentricity was 15 in. In each case, a double set of observations was obtained, by moving to the right and then to the left.

The results of these tests were quite consistent, so a larger apparatus with a loading capacity of 6 tons was built. An average of all the tests with the smaller apparatus shows that for a depth of less than 12 in. of sand, the percent of transmission to areas not directly covered by the load is always less than sixteen. There is a sharp change in the percent transmission on areas directly underneath the load or any part of it and those not underneath any part of the load.

The large apparatus shown in fig. 4, was designed and constructed for a loading capacity of six tons. With the exception that the load was applied by means of a hydraulic jack instead of bricks, the device was the same as the one previously

Lateral Earth Pressure.

described. The loading and measuring areas were always the same. The areas used were 12 x 12 in., 6 x 24 in., and 6 x 6 in. The platform scale used had a scale ratio of 200 to 1, so that the maximum weighing strip movement was 1/1600 in.

The tests were run on depths of sand varying from 3 in. to 30 in., and the eccentricity from 12 in. to the right to 12 in. to the left. The loads were varied from 300 lbs. to 5400 lb. The curves in fig. 5, show the quantitative results obtained. Thefollowing observations and conclusions are noted by the authors:

The loading strip (12 x 12 in.) began to sink in the sand at loads of approximately 2500 lb. where the sand exceeded 12 in. in depth. This effect was not noted for smaller depths until a much higher load was reached. After sinking began, additional load caused an additional sinking; readings were taken after equilibrium had been reached. A distinct heaving and flowing of the sand occurred for some distance on each side of the loading strip. The sand usually assumed a water ripple formation surrounding the block. As many as three ripples of 1/2 in. in height occurred. In every case the percentage of transmission appeared to show a uniform increase as greater loads were applied. This may be due to the flowing of the sand as well as the slight decrease in depth due to the imbedding of the loading strip. A maked decrease in the percentage of load transmitted occurs when the eccentricity of the loading strip equals its width, this decrease becoming lessmarked as the depth increases. The conclusions are: (1) that at depth of 12 in. for loads upto 300 lbs. per sq. ft. and 23 in. for loads upto 5400 lbs. per

Lateral Earth Pressure.

sq. ft., the percentage of transmission has a maximum value of twenty. (2) The maximum transmission over areas whose eccentricity is greater than the width of the loading strip is never greater than sixteen percent.

In the second set of tests run by Fehr, the box was changed from 3 ft. by 4 ft. deep to 4 ft. 6 in. by 9 ft. 4 in. by 5 ft. deep. A test with sand gave the same results as with the smaller box. A material was made up to give the greatest possible cohesion. It consisted of 85% clay, 10% sand, and 5% loam, weighed 83 lbs. per cu. ft. loose, and 103.5 lbs. when packed. Its moisture content was 11:3%, natural slope 38½°. The first run was on 6 in. of loose material. The application of pressure formed a "Brick" which was not broken up but was covered by loose soil when it was desired to increase the depth. This procedure was unfortunate, for the mass was no longer homogeneous. It was adopted to save labor, because the "brick" so formed could only be broken up with a pick. The results are therefore for an intensely packed soil. The curves have the same characteristics as the sand curves. There was a regular increase in the percent of transmission as the load varied from 600 to 10,000 lbs. per sq. ft., the maximum loads causing an average increase of 36 per cent in the transmission as produced by the minimum loads (see fig. 6).

The tests were repeated with loam, containing 22% gravel, 13.6% moixture, and weighing 73 lbs. per cu. ft. when loose and 94 lbs. when packed; the natural slope was 40½°. The average increase from minimum to maximum load was 47% in the transmission. (see fig. 7).

Lateral Earth Pressure.

The distribution curves show a marked flattening with an increase in depth of soil, and a rather sudden bend where the loading strip just clears the weighing strip. This bend is more marked in the cohesionless materials. Cohesionless materials give the greater percentage of transmission of loads. The percentage of transmission increases with increase of load. For depths greater than two feet, the percentage of transmission is always less than twenty. For eccentric loads, the percent of transmission is always less than twenty when the loading strip is not over any part of the weighing strip.

A.T.Goldbeck (1916) (278-281)

American Society for Testing Materials.

A.T.Goldbeck working in conjunction with E.B.Smith in the Bureau of Roads and Rural Engineering, Department of Agriculture, developed an apparatus which was to "be applicable to the measurement of earth pressures against structures, as well as for use in the laboratory for the study of the laws of pressure distribution thru particular kinds of material, under different conditions of compaction." Noting that a movement of any part of the apparatus affects the value of the recorded pressures, decreasing it if the movement is away from the material and vice versa, the apparatus was designed to cause no disturbance of the natural state of the fill.

The testing apparatus, consists of a diaphragm cell, see Fig. 6, a brass diaphragm J, resting on a cast-iron base G. The diaphragm is stiffened by a disk on each side, A and B, leaving an annular clearance of 0.03 in., which is the only portion of the diaphragm J that is flexible. A surface diaphragm H serves as a protection and stiffener. In the base G is a slightly crowned support F, electrically insulated from the base, and connected electrically to the outside apparatus by an insulated wire. Air pressure is slowly admitted until it equilibrates the external pressure on the disk, causing the br breaking of contact between the base B and the support F. This is indicated immediately by means of an ammeter, and the air pressure required is taken as the pressure of the material on the disk. The materials used were the result of considerable investigation, in order to cut down the elastic deformations. In the latest type, the deformation is about 0.0003 in.; the movement necessary to break electrical contact is not over 0.00001 in.

Using a mercury gage to determine the air pressure, cells were calibrated horizontally and vertically under hydrostatic load. They were found to be remarkably accurate, the average air pressure for loads varying from 2 to 10 lb. per sq. in. was 6.56; the average hydrostatic pressure being 6.506. Similar accuracy was found under dead load readings, the load being applied on a square inch, the area of the diaphragm being 10 sq. in. Several trial tests were run in sand and earth fills. These cells have been extensively used; to determine the lateral pressure of concrete in forms at the Lincoln Memorial, of clay sand mixtures at the Miami Conservancy dams and elsewhere. The apparatus is small and easily installed, as well as quite inexpensive. Altho pressure is measured by moving the diaphragm against the fill, the amount of motion is so small, that very little opportunity is given for the development of passive resistances in the fill.

Lateral Earth Pressure.

Prof. Melvin Enger (1916)(11)

University of Illinois.

American Society of Civil Engineers.

The experimental work of the "Committee to Report on Stresses in Railroad Track," of the A.S.C.E. included tests on the Transmission of pressure in ballast. The final report of the Committee includes a detailed analytical and experimental discussion as section V, p. 251. The work was carried on in connection with undergraduate thesis work, covering a space of several years. The material used were chiefly railroad track ballast, the difficulty of taking measurements in the field caused the construction of a laboratory apparatus.

A section of track, consisting of three ties and ballast rested on a concrete platform. The load, applied by a jack on the ties, was measured by means of a calibrated steel spring. The intensity of pressure in the ballast was determined by a specially designed pressure capsule, consiting of a bearing plate transmitting the pressure by means of the screw S, see fig. 7, to a bell crank lever, and recorded by an indicating dial micrometer. The area of the diaphragm was 5 sq. in; the material used was hard steel. The cells were placed on the concrete floor and the required height of ballast placed over them. On this were set the ties and loads were applied.

The results indicate that the distribution of pressure at a given depth is independent of the material; the tests included sand, broken stone and pebble ballasts. The laws of distribution of pressure thru non-cohesive granular material are therefore not dependent upon the characteristics of the material.

The pressure at any point directly underneath the tie,

Lateral Earth Pressure.

at a depth h; is

$p_c = \frac{16.8\ p_a}{h^{1.25}}$, where pa is the average pressure over the tie, both pressures being in lb. per sq. in., and h in inches is measured from the bottom of the tie; which in all cases was 8 in. wide. For depths less than 4 in. and more than 30 in. this formula gives results which are too high. At any point in the ballast, at the depth h, and distant x in. from the center line of the tie, $p = \frac{16.8\ p_a}{h^{1.25}} (10)^{-6.05\frac{x^2}{h^{2.5}}}$ This formula is accurate under the same conditions as the previou one.

Special Committee to Codify Present Practice.

on the Bearing Value of Soils for Foundations(1915-1921

American Society of Civil Engineers.

Since 1914, this Committee has been discussing two problems, (1) the present practice on the bearing value of soils and (2) the physical characteristics of soils in relation to Engineering structures. Most of the experimental work has been performed by the sub-committee of the U.S.Bureau of Standards. The sub-committee in 1916 improved the Goodrich type of pressure cell to give a deflection of only 0.00005 in., the original apparatus had a maximum displacement of from 0.001 to 0.002 in. The gauge, however, was abandoned on account of the pressure discontinuity introduced at the rim of the piston as it moves under pressure. Various other types were tried, and a careful study made of all the previous types of apparatus. Numerous preliminary tests to determine the ratio of transmitted pressure in sand bore out Darwin's Statement that the internal friction is a function of the pressure and of the density.

Lateral Earth Pressure.

The first apparatus to measure conjugate pressure consisted of a steel case, 12 in. diameter and 12 in. deep, with a solid cover having two openings, both off center. Pressure was applied thru one opening and a measuring plate covered the other. This plate bore against a short strut operating a lever, and measurements were made by a cathetometer reading on a mirror attached to the lever. Since the law of variation of the wall friction was not known, the results are not conclusive. The coefficients of a particular sand was shown to vary with the density. A few tests with clay showed that the viscosity was an important factor.

In 1917, a careful study of the strain-stress diagrams for granular materials was made. The work however is not yet completed. In conclusion to these tests as well as a number on dilatancy, the statement is made that the bearing value of soils cannot be codified upon the basis of a purely frictional theory of earth resistance as the Coulomb and Rankine theories. It is suggested that the basis be a theory of elastic equilibrium with frictional equilibrium as a limiting state determining the discontinuities of the field. The laws of elastic equilibrium are not to be imposed on the materials as in the older theories, but must be determined experimentally and interpreted in terms of the general theory of stress-strain distribution.

Five progress reports have appeared to date (Feb. 1, 1922), dealing mainly with definitions, theoretical discussion and preliminary tests. The final report will probably be a valuable contribution to the literature on this subject.

Lateral Earth Pressure. Pressures in Bins.

Various men had made some attempt to determine the effect of side walls on the lateral pressure of granular materials. The investigations which deal entirely with the case of grain bins are included in this section:

Isaac Roberts (1882) [538]

London.

Robert's report in "Engineering" is entitled "on the Pressure of Wheat stored in Elongated Cells or Bins." He notes that sand, wheat, etc. in bins does not obey the laws of hydraulics. He constructed several model bins with no bottom. The false bottom rested on a weighing machine. Before filling the bin, the beam of the weighing machine was overbalanced. After filling, the rider was moved back to a point of balance. In this way he was sure to obtain the active pressure.

Cell No. 1 was a hexagonal prism, 4 in side and 60 in. high. The filling material was wheat weighing 7.8 lbs. per gal. (58.5 lbs per cu. ft.) 1 gal. of wheat - 7.8 lbs - 6¼ in. gave 5½ lbs.

2 " " - 7½ lbs.

9 " " - 70.2 lbs - 56 in. gave 7½ ".

Of 26 sets of readings taken, the highest value was 7½ lbs. This shows the great influence of the wall friction in reducing the pressure on the bottom of a bin.

Cell No. 2, was square, 7 in. side and 36 in. high. 1 gal. of wheat filled 5½ in. of the height. 17 sets of tests were run, with results similar to those of cell 1.

Cell No. 3, was hexagonal, 6 7/8 in. side, 60 in high. 18 sets of readings were taken, with same results.

Cell No. 4, was hexagonal, 12 in. side, 96 in. high, and 19 sets of readings were taken, with same results.

Lateral Earth Pressure.

Cell No.	Diam. of Inscribed Circle.	Area of Cell.	Height of Fill of Max. Pressure.	H	Press. on Bottom.	Total Weight of Fill
1	7 in.	41.6 sq.in.	13.5	60	7 lb.	70.2
2	7	49.0	11	36	$8\frac{1}{2}$	46.8
3	12	122.8	24	60	48	202.8
4	20.75	374.1	36	96	224	1014

The pressure on base against volume of fill will give a curve which is either parabolic or hyperbolic.

$$\frac{P}{A} = DCW,$$

$$P = 1.03\,DWA.$$

P = total pressure on bottom of cell.

A = area of the bottom

D = diameter of inscribed circle.

W = weight of fill in lbs. per cu ft.

C = 1.03, a constant.

Cell No.	1	2	3	4	Average.
P(Experiment)	7	8.5	48	224	91.9
P(Formula)	8.4	9.7	42.8	225.8	91.7

a fairly close agreement.

H. A. Jannsen (1895) (34?)

Vereins deutscher Ingenieure.

Jannsen experimented with model bins, 20, 30, 40 and 60 cm. square sections. The bottom rested directly on a scale; the walls were on jack screws, by which he could raise the walls and force the wall friction to act. He determined the coefficient of wall friction by measuring the pull required to start a bottomless box filled with grain as it rested on the same material as was used for the wall.

He finds that the lateral pressure is a function of the vertical pressure. L = k V; where the vertical force on the bottom of the bin is given by the formula

$$V = \frac{wR}{cu'}\left(1 - e^{-\frac{cu'h}{R}}\right).$$

for wooden bins u' = 0.3; c = 0.67

Prante(1896)

Vereins deutcher Ingenieue.

Prante investigated the lateral pressure in two circular iron bins, 1.5 and 3.8 meters diameters and 19 meters high. The measuring area was a diaphragm on knife edges connected by levers to a scale platform. He uses Jannsen's formula for static pressure; but finds that the lateral pressure when the wheat is in motion at the rate of 1 mm per second), is four times the static pressure. However, the location of the discharge gate influenced this result, being opposite to the diaphragm.

Max Tolz(1903)(613)

Canadian Soc. of Civil Engs.

Tolz's tests on the lateral pressure in grain bins are considered the most reliablo by Ketchum. The test bin was of wood, 14 ft. square and 65 ft. high. A steel plate was rigidly connected over an opening in the side of the bin near the base. The opening was 1 ft. 6 in. x 3 ft. 0 in. The deflections of the plate were measured and the pressures calculated. The maximum observed deflection corresponded to a pressure of 3 lbs. per sq. in. The objection to this apparatus is the change in shape of the fill caused by the deflection of the pressure area. This is especially true in a bin, where the stresses in the fill are localized.

Jamieson(1903)(75)

Canadian Soc. of Civil Engs.

The bin pressure problem was also investigated by Jamieson, of the Canadian Engineers, by means of a pressure gauge. A rubber diaphragm was inserted in the wall of the bin and connected to a vessel of mercury. The diaphragm was filled with water. A graduated gage glass, 1/16 in. diameter, gave the pressures directly. The apparatus was found very sensitive and accurate, in tests on models and in actual bins. Just how he determined the accuracy is hard to tell. Two types of diaphragms were used, a 6 in. square and a 12 in. circular shaped test area. In the calibration, it was found that for 1 lb. of pressure, the maximum deflection of the square diaphragm was 0.00236 in., and for the round diaphragm 0.00753 in. The pressures recorded were in the neighborhood of 5 lbs. per sq. in. This means a total load of 180 lbs. on the square diaphragm and 565 lbs. on the round diaphragm. Assuming that the deflection was proportional to the total load, this means a total deflection of 0.425 inch for each diaphragm, altogether too great a deflection for the accurate determination of pressures. A movement of 0.425 in. causes considerable change in the shape of the filling material near the pressure area, with consequent changes in stress.

Using a circular steel bin, 12 in. diameter and 6 ft. 6 in. high, he measured the ratio of the pressure on the bottom to the weight of the contained material. In this test, sand was used; the density being 100 lbs. per cu. ft., natural slope 34°. With a height of fill of 36 in., the bottom pressure is 37% of the total weight; height of 60 in., 23%; and height of 78 in., 18%. The rest of the sand is held by the friction on the side walls.

Cain in discussing these tests, shows that in determining the ratio of horizontal to vertical pressure the Rankine formula: $K = \frac{1-\sin\phi}{1+\sin\phi}$ is very reliable, if the wall friction is taken into account, and the normal component of the lateral pressure considered.

Bovey plotted curves based upon Jamieson's tests and showed that the maximum horizontal and vertical pressures did not occur at the same height of fill. In a bin 12 ft. by 14 ft. section the maximum horizontal pressure of 2 lbs. per sq. in. occured when 5500 bushels of grain were in the bin. In each case, the pressure started to decrease very slowly upon increasing the height of fill.

Eckhardt, Lufft. (1914) (406)

"Eng. News," N.Y.

Lufft tested the lateral pressure of wheat in full sized bins by inserting a pressure cell into the side wall. The cell consisted of a diaphragm laid flush with the inner wall of the bin, which was 23 ft. 10 in. diameter and 55 ft. high, bearing against a vessel of glycerine. The pressure was transmitted to a mercury tube, where it was read. The wheat used was 48 lbs. per cu. ft.; the bin had the inner surface faced with cement. The pressure height of fill curve is parabolic. At a height of fill of 44 ft., the lateral pressure was 5 lbs. per sq. in. For greater heights, the pressure remained fairly constant, reaching a value of 5.2 when the height of fill was 55 ft. Lufft tried to obtain the variation in pressure as the bin was emptied, but could get no uniform results.

Lateral Earth Pressure.

J. Pleissner (1906) 493

Vereins deutscher Ingenieure.

Five model bins were used by Pleissner in his tests.

A. was 1.57 meters square and 18 meters high (5 ft. by 57½ ft.) made of cribbed timbers.

B. was the same size as A, but had rings spaced 1.6 meters vertically (5.12 ft.), projecting 0.045 meters into the bin.

C. was a plank bin of the same size.

D. was a plank bin, 2.15 m. by 2.9 m. by 9 m. (69 ft. by 9.3 ft. by 28.8 ft.)

E. was a reinforced concrete bin, 2.5 m. by 3.15 m. by 17.24 m. (8.0 ft. by 10.2 ft. by 55.0 ft.), with rings 2.93 m. apart, projecting 0.080 meters into the bin.

In the first three the pressure on the bottom was measured by supporting the bottom with rods running thru the bin and connected overhead to scale beams. This method is objectionable because it breaks up the continuity of the fill volume. The last two had loose planks in the bottom, 12 in. wide, 3 and 6 in. thick and a span of 8.2 ft. The deflection of these was measured to obtain the acting pressures. To check these reading, bags of water were placed underneath the test planks, and connected to pressure tubes.

The lateral pressures were measured by the deflection of vertical planks, the lower edges being at the bottom of the bin. In bins E and D, the pressure areas were 10.33 ft. high and 8.2 ft. wide, made up of 12 in. by 2.8 in. planks, with a span of 8.2 ft. In bins A,B

Lateral Earth Pressure.

and C, the pressure areas were 4.5 ft. wide and 2.95 ft. high, made up of 7.8 in. by 1 in. planks, with a span of 2.82 ft. Three rubber bags filled with water were placed against each test area, spaced equally at the midpoint of the planks.

There were in all 126 separate tests with wheat, rye and flaxseed, at rest and in motion. The reading for pressure at rest are in close agreement with those computed by Jannsen's formulae, the horizontal component checking exactly. When the velocity of the grain was 1 mm. per second, the deflection of the planks were greater than before, showing that the pressures were greater, but the rubber diaphragms did not check the results at all. The value of K is not a constant. The following table gives the average results; note that K is greater for the smaller bins.

Bin			A	B	C	D	E
u'	tan	, Wheat:	0.43	0.58	0.35	0.45	0.71
		Rye:	.54	.78	.37	.55	.85
K	L/V ,	Wheat:	0.4-0.5	0.4-0.5	.34-.46	.30	.30-35
	-	Rye:	.23-32	.3-.34	.3-.45	.23-.28	.30

Ketchum, in discussing the grain bin tests, draws the following general conclusions.

1. The laws of grain pressure are not the same as the laws of liquid pressure.

2. The lateral pressure is less than the vertical pressure, and increases very little after the height of the fill is over $2\frac{1}{2}$ to 3 times the width of the bin.

3. The ratio of lateral to vertical pressure is variable.

4. The pressure of grain in motion is less than the pressure of grain at rest; the increase is about 10 percent.

5. Discharge gates should be at the center of the bin.

Lateral Earth Pressure.

Pressure of Concrete in Forms.

This very important subject has received but little attention, even tho the opportunity for experimental research is probably more common in this type of lateral pressure than in any other. Only two men, Maj. F.H.Shunk and E.B. Smith have exp rimentally investigated the actual pressure of concrete in forms, before the mix has set. In addition to the reports of these two men, there is only one other reference to the subject - a discussion of Shunk's results by Paaswell.

Francis R. Shunk(1909)(573)

Major, U.S.Army, Corps. of Engineers.

The experiments were conducted in 1908 on Lock #1, Mississippi River, at St. Paul. The mix used was 1:3:5½, very wet consistency having a density of 152 lbs. per cu. ft. The blocks were cast 25 ft. 9 in. in height. It was noticed that the workmen often sank 18 in. into the mix as it was poured.

A metal plate, 9.23 in. diameter, was inserted into the side of the form at a height of 18 in. from the bottom. The plate was protected by a steel cylinder set thru the form, and bore against a weighted lever. To avoid obtaining passive pressures, the lever was overbalanced before any concrete was poured. After the concrete reached the desired level, the rider on the lever was run back to a point of balance and the reading taken. The time element was of prime importance because of the setting of the mix. The diagrams giving the results were plotted as pressure against height of concrete at a given time interval after mixing.

Lateral Earth Pressure.

The pressure at first is that of a liquid of a density of 152 lbs. per cu. ft.

T= time, in minutes, after filling, that concrete shows strength.
R= rate of filling per hour.

$$T = C + \frac{150}{R}$$

, the values of C are given below for various temperatures of the form in degrees Fahrenheit.

Temperature	80°	70	60	55	50	40°F
C	20	25	35	42	50	70.

The results of the tests are in the following table.

