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I hereby recommend that the thesis prepared under my supervision by Frederic Edwin Fuller

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be accepted as fulfilling this part of the requirements for the degree of Doctor of Philosophy

Approved by:

S. J. M. Allen

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A STUDY OF X-RAY SCATTERING

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DOCTOR OF PHILOSOPHY

1936

by

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INTRODUCTION

The present work was undertaken in continuation of an investigation by Charles H. Perry, Doctor's Thesis, 1935, and by Frederic E. Fuller, Master's Thesis, 1935, both presented to the University of Cincinnati. The purpose of the above investigations and likewise of this one, finally, is to determine experimentally the ratios of the scattering coefficients of the light elements.

The work previous to Perry's [1-14] is unsatisfactory in either one or both of two respects. Namely, the total scattered radiation emergent from the samples was not measured, and the effect of the absorption of the scattered radiation by the sample was not accounted for. Perry eliminated the first objection by surrounding the sample, except for the path of the incident radiation, by a large spherical ionization chamber. He proposed to eliminate the second objection by plotting the measured scattering of progressively smaller samples against their masses and by extrapolating the resulting curve through

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the origin to obtain the ratio of scattering to mass for zero mass and thus to eliminate the effect of absorption in the sample.

THEORY OF INTERPRETATION OF MEASUREMENTS

In this work a method of performing this extrapolation will be given and an interpretation of the result so obtained will be derived. For the purpose of solving the latter problem let the scattering body be a circular cylinder of radius R, length l, in which points are to be located by the cylindrical coordinates r, α, z, the z-axis of which coincides with the axis of the cylinder parallel to which the incident radiation passes in the sense of z increasing. Let I_0 be the intensity of the incident beam. Then by definition of μ where I is the intensity at depth z of the sample,

$$dI = -I\mu dz \tag{1}$$

which integrated from $z = 0$ to $z = z$, gives

$$I = I_0 e^{-\mu z} \tag{2}$$

for the intensity of the incident radiation at depth z. Now let A be defined by the relation

$$dI = -dA \tag{3}$$

Then,

$$dA = I\mu dz = I_0 e^{-\mu z}\mu dz$$

Of this radiation dA absorbed from the incident beam, a part designated by dS will be scattered. For a given sample and incident radiation, let p, a constant in this case, be defined by

$$dS = pdA \tag{4}$$

Then,

$$dS = pI_0 e^{-\mu z}\mu dz$$

and

$$S = \int_0^z pI_0 e^{-\mu z}\mu dz = pI_0(1 - e^{-\mu z}) \tag{5}$$

Observe that S as defined above refers to the energy scattered, and not to the energy emergent from the sample.

Also we have by definition

$$\mu = \sigma + \tau \tag{6}$$

where

$$dI = -I\mu dz = -I\sigma dz - I\tau dz \tag{7}$$

and where

$$dS = I\sigma dz \; ; \; dT = I\tau dz \tag{8}$$

where T refers to the energy totally absorbed by the sample, and S, as before, to the energy scattered by the sample. But from the conditions defining p, and from the relation $dI = -dA$, we have

$$\begin{aligned} dS &= pdA & \qquad dT &= (1 - p)dA \\ &= -pdI & &= -(1 - p)dI \\ &= pI\mu dz & &= (1 - p)I\mu dz \end{aligned}$$

Thus,

$$I\sigma dz = pI\mu dz \; ; \quad I\tau dz = (1 - p)I\mu dz$$

so that finally

$$\sigma = p\mu \qquad ; \quad \tau = (1 - p)\mu. \tag{9}$$

In words, p may be defined as the probability that a photon which undergoes a scattering-absorption process will be scattered.

Now, let us calculate

$$\left(\frac{dS_1}{dW}\right)_0$$

where S_1 is the energy of the emergent scattered radiation composed of photons which have undergone but a single scattering-absorption process and where the zero subscript refers to the value of $\frac{dS_1}{dW}$ when W, the sample weight, is zero. The energy of a monochromatic parallel beam of incident radiation of frequency ν_0 falling on area $r dr d\alpha$ at depth z of a sample is

$$h\nu_0 Jr\ dr\ d\alpha\ e^{-\mu_0 z}$$

where μ_0 is the absorption coefficient of the sample for the incident radiation and where J is the number of photons passing through unit area of cross-section of incident beam per unit time. The energy absorbed in element of volume $r\ dr\ d\alpha\ dz$ is

$$\mu_0 h\nu_0 Jr\ dr\ d\alpha\ e^{-\mu_0 z}\ dz$$

and the energy scattered in this element of volume is

$$p\mu_0 h\nu_0 Jr\ dr\ d\alpha\ e^{-\mu_0 z}\ dz.$$

The energy emerging from sample in direction of $d\varphi$, $d\theta$ from element of volume $r\ dr\ d\alpha\, dz$ is

$$f(\theta)\sin\theta\, p\mu_0 h\nu_0 Jr\ dr\ d\alpha\, dz\ e^{-\mu_0 z}\ e^{-\mu_1 d}\ d\theta\ d\varphi$$

where

$$\int_0^{2\pi}\int_0^{\pi} f(\theta)\sin\theta\, d\theta\, d\varphi = 2\pi\int_0^{\pi} f(\theta)\sin\theta\, d\theta = 1,$$

and where μ_1 is the absorption coefficient of the sample for radiation of absorption coefficient μ_0 which has undergone one and only one absorption-scattering process, and where d is the distance from element of volume $rdrd\alpha dz$ in direction of $d\varphi$, $d\theta$ to surface of sample. Finally,

$$S_1 = p\mu_0 K \int_0^{l=R}\int_0^{\alpha=2\pi}\int_0^{\varphi=2\pi}\int_0^{\theta=\pi}\int_0 rdrd\alpha d\varphi f(\theta)\sin\theta d\theta \cdot e^{-\mu_1 d}\, e^{-\mu_0 z}\, dz$$

$$K = h\nu_0 J \tag{10}$$

$$S_1 = p\mu_0 K\int_0^R\int_0^{2\pi}\int_0^{2\pi} rdrd\alpha d\varphi \int_0^1 e^{-\mu_0 z}\, dz \left\{ \int_0^{\tan^{-1}\frac{\beta}{1-z}} f(\theta)\sin\theta e^{-\mu_1(1-z)\sec\theta}\, d\theta \right.$$

$$\left. - \int_{\pi-\tan^{-1}\frac{\beta}{z}}^{\pi} f(\theta)\sin\theta e^{-\mu_1 z\sec\theta}\, d\theta + \int_{\tan^{-1}\frac{\beta}{1-z}}^{\left(\pi-\tan^{-1}\frac{\beta}{z}\right)} f(\theta)\sin\theta e^{-\mu_1\beta\csc\theta}\, d\theta \right\} \tag{11}$$

where β is the distance from the point r, α, z in the plane of z in the direction of φ to the cylindrical surface of the cylinder. The integration over θ has been divided between three regions formed by the conoids generated by a line drawn from the element of volume $rdrd\alpha dz$ to the circular edges of the cylinder.

Now in general, where

$$F(\alpha) = \int_{x_0}^{X} f(x,\alpha)\, dx$$

we have

$$\frac{dF}{d\alpha} = \int_{x_0}^{X} \frac{\partial f(\alpha,x)}{\partial\alpha}\, dx + \frac{dX}{d\alpha} f(X,\alpha) - \frac{dx_0}{d\alpha} f(x_0,\alpha)$$

And in this case,

$$F(l) = p\mu_0 K\int_0^R\int_0^{2\pi}\int_0^{2\pi} rdrd\alpha d\varphi\; F'(l)$$

Then,

$$\frac{dS}{dl} = p\mu_0 K\int_0^R\int_0^{2\pi}\int_0^{2\pi} rdrd\alpha d\varphi\; \frac{\partial F'(l)}{\partial l}$$

$$\frac{\partial F'}{\partial l} = \int_0^l \left\{\;\right\} dz + f(l,z)_{z=l}$$

$$\left(\frac{\partial F'}{\partial l}\right)_{l=0} = \Big[f(l,z)_{z=l} \Big]_{l=0}$$

$$= \left[e^{-\mu l}\left\{ \int_0^{\frac{\pi}{2}} f(\theta)\sin\theta d\theta - \int_{(\pi-\tan^{-1}\frac{\rho}{l})}^{\pi} f(\theta)\sin\theta\, e^{-\mu l\sec\theta}\, d\theta + \int_{\frac{\pi}{2}}^{(\pi-\tan^{-1}\frac{\rho}{l})} f(\theta)\sin\theta\, e^{-\rho\csc\theta}\, d\theta \right\}\right]_{l=0}$$

$$= \int_0^{\pi} f(\theta)\sin\theta d\theta = \frac{1}{2\pi}$$

$$\left(\frac{dS}{dl}\right)_0 = \frac{p\mu_0 K}{2\pi}\int_0^R\int_0^{2\pi}\int_0^{2\pi} r\,dr\,d\alpha\,d\varphi = p\mu_0 K\pi R^2 = p\mu_0 KA$$

and finally,

$$\left(\frac{dS}{dW}\right)_0 = p\frac{\mu}{\rho}K \quad ; \qquad W = \rho A l$$

