A FUNDAMENTAL STUDY OF
LUBRICATING FILMS

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by

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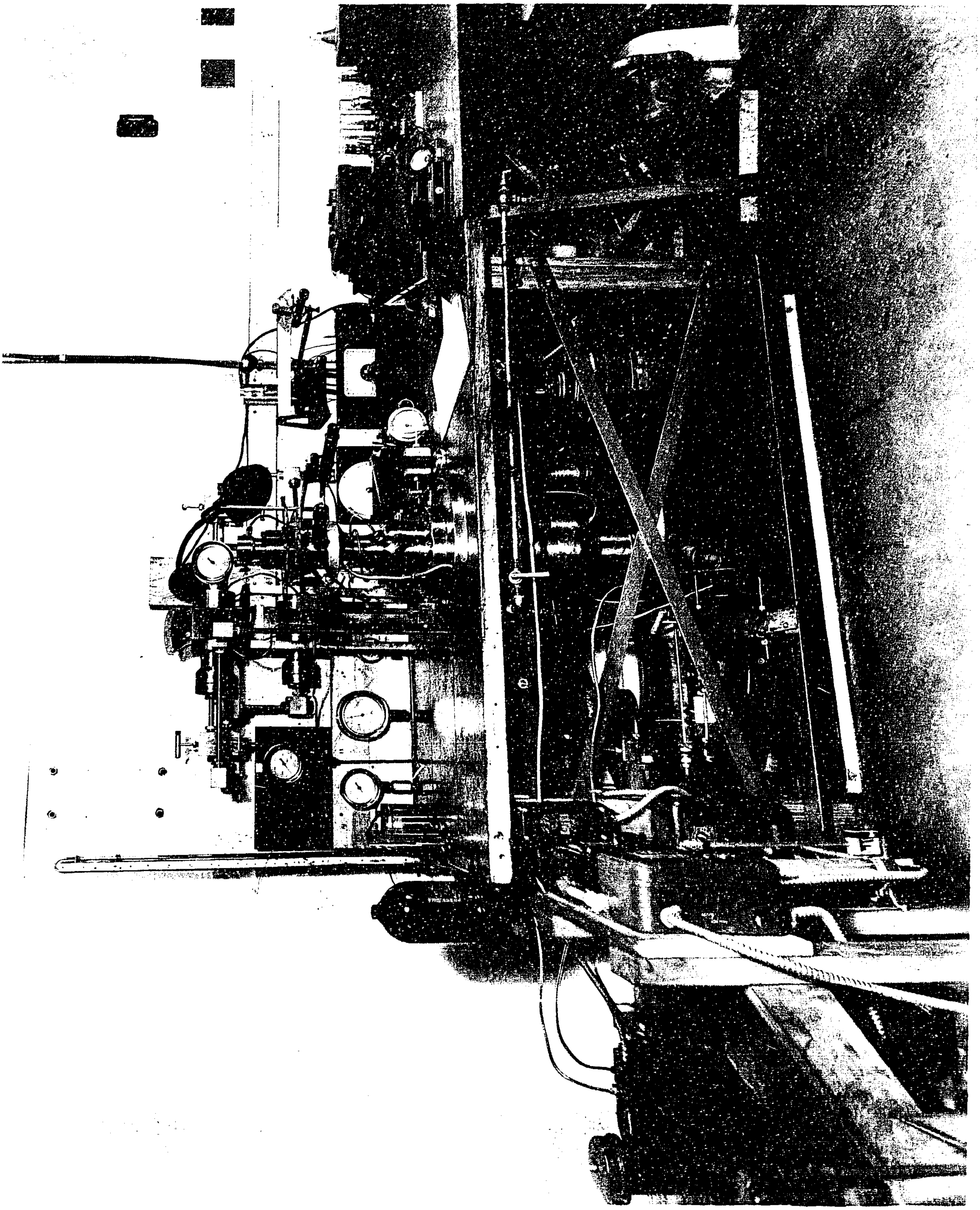
SURFACE QUALITY MACHINE



INTRODUCTION

Sir Isaac Newton (1687) formulated the fundamental law of viscous flow which states that the shearing stress in a viscous fluid is proportional to the rate of shear. Osborne Reynolds was the first to realize that lubrication was inherently hydrodynamical. The classic experiment of Beauchamp Tower's prompted Reynolds to apply hydrodynamic principles to lubrication. From this time to the present a large quantity of theoretical and experimental work has been done.

The field of Lubrication divides itself into two parts: "Thick F ilm" or "Hydrodynamical Lubrication" and "Boundary Lubrication". Thick F i lm Lubrication is the condition encountered in high speed, lightly loaded journals in which there is a complete film of lubricant separating the two moving members, and in which results, calculated by Hydrodynamics, agree fairly well with experimental results. Boundary Lubrication is the condition encountered in a slow running, heavily loaded journal with a film of rather indefinite thickness between the two members, and in which the results as calculated by the Hydrodynamical Theory differ from experimental results. Under conditions of Boundary Lubrication wear may take place and the friction may vary with the chemical nature of the lubricant which is in contrast to Thick Film Lubrication where the friction is determined by the viscosity only.

The work done in the field of Thick Film Lubrication is too extensive to be discussed in detail here. A large portion of this work deals with cylindircal journal bearings. The experimental work usually involves the relation between the coefficient of friction and the Sommerfeld Variable (dimensionless) $\frac{UN}{P}$ where U = viscosity, N = journal speed, P = pressure (lbs/in^2 of projected area). This experimental relationship is compared with theory. The theoretical relationship is based on Hydrodynamics and the boundary conditions, i.e., a solid cylinder coaxial or offset with respect to a hollow cylinder of slightly larger diameter. Usually these cylinders (the bearing) are considered infinitely long although there are some treatments considering bearings of finite length. The treatment for the finite length bearing is rather complicated mathematically.

There has been a large quantity of work done on Boundary Lubrication. Usually, some sort of experiment is run using different lubricants and observing the rate of wear, seizure time, seizure load, or friction. Most of the experimental equipment involve two or more members whose geometry are such as to produce extremely high unit pressures at a point or at a line of contact. The center of curvature of the members are usually located on opposite sides of the part or line of contact, for instance, two crossed cylinders, or four balls packed together, one of which rotates. S tudies in boundary lubrication usually have a definite chemical aspect. There is no mathematical theory developed for boundary lubrication.

OBJECT

The experiments, to be described later, were run with the purpose of investigating (1) the effects of surface finish on bearing performance and (2) the deviation of the behavior of the lubricant from the Laws of Fluid Mechanics. That is, to be more specific, to observe the effect of changing the surface finish on such variables as, shearing stress, rate of flow or resistance to flow. After observing the effects on these variables the conclusions can then be applied to actual bearings and loose terms such as "Load Carrying Capacity" can be discussed in a fundamental manner.

METHOD OF ATTACK

A large portion of the experimental work done in the field of Thick Film lubrication has been concerned with tests on journal bearings in which such items as friction, load carrying capacity, attitude or running position of the journal, etc. were studied. In tests of these sorts the film thickness, pressure, flow, temperature all vary around the journal.

The following experiment, in contrast with the above, employs not only a uniform film thickness but a controllable film thickness. This is a unique feature about the equipment. The necessity of having control of all the important variables is evident if a fundamental investigation is to be carried out.

The use of annular plane surfaces permits a high degree of precision with regard to flatness and parallelism to be obtained. By making the annular ring narrow in width, treatment of the surface approximately as an increment is permitted.

In view of the above the data obtained is valid for any increment of area under similar conditions of speed, surface roughness, film thickness, etc. Consequently, it may be applied to any geometrical shape by integrating over that shape (at least in theory).

EXPERIMENTAL EQUIPMENT

A. General Description of Main Unit

Sketch 1 shows a cross section of the machine. The lower test specimen or lower cup is bolted to the lower spindle which is rotated by a Vee belt drive and a variable speed d c motor. The alignment of the lower spindle is maintained by two long bronze bushings. The pressure thrusts on the lower spindle are taken up by a pair of Fafnir 7412 DF Super Precision Angular Contact Ball Bearings. Oil leakage by the lower bronze bearing is carried up to lubricate the ball bearings. Oil is pumped in at the bottom of the unit. It flows up the hollow lower spindle into the space between the test specimens (See S-2). From here it flows approximately radially outward between the upper and lower test surfaces into the chamber surrounding the specimens. The oil leaves this chamber by a pipe and passes through a valve to atmospheric pressure. The upper or torque spindle is mounted in a tapered bronze bushing which in turn is pressed into a tapered cloth base bakelite bushing, and is therefore insulated from the machine, the compensator piston being similarly constructed. The purpose of the compensator piston is to balance out the unbalanced forces arising on the test specimen. In this way the torque spindle is made to float on the oil film between the annular test surfaces.

In operation the lower spindle is rotated at a constant speed. Torque is transmitted through the oil film to the upper or "torque measuring" spindle, where it is balanced out by displacing a pendulum from its equilibrium position (to be described in detail later). In this way, two surfaces with relative motion are in close proximity but are separated by a film of oil which can be regulated by adjusting the pressures and the rate of flow. The test surfaces are narrow annular strips and may be regarded approximately as an increment of surface.

The speed is measured by a stroboscopic method. The thickness of the oil film between the test surfaces is measured by a capacitance method. The oil temperature is measured by thermocouples.

B. Hydraulic Circuit

Sketch S-3 shows the hydraulic circuit. There are two distinct oil systems, one is the system supplied by the Vickers pump which is capable of supplying oil at 1000 psi. The other system is supplied by the Super Pressure pump (American Instrument Co.) and is capable of supplying oil at 5000 psi. Actually it is capable of producing much higher pressures, but the other equipment will not stand higher pressures.

The oil from the Vickers pump never enters the test specimens. It is used for balancing the torque spindle, i.e., it is normally used to maintain pressure in the compensator cylinder.

The oil from the super pressure pump passes through the test specimens and its properties are of importance. The path of this oil is indicated by the heavy line in S-3. It is pumped into the high pressure receiver which is about one half full of Nitrogen. This receiver damps out the surge from the pump. The pressure in the receiver runs around 5000 psi. The oil from the receiver is throttled by valve 1 to something less than 5000 psi. It flows freely through valve 3 (which has no control over this line) and then through a heavy resistance coil and into the bottom of the machine. The oil flows up the hollow spindle, through the test samples and out through throttle 10 where it is dropped to approximately atmospheric pressure. Valve 11 must be open and valve 12 closed during normal operation.

The remainder of the system is used for building up the air or nitrogen pressure in the receiver. (At first air was used but this was considered dangerous, hence N_2 is now used.)