Temperature.	Time after filling.	Height of fill.	Pressure.	Notes.
70°F	0-80 min.	5 ft.	that of a fluid of 152 lb. density.	
76	70 "	6.5	" " " "	
41	130 "	10	" " " "	
			initial set in 2.5 hrs.	
	95 "	10	1530 lb.per sq.ft.	
	120 "	12.5	1700 " " "	and remained constant
	130 "	14	1700 " " "	" " "
54	70 "	3.5	450 " " "	, a fluid of 152 lb.
	118 "	7	800 " " "	
	138 "	8.5	860 " " "	
	152 "	9.5	860 " " "	
59	0-30 "	0-5	that of a fluid of 152 lb. density.	
		5	770 lb.per sq.ft.	
	60	6.5	850 " " "	,includes a 20 min.res
	70	7	850 " " "	
	88	9.5	1120 " " "	
	103	12	1220 " " "	
	134	15	1220 " " "	
	140	15.5	1320 " " "	
40	0-100	0-6	that of a fluid of 152 lb. density.	
		6	860 lb.per sq.ft.	24 min. rest.
	114	7.5	970 " " "	
	144	9.5	1250 " " "	
	170	11	1370 " " "	

Lateral Earth Pressure.

Paswell made a theoretical study of the results. He assumes the form $P = whe^{-kt^2}$, where **w** is the density, **h** the height and **t** the time after filling in hours. **e** is the Naperian base and **k** is an empirical constant. The time after filling, in hours, after which the concrete is no longer liquid is given by Shunk as $t_0 = C + \frac{2.5}{R}$ where R isthe rate offilling or $\frac{dh}{dt}$. Assuming that the lateral pressure at t_0 is 95 percent of the hydrostatic pressure, the value of K is $\frac{1}{t^2}\log_e\left(\frac{20}{19}\right)$; substituting we obtain the formula for the intensity of pressure

$$p = Kwh, \text{ where } K = \left\{\frac{19}{20}\right\}^{\left(\frac{t}{t_0}\right)^2}$$

The total pressure on the form per foot width is

$$P = w\int_0^h ue^{-a^2u^2}\,du = \left(\frac{1 - e^{-Kt^2}}{Kt^2}\right)\frac{wh^2}{2} = A\frac{wh^2}{2}.$$

The values of K and A for various values of t/t_0

t/t_0	K	A	t/t_0	K	A	t/t_0	K	A
0	1.00	1.00	2	.81	.90	9	.02	.24
1/4	1.00	1.00	3	.63	.80	10	.01	.19
1/2	.99	.99	4	.44	.68			
						20	--	.05
3/4	.97	.99	5	.16	.46	∞	.00	.00
1	.95	.97	7	.08	.37			
1 1/2	.89	.94	8	.04	.29			

To find $t_0 = C + \frac{2.5}{R}$, the following values are given for C.

Temperature	80°F	70	60	55	50	40
C	.35	.42	.58	.70	.83	1.17

E.B.Smith (1920)

American Concrete Institute.

The tests were in a special form 8 in. by 9¼ in. by 10 ft. inside dimensions. Readings were obtained by the use of a Goldbeck pressure cell in the bottom to get the vertical component and three cells in the sides about 6 in. from the bottom, to read the horizontal component. The concrete was tamped and joggled before taking any readings. A detailed description of the Goldbeck cell is found in another section of this paper.

The results obtained show a hydrostatic pressure due to a liquid of 145 lbs. per cu. ft. from the beginning of the test to a height of 1 ft. The pressure increases slower than the hydrostatic till a maximum is reached. Addition height of concrete will not increase either component, after the maximum occurs.

1. The maximum pressure increases as the rate of filling increases; for a rate over 1 ft. per hr. the pressure increases as the 0.3 power of the rate.

2. For a sloppy mix, the maximum pressure is greater for the greater widths of forms; this is not true for dry mixes.

3. The drier the mix, the greater the maximum. This is probably due to the arching action developed by tamping. For low heads, 4 ft. or less, the dry mix gives a greater maximum than the wet mix, up to the time of set. Sloppy mixes act like fluids and show no wedging action.

4. The richer the mix, the greater the maximum pressure. An increase of 0.12 lbs. per sq. in. for each percent increase in the ratio of cement to aggregate, for ratios over 12%, is given.

5. The maximum pressure occurs in about 30 mins., before the mix starts to set.

P = lateral pressure in lbs. per sq. in.

h = head of concrete in ft.

c = percent of cement to aggregate in the mix.

s = consistency of the mix in inches slump.

r = rate of filling per hour

$$p = h^{0.2} r^{0.3} + 0.12\,c - 0.3\,s$$

Lateral Earth Pressure.

This formula holds true if h is half the rate of filling for the ordinary mix and 3/4 the rate for sloppy mixes.

Vertical pressure = 0.25 h + p.

If the inside distance between forms is more than half the head of concrete, the vertical pressure equals the total weight of the concrete. In designing forms too great accuracy is useless, so approximate values should be used.

The subject is fairly well covered, both from the experimental and from the theorètical point of view, in the above discussions. Further investigation, especially the development of a simpler apparatus, will probably lead to some more simple formulae than the ones given above.

DESIGN OF THE APPARATUS.

In 1915, Professors G.M. Braune and C.C. Meyers of the University of Cincinnati conceived the idea of a large scale apparatus to accurately determine the lateral pressure of earth. The problem was assigned to B.H. Wulfekoetter (Fellow in Civil Engineering for 1916-1917). A preliminary design was made, but changes in the Engineering College caused by the war conditions prevented any considerable progress. The design was based on that of Mueller-Breslau (1906), in that the same types of wall and bin were used. The readings, however, were to be obtained by means of scales. The plans were submitted for correction and criticism to a number of men, well known in the Engineering profession, and especially in the subject of Earth Pressure and Retaining Wall design. The design called for a steel bin, 5 ft. high, with movable side walls, which walls could move in a direction parallel to the test wall, thereby attempting to eliminate the side wall effect. This part of the design was objected to by Professpr J.H. Smith of the University of Pittsburgh. Careful analysis by Professor Braune and the writer pointed out the error in this method pf trying to eliminate the side wall effect. For any motion of the side walls, caused by forces existing in the fill, changes the volume and shape of the fill, and prevents accurate determination of the existing pressures. Professor Wm Cain of the University of North Carolina warned against too weak a test wall, since deflections of the wall will cause a dissipation of the forces in the fill. The original design called for a thin steel wall, which was to be counterbalanced vertically. Dr. D. B. Steinman, Consulting Engineer, late Professor of Engineering at the College of the City of New York, suggested that the test-wall be left entirely free, since an actual retaining wall is not counterbalanced, but actually rests on the foundation. Since it is advisable to approximate actual conditions as closely as possible, the wall is not counterbalanced. He also suggested that the effect of different methods of placing the fill behind the wall be determined. Mr. Allen Hazen, Consulting Engineer, a member of the Soils Committee of the A.S.C.E., suggested a careful study of the effect of vibration and shock, as well as the question of packing and water content. Mr. Robert A. Cummings, Chairman of the Soils Committee of the A.S.C.E., offered the aid of the Committee in the proposed investigation, and suggested cooperation to prevent unnecessary duplication of work. Mr. A.T. Goldbeck, of the U.S. Dept. of Agriculture, Bureau of Highways and Rural Engineering, personally showed the writer the methods employed by his Bureau in conducting large scale tests in connection with highway research, and gave some valuable suggestions for the actual testing.

The writer, under the guidance of Professor Braune, as Baldwin Fellow in Civil Engineering (1919-1921) completed the design of the apparatus and supervised its construction and installation. Each part was carefully studied, numerous preliminary ideas were developed and changed in accordance with suggestions made by the above mentioned men and by others. In all, eighteen months were spent in completing the final design. The writer believes this rather long period well spent, for the apparatus, as constructed, has proven entirely satisfactory. After completing the tests, he has no suggestion to make, which would change the apparatus. It has proven very accurate, quite easy to manipulate, remembering the large amount of work required to conduct such tests, and still flexible enough, so that changes to take care of various tests could be easily made.

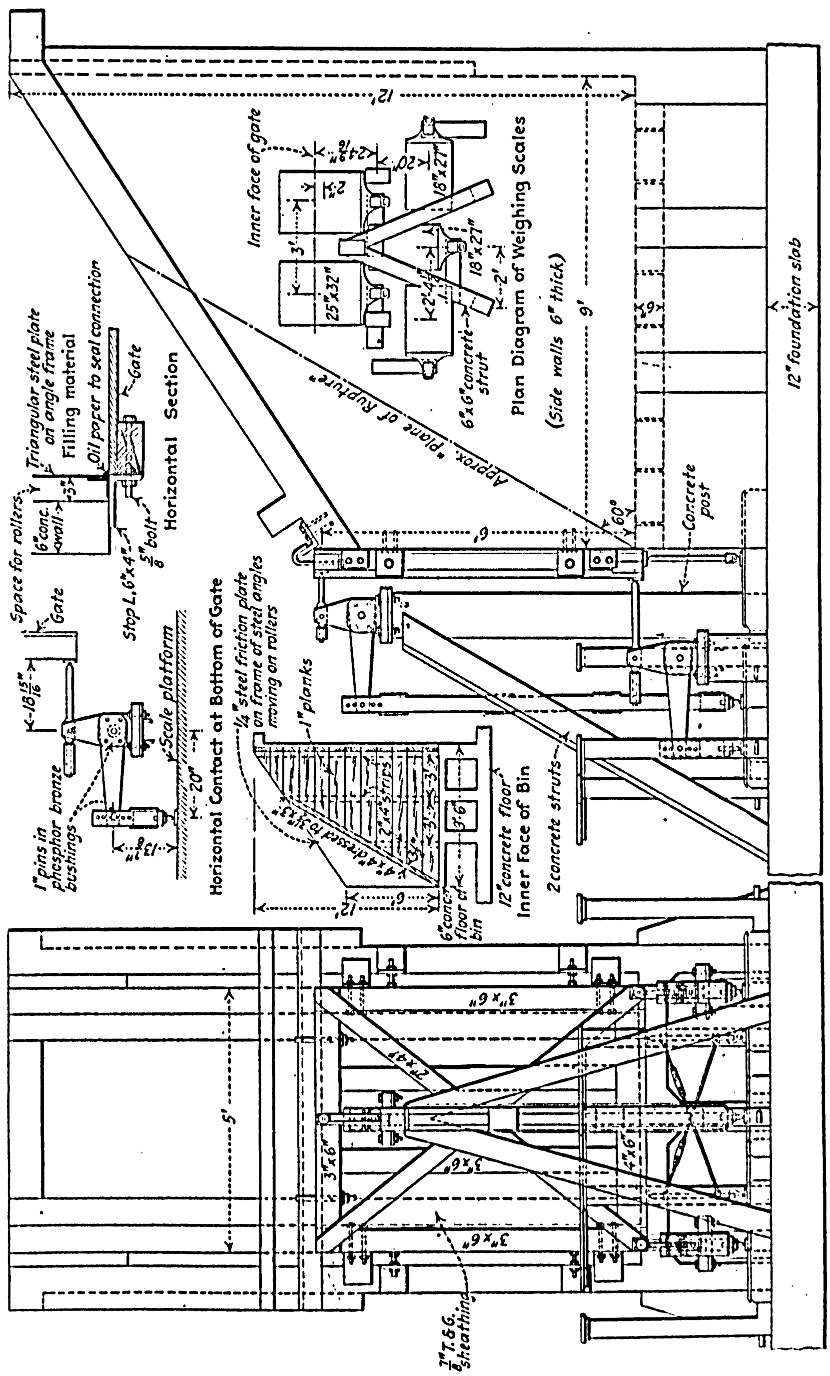
The final design takes into consideration the following points, as outlined in the statement of the accurate determination of the lateral pressures of granular materials. The construction is the most economical, consistent with the requirements of rigidity and sensitiveness. The pressures are measured by the action of a material in a bin upon a "free" wall closing one side. This test-wall is supported by two vertical and three horizontal contacts, measuring respectively the horizontal and vertical forces. Provision is made for experiments with vertical and sloped walls, also for variously surfaced walls. The size of the apparatus is such that the arching action of the filling material may be neglected and that it will give values not of minature size, and yet not require excessive labor. The method used for eliminating the side wall resistances is that suggested by Professor Braune: that is, to have that part of the fill which may tend to move rest against steel plate, which are supported on roller wheels bearing against the sides of the bin. The bin is large enough to allow a full plane of repose, has provision for sloping surfaces and placed so that the minimum labor is required to fill and empty it. The bin is well drained and rests upon a solid foundation, and so located, that it is free from vibrations and external shocks.

The work of installing as well as designing the apparatus was done under the guidance of Professor G.M. Braune. Since Sept. 1921, the testing and the computation has been carried on with the advice and under the guidance of Professor E.D. Gilman, of the Civil Engineering Dept. of the University of Cincinnati.

DESCRIPTION AND CONSTRUCTION.

Dimensions. The bin is of reinforced concrete, 5 ft. 6 in. wide, 9 ft. long and 6 ft. high in front and 12 ft. high in back. The front is entirely open, this being the location of the "free" test-wall. The back has an opening three feet wide, which is closed by planks to any required height. This opening decreases the difficulty of emptying the bin. The top of the bin slopes approximately at the natural repose of sand, 34°, thereby permitting the maximum case of sloping surface. The length of the bin is enough to allow a full plane of repose inside the bin, even when full. Assuming maximum conditions, a 6-in. wall was found sufficient. A reinforced concrete beam ties the side walls together in front and in back, and the side walls are supported between a longitudinal beam at the top and the floor at the bottom. The floor of the bin is 2 ft. 6 in. above the foundation, thus providing ample room for testing apparatus and at the same time simplifying the problem of filling and emptying the bin. The back of the bin faces a road, the level of which is halfway up the back. The foundation is a 12-in. reinforced concrete slab on packed cinders. In this way drainage and elimination of vibrations is assured. The floor is 6in. thick and rests on three longitudinal walls, one on each side and one on the center line. These supporting walls are stiffened by three cross walls, one at the back and two at the third points. The floor is therefore supported on a cellular affair of six cells, the front two being open in front. These cells are all connected by drain pipes to prevent any

Lateral Earth Pressure.



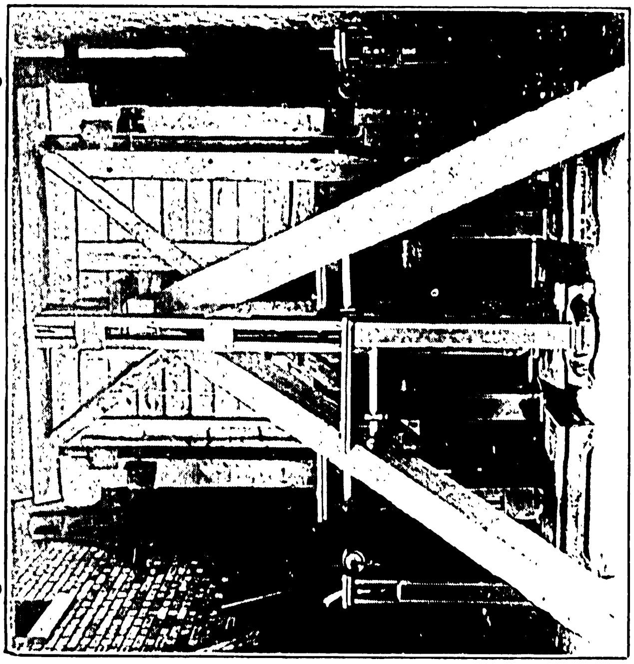
GENERAL DRAWING
EARTH PRESSURE APPARATUS

collection of water. Drain holes were left in the floor. The entire structure above the foundation was poured monolithic. The design was in accordance with the specifications of the Joint Committee on Concrete and Reinforced Concrete. The bin was made of concrete rather than of steel because of economy. A design for a steel bin had been completed, but was discarded. A steel bin might have caused trouble because of the continual presence of water in the fill. Painting would not have prevented rusting, because the process of filling and emptying the bin would have practically rubbed off all paint.

The "free" test-wall is of wood, of very stiff construction, 6 ft. 4 in. high and 5 ft. wide. A wooden wall is easier to handle and easier to change to suit various types of experiments, than a steel wall. A steel wall was designed and found to weigh 860 lbs., the wooden wall weighs 290 lbs. The latter consists of 7/8-in. sheating, well fitted, and stiffened by four 3 x 6 timbers vertically and a 3 x 6 on top and 4 x 6 on bottom, horizontally. The bin being 6 ft. high, there is a 2-in. lap of wall at top and bottom. The inside faces of the bin are 5 ft. 6 in. apart; the wall is 5 ft. wide. To take up the excess space, the bin is faced with rough timber to give a clear width of 5 ft., except in front, where the friction plates (see below) occupy the excess space.

Two bent bolts are inserted thru the top horizontal timber. These can be hooked over a 6 x 6 x 3/4 angle rigidly fixed into the top of the concrete bin. In this way the test wall is safely held when the apparatus is not being used. Two short angles are bolted to each side stiffener, so that the face of the

Lateral Earth Pressure.



FRONT VIEW
EARTH PRESSURE APPARATUS

angles is in the plane of the inner face of the wall and the legs of the angles bear against the front of the bin walls. In this way, the wall cannot move towards the bin or against the filling material. Two short angles are fixed into the front faces of the bin walls, so that the faces of the out-standing legs are flush with the inner faces of the bin walls. Bolts pass thru these projecting legs, with the heads towards the test wall. By adjusting these bolts, side or lateral motion of the gate is prevented. Such lateral movements may be caused by eccentricity of filling setting up unbalanced forces in the plane of the wall. By carefully watching these possible contacts, such eccentricity may immediately be detected. The method of changing the wall to take care of special cases is described in the section on tests.

Several methods for measuring the pressures were investigated. The simplest and, at the same time, accurate and sensitive, apparatus for measuring the forces acting upon the test-wall was chosen; namely, platform scales. The deciding factor was the amount of motion necessary to get readings. An indirect method, such as measuring deformations in struts, changes in resistance in a resistance pile, or the friction dynamometer is too uncertain, and requires continual calibration. Springs, tho they may be carefully calibrated, have too much deflection under the loads which act in these tests. The platform scales used were carefully calibrated to determine the deflection of the platform under various loads. For the vertical components, two 2000 lb. scales were used; for the horizontal components, three 1000 lb. scales were used. The scales are equipped with a single beam having a sliding poise with set-screw. The beam, which reads to 100 lbs. and has $\frac{1}{2}$ lb. graduations, passes thru a trig loop, limiting its range of motion to about 1 in. This means a movement of 1/100 in. of the platform. To decrease this, special stops were inserted into the scale cap over the end of the beam. These stops were shaped like a "T" with the horizontal arm above the beam and the vertical arm, threaded, passing thru lock nuts on the cap of the scale. By adjusting the stop, the movement of the beam can be cut down to 1/40 in.; the platform movement then being 1/4000 in. It was later decided to double this value, because of the difficulty of detecting the point of balance when the range of motion of the beam was so small.

The Contacts. The "free" gate rests on two vertical and three horizontal supports, The two vertical supports are symmetrically placed, at about the quarter points, and rest directly upon the platform scales. By inserting the platform into the open cell of the bin foundation, underneath the bin floor, these supports are placed in the centers of the platforms. (See detail in plans). Each vertical support consists of a 1-in. steel rod, the upper end pointed and the lower end threaded. The point bears into a steel bearing plate screwed to the wooden wall. The threaded end is screwed into a steel contact point which rests on a steel plate placed on the scale platform. These contact points allow a certain amount of motion of the test-wall vertically. By raising one or the other, the ax-is of the wall may be tipped in either direction. In this way, the wall may be adjusted for eccentricity. When the wall was exactly vertical, the two scales had the same zero reading.

Three horizontal contacts are used to give a stable system of support. Since the stress on the wall is a function of the depth, the three supports are placed, one at the top in the center, and one in each of the lower corners. Steel plates are screwed onto the wall at these points to form bearing plates. Each horizontal support consists of a strut pivoted to a bell crank. This arrangement changes the horizontal force to a vertical load on the scale platform. The bell crank arms are in the ratio of 1 to 2, thereby decreasing the reading to half the actual pressure. The vertical strut consists of two 5-in. channels, and at the lower end carries an adjustable contact point, a threaded 1-in. rod in a cast shoe. All parts are made very rigid to cut deformations to negligible values. All pivots are fitted in phosphor-bronze bushings to prevent frictional resistances. By adjusting the threaded contact points, the test-wall may be moved or tipped about its lower edge, and thereby adjusted for any deviation from the vertical. The bell cranks rest on steel plates fixed into stiffened reinforced concrete posts. Four 7/8 in. bolts are used to hold the bell crank in place. The lower steel plates have 1 1/8 in. holes for these bolts. This construction was considered necessary for the initial installation. A certain amount of flexibility is desirable, because of the size of all the parts. By using lock washers, there is no possibility of the crank arm being moved.

The scales were placed so as to make simultaneous readings possible. This required the compact arrangement shown in the details. Later, to prevent any possible wobbling of the vertical supports, two lateral cross braces were made and hooked to the rods. Turnbuckles were inserted in these braces for adjustment. These were found very useful in adjusting either vertical contact for deviation from the vertical.

All the steel parts were made by the Stacey Bros. Gas Construction Co., Elmwood, Ohio, during the summer of 1920. The company is to be commended for its generosity in providing these parts at a cost far below the actual value of the materials used in their manufacture.

The retarding effect of the side walls has always introduced an element of uncertainty into earth pressure measurements made in boxes and bins. Winkler tried to determine the resistance of each wall by duplicating his tests with a center partition. The second set of tests was affected by four wall friction losses. The first set of tests was affected by two friction losses. This method has been shown inaccurate. In this apparatus, it was decided to eliminate the resistance effect of the side walls. The first method decided upon, mentioned above, was to have the side walls on rollers, permitting slight motion in a direction parallel to the wall. It was thought that this flexibility would remove the effect of forces along the sides of the fill. The objection that any such motion would cause a change in the volume and shape of the fill, as well as alter the stresses acting therein, caused the abandonment of this design. Professor Braune then suggested a method approximating actual condition. Behind a retaining wall, a certain portion of the fill tends to move towards the wall. Taking any length of wall, the earth in back of it is not retarded by the earth on each side, because that too tends to move. Hence it was decided to have part of the side walls of the bin movable in the direction in which the fill tends to move, i.e. in a plane normal to the test wall.