Further, if we replace $e^{-\mu_1 d}$ in (11) by $(1-e^{-\mu_r d})$ we get

$$S_1' = p\mu_0 K\int_0^R\int_0^{2\pi}\int_0^{2\pi} r\,dr\,d\alpha\,d\varphi\int_0^1 e^{-\mu_0 z}\,dz\left\{\int_0^{\tan^{-1}\frac{\rho}{1-z}} f(\theta)\sin\theta\left[1-e^{-\mu_1(1-z)\sec\theta}\right]d\theta - \int_{(\pi-\tan^{-1}\frac{\rho}{z})}^{\pi} f(\theta)\sin\theta\left[1-e^{-\mu_1 z\sec\theta}\right]d\theta + \int_{\tan^{-1}\frac{\rho}{1-z}}^{(\pi-\tan^{-1}\frac{\rho}{z})} f(\theta)\sin\theta\left[1-e^{-\mu_1\rho\csc\theta}\right]d\theta\right\} \quad (12)$$

where S_1' is that part of the scattered radiation which having undergone a first scattering-absorption process undergoes a second scattering-absorption process. Of this energy S_1' a part is totally absorbed in the sample while the remainder appears as radiation which has been scattered a second time and which may emerge from the sample as scattered radiation. Now in all cases there exists a finite number ϵ such that

$$S_T = S_1 + \epsilon S_1'$$

where S_T is the total scattered radiation emerging from the sample. Then,

$$\left(\frac{dS_T}{dW}\right)_0 = p\left(\frac{\mu}{\rho}\right)_0 K + \epsilon_0\left(\frac{dS_1'}{dW}\right)_0 + (S_1')_0\left(\frac{d\epsilon}{dW}\right)_0$$

We observe in (12) that

$$\left(\frac{dS_1'}{dW}\right)_0 = 0 = (S_1')_0$$

Thus we have the important result that

$$\left(\frac{dS_T}{dW}\right)_0 = p\left(\frac{\mu}{\rho}\right)_0 K \qquad (13)$$

Let us observe that this result (13) is independent of the distribution in angle of the scattered radiation, of the wave-length and absorption coefficient of the scattered radiation; but depends only upon: p, the scattering-total absorption ratio; $\left(\frac{\mu}{\rho}\right)_0$, the mass absorption coefficient of the sample for the incident radiation; and linearly upon the area of the sample.

The experimental results for determining the ratios of scattering coefficients for the two elements magnesium and aluminum are as shown in Figures 1 and 2. The areas of the samples and the intensities of the sensibly monochromatic incident radiation for the two sets of samples in each of the cases are the same. Therefore, having determined the slopes of these curves at the origin, we have by means of (9) and (13),

$$\frac{\left(\frac{\sigma}{\rho}\right)_1}{\left(\frac{\sigma}{\rho}\right)_2} = \frac{p_1}{p_2}\,\frac{\left(\frac{\mu}{\rho}\right)_1}{\left(\frac{\mu}{\rho}\right)_2} = \frac{\left(\frac{dS_1}{dW}\right)_0}{\left(\frac{dS_2}{dW}\right)_0} \qquad (14)$$

To obtain the ratios of the scatterings per electron we have when n is the number of electrons, when Z is the atomic number and A is the atomic weight, and when N is Avogadro's number,

$$\left(\frac{dS}{dW}\right)_0 = \left(\frac{dS}{dn}\right)_0 \frac{dn}{dW} \quad ; \quad \left(\frac{dS}{dn}\right)_0 = \frac{1}{\left(\frac{dn}{dW}\right)_0} \left(\frac{dS}{dW}\right)_0$$

$$n = Z\frac{W}{A}N \quad ; \quad \frac{dn}{dW} = \frac{Z}{A}N$$

$$\left(\frac{dS}{dn}\right)_0 = \frac{A}{ZN}\left(\frac{dS}{dW}\right)_0$$

Thence,

$$\frac{\left(\frac{dS}{dn}\right)_1}{\left(\frac{dS}{dn}\right)_2} = \frac{A_1 Z_2}{A_2 Z_1} \frac{\left(\frac{dS}{dW}\right)_1}{\left(\frac{dS}{dW}\right)_2} = \frac{P_1\left(\frac{\mu}{\rho}\right)_1 A_1 Z_2}{P_2\left(\frac{\mu}{\rho}\right)_2 A_2 Z_1} \tag{15}$$

Now we must consider a method of determining $\left(\frac{dS}{dW}\right)_0$ from the experimental data. We have shown that the energy of the scattered radiation is given by a function of the form

$$\alpha(1-e^{-\beta W})$$

However, the scattered radiation which emerges from the sample is given by a function of the above form diminished by means of some factor f(W) which is determined by the absorption of the scattered radiation in the sample. Thus,

$$S = \alpha f(W)(1-e^{-\beta W}) \tag{16}$$

The function f(W) is a maximum >0 at $W = 0$ and approaches a lower bound >0 as $W \rightarrow \infty$. Let us try a function of the form

$$f(W) = a + be^{-cW} \tag{17}$$

Since the absorption coefficient of the incident radiation for both sets of samples is known, we write

$$F = \frac{S}{(1-e^{-\frac{\mu}{\rho}\frac{W}{A}})} \tag{18}$$

Thence we have

$$F = a + be^{-cW} \tag{19}$$

where a, b, c are to be determined from the experimental data. We have

$$\frac{dS}{dW} = (a + be^{-cW})\beta e^{-\beta W} - cbe^{-cW}(1-e^{-\beta W})$$

$$\left(\frac{dS}{dW}\right)_0 = \beta(a + b) \tag{20}$$

We have for the curve fitting

$$F_i - \epsilon_i = a + be^{-cW_i}$$

$$\sum \epsilon_i^2 = \sum\{F_i - a - be^{-cW_i}\}^2$$

We seek to determine a, b, c so that $\sum \epsilon_i^2$ is a minimum.

$$\frac{\partial \sum \epsilon_i^2}{\partial a} = -2\sum\{F_i - a - be^{-cW_i}\} = 0$$

$$\frac{\partial \sum \epsilon_i^2}{\partial b} = -2\sum\{F_i - a - be^{-cW_i}\}e^{-cW_i} = 0$$

$$\frac{\partial \sum \epsilon_i^2}{\partial c} = 2\sum\{F_i - a - be^{-cW_i}\}bW_i e^{-cW_i} = 0$$

Thus,

$$\left.\begin{aligned} &\sum F_i - an - b\sum e^{-cW_i} = 0 \\ &\sum F_i e^{-cW_i} - a\sum e^{-cW_i} - b\sum (e^{-cW_i})^2 = 0 \\ &\sum F_i W_i e^{-cW_i} - a\sum W_i e^{-cW_i} - b\sum W_i (e^{-cW_i})^2 = 0 \end{aligned}\right\} \tag{21}$$

On eliminating a and b we have

$$\begin{vmatrix} \sum F_i & -n & -\sum e^{-cW_i} \\ \sum F_i e^{-cW_i} & -\sum e^{-cW_i} & -\sum (e^{-cW_i})^2 \\ \sum F_i W_i e^{-cW_i} & -\sum W_i e^{-cW_i} & -\sum W_i (e^{-cW_i})^2 \end{vmatrix} = 0 \tag{22}$$

which when expanded is

$$\sum F_i\left\{\sum e^{-cW_i}\cdot\sum W_i (e^{-cW_i})^2 - \sum (e^{-cW_i})^2 \sum W_i e^{-cW_i}\right\}$$
$$+ n\left\{-\sum F_i e^{-cW_i}\sum W_i (e^{-cW_i})^2 + \sum (e^{-cW_i})^2 \sum F_i W_i e^{-cW_i}\right\}$$

$$-\sum e^{-cW_i}\left\{-\sum F_i e^{-cW_i}\sum W_i e^{-cW_i} + \sum F_i W_i e^{-cW_i}\sum e^{-cW_i}\right\} = 0 \qquad (23)$$

Let (23) be designated by $f(c)$. Then

$$f(c) = f(c_0) + \partial c\, f'(c_0) = 0, \text{ approximately}$$

$$\partial c = -\frac{f(c_0)}{f'(c_0)}, \text{ approximately} \qquad (24)$$

By this means c is calculated by successive approximations until a value of c is obtained for which $f(c)$ vanishes.