In order to charge the receiver with N_2 the whole system is first purged with N_2. About 75 psi. of N_2 pressure is built up in the large compression tank. Then the nitrogen in this tank is compressed by pumping oil into the tank with the Vickers pump. The nitrogen is compressed to 1000 psi. this way. The receiver at this point would have nitrogen only in it at 1000 psi. The Vickers pump is then stopped and oil is pumped into the Compression tank by the Super Pressure Pump to raise the pressure to 2500 psi. Then oil is pumped directly into the receiver until the pressure is about 5000 psi.

The resistance in the line entering the unit from the receiver and the resistance in the line to the compensator cylinder are necessary for the stable operation of the machine. These resistances produce stabilization in the following way: if the torque spindle moves downward the flow through the lower spindle is restricted and the pressure rises because of less pressure drop in the external resistance. The pressure in the compensator cylinder decreases for a similar reason. Both of these effects bring forces into play which oppose the original assumed displacement. At thick films (.002" or more) a small displacement of the torque spindle does not restrict the flow appreciably through the lower spindle and accordingly at these film thicknesses the spindle is not stable, but these film thicknesses are of no interest. There is an instability effect at very small films also, but the stabilizing effects overpower it. When the torque spindle is in contact with the lower spindle, the area on which the pressure in the lower spindle acts is less than when a definite film of oil exists between the test surfaces. This change in effective area tends to produce instability, but as mentioned before is not the controlling factor.

When the machine is operating normally, oil is flowing along the heavy path shown on S-3, as remarked before. The thickness of the film may be regulated in two ways. One method (the standard method in the tests to follow) is to regulate the film thickness by bleeding part of the oil through valve 4 (S-3) to atmospheric pressure. The other method is simply to increase the compensator pressure P_c by regulating the relief valve on the Vickers Pump.

C. Torque Spindle Bearing

One of the unique features of the machine is that it permits very delicate torque measurements to be made in spite of very heavy axial loads on the torque spindle. The torque spindle is $\frac{3}{4}$" in diameter and about 12" long and may carry 3000 lbs. or more axial load. Under these conditions a torque of .1 in.-gr. can be detected. The essence of the situation is that the torque spindle is subjected to only viscous friction so that the frictional force is proportional to the velocity. Torque measurements are made at zero velocity, thus the friction is practically zero. The complete lack of metallic friction is due to the use of the "Hydrofilm" bearing developed by the Cincinnati Milling Machine Co.

Sketch S-4 illustrates the principle of the bearing. A_1, A_2, A_3, and A_4 are pockets to collect oil at ½ the oil pressure entering the bearing. The oil in these pockets is then transmitted freely to the pressure areas B_3, B_4, B_1, and B_2 respectively. The oil entering the A pockets has passed through resistances R (which is the clearance space between journal and bearing). The oil leaves the B areas by flowing through resistances R' to atmospheric pressure. If the journal moves to the right say, R_1 and R_1' will increase and R_3 and R_3' will decrease. Consequently the pressure in A_1 will decrease on two counts (1) because of the increase in R_1 (2) because of the decrease in R_3'. Also the pressure in A_3 will increase on two counts (1) because R_3 has decreased and (2) because R_1' has increased. The result of a movement to the right then causes the pressure in B_1 to increase and that in B_3 to decrease which tends to center the journal in the bearing.

This bearing was not incorporated in the machine when it was originally built so that it had to be altered. The pressure pockets A and areas B were ground on the spindle instead of being in the bushing, because of structrual difficulties. Three sets of pockets and areas were used instead of four because four could not be applied to the machine. The radial clearance between the spindle and bronze bushing is about .0003 in. and that between the compensator piston and cylinder is about .0005 in. Consequently a precision alignment is needed between the torque spindle bearing and the compensator cylinder. This alignment is difficult to maintain because of the fibre insulating bushing around the bronze bushing. Frequently the compensator cylinder has to be realigned.

If there is much hydraulic "capacitance" in the piping connecting the pockets A with the areas B hunting of the spindle for the central position is likely to occur. Accordingly it is undesirable to have any air entrapped in these lines.

D. Measurement of Variables

The following table shows the important variables and their approximate range.

Variable	Symbol	Range
Film thickness	h	.0015 to .00015 in.
Torque	T	0 to 350 in.-gr.
Speed	N	60 to 350 rpm
Pressure inside samples	Po	0 to 3000 psi

Variable	Symbol	Range
Pressure outside samples	P_1	0 to 3000 psi
Rate of Oil Flow	Q	0 to 2 cc/sec.
Oil temperature inside samples	T_O	20°C to 60°C
Oil temperature outside samples	T_F, T_R	20°C to 60°C

(1) Film Thickness

As mentioned earlier the film thickness is measured by a capacitance method. This method was used rather than an ordinary indicator because it registers changes directly at the test samples whereas an indicator would have to be mounted outside somewhere with metal parts between it and the actual test surfaces. These metal parts would expand and contract because of thermal changes and because of loads applied to them. The deflection of the lower spindle would also cause relatively large errors. With these difficulties at hand the advantages of the capacitance method are evident.

The test surfaces act as a condenser with the oil between them as a dielectric. Essentially they behave like a parallel plate condenser where the capacity is inversely proportional to the spacing. This capacity is measured by an oscillator constructed for the purpose (see S-5 and S-6). The oscillator is tuned to a fixed frequency of 380 KC. The output is coupled to a parallel resonant circuit, one element of which is the capacity of the machine. As the film thickness decreases more capacity is put into the parallel resonant circuit and hence an equal amount must be removed by adjusting the tuning condenser which is in parallel with the machine. In this way the capacity of the machine is measured. Originally the oscillator was built to measure capacitances up to about 2000 uuf. but later an extra unit was made covering a range from around 1000 to 10000 uuf.

The oscillator was calibrated against a Precision Condenser (General Radio Co. Type 722H #198). The calibration curves are shown on M-35, M-36, M-37, and M-38.

Several methods were used to calibrate the machine and oscillator together giving capacity vs film thickness relations.

a. First Method

The first calibration of the oscillator was made in November 1942. The compensator cylinder head was removed and a steel ball was placed in the center hole of the torque spindle so that there would be a definite point for the indicator to contact. The spacing between the test surfaces was adjusted by holding the threaded disc which screws on the torque spindle, and turning the torque spindle in the disc. In this way the spindle was either raised or lowered. The hand holes were opened and all oil was removed from between the samples and surrounding the samples. The test surfaces were cleaned very carefully with cotton and carbon tetrachloride or alcohol and ether. This was necessary because small particles of dirt or lint would short the surfaces at small film thicknesses. The "zero" reading of the indicator was set by screwing the spindle down, until the surfaces were together. With air between the surfaces it was possible to raise the spindle .0001 in. and have an open circuit electrically. This indicates the parallelism of the surfaces is something better than .0001 in. in 2-1/2 in. Next oil was introduced between the surfaces and the capacity was measured for various film thicknesses. At films of .0005 in. or less considerable difficulty was encountered because the surfaces were being shorted. If the lower spindle was oscillated through a small angle the short would sometimes disappear. The lower spindle was not rotated completely because it had at that time a little more than .0001 in. axial runout. The difference in stray capacity when air surrounded the samples and when oil surrounded them was at this time considered negligible because the total stray capacity is about 300 uuf and the clearance between the samples and the chamber walls is large. The difference in fringing of the lines of force at the edge of the test surfaces, and the change in stray capacity due to the removal of the compensator cylinder head were also neglected.

The result, using the above procedure is shown on Curve M-19. Readings with air between the samples were obtained with much more certainty for small films than those with oil because of the shorting with oil. An effort was made to find the dielectric constant and multiply the curve for air accordingly. However, the dielectric constant appeared to vary with film thickness (it was probably the end effects which really varied.) The curves for air and oil were plotted on log-log paper. (See M-21) Here it is seen that the curve for air (C-Co) vs film thickness is not a straight line. C is the total capacity and Co is the stray capacity. This indicates that fringing should not be neglected.

b. Second Method of Calibration

In April 1943, the second pair of ball bearings were installed. The lower spindle now has an axial runout slightly less than .0001 in. However it was decided in view of the above to make calibration tests at low pressures with oil completely surrounding the surfaces as in actual tests. Low pressures were used so that elastic deflections would be negligible. The lower spindle was rotated and average values of film thickness were read.

The indicator now registers the movement of a pin which is fastened securely to the threaded disc on the torque spindle. This threaded disc is locked to the spindle by a lock nut and the film thickness is regulated by the flow of oil between the test surfaces. The "zero" setting of the indicator was determined by shutting off the oil flow thereby allowing the torque spindle to settle. When the torque spindle first offered resistance to turning, the indicator reading was noted. Curves M-32 and M-33 were obtained in this way. Curve M-33 shows the data plotted against reciprocal film thickness, and also the data from the first method of calibration. It is evident that there is considerable discrepancy. According to these later data, 2000 uuf corresponds to about .00027 in. Two thousand micro micro farads was the limit of the oscillator.

c. Extension of Range of Oscillator

An additional tank circuit was made for the oscillator. Another coil and a bank of fixed condensers and a variable air condenser were mounted on a separated chassis. The original tank circuit was unaltered except that a means was provided for switching the original one out and the new one in.

The new unit ranges from 0 to 10,000 $\mu\mu f$ by steps of 1000 $\mu\mu f$ with a 1250 $\mu\mu f$ variable air condenser to tune between the steps of the fixed condensers.

d. Modification of Second Method of Calibration

In the second method of calibration the film thickness was varied by changing the flow. As the flow was decreased the pressure inside the test samples would drop (from 45 psi to about 17 psi). This method was modified by readjusting the valve between the chamber surrounding the samples and atmosphere. By adding more or less resistance here the pressure inside and outside the samples could be kept constant over a complete run. The compensator pressure was constant also, so all deflections should remain the same. The "zero" reading of the indicator was obtained again by shutting off the flow, but with the pressures locked in the machine. The spindle would settle and finally come to rest and the pressures would remain approximately the same. In these tests it was considered better to locate the "zero" of the indicator when the spindle offered a large resistance to turning, because dirt on the test specimens might cause a small drag. In determining the "zero" the lower spindle was oscillated slightly. The difference in "zeros" as determined by the two methods amounts to about .0001 in.

Curves M-39 and M-40 show calibration data obtained by the above procedure. There is considerable spread of this data from .0005 in. on down. However, the data reproduces itself within about .00005 in. It will be readily understood that the reciprocal scale of film thickness magnifies errors at the small films, and that, theoretically, the data should be a straight line. Curve M-41 shows calibration data taken with 500 psi inside the test samples. The higher the pressure, the faster the oil leaks through the bearings, and the more difficult it is to get a reliable "zero" reading because the pressure leaks out and changes in deflection occur. Inaccuracies in the determination of the "zero" change the slope of the curves. Curves M-41 and M-42 show the runs at 500 psi and 2000 psi respectively.