The part of the fill which tends to move is approximately a triangular prism, with the bases formed by the surface of the fill, the back of the wall and the "plane of rupture". For sand, the wedge angle is approximately 30°. Two triangular steel plates were made, of the required shape. They rest on three phosphor-bronze bushed rollers bearing against the side walls of the bin. At the points of contact, the concrete was finished very smooth. The clear width inside the bin between the concrete walls is 5 ft. 6 in. Each of these stiffened plates, occupies 3 in., giving a clear width of 5 ft., the width of the test wall. That part of the side walls not covered by these plates is faced with timber, held to the sides by bolts run thru the walls, so that the bin is of uniform width at all points. These steel plates are accurately counterbalanced vertically. The preliminary tests, run before these plates were counterbalanced, showed the necessity of eliminating the pressure due to the tendency of these plates to slip down. Two rollers are placed at third points along the plane of rupture to eliminate the friction between the plates and the wooden facing of the walls.

Careful workmanship was required to make all joints as close as possible, to prevent sand flowing into the cracks. Thin strips of oiled paper, bent into angles, were placed along all the edges of the test wall. The joint between the steel plates and the timber was also covered with oiled paper. The area covered by this paper was so small compared to the total exposed area that no account of its effect was taken.

In these tests, the width of fill being 5 ft., losses due to the frictional resistances of the rollers back of the friction plates, may be disregarded. No attempt was made to alter the shape of these plates - since the wedge angle probably averaged very close to 30°. Some of the tests were run with these plates overbalanced, i.e. practically fixed. Altho the the resulting pressures are probably less, the amount of decrease is not very large. The error so incurred does not exceed the errors due to non-homogeneous fill or to slight differences or speed of filling. It was at first suggested to make these plates of wood, but $\frac{1}{4}$ in. plates backed by $2\frac{1}{2} \times 2\frac{1}{2}$ angles along the perimeter and three bracing angles across the plates, were found to be much stiffer.

METHOD OF OPERATION.

The most important factor in the method of operation was to guard against getting passive instead of active pressures. Before each test, every contact point was loosened until the test-wall was entirely free from the bin. This was to insure that the wall was not bearing against the bin. By adjusting the vertical supports until the two scales reading vertical components read exactly alike, the axis of the wall was put into a vertical position. A plumb bob hung on the wall showed whether the plane of the wall was vertical. By adjusting the upper horizontal support, the wall can be tipped forward or backward. All the horizontal contacts were then tightened till the increase in readings on the scales showed that the wall was bearing against the bin. The contacts were fixed so asto give the minimum readings on the scales, with the wall as close to the bin as possible. In this way a minimum of opening between the edges of the wall and the bin was obtained. These openings, as mentioned above, were closed by the thin strips of oiled paper. The apparatus was then left for at least one hour, usually over night, and the zero readings taken again. Readings could be taken to $\frac{1}{4}$ lb. and were so taken at the beginning and end of each set - all other readings were taken to $\frac{1}{2}$ lb.

The stops on the scale beams were adjusted to allow a total motion of 1/40 in. of the beam. For the vertical supports, this meant a total possible movement of the wall of 1/4000 in. The bell crank multiplication used on the horizontal supports reduced this maximum movement to 1/8000 in. Before any fill was placed into the bin, the riders on the scale beams were moved out, i.e. the scales were overbalanced, to a point more than the next reading. After a definite height of fill had been placed, the riders were slowly moved back, and the readings were taken when the beam no longer touched the lower support of the trig loop. As soon as the reading was taken, the rider was run out again to a point beyond the next possible reading. This method probably cut down the motion of the gate to a maximum of ½ its possible movement, or to 1/8000 in. vertical and 1/16000 in. horizontal-values below the elastic deformation of the wall, or of any actual retaining wall. The pressure recorded, therefore, is not the passive resistance of the fill caused by the movement of the wall.

To test the sensitiveness of the method, readings were often repeated in as short an interval of time as possible. The results were usually identical, readings very seldom varying by ½ lb. from the previous. The average readings being in the neighborhood of 500 lbs,, sometimes as high as 1500 lbs., this variation is negligible.

To determine the variation of pressures caused by greater movements of the scale beams and therefore of the wall, readings of one fill were taken for the scale beams free to move the entire height of the trig loop. In the following readings and all others given, the scales are numbered as follows:

No. 1 - the upper horizontal reading.

No. 2 - the lower left (facing the bin) reading.

No. 3 - the lower right (facing the bin) reading.

No. 4 - the left vertical reading.

No. 5 - the right vertical reading.

Note that the vertical readings less the zero readings give the the actual vertical pressures, while the horizontal readings must be doubled to take into account the bell crank device.

Scale Readings

	#1	#2	#3	#4	#5
Scale Beam in restricted position	554	464	528	985	701
Scale Beam at highest point of trig loop	533	451	510	951	675
Scale Beam at middle point of trig loop	541	459	518	965	683
Scale Beam at lowest point of trig loop (same position as when restricted)	553	465	527	985	701

The total movement of the beams was ½ in., corresponing to 1/200 in. vertical and 1/400 in. horizontal movement of the wall. This amount of motion had no effect upon the recorded pressures. Similar results were obtained in the other two tests to determine the effect of movements of the wall. We may therefore conclude that for values of such size, the pressures are unaffected. The effect of larger movements could not be determined.

In addition to the readings of the scales, record was kept of the temperature, time of day, state of the weather and age of the fill in the bin. In the first few tests, readings were taken for every 3 in. increase in height of fill. This was changed to 6 in. increments, since too much time was required, both during the test and in computing results.

The five readings give the magnitude, direction and point of application of the resultant earth pressure. The vertical component is the sum of the readings on scales 4 and 5, less the zero readings on those two scales. The total horizontal component is twice the sum of the readings on scales 1, 2 and 3, after the zero readings on these scales have been subtracted. The magnitude of the resultant can then be found. The direction is the angle whose tangent is the ratio of the total vertical to the total horizontal component. By taking moments about the intersection of a plane thru the vertical supports and a plane thru the lower horizontal supports, we eliminate the effect of these four actions, and have moments caused by the upper horizontal support and by the total resultant. The upper horizontal support is 6.04 ft. above this line. Resolving the resultant into a vertical component acting along the back of the wall and a horizontal component acting at a distance $\underline{x}$ above this axis of moments, and equating moments, we obtain $\underline{x}$, the height of the resultant pressure. In the case of a vertical wall, the wall itself has no moment, because the vertical supports are placed under its center of gravity. When additional parts were attached to the wall, for the cases of glass backed wall or oblique walls, the moment of these additional parts was taken into account.

Tests with glass and sheet metal backed walls were performed with the covering directly attached to the back of the wooden wall. For the cases of walls sloping towards the fill, a "false" wall was built and attached to the wall at three points. At the foot the "false" wall spiked to the main wall. At the mid-point and at the top, the two walls were connected by heavy struts. Similar methods of connection were used for the wall sloping away from the fill. Because of the large vertical component in this case, sixteen wires were also run from the lowest point of the false wall to various points in the main wall to help transmit the vertical component and decrease the deflection of the "false" wall. In both of those cases, the additional parts were inside the bin, and care was taken to prevent binding along the sides or contact on the floor. For the walls with considerable slope, results are not as accurate as for the tests with the vertical walls. Some of these tests were repeated to obtain check values.

In each test, a sample of the material used was taken. Air tight cans, holding about 15 lbs. of material were filled, at about the middle of each test. The material was later carefully tested for density, natural slope, coefficient of internal friction, coefficient of internal resistance and moisture content The coefficients of friction and resistance (kinetic and static friction) of the material of the fill and the material on the back of the wall was also determined. The method used to determine these coefficients was to measure the force required to start a weighed amount of material resting on the same material. This gave the coefficient of internal resistance. The force required to keep the mass in uniform motion gave the coefficient of internal friction. The mass was in a bottomless box with the lower edges bevelled off so that, practically, only the sand was in contact. The box was cubical in shape and contained 5½ lbs. of sand. The coefficient of internal resistance was taken as a measure of the combined effect of friction and cohesion; while the coefficient of internal friction was due to friction only, since there can be no cohesion during motion. The angles corresponding to these coefficients are respectively called the angles of internal friction and internal resistance. The angle of internal resistance in all cases (for sand) was found to be about 90 per cent of the angle of natural repose.

Outline of Tests

I. Variation in physical properties of the sand under various conditions.

II. Lateral pressure of horizontal fills against a vertical wooden wall.

III. Lateral pressure of horizontal fills against a glass backed vertical wall.

IV. Lateral pressure of horizontal fills against a sheet metal backed vertical wall.

V. Lateral pressure of sloping fills against a vertical wooden wall.

VI. Lateral pressure of sloping fills against a glass backed vertical wall.

VII. Lateral pressure of sloping fills against a sheet metal backed vertical wall.

VIII. Lateral pressure of horizontal fills against walls with positive back batter.

IX. Lateral pressure of horizontal fills against walls with negative back batter.

X. Lateral pressure of sloping fills against a wall with a positive batter of 1 : 4.

XI. Lateral pressure of sloping fills against a wall with a negative back batter of 1 : 6.

XII. Lateral pressure of irregular fills against a vertical wooden wall.

XIII. Effect of settling and changes of temperature upon the lateral pressure of horizontal fills against a vertical wall.

XIV. Effect of static loads on the lateral pressure of horizontal fills against a vertical wall.

XV. Effect of moving loads on the lateral pressure of tamped horizontal fills against a vertical wall.

XVI. Effect of static loads on the lateral pressure of horizontal fills against a wall with positive back batter of 1 : 4.

A few of these tests were performed in the spring of 1920; the remainder in the fall of 1920. Filling and emptying the bin 18 times required the moving of approximately 150 tons of sand back and forth. All this was by shovelling, there being no mechanical devices available.

I. Physical Properties of the Sand.

The material used was a river sand excavated from the subway cut on Canal St., Cincinnati, Ohio. When first obtained its specific gravity was about 2.6, its moisture content 9%. It weighed 100 lbs. per cu. ft., shovelled into a box, and slightly tamped. Continual exposure caused a slow drying out, and during the last tests, the moisture content was as low as 3%. The first tests are therefore with a "damp" sand, the rest with a "humid" sand, in accordance with the definitions set up by the A.S.C.E. Soils Committee. The moisture had considerable influence on the other physical properties. A mechanical analysis showed 92% passed the #4 sieve; 2½% passed the #100 sieve.

Sieve # :	4	8	10	14	28	48	100
% passed :	92	85	81	78	63	10	2½

The material contained very little clay, and showed very little dirt under the Abrams' Colorimetric Cleanliness test.

To measure the natural slope, the sand was poured on a table to form a cone about 12 in. high and the slope along six elements was measured. Four determinations of each coefficient of friction and resistance were made for every sample. At first the coefficient of resistance was measured by using a spring scale and also by weights over a pulley. The results were practically identical, so that the spring scale method only was used in the later tests. A complete set of physical tests was also made on dry sand.

The density of the material was a minimum, 90 lbs. per cu. ft. when the moisture content was between 4 and 5 per cent. The natural slope was greater for the more moist state; perfectly dry sand had a slope of 30°15'; sand with 9% moisture was about 10° higher. The same holds true for the angle of internal friction, 28°15' for dry sand and 34° for the very moist sand; and also for the angle of internal resistance, 28°15' for dry sand and 37° for the very moist sand. Note that for the dry sand, there is no difference between the angles of friction and resistance. This difference was 2° for a moisture content of 3%; and 3° for a moisture content of 9%. The intermediate values are fairly consistent. The difference between the internal friction and internal resistance is caused by the adhesion due to the presence of a water film around each particle of sand. This property was called "cohesion" by Coulomb and the name is still used, tho it is hard to see how any cohesive forces can act between particles or grains of a fill separated by a water film. This film is always present for there are no perfectly dry materials in back of retaining walls.

To determine whether the physical properties of the material varied with the height of fill, two loosely built wooden boxes, with no covers were imbedded in the sand for a period of three months, from June to September. One cube was 6 ft. below the surface; the other, 10 ft. below the surface.

Physical tests on the material in these cubes and of the surface material are as follows:

	Surface material	at 6-ft.depth	at 10-ft.
Density	100 lbs.	98 lbs.	101 lbs.
Moisture	9%	9%	9%
Natural slope	40°0'	40°40'	43°40'
Internal friction	34°0'	34°30'	35°30'

Very little can be concluded from these tests, because of the small samples used. Further investigation in this matter is necessary and should be carried out.

II. Lateral Pressure of Horizontal Fills against a Vertical Wooden Wall.

Several tests were performed in April and May, 1920, while the apparatus was being adjusted and calibrated. The following data was obtained after it was found that results could be closely duplicated and the apparatus was completed and satisfactory. In the first test, all figures and computations are given, to show the method. Columns contain the following:

1. Height of fill in bin.

2, 3, 4, 5, 6. The actual readings on scales #1, #2, #3, #4, #5.

7, 8, 9, 10, 11. The pressure recorded, obtained by subtracting the zero readings from the scale readings.

12. The top Horizontal pressure, H_1, double the reading.

13, 14. The bottom Horizontal pressures, H_2 and H_3; double the readings.

15. The total Horizontal component; the sum of H_1, H_2 and H_3.

16. The total Vertical component; the sum of the vertical scale readings, found in columns 10 and 11.

17. The total pressure, the square root of the sum of the squares of the two total components.

18, 19, 20. The horizontal component, the vertical component and the total pressure per foot length of wall. The test wall is 5 ft. wide.

21. The ratio of the total vertical to the total horizontal component.

22. The angle whose tangent is the above ratio.

The next few columns deal with finding the height of the total pressure, found by the formula developed for each type of test-wall. The results are compared with three theories. The Coulomb theory, where the wall friction is disregarded, is shown by the coefficient of ($\frac{1}{2}\gamma h^2$) as N. In the case of a vertical wall it is equal to the horizontal component, and takes no account of a vertical component. In the cases of inclined walls, the horizontal component, as given by this theory, is denoted by N_H. The general wedge theory, where the wall friction is taken into account, is shown by the coefficient of ($\frac{1}{2}\gamma h^2$) as C. The horizontal component is H.

The Rankine theory is shown by the coefficient of $(\frac{1}{2}yh^2)$ as R. The horizontal component is K.

It was decided to compare the theories by the coefficients of the quantities $(\frac{1}{2}yh^2)$, in order to compare the results with hydrostatic pressures. y is the density and h the height of fill.

To find the height of the total pressure, we note that the intersection of the plane thru the vertical contacts and the plane thru the lower horizontal contacts is level with the bottom of the fill. The upper horizontal contact is 6.04 ft. above this axis; the back of the wall, along which the vertical component, V, acts is 0.23 ft. from the axis. Calling x the height of the horizontal component,

$$Hx = 6.04H_1 + 0.23V, \quad \text{or} \quad x = \frac{6.04H_1}{H} + \frac{0.23V}{H}$$

The weight of the wall has no effect, because the vertical supports are below the center of gravity. The large amount of data in each test makes it impractical to give all the tests in full. In each case after the first, only the essential data is given.

See tables for Tests #1 & #2 below.

Lateral Earth Pressure.

Test #1.

May 12, 1921. 2-5 PM.
Wea ther : Rain.
Temperature : 10-12°C
(50-55°F)
Moisture Content : 9%

Physical Properties:
Density : 100 lb.
Angle of Internal Resistance: 37°
" " Internal Friction: 34°
" " Natural Slope: 40°
" "Friction on Wall:
(static friction) dry : 30°
" " damp: 32°30'.

1.	2.	3.	4.	5.	6.	7.	8.
	Scale		Readings.			Scale Measurements.	
h	H#1	H#2	H#3	V#4	V#5	#1	#2
in.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
0	95	60.5	60	150.5	157	---	---
6	96	62	64	152.5	160	1	1.5
9	97	66	70	159	165	2	5.5
12	97.5	78	82	178	183	2.5	17.5
15	99	90	95.5	199,5	194	4	29,5
18	100	100	104	214	205	5	39.5
21	102	112	117	234.5	222	7	51.5
24	106	124.5	132.5	259	247	11	60
27	113	137	146	284	269	18	76.5
30	122	146.5	163	296.5	303	27	86
33	138	17 1.5	188	345	333	43	111
36	148	194	212	377	372	53	133.5
39	161	200.5	231	403	412	66	140
42	178	215	248	422.5	432	83	154.5
45	187	232.5	268	465	482	92	172
48	209	246	287	494.5	532	104	185.5
51	229	263	305	526.5	560.5	134	202.5
54	250.5	278	326.5	561	588	155.5	217.5
57	274.5	295	349.5	610.5	632	179.5	234.5
60	303	313	371	657	676	208	252.5
63	332	326.5	385	699	721	237	266
66	386	362	439	791	785	291.	301.5
69	442	379	459	845.5	842.5	347	318.5
72	471	397	487	907	896	376	336.5

h: height of fill
H#1: top horizontal contact -scale reading.
H#2: bottom " " " "
H#3: " " " " "
V#4: vertical contact " "
V#5: " " " "

Test #1 (cont.)

1. in. h in.	9. Scale #3 lb.	10. Measurements. #4 lb.	11. #5 lb.	12. Horizontal Pressures. H#1 lb.	13. H#2 lb.	14. H#3 lb.	15. Total Comp. Horizontal lb.
0	----	----	----	----	----	----	----
6	4	2	3	2	3	8	13
9	10	8.5	8	4	11	20	35
12	22	27.5	26	5	35.	44	84
15	35.5	49	37	8	59	71	138
18	44	63.5	48	10	79	88	167
21	57	84	65	14	103	114	231
24	72.5	108.5	90	22	120	145	287
27	86	133.5	112	36	153	172	361
30	103	146	145	54	172	206	432
33	128	194.5	176	86	222	256	564
36	152	226.5	215	106	267	304	657
39	171	252.5	255	132	280	342	754
42	188	272	274	166	309	376	851
45	208	314.5	325	184	344	416	944
48	227	344	365	208	371	454	1033
51	245	376	403.5	268	405	490	1163
54	266.5	410.5	431	311	435	533	1279
57	289.5	460	475	359	469	579	1407
60	311	506,5	519	416	505	622	1543
63	325	548.5	564	474	532	650	1656
66	379	640.5	628	582	603	758	1973
69	399	695	685.5	694	637	798	2129
72	427	756.5	739	752	673	854	2273

Test #1 (cont.)

1. h	16. Total Comp. Vertical lb.	17. Total P. lb.	18. Pressures per ft. H lb.	19. V lb.	20. P lb.	21. V/H or tan ϕ'	22. direction of P ϕ'
0	-----	-----	-----	-----	-----	-----	-----
6	5	14	2.6	1.0	2.8	.385	21°00'
9	16.5	39	7.0	3.5	7.5	.472	25°15'
12	53.5	100	16.8	10.7	19.9	.637	32°30'
15	86	163	27.8	17.2	30.5	.623	31°55'
18	111.5	198	33.4	22.3	39.9	.667	33°40'
21	149	275	46.2	29.8	55.0	.645	32°50'
24	198.5	349	57.4	39.7	69.8	.692	34°40'
27	245.5	437	72.2	49.1	87.3	.680	34°10'
30	291	521	86.4	58.2	104.2	.674	34°00'
33	370.5	675	112.8	74.1	134.9	.658	33°20'
36	441.5	792	131.4	88.3	158.3	.673	33°55'
39	507.5	909	150.8	101.5	181.8	.673	33°55'
42	546.5	1011	170.2	109.2	202.2	.642	32°40'
45	639.5	1140	188.8	127.9	225.0	.678	34°10'
48	709	1253	206.6	141.8	250.6	.685	34°25'
51	779.5	1400	232.6	155.9	280.0	.670	33°50'
54	841.5	1531	255.8	168.3	306.2	.659	33°20'
57	935	1690	281.4	187.0	338.0	.665	33°40'
60	1025.5	1853	308.6	205.1	370.6	.665	33°40'
63	1112.5	1995	331.2	222.5	399.0	.673	33°55'
66	1268.5	2346	394.6	253.7	469.2	.642	32°40'
69	1370.5	2532	425.8	274.1	506.4	.644	32°50'
72	1495.5	2721	454.6	299.1	544.2	.659	33°20'

Test #1.

1.	23.	24.	25.		
	Point of	Application.		x	x/h
h	$6.04H_1$	0.23V	Sum of #s 23&24	#25/H	
0	------	------	-----	-----	-----
6	12	1	13	1.	---
9	24	4	28	0.80	---
12	30	12	42	.50	---
15	48	20	68	.44	.35
18	60	26	86	.52	.344
21	84	34	118	.51	.292
24	133	46	179	.625	.312
27	217	56	273	.757	.336
30	326	67	393	.910	.364
33	519	85	604	1.07	.389
36	640	102	742	1.13	.378
39	797	117	914	1.21	.373
42	1001	126	1127	1.33	.380
45	1111	147	1258	1.33	.355
48	1258	163	1421	1.38	.345
51	1620	179	1799	1.55	.365
54	1880	193	2073	1.62	.360
57	2152	215	2367	1.68	.354
60	2515	236	2751	1.78	.356
63	2862	256	3118	1.88	.358
66	3508	292	3800	1.93	.351
69	4190	315	4505	2.12	.369
72	4540	344	4884	2.15	.359

Lateral Earth Pressure.

Test #2.

May 4,1921; 2-3 PM. 0-3 ft.
May 5,1921; 2-4 PM. 3-6 ft.
Weather: Clear.
Temperature: 60-65° F.

Physical Properties:

same as in Test #1.

1. h in.	2. Top. horizontal H_1	3. Total-Pressures: H	4. Vert. V	5. Total P	6. Pressures per foot. #s 3,4,5/ 5 ft. H	7. V	8. P	9. ϕ' Direction of P	10. x/h Ratio of height of P:h
6	20	66	4	66	13.2	0.8	13.2	3°	
9	22	78	9	78	15.6	1.8	15.6	6°	
12	24	100	23	103	20.0	4.6	20.6	13°	
15	26	128	47	136	25.6	9.4	26.3	20°	
18	30	170	80	188	34.0	16.0	37.6	25°	
21	32	206	111	234	41.2	22.2	46.8	23°40'	
24	38	268	160	312	53.6	32.0	62.4	30°50'	
27	42	334	204	391	66.8	40.8	78.2	31°30'	
30	50	386	253	462	77.2	50.6	92.4	33°10'	
33	60	476	300	563	95.2	60.0	112.2	32°20'	
36	115	650	387	757	130.0	77.3	151.4	29°55'	
39	154	762	439	880	152.4	87.8	176.0	30°00'	
42	166	798	486	934	159,6	97.2	186.8	31°20'	
45	190	878	555	1040	175.6	111.0	208.0	32°20'	.398
48	244	1026	654	1215	205.0	130.8	242.3	32°50'	.404
51	318	1222	768	1443	244.4	153.6	288.6	32°10'	.395
54	352	1302	823	1541	260.4	164.6	308.2	32°10'	.389
57	396	1400	893	1661	280.0	176.6	332.2	32°30'	.391
60	469	1562	992	1851	312.4	198.3	370.1	32°20'	.391
63	556	1706	1071	2016	341.2	214.2	403.2	32°10'	.402
66	700	1986	1224	2333	397.2	244.8	466.6	31°40'	.404
69	752	2112	1344	2502	422.4	268.8	500.4	32°30'	.400
72	834	2292	1443	2708	458.4	288.6	541.6	32°10'	.390

Notes: The bin was filled in two days. The readings for the height of 36 in. are the averages of the readings at the end of the first day, and at the beginning of the second. The high values for x/h are due to the settling of the fill over night; see test XIII for the effect of settling. The low values of ϕ' at the beginning of the test are caused by the dry wall.