$$\begin{aligned} f'(c) = \sum F_i \Big\{ & -2\sum e^{-cW_i}\sum W_i^2 (e^{-cW_i})^2 + \sum (e^{-cW_i})^2 \sum W_i^2 e^{-cW_i} \\ & + \sum W_i e^{-cW_i}\sum W_i (e^{-cW_i})^2 \Big\} \\ + n\Big\{ & 2\sum F_i e^{-cW_i}\sum W_i^2 (e^{-cW_i})^2 - \sum (e^{-cW_i})^2 \sum F_i W_i^2 e^{-cW_i} \\ & - \sum F_i W_i e^{-cW_i}\sum W_i (e^{-cW_i})^2 \Big\} \\ - \sum e^{-cW_i} & \Big\{\sum F_i e^{-cW_i}\sum W_i^2 e^{-cW_i} - \sum e^{-cW_i}\sum F_i W_i^2 e^{-cW_i}\Big\} \\ - \sum W_i e^{-cW_i} & \Big\{\sum F_i e^{-cW_i}\sum W_i e^{-cW_i} - \sum F_i W_i e^{-cW_i}\sum e^{-cW_i}\Big\} \end{aligned} \qquad (25)$$

From the first two of equations (21) we have

$$an + b\sum e^{-cW_i} = \sum F_i$$

$$a\sum e^{-cW_i} + b\sum (e^{-cW_i})^2 = \sum F_i e^{-cW_i}$$

Hence,

$$a = \frac{\sum F_i \sum (e^{-cW_i})^2 - \sum F_i e^{-cW_i}\sum e^{-cW_i}}{n\sum (e^{-cW_i})^2 - (\sum e^{-cW_i})^2}$$

$$b = \frac{n\sum F_i e^{-cW_i} - \sum F_i \sum e^{-cW_i}}{n\sum (e^{-cW_i})^2 - (\sum e^{-cW_i})^2} \qquad (26)$$

However, since, as was found, equations (26) involve small differences of large numbers, it is more convenient to use the last two of equations (21).

$$a\sum e^{-cW_i} + b\sum (e^{-cW_i})^2 = \sum F_i e^{-cW_i}$$

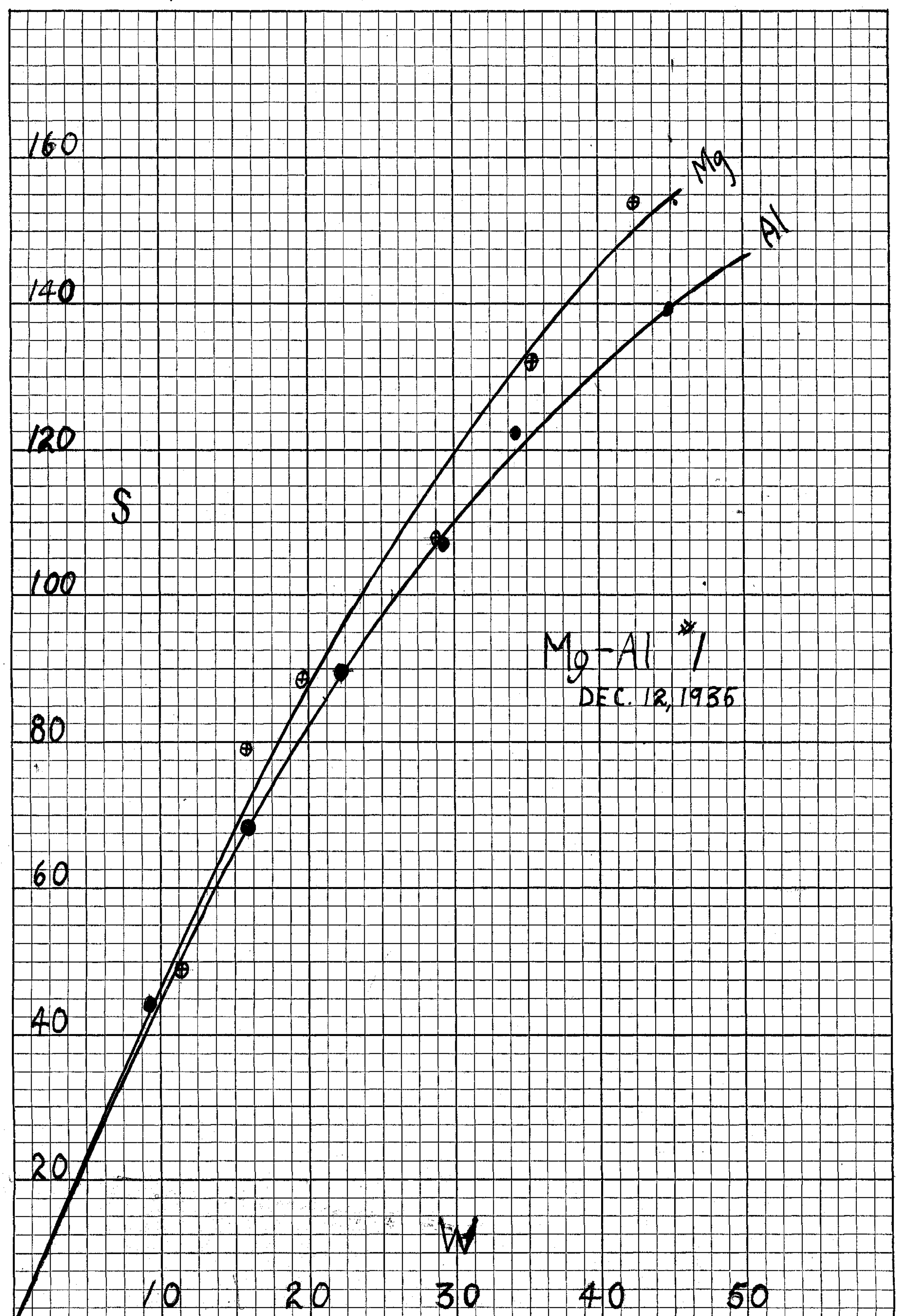
$$a\sum W_i e^{-cW_i} + b\sum W_i (e^{-cW_i})^2 = \sum F_i W_i e^{-cW_i}$$

