If Black Holes Are Superficial

Javier Viaña,[1★] Katherine L. O'Grady[2]
[1,2]*College of Engineering and Applied Science, University of Cincinnati, Cincinnati, OH 45219, USA (vianajr@mail.uc.edu)*

ABSTRACT
Prior work on black hole (BH) thermodynamics suggests the entropy depends not on the volume, but rather the surface area of the event horizon. Such findings highlight the intriguing nature of BHs and give rise to the idea that information may be entirely encoded on the surface. We study the case of a superficial Schwarzschild BH, and calculate the net force (F_{net}) exerted on the surface of its spherical shell from the self-gravitational pull. We demonstrate that the F_{net} is exactly $\frac{c^4}{4G}$, $3.025 \cdot 10^{43}$ Newtons, a force that is constant and independent of the size and the mass of the BH, meaning all such Schwarzschild BHs share the same F_{net}. Surprisingly, the F_{net} matches F_{max}, the limit of the maximum force conjecture. This establishes a new potential connection between the formation of BHs and the F_{max}. We demonstrate that under the validity of this F_{max}, the mass of the superficial BH is contained at precisely the Schwarzschild radius. Finally, we provide further evidence to reject the concept of a point mass singularity and we theorize on the creation of a BH given the findings.
Key words: black hole physics – gravitation – cosmology: theory

1 INTRODUCTION

In the 1970s, Bekenstein (1973) identified a peculiar relationship between entropy and the surface area of a black hole, which increases to a greater degree than would be expected when absorbing a certain amount of matter. Bekenstein's discovery gave rise to the study of black hole thermodynamics, and today his claims continue to be elaborated on by others in the field (Almheiri 2021; García-Compeán 2021; Hashimoto 2020; Bianchi 2020). Hawking (1975) later determined the precise proportionality of the relationship between entropy and area, which then led to the Bekenstein-Hawking entropy formula, in dimensionless form $S_{BH} = \frac{c^3 A}{4G\hbar}$.

The fact that BHs obey the laws of thermodynamics suggests that underlying microstates could be described using statistical means. Breakthroughs on how to compute such probabilities remained elusive until others expanded on the proof by Hawking (1974) that BHs do indeed create and release small amounts of matter in the form of particles via black body radiation, coined Hawking radiation, in a manner dependent on the Hawking temperature, $T_H = \frac{\hbar c^3}{8\pi G M k_B}$. Hawking (1976) believed that the quantum information of matter used to create BHs would be transformed incoherently, making it impossible to relate the prior quantum states of particles entering to those of particles leaving by way of Hawking radiation, which may be governed by virtual particle interactions on the event horizon. This concept became known as the BH information paradox.

However, if all information is encoded entirely on the surface of a BH, it may be possible for the information to remain indefinitely, even in light of Hawking radiation. A seminal paper by 't Hooft (2001) showed that one can compute the density of quantum states of a BH by deriving the spectral density from the Hawking temperature, and avoid thermodynamics altogether by instead considering BHs quantum objects. To delineate a proper description of particle states in the vicinity of a BH, 't Hooft (2001) argued that a two-dimensional function is required. This novel idea was deemed the holographic principle, and states that information which originates in 3-dimensional space may be encoded on the 2-dimensional surface of a BH, like that of a 3-dimensional holographic image which can be derived from a 2-dimensional encoding.

Further work by Susskind (1995) to describe a detailed, theoretical implementation of t' Hooft's holographic principle in the context of string theory proved immensely thought-provoking and paved way for additional research in string theory. Susskind elaborated on t' Hooft's claim and demonstrated that if the entropy of a BH is confined to the event horizon, no distribution of matter will ever require more than one bit of information per Planck area, l_p^2, to be completely described. Subsequent efforts by Mathur (2005) in string theory related the entropy and microstates of a BH with that of a fuzzball, an interwoven network of strings which encompass the total surface area of the event horizon. This interlaced structure confining the shell of the BH, at precisely the Schwarzschild radius, could theoretically encode the quantum information required to describe all matter that went into the BH. In this sense, a BH *is* a fuzzball. Should additional matter merge with the BH, particles would be converted to their respective string representations and overlay the existing mesh, coalescing with it and capturing the informational content directly on the surface. Mathur (2005)'s fuzzball premise supports the idea that there is, in fact, no singularity at the center of a BH, but rather, matter in the form of strings spans the boundary of the event horizon. The matter in a BH would no longer be compressed to an infinitely dense point, but would alternatively comprise the border of the event horizon, which may be the fuzzball surface itself.