Lateral Earth Pressure.

Table showing comparison between experiment and theories for the case of a horizontal fill and a vertical wall. See Curves "A".

h in.	Test #1. H	(Pressures per foot of wall width.) P	Test #2. H	P	Average H	P	$\frac{1}{2}yh^2$ y:100	H/ /$\frac{1}{2}yh^2$	P/ /$\frac{1}{2}yh^2$
12	16.8	19.9	20.0	20.6	18.4	20.3	50	.368	.406
15	27.6	30.5	25,6	26.3	26.6	28.4	78	.341	.369
18	33.4	39.9	34.0	37.6	33.7	38.8	112.5	.300	.345
21	46.2	55.0	41.2	46.8	43.6	50.9	153	.284	.332
24	57.4	69.8	53.6	62.4	55.5	66.1	200	.278	.331
27	72.2	87.3	66.8	78.2	69.5	82.9	253	.275	.328
30	86.4	104.2	77.2	92.4	81.7	98.3	312	.262	.315
33	112.8	134.9	95.2	112.2	104.0	123.6	378	.276	.327
36	131.4	158.3	130.0	151.4	130.7	154,9	450	.290	.344
39	150.8	181.8	152.4	176.0	151.6	178.9	529	.287	.338
42	170.2	202.2	159.6	186.8	164.9	194.5	611	.270	.318
45	188.8	225.0	175.6	208.0	182.2	216.5	702	.260	.309
48	206.6	250.6	205.2	242.3	205.9	246.5	800	.257	.308
51	232.6	280.0	244.4	288.6	238.5	284.3	902	.265	.316
54	255.8	306.2	260.4	308.2	253.1	307.2	1012	.250	.301
57	281.4	338.0	280.0	332.2	2 80.7	335.1	1128	.249	.297
60	308.6	370.6	312.4	370.1	310.5	370.3	1250	.248	.296
63	331.2	399.0	341.2	403.2	336.2	401.1	1380	.244	.291
66	394.6	469.2	397.2	466.6	395.9	468.9	1510	.262	.310
69	425.8	506.4	422.4	500.4	424.1	503.4	1650	.257	.305
72	454.6	544.2	458.4	541.6	456.5	542.9	1800	.254	.302
					Averages from 24in.:			.264	.314

Coulomb and Rankine formulae are:

$$P = \tfrac{1}{2}yh^2 \tan^2 \tfrac{1}{2}(90^\circ - \phi) = N(\tfrac{1}{2}yh^2) = R(\tfrac{1}{2}yh^2).$$

Both theories give a Horizontal resultant.

If we take $\phi = 40^\circ$, the natural slope, $R = N = \tan^2 25^\circ = .218$
$\phi = 37^\circ$, the internal resistance, $N = \tan^2 26^\circ\tfrac{1}{2} = .248$
$\phi = 34^\circ$, t he internal friction, $N = \tan^2 28 = .283$

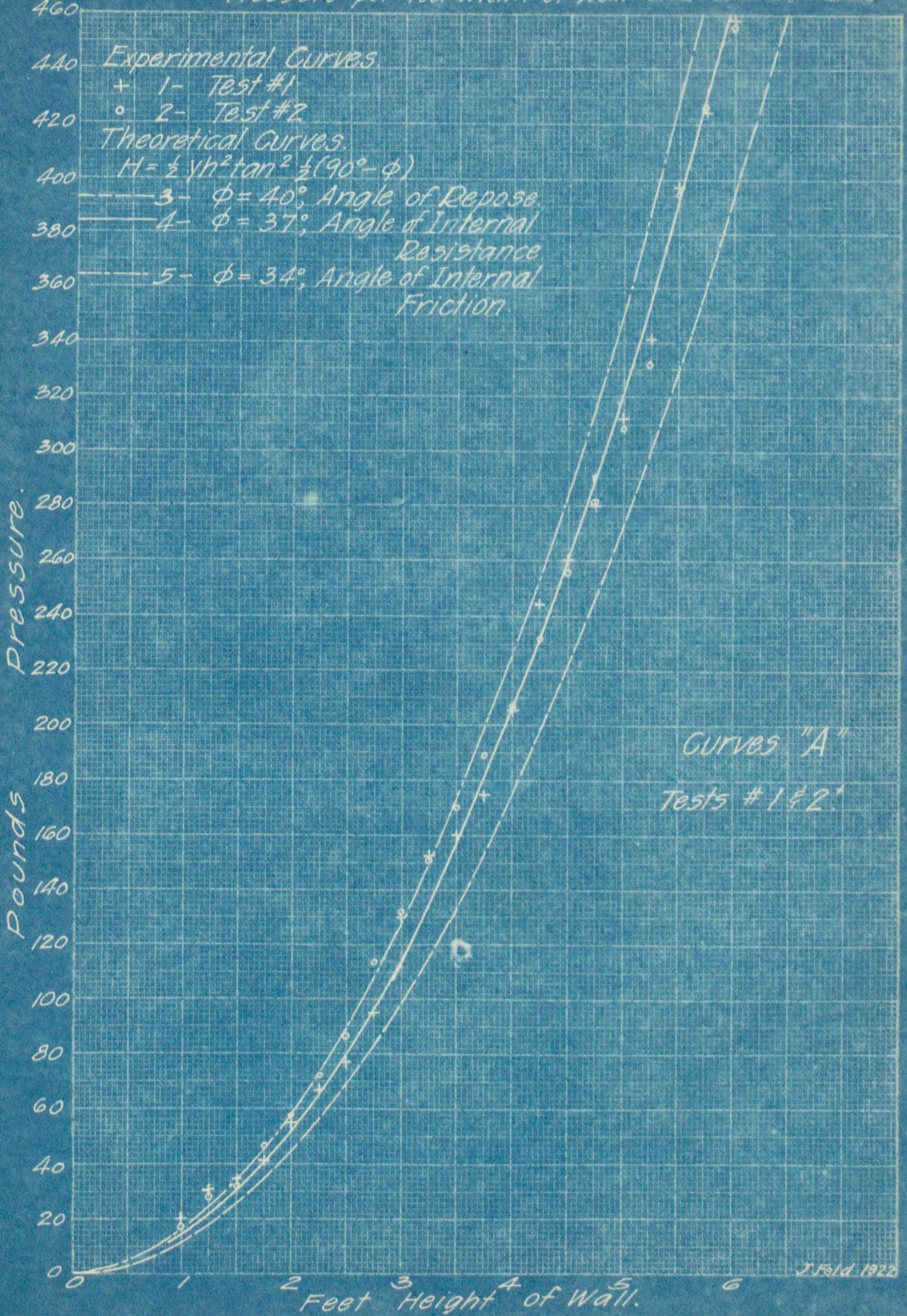
The total pressure in the general wedge theory is, for this case: $\frac{N}{\cos\phi'} = C_1 = \frac{.248}{\cos 32^\circ 30'} = .294$; for ϕ = internal resistance; ϕ' = friction along the wall. In C_1, we have assumed that the nature of the wall has no effect on the horizontal component. Assuming that it has, for $\phi = 37^\circ$, $\phi' = 32^\circ 30'$, $n = 0.818$, $C_2 = .230$.

$H = C_2 \cos\phi' = .193$; $n = \sqrt{\frac{\sin(\phi+\phi')\sin\phi}{\cos\phi'}}$

Curves for Vertical Wall and Horizontal Fill.

Lateral Earth Pressure.

Pressures plotted are the Horizontal Component of the Lateral Pressure per foot width of wall.



<u>Conclusions</u>: The two tests in this case give the same results, and agree quite closely. It is very evident that a vertical component exists. The amount is very closely given by the product of the horizontal component times the coefficient of friction along the wall. The average of all the readings gives for the angle of inclination of the resultant pressure, the value 33°. The fill was quite moist and the wooden wall, when exposed again was quite damp. The coefficient of friction between the sand and a wet yellow pine board, across the grain (identical with the case of the test wall) was found to be 32°30!

The point of application is above the third point, varying between 0.350h to 0.400h, depending on conditions. The cause and nature of this variation; as well as that of the inclination is taken up in later tests.

The Rankine and Coulomb theories give the same formula for this case. The formula seems to give the amount of the <u>horizontal</u> component very closely, if the value given to ϕ is the angle of <u>internal resistance</u>. Using ϕ as the natural slope gives low values. Using ϕ as the internal friction angle gives high values, which is to be expected, for we then disregard the effect of cohesion. The average value for $H/\frac{1}{2}\gamma h^2 = 0.264$, corresponds almost exactly to $N = \tan^2 \frac{1}{2}(90^\circ - \phi)$; where $\phi = 36^\circ$. The experimental value of the angle of internal resistance was 37°. The general wedge theory gives low values both for the horizontal component and for the total pressure, but the ratio is the same as found. The curves shown give a graphical comparison between the two experimental results, the Coulomb-Rankine theory for ϕ natural slope; and the formula $H = \frac{1}{2}\gamma h^2 \tan^2 \frac{1}{2}(90-\phi_1)$, where ϕ_1 is the angle of internal resistance. It had first been assumed that ϕ should be the angle of internal friction. Graphical comparison with the experimental curves, showed a constant difference between theory and experiment. It was evidently due to the disregarding of the cohesion effedt. Introducing the effect of the cohesion into the value of ϕ has solved the difficulty.

III Lateral Pressure of Horizontal Fills Against a Glass Backed Vertical Wall.

Closely fitting sheets of glass were fastened to the back of the wooden wall to a height of 24in. above a strip of wood nailed to the wall at the bottom. This 2-in. strip held the glass in place and prevented any slipping. Because of the small heights used, and this method of fastening, accuracy could not be expected.

Test #3.

Oct. 8, 1921.
Weather : Clear.
Temperature : 12°C
Moisture Content : 4.9%

Physical Properties:

Density :	93 lb.
Natural Slope	35°30'
Internal Resistance	33°50'
Internal Friction	31°50'
Wall Friction	28°10'

h in.	Top Hor. H_1	Total H	Comp. V	Total Press. P	Pressures per foot. H	V	P	V/H $\tan\phi'$	ϕ'	$H/\frac{1}{2}\gamma h^2$
12	2.5	61.5	46.5	77	12	9	15	.755	37°	.528
15	5.5	107.5	74.5	153	22	15	31	.694	35°	.304
18	9.5	144.5	98.5	175	29	20	35	.681	34°	.2 76
21	14	172	122.5	211	34	25	42	.712	35°	.240
24	23.	263.5	172.5	314	53	35	63	.655	33°	.285
									Average:	.273

$C_1 = \tan^2 \frac{1}{2}(90-\phi_1) = \tan^2 28°05' = 0.284.$
$C = \tan^2 \frac{1}{2}(90-\phi.) = \tan^2 27°15' = 0.265$

Very little can be concluded from this test except that, in general, the conclusions of the test with a wooden wall hold also for a smoother wall. The large value of the angle of inclination is due to the method of holding the glass in place; the bottom clamp gives the fill a chance to rest on the wall directly. Note that ϕ' decreases as the height of fill increase and a greater portion of glass surface acts. This is shown very well in the test of sloped surfaces against a glass wall.

IV Lateral Pressure of Horizontal Fills Against a Sheet Metal Backed Vertical Wall.

To determine whether the nature of the wall had any effect upon the lateral pressure, the wooden wall was covered with sheet metal to a height of 3 ft. The sheet metal was nailed directly to the wall. The surface was clean and polished.

Test #4.

Oct. 9, 1921
Weather : Clear
Temperature : 17°C
Moisture in Fill : 4.2%

Physical Properties :

Density : 94 lb.	
Natural Slope	38° 30'
Internal Resistance	37° 30'
Internal Friction	34° 50'
Wall Friction	28° 00'

h. in.	Top Hor. H_1	Total H	Comp. V	Tot. Press. P	Pressures per foot H	V	P	Inclination. ϕ'	$H/\frac{1}{2}\gamma h^2$	$V/\frac{1}{2}\gamma h^2$
14	4	81	51.5	96	16	10	19	32° 25'	.250	.297
17	5.5	106.5	74.5	128	21	15	26	35° 00'	.224	.277
20	8.5	160.5	97.5	188	32	20	38	31° 15'	.244	.290
23	13	220	135	258	44	27	52	31° 35'	.255	.301
26	20.5	279.5	168	326	56	34	65	31° 00'	.254	.295
29	31	354	210.5	412	71	42	82	30° 45'	.258	.298
32	41.5	415.5	245.5	483	83	49	95	30° 30'	.248	.284
35	58	501	293.5	580	100	59	116	30° 20'	.251	.291
38	78	584	340	675	117	68	135	30° 15'	.248	.286
							Averages:	31° 00'	.248	.291

$N = R = \tan^2 \frac{1}{2}(90-\phi) = \tan^2 25°45' = .232.$

$N_1 = R_1 = \tan \frac{1}{2}(90-\phi) = \tan^2 26°15' = .243.$

$C = \frac{N_1}{\cos\phi'} = \frac{.243}{.883} = .275$ (taking $\phi' = 28°$)

$C = .243/.857 = .284$ (taking $\phi' = 31°$).

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The results check the conclusions given above. The lateral pressure is therefore independent of the nature of the wall. The high value for the angle of inclination was expected. After the fill was removed, the surface of the sheet metal was found to have been rusted by the contact and rubbing of the damp sand.

V. Lateral Pressure of Sloping Fills against a Vertical Wooden Wall.

To determine the lateral pressure of non-horizontal fills against a vertical wall, two tests were run. In the first (test #5), the bin was filled to a height of 5 ft. with the surface sloping at the maximum possible angle below the horizontal. The fill was then changed, so that various sloped surfaces were obtained. The change in slope was in about 3^{o} intervals, until the greatest possible slope above the horizontal was obtained. In this way, we have the comparison of all possible surfaces with a fixed height of wall. In the second, (test #6) the surface slope was kept constant and different heights of fill were tested. It was found impractical to use the angle of repose as the slope to be tested, because the material would slip and the larger particles would roll down to the wall. As large a slope as possible was used. Further tests with sloped surfaces and broken surfaces are described in later sections.

The table showing test #5 gives the following data: The value of h= 5 ft; α, the wall inclination is 0. Column 1 gives the surface inclination, ε, negative if below the

Lateral Earth Pressure.

horizontal, positive when above the horizontal. It was difficult to obtain plane surfaces without considerable disturbance of the fill, so that a rod was fixed at the proper height, across the width of the bin, and 8 ft. in back of the wall; and the sand filled to a plane surface by eye. The surfaces below the horizontal were probably somewhat concave, while those above the horizontal were convex.

Column 2 gives the top horizontal pressure. The first two readings were probably a little too high, because of the impact of the fill as it was thrown against the wall.

Columns 3, 4, and 5 give the total horizontal and vertical components and the total pressure on the wall.

Column 6 gives the angle of inclination of the resultant.

Columns 7, 8, and 9 give the horizontal and vertical components and the total pressure per foot width of the wall.

Column 10 gives the ratio of the height of inclination to the total height of fill. This value is found the same way as in tests 1 and 2.

Column 11 gives the coefficient of $(\frac{1}{2}\gamma h^2)$ for the experimental horizontal component per foot of wall.

Column 12 gives the coefficient J of the formula Horizontal Component = $J(\frac{1}{2}\gamma h^2)$, where the general wedge theory is used, disregarding friction on the wall, and taking ϕ as the internal resistance.

$$J = \left(\frac{\cos\phi}{n+1}\right)^2 \quad ; \quad n = \sqrt{\frac{\sin\phi \, \sin(\phi - \varepsilon)}{\cos\varepsilon}}.$$

Lateral Earth Pressure.

Column 13 gives the coefficient K, of the formula: Horizontal Component $= K(\frac{1}{2}\gamma h^2)$, where the Rankine theory is used. The theory does not apply for negative slopes. ϕ is taken as the angle of internal resistance.

$$K = \cos^2\varepsilon \, \frac{\cos\varepsilon - \sqrt{\cos^2\varepsilon - \cos^2\phi}}{\cos\varepsilon + \sqrt{\cos^2\varepsilon - \cos^2\phi}} .$$

By comparing the horizontal components (in this case, also the normal components), we eliminate the different assumptions as to the direction of the total pressure.

Column 14 gives the coefficient H of the formula: Horizontal Component $= H(\frac{1}{2}\gamma h^2)$, where the general wedge theory is used, taking into account the friction of the wall. ϕ is taken as the angle of internal resistance.

$$H = \left(\frac{\cos\phi}{n+1}\right)^2 ; \qquad n = \sqrt{\frac{\sin(\phi+\phi')\,\sin(\phi-\varepsilon)}{\cos\phi' \quad \cos\varepsilon}} .$$

Column 15 gives the percentage ratio of the area of the theoretical wedge of rupture to the area of the wedge when the surface is horizontal. The plane of rupture was assumed to bisect the angle between the vertical and a line drawn at the angle of internal resistance to the horizontal.

$$A = \frac{1}{2}h^2 \, \frac{\sin\omega \, \cos\varepsilon}{\sin(90^\circ - \varepsilon - \omega)} , \qquad \omega = \text{angle of the wedge.}$$

In this case $\phi = 38^\circ$, $\omega = 26^\circ$.

Column 16 gives the percentage ratio of the horizontal component with respect to the horizontal component when the fill is level.

The same filling material was used in test #6, and the fill was placed at an angle of 34° above the horizontal, upto 3 ft.

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Test #5.

Sept. 25, 1921.
Sept. 27, 1921.
Weather: Cloudy and rain.
Temperature: 22°C.
$h = 5$ ft., $\alpha = 0$.

Physical Properties: Density: 95 lb.
Natural Slope: 39°30'
Internal Resistance: 38°
Internal Friction: 35°30'
Wall Friction: 32°30'

1. Surf. Slope ε	2. H_1 lb.	3. Total Pressure H lb.	4. Total Pressure V lb.	5. Total Pressure P lb.	7. Press./ft. H lb.	8. Press./ft. V lb.	9. Press./ft. P lb.	6. Inclin. of P. ϕ'	10. x/h	11. $\frac{H}{\frac{1}{2}\gamma h^2}$	12. J (Wedge) Theories	13. K (Rankine) Theories	14. H	15. % Area Wedge	16. % H (Exp.)
-33°40'	255	1064	679	1265	213	136	253	32°30'	.33	.179	.179	----	.163	76	78
-31°	253	1058	678	1260	212	136	252	"	"	.179	.183		.163	77	78
-28°05'	251	1052	679	1252	210	136	250	33°	"	.177	.187		.171	79	77
-25°	251	1052	685	1255	210	137	251	"	"	.177	.191	----	.176	81	77
-21°50'	255	1063	689	1270	213	138	254	"	"	.179	.196		.181	84	78
-18°30'	257	1089	719	1305	218	144	261	"	"	.184	.201		.186	86	80
-14°55'	258	1101	730	1320	220	146	264	33°30'	"	.185	.208	----	.192	89	80
-11°20'	287	1144	778	1385	229	156	277	34°	"	.193	.213		.198	93	84
- 7°40'	299	1229	810	1470	246	162	284	33°30'	"	.204	.217		.205	94	90
- 3°50'	331	1314	868	1570	263	174	314	33°	.34	.221	.225	----	.212	97	96
0	365	1412	914	1680	287	183	336	32°30'	"	.237	.237	.237	.220	100	100
+ 3°50'	399	1487	946	1760	297	189	352	33°	.35	.250	.253	.239	.228	102	108
+ 7°40'	416	1547	1011	1850	309	202	370	32°30'	.35	.260	.264	.241	.239	107	113
+11°20'	517	1679	1080	1945	336	216	389	"	.40	.282	.273	.244	.250	111	123
+14°55'	532	1712	1100	1990	342	220	398	"	.40	.288	.285	.249	.264	115	125
+18°30'	569	1833	1170	2160	367	234	432	"	.40	.308	.296	.253	.279	119	134
+21°50'	608	1939	1240	2305	388	248	461	"	.42	.327	.314	.266	.295	124	142
+25°	680	2079	1330	2480	416	266	496	"	.40	.351	.332	.278	.314	129	152
+28°05'	796	2319	1500	2610	464	300	550	33°	.44	.391	.352	.296	.344	135	169
+31°	804	2344	1530	2650	469	306	556	"	.44	.395	.391	.319	.377	142	203
+34°	834	2504	1600	2900	501	320	594	32°30'	.43	.423	.425	.362	.432	149	216
Averages:										.257	.253		.242		

Lateral Earth Pressure.

Test #6.

Date: Sept. 22, 1921.
Weather: Clear.
Temperature: 21°C.

Physical Properties, same as for test #5.
Fill at $\varepsilon = 34^{\circ}$.

h	H_1	H	V	P	H	V	P	ϕ'	x/h	$H/\frac{1}{2}yh^2$	$P/\frac{1}{2}yh^2$
in.		Total Pressure.			Pressure/ft.			Inclin. of P			
12	13	112	43	120	24	9	24		---	.53	.53
18	$15\frac{1}{2}$	163	103	192	33	21	38	32°	.48	.33	.36
24	25	373	211	429	75	42	86	30°	.28	.419	.480
30	$40\frac{1}{2}$	501	300	584	100	60	117	31°	.33	.356	.416
36	102	756	473	891	151	95	178	32°	.33	.373	,440
						Averages 2 ft. to 3ft:				382	.444

The theoretical coefficients of $(\frac{1}{2}yh^2)$ are:

H = 0.334; C = 0.396 for the wedge theory considering wall friction.

N = 0.384 for the wedge theory disregarding wall friction.

K = 0.333, R = 0.402 for the Rankine theory.

The coefficient for the total pressure, corresponding to N = 0.384, and taking $\phi' = 32°30'$ (as in test #5), is 0.455

taking $\phi' = 31°$ (as seems to be case in this test) is 0.445.