At high pressures with a constant oil flow between the samples the film thickness as read by the oscillator will remain the same as it should even if the pressures fluctuate slightly. The indicator, however, will show a definite movement, caused by elastic deflections. The fluctuations of the pressure at 2000 psi are on the order of several hundred lbs. per sq. in. This causes deflections making an appreciable error. Curve M-42 has a different slope than the other curves and this is probably caused by the inability to determine the "zero" as described above, whereas at 100 psi the pressure can be held within a few lbs. per sq. in., giving much greater accuracy.

Curves M-45, M-47, and M-48 are the final curves obtained by the method of part d above.

It should be mentioned that the variation of the dielectric constant was investigated for variations in both temperature and pressure. From 0 to 5000 psi, no change in dielectric constant could be observed. Neither was there any appreciable change with temperature from 20°C to 60°C.

This method of measuring film thickness is very sensitive to small changes in film thickness at thin films. A change of thickness of .000005 in. around .0003 in. is readily observable.

Another feature of this method which is of passing interest is the change in "Q" for the machine as the film thickness is decreased. Curve C-57 illustrates this. As the film thickness is decreased the parallel resistance of the film decreases, hence the equivalent series resistance increases. Consequently the ratio of reactance to equivalent series resistance decreases because of the increase in series resistance and the decrease in reactance.

(2) Torque

The torque transmitted through the oil film is measured by applying an equal external torque and measuring this external torque. By using a null method viscous friction does not enter. Since any dry friction will cause an error, the system for balancing the transmitted torque must be practically frictionless. The system shown in S-7 was used. The weight on the end of the silk thread is pulled from its equilibrium position causing a reaction at the point of suspension. The tangential component of this reaction balances the internally applied torque. This system works very satisfactorily because of the complete absence of Coulomb friction.

(3) Speed

The speed of the lower spindle is measured stroboscopically. Strips of tracing cloth with properly spaced lines are clipped to the sheave (see S-1). These strips are observed under a neon tube. One plate of the neon tube is rendered ineffective by connecting a diode (11726-GT) as a half wave rectifier. Then the neon tube flashes 60 times a second instead of 120 times a second. By making strips with 30, 20, 15, 12, 10, and 9 lines the spindle speed could be set to 120, 180, 240, 300, 360, and 400 rpm.

(4) Pressures

The pressure inside the test samples, P_o is measured by a standard bourdon tube gage connected at the bottom of the unit (see S-3). The hole in the lower spindle is large so that there is a negligible drop from the bottom to the test samples.

The pressure outside the test samples, P_1 is measured by a bourdon tube gage also, which is connected to the chamber around the samples. (See S-3)

For the high pressure tests the above gages are 0 to 10000 psi gages. Consequently the difference between P_o and P_1 which is small compared to P_o, cannot be read very accurately from these two gages. Two differential gages were designed and built. They are located as shown in S-3. One has a range of pressure drops from 0 to 250 psi and the other from 0 to 1000 psi. S-8 shows the construction of these gages. There are two plungers, one for the high pressure and one for the low pressure, which are supported in Hydrofilm Bearings described earlier. Consequently with no friction along the plungers the gages are reasonably sensitive to small pressure changes.

(5) Rate of Oil Flow

The rate of oil flow between the samples is measured by catching the oil from the heavy line in S-3 in a graduate before it enters the reservoir. This is called the main flow. In order to get the total flow between the samples the flow which supplies the torque spindle bearing and the differential gages must be added to the main flow. These are called leakage flows and are small compared to the main flow as long as the main flow is large. They are measured by catching the oil from the drains which are in question.

(6) Oil Temperatures

The temperature of the oil inside the samples, T_o is measured by an Alumel-Chromel thermocouple whose leads enter at the bottom of the unit and run up the hollow spindle. They are insulated from each other and the machine by porcelain spacers.

The temperature in the chamber surrounding the test samples is measured by two thermocouples whose leads are brought out through insulated bushings in the walls of the chamber. The arrangement of the couples is shown in S-9.

The cold junction of the couples is at room temperature, and is measured by a thermometer. These junctions are surrounded by a box which protects them from drafts.

The emf's of the thermocouples are measured with a Leeds and Northrup Student's Potentiometer, the necessary standard cell and working cell and galvanometer.

EXPERIMENTAL PROCEDURE

A. Test Procedure

The method of running the tests is very simple. Soon after the machine is started up in the morning readings are taken at short intervals of time. As the machine approaches its steady state temperature, readings are taken at longer intervals. These readings are taken at a relatively large film thickness during the heating period which is 4 or 5 hours. The inside pressure P_0 and the speed have to be adjusted frequently during this period in order to maintain them constant. Actually the film thickness does vary but only slightly. It may vary from .0012 in. to .0010 in. during the heating up period. The variation of the torque and the temperature are of importance because the changes in torque are caused by changes in viscosity. Consequently these data at thick films serve to determine the viscosity-temperature curve for the oil. At films of .001 in. the effect of surface finish is nil, especially with a lapped surface of 1-2 MI (see C-26) and the oil behaves as a viscous fluid according to theory.

After the machine has reached a steady temperature the film thickness is varied by by-passing some of the oil through valve 4 (see S-3) to atmospheric pressure. The throttle 10 then has to be readjusted to hold the pressure in the machine constant. This procedure is carried on until throttle 10 has been completely closed in which case the oil flow into the machine is just supplying the leakage by the torque spindle and differential gages. If high pressures are to be maintained in the machine the leakage will be greater than when low pressures are maintained and consequently the film thickness at leakage flow will be greater at the higher pressures than at lower pressures where the leakage flow is very small. In order to go to smaller films the compensator pressure is increased by small increments.

The procedure indicated above is the one commonly used, however in some cases the order of by-passing oil and then increasing the compensator pressure has been changed, i.e., the film thickness is varied by first increasing the compensator pressure and then by-passing oil. This procedure produces somewhat different velocity and temperature distributions in the film.

B. Preparation of Samples

The test samples are listed in Table I, page .
Some of these samples were not tested because of inaccuracies or other reasons. The samples must be made to a rather high degree of precision. It is felt that the test surfaces should be plane within .000020 in., i.e., they should have no chatter or waviness and should be flat. This degree of precision is necessary because the effects of surface finish should not be complicated by the effects of geometrical inaccuracies. These surfaces had to be ground very carefully in order to satisfy the precision requirements above. None of the ground surfaces prior to those ground on dead centers really met the precision requirements. Samples 4a, 3a, 3b, 4b, 5, 2b, 6, and 7 were ground by mounting them on a special mandrel (see S-10).

TABLE I

Sample No.	Type of Surface	Approx. M.I.	Prof. Rec. No.	Photo	Remarks
1	Ground	about 20			Original Sample Chatter 3/16" apart
1a	Lapped	1-2			Lapped at Cincinnati Ordinance Gage Laboratory Check with optical flats
1b	Lapped	1-2			Relapped because surface became damaged
2	Ground	16-17			Made by Heald Machine Co. Low spot on one piece
2a	Ground	50-55	R-1 R-2	1 and 2	Refinished 1-3-44 by U.S. Ordinance Gage Lab
3	Ground	6-9			Made by H. M. Co.
4	Lapped	1-2			Made by H. M. Co.
4a	Ground	4-7	R-9		By U.S. Ordinance Gage Laboratory
3a	Ground		R-5 R-6	3-4	Ground on dead centers Norton 3846-K wheel (but was not flat because centers were out of round.) Circumferential scratch
3b	Ground		R-7 R-8	same as 3a4	Ground on dead centers Circumferential scratch 20 MI radial 5 MI tangential Same appearance as 3a
4b	Ground	3-5	R-10 R-11	P-5	Ground on dead center with cup wheel ⊥ to work

TABLE I (con't.)

Sample No.	Type of Surface	Approx. M.I.	Prof. Rec. No.	Photo	Remarks
5	Ground	30-50	R-12 R-13	P-6	Ground on dead center with cup wheel $\perp$ to work
2b	Ground	20-30	R-3 R-4	P-7	" " " "
6	Ground	20-30	R-14	P-8	" " " "
7	Ground	20-30	R-15		" " " "

This mandrel was then placed between dead centers and driven by a belt from a Variable Speed D. C. Motor. The locating face A (S-10) had to be lapped flat and square with the axis of rotation. The locating faces on the test samples also had to be lapped flat if they were not already flat in order to prevent distortion when they are bolted to the lower spindle. Photographs and Profilometer Recordings are listed in Table I. The profilograms were taken on a Brush Analyser. Samples No. 6 were ground so that the scratches on one are the mirror image of those on the other, i.e., when they are placed face to face the scratches are parallel. Whereas Samples No. 7 or 2a are ground so that when they are placed face to face the scratches cross as indicated in S-11.

EXPERIMENTAL WORK

As mentioned before viscosity data is obtained by running at relatively thick films and allowing the machine to heat up. During this heating up period the film thickness remains reasonably constant. Curve C-26 shows the viscosity plotted against temperature for two runs made at different film thickness, one at about .0018 in. and the other at .00115 in. This shows that at films around .001 in. the viscosity is a property of the fluid and small variations in film thickness have no effect (except of course in computing the viscosity from the torque using $u = .01595 \frac{Th}{N}$, see page). In the tests the variation of film thickness around .001 in. is normally about .0002 in.

Curve C-34 shows absolute viscosity vs temperature at 50, 1000, 2000, and 3000 psi. Here the viscosity is plotted against the average of the three temperature readings T_I, T_F, and T_R (see S-9). The majority of tests were run with Sohio No. 10 oil; however, it was found that as time went on the viscosity might vary a few percent from that in C-34. This may be caused by the amount of air (in the earlier tests) or N_2 dissolved in the oil. Consequently it has been found advisable to determine the viscosity for each run.

Curve C-28 shows a heating up curve and the region where the viscosity data was obtained. The portion of the curve where the film thickness was varied is used to get information on the effect of surface finish and the effect of thin films.

Curves C-35, C-36, C-40, and C-41 show the shearing stress plotted against the viscosity times the rate of shear, for 50, 1000, 2000, and 3000 psi. The shearing stress is calculated directly from the observed values of torque. The viscosity times the rate of shear is the theoretical shearing stress. The rate of shear has been assumed constant through the film and independent of the radial motion of the oil. The equations of flow (page   ) indicate that the tangential and radial velocities are independent which, in conjunction with the expressions for shearing stress lead to the result of $\frac{\mu V}{h}$ where μ, V, and h are defined on page . In computing $\frac{\mu V}{h}$ μ has been taken as the viscosity that corresponds to the average of the three temperatures, T_I, T_F, and T_R. This procedure of basing the viscosity on the average temperature will be discussed further. Originally the thermocouples outside the test samples and the one inside were made of Alumel-Chromel wires .083 in. in diameter. It became apparent that the couples outside were not reading the temperature of the oil as it left the film. Consequently very small thermocouples made from Copper-Constantan wires .0005" in. diameter were substituted for the two outside couples. The junctions were made by soldering the two wires. The wires were crossed very near their ends so that the junctions approximate a parallelapiped whose sides are .0005, .0005, and .001 in. Even these couples were too large to measure the oil temperature at the exit of the film. Actually they measured the bulk temperature of the oil in the chamber. They probably would have been satisfactory for a .001 in. film but at this film thickness the energy dissipated in the film per unit volume of oil is small and produces temperature rises of the order of .3°C. It is at thin films of the order of .0003 in. or smaller where the rise in temperature is appreciable. For instance Curve C-23 shows the calculated temperature rise in the film, neglecting conduction, (see page for derivation) for test No. 44 shown on C-20.