The publications discussed previously advocate for the notion that a BH may not be a point singularity, but rather, may be a physical surface with matter exclusively contained on the event horizon at the Schwarzschild radius, and empty space inside.

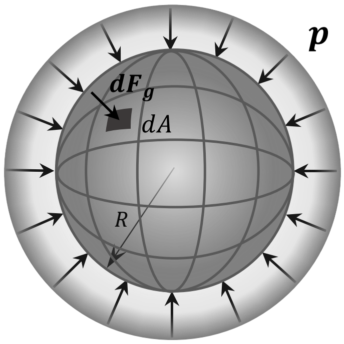

Figure 1. Pressure field exerted on the BH from its self-gravitational attraction.

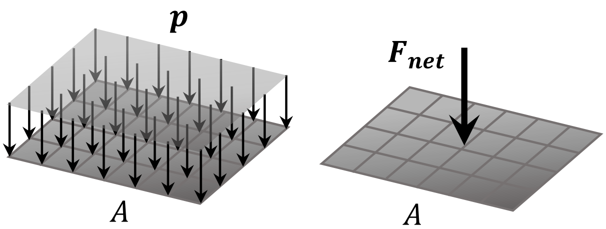

Figure 2. Self-gravitational attraction pressure field on a horizontal plane with the same area as the spherical surface and the net gravitational compression force exerted on the BH.

2 METHODOLOGY

We start considering a superficial Schwarzschild BH, which we denote as a hypothetical system whose mass is not located at the BH's center of gravity but scattered all over the spherical shell of radius R (where R is not necessarily the Schwarzschild radius). Note that both the regular Schwarzschild BH and the superficial Schwarzschild BH share the same centers of gravity despite their difference in the distribution of matter.

Due to the negligible thickness of the shell, mathematically, we can understand the self-gravitational pull of the BH as an equivalent pressure exerted on it's surface, as shown in Fig. 1.

The net force on this sphere is null, because every differential force, $\boldsymbol{dF_g}$, is counterbalanced by its opposite. In order to account for the net compression that results from the gravitational pull, $\boldsymbol{F_{net}}$, we consider the flattened theoretical surface of the sphere (Fig. 2 as an auxiliary construction to visualize the compression force exerted on the mass of the BH). Thus, $F_{net} = p \cdot A$.

To determine the value of the net compression force of the BH, we first need a formula that defines $\boldsymbol{dF_g}$. We resort to Fig. 3 for this task, where we consider a differential mass of the superficial BH (dM).

The force $\boldsymbol{dF_g}$ exerted on dM from the self-gravitational pull of the BH is pointing towards the center of gravity of the system and has a value of $\frac{G \cdot M \cdot dM}{R^2}$.

At this point, we need to make the assumption that the pressure is uniform across the BH, i.e., the mass is equally distributed, in a Schwarzschild BH this assumption holds. This implies the following two equivalences $\frac{F_{net}}{A} = \frac{dF_g}{dA}$, and $\frac{dM}{dA} = \frac{M}{A}$. Using these two equations, and the definition of dF_g, the pressure can be defined as

$$p = \frac{F_{net}}{A} = \frac{dF_g}{dA} = \frac{G \cdot M}{R^2} \cdot \frac{dM}{dA} = \frac{G \cdot M^2}{R^2 \cdot A}. \qquad (1)$$

We now retake the definition of F_{net} and insert the result obtained

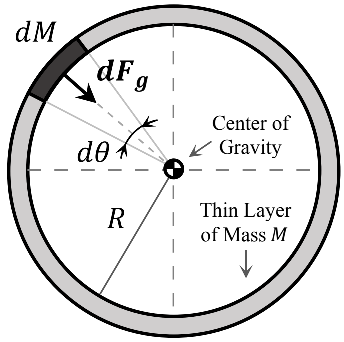

Figure 3. Cross section view of the superficial BH, schematic representation of the gravitational force exerted on a differential mass on the surface of the BH by the BH's total mass.

in equation (1),

$$F_{net} = p \cdot A = \frac{G \cdot M^2}{R^2}. \qquad (2)$$

Using the Schwarzschild radius we can differentiate four cases:

(i) If $R > R_s$: This is an impossible scenario. It would imply that the radius of the shell is bigger than the event horizon.