Conclusions.

For a vertical wall, with any inclination of the surface of the fill, the value of the lateral earth pressure is given in

a. Magnitude, by the formulae derived by the wedge theory for the horizontal component assuming the pressure to be normal to the wall, i.e. disregarding the wall friction. This might be expected since the previous tests showed that the nature of the wall surface had no effect on the horizontal component. The vertical component is equal to the horizontal times the coefficient of friction on the wall.

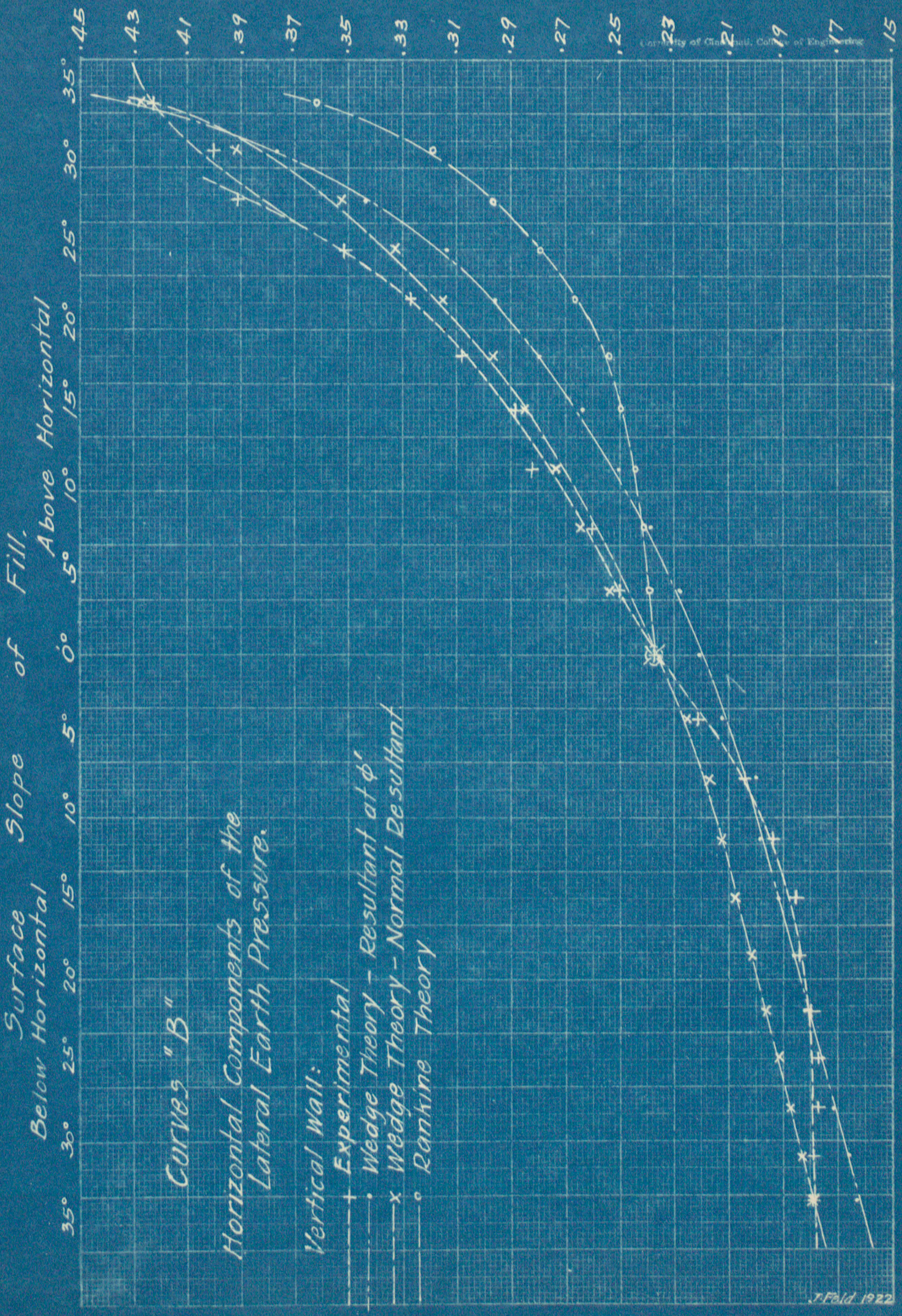
Lateral Earth Pressure.

b. direction, by the angle of friction between the fill and the surface of the wall.

c. point of application, by the value 0.33-0.40 h, depending upon conditions. These conditions are given in later tests.

The point of application of the resultant for fills below the horizontal is at 1/3h. For fills above the horizontal it is higher, reaching 0.40 h at the greatest sloped fills. This test, absolutely proves the presence of a vertical component for all cases. Rankine's theory would give an upward component for all the cases of negative ε. Columns 15 and 16 were computed to compare the variation of pressure with that of the wedge area. The pressures seem to vary much faster than the area of the wedge, especially at the greater slopes. The value of the pressure is therefore a function of a number of variables, including the area of the wedge, and not upon the area of the wedge itself, as is assumed by Rebhann in his wedge theory. Obtaining the maximum pressure by differentiating with respect to the area of the wedge is therefore incorrect. This is based on the assumption of a plane of rupture, which bisects the angle between the vertical and the line drawn at the angle of internal resistance to the horizontal. The general wedge theory gives low values, because it assumes the friction of the wall to diminish the pressure.

Curves for Sloped Fills & Vertical Wall
Coefficients of $\frac{1}{2}\gamma h^2$



Lateral Earth Pressure.

VI. Lateral Pressures of Sloping Fills against a Vertical Glass-backed Wall.

A few readings were taken of the pressure of a fill 2 ft. high and inclined at 31° to the horizontal against a glass-backed vertical wall.

The pressure per foot of width was 107 lbs, the horizontal component was 92 lbs., or a coefficient of 0.500. The theory for this case gives $N = 0.495$, for $\phi_1 = 33°50'$, $\varepsilon = 31°$. The angle of inclination was about 30°, somewhat higher than the coefficient of friction on the glass wall, due to the obstructions on the wall in the shape of clamps. The results agree with the conclusions given in section V.

VII. Lateral Pressures of Sloping Fills Against a Sheet Metal Backed Vertical Wall.

To determine whether the conclusions of section V were affected by changing the wall, the test for fills above the horizontal was repeated with sheet-metal wall. Height of fill was 3 ft.

Test #7.

Date : Oct. 9, 1921.
Weather: Clear.
Temperature: 17°C.
Height of fill : 3 ft.

Physical Properties: Density : 94 lb.
Natural Slope: 38°
Internal Resistance: 34°30'
Internal Friction: 32°
Wall Friction: 28°

Surface Slope ε	H_1	Total Pressures. H	V	P	Pressures /ft. H	V	P	Inclination of P ϕ'	x/h	$H/\frac{1}{2}\gamma h^2$
0°	122	585	340	675	117	68	135	30°	.333	.280
+ 4°45'	127	601	350	695	120	70	139	30°	.339	.297
+11°20'	143	669	397	779	134	79	156	30°30'	..348	.316
+16°40'	167	764	434	884	153	87	177	28°30'	.358	.337
+21°50'	266	895	495	1045	175	99	209	28°	.375	.366
+26°30'	237	996	544	1138	199	109	228	28°30'	.402	.409
+27°40'	270	1105	592	1352	221	118	250	28°	.415	.423
							Averages:	28°45'		.347

Lateral Earth Pressure.

To compare the coefficient of $(\frac{1}{2}\gamma h^2)$ as found experimentally with those found by the various theories, for horizontal components

Experiment:	.280	.297	.316	.337	.366	.409	.423
N (wedge theory)	.277	.288	.311	.336	.363	.403	.416
K (Rankine theory)	.277	.277	.284	.296	.317	.349	.362
H (wall friction theory)	.220	.228	.254	.279	.308	.347	.363
(surface slope)	0	$4^o45'$	$11^o20'$	$16^o40'$	$21^o50'$	$26^o30'$	$27^o40'$

The above results show the same conclusions as those of test #6. The resultant acts at the angle of friction on the wall, and above 1/3 h, especially for the greater slopes. The amount of the horizontal component is given by the wedge theory which disregards the effect of the friction of the wall. Sections V, VI and VII show that the horizontal component of the pressures of any kind of fill on a vertical wall is independent of the nature of the wall surface.

VIII. Lateral Pressures of Horizontal Fills against Wooden Walls with Positive Back Batter.

Four walls were used, with backs sloping at 1:12 ($4^o45'$), 1:8 ($7^o15'$), 1:6 ($9^o30'$) and 1:4 ($14^o00'$). The fixed parts of the recording apparatus required the whole test wall to be placed inside the bin. The desired shape of wall was constructed and nailed to the inside face of the wooden vertical wall. In the tests of this set, considerable trouble was given by sand leaking thru cracks and getting under the wall. Several of the tests had to be repeated because the vertical component would be decreased by sand getting underneath the wall. In the repeated tests, it was found that this had little effect on the horizontal readings, for the friction under the wall was quite small, the sand grains being fairly spherical.

A preliminary test to determine the direction of the resultant was run with a 1:4 wall. In this test, special care was taken to have the wall perfectly free. The fill was approximately 4 ft. high, but because of the several changes in its shape made to prevent leakage, no attempt is made to deduce the magnitu~~t~~de of the pressure from the data of this test. The height of fill was always measured vertically.

TEST SCALE PRESSURES.

	Top Horiz.	Bott. Horiz.	Vertical.	Total Horiz.	V/H	($\phi + \alpha$)
a.	87	708	714	795	.895	41°55'
b.	85	714	739	799	.925	42°45'
c.	85	712	716	797	.898	41°55'
d.	81	716	749	797	.940	43°10'
e.	83	714	737	797	.925	42°45'
f.	83	714	742	797	.930	42°00'
g.	84	715	727	799	.910	42°20'
				Average:	.918	42° 25'

After each reading, the space underneath the wall was cleaned out. This necessitated raising the wall about 1/100 in. each time, and lowering it again. The readings are quite consistent after such treatment of the wall.

The resultant is inclined at an angle of 42°25' to the horizontal. The wall slopes at 14°00', so that the resultant makes an angle of 28°25' to the normal to the wall. The angle of wall friction as experimentally determined for this sand (93 lbs. per cu. ft., moisture content 3.2%, $\phi_1 = 31°$) was just below 29°. This affords absolute proof

Lateral Earth Pressure.

that the resultant on a wall inclined with the base towards the fill acts at an angle from the normal, which angle is the angle of friction between the wall and the fill.

To determine the magnitude and point of application of the resultant, a test was run on each wall with horizontal fills, and also a test with oblique fills (described later) on the 1:4 wall.

Readings were taken as in the tests with the Vertical walls. The computation of the components and of the total pressure is also the same. In addition, there is given the value of the normal and tangential components (N and T) on the wall. The relations which exist between the P (total pressure), H (Horizontal component), V (vertical component), N, T, ϕ' (angle of wall friction) and α (angle of inclination of the wall from the vertical) are:

$$P = \sqrt{H^2+V^2}; \qquad \tan(\phi'+\alpha) = V/H.$$

$$H = P\cos(\phi'+\alpha); \quad N = P\cos\phi' = \frac{H\cos\phi'}{\cos(\phi'+\alpha)}$$

$$V = P\sin(\phi'+\alpha); \quad T = P\sin\phi' = \frac{H\sin\phi'}{\cos(\phi'+\alpha)}$$

In computing the height of the resultant, the moment of the wall about the vertical supports had to be taken into account, for the wall was no longer absolutely balanced.

The height of the resultant $\underline{x} = \dfrac{6.04H_1 + V(0.48+\frac{6}{a}) + V_0(0.48+\frac{2}{a})}{H+\frac{V}{a}}$.

for taking moments about the vertical supports

$$Hx = H_1(6.04) + 0.48V + V(6-x)\frac{1}{a} + V_0(0.48+\frac{6}{3a}).$$

where H = total horizontal component

Lateral Earth Pressure.

H_1 = top horizontal component with lever arm of 6.04

V = total vertical component, taken to act at the same point on the back of the wall as H.

V_0 = additional weight of the wall due to false wall

a = batter ration, (1:a)

V acts at a distance $(0.48 + \frac{6 - x}{a})$ ft. from the axis.

V_0 acts at a distance $(0.48 + \frac{6}{3a})$ ft. from the axis.

assuming the additional piece to be a solid triangular wedge of base 6/a ft.

Test #8.

Date : Nov. 17, 1921.
Weather : Clear.
Temperature : 14°C.
Wall Slope :+ L : 12 (4°45')
Moisture Content : 3.5%

Physical Properties : Density: 96 lb.
Natural Slope : 32°45'
Internal Resistance: 29°40'
Internal Friction : 27°30'
Wall Friction : 29°.

Height of Fill. (ft.)	Pressures per ft. of Wall. H	V	P	N	T	Height of P. x/h	Coefficients of $(1/2\gamma h^2)$ H	P	N
2.5	70	47	84	73	44	.39	.234	.280	.244
3	92	62	112	93	54	.39	.213	.260	.215
3.5	125	84	151	132	74	.405	.213	.257	.225
4	154	104	187	163	90	.396	.203	.247	.215
					Averages:	.395	.216	.271	.225

$V = H \tan(\phi' + \alpha)$, $T = N \tan(\phi')$.

Test #9.

Date : Nov.20, 1921.
Weather : Clear.
Temperature : $9\frac{1}{2}$°C.
Wall Slope :+1:8 (7°15')
Moisture Content : 3.5%

Physical Properties : Density: 96 lb.
Natural Slope : 32°45'
Internal Resistance: 29°40'
Internal Friction : 27°30'
Wall Friction : 29°.

Height of Fill.	Pressures per ft. of Wall. H	V	P	N	T	Height of P. x/h	Coefficients of $(\frac{1}{2}\gamma h^2)$ H	P	N
2.5	69	49	85	74	41	.345	.230	.283	.247
3	88	64	108	95	53	.337	.204	.250	.220
3.5	115	84	142	124	69	.378	.195	.241	.211
4	148	108	184	160	89	.420	.193	.239	.208
					Averages:	.370	.208	.253	.222

Lateral Earth Pressure.

Test #10.

Date : Nov. 21, 1921.
Weather : Clear.
Temperature : 10°C.
Wall Slope : +1:6 (9°30')
Moisture Content : 3.7%

Physical Properties: Density : 97 lb.
Natural Slope: 36°
Internal Resistance: 32°
Internal Friction : 29°40'
Wall Friction : 29°

Height of Fill. H	Pressures per ft. of Wall. V	P	N	T	Height of P. x/h	Coefficients of $(\frac{1}{2}\gamma h^2)$ H	P	N
2.5	61	48	78	73	34	.388	.202	.254	.241
3	81	65	104	91	50	.373	.186	.238	.208
3.5	121	97	155	136	75	.392	.204	.261	.229
4	157	125	194	176	97	.405	.203	.250	.227
			Averages:			.390	.199	.251	.226

Test #11.

Date : Nov. 23, 1921.
Weather : Clear.
Temperature : 14°C.
Wall Slope : +1:4 (14°00')
Moisture Content : 3.7%

Physical Properties: Density: 97 lb.
Natural Slope : 35°
Internal Resistance : 31°
Internal Friction : 29°15'
Wall Friction : 29°.

Height of Fill. H	Pressures per ft. of Wall. V	P	N	T	Height of P. x/h	Coefficients of $(\frac{1}{2}\gamma h^2)$ H	P	N
2.5	93	86	127	111	61	.390	.308	.420	.367
3	125	117	172	150	83	.388	.287	.394	.344
3.5	152	142	209	182	101	.409	.256	.352	.306
			Averages :			.396	.284	.389	.339

Lateral Earth Pressure.

To compare the experimental results with the various theories, the following coefficients were computed for each type of wall. The Rankine Theory, as originally given, does not apply to a sloped wall. The general wedge theory, assuming the resultant to act at ϕ' to the normal to the wall, i. e. $(\phi'+\alpha)$ to the horizontal gives

$$P = 1/2h^2y(C) \quad \text{and} \quad H = 1/2yh^2(H)$$

for the total and horizontal components. For the case of a level fill,

$$(C) = \left(\frac{\cos(\phi-\alpha)}{(n+1)\cos\alpha}\right)^2 \frac{1}{\cos(\phi'+\alpha)}, \quad n = \sqrt{\frac{\sin(\phi+\phi')\sin(\phi)}{\cos(\alpha)\cos(\phi'+\alpha)}}.$$

$$(H) = C\cos(\phi'+\alpha).$$

The wedge theory which assumes the resultant pressure to act normal to the wall, gives $N = \frac{1}{2}yh^2(N)$ as the normal component, where

$$(N) = \left(\frac{\cos(\phi-\alpha)}{(n+1)\cos\alpha}\right)^2 \frac{1}{\cos\alpha} = \left(\frac{\cos(\phi-\alpha)}{\sin\phi+\cos\alpha}\right)^2 \frac{1}{\cos\alpha}, \quad n = \frac{\sin\phi}{\cos\alpha}.$$

Table Comparing Experiment with Theory- Inclined Walls.
See Curves "C". Horizontal Fill, Heel of Wall in Fill.

				Experimental.			Theoretical-	-----	-----	-Experiment.		
Wall	α	ϕ'	y	H	C	N	C	H	N	J	x/h.	ϕ
+1:12	4°45'	29°	96	.216	.271	.225	.338	.281	.371	.370	.395	29°40'
+1: 8	7°15'	29°	96	.208	.253	.222	.360	.290	.383	.380	.370	29°40'
+1:6	9°30'	29°	97	.199	.251	.226	.255	.278	.374	.368	.390	32°00'
+1:4	14°00'	29°	97	.284	.389	.339	.413	.302	.428	.415	.396	31°00'
Averages:				.227	.291	.253	.367	.288	.389	.383	.388	

The ratio between the experimental and theoretical values of H = .790; C = .795, N = .65, so that the tests seem to show that the general wedge theory taking into account the wall friction applies to the case in battered walls and a horizontal fill. The resultant is inclined to the normal at an angle equal to the angle of wall friction. The resultant acts at about 3/8 h. The error in these tests is quite large, unless we disregard all theories, as giving too high results.

IX. Lateral Pressure of Horizontal Fills against a Wooden Wall with Negative Back Batter.

Five walls were used, with backs sloping at 1:12 ($4^{o}45'$), 1:6 ($9^{o}30'$), 1:4 ($14^{o}00'$); 1:3 ($18^{o}25'$); 1:2 ($27^{o}15'$). The wall was constructed in the same manner as described in sec. VIII. Since the lower end was a point, the upper end now overhanging, there was no trouble in attaching the extra wall to the original vertical wall, and no possibility of sand leaking under the wall.

Readings and the computation of the pressures are the same as for the vertical walls. In computing the height of the resultant, the moment of the wall about the vertical supports had to be taken into account, for the wall was no longer absolutely balanced. The height of the resultant x is given by the formula

$$x = \frac{6.04\,H_1 + 0.48V + V_0\left(0.48 + \frac{4}{3a}\right)}{H - \frac{V}{a}};$$

the derivation of this formula is the same as that given in the previous section.

Lateral Earth Pressure.

Test #12.

Date : Oct. 30, 1921.
Weather : Clear.
Temperature : 18° C.
Wall Slope : -1:12, (4°45'); $(\phi' + \alpha)$ = 25°.
Moisture Content : 3.65%

Physical Properties: Density: 97 lb.
Natural Slope : 34°50'
Internal Resistance: 30°30'
Internal Friction : 27°55'
Wall Friction : 29°45'

Height of Fill. h(ft)	Top H_1	Total Pressures. H	V	P	$(\phi' + \alpha)$	Pressures/foot. H	V	P	x/h	$\frac{H}{1/2yh^2}$	$\frac{P}{1/2yh^2}$
2	45	204	84	220	22°	41	17	44	.370	.216	.232
2.5	67	298	127	308	23°	60	25	62	.344	.201	.208
3	105	458	207	503	24°	92	41	101	.343	.214	.235
3.5	152	639	307	710	26°	128	61	142	.340	.218	.246
4	219	892	441	995	26°20'	178	88	199	.377	.234	.262
4.5	238	1067	532	1190	26°30'	213	106	238	.375	.222	.248
			Averages :		24°38'				.358	.221	.239

Test #13.

Date : Nov. 6, 1921.
Weather : Clear.
Temperature : 14°C.
Wall Slope: -1:6, (9°30'); $(\phi' + \alpha)$ 19°30'.
Moisture Content: 3.25%

Physical Properties : Density: 94 lb.
Natural Slope : 38°20'
Internal Resistance: 31°50'
Internal Friction: 28°30'
Wall Friction : 29°

Height of Fill. h(ft)	Top H_1	Total Pressures. H	V	P	$(\phi' + \alpha)$	Pressures/foot. H	V	P	x/h	$\frac{H}{1/2yh^2}$	$\frac{P}{1/2yh^2}$
2.5	26	306	91	320	16°30'	61	18	64	.40	.208	.218
3	43	424	139	445	18°10'	85	28	89	.35	.201	.210
3.5	74	565	199	599	19°20'	113	40	120	.34	.196	.208
4	126	645	256	695	21°40'	129	51	139	.40	.172	.185
		Averages:			19°00'				.37	.195	.205

Lateral Earth Pressures.

Test #14.

Date :Nov.10,1921. Physical Properties: Density: 97 lb.
Weather :Clear. Internal Resistance : 31°
Temperature: 7°1/2. Natural Slope : 34°
Wall Slope:-1:4,(14°); $(\phi'+\alpha)$=13°. Internal Friction : 29°
Moisture Content: 3.5% Wall Friction : 27°

Height of Fill. h(ft)	Top H_1	Total Pressures. H	V	P	$(\phi'+\alpha)$	Pressures/foot. H	V	P	x/h	$\frac{H}{1/2yh^2}$	$\frac{P}{1/2yh^2}$
2.5	20	260	58	266	12°25'	52	12	53	.48	.173	.177
3	39	372	84	380	12°35'	74	17	76	.32	.167	1171
3.5	88	545	123	560	12°45'	109	25	112	.36	.185	.190
4.	134	695	154	710	12°30'	139	31	142	.36	.183	.187
			Averages :		12°34'				.36	.177	.181

Test #15.