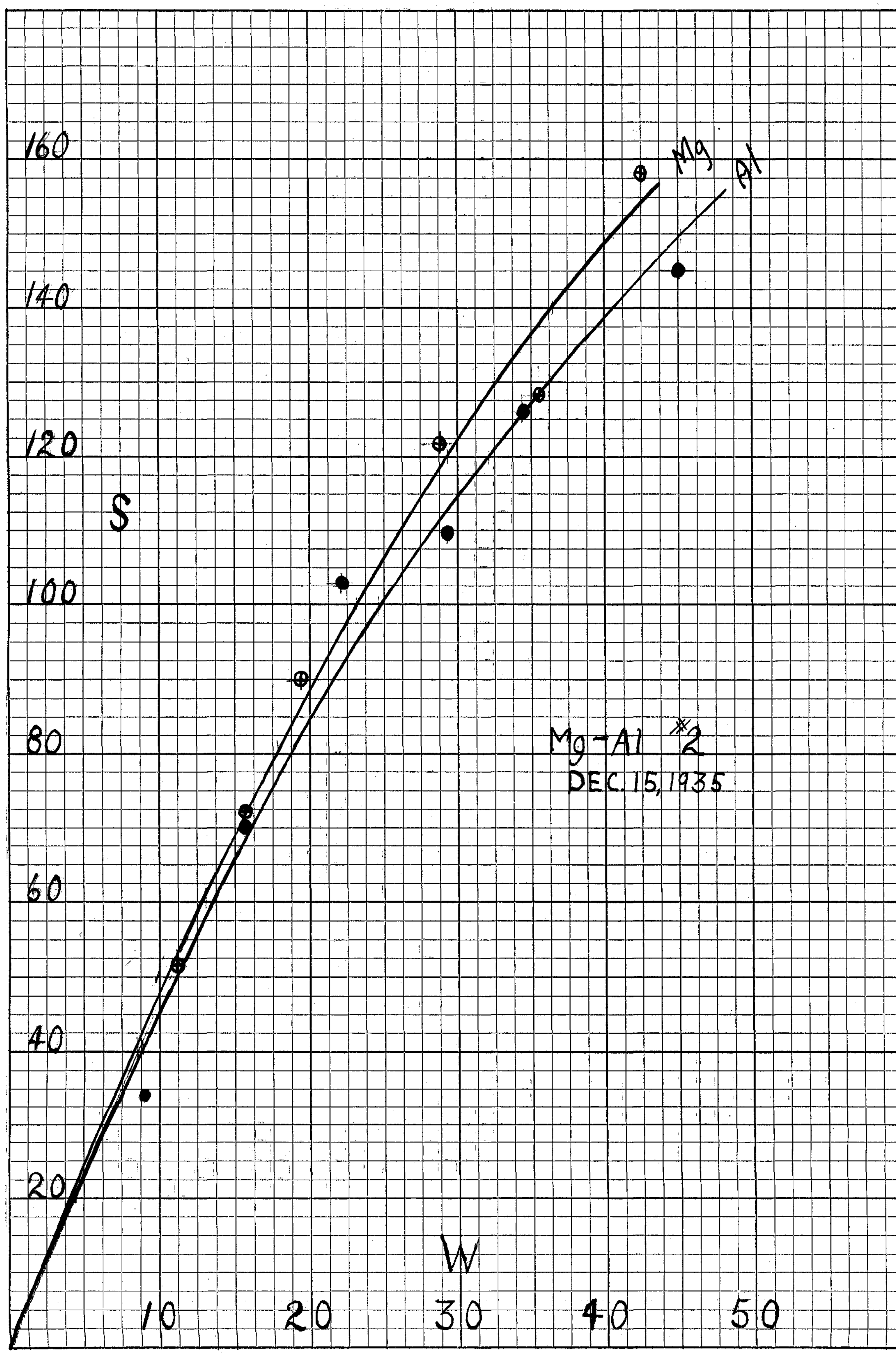
Hence,

$$a = \frac{\sum F_i e^{-cW_i} \sum W_i (e^{-cW_i})^2 - \sum (e^{-cW_i})^2 \sum F_i W_i e^{-cW_i}}{\sum e^{-cW_i} \sum W_i (e^{-cW_i})^2 - \sum (e^{-cW_i})^2 \sum W_i e^{-cW_i}}$$

$$b = \frac{\sum e^{-cW_i} \sum F_i W_i e^{-cW_i} - \sum F_i e^{-cW_i} \sum W_i e^{-cW_i}}{\sum e^{-cW_i} \sum W_i (e^{-cW_i})^2 - \sum (e^{-cW_i})^2 \sum W_i e^{-cW_i}} \quad (26)'$$

University of Cincinnati, College of Engineering.




160
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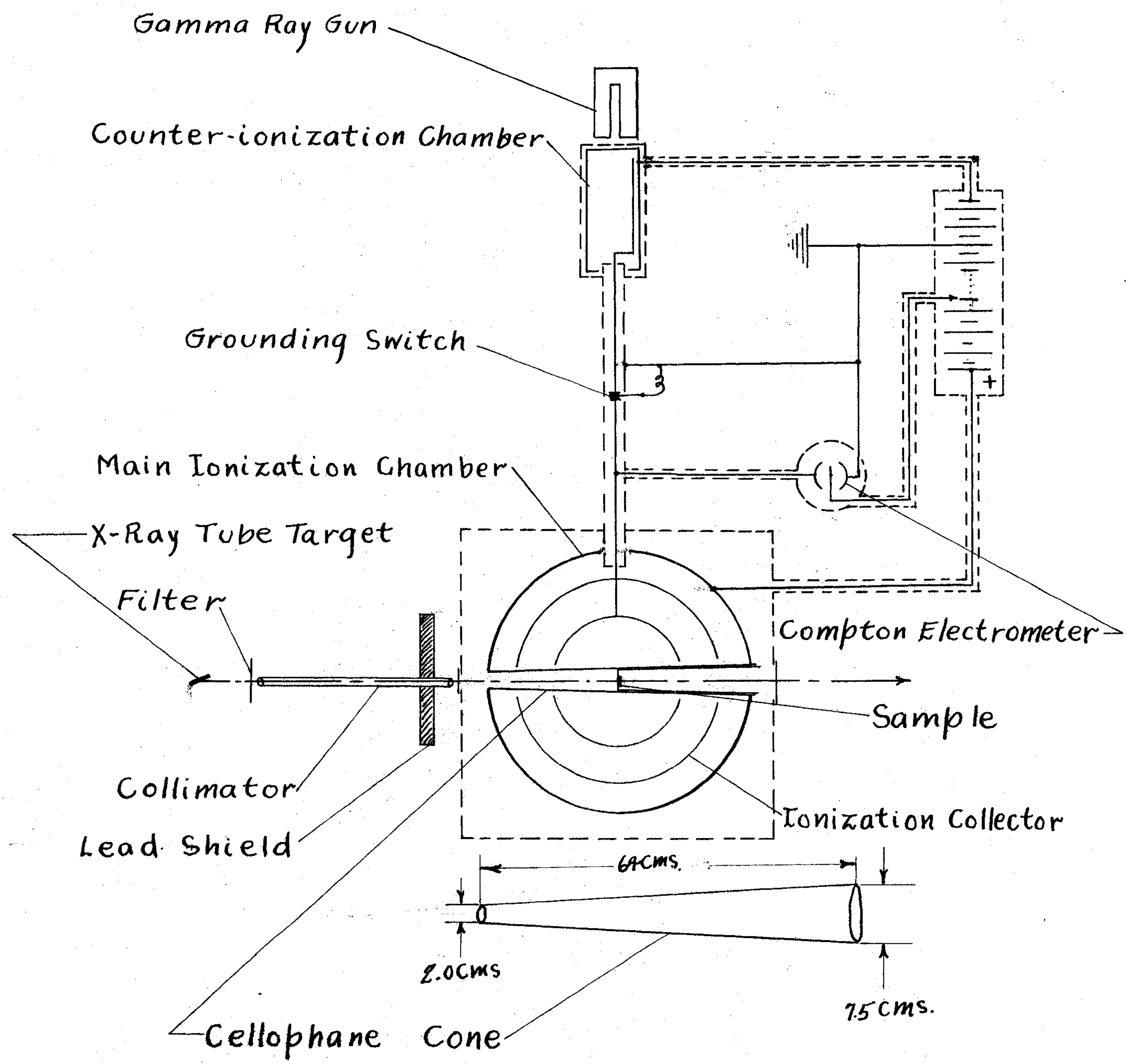

Gamma Ray Gun
Counter-ionization Chamber
Grounding Switch
Main Ionization Chamber
X-Ray Tube Target
Filter
Compton Electrometer
Sample
Collimator
Lead Shield
Ionization Collector
64 CMS.
2.0 CMS
7.5 CMS.
Cellophane Cone

Fig 3

APPARATUS AND EXPERIMENTAL PROCEDURE

The apparatus used in this work, the schematic diagram of which is shown in Figure 3, was essentially the same as that devised and used by Perry. The ionization chamber used for measuring the intensity of the scattered X-rays emerging from the sample was a hollow, copper sphere, 63 cms. in diameter, maintained at a positive potential of 1300 volts with respect to ground by means of "B" batteries. It was found that, even with the extreme care taken in insulating the ionization collector, a large "natural leak" obtained. Also, the column of air in the incident beam scattered radiation into the ionization chamber greatly to increase the extraneous current. The counter ionization chamber, connected as shown, served to provide a counter-leak which could be adjusted by suitably moving the gamma-ray gun. The entire apparatus was shielded against stray electrical and X-ray radiation as shown by the dotted lines in Figure 3.

The incident beam was collimated by means of a suitably shielded, heavy, brass tube, 30 cms. in length and 1.25 cms. inside diameter; and passed through the sphere on a diameter of the sphere on the axis of a slender, thin-walled, cellophane cone. This cellophane cone served to isolate from the ionization chamber the ionization in the air column of the incident beam, while permitting scattered radiation emerging from the samples to pass into the ionization chamber. Careful adjustments were made so that the incident beam after emerging from the collimator passed through the spherical ionization chamber without encountering any matter other than two thin cellophane windows (one on entering, the other on leaving); a sample support formed of thin, light, cellophane strips; the samples; and the air in the path of the beam. The sample support was mounted on a second cellophane cone which fitted snugly into the first and held the sample on the axis of the first cone at the center of the sphere. The sample-holder was removable to allow rapid and easy change of samples. The assembly of these cones and their dimensions is shown in Figure 3. The dimensions of the incident beam are as follows: 1.25 cms. diameter on entering the sphere; 2.5 cms. at the sample; and 3.75 cms. on leaving the sphere. The diameter of the samples was 1.127 cms.

The X-ray tube was a General Electric, type CA crystal-analysis tube with molybdenum, water-cooled target. The transformer, kenetron, and control equipment providing single-wave rectification serving the tube was a Kelley-Koet, type KBRA X-ray apparatus. The cooling water for the target passed through a series of glass tubes before entering and after leaving the target. The current through these tubes was less than one milliampere at 40 KV. The X-ray tube was placed in an oil bath with container of heavy lead sheet provided with two suitable openings so located in relation to the target as to provide two X-ray beams in a horizontal plane, one on each side of the tube. The one beam was the incident beam, the other was directed directly into a long, auxiliary ionization chamber, the current output of which was measured by a sensitive galvanometer. The intensities of the X-ray beams generated by the tube were maintained constant by adjustments which maintained constant the deflection of this galvanometer. The procedure in this respect was to hold the X-ray tube current constant at 36 milliamperes and to adjust the primary voltage on the high tension transformer serving the X-ray tube in a manner that maintained the galvanometer deflection constant.