From the above discussion it is apparent that the discrepency between the Calculated Shearing Stress and the Observed Shearing Stress may be caused by heating in the film. At thin films all of the experimental points are low which is just what would be expected if the actual viscosity were less than the viscosity used. It is evident that the true effects of thin films (where the bulk properties of the oil no longer hold) and surface finish are hidden because of the question of the viscosity of the film.

Curves C-31, C-32, C-37, and C-38 show the variation of the difference in temperature between inside and outside of the test samples plotted against time for 50, 1000, 2000, and 3000 psi respectively. These data were taken during the heating up period. The inside temperature is consistently higher than the cutside temperature for the 60 RPM runs. (At 3000 psi the 60 RPM run could not be made because the motor would not run smoothly under that load.) Apparently at low speeds the outside temperatures are relatively lower then the inside because less heat is generated in the bearings, this heat being conducted to the metal surrounding the chamber, while the inside temperature is influenced by the heat generated in the oil as it passes through the large resistance coil just before entering the machine. (The pressure drop in this resistance may be 2000 or 3000 psi.) At higher speeds the heat generated in the bearings tends to raise the outside temperatures thus decreasing $(T_I - T_F)$. As a result it does not seem advisable to base the viscosity on the average of the inside and outside temperatures. Accordingly it is felt that the film temperature depends more on the inside or inlet temperature than on the outside temperatures.

Two tests were run with the purpose of having different heat conditions in the film. The two methods of running tests outlined under Procedure (page) produce different heating effects in the film. The calculated temperature rise, neglecting conduction is plotted against film thickness on C-51. Curve C-47 shows the Observed Shearing Stress plotted against the Calculated Stress. The viscosity in this case has been taken as that corresponding to the inside temperature. From C-51 it is seen that the greatest difference in temperature occurs at about .00065 in. Assuming the average film temperature is equal to the inlet temperature plus ½ rise in temperature gives an average rise of $1^{\circ}F$ and $\frac{1}{2}{}^{\circ}F$ for the two tests at .00065. One half of a degree F will change the viscosity by about 1% in this case, which is hardly perceptible. Consequently these two methods of running tests do not make an appreciable difference in the film temperature. Curve C-50 shows the average radial velocity plotted against film thickness for the above two tests. The largest and smallest velocities are about

24 in/sec. and 6 in./sec. At these speeds the average time for the oil to pass through the test surfaces are .0052 sec. and .021 sec. However it is possible for the conduction of heat into the metal samples to occur because even though the temperature differences be small, the temperature gradient may be sizeable since the film is so thin.

Curve C-54 shows Calculated Shearing Stress vs Observed Shearing Stress for 50, 500, and 2000 psi. In this case the viscosity is based on the inlet temperature plus ½ rise in temperature as calculated neglecting conduction. The deviation is greatest for the 50 psi run and in this run the rate of flow at the thin films was least and the temperature would therefore be greatest. The deviation for the 2000 psi run is relatively small and the rate of oil flowat the thin films was comparatively large. Accordingly the temperature rise would be expected to be the least in this case.

Again the necessity of knowing the actual film temperature is illustrated.

Curve C-49 shows the Observed Stress plotted against the Calculated Stress for Sample No. 2a. Tests Nos. 75, 76, 77 were run consecutively. The samples before they were installed had a dull "mat" appearance. After the tests the peaks of the surface roughness were polished off. (See P-1 and P-2.) This indicates that the surface changed somewhat during the tests. Test No. 80 is identical with the others and was run to see if any difference could be observed between this and the first test. Apparently the polishing off of the peaks had negligible effect in changing the stress. The last points (corresponding to about .35 lb/in^2 calculated shearing stress) represent the observed stress at about .00035 or .0004 in. It was not possible to go any closer than this without shorting the oscillator. When the samples were first put in (Test No. 75) .00045 in. was the minimum film thickness that could be attained. However on the other tests the film thickness was decreased below .00045 in. causing the oscillator to short. The machine was allowed to run in this condition and the output of the oscillator gradually climbed up to its normal value. Apparently the peaks were being worn off until there was no intermittent contact. It should be observed that the deviation of this curve from the theoretical straight line is probably even more than is shown on C-49 because of the heating effect, which would tend to lower the curve toward the theoretical line since the viscosity for C-46, 48, and 49 was based on the inlet temperature only.

There is another effect which needs to be considered. This is the meaning of the film thickness reading of the oscillator. The surfaces being rough, will have various radii of cruvatures at different points. At the peaks these radii will be the smallest and the charge density will be greatest. Because of this and the inverse relationship of capacity with distance a smooth parallel plate condenser would have to be closer than the plates of a rough condenser (as measured from the geometric average of the peaks and valleys) in order to give the same capacity. Or the rough plates of a condenser would not have to be brought as close as a smooth plate condenser to give the same capacity. Accordingly, if the surface roughness has any effect on the capacity, the curve of C-49 should deviate even more from the theoretical straight line.

Curve C-48 shows three tests run at different speeds. On each curve there is a decided break at the stress corresponding to .00040 in. which probably corresponds to interference between the peaks. The accuracy indicated by the digits in the film thickness reading may seem unwarrented, however a change in film thickness of .000010 in. in the region around .0004 in. produces a readily measureable change in torque and oscillator reading. The oscillator was calibrated against a .0001 in. indicator. Accordingly one cannot be sure of the absolute value of the film thickness to this degree but changes of these magnitudes are readily observed on the oscillator. It should again be remembered that these curves probably deviate more than is shown because of heating and perhaps the effect of surface roughness on capacity.

Curve M-58 shows Static Torque vs Compensator Pressure. This curve was obtained by stopping the lower spindle and observing the torque on the upper spindle. With the lower spindle stopped there is a small (about

2 in.-gr.) torque on the torque spindle which is probably caused by slight nonuniformities in the flow around the piston and spindle. This torque remains approximately constant as the film thickness is decreased. When the samples were put in and when the compensator pressure was increased, the static torque varied as shown by M-58. This is caused by the surface roughness. The samples were ground in such a way that curved grinding marks were produced on the surface. These marks are not symmetrical about a radial line. The oil flowing between the samples is deflected by these surface scratches and thereby produces a torque similar in principle to a reaction turbine. As the compensator pressure is increased the film thickness decreases, the velocity of flow increases and the reactionary torque is greater. This static torque is not superimposable on the torque caused by rotation. The surface scratches on the upper and lower samples are situated such that there exists an angle of about 60° between a scratch on the upper surface and the scratch opposite on the lower surface. (See S-11) The point A will move outward or inward depending on the direction of rotation. It is possible that this would produce a pumping action either outward or inward.

Curve C-46 is the same as C-48 except this static torque has been subtracted from the observed torque. As mentioned above this procedure is not justified, however C-46 shows that there is still a difference between the observed stress and the calculated.

Curve C-56 shows a comparison of the stresses for Samples No. 1b and No. 3b. The viscosity on both of these runs was based on the inlet temperature plus the average calculated rise in temperature. Sample No. 3b had a greater roughness in the radial direction than in the tangential direction (see R-7 and R-8).

B. Rate of Flow

Curve C-55 shows the Rate of Flow plotted against film thickness for runs at 2000 psi and 50 psi. The calculated curve is based on the theory developed on page . It is evident that the calculated values do not agree very well with the measured values. There are several possible causes for this discrepency. One is that there may be a small leakage by the screws and locating surfaces of the test samples shown in S-2, however, the screws have always been pulled down firmly with copper washers under their heads and the locating surfaces have in some cases been lapped flat so that it is rather troublesome to pull the test sample off the ends of the spindles even with all the screws removed. Another cause for disagreement and the most important one is that in order to calculate the rate of flow the pressure drop (difference of large quantities) is needed. This drop is read from the differential gages. An error of 5 psi can easily be made in 50 or 60 psi pressure drop. Also at high pressures there may be some error in these gages because of the pressure drop in the lines feeding them. The rate of flow to these gages is comparatively small but nevertheless exists. The pipes have a 3/16" bore. A third possible cause of error is the use of an incorrect viscosity, but this would not account for the entire differences between the observed and calculated curves. Another possibility lies in the assumptions made in the derivation of the expression for Rate of Flow (see page). No account has been taken of the rotational motion nor of the centrifugal force on the oil. However a simple calculation shows that the centrifugal force per unit volume is very small as compared to the pressure and viscous forces.

Curve C-56 shows the Observed Rate of Flow plotted against the Calculated Rate of Flow for various pressures. This is a better way to handle the data of C-55 since the Rate of Flow depends on the pressure drop, and viscosity besides film thickness. By plotting the Observed Flow against the Calculated Flow these factors are taken account of.

Curve C-55 shows the Rate of Flow plotted against film thickness for Sample No. 2a (50-55 MI) and Sample No. 1b (1-2 MI). Curve C-61 shows these data plotted by the second method as mentioned above. Apparently the Rate of Flow is greater for the smooth sample although the data is not very consistent. At this point it may be recalled that the orientation of the scratches (see S-11) might produce a pumping action either inward or outward depending on the direction of rotation. For the data of C-61 the rotation was such as to produce an inward pumping action for the 50-55 MI samples which would tend to produce the results obtained. The effect of rougher surfaces might well also tend to produce this result. These rough samples were definitely concave, perhaps .00005 in. in the width of the ring. This fact would tend to oppose the above.

As mentioned before the treatment of the flow data is rather unsatisfactory because of the inaccurate differential pressure readings. The actual rates of flow observations plot into a fairly smooth curve as shown on C-55.

C. Pressure Relations

The condition for equilibrium in the vertical direction, of the torque spindle may easily be written, assuming a linear pressure drop across the test surfaces. This expression will involve P_o, P_1, P_c and the weight of the spindle. The total force on the test surface (the integrated pressure over this surface) may be computed from the equilibrium equation using observed values of P_o, P_1, P_c and the weight of the spindle. It would be interesting to see how this experimental value of the normal force varies with film thickness, and theory. A simple theory is shown on page . With the equipment as it is, it is impossible to investigate this effect because the difference of large quantities (pressures) is needed. These differences cannot be read with the required degree of precision necessary.