Date: Nov.11, 1921. Physical Properties: Density: 97 lb.
Weather:Cloudy and Rain. Internal Resistance: 30°20'
Temperature: 8°C. Natural Slope : 36°20'
Wall Slope:-1:3,(18°25'); $(\phi'+\alpha)$= Internal Friction : 27°30'
Moisture Content:3.55% =11°35'.Wall Friction : 30°

Height of Fill. h(ft)	Top H_1	Total Pressures. H	V	P	$(\phi'+\alpha)$	Pressures/foot. H	V	P	x/h	$\frac{H}{1/2yh^2}$	$\frac{P}{1/2yh^2}$
2.5	18	269	67	276	14°	54	13	55	.40	.181	.185
3	38	365	80	373	12°20'	73	16	75	.39	.171	.175
3.5	79	488	101	499	11°40'	98	20	100	.39	.167	.170
4	122	619	128	630	11°10'	124	26	126	.40	.163	.166
			Averages:		12°53'				.395	.171	.174

Test #16.

Date: Nov.13, 1921. Physical Properties: Density: 97 lb.
Weather : Cloudy. Internal Resistance: 30°20 '
Temperature: 6°1/2 C. Natural Slope : 36°30'
Wall Slope :-24.75 : 48,(27°15') Internal Friction: 27°30'
Moisture Content:3.55% $(\phi'+\alpha)$= 2°45'. Wall Friction: 30°

Height of Fill. h(ft)	Top H_1	Total Pressures. H	V	P	$(\phi'+\alpha)$	Pressures/foot. H	V	P	x/h	$\frac{H}{1/2yh^2}$	$\frac{P}{1/2yh^2}$
2.5	11	184	14	185	4°30'	37	3	37	.40	.124	.124
3	15	235	21	236	5°	47	4	47	.35	.110	.110
3.5	30	300	29	301	5°30'	60	6	60	.40	.102	.102
4	55	378	41	381	8°20'	76	8	76	.40	.100	.100
			Averages:		5°50				.39	.109	.109

Lateral Earth Pressure.

Note: The sand seemed to "creep" away from the wall. There seemed to be a space between the sand and the wall, at least one foot deep, and about 1/2 in. wide. As the depth of fill increased, the lower portion probably got a better grip on the wall, hence the increase in $(\phi' + \alpha)$.

To compare the experimental results with the various theories, the same coefficients were computed as in the previous section. The formulae are the same, but the value of α is now negative. Instead of N, the normal pressure, J, the horizontal component of N is given. $J = N \cos\phi'$

Table Comparing Experiment with Theory. Inclined Walls.
See Curves "C". Horizontal Fill. Heel of Wall Overhanging.

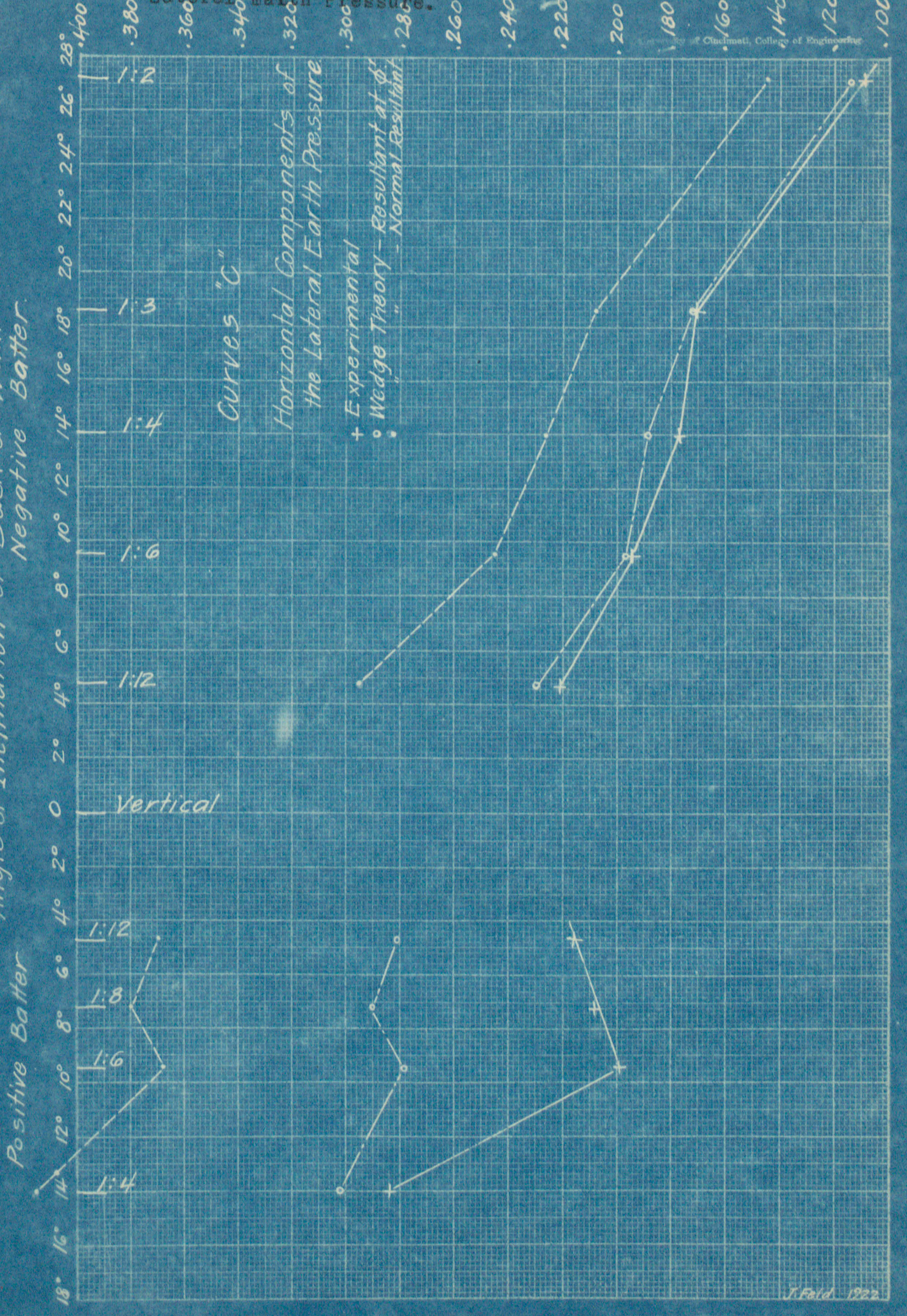
Wall	ϕ	ϕ'		Direction of P.		Exper.		Theory.		Exper.	
Slope			y	Exper.	Theory.	H	P	H	P	J	x/h.
1:12	30°30'	29°45'	97	24°38'	25°	.221	.239	.229	.256	.295	.358
1:6	31°50'	29°	94	19°00'	19°30'	.195	.205	.197	.210	.245	.37
1:4	31°00'	29°	97	12°34'	13°	.177	.181	.189	.194	.226	.36
1:3	30°20'	30°	97	12°53'	11°35'	.171	.174	.172	.176	.208	.395
1:2	30°20'	30°	97	5°50'	2°45'	.109	.109	.113	.113	.144	.39
Averages:				14°59'	14°22'	.175	.182	.180	.190	.224	.375

The total pressure is very closely given by the general wedge formula taking into consideration the wall friction. Its direction is always at an angle ϕ' to the normal, and its height about 3/8 h. The height of application seems to rise with an increase in the inclination of the wall.

X. Lateral Pressure of Sloping Fills against a Wall with a Positive 1:4 Batter.

As in the tests with a vertical wall, various surfaces with a constant height of wall of 3 ft, were tested. For the method of computation see sec. VIII. In calculating the theoretical values, the following formulae were used:

Curves for Sloped Walls & Horizontal Wall.
Coefficients of $\frac{1}{2}\gamma h^2$.
Lateral Earth Pressure.



Lateral Earth Pressure.

General wedge theory - taking into consideration the wall friction,

$$E = 1/2yh^2C, \quad C = \left(\frac{\cos(\phi'-\alpha)}{(n+1)\cos\alpha}\right)^2 \cdot \frac{1}{\cos(\phi'+\alpha)}, \quad n = \sqrt{\frac{\sin(\phi+\phi')\sin(\phi-\varepsilon)}{\cos(\phi'+\alpha)\cos(\alpha-\varepsilon)}}$$

Horizontal component coefficient $H = C\cos(\phi'+\alpha)$.

Normal pressure wedge theory, disregarding the wall friction,

$$E = 1/2yh^2N, \quad N = \left(\frac{\cos(\phi-\alpha)}{(n+1)\cos\alpha}\right)^2 \frac{1}{\cos\alpha}, \quad n = \sqrt{\frac{\sin\phi\,\sin(\phi-\varepsilon)}{\cos\alpha\,\cos(\alpha-\varepsilon)}}.$$

Test #17. See Curves "D".

Date: Nov. 24, 1921.
Weather: Clear.
Temperature: 14°C.
Wall Slope: 1:4, (14°).
Moisture Content: 3.7%.

Physical Properties: Density: 93 lb.
Natural Slope: 35°
Internal Resistance: 31°
Internal Friction: 29°15'
Wall Friction: 29°

$(\phi'+\alpha) = 43°$. $1/2yh^2 = 419$.

Surface Slope. ε	Pressures per foot. H	V	P	N	T	Height of P. x/h	Coefficients of $1/2yh^2$. Experimental. H	P	N	Theoretical. H	N	J
-41°10'	63	58	86	75	42	.33	.151	.206	.179	.168	.266	.258
-32°00'	77	72	106	93	50	.33	.184	.253	.222	.195	.299	.290
-18°25'	90	84	124	108	60	.34	.215	.296	.258	.235	.351	.340
-12°30'	100	94	139	120	66	.34	.239	.332	.287	.255	.373	.362
- 6°20'	110	102	150	131	73	.35	.263	.358	.313	.276	.400	.388
Level	124	115	170	148	82	.376	.296	.405	.354	.302	.428	.415
+ 6°20'	132	123	180	157	87	.396	.315	.430	.375	.336	.464	.450
+12°30'	140	131	192	168	95	.415	.335	.459	.401	.380	.514	.494
+18°25'	155	145	213	186	103	.416	.370	.509	.445	.425	.557	.540
+20°30'	160	149	219	191	106	.420	.382	.523	.456	.451	.583	.565
+26°30'	166	155	228	199	110	.439	.397	.545	.474	.571	.685	.665
+30°15'	178	166	244	213	118	.446	.425	.583	.510	.765	.863	.837
Averages:							.298	.408	.357	.363	.482	

Direction of P is $\cos^{-1}\frac{H}{P} = \frac{298}{408} = .732$ or 43°.

Point of application varies from 1/3 to 4/9 h, depending on slope. The experimental value of the resultant is 82% of the theoretical as given by the general wedge theory, and 74% of

Lateral Earth Pressure.

the normal pressure wedge theory.

XI. Lateral Pressure of Sloping Fills against a Wall with a Negative Back Batter of 1:6.

A test similar to the previous one was run with a wall having an overhanging top, i.e. a negative batter. The computations and theoretical formulae are the same as in the previous section, and in section IX.

Test #18. See Curves "D".

Date : Nov. 8, 1921.
Weather : Clear.
Temperature: $12^{\circ}C$.
Wall Slope: -1:6, ($9^{\circ}30'$).
Moisture Content: 3.3%.

Physical Properties: Density: 94 lb.
Natural Slope : $38^{\circ}20'$
Internal Resistance: 32°
Internal Friction: $28^{\circ}20'$
Wall Friction : 30°

$h = 3$ ft., $(\phi' + \alpha) = 20^{\circ}36^{\circ}$, $1/2\gamma h^2 = 423$.

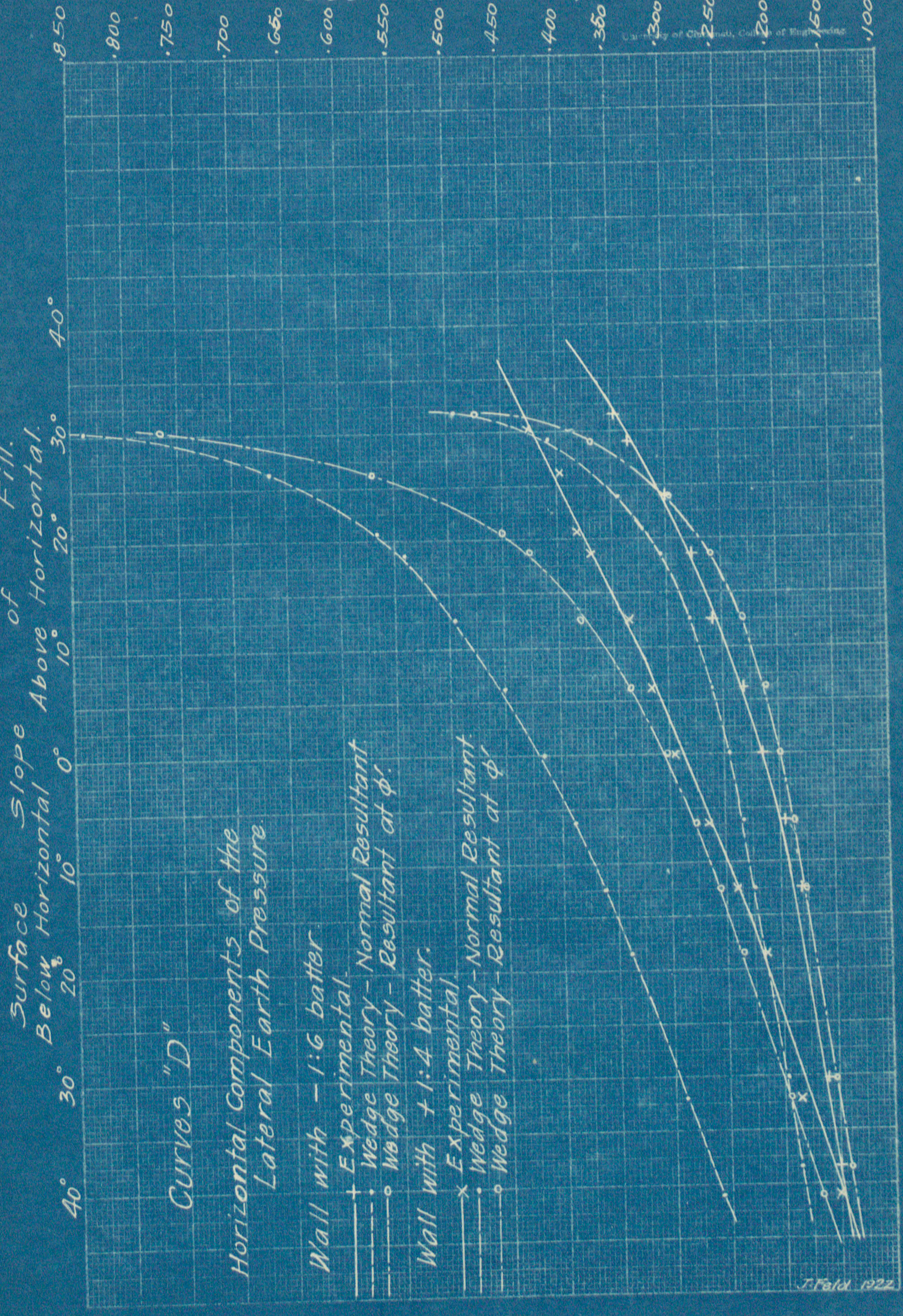
Surface Slope. ε		Pressures per ft. H	V	P	Inclination of P $(\phi'+\alpha)$	x/h	$\frac{H}{1/2\gamma h^2}$	$\frac{P}{1/2\gamma h^2}$	Theoretical Coefficients. H	C	J
$-38^{\circ}20'$	19:24	63	23	67	$20^{\circ}00'$	.398	.149	.158	.139	.148	.187
$-30^{\circ}15'$	1:2	67	24	72	$19^{\circ}40'$	.382	.158	.170	.153	.164	.198
$-12^{\circ}30'$	2:9	76	26	80	$19^{\circ}10'$	.353	.180	.189	.176	.187	.223
$-6^{\circ}20'$	1:9	82	31	87	$20^{\circ}50'$	.358	.194	.205	.186	.199	.233
Level	---	91	33	97	$19^{\circ}55'$	.358	.215	.229	.198	.211	.245
$+6^{\circ}20$	1:9	98	36	104	$20^{\circ}10'$	.385	.231	.246	.211	.225	.260
$+12^{\circ}30'$	2:9	109	40	117	$20^{\circ}20'$	.365	.258	.277	.232	.248	.280
$+18^{\circ}25'$	1:3	117	44	125	$20^{\circ}40'$	.374	.276	.296	.258	.276	.305
$+24^{\circ}00'$	4:9	128	48	136	$20^{\circ}40'$	.385	.300	.321	.297	.317	.343
$+29^{\circ}00'$	5:9	141	54	151	$21^{\circ}00'$	.368	.333	.357	.369	.394	.407
$+31^{\circ}30'$	11:18	147	57	158	$21^{\circ}10'$	.370	.347	.374	.475	.507	.495
	Averages:				$20^{\circ}20'$		.240	.257	.245	.261	.289

The direction of the resultant is at $(\phi'+\alpha)$ to the horizontal, or the resultant is at an angle ϕ' to the normal. The point of application varies, but is always above the third point and rises with increased slopes of fill. The magnitude, as an average, is given very closely by the general wedge theory, the theoretical values being about 5% lower than the ex-

Curves for Sloped Fills & Sloped Wall

Lateral Earth Pressure.

Coefficients of $\frac{1}{2}\gamma h^2$



perimental values, except for the very steep slopes above the horizontal, where the experimental are much lower than the theoretical values.

XII. Lateral Pressure of Irregular Fills against a Vertical Wooden Wall.

The object of this test was to determine the presence or absence of a wedge of rupture. The bin was filled to a level of 6 ft. and several readings taken. Starting at the back, 9 ft. from the wall, the fill was raised 1 ft. Readings were taken as the increased height of fill approached the wall. In this way, the rate of increase in lateral pressure was determined. If there is a fixed prism of rupture, the increase in fill beyond the plane of rupture will have no effect on the lateral pressure, but an increase in fill between the wall and the plane of rupture will cause an increase in pressure. The increase in fill was governed by placing a board across the width of the bin and at the required distance from the back, 1 ft., 2 ft., etc., successively. The sand was shovelled behind this board to a height of 1 ft., and readings were taken. This practically corresponded to a uniform load of 100 lbs. per sq. ft. moving towards the wall. The board was removed, and the sand naturally changed its shape, taking a position of natural repose. The distance between the wall and the toe of the slope was about 6 in. less than the distance between the wall and the previous vertical face of the increased fill. Readings were taken after this change.

The report of the test includes the horizontal and vertical components and the resultant pressure per foot width of wall for each increment of fill in both shapes. The next columns give the percentage increases in each value. Taking the plane of repose to be at 36° to the horizontal, and the plane of rupture to bisect the angle between the vertical wall and the plane of repose, the areas of the wedges between the wall and the two inclined planes were computed. These values include the additional fill. The percentage increases in these is also given.

Conclusion: The test shows the presence of a "wedge" or "prism" of rupture, very closely approximating the theoretical wedge. Loading beyond the theoretical plane of rupture had practically no effect on the pressure. As soon as the load crossed the plane of rupture, the horizontal and vertical components increased, the vertical somewhat faster than the horizontal. The variation in the total pressure is very close to the variation in the theoretical wedge of rupture. The height of the resultant remained practically unchanged. It is very evident that all the material above the plane of repose does not act as the wedge of rupture. Whether the rupture surface is plane or not is hard to determine. Altho carefully watched for there was no indication of a break in the top surface of the fill in any of the tests. Such a break could hardly be expected from the small movements of the wall.

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Lateral EARTH Pressure.

Test # 19.

Date : May 19,1921'.
Weather : Clear.
Temperature: 28° C.

Physical Properties : same as in Test 1.
Original Fill : 6 ft. level.

1 ft. high

Additional Fill Face	Dist. from wall	Pressures per ft. H	V	P	V/H tan φ'	x/h Height of P.	Percent Increases. H	V	P	r.	u.	x/h	V/H.
0	(0 wall)	467	296	552	.633	.368	0.0	0	0	0	0.0	0	0.0
Vert.	8	468	296	554	.633	.367	0.2	0	0	3.5	0	0	0
Sloped	7.5	468	297	555	.633	.367	0.2	0	1	3.5	0	0	0
Vert.	7	468	297	555	.633	.367	0.2	0	1	7.0	0	0	0
Sloped	6.5	468	297	555	.633	.367	0.2	0	1	7.0	0	0	0
Vert.	6	470	297	556	.633	.368	1.0	0	1	11.0	0	0	0
Sloped	5.5	470	298	556	.633	.368	1.0	1	1	11.0	0	0	0
Vert.	5	471	299	558	.634	.369	1.0	1	1	15	0	0	0
Sloped	4.5	471	299	558	.634	.368	1.0	1	1	15	0	0	0
Vert.	4	473	301	561	.635	.369	1	2	2	19	0	0	0
Sloped	3.5	473	301	561	.636	.370	1	2	2	19	0	0	0
Vert.	3	481	316	576	.655	.369	3	7	4	23	3.5	0	3.5
Sloped	2.5	484	320	580	.662	.368	4	8	5	23	3.5	0	4.5
Vert.	2	499	341	605	.684	.361	7	15	9	27	14.5	-2	8
Sloped	1.5	500	341	605	.682	.360	7	15	9	27	14.5	-2	8
Sloped	0	569	374	680	.656	.380	22	26	23	36	28	+3	4
						Sums:	50	78	61	250	64	---	28

(r) Area between wall and plane of repose for 6 ft. level: 9.18 sq.ft.

(u)Area between wall and plane of rupture for 6 ft. level: 24.78 sq. ft.

Lateral Earth Pressure.

XIII. Effect of Settling and Changes of Temperature upon the Lateral Pressure of Horizontal Fills against a Vertical Wall.

Several tests were conducted to determine the effect of changes in temperature, humidity, etc., and to investigate the effect of settling. Readings were taken about one hour apart. One test consisted of readings over a period of eight days, during a temperature variation of 13°C. (23°F). During this period there were several rains.

A "Blank test", where readings on the scales were caused by a mechanically applied pressure against the walls, showed very little variation, thereby proving that the variations in the following tests are due to changes in the fill and not to changes in the apparatus. Not all the readings are given in these tests, only those where changes occurred. In the eight day test mentioned above, 80 sets of readings were taken, distributed in such a way, that the effect of extremes of temperature and humidity could be investigated.

The Maximum (x) and minimum (n) values are given below.