(ii) If $R = R_s = \frac{2G \cdot M}{c^2}$: Then the mass of the BH is exactly scattered at the R_s, thus at the event horizon. In this case, the value of the F_{net} would be

$$F_{net} = \frac{c^4}{4G}. \qquad (3)$$

There are two important lessons that we can extract from this result: 1. The F_{net} of all superficial Schwarzschild BH's is constant regardless of the size and the mass of the black hole. 2. The value of the F_{net} matches exactly the F_{max}, $3.025 \cdot 10^{43}$ Newtons, which is the theoretical maximum force. In general relativity, it has been conjectured that there exists an upper bound for forces acting between two bodies. The main idea is that there should exist an $F_{max} = \frac{c^4}{4G}$, just like in special relativity there is a maximum allowed speed c. This concept of a maximum force was first postulated by Gibbons (2002) and Schiller (2005). We find it interesting that the result we obtained for the F_{net} is precisely the F_{max}. Several authors have studied and supported the concept of F_{max} (Barrow 2014; Ong 2018; Faraoni 2021; Barrow 2020; Good 2015). It is believed that the F_{max} exists in the context of binary black hole systems with touching horizons just prior to merging (Barrow 2020; Atazadeh 2021). The presence of F_{max} has also been argued in the confines of a single BH by Schiller. However, we provide an alternative connection between the F_{max} and the definition of a BH. Perhaps even more curious is that this $F_{net} = F_{max}$ does not depend on the mass nor radius of the BH. That is, the F_{net} is exactly F_{max} throughout the life of the BH, as well as for distinct BHs of various sizes. This leads us to believe that the F_{max} could be profoundly related to the origin and the definition of a BH.

(iii) If $R_s > R > 0$: Then the F_{net} is bigger than F_{max}, which under the validity of F_{max} would be impossible.

(iv) If $R = 0$: In the extreme case where the inner radius is null, the BH would no longer be superficial, but rather compact, i.e., a point mass. However, in the vicinity of this situation, where $R \approx 0$, the definition of the F_{net} would still be valid, and as it happened in the previous case, the F_{net} would surpass the F_{max}, which would imply that this configuration is not possible. This leads the authors to believe that a point mass singularity is not feasible, and that all

Schwarzschild BH's should be superficial. Of course, making such a claim with only this evidence is a bold statement, thus, we leave this research open to future work.

3 DISCUSSION

Assuming the validity of the F_{max}, the only feasible superficial Schwarzschild BH is that which has its mass at a radius of $R = R_s$ and a net force of $F_{net} = \frac{c^4}{4G}$. The net force derived in the paper appears to indicate that F_{max} exists within the single body BH, as the net self-exerted compressing force.

Such findings suggest that the maximum force limit is not only present in dual systems as thought before, but also within the self exerted forces of a mass defining a BH.

The fact that F_{net} is constant and equal to F_{max} throughout the life of a BH poses the following questions concerning the origin of a BH: Could a BH be created when a certain set of particles are compressed to a degree that attempts to surpass this F_{max} limit? Is the BH configuration the means that nature has devised to prevent any matter from crossing this limit? It seems reasonable to ask these questions, considering the BH grows precisely in a way that maintains the relation between R and M to keep this force constant. Thus, we theorize that reaching the F_{max} may as a consequence spark the creation of a BH.

Perhaps a less audacious statement is to assume that if there exists a limiting value on the force, then a black hole (given its extreme environment) is one of the best candidates to reach such value. This aligns with the findings of this paper, in which we proved with simple math the connection between F_{max} and the nature of a BH.

Additionally, we would like to note that the combination of variables $\frac{c^4}{G}$ is equivalent to the Planck Force ($F_P = \frac{E_P}{l_P}$) of $1.21{\cdot}10^{44}$ Newtons, given by the Planck energy (E_P) and the Planck length (l_P). Some authors believe that, should BHs be superficial, the thickness may be the Planck length. We believe that this new connection between $F_{net} = \frac{c^4}{4G}$ and black holes might provide further food for thought in this context and may help prove if such thickness is, indeed, l_P.

4 CONCLUSIONS

(i) We have been able to make a new connection between the F_{max} and black holes.

(ii) We have demonstrated that if the Schwarzschild BH is superficial, the mass must be scattered exactly at R_s.

(iii) The F_{net} of all superficial Schwarzschild BH's is constant and exactly $\frac{c^4}{4G}$, independent of the mass and the size of the black hole, and matches the F_{max}.

(iv) We have also seen further evidence to believe that there cannot exist point mass singularities and that all the Schwarzschild black holes could be hollow and superficial.

(v) We also suggest a theory on how the F_{max} could be a reason for the creation of a BH.

(vi) We provide further evidence to prove if the thickness of a superficial BH's shell is the Planck length.

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DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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