It is believed that in this way not only the output of the tube was kept constant but also the voltage on the tube. The reading of the X-ray tube current was believed to be a correct reading at all times. However, the pre-voltage for a given tube-voltage depended upon the temperature of the apparatus, and was thus not a dependable criterion of the tube-voltage. The tube-voltage used was about 30 KV. By means of a suitable motor-generator set the voltage serving the X-ray apparatus was rendered constant except for slow variations to within one-tenth volt in 60 volts. Not withstanding the improvements over Perry's method (the use of the motor-generator set, the auxiliary ionization chamber, and the oil bath for the X-ray tube), the readings obtained were by no means precise as may be seen from the record of experimental data and from the Figures 1 and 2.

The measurements of ionization current were made by timing the drift of a Compton electrometer between two fixed points on a scale. The procedure in taking measurements was to adjust the gamma-ray gun so as to give a leak current corresponding to a time reading of about 40 seconds. This leak current was the result of the natural leak of the system and of the radiation scattered by the air in the incident beam. Five readings of this leak were taken with no sample in place. A sample, say aluminum, was then put in place and five readings were taken. This sample was then removed and five leak readings were taken. In this manner, alternating aluminum samples with magnesium samples, the data was obtained.

The lack of precision obtained is observed to be derived from two sources: one, an uncertainty as to any particular reading; the other, a slow change of the leak readings. This latter effect may be accounted for by arguments involving changes in the room temperature, which were large, and the disparity between the sizes of the spherical and counter-leak ionization chambers. With regard to the uncertainty in any particular reading, I have no suggestions to offer. However, I wish to point out that having reduced the tube voltage from Perry's value of 45 KV to 30 KV, the intensity of the incident beam was thereby greatly reduced, in fact, by the ratio of : 1. Thus a high electrometer sensitivity was required and the ionization currents corresponding to the scattering from the samples were relatively small quantities as compared with the "natural leak."

The X-ray radiation used in this work was that resulting from filtering by means of zirconium oxide, radiation generated at a molybdenum target operating at 30 KV. The pertinent information obtained with regard to the filter used is as follows:

$-t = .52$, see Figure 10, author's Master's Thesis;

transmission of Mo $K\alpha$ radiation $\doteq$ 30 per cent;

effective mass absorption coefficient of transmitted radiation in aluminum = 6.32.

The effective mass absorption coefficient recorded above was determined for X-ray tube operating conditions and absorption sheet thicknesses the same as obtained in the scattering experiments. Therefore, this value for aluminum and the corresponding value for magnesium were used in computing the values of β used in the curve fitting. In Figure 4 is shown the absorption curve from which the effective absorption coefficient in aluminum was computed. We observe that for the thicknesses of absorption sheets used the absorption is "linear" in accordance with the assumptions underlying the form of the factor $(1-e^{-\beta W})$ in the curve-fitting formula. The mass absorption coefficient of Mo K_α radiation in aluminum is 5.22. Therefore, we conclude that the radiation used included a not inconsiderable portion softer than the Mo K_α.

The many diverse factors involved in the choice of the proper tube voltage are discussed in the author's Master's Thesis, pp. 23-30. Some discussion is also given there with

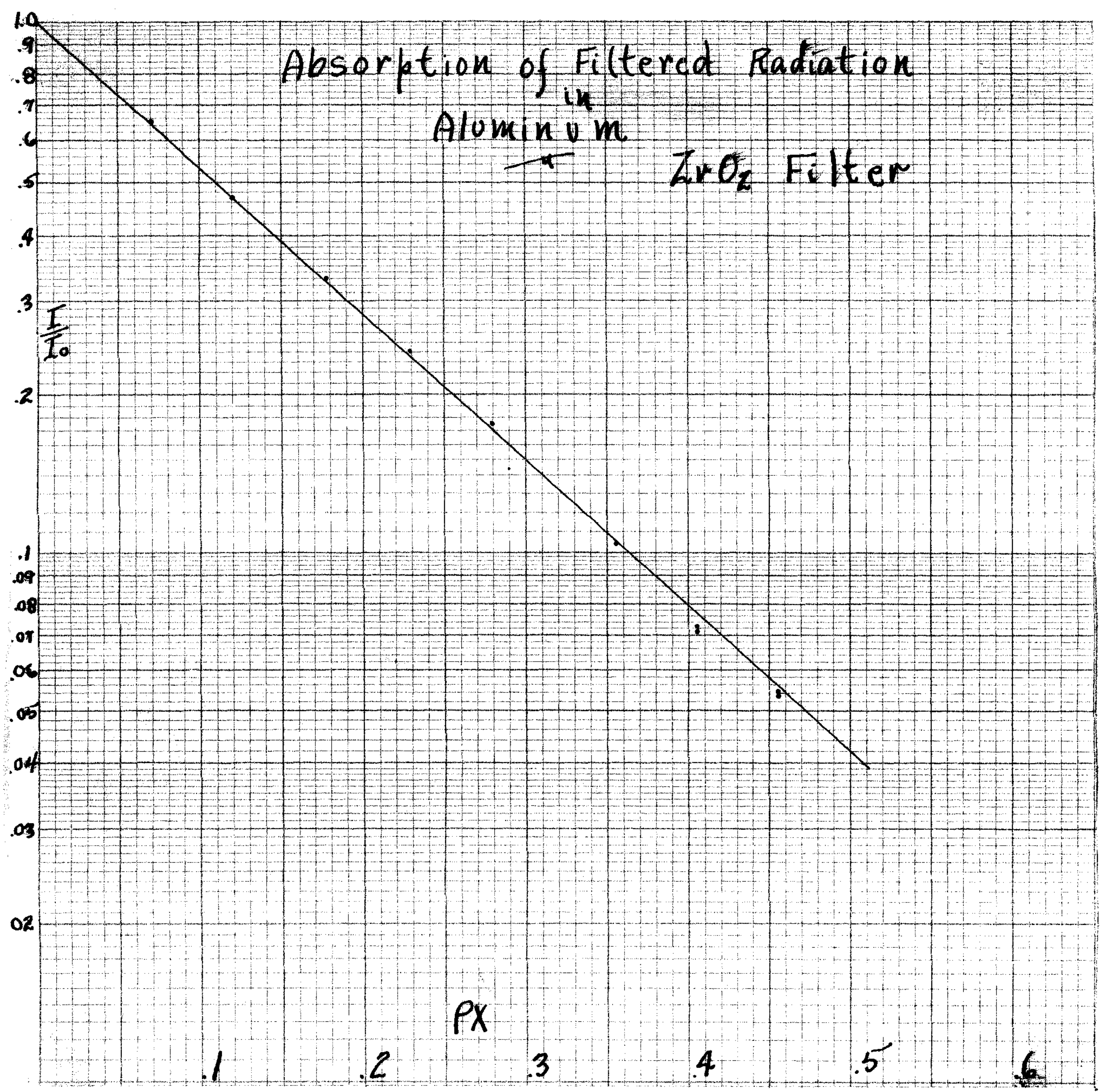
Absorption of Filtered Radiation in Aluminum
ZrO_2 Filter
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.09
.08
.07
.06
.05
.04
.03
.02
$\frac{I}{I_0}$
ρX
.1
.2
.3
.4
.5
.6

FIG. 4

respect to proper filter thickness. The following should be added. There is in the filtered radiation, in addition to that part of the incident radiation which is transmitted, radiation emitted from the filter which is characteristic of the filter substance. Let us consider the case where the incident radiation and the filtered radiation are each single lines. Then, where F is the intensity of the fluorescent radiation,

$$dF = f\mu_1 I_0\, e^{-\mu_1 z}\, dz\, e^{-\mu_2(l-z)}$$
$$= f\mu_1 I_0\, e^{-\mu_2 l}\, e^{-(\mu_1-\mu_2)z}$$

where f is that number which multiplied by μ_1, the absorption coefficient of the incident radiation of frequency ν_1, gives that part of the absorption coefficient which may be attributed to the excitation of fluorescent radiation of frequency ν_2 and absorption coefficient μ_2; and where l is the thickness of the filter.