D. Coefficient of Friction

The coefficient of friction may be easily obtained by dividing the shearing stress (tangential) by the pressure. Thus, instead of plotting Observed Stress vs $\frac{\mu V}{h}$, the curve would show Friction Coefficient vs $\frac{\mu V}{hP}$ and would be essentially the same as the shearing stress curves except with a different slope. This method of treatment is similar to the usual treatment of journal bearing data where the coefficient of friction is plotted against $\frac{\mu N}{P}$ where N is the rotary speed, P is the load, per sq. in. of projected area, and μ is the viscosity.

E. Additional Work Concerning Film Temperatures

A Silicone Oil was obtained whose viscosity changes very slowly with temperature. Curve C-62 shows the viscosity - temperature relation for this oil at 500 psi. The viscosity for the Sohio No. 10 oil is also plotted for comparison purposes.

Curve C-63 shows a comparison between experiment and theory of the shearing stress. In this case the shearing stress was calculated using the viscosity corresponding to the inlet temperature T_I. For thin films (.0003 in. or less) the experimental stress is definitely greater than calculated. By using the Silicone Oil the error caused by the heating of the film has been minimized. A similar test using Sohio No. 10 is also shown and lies below the theoretical line, as would be expected, considering the heat generated in the film.

A method of determining the film temperatures by comparing the results of two tests run on oils which have widely different viscosity indices has been devised. This method is developed in part (m) of the Analytical Treatment Section. Curve C-64 shows the temperature rises which existed at various film thicknesses of Test No. 82. The several methods of estimating the film temperature rise are plotted for comparison. The ratio of torque method (see part (m)) indicates a cooling in the thicker films which is caused by a loss of heat from the film to the surrounding metal. This is not necessarily a general result but applies to this test only. As the film thickness is decreased more and more heat is generated in the film and causes the temperature to stop falling and begin to rise. There is no doubt about the drop in temperature at the thicker films because Curve C-63 verifies this. It will be noticed that the points designated by crosses on C-63 lie slightly above the theoretical line. This corresponds to a cooling in the film. These data have been corrected for the temperature rise indicated on C-64 and have been replotted on Curve C-65. This again throws the data for thin films above the theoretical line and the data checks quite well with that of C-63 for the Silicone Oil where the heating effect in the film is minimized. If the test on Silicone Oil (#93) is corrected for the temperature rise these two curves superimpose as indicated by the crosses on C-65. Thus, one conclusion is that there is no difference in friction for a poor lubricating oil (the Silicone Oil) and a normal lubricating oil (Sohio No.10) down to films of the order of .00015 in.

The method of determining the film temperature used above solves the most troublesome variable encountered. This method does have some shortcomings, however. These are discussed on page . In spite of these, this method has good possibilties.

PROPOSED IMPROVEMENTS

As mentioned earlier the method of definitely maintaining and controlling a film of oil of uniform thickness is new and affords a means of making a fundamental investigation of the laws of fluid flow in small films under the conditions encountered in bearings, and the effect of Surface Finish, and the physical and chemical nature of the lubricant thereon. It is felt that further work using this method with refinements in design will lead to important information regarding the intimate details of bearing operation. Experience with the present machine has pointed out difficulties and improvements that should be considered in future work. The following list of features are proposed to eliminate the difficulties encountered in the present machine, and to permit the extension of the range of operation.

Proposed features to be discussed:

1. Supplementary Method of Film Thickness Measurement
2. Refined Method of Calibrating Oscillator
3. Auxiliary source of oil for supplying leakage by the bearings
4. Higher Spindle Speeds
5. Elimination of fibre bushing and alignments based on these bushings
6. Thrust bearing for lower spindle
7. Filtering of Oil
8. Elimination of necessity of reading absolute pressures to a high degree of precision. It is only necessary to measure the differential pressures accurately.

9. Consideration of measuring the temperature gradient across the film.

10. Method of checking parallelism of samples.

11. Elimination of the possibility of small leaks from inside the samples to outside.

12. Construction that will enable the machine to be operated up to say 40000 psi.

13. Convenient and Accurate Method of measuring speed.

14. Consideration of dissolved Nitrogen in oil.

15. Consideration of the break down voltage across the test surfaces.

1. Supplementary Method of Film Thickness Measurement

The capacitance method of measuring film thickness is desireable because it eliminates errors caused by expansions and contractions of the spindle. However the dielectric constant is involved in it and this may change at thin films with various conditions of flow, speed, and so forth. The change of dielectric constant with pressure and temperature can readily be measured independently of the machine as was done for the present tests. The capacitance method could be used to measure very accurately the film thickness at say .0005 in. Then with the machine at constant temperature relative changes could be measured with a very sensitive electrolimit gage or other means. These relative changes should be measured very accurately at least to .000010 in. By comparing the increments of film thickness as the spindle is progressively lowered, as found by the two methods, information regarding the change in dielectric constant and resistance may be obtained. It would be interesting to see if the resistance approached zero as a linear function of the film thickness. The voltage should be kept low across the film to prevent arcing.

2. Refined Method of Calibrating the Oscillator

The oscillator should be calibrated (capacity vs film thickness) as accurately as possible. Consideration of how this is to be done should be taken into account in the design of the machine. The thrust bearing for the lower spindle can be constructed so that the lower spindle will run "dead" true (to be discussed later) and will have a very definite position axially (no ball bearings). The torque spindle should then be wrung to the lower spindle. With the samples wrung together the "zero" reading should be taken on a suitable gage or electric comparator. The comparator should indicate on a removable block (say a gage block) which is wrung to an accurately lapped surface of the torque spindle. To work through a block is necessary because the block can be removed at which time the spindle may be raised without danger of changing the "zero" setting or damaging the indicator. The spindle can then be raised a definite amount by pumping a small quantity of oil through the samples or by providing a means of setting the spindle at definite film thicknesses. It would be best to have facilities for using both methods since they could be used to check each other. S-12 shows a removable plate screwed onto the end of the spindle. Gage blocks can be put in on the accurately lapped surfaces as indicated. Using a setup of this type the oscillator could be calibrated very accurately.

3. Auxiliary Source of Oil for supplying leakage by the Bearings

As pointed out earlier at high pressures the Leakage Flow through the bearings is greater than at low pressures. This means that the chamber surrounding the samples with pressure P_1 in it must supply the leakage by the torque spindle and by the lower spindle. As a result there is a certain minimum rate of flow required through the test samples which limit the range of film thickness and oil temperature in the film. Consequently there should be an independent supply of oil for the torque spindle bearing and the lower thrust bearing (see S-12) to partially make up the leakage flow.

4. Higher Spindle Speed

The design should take into account the power required to drive the lower spindle say up to a speed of 2000 RPM. The bearings must be long and the clearance close to offer a high resistance to the flow of oil (especially at 30000 or 40000 psi). However the power required can be calculated fairly simply, at least approximately. The relative thermal expansions of the spindle and bearings should be given consideration. The spindle might be made of Invar or an alloy with a small coefficient of expansion.

5. Elimination of Fibre Bushings

It has been found very difficult to maintain precision alignments across fibre bushings expecially when under pressure. One way of insulating the upper test samples is shown in S-13. Here a quartz disc is lapped flat and separates the test sample from the torque spindle.

6. Thrust Bearing for Lower Spindle

This bearing must allow the lower spindle to run absolutely true. Ball or Tapered Roller Bearings are not sufficiently accurate. One possibility is to have a shoulder whose faces are accurately lapped flat and square to this shoulder. A small amount of vertical movement of the spindle is tolerable provided the movement takes place with changes in pressure or oil flow and not with every revolution of the spindle. Only at the time of calibration is it necessary to have the thrust shoulder well in contact with the lower locating surface. All of these surfaces should be hardened, ground, and lapped until they are absolutely flat and square. Provision should be made so that an indicator can be mounted on the bottom spindle somewhere for instance on the sheave to check the axial movement of the lower spindle as pressure is applied. Knowing this and the two readings of film thickness from the upper spindle the actual film thickness should be relatively certain. The clearance between the thrust faces and the mating surfaces of the thrust bearing should be kept very small. The Hydrofilm bearing may be applied by putting two annular channels in the locating surfaces and feeding the oil from the periphery toward the center. Each "pressure pick up" channel is connected to the opposite "pressure area."

7. Filtering of Oil

This problem is self explanatory. The equipment to do this is another story. However there are types of absorbent and magnetic filters which could be designed to operate at high pressures. There are also types of porous rock filters which may be the solution to this problem.

8. Elimination of necessity of measuring absolute pressures to a high degree of precision

In order to get reliable data pertaining to the pressure distribution across the test surfaces it is necessary in the present machine to measure not only the difference of large pressures accurately but also their absolute magnitude. The measurement of the difference in pressure to say 1 psi is a much easier problem than measuring the absolute value of a large pressure to 1 psi. By making the torque spindle as shown in S-12 the equilibrium equation for the spindle can be expressed in terms of pressure differences only. (See page .) It is quite possible to make very sensitive pressure differential gages to operate at high pressures. As mentioned before the differential gages on the present machine are not sufficiently accurate to get reliable data for comparing the rates of flow. This is important because the surface finish undoubtedly affects the rate of flow.

9. Consideration of the measurement of the temperature gradient across the film

The actual temperature of the metal on each side of the film has an effect on the temperature of the film itself. Data of this sort is necessary to use the expression derived for the film temperature on page . The surface finish probably has an effect on the heat transfer properties of this film. This is also an important question. It would be no trouble to put a small thermocouple in the metal of the upper sample but one in the lower sample would have to have its leads brought out through slip rings (which apparently aren't satisfactory) or perhaps electrodes in a channel of mercury.

10. Method of Checking Parallelism of Samples

In the present machine the torque spindle bearing is mounted in a fibre bushing and the spindle is $\frac{3}{4}$" in diameter. There is enough flexibility in the spindle to absorb errors in parallelism between the sample of .0001 in. or less without making appreciable error in the proper functioning of other parts of the machine. The surface of the lower sample can be checked by putting a sensitive indicator (better than .0001 in. indicator) on it and by "sweeping" a large diameter circle on the lower housing with the indicator fastened to the lower spindle. A special bar may be required to indicate from in order to check the squareness of the locating face of the upper housing. These mating surfaces should be hardened and lapped until they are dead true. The torque spindle should have an accurately lapped shoulder so that the upper housing and spindle could be turned upside down and the test surface indicated from the locating surface on the upper housing.

11. Elimination of the possibility of leaks from inside the samples to outside

In the present machine the upper sample is bolted to the torque spindle. Two of these bolt holes extend clear through the shoulder on the end of the torque spindle. The design shown in S-13 eliminates these holes. The mating surfaces should be accurately lapped to prevent any leakage besides for the maintenance of parallelism.

12. Construction that will enable machine to operate at high pressure

With the elimination of fibre bushing there is no particular trouble in withstanding 40000 psi.