Age of Fill.	Time of Day.	H	V	P	V/H.	x/h
0	5PM	456	299(x)	545	.658	.355(n)
14	12 M	419(n)	286	508(n)	.680(x)	.363
88-90	9AM	465	278(n)	542	.597	.370
90	11AM	468	278	544	.594(n)	.371
114	11AM	473(x)	286	553(x)	.603	.374(x)
Percent variations:						
of maximum	:	3.7	0	1.5	3.3	5.4
of minimum	:	8.1	6.3	6.8	9.7	0
total variations	:	11.8	6.3	8.3	13.0	5.4

Lateral Earth Pressures.

Test # 20.

Date : May 12-19, 1921. Physical Properties: same as in Test No. 1.
Wooden Wall- Vertical. Time Test.
Horizontal Fill- 6 ft. high.

Day	Time	Age of Fill (hrs.)	Temp. °C.	Pressures per ft. H	V	P	V/H tan ϕ'	X/h	Per cent Changes. H	V	P	X/h	Weather. and Humidity
1st	5PM	0	12	456	299	545	.658	.355	——	——	——	——	Rain.
2nd	12 M	19	14	419	286	508	.680	.363	-8.1	-3.9	-6.8	+2.3	Storm.
	2PM	21	12	422	286	509	.678	.364	-7.4	-3.9	-6.6	+2.5	Clear.
	6PM	25	12	422	286	509	.679	.361	-7.4	-3.9	-6.6	+1.7	"
3rd	9AM	40	19	426	280	510	.660	.363	-6.6	-5.7	-6.4	+2.3	"
	12 M	43	20	432	284	517	.658	.365	-5.3	-4.5	-5.1	+2.8	"
	4PM	47	20	432	286	518	.662	.364	-5.3	-3.9	-5.0	+2.5	"
4th	11AM	66	16	450	279	530	.620	.364	-1.3	-6.0	-2.8	+2.5	"
	12 M	67	17	451	279	530	.620	.365	-1.1	-6.0	-2.8	+2.8	"
	12:30PM	67.5	17	453	279	532	.616	.365	-0.7	-6.0	-2.4	+2.8	"
5th	8AM	87	13	460	279	538	.607	.366	+0.9	-6.0	-1.3	+3.1	"
	9AM	88	15	465	278	542	.597	.370	+2.0	-6.3	-0.6	+4.2	"
	10AM	89	15	466	278	543	.599	.371	+2.2	-6.3	-0.4	+4.5	"
	11AM	90	17	468	278	544	.594	.371	+2.6	-6.3	-0.2	+4.5	"
	12 M	91	17	469	280	545	.595	.372	+2.8	-5.7	0.0	+4.8	"
	1PM	92	17.5	469	280	547	.596	.372	+2.8	-5.7	+0.4	+4.8	"
	2PM	93	17.5	469	282	547	.599	.371	+2.8	-5.1	+0.4	+4.5	"
	3PM	94	17.5	469	282	547	.601	.370	+2.8	-5.1	+0.4	+4.2	"
	5PM	96	17	467	283	546	.606	.370	+2.4	-4.8	+0.2	+4.2	"
	6PM	97	16.5	466	283	545	.608	.370	+2.2	-4.8	0.0	+4.2	"
	10PM	101	14	469	283	548	.604	.396	+2.8	-4.8	+0.6	+4.0	"
6th	8AM	111	14	461	281	540	.608	.369	+1.1	-5.4	-0.9	+4.0	"
	9AM	112	16	463	280	541	.605	.371	+1.5	-5.7	-0.7	+4.5	"
	10AM	113	17	466	282	545	.605	.373	+2.2	-5.1	0.0	+5.1	"
	11AM	114	19	473	286	553	.603	.374	+3.7	-3.9	+1.5	+5.4	"
	12 M	115	19	471	288	552	.612	.374	+3.3	-3.3	+1.3	+5.4	"
	2PM	117	19	469	290	551	.617	.373	+2.8	-2.7	+1.1	+5.1	"
	5PM	120	18	465	292	549	.628	.372	+2.0	-2.1	+0.7	+4.8	"
7th	8AM	135	17	450	287	534	.638	.368	-1.3	-3.6	-2.0	+3.1	"
	12 M	139	20.5	452	289	536	.640	.372	-0.9	-3.0	-1.7	+4.8	"
	8PM	147	20	452	294	539	.650	.366	-0.9	-1.5	-1.1	+3.1	"
8th	8AM	159	19	443	292	530	.659	.363	-2.8	-2.1	-2.8	+2.3	"
	11AM	162	23	446	291	533	.652	.366	-2.2	-2.4	-2.2	+3.1	"
	12 M	163	23.5	451	291	534	.650	.366	-1.8	-2.4	-2.0	+3.1	"
	2PM	165	26	451	294	539	.651	.364	-1.1	-1.5	-1.1	+2.5	Warm.

Lateral Earth Pressure.

1. The pressure just after filling is practically the maximum.

2. A rise in temperature is accompanied by an increase in both components, and an increase in the height of the resultant.

3. A drop in temperature is accompanied by a decrease in both components and a decrease in the height of the resultant.

4 . The ratio of vertical to horizontal components seems to be unaffected by the age of the fill or by changes in temperature. It is, however, affected by the humidity, because of changes in the water content.

5. The height of the resultant varies as noted above, and at the same time slowly rises as the age of the fill increases. For this type of fill, horizontal top surface, there seems to be a maximum of 3/8 h.

6. Settling of fill is accompanied by a slight decrease in pressure, with intermittent sudden increases, due to small ruptures in the fill. These increases soon vanish.

7. The minimum pressures occur very shortly after the filling. (Tests described below show this more clearly than the tests given in this section).

8. Curves plotted for the above test give an average variation of the pressure of 0.15 per cent per degree Centigrade, taking the average for each day.

The curves of pressure against time are very similar to the curve of "damped vibrations". The total variation in the horizontal component of 11.8 per cent is noteworthy.

Lateral Earth Pressure.

Test # 21.

Date: May 19-23, 1921. Physical Properties : same as Test 1.

Time Test on a Tamped Fill; 7 ft. High; 6 ft. vertical Wall. The fill was 168 hours old when the test began.

Day.	Time.	Age of Fill hr.	Weather. Clear.	Pressure/foot. H	V	P	V/H. tan ϕ'	X/h	Percent Changes. H	V	P	x/h
1st	4PM	0	26°C,	569	374	680	.659	.380	---	--	--	---
	5PM	1	25°	570	378	684	.664	.381	+0.2	+1.1	+0.6	+0.3
2nd	8AM	16	21°	560	373	682	.665	.383	-1.6	-0.3	+0.3	+0.8
	12 M	20	24°	566	373	678	.659	.385	-0.5	-0.3	-0.3	+1.3
	1PM	21	25°	565	373	677	.660	.385	-0.7	-0.3	-0.4	+1.3
	2PM	22	26°	561	375	682	.658	.379	-1.4	+0.3	+0.3	-0.3
	5PM	25	25°	583	376	690	.645	.379	+2.5	+0.5	+1.5	-0.3
3rd	10AM	42	25°	568	371	678	.654	.378	-0.2	-0.8	-0.3	-0.5
	1PM	45	25°	569	370	678	.651	.377	0	-1.1	-0.3	-0.8
5th	1PM	93	27°	563	366	671	.651	.377	-1.1	-2.1	-1.3	-0.8

The variation in tamped fills seems to be very much less than in ordinary fills. The temperature range in this test was only 5°C., so that large variations cannot be expected.

Test # 22a. Time Tests. Test # 22b.

Age of Fill. hr.	Top H_1	Total Pressures. H	V	Weather.
0	834	2504	1458	Clear, 20°C.
1/4	842	2539	1509	"
1/2	844	2545	1522	"
3/4	846	2552	1532	"
1	845	2552	1535	"
1 1/4	845	2551	1536	"
1 3/4	845	2554	1541	"
2	844	2552	1541	"

Fill is 5 ft. high, Top surface at +31°. Vertical wooden wall.

Age of Fill. hr.	Top H_1	Total Pressures. H	V	Weather.
0	1100	2962	2033	Clear
1	1114	3021	2036	"
1 1/4	1108	2991	2013	"
43 1/4	1088	2960	2004	Rain.
67	1079	2890	1987	"
73	1098	2922	1993	Cloud

Fill is 7 ft. 9 in. high against a vertical wooden wall 6 ft. high.

The maximum pressure occurs soon after filling.

The fill undergoes a settling as soon as it is placed, causing an increase in pressure in about 1 hour. The pressure then slowly decreases, reaching a low point within 24 hours.

It then oscillates each day as shown in the previous test.

XIV. Effect of Static Loads on the Lateral Pressure of Horizontal Fills against A Vertical Wall.

The horizontal section of the bin is 5 ft. by 9 ft. so forty-five 1 in. boards 11¾ x 11¾ were used to cover the surface of a 6 ft. horizontal fill of sand. A single load of 100 lbs. (a keg of nails) was successively placed on each square, starting with the one furthest from the wall, and readings were taken while the load rested on the fill and after it had been removed. The effect was that of a localized surcharge of 100 lbs. per sq. ft. The averages of the five positions of the load at each distance back of the wall gave the values shown below. The readings for the five positions of the load at each distance back of the wall were about the same. The surprising feature of this test was, that after the load was removed, the pressure did not diminish immediately. The fill returned to its original condition several days later.

TEST # 23.

Date: May 6, 1921. Physical Properties: same as in Test 1'
Weather: Clear. d = distance of surcharged load of
Temperature : 15°C. 100 lb. from back of wall.

Readings are the averages of five positions of load.

d ft.	Pressures per ft. H	V	P	V/H tan φ'	x/h	Percent Changes in H	V	P	V/H	x/h
No Load.	469.2	286.8	551.6	.610	.367	----	----	----	----	----
8.5	472.5	288.6	553.6	.610	.367	+0.7	+0.6	+0.4	0	0
7.5	473.4	288.9	554.6	.611	.367	.9	.7	.5	+0.2	0
6.5	474.4	289.4	555.8	.611	.367	1.1	.9	.8	.2	0
5.5	475.3	290.0	556.8	.611	.368	1.3	1.1	.9	.2	.3
4.5	476.3	290.5	557.9	.611	.367	1.5	1.3	1.1	.2	0
3.5	478.1	291.7	560.1	.611	.368	1.9	1.7	1.5	.2	.3
2.5	479.7	293.8	562.7	.612	.368	2.2	2.4	2.0	.2	.3
1.5	483.1	297.5	567.6	.617	.380	3.0	3.7	2.9	.3	3.5
After Test	483.5	299.3	586.6	.620	.380	3.0	4.5	3.1	1.6	3.5

Lateral Earth Pressure.

The theoretical plane of rupture, $\phi = 36^\circ$, cuts the top surface at 3.06 ft. The plane of slope cuts the top surface at 8.25 ft.

Conclusions: There is an appreciable increase in pressure due to loads not on the wedge of rupture. The increase is much faster, however, as the load moves on the theoretical wedge. The effect of a static load is to change the physical characteristics of the fill, for the changed pressure remained after the load has been removed. Assuming that the density has not been changed, and is 100 lbs. thruout the fill, the final horizontal component requires a ϕ of $35^\circ 10'$. Assuming that the concentrated load compresses the entire fill to 7/6 of its density, or 116.7 lbs., ϕ is then $38^\circ 40'$. Assuming that the final density is the average of these two extreme values, or 108.3 lbs., ϕ is then $37^\circ 00'$. The formula used is $H = \frac{1}{2} \gamma h^2 \tan^2 \frac{1}{2}(90^\circ - \phi)$. The experimental value for ϕ of the material used is close to 37°. It is very probable, however, that both the density and the coefficient of resistance of the fill are changed.

The present method, employed in practice to take account of static surcharges, assumes that ϕ is not changed, but that the density is changed. It requires that the readings for a concentrated load at any point on the wedge be the same. The test does not show this. It was impossible to bring the load closer to the wall than 1 ft. The results clearly show that the effect of a load depends on its distance from the wall.

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XV. Effect of Moving Loads on the Lateral Pressure of Horizontal Fills against a Vertical Wall.

After completing the previous test, an attempt was made to measure the effect of loads rolling across the fill, parallel to the wall. The keg of nails was rolled three times across the width of the fill, at various distances from the wall, starting at the greatest distance. Readings were taken directly after this rolling, the load having been removed from the fill. The previous test had tamped or compacted the fill fairly well, so that the effect was that of a moving load on a settled or tamped fill.

Test #24. Physical Properties: same as for test 23.

Date: May 6th, 1921.

Pressures given are per ft. width of wall exerted by a 6 ft. horizontal fill against a wooden vertical wall, after a load of 100 lbs. has been rolled across the fill at a distance d from the wall.

d	Pressures per ft. H	V	P	V/H, tan ϕ'	x/h	Percent Changes in H	V	P
No Load.	483.6	299.4	568.6	.620	.369	---	---	---
8.5	484.4	299.8	569.6	.620	.368	+0.2	+0.1	+0.2
7.5	484.6	300.0	570.0	.619	.368	.2	.2	.2
6.5	484.6	300.1	570.0	.620	.368	.2	.2	.3
5.5	485.0	300.1	570.4	.619	.369	.3	.2	.4
4.5	485.4	300.4	570.8	.619	.369	.4	.3	.5
3.5	485.8	300.6	571.2	.619	.369	.5	.4	.5
2.5	485.9	300.9	571.4	.619	.369	.5	.5	.7
1.5	486.6	301.6	572.4	.619	.370	.6	.7	1.0

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Rolling or moving loads on a tamped fill have no effect on the pressure. To investigate the action of this compressed fill as it aged, it was allowed to stand for seven days. The following summary of pressures exerted by this fill is interesting. The height of fill is 6 ft., original γ = 100 lbs. $\phi = 36^{o}$.

	Pressures per foot. H	V	P	V/H Tan ϕ'	X/h	Percent Changes in H over original values	V	P	x/h
Just after filling:	458.4	288.6	541.6	.652	.367	——	——	——	
48 hrs. later :	469.2	268.8	551.6	.611	.367	+2.4	-0.6	+1.8	0
After Static Test :	483.5	299.3	568.5	.620	.380	+5.5	+3.7	+5.0	+3.5
1 hr. later :	483.6	299.4	568.6	.620	.369	+5.5	+3.7	+5.0	+0.5
After Dynamic Test:	486.6	301.5	572.4	.619	.370	+6.2	+4.5	+5.7	+0.8
96 hr. later (144 total)	499.0	287.0	575.6	.575	.384	+8.9	-0.1	+6.3	+4.6
118 " " 166 "	464.8	268.2	537.0	.577	.392	+1.4	-7.1	-0.8	+6.8

Note-maximum values occurred at 96 hrs. (total 144 hr.)
minimum " " " 118 " " 166 "

The fill comes back almost to its original state, except that the resultant rises 6.8%. The decrease in the vertical component is due to the drying out of the sand along the wall. The maximum at 96 hrs. is due to a slight rupture in the fill. It is very interesting to note that this packing and loading of the fill caused a variation of only 8.9% in the horizontal component, less than the variation in a similar fill caused by a 13^{o}C. change in temperature over a similar length of time of 166 hours.

XVI. Effect of Static Loads on the Lateral Pressure of Horizontal Fill against a Wall with Positive Back-Batter of 1:4.

A test similar to that in Sec. XIV was run with a

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4 ft. fill and a wall sloping at + 1:4. The distance of the load is measured from the top of the wall. The heel of the wall is 1 ft. closer to the back of the bin; so that, in the last position, the load was directly above the back of the wall.

Test #25	Physical properties - same as in test 17
Date: Dec. 4, 1921	Height of fill: 4 ft.
Weather: Clear	Wall batter +1:4 (14°00').
Temperature $3\frac{1}{2}$°C.	Load - 100 lbs. on a sq. ft.

d = distance of load from top of wall.

d	Total Pressures on Wall. H	V	V/H tan φ'	Percent Increases in H	V
No Load	823	746	.907	0	0
8 ft.	823	746	.907	0	0
7	823	748	.909	0	+0.3
6	823	748	.909	0	+0.3
5	823	748	.909	0	.3
4	825	748	.908	+0.2	.3
3	828	748	.905	.6	.3
2	831	756	.910	1.0	1.3
1	840.5	761	.907	2.1	2.0
No Load	840	763	.908	2.1	2.3
1 hr. later:	840	747	.890	2.1	0.1

The results are very similar to the case of a vertical wall. The change in vertical component disappears soon after the load is removed while the horizontal component remains undiminished. In the last position, the 100 lb. load is directly over the wall, yet the increase in vertical component is only 17 lbs. The fill is 4 ft. high.

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General Conclusions on the Pressure of Sand Fills, Deduced from the Cincinnati Experiments.

The material used was sand under varying conditions; the moisture varying from 9 to 3 per cent. The walls used were backed with wood, glass and sheet metal. The back of the wall varied from a negative 1:2 to a positive 1:4 batter. The surface of the fill varied from the maximum below the horizontal to the maximum above the horizontal. The greatest height of fill with a horizontal surface was 6 ft. In surcharge tests, a concentrated load of 100 lbs. was used.

1. The fill does not act like a liquid. The transmission of pressure does not obey the Pascal law.

2. The fill does not act like an elastic solid. Loads on the fill cause a change in physical properties as well as a deformation. These changes slowly disappear, but not according to any definite law.

3. There is no sharply defined wedge of rupture, nor can any surface of rupture be detected on the surface of the fill. However, loads on the fill beyond the wedge of rupture have practically no effect on the lateral pressure. Loads on the wedge of rupture do not increase the pressure by as much as would be required by a well defined and distinct wedge.

4. The lateral earth pressure for all types and kinds of wall and for all types of loading and fill, acts at an angle to the normal to the wall, which angle is equal to the angle of static friction between the fill and the back of the wall. The height of application is above the 1/3 point, but below the 4/10 point of the height.

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5. For the case of vertical wall and horizontal fill, the value of the horizontal pressure is given by the formula $E=\frac{1}{2}yh^2\tan^2\frac{1}{2}(90°-\phi)$, where ϕ is the angle of internal resistance of the fill, corresponding to the static friction of the filling material on itself. This formula is a special case of both the Rankine and Coulomb theories. The value of the vertical component is easily determined from a consideration of Conclusion 4.

6. For all general cases, the pressure is obtained closest by the general wedge theory

$$E=\frac{1}{2}yh^2C; \qquad C=\left(\frac{\cos(\phi-\alpha)}{(n+1)\cos\alpha}\right)^2\frac{1}{\cos(\phi'+\alpha)}.$$

$$H=E\cos(\phi'+\alpha); \qquad n=\sqrt{\frac{\sin(\phi+\phi')\sin(\phi-\varepsilon)}{\cos(\phi'+\alpha)\cos(\alpha-\varepsilon)}}.$$

The wedge theory which disregards wall friction gives results which are too high, except in the case of a vertical wall, when it is closer to the experimental results than the general wedge theory.

7. For the case of a vertical wall, the general wedge theory is somewhat low, and it is better to use the wedge theory disregarding the wall friction

$$H=\frac{1}{2}yh^2N; \qquad N=\left(\frac{\cos(\phi-\alpha)}{(n+1)\cos\alpha}\right)^2\frac{1}{\cos\alpha}.$$

$$E=H\sec\phi'. \qquad n=\sqrt{\frac{\sin\phi\sin(\phi-\varepsilon)}{\cos\alpha\cos(\alpha-\varepsilon)}},$$

8. The experimental results are not in agreement with the Rankine theory, except as noted in Conclusion 5.

9. The effect of settling is to increase the pressure exerted by the fill until a maximum is reached, within two hours of placing the fill. The pressure then decreases to a minimum within 24 hours. After that period, the pressure oscillates between the maximum and minimum so found.

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10. The pressure is a direct function of the temperature.

11. The pressure is but slightly affected by humidity, except that dry weather will dry out the fill along the wall, causing a greater horizontal and a less vertical component.

12. The effect of surcharges, both static and dynamic, is to compress the fill and increase the resultant pressure. Such an increase disappears in time, usually in about seven days.

APPENDIX "A"

Solution of Couplet's Equation for the Base Width of Walls.

Couplet's theory gives the equation of equilibrium as:

(1) $$\frac{(b - x)^3}{2\,b\,E} \cdot \frac{F\,h^2}{3\,c} = \frac{w}{y} \cdot \frac{h\,x^2}{2}$$

From his tetrahedral arrangement of spherical grains, he concludes that $F/E = 2$, $b = h/2\sqrt{2}$, $c = 3h/2\sqrt{2}$; substituting :

(2) $$\frac{\left(\frac{h}{2\sqrt{2}} - x\right)^3 \cdot \sqrt{2} \cdot h^2}{2 \cdot \frac{h}{2\sqrt{2}} \quad 3 \cdot \frac{3h}{2\sqrt{2}}} = \frac{w}{y} \cdot \frac{h\,x^2}{2}$$

(3) $$\left(\frac{h}{2\sqrt{2}} - x\right)^3 = \frac{w}{y} \cdot \frac{h\,x^2}{2} \cdot \frac{9\,h^2}{4} \cdot \frac{1}{\sqrt{2}\,h^2} = \frac{9}{8\sqrt{2}} \cdot \frac{w\,h\,x^2}{y}$$

(4) $$\left(\frac{h}{2\sqrt{2}} - x\right)^3 - \frac{9}{8\sqrt{2}}\,\frac{w\,h\,x^2}{y} = 0.$$

(5) $$\frac{h^3}{16\sqrt{2}} - \frac{3\,h^2\,x}{8} + \frac{3\,h\,x^2}{2\sqrt{2}} - x^3 - \frac{9}{8\sqrt{2}}\,\frac{w\,h\,x^2}{y} = 0.$$

(6) $$x^3 + x^2\left(\frac{9\,w\,h}{8\sqrt{2}\,y} - \frac{3\,h}{2\sqrt{2}}\right) + x\left(\frac{3\,h^2}{8}\right) - \frac{h^3}{16\sqrt{2}} = 0.$$

(7) Let $m = w/y$: Equation (3) is approximately

$$0.796\,m\,h\,x^2 = (.354\,h - x)^3 .$$

The following discussion of the equation resulted from an attempt to find a simple solution, which could be compared with the results of other theories.

I/ Approximate solution by trial, for ordinary values of w/y. The ratio of density of wall to density of fill may vary from 1.0 to 2.0. A value for x is found for m changing by 0.1 .