$$F = \frac{f\mu_1 I_0\, e^{-\mu_2 l}}{-(\mu_1-\mu_2)} \int_0^l e^{-(\mu_1-\mu_2)z} \cdot -(\mu_1-\mu_2)\,dz$$
$$= \frac{f\mu_1 I_0\, e^{-\mu_2 l}}{\mu_1-\mu_2}\left[1 - e^{-\mu_1 l}\, e^{+\mu_2 l}\right]$$
$$= f\mu_1 I_0\, \frac{e^{-\mu_2 l} - e^{-\mu_1 l}}{\mu_1-\mu_2} \qquad (27)$$
$$\frac{dF}{dl} = f\mu_1 I_0\, \frac{\mu_1 e^{-\mu_1 l} - \mu_2 e^{-\mu_2 l}}{\mu_1-\mu_2}$$

Thus, F, the intensity of the fluorescent radiation emitted by the filter, has a maximum value given by

$$\mu_2 e^{-\mu_2 l} = \mu_1 e^{-\mu_1 l}$$
$$\log\mu_2 - \log\mu_1 = (\mu_2-\mu_1)l$$
$$l_m = \frac{\log\left(\frac{\mu_2}{\mu_1}\right)}{\mu_2-\mu_1} \qquad (28)$$

For a molybdenum filter and M_o K_α incident radiation,

$$\frac{\mu_1}{\rho} = 19.5 \quad ; \quad \frac{\mu_2}{\rho} = 26.0$$

$$\rho l_m = \frac{\log\left(\frac{26.0}{19.5}\right)}{26 - 19.5} = .205$$

Also, by means of (27) and the fact that $I = I_o e^{-\mu_1 l}$,

$$\frac{F}{I} = \frac{f\mu_1 I_o \dfrac{e^{-\mu_2 l} - e^{-\mu_1 l}}{\mu_1 - \mu_2}}{I_o \; e^{-\mu_1 l}} = f\frac{\mu_1}{\mu_1 - \mu_2}\left\{e^{(\mu_1 - \mu_2)l} - 1\right\}$$

and for $\mu_2 > \mu_1$, as is usually the case,

$$\frac{F}{I} = f\frac{\mu_1}{\mu_2 - \mu_1}\left\{1 - e^{-(\mu_2 - \mu_1)l}\right\} \qquad (29)$$

Thus $\frac{F}{I}$, the ratio of the fluorescent radiation emitted to the incident radiation transmitted by the filter in a monotonically increasing function of l. Without definite knowledge of the value of f we cannot draw practical conclusions from (29) further than to preclude the conclusion that might be drawn from Figure 10 of the author's Master's Thesis, that the quality of the radiation is improved with indefinite increase of filter thickness.

RECORD OF EXPERIMENTAL DATA AND COMPUTATION

Weights of Samples

(Diameter of samples = 1.127 cms.)

Mg No.	Mg Wt. gs.	Al No.	Al Wt. gs.
1	.0112	1	.0091
2	.0157	2	.0156
3	.0197	3	.0224
4	.0289	4	.0295
5	.0354	5	.0345
6	.0424	6	.0449

Experiment No. 1, Dec. 12, 1935

36 milliamperes - 30 KV, ZrO_2 filter

Li	Mg_3	L_2	Al_5	L_3	Mg_1	L_4	Al_4
28.9	8.8	25.6	6.4	27.7	10.8	24.6	6.9
37.0	8.6	36.4	6.6	24.0	11.2	24.0	7.0
28.4	8.6	23.0	6.4	26.0	10.4	26.0	6.8
39.4	7.6	33.8	6.2	29.0	12.4	25.1	6.6
27.0	8.0	32.2	6.0	23.3	11.8	26.0	7.1
L_5	Mg_4	L_6	Al_2	L_7	Mg_5	L_8	Al_3
24.3	6.5	25.1	9.2	21.8	5.4	17.0	6.9
27.2	6.4	22.3	8.6	21.0	5.5	18.0	7.0
23.9	6.8	19.6	8.2	20.8	5.4	15.6	6.8
23.0	7.2	17.8	8.6	23.5	5.6	19.8	6.6
24.3	6.2	22.8	8.8	19.0	5.3	18.4	7.1

L_9	Mg_6	L_{10}	Al_6	L_{11}	Mg_2	L_{12}	Al_1
17.7	4.6	17.4	4.7	14.4	7.0	15.7	8.8
18.5	4.7	16.6	5.2	15.2	6.8	16.5	9.4
16.6	4.8	16.0	4.8	14.2	7.0	14.8	9.0
17.9	4.7	17.5	5.0	17.0	6.8	16.1	8.2
18.7	4.8	16.5	5.1	14.0	7.1	17.0	9.5

L_{13}
15.2
12.8
13.6
12.8
14.4

On taking the proper average values, there results

L	T	Sample
31.1	8.32	Mg_3
28.1	6.32	Al_5
25.6	11.32	Mg_1
24.8	6.80	Al_4
23.0	6.62	Mg_4
21.3	8.68	Al_2
19.5	5.44	Mg_5
17.8	6.88	Al_3
17.3	4.72	Mg_6
15.9	4.96	Al_6
15.5	6.94	Mg_2
14.4	8.98	Al_1

The leak and leak and sample times recorded above when converted into scattering currents due to samples alone, give

Mg Wt. gs.	Mg S	Al Wt. gs.	Al S	Sample Nos.
.0112	49.3	.0091	44.0	1
.0157	79.5	.0156	68.3	2
.0197	88.1	.0224	89.1	3
.0289	107.5	.0295	106.7	4
.0354	132.6	.0345	122.6	5
.0424	154.2	.0449	139.1	6

Experiment No. 2, Dec. 15, 1935

36 milliamperes - 30 KV, ZrO_2 filter

L_1	Mg_2	L_2	Al_1	L_3	Mg_6	L_4	Al_6
33.3	9.8	35.0	13.6	34.4	5.1	30.0	5.6
28.4	8.6	26.2	15.6	28.6	5.0	29.2	5.2
34.7	10.0	34.6	15.2	25.5	5.5	30.7	5.8
35.0	10.6	36.7	16.2	25.2	5.1	30.5	5.7
40.6	10.3	34.2	12.2	30.5	5.3	31.0	5.9

L_5	Mg_5	L_6	Al_3	L_7	Mg_4	L_8	Al_2
30.0	6.2	23.6	7.0	24.1	6.0	24.5	9.6
28.8	6.2	34.2	7.1	22.7	6.1	18.2	8.3
26.7	6.1	28.0	7.6	24.6	6.1	22.6	8.5
29.0	6.2	23.9	7.0	26.3	6.1	21.3	9.0
33.3	6.2	24.8	6.9	29.4	6.3	21.2	8.8

L_9	Mg_1	L_{10}	Al_4	L_{11}	Mg_3	L_{12}	Al_5	L_{13}
22.6	10.4	22.4	6.5	20.0	7.2	22.4	5.7	18.0
17.0	10.3	22.5	6.2	18.0	7.0	22.6	5.5	21.6
20.0	9.8	27.2	6.5	20.5	7.6	18.0	5.7	20.8
18.4	10.0	23.9	6.3	19.4	6.6	18.2	5.8	22.0
21.4	11.0	20.5	6.4	17.3	7.2	18.0	5.7	21.6

On taking the proper average values, there results

L	T	Sample
33.9	9.9	Mg_2
31.1	14.5	Al_1
29.5	5.2	Mg_6
29.9	5.6	Al_6
28.3	6.2	Mg_5
26.1	7.1	Al_3
23.5	6.1	Mg_4
20.6	8.8	Al_2
21.6	10.3	Mg_1
21.1	6.4	Al_4
19.4	7.1	Mg_3
20.3	5.7	Al_5

The leak and leak and sample times recorded above when converted into scattering currents due to samples alone, give

Mg		Al		
Wt. gs.	S	Wt. gs.	S	Sample Nos.
.0112	50.8	.0091	36.8	1
.0157	71.5	.0156	65.1	2
.0197	89.5	.0224	102.7	3
.0289	121.3	.0295	108.8	4
.0354	125.8	.0345	126.0	5
.0424	158.4	.0449	145.3	6

CURVE FITTING

Mg, experiment No. 1

$$\beta = \frac{\mu}{\rho} \times \frac{1}{1000} = \frac{5.29}{.997 \times 1000} = .00531$$

S_i	W_i	βW_i	$e^{\beta W_i}$	$e^{-\beta W_i}$	$1-e^{-\beta W_i}$	F_i
49.3	11.2	.05947	1.06128	.94226	.05774	853.83
79.5	15.7	.08337	1.08695	.92000	.08000	993.75
88.1	19.7	.10461	1.11028	.90067	.09933	886.94
107.5	28.9	.15346	1.16586	.85773	.14227	755.60
132.6	35.4	.18797	1.20680	.82864	.17136	773.81
154.2	42.4	.22514	1.25250	.79840	.20150	765.26