13. Convenient and Accurate way of measuring speed

The problem here is self evident and there are many ways of accomplishing this in a satisfactory way.

14. Consideration of dissolved Nitrogen in the oil

In the receiver oil comes into contact with Nitrogen at high pressures and some dissolves. This dissolved nitrogen comes out of solution when the pressure is reduced. This fact makes the control of uniformity of the oil difficult. The receiver with nitrogen and oil in it is very necessary. One possible cure would be to separate the oil and nitrogen by a flexible membrane of neoprene as shown in S-14. The membrane and mounting should be free to slide in the cylinder.

15. Consideration of the Break-down voltage across the test samples

There is a good possibility that arcing across the film may occur especially with rough samples when the film is thin. A simple experiment was run which showed that the oil broke down at 15000 v. and about 1/8 in., i.e., about 100000 volts/inch. Assuming this break down voltage to vary linearly with spacing, 10 v. would break down a film of .0001 in. This may explain the "shorting" of the oscillator at thin films with the rougher surfaces. The voltage across the test samples varies from about 11 volts at very thick films to 3 volts at about .0002 in. The variation of voltage across the condenser with film thickness is caused by variations in the effective impedance or a dropping off of the "Q" of the circuit.

ANALYTICAL TREATMENT

On the following pages certain problems pertaining to the machine are treated. A list of these problems follows:

a. Coefficient of Friction vs $\frac{\mu V}{nP}$

b. Simple expression for torque transmitted by oil film

c. Shearing stress, observed and calculated

d. Simple calculation for temperature rise in film

e. Equilibrium expression for spindle of old and proposed machine

f. Simple treatment of rate of flow and pressure distribution

g. Solution of Equations of Viscous Flow in Cylindrical coordinates

h. Development of Equations of Flow for variable viscosity

i. Development of Heat Equation for variable viscosity

j. Attempted solution of h and i by one method

k. Attempted solution of h and i by separation of variables

l. Simplified treatment of film temperatures using Newtonian Cooling

m. Method of Measuring Film Temperature

Nomenclature

Some of these problems, notably j and k, have not been carried to a successful conclusion. However what has been done will be indicated.

Section (a) shows the simple derivation of Petroff's

Equation for journal bearings involving the coefficient of friction as a linear function of the dimensionless "Sommerfeld Variable" $\frac{\mu N}{P}$. This equation is good for a journal running concentric with its bearing. Of course as a journal is loaded it ceases to run concentric. Thus the circumferential variations of pressure, oil velocity, film thickness, stress, and temperature, effect the Sommerfeld Variable in a composite way. The friction coefficient for the plane annular surfaces is derived in an analgous way. As mentioned earlier if the expression for the friction coefficient is multiplied by the average pressure on the surface the expression is reduced essentially to shearing stress as a linear function of $\frac{\mu V}{h}$. Values of $\frac{\mu V}{hP}$ for the annular surfaces should not be compared with values of $\frac{\mu N}{P}$ without caution. One variable takes account of the changes in viscosity with pressure and the film thickness while the other does not do so specifically. The point is to separate these variables and observe their effect as one is varied with the others fixed or known.

Section (b) shows a simple determination of the Torque transmitted by the oil film. This expression when solved for the viscosity is what was used in measuring the viscosity, the data for thick films being used for the viscosity determination.

Section (c) needs no comment.

Section (d) shows the derivation of the temperature rise in the film caused by the tangential shearing of the oil and also the radial shear. From equations 3 and 4 of part (g) it is obvious that the pressure drop is independent of the tangential motion thus it seems reasonable to simply add the heats generated by the two motions. This is also the basis for neglecting the radial motion in computing the torque or viscosity, and the tangential shearing stress, usually referred to simply as shearing stress.

Section (e) needs no comment.

Section (f) shows a very simple treatment of the pressure and velocity relations for the flow between the test samples. Here as before the tangential motion is neglected. The expression for the pressure turns out to be very nearly linear although it doesn't appear so. This pressure distribution is shown on C-25. The rate of flow as computed from these calculations is shown on C-24. This theory agrees quite well with the more general theory in part (g).

Section (g) was worked out with the idea of substantiating the elementary treatment (f). The expression for the rate of flow compares fairly well with that of part (f). The expression for the torque turned out to involve an extremely lengthy computation and so was not carried through.

Section (h) shows the general equations for viscous flow considering variable viscosity. These equations are much too complicated to solve in any case except the most simple. The case of oil flowing between two flat plates was chosen. There are several conditions that are somewhat analogous to the state of affairs in the machine. One of the plates can be considered as moving relative to the other, or they can be considered as fixed with oil being pushed between them.

Section (i) needs no comment.

Section (j) shows one method of arriving at a differential equation for the velocity. Other methods were tried, but they, in all cases, became very complicated. It is not clear just how to satisfy the boundary conditions chosen. Perhaps progress can be made under other boundary conditions.

Section (k) shows another equation for the velocity. It is not clear how to select the δ's and the A and B to satisfy the boundary conditions.

Section (l) shows a simplified treatment of the film temperature. The assumption of a uniform temperature distribution across the film is made. Difficulty in applying this theory is caused by the fact that the film coefficient of heat transfer k is not known and that the temperature of the metal on each side of the film is not known.

Section (m) shows a method of determining the effective temperature of the film. This method takes account of the heat generated in the film alright, but it only partially takes account of the heat lost by conduction. In order to determine the the temperature which corresponds to the ratio of torques some relation between the temperature rises is needed. The relation used is essentially that derived in part (d). This relation neglects conduction. A more accurate expression for the rise in temperature would be

$$\Delta\theta = k_1 \frac{T}{Q} - \frac{k_2 (\theta - \theta_m)}{Q}$$

where the first term on the right is due to the heat generated and the second term due to the heat lost (or gained) by "Newtonian Cooling". The trouble with using this expression in the method is that k_2 and θ_m are unknown. They cannot be measured readily on the present machine because of constructional details. The effect of heat generated and of heat lost is reflected by the measured ratio of torques, and it is in this way that the method of torque ratios takes account of conduction.

a) DIMENSIONLESS FRICTION COEFFICIENTS

For Journal Brg's

$$f_t = \frac{T}{\frac{D}{2}} \qquad ; \qquad f_n = W$$

$$f \equiv \frac{f_t}{f_n} = \frac{2T^*}{DW}$$

$$= \frac{2\pi^2 \frac{D}{C} \frac{L}{D} \mu N D^3}{DW}$$

$$= \frac{2\pi^2 \mu N D^3}{C D^2 P}$$

$$f = 2\pi^2 \frac{D}{C} \frac{\mu N}{P} \qquad \left\{ \begin{array}{l} \text{Petroff's} \\ \text{Equation} \end{array} \right.$$

Where

f_t = tangential force

f_n = normal force = W

f = coefficient of friction

W = normal load

T = torque on journal

D = diameter of journal

C = radial clearance

μ = coefficient of viscosity

P = $\frac{W}{DL}$ = load per in^2 of projected area

N = speed

*T $= \pi^2 \frac{D}{C} \frac{L}{D} \mu N D^3$ for a centrally located journal

DIMENSIONLESS FRICTION COEFFICIENTS

For Plane Annular Surfaces

$$f_t = \frac{T}{\frac{D}{2}} = \frac{2T}{r_1+r_0} = \frac{2}{r_1+r_0}\left[\frac{\pi^2\mu N}{60h}\,(r_1^4-r_0^4)\right]$$

$$f_n = \left(\frac{P_0+P_1}{2}\right)\pi\,(r_1^2-r_o^2) = P_{avg}\,\pi\,(r_1^2-r_o^2)$$

$$f = \frac{f_t}{f_m} = \frac{\frac{2}{r_1+r_o}\left[\frac{\pi^2\,(r_1^4-r_o^4)}{60h}\,\mu N\right]}{P_{avg}\,\pi\,(r_1^2-r_o^2)}$$

$$= 2\pi\left[\frac{r_1^2+r_o^2}{60h\,(r_1+r_o)}\right]\frac{\mu N}{P_{avg}}$$

$$= \frac{2(r_1^2+r_o^2)}{60(r_1+r_o)^2}\;\frac{\mu\,2\pi\left(\frac{r_o+r_1}{2}\right)N}{h\,P_{avg}}$$

$$f = \frac{r_1^2+r_o^2}{30(r_1+r_o)^2}\;\frac{\mu V}{h\,P_{avg}}$$

Where

T = torque

$\frac{r_1+r_o}{2}$ = average radius of test samples

P_o = pressure inside samples

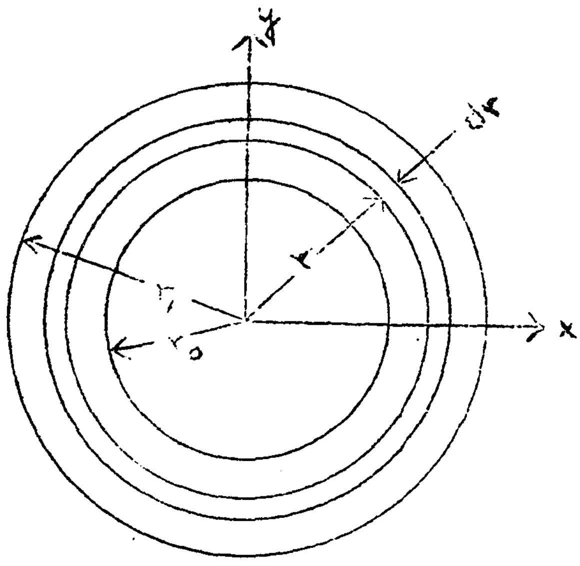
P_1 = pressure outside samples

h = film thickness

V = surface speed

$$T = \frac{\pi^2\mu N}{60h}\,(r_1^4-r_o^4)$$ by a simple integration

b) TORQUE TRANSMITTED BY OIL FILM



By Newton's Law

$$F = \mu A \left(\frac{v}{h}\right)$$

$$dF = \mu 2\pi r dr \frac{v}{h}$$

$$dT = r dF = \mu 2\pi r^2 dr \frac{v}{h}$$

$$T = \int_{r_o}^{r_1} \frac{\mu 2\pi r^2 \; 2\pi r N}{60h}\, dr$$

$$= \frac{4\pi^2 \mu N}{60h} \int_{r_o}^{r_1} r^3 dr$$

$$= \frac{4\pi^2 \mu N}{4 \times 60h} \left[r_1^{\,4} - r_o^{\,4} \right]$$

$$= \frac{\pi^2 \mu N}{60h} \left[\left(\frac{5}{4}\right)^4 - \left(\frac{9}{8}\right)^4 \right]$$

$$= .84 \frac{\pi^2 \mu N}{60h}$$

NOMENCLATURE

$N \equiv$ Speed (RPM)
$\mu \equiv$ Viscosity (lb-sec/in^2)
$F \equiv$ Force (lb.)
$r \equiv$ Radius (in.)
$h \equiv$ Film Thickness (in.)
$r_o =$ 9/8 in.
$r_1 =$ 5/4 in.

$$\mu = \frac{60}{.84\pi^2} \frac{Th}{N} \quad \text{with T in in-lbs}$$

$$= \frac{60}{.84\pi^2 \times 454} \frac{Th}{N} \quad \text{with T in in-gr}$$

$$= .01595 \times 10^{-2} \frac{Th}{N}$$

c) SHEARING STRESS

Observed Stress

The observed stress σ is obtained from the torque as follows:

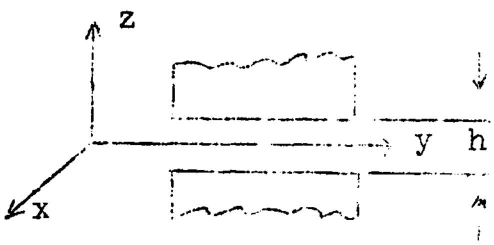
$$\text{Stress} = \frac{\text{Tangential force}}{\text{Area}}$$

$$\sigma = \frac{F_t}{A} = \frac{T}{A\, r_{avg.}} = \frac{T}{\frac{(r_1+r_0)}{2}\, \pi\, (r_1^2 - r_0^2)}$$

$\sigma = .903\ T$ where T is in in-lbs, σ in lb/in^2

$\sigma = .00199\ T$ where T is in in-gr, σ in lb/in^2

Calculated Stress



Consider the axes situated as shown and the upper test surface moving in x direction with the velocity V.