If $m = 1.0$,	$0.796\,h\,x^2 = (.354\,h - x)^3$;		$x = 0.1239\,h$.
1.1	0.884 "	"	0.1201 h .
1.2	0.956 "	"	0.1175 h .
1.3	1.036 "	"	0.115 h .
1.4	1.115 "	"	0.1125 h .
1.5	1.196 "	"	0.1101 h .

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If $m = 1.6$	$1.274\ h\ x^2 = (.354\ h - x)^3$	;	$x = 0.1080\ h$.
1.7	1.354 "	"	0.106 h.
1.8	1.434 "	"	0.104 h.
1.9	1.512 "	"	0.1025 h.
2.0	1.592 "	"	0.101 h.

If we let $x = b\ m^a\ h$, then for $m = 1$, $b = 0.1240$;

$$x = 0.1240\ m^a\ h, \quad \text{or} \quad m^a = \frac{x}{0.1240\ h}$$

Substituting all the values of m, and averaging, we get the value of a, from t he formula $a = \log \frac{a}{0.1240\ h} \div \log m$..

m	x/h	m^a	log m^a	log m^a	log m	a
1.0	.1239	1.000	0	0	0	- - -
1.1	.1201	.970	9.986 - 10	- .014	.042	- .333
1.2	.1175	.949	9.979 - 10	- .021	.079	- .266
1.3	.115	.928	9.967 - 10	- .033	.114	- .290
1.4	.1125	.908	9.958 - 10	- .042	.146	- .288
1.5	.1101	.890	9.949 - 10	- .051	.176	- .290
1.6	.108	.871	9.940 - 10	- .060	.204	- .294
1.7	.106	.855	9.932 - 10	- .068	.230	- .296
1.8	.104	.839	9.924 - 10	- .076	.255	- .298
1.9	.1025	.826	9. 917 - 10	- .083	.278	- .302
2.0	.101	.815	9.911 - 10	- .089	.301	- .296
					Average:	- .294.

Empirical Result : $x = \dfrac{0.124\ h}{m^{0.294}}$

Investigation into the nature of the roots by DesCartes' Rule of Signs gives the following :

$$(6) \qquad x^3 + x^2 \left(\frac{9\ m\ h}{8\sqrt{2}} - \frac{3\ h}{2\sqrt{2}}\right) + x\left(\frac{3\ h^2}{8}\right) - \frac{h^3}{16\sqrt{2}} = 0.$$

If $m = 4/3$, then the coefficient of x^2 vanishes for all values of h and x.

If $m > 4/3$, the sign of the coefficient is plus and the rule shows the existance of one positive root and a possibility of two negative roots.

If $m < 4/3$, the sign of the coefficient is negative and the rule shows the possible existance of three positive roots and no negative roots. From this we may conclude that there is but one real root, a positive value. But since the critical point occurs at $m = 4/3$, a possible value as well as a frequent actual value of m, an extra complication occurs in the solution.

A rigid solution is given by the Cardan method for cubic equations:

(6) If $x^3 + x^2\left(\frac{9\ m\ h}{8\sqrt{2}} - \frac{3\ h}{2\sqrt{2}}\right) + x\left(\frac{3\ h^2}{8}\right) - \frac{h^3}{16\sqrt{2}} = 0.$

be written as $x^3 + x^2(3\ a) + x(3\ b) + 2\ c = 0,$

then $B = -a^2 + b$; $C = a^3 - 3/2\ a\ b + c$;

by letting $x = y - a$; $y^3 + 3\ B\ y + 2\ C = 0.$

$S_1 = (-C\ \sqrt{B^3 + C^2})^{1/3}$; $S_2 = (-C - \sqrt{B^3 + C^2})^{1/3}$, then

the roots are $x_1 = -a + (s1 + s_2)$;

$x_2 = -a - (1/2)(s_1 + s_2) + 1/2\ \sqrt{-3}\ (s_1 - s_2)$;

$x_3 = -a - (1/2)(s_1 + s_2) - 1/2\ \sqrt{-3}\ (s_1 - s_2)$.

In this equation we are looking only for the real root x_1,

(9) Letting $x = d\ h$ and dividing by h^3, we obtain

$$d^3 + d^2\left(\frac{9\ m}{8\sqrt{2}} - \frac{3}{2\sqrt{2}}\right) + \frac{3\ d}{8\sqrt{2}} - \frac{1}{2\sqrt{2}} = 0.$$

(10) Comparing Cardan's form with the above,

$$a = \frac{1}{3}\left(\frac{9\ m}{8\sqrt{2}} - \frac{3}{2\sqrt{2}}\right) + \frac{3\ m}{8\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{3\ m - 4}{8\sqrt{2}}$$

$$b = \frac{1}{3}\left(\frac{3}{8}\right) = \frac{1}{8} \quad . \quad c = \frac{1}{2}\left(\frac{-1}{16\sqrt{2}}\right) = \frac{-1}{32\sqrt{2}}$$

(11) $B = -a^2 + b = -\left\{\frac{3\ m - 4}{8\sqrt{2}}\right\}^2 + \frac{1}{8} = \frac{-9\ m^2 + 24\ m - 16 + 16}{(8\sqrt{2})(8\sqrt{2})}$

$$= \frac{24\ m - 9\ m^2}{(8\sqrt{2})^2} = \frac{3\ m\ (8 - 3\ m)}{(8\sqrt{2})^2} = \frac{3\ m\ (8 - 3\ m)}{128}$$

(12) $C = a^3 - 3/2\ a\ b + c = \frac{(3\ m - 4)^3}{8\sqrt{2}} - \frac{3}{2}\left(\frac{3\ m - 4}{8\sqrt{2}}\right)\left(\frac{1}{8}\right) + \left(\frac{-1}{32\sqrt{2}}\right)$

$$= \frac{(3m - 4)^3 - 24\ (3\ m - 4) - 32}{(8\sqrt{2})^3}$$

$$= \frac{27\ m^3 - 108\ m^2 + 144\ m - 64 - 72\ m + 96 - 32}{(8\sqrt{2})^3}$$

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$$= \frac{27\ m^3 - 108\ m^2 + 72\ m}{1024\sqrt{2}} = \frac{9\ m\ (3\ m^2 - 12\ m + 8)}{(8\sqrt{2})^3}$$

(13) $$\sqrt{B^3+C^2} = \left[\left\{ \frac{3\ m\ (8 - 3\ m)}{128} \right\}^3 + \left\{ \frac{9\ m\ (3\ m^2 - 12\ m + 8)}{(8\sqrt{2})^3} \right\}^2 \right]^{1/2}$$

$$\left[\left\{ \frac{3\ m}{2^7} (2^3 - 3\ m) \right\}^3 + \left\{ \frac{3^2\ m\ (3\ m^2 - 2^2 \cdot 3\ m\ 2^3)}{(2^3\sqrt{2})^3} \right\}^2 \right]^{1/2}$$

$$= \left\{ \frac{3^3 m^3}{2^{21}} (2^9 - 2^6 \cdot 3^2\ m + 2^3 \cdot 3^3\ m^2 - 3^3\ m^3) \right.$$

$$\left. + \frac{3^4\ m^2}{2^{21}} (3^2\ m^4 + 2^4\ 3^2\ m^2 + 2^6 - 2^3\ 3^2\ m^3 + 2^4\ 3\ m^2 - 2^6\ 3\ m) \right\}^{1/2}$$

$$= \frac{3^{3/2}\ m}{2^{15/2}} \left\{ \left(\frac{1}{2^6}\right) (2^9\ m - 3^2 2^6\ m^2 + 3^3\ \cancel{2^3}\ m^3 - \cancel{3^3}\ m^4 + 3\ \cancel{3}\ m^4 + 2^4\ 3^3\ m^2 \right.$$

$$\left. + 2^6\ 3 - 2^3\ \cancel{3^3}\ m^3 + 2^4\ 3^2\ m^2 - 2^6\ 3^2\ m) \right\}^{1/2}.$$

$$= \frac{3^{3/2}\ m}{2^{15/2}} \left\{ \frac{(2^9 m - 2^6\ 3^2\ m^2 + 2^4\ 3^3\ m^2 + 2^6\ 3 + 2^4\ 3^2\ m^2 - 2^6\ 3^2\ m)}{2^6} \right\}^{1/2}$$

$$= \frac{3^{3/2}\ m}{2^{15/2}} \left(\left(\frac{1}{2^6}\right) (2^6\ 3 \quad m\ (2^9 - 2^6\ 3^2) \quad m^2 (2^4\ 3^3 \quad 2^4\ 3^2 - 3^2\ 2^6)) \right)^{1/2}$$

$$= \frac{3^{3/2}\ m}{2^{15/2}} \left(\left(\frac{1}{2^6}\right) (2^6\ 3 \quad 2^6\ m\ (2^3 - 3^2) \quad 2^4\ 3^2\ m^2\ (3 \quad 1 - 2^2)) \right)^{1/2}$$

(13) $$(B^3 + C^2)^{1/2} = \frac{3^{3/2}\ m}{2^{15/2}} (3 - m)^{1/2} = \frac{3\sqrt{3}\ m\sqrt{3 - m}}{2^7\sqrt{2}}$$

(14) $$\sqrt{B^3 + C^2} = \frac{3\sqrt{3}\ m\ \sqrt{3 - m}}{128\sqrt{2}} \qquad \frac{3\ m\sqrt{9 - 3\ m}}{128\sqrt{2}}$$

(15) $$-C = -\frac{9\ m\ (3\ m^2 - 12\ m + 8)}{1024\sqrt{2}} = \frac{3\ m}{128\sqrt{2}} \cdot \frac{3}{8} (-3\ m^2 + 12\ m - 8)$$

(16) $$(-C + \sqrt{B^3 + C^2})^{1/3} = \left\{ \frac{3\ m^2}{128\sqrt{2}} \cdot \frac{3}{8} \cdot (-3\ m^2 + 12\ m - 8) + \frac{3\ m\ (9 - 3m)^{1/2}}{128\sqrt{2}} \right\}^{1/3}$$

$$= s_1 = \left(\frac{3\ m}{128\ 2}\right)^{1/3} \left\{ \frac{-9\ m^2 + 36\ m - 24 + (9 - 3\ m)^{1/2}}{8} \right\}^{1/3}$$

$$= \frac{3^{1/3}\ m^{1/3}}{2^{7/3} \cdot 2^{1/6}} \cdot \frac{1}{2} (-9\ m^2 + 36\ m - 24 + 8\sqrt{9 - 3\ m})^{1/3}$$

$$= \frac{3^{1/3}\ m^{1/3}}{2^{7/2}} (-9\ m^2 + 36\ m - 24 + 8\sqrt{9 - 3\ m})^{1/3}$$

(17) $$s_2 = (-C - \sqrt{B^3 + C^2})^{1/3} = \left\{ -9\ m^2 + 36\ m - 24 - 8\sqrt{9 - 3m} \right\}^{1/3} \frac{3^{1/3} m^{1/3}}{2^{7/2}}$$

(18) $$s_1 + s_2 = \frac{3^{1/3}\ m^{1/3}}{2^{7/2}} \left[(-9\ m^2 + 36\ m - 24 + 8\sqrt{9 - 3\ m})^{1/3} \right.$$

$$\left. + (-9\ m^2 + 36\ m - 24 - 8\sqrt{9 - 3\ m})^{1/3} \right]$$

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(19) $$s_1 - s_2 = \frac{3^{1/3} m^{1/3}}{2^{7/2}} \left(\left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.$$
$$\left. \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)^{1/3} \right).$$

(20) $$-a = \frac{4 - 3m}{8\sqrt{2}} = \frac{4 - 3m}{2^{7/2}} .$$

(21) $$x_1 = \left(-a + (s_1 + s_2)\right) h .$$
$$= h \left(\frac{4 - 3m}{2^{7/2}} + \frac{3^{1/3} m^{1/3}}{2^{7/2}} \left(\left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.\right.$$
$$\left.\left. \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)^{1/3} \right)\right)$$
$$= \frac{h}{2^{7/2}} \left((4 - 3m) + 3^{1/3} m^{1/3} \left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.$$
$$\left. + 3^{1/3} m^{1/3} \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)^{1/3} \right)$$
$$= \frac{h}{2^{7/2}} \left((4 - 3m) + \left(3m \left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)\right)^{1/3} \right.$$
$$\left. + \left(3m \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)\right)^{1/3} \right)$$

(22) Calling $E = \left(3m \left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)\right)^{1/3}$

$F = \left(3m \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)\right)^{1/3}$

$$x_1 = \frac{h}{2^{7/2}} \left(4 - 3m + E + F \right).$$

(23) $$x_2 = \left(-a - 1/2 (s_1 + s_2) + 1/2 \sqrt{-3} \, (S_1 - s_2)\right) h.$$
$$= h \left\{ \frac{4 - 3m}{2^{7/2}} - \frac{3^{1/3} m^{1/3}}{2^{9/2}} \left(\left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.\right.$$
$$\left. + \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)^{1/3} \right)$$
$$+ \frac{\sqrt{-3}}{2} \cdot \frac{3^{1/3} m^{1/3}}{2^{7/2}} \left(\left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.$$
$$\left.\left. - \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)^{1/3} \right)\right\}$$
$$= h \left\{ \frac{4 - 3m}{2^{7/2}} - \frac{1}{2^{9/2}} \left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)^{1/3} \right.$$
$$+ \left(3m \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)\right)^{1/3})$$
$$+ \frac{\sqrt{-3}}{2} \cdot \frac{1}{2^{7/2}} \left(\left(3m \left(-9 m^2 + 36 m - 24 + 8\sqrt{9 - 3m}\right)\right)^{1/3} \right.$$
$$\left.\left. - \left(3m \left(-9 m^2 + 36 m - 24 - 8\sqrt{9 - 3m}\right)\right)^{1/3} \right)\right\}$$
$$= h \left\{ \frac{4 - 3m}{2^{7/2}} - \frac{1}{2^{9/2}} (E + F) + \frac{\sqrt{-3}}{2} (E - F) \right\}$$

Late ral Earth Pressure.

$$x_2 = \frac{h}{2\sqrt[3]{2}}\left(4 - 3m - \frac{E+F}{2} + \sqrt{-3}\left(\frac{E-F}{2}\right)\right)$$

$$= \frac{h}{2\sqrt[3]{2}}\left(4 - 3m + \frac{E}{2}(\sqrt{-3} - 1) + \frac{F}{2}(1 - \sqrt{-3})\right)$$

$$= \frac{h}{2\sqrt[3]{2}}\left(4 - 3m + \left(\frac{E-F}{2}\right)(\sqrt{-3} - 1)\right)$$

(24) $$x_3 = h\left(- a - 1/2\,(s_1 + s_2) - \frac{\sqrt{-3}}{2}(s_1 - s_2)\right).$$

$$= \frac{h}{2\sqrt[3]{2}}\left(4 - 3m + \frac{(E+F)}{2}(\sqrt{-3} - 1)\right)$$

(25) Valuation of the roots:

$$E = \left(3m\left(- 9m^2 + 36m - 24 + 8\sqrt{9 - 3m}\right)\right)^{1/3}$$

$$F = \left(3m\left(- 9m^2 + 36m - 24 - 8\sqrt{9 - 3m}\right)\right)^{1/3}.$$

Let $G = 3m(-9m^2 + 36m - 24) = 9m(-3m^2 + 12m - 8)$

$H = 24m\sqrt{9 - 3m}$

Then $E = (G + H)^{1/3}$; $F = (G - H)^{1/3}$.

m.	9 - 3 m.	$\sqrt{9 - 3m}$.	H	$-3m^2$.	12 m.	J $(-3m^2+12m-8)$	G = (9mJ)
1.0	6.0	2.45	58.8	- 3.00	12.00	1.00	9.00
1.1	5.7	2.39	63.0	3.63	13.20	1.57	15.55
1.2	5.4	2.32	66.8	4.32	14.40	2.08	22.5
1.3	5.1	2.26	70.5	5.07	15.60	2.53	29.6
1.4	4.8	2.19	73.5	5.88	16.80	2.92	36.8
1.5	4.5	2.12	76.3	6.75	18.00	3.25	43.9
1.6	4.2	2.05	78.6	7.68	19.20	3.52	50.7
1.7	3.9	1.97	80.5	8.66	20.40	3.74	57.1
1.8	3.6	1.90	82.1	9.73	21.60	3.87	62.6
1.9	3.3	1.82	83.0	10.81	22.80	3.99	68.1
2.0	3.0	1.73	83.0	12.00	24.00	4.00	72.0

Lateral Earth Pressure.

m	G+H	G-H	E	F	E+F	$\frac{E-F}{2}$	$\frac{E+F}{2}$	4-3m	K = (4-3m+E+F)	$x_1/h = \frac{K}{2^{7/2}}$	Empirical x_1/h
1.0	67.8	-49.8	4.08	-3.68	0.40	3.83	0.20	1.0	1.40	.1239	.1239
1.1	78.6	-47.4	4.29	-3.62	0.67	3.96	0.34	0.7	1.37	.121	.1201
1.2	89.3	-44.3	4.47	-3.54	0.93	4.01	0.47	0.4	1.33	.118	.1175
1.3	100.1	-40.9	4.65	-3.45	1.20	4.05	0.60	0.1	1.30	.115	.115
1.4	110.3	-36.8	4.80	-3.33	1.47	4.07	0.74	-0.2	1.27	.1125	.1125
1.5	120.2	-32.4	4.95	-3.19	1.76	4.07	0.88	-0.5	1.26	.111	.1101
1.6	129.3	-27.9	5.06	-3.03	2.02	4.05	1.01	-0.8	1.22	.108	.1080
1.7	137.6	-23.4	5.16	-2.86	2.30	4.01	1.15	-1.1	1.20	.106	.106
1.8	144.7	-19.5	5.25	-2.66	2.59	3.96	1.29	-1.4	1.19	.105	.104
1.9	151.1	-14.9	5.32	-2.46	2.86	3.89	1.43	-1.7	1.16	.1025	.1025
2.0	155.0	-11.0	5.37	-2.23	3.14	3.80	1.57	-2.0	1.14	.101	.101

m	L	M	N	O	P	Q	x_2/h	x_3/h
1.0	-2.83	0.80	-.250	.091	.575	.030	-.250+.575i	.091+.030i
1.1	-3.26	0.36	-.288	.031	.595	.051	-.288+.595i	.031+.051i
1.2	-3.61	-0.07	-.319	-.001	.604	.071	-.319+.604i	-.001+.071i
1.3	-3.95	-0.50	-.349	-.004	.610	.090	-.349+.610i	-.004+.090i
1.4	-4.27	-0.94	-.378	-.009	.613	.111	-.378+.613i	-.009+.111i
1.5	-4.57	-1.38	-.404	-.012	.613	.132	-.404+.613i	-.012+.132i
1.6	-4.85	-1.81	-.429	-.016	.610	.152	-.429+.610i	-.016+.152i
1.7	-5.11	-2.25	-.452	-.020	.604	.173	-.452+.604i	-.020+.173i
1.8	-5.36	-2.69	-.475	-.024	.595	.194	-.475+.595i	-.024+.194i
1.9	-5.59	-3.13	-.494	-.028	.585	.214	-.494+.585i	-.028+.214i
2.0	-5.80	-3.57	-.512	-.032	.575	.236	-.512+.575i	-.032+.236i

$$L = 4-3m-\frac{E-F}{2},\quad M = 4-3m-\frac{E+F}{2},\quad N = \frac{L}{2^{7/2}},\quad O = \frac{M}{2^{7/2}},\quad i = \sqrt{-1},$$

$$P = \frac{\sqrt{3}(E-F)}{2\cdot 2^{7/2}},\quad Q = \frac{\sqrt{3}(E+F)}{2\cdot 2^{7/2}},\quad x_2 = \frac{L}{2^{7/2}} + iP,\quad x_3 = \frac{M}{2^{7/2}} + iQ.$$

Comparison between Empirical and Complete Solutions.

Empirical : $x_e = 0.124\, m^{-.294}\, h.$

Complete : $$x_c = \frac{h}{2^{7/2}}\Big((4-3m) + \big(3m(-9m^2+36m-24+8\sqrt{9-3m})\big)^{1/3} + \big(3m(-9m^2+36m-24-8\sqrt{9-3m})\big)^{1/3}\Big)$$

$$\frac{h}{2^{7/2}} = 0.885\ h.$$

$$x_e = 0.885\ h\ (0.1401\ m^{-.294}\ h).$$

$$.1401\ m^{-.294} = 4-3m + (\quad)^{1/3} + (\quad)^{1/3}$$

$$(.1401\ m^{-.294} + 3m - 4) = (\quad)^{1/3} + (\quad)^{1/3} = E + F.$$

Lateral Earth Pressure.

Other methods used to approximate the solutions were :

<u>Newton's Rule</u> (1779). If $f(x) = 0$. and x_1 is the first approximation,

then $x_2 = x_1 - \frac{f(x)_1}{f'(x)_1}$. $x_3 = x_2 - \frac{f(x)_2}{f'(x)_2}$

where $f'(x)$ is $\frac{d}{dx} f(x)$.

<u>Regula Falsi</u> of Abraham ben Ezra (1130).

If $y = f(x) = 0$, then $x = x_1$, makes $y < 0$, and
$x = x_2$, makes $y > 0$,

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)} = x_1 + \frac{(x_2 - x_1) f(x_1)}{f(x_1) - f(x_2)} = x_2 + \frac{(x_2 - x_1) f(x_2)}{f(x_1) - f(x_2)}$$

$x_4 =$ the same with x_1 and x_2 replaced by x_1 and x_3 .

<u>Trigonometric Method</u>. If $y^3 + 3By + 2C = 0$, let $y = 2r \sin \theta$,

then $8 \sin^3\theta + \frac{6 B \sin\theta}{r^2} + \frac{2 C}{r^3} = 0$.

But $8 \sin^3\theta - 6 \sin\theta + 2 \sin 3\theta = 0$. hence $r = \sqrt{-B}$ and $\sin 3\theta = \frac{C}{\sqrt{-B^3}}$.

$y_1 = 2r \sin\theta$.
$y_2 = 2r \sin(120^\circ + \theta)$.
$y_3 = 2r \sin(240^\circ + \theta)$.

This solution holds only if there are 3 real roots.

If B^3 is < 0 and $|B^3| < C^2$, or if

$B^3 > 0$, the solution fails, as happens in this eqation,

for there is then one real root and two imaginaries. A solution by hyperbolic sines is then possible.

---------------------The End.---------------------