Try c_0 = .01100

cW_i	e^{cW_i}	e^{-cW_i}	$(e^{-cW_i})^2$	$W_i e^{-cW_i}$	$W_i(e^{-cW_i})^2$
.12320	1.1311	.88408	.78161	9.902	8.754
.17270	1.1885	.84139	.70793	13.210	11.115
.21670	1.2420	.80517	.64829	15.862	12.771
.31790	1.3742	.72767	.52951	21.030	15.303
.38940	1.4909	.67073	.44987	23.744	15.925
.46640	1.5942	.62726	.39345	26.596	16.682
		4.55630	3.51067	110.343	80.550

$W_i^2 e^{-cW_i}$	$W_i^2(e^{-cW_i})^2$	$F_i e^{-cW_i}$	$F_i W_i e^{-cW_i}$	$F_i W_i^2 e^{-cW_i}$
110.9	97.2	754.9	8454	94700
207.4	174.5	836.1	13127	206100
312.5	251.6	714.1	14068	277100
607.7	442.2	549.8	15890	459220
840.5	563.8	519.0	18373	650400
1127.7	707.3	480.0	20353	862900
	2236.6	3854.0	90266	2550500

$$f(c_o) = 5029\left\{4.556 \times 80.55 - 3.5107 \times 110.3\right\}$$
$$+ 6\left\{- 3854 \times 80.55 + 3.5107 \times 90260\right\}$$
$$- 4.556\left\{- 3854 \times 110.3 + 90260 \times 4.556\right\}$$
$$= + 54{,}170$$

$$f'(c_o) = 5029\left\{- 2 \times 4.556 \times 2237 + 3.5107 \times 3207\right.$$
$$\left.+ 110.3 \times 80.55\right\} + 6\left\{2 \times 3854 \times 3207 -\right.$$
$$\left.- 3.5106 \times 2550000 - 90270 \times 80.55\right\}$$
$$- 4.556\left\{3854 \times 3207 - 4.556 \times 2{,}550{,}000\right\}$$
$$- 110.3\left\{3854 \times 110.34 - 90270 \times 4.556\right\}$$

$$f'(c_o) = 149{,}837{,}000$$
$$\delta c_o = - \frac{54{,}170}{149{,}837{,}000} = .000363$$

c_1 = .01064

cW_i	e^{cW_i}	e^{-cW_i}	$(e^{-cW_i})^2$	$W_ie^{-cW_i}$	$W_i(e^{-cW_i})^2$
.11917	1.12686	.88742	.78751	9.939	8.820
.16705	1.18146	.84641	.71641	13.288	11.247
.20961	1.23320	.81090	.65756	15.975	12.954
.30749	1.36001	.73529	.54065	21.250	15.625
.37665	1.45739	.68616	.47081	24.290	16.667
.45114	1.57009	.63690	.40564	27.004	17.199
		4.60308	3.57858	111.746	82.512

$W_i^2e^{-cW_i}$	$W_i^2(e^{-cW_i})^2$	$F_ie^{-cW_i}$	$F_iW_ie^{-cW_i}$	$F_iW_i^2e^{-cW_i}$
111.3	98.8	757.7	8486	95040
208.6	176.6	841.1	13205	207320
314.7	255.2	719.2	14169	279130
614.1	451.5	555.6	16056	464020
838.0	590.0	530.9	18796	665400
1144.9	729.2	487.4	20665	876190
3231.6	2301.3	3891.9	91377	2587100

$$f(c_{,}) = 5029.19 \left\{ 4.60308 \times 82.512 - 3.57858 \times 111.75 \right\}$$
$$+ 6 \left\{ - 3891.9 \times 82.512 + 3.57858 \times 91377 \right\}$$
$$- 4.60308 \left\{ - 3891.9 \times 111.75 + 91377 \times 4.60308 \right\}$$
$$= + 7.$$

$$D = 4.60308 \times 82.512 - 3.57858 \times 111.746 = - 20.083$$

$$Ma = 3891.9 \times 82.512 - 3.57858 \times 91377 = - 5871.5$$

$$Mb = 4.60308 \times 91377 - 111.746 \times 3891.9 = - 14{,}288.6$$

$$a = 292 \; ; \quad b = 712$$

$$(a + b)\beta = 5.33 \pm .17$$

Wi	Si	S_e	ϵ_i	ϵ_i^2	$\left(\frac{\epsilon_i}{1-e^{-\beta W_i}}\right)^2$
11.2	49.3	53.3	-4.0	16.00	4800
15.7	79.5	71.5	+8.0	64.00	100
19.7	88.1	86.3	+1.8	3.20	330
28.9	107.5	116.0	-8.5	72.30	3600
35.4	132.6	133.8	-1.2	1.40	50
42.4	154.2	150.2	+4.0	16.00	400
					9380

Similar computations as above were made for the data of Al, No. 1; Mg, No. 2; Al, No. 2. The results were as tabulated below:

	a	b	$(a + b)\beta$
Mg 1	292	712	$5.33 \pm .17$
Al 1	192	656	$5.39 \pm .05$
Mg 2	408	564	$5.16 \pm .14$
Al 2	487	248	$4.66 \pm .25$

For the purpose of calculating probable error

$$a + b = \frac{\sum Fi}{n - 3} \quad \text{, approximately.} \tag{30}$$

Hence by the well known result; that if

$$Q = f(q_1 , q_2 , \ldots q_n)$$

and the probable errors of the q's are ϵ_1 , ϵ_2 , ... ϵ_n, then R, the probable error in Q, is given by

$$R = \sqrt{\sum \left(\frac{\partial Q}{\partial q_i}\right)^2 \epsilon_i^2} \quad ;$$

we have

$$R(a + b) = \frac{\sqrt{\sum_i \left(\frac{\epsilon_i}{1 - e^{-\beta W_i}}\right)^2}}{n - 3} \tag{31}$$

By this means the probable errors as shown were computed.

Finally, there result the two values

$\dfrac{\left(\frac{dS}{dW}\right)_{Mg}}{\left(\frac{dS}{dW}\right)_{Al}}$	Experiment
$.989 \pm .03$	No. 1
$1.11 \pm .07$	No. 2

The mean of these two gives

$$\frac{\left(\frac{dS}{dW}\right)_{Mg}}{\left(\frac{dS}{dW}\right)_{Al}} = 1.05 \pm .06 \tag{32}$$

However, this result is subject to a correction due to the fact that the air in the column of air intercepted by the sample in the incident beam scatters according as the intensity in this part of the beam varies with the sample thickness. In order to compute this correction, let

A = area of sample

σa = scattering coefficient of air

z = distance along air column from sample.

Then energy scattered in layer dz of air column with sample in place is

$$IA\sigma a \, dz \qquad (33)$$

$$I = Ke^{-\mu_s l} e^{-\mu_a z} \qquad (34)$$

where,

K = intensity of incident beam at sample

μ_s = absorption coefficient of sample

μ_a = absorption coefficient of air

l = thickness of sample.

On substituting (33) in (32), we have

$$KAe^{-\mu_s l} e^{-\mu_a z} \sigma a \, dz$$

The energy scattered in layer dz of air column without sample in place is

$$KAe^{-\mu_a z} \sigma a \, dz$$

Thus the correction to be added to the measured values is

$$\int d\Delta S = - \frac{CKA\sigma a}{\mu_a} (1-e^{-\mu_s l}) \int_0^z e^{-\mu_a z} \, -\mu_a dz$$

where c is taken as a constant depending upon the relative efficiencies with which scattering originating at the sample and scattering originating along the air column are measured by the ionization chamber.

$$\Delta S = \frac{CKA\sigma_a}{\mu_a}(1 - e^{-\mu_s l})(1 - e^{-\mu_a m})$$

where m is the radius of the spherical ionization chamber. But $\mu_a m$ is small. Therefore,

$$\Delta S = CKA\sigma_a m \, (1 - e^{-\mu_s l})$$

$$= CKA\left(\frac{\sigma}{\rho}\right)_a \rho_a m \left[1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W_s}{A}}\right]$$

The corrected value of the scattering is

$$S = S^{*} + CKA\left(\frac{\sigma}{\rho}\right)_a \rho_a m \left[1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W_s}{A}}\right]$$

where S^{*} refers to experimental values. Now place

$$B = c\left(\frac{\sigma}{\rho}\right)_a \rho_a m$$

$$S = S^{*} + KAB\left\{1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W_s}{A}}\right\}$$

$$\frac{dS}{dW} = \frac{dS^{*}}{dW} + KB\left(\frac{\mu}{\rho}\right)_s e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W}{A}}$$

$$\left(\frac{dS}{dW}\right)_0 = \left(\frac{dS^{*}}{dW}\right)_0 + KB\left(\frac{\mu}{\rho}\right)_s$$