The components of stress are given by:

$$\sigma_{zx} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$$

$$\sigma_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

Assuming

$$v_z = 0$$

$$\frac{\partial v_y}{\partial x} = 0 \qquad \sigma_{zx} = \mu \frac{\partial v_x}{\partial z} = \mu \frac{V}{h}$$

$$\frac{\partial v_x}{\partial y} = 0$$

where v_x is a linear function of z

d) THEORETICAL HEATING OF OIL FILM NEGLECTING CONDUCTION

A.) Heating by Tangential Shear of Oil

By the Conservation of Energy

Energy Absorbed/cc = Mass x Sp. Ht. x ΔT

$$\text{Energy Absorbed/sec} = F_t \times 2\pi r \times N_{sec.} \quad \left[\text{in-gr/sec}\right]$$

$$= T_{in-gr} \times 2\pi \times N_{sec.}$$

$$= T_{in-gr} \times \frac{2\pi N_{sec.}}{12 \times 454 \times 778} \quad \left[\text{BTU/sec}\right]$$

$$= T_{in-gr} N_{RPM} \frac{2\pi}{12 \times 454 \times 778 \times 60} \quad \left[\text{BTU/sec}\right]$$

$$= .247 \times 10^{-7}\, TN$$

$$\text{Energy Absorbed/cc} = \frac{\text{Energy Absorbed/sec}}{\text{Rate of Flow (cc/sec)}}$$

$$= .247 \times 10^{-7} \frac{TN}{R} \quad \left[\text{BTU/cc}\right]$$

Then

$$\Delta T = \frac{\text{Energy Absorbed}}{\text{Mass x Sp. Ht.}}$$

$$\Delta T = \frac{.247 \times 10^{-7}}{.877 \times Sp.Ht.} \quad \frac{TN}{R}$$

$$Sp.Ht^{1} = \frac{1}{\sqrt{d}} \; \frac{(.388 + .00045t)}{454} \text{ BTU/GR/}^{o}F$$

For $d = .877$

$t = 100^{o}F$

$SP.HT. = .00109$ BTU/GR/^{o}F

$$\Delta T = \frac{.247 \times 10^{-7}}{.877 \times .00109} \quad \frac{TN}{R}$$

$$\Delta T = .258 \times 10^{-4} \quad \frac{TN}{R} \quad (^{o}F)$$

B.) HEATING BY RADIAL SHEAR OF OIL

Starting with Bernouilli's Equation.

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} + \text{Losses (ft.)(Per Lb. basis)}$$

$$\frac{p_1}{d_1} = \frac{p_2}{d_2} + Q \qquad \text{Since } v_1 = v_2 = 0$$

$$Q = \frac{p_1 - p_2}{d} \text{ (ft-lbs)/lb.} \qquad \text{Since } d_1 = d_2 \text{ (Incompressible)}$$

$$Q = \frac{(p_1 - p_2) \times .877}{778 \times 454 \times d} \qquad \text{(BTU/cc)}$$

Where p (lb/ft^2)

d (lb/ft^3)

$$Q = \frac{.877 \; (p_1 - p_2)}{12 \times 778 \times 454 \times d} \qquad \text{Where } p \text{ (lb/in}^2\text{)}$$

d (lb/in^3)

1 Chem. Eng'rg. Handbook - Perry P-2373

$$\Delta T = \frac{\text{Energy Absorbed}}{\text{Mass x Sp. Ht.}}$$

$$= \frac{(p_1 - p_2) \times \cancel{.877}}{778 \times 454 \times d \times \cancel{.877} \times .00109 \times 12}$$

Where d (lb/in^3)

$$= \frac{(p_1 - p_2)}{778 \times 454 \times \frac{(2.54)^3}{454}\, d \times .00109 \times 12}$$

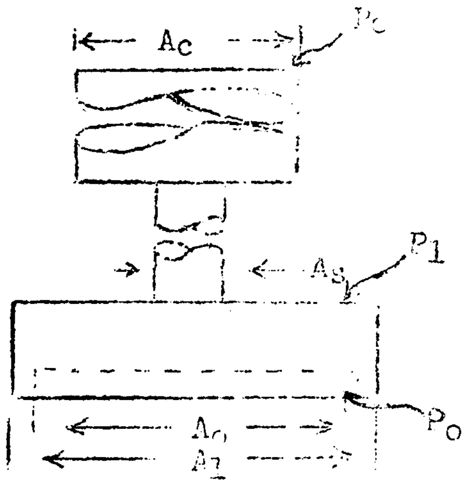
Where d (Gr/cc)

$$= \frac{p_1 - p_2}{778 \times (2.54)^3 \times .877 \times .00109 \times 12}$$

$$\Delta T = .00687\ (p_1 - p_2) \qquad (^{\circ}F)$$

e) EQUILIBRIUM CONDITIONS FOR SPINDLE

Present Machine



Areas on Spindle

$\sum F = 0$

$$P_c A_c + P_1(A_1 - A_s) + W = P_o A_o + \frac{P_o + P_1}{2}(A_1 - A_o)$$

The absolute values of the P's are needed here.

W is the weight of the spindle.

Proposed Machine

The problem is to find the conditions on the A's that will permit the equilibrium equation to be expressed in terms of pressure differences only.

The above equation may be written

$$P_c A_c + P_1(A_1 - A_s) + W = P_o A_o + \frac{A_1 - A_o}{2}(P_1 - P_o) + (A_1 - A_o)P_o$$

$$P_c A_c + P_1(A_1 - A_s) + W - A_1 P_o = \frac{A_1 - A_o}{2}(P_1 - P_o)$$

$$P_c A_c - P_o A_c + P_o A_c + P_1(A_1 - A_s) + W - A_1 P_o = \frac{A_1 - A_o}{2}(P_1 - P_o)$$

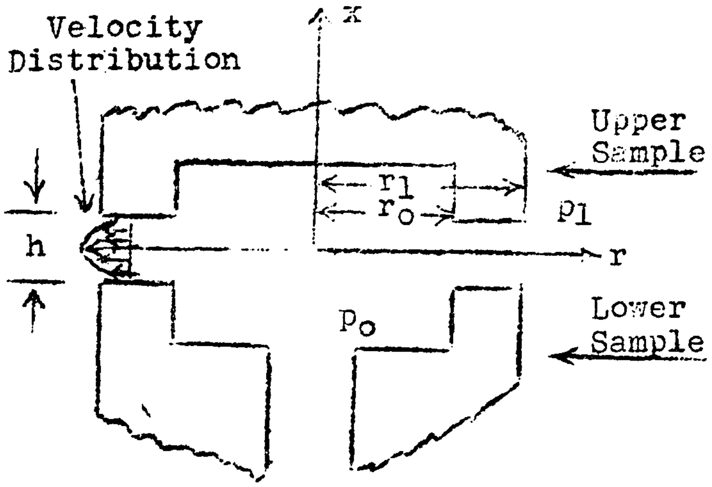
$$W + A_c(P_c - P_o) + (A_1 - A_s)P_1 - (A_1 - A_c)P_o = \frac{A_1 - A_o}{2}(P_1 - P_o)$$

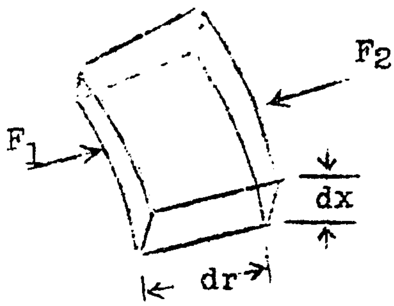
If $(A_1 - A_s) = (A_1 - A_c)$ only pressure difference will be involved.

Thus $A_s = A_c$ will permit this.

f) PRESSURE AND VELOCITY RELATIONS

For Radial Flow Thru Test Specimens





Taking the axes as shown and considering the equilibrium of the element of volume

Force downstream due to pressure = $F_1 - F_2$

$$= (p + \partial p) r\, \partial\theta\, \partial x - p\,(r + \partial r)\partial\theta\, \partial x$$

$$= r\, \partial p\, \partial\theta\, \partial x - p\, \partial r\, \partial\theta\, \partial x$$

Shearing Resistance $= -s\, r\, \partial\theta\, \partial r + (s + \frac{\partial s}{\partial x} \partial x) r\, \partial\theta\, \partial r$

$$= \frac{\partial s}{\partial x}\, r\, \partial\theta\, \partial r\, \partial x$$

Force Downstream = Shearing Resistance
(Neglecting Inertia Forces)

$$r\, \partial p\, \partial\theta\, \partial x - p\, \partial r\, \partial\theta\, \partial x = \frac{\partial s}{\partial x}\, r\, \partial\theta\, \partial r\, \partial x$$

$$r\, \partial p - p\, \partial r = \frac{\partial s}{\partial x}\, r\, \partial r$$

But $s = \mu R$

$$= \mu \frac{\partial v}{\partial x}$$

$$r\, \partial p - p\, \partial r = \mu r \frac{\partial^2 v}{\partial x^2}\, dr$$

$$\frac{\partial s}{\partial x} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial p}{\partial r} - \frac{p}{r} = \mu \frac{\partial^2 v}{\partial x^2}$$

If we assume $v = f(x)$ only

and $p = f(r)$ only

the above equation may be separated into 2 parts (i)(ii)

Since the left hand side is a function of r only and the right hand side a function of x only both sides must be constant.