But

$$\left(\frac{dS}{dW}\right)_0 = p_s\left(\frac{\mu}{\rho}\right)_s K$$

Thus,

$$\left(\frac{dS}{dW}\right)_0 = \left(\frac{dS^{*}}{dW}\right)_0 + \frac{B}{p_s\left(\frac{\mu}{\rho}\right)_s}\left(\frac{dS}{dW}\right)_0\left(\frac{\mu}{\rho}\right)_s = \left(\frac{dS^{*}}{dW}\right)_0 + \frac{B}{p_s}\left(\frac{dS}{dW}\right)_0$$

$$\left(\frac{dS}{dW}\right)_1 = \frac{1}{\left(1 - \frac{B}{p_1}\right)}\left(\frac{dS^{*}}{dW}\right)_1$$

$$\frac{\left(\frac{dS}{dW}\right)_1}{\left(\frac{dS}{dW}\right)_2} = \frac{\left(1 - \frac{B}{p_2}\right)}{\left(1 - \frac{B}{p_1}\right)} \frac{\left(\frac{dS^{*}}{dW}\right)_1}{\left(\frac{dS^{*}}{dW}\right)_2} \quad ; \quad f_{12} = \frac{\left(1 - \frac{B}{p_2}\right)}{\left(1 - \frac{B}{p_1}\right)} \tag{35}$$

For computing f_{12} we have

$$p = \frac{\sigma}{\mu}$$

$$\frac{\sigma}{\rho} = .023\, Z^{1.16} \lambda$$ [15]

$$\left(\frac{\mu}{\rho}\right)_a = .18$$

$$\rho_a = .00124 \text{ gs/cm}^3$$ [16]

$$f\,\frac{Mg}{Al} = .99 \tag{36}$$

where it has been assumed that $c \cong 1$.

15. Heil, Phys. Rev., 46, 58
16. Int. Crit. Table, vol. 5.

If, however,

$$r = cK\left(\frac{\sigma}{\rho}\right)_a \rho_a \, m$$

is determined experimentally, then

$$\left(\frac{dS}{dW}\right)_0 = \left(\frac{dS^*}{dW}\right)_0 + r\left(\frac{\mu}{\rho}\right)_s$$

and

$$\delta\left(\frac{\left(\frac{dS}{dW}\right)_1}{\left(\frac{dS}{dW}\right)_2}\right) = \frac{\left(\frac{dS^*}{dW}\right)_2\left(\frac{\mu}{\rho}\right)_1 - \left(\frac{dS^*}{dW}\right)_1\left(\frac{\mu}{\rho}\right)_2}{\left(\frac{dS}{dW}\right)_2^2}\, r$$

Also we have

$$S = S^* + rA\left\{1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W}{A}}\right\}$$

$$S = (a + be^{-cW})\left(1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W}{A}}\right) \qquad (37)$$

$$S^* = \left\{(a - rA) + be^{-cW}\right\}\left(1 - e^{-\left(\frac{\mu}{\rho}\right)_s \frac{W}{A}}\right)$$

which shows that if S fits a function, (37), then S^* also fits a function of the same form. Thus it is a valid procedure to curve fit the experimental values of scattering and apply the correction later.

We observe that for elements whose atomic numbers differ but slightly the correction is small; but when the difference is large, as for example for boron and aluminum, the correction is not small. It is especially to be noted that if that part of the incident beam beyond the sample which is intercepted by the sample falls, for example, upon a metal window, a large error is introduced in the measurements.

On applying the correction (36) to the result (32), there results

$$\frac{\left(\frac{dS}{dW}\right)_{Mg}}{\left(\frac{dS}{dW}\right)_{Al}} = 1.04 \pm .06 = \frac{\left(\frac{\sigma}{\rho}\right)_{Mg}}{\left(\frac{\sigma}{\rho}\right)_{Al}} \qquad (38)$$

The result (38) depends upon the assumption that the ionization currents measured are proportional to the energy scattered by the samples. For an ionization chamber operating at a voltage well upon the flat portion of its saturation curve, the ionization currents depend only upon the total energy scattered for scattered radiation of a given quality. Even for voltages below the saturation value the ionization currents for low intensities of radiation, as those in this work, are proportional to the total energy scattered[17] but are not independent of the angular distribution of the scattered radiation for asymmetrical dispositions of the chamber and collecting system. This is true in this case because the field is asymmetrical and the efficiencies of the chamber for beams of different directions are different. The state of saturization of the ionization chamber is not known since the saturation effect sought was masked by a large "natural leak" which varied with the chamber voltage and other conditions the nature of which was not apparent. It was also found that on changing the chamber voltage, several hours were required for the state of saturation to become constant. Since great care was taken in placing the collecting system symmetrically with respect to the ionization chamber and since some indication of saturation was discerned in the measurements taken,

17. Compton and Allison, X-Ray in Theory and Experiment, Chap. VII.

it is assumed that the ionization currents were closely proportional to total scattered energy and practically independent of its angular distribution. It is also assumed that the angular distributions of the scattered radiation were not so different that the scattered radiation falling in the small portion of the sphere isolated from the ionization chamber by the cellophane cone was of consequence. In the author's Master's Thesis, pp. 3-18, it was established that of all the radiation scattered by the samples in the case of the light elements only the Thomson and Compton radiation reached the ionization chamber. Also, experimental works were cited from the literature [18] which established that for the light elements with neighboring atomic numbers, the quality of the radiations scattered by each are not appreciably different; but for, say boron and aluminum, the difference in quality is small but not inappreciable for the purposes here.

To recapitulate the conclusions of the discussion above we have: the method as described gives a correct result for the ratio of the scattering coefficients for the Thomson and Compton scattering only.

THEORETICAL RESULTS FOR THE RATIO OF SCATTERING COEFFICIENTS

According to Wentzel, Woo, Waller, Hartree, Jauncey and others[18] the intensity of scattering from an atom of atomic number Z on a sphere of unit radius at the scattering angle φ is given by

$$I_\varphi = I_e \left\{ F^2 + R(Z - \sum f^2) \right\}$$

where

$$I_e = I_0 \frac{e^4}{2r^2 m^2 c^4}(1 + \cos^2 \varphi)$$

F = coherent scattering factor

$\sum f^2$ = incoherent scattering function

$$R = \frac{1}{(1 + \frac{h}{mc\lambda} \operatorname{vers} \varphi)}$$

Hence the total scattering is given by

$$S = I_0 \frac{\pi e^4}{m^2 c^4} \int_{-1}^{+1} (1 + \cos^2\varphi) d(\cos\varphi) \left\{ F^2 + R(Z - \sum f^2) \right\}$$

and

$$\frac{\sigma}{\rho} = \frac{\pi N_0 e^4}{m^2 c^4}(1 + \cos^2\varphi) d(\cos\varphi) \left\{ F^2 + R(Z - \sum f^2) \right\} \qquad (39)$$

where $N_0 = \frac{N}{A}$ is the number of scattering atoms per unit mass. Values of F and $\sum f^2$ have been computed[19] for a suitable range of values of $\frac{1}{\lambda} \sin \frac{\varphi}{2}$. By means of these results and (39) the following scattering coefficients were computed:

$$\lambda = .76A^\circ \; ; \quad \left(\frac{\sigma}{\rho}\right)_{Mg} = .391; \quad \left(\frac{\sigma}{\rho}\right)_{Al} = .405 \; ; \quad \frac{\left(\frac{\sigma}{\rho}\right)_{Mg}}{\left(\frac{\sigma}{\rho}\right)_{Al}} = .965 \qquad (40)$$

In computing the results (40) from (39) it was evident that although the scattering coefficients varied with wave-length their ratios varied but slightly in the interval $\lambda = (.70 - .80)A^\circ$.

18. Compton and Allison, X-Rays in Theory and Experiment, Chap. III.
19. " " " , X-Rays in Theory and Experiment, Appendix IV.

Heil[20] computed by the method above the scattering coefficients for carbon, aluminum, iron, tin and gold in the range of wave-length $\lambda = (.4 - 1.1)A^{\circ}$. For representing his results approximately he offers

$$\frac{\sigma}{\rho} = .0230\ Z^{1.16}\lambda \tag{41}$$

from which we obtain

$$\lambda = .76A^{\circ}\ ; \quad \left(\frac{\sigma}{\rho}\right)_{Mg} = .312\ ; \quad \left(\frac{\sigma}{\rho}\right)_{Al} = .342\ ; \quad \frac{\left(\frac{\sigma}{\rho}\right)_{Mg}}{\left(\frac{\sigma}{\rho}\right)_{Al}} = .912 \tag{42}$$

As to the differences between the results of (40) and (42) I have nothing to offer.

CONCLUSIONS

The theory predicts the ratio of the scattering of magnesium to that of aluminum to be less than one, but the experiment shows this ratio to be greater than one. It is to be noted that the theory refers to all the scattered radiation and that in the experiment only Thomson and Compton scattering is measured.

20. loc. cit.

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