(i) $$\frac{dp}{dr} - \frac{p}{r} = \alpha \text{ (a constant)}$$

Multiplying by $\frac{1}{r}$

$$\frac{1}{r}\frac{dp}{dr} - \frac{p}{r^2} = \frac{\alpha}{r}$$

$$\frac{d}{dr}\left(\frac{p}{r}\right) = \frac{\alpha}{r}$$

$$\frac{p}{r} = \alpha \ln r + C_1$$

Applying the Boundary Conditions

$p = p_o$ when $r = r_o$

$p = p_1$ when $r = r_1$

Give finally

$$p = \frac{p_o}{r_o} r - \frac{p_o r_1 - p_1 r_o}{r_1 r_o \ln \frac{r_1}{r_o}} r \ln \frac{r}{r_o}$$

(ii) $$\mu \frac{d^2 v}{dx^2} = \alpha$$

$$\frac{dv}{dx} = \frac{\alpha}{\mu} x + C_2$$

$$v = \frac{\alpha}{\mu} \frac{x^2}{2} + C_2 x + C_3$$

(ii Cont.)

Applying the boundry conditions

$$v = 0 \quad \text{When } x = \frac{h}{2}$$

$$\frac{\partial v}{\partial x} = 0 \quad \text{When } x = 0$$

Give finally

$$v = \frac{p_o r_1 - p_1 r_o}{2\mu\, r_1 r_o \ln \frac{r_1}{r_o}} \left(\frac{h^2}{4} - x^2\right)$$

Now the rate of flow is

$$R = \int_{-\frac{h}{2}}^{+\frac{h}{2}} v\, dA = 2\int_{o}^{\frac{h}{2}} v\, dA$$

$$= 2\int_{o}^{\frac{h}{2}} v\, 2\pi \frac{(r_o + r_1)}{2}\, dx$$

$$= 2\pi(r_o + r_1) \frac{(p_o r_1 - p_1 r_o)}{2\mu\, r_1 r_o \ln \frac{r_1}{r_o}} \int_{o}^{\frac{h}{2}} \left(\frac{h^2}{4} - x^2\right) dx$$

$$R = \pi (r_o + r_1) \frac{p_o r_1 - p_1 r_o}{12\mu\, r_1 r_o \ln \frac{r_1}{r_o}}\, h^3$$

g) EQUATIONS FOR VISCOUS FLOW BETWEEN SAMPLES

1. $\nabla p = \mu \nabla^2 q$
2. $\nabla \cdot q = 0$

} General eq's. of viscous flow for an incompressible fluid. (Neglects inertial forces and assumes constant viscosity.)

No.1 Becomes in cylindrical coordinates

$$\frac{\partial p}{\partial r} = \mu \left(\frac{\partial^2 q_r}{\partial r^2} + \frac{1}{r}\frac{\partial q_r}{\partial r} + \frac{1}{r^2}\frac{\partial q_r}{\partial \theta} + \frac{\partial^2 q_r}{\partial z^2}\right)$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \mu \left(\frac{\partial^2 q_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial q_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial^2 q_\theta}{\partial z^2}\right)$$

$$\frac{\partial p}{\partial z} = \mu \left(\frac{\partial^2 q_z}{\partial r^2} + \frac{1}{r}\frac{\partial q_z}{\partial r} + \frac{1}{r^2}\frac{\partial q_z}{\partial \theta} + \frac{\partial^2 q_z}{\partial z^2}\right)$$

From the symmetry of the problem

$$\frac{\partial q_r}{\partial \theta} = 0 \quad \frac{\partial q_\theta}{\partial \theta} = 0 \quad \frac{\partial p}{\partial \theta} = 0 \quad \text{and } q_z = 0 \text{ (assumed)}$$

Hence the equations become

3. $$\frac{\partial p}{\partial r} = \mu \left(\frac{\partial^2 q_r}{\partial r^2} + \frac{1}{r}\frac{\partial q_r}{\partial r} + \frac{\partial^2 q_r}{\partial z^2}\right)$$

4. $$0 = \frac{\partial^2 q_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial q_\theta}{\partial r} + \frac{\partial^2 q_\theta}{\partial z^2}$$

5. $$\frac{\partial p}{\partial z} = 0$$

Solution of Eq. 3

Using $\nabla \cdot \bar{q} = 0$ and the symmetry properties.

(3) becomes: $$\frac{\partial p}{\partial r} = \mu \left[\frac{q_r}{r^2} + \frac{\partial^2 q_r}{\partial z^2}\right] \quad 3a$$

Since $\frac{1}{r} \frac{\partial}{\partial r}(r\, q_r) = 0$ (By $\nabla \cdot q = 0$)

$$r\, q_r = f\ (z)$$

$$q_r = \frac{f(z)}{r}$$

(3a) becomes $$\frac{\partial p}{\partial r} = \mu \left[\frac{f(z)}{r^3} + \frac{1}{r}\frac{d^2 f(z)}{dz^2}\right]$$

This may be integrated immediately i.e.

$$\int_{p_o}^{p_1} \partial p = -\frac{\mu}{2} f(z) \left[\frac{1}{r^2}\right]_{r_o}^{r_1} + \mu \frac{d^2 f}{dz^2}\left[\ln r\right]_{r_o}^{r_1}$$

or $$-\Delta p = \frac{\mu}{2}\left[\frac{r_1^2 - r_o^2}{r_1^2\ r_o^2}\right] f(z) + \mu \ln \frac{r_1}{r_o} \quad \frac{d^2 f(z)}{dz^2}$$

where $\Delta p = p_o - p_1$

(3b) $$\text{or} - \Delta p = K^2 f(z) + L^2 \frac{d^2 f(z)}{dz^2}$$

Where $$K^2 = \frac{\mu}{2}\left[\frac{r_1^2 - r_o^2}{r_1^2\ r_o^2}\right]$$

$$L^2 = \mu \ln \frac{r_1}{r_o}$$

Eq. 3b is a non homogeneous 2nd order linear equation with constant coefficients

The reduced equation is

$$\frac{d^2 f}{dz^2} + \frac{K^2}{L^2} f = 0$$

The solution of which is

$$f = A \begin{matrix}\sin \\ \cos\end{matrix} \frac{K}{L} z$$

A particular solution of 3b is $f = -\frac{\Delta p}{K^2}$

The complete solution of 3b is

$$f = A \begin{matrix} \sin \\ \cos \end{matrix} \frac{K}{L} z - \frac{\Delta p}{K^2}$$

$$\text{(3c)} \quad \text{or } q_r = \frac{1}{r} A \begin{matrix} \sin \\ \cos \end{matrix} \frac{K}{L} z - \frac{\Delta p}{K^2}$$

Placing the origin midway between the test surfaces gives the following boundary conditions

$$\text{when} \quad z = 0 \quad \frac{\partial q_r}{\partial z} = 0$$

$$z = \frac{h}{2} \quad q_r = 0$$

Using these to determine A in (3c) and selecting the cos. solution gives

$$\text{(3d)} \quad q_r = \frac{\Delta p}{rK^2} \left[\frac{\cos \frac{K}{L} z}{\cos \frac{Kh}{L2}} - 1 \right]$$

The rate of flow is

$$Q = 2\int_0^{\frac{h}{2}} q . 2\pi r \, dz$$

Using (3d) and integrating gives

$$Q = \frac{4\pi \Delta p}{K^2} \left[\frac{L}{K} \tan \frac{Kh}{2L} - \frac{h}{2} \right]$$

Expanding the tan $\frac{Kh}{2L}$ into its series gives

$$\text{(3e)} \quad Q = \frac{4\pi \Delta p \, h^3}{24 \, L^2} \left[1 + \frac{2}{5} \left(\frac{K}{L}\right)^2 \left(\frac{h}{2}\right)^2 + \frac{17}{105} \left(\frac{K}{L}\right)^4 \left(\frac{h}{2}\right)^4 + \text{---} \right]$$

Putting in the values of K and L
and dropping negligible terms of the series gives

$$Q = 4.96\,\frac{\Delta p\, h^3}{\mu} \quad (in^3/sec)$$

$$K^2 = \frac{\mu}{2}\left(\frac{r_1^2 - r_o^2}{r_1^2\, r_o^2}\right) \qquad r_1 = \frac{5}{4}\text{ in} \qquad r_o = \frac{9}{8}\text{ in}$$

$$= .075\,\mu$$

$$L^2 = \mu \ln \frac{r_1}{r_o} = .1055\,\mu$$

SOLUTION OF EQ. 4

The variables may be separated by assuming

$$q_\theta = R\,(r)\; Z\,(z)$$

Substituting this into 4 gives

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = -\frac{Z''}{Z} = -\gamma^2$$ where γ is a constant

This gives two ordinary equations

(4a) $r^2R'' + rR' + \gamma^2 r^2 R = 0$

Eq. (4a) is Bessel's Eq. of order zero and Bessel Functions of order zero are a solution i.e.

$$R = J_o\,(\gamma r)$$

(4b) $Z'' - \gamma^2 Z = 0$

This eq. is easily solved by assuming $Z = e^{\gamma z}$ giving

$$Z = A \begin{matrix}\sinh\\ \cosh\end{matrix}\ \gamma z$$

Choosing the origin on the stationary sample gives the Boundary Conditions

1. When $z = 0$ $\qquad q_\theta = 0$

2. When $z = h$ $\qquad q_\theta = r\omega$

3. When $r = r_1$ $\qquad q_\theta = \dfrac{\omega r_1}{h}\, z$

A particular solution satisfying B.C. 1 is

$$q_\theta = A J_0(\gamma r) \sinh \gamma z + \frac{\omega r_1}{h} z$$

B.C. 3 is satisfied if $J_0(\gamma r_1) = 0$ i.e. if the γr_1's are the zeros of $J_0(\gamma r_1)$

This gives a set of γ's say γ_j where j is 1,2,3 ----

B.C. 2 will be satisfied by taking a sum and determining the coefficients i.e.

$$(4c)\quad q_\theta = \sum_{j=1}^{\infty} A_j J_0(\gamma_j r) \sinh \gamma_j z + \frac{\omega r_1}{h} z$$

when $z = h$ $q_\theta = f(r)$ where $f(r) = \begin{cases} 0 \text{ for } 0 < r < r_0 \\ \omega r \text{ for } r_0 < r < r_1 \end{cases}$

$$\text{thus}\quad f(r) = \sum_{j=1}^{\infty} (A_j \sinh \gamma_j h) J_0(\gamma_j r) + \omega r_1$$

$$\text{or}\quad F(r) = f(r) - \omega r_1 = \sum_{j=1}^{\infty} (A_j \sinh \gamma_j h) J_0(\gamma_j r)$$

The A_j's are determined by

$$(4d)\quad A_j \sinh \gamma_j h = \frac{2}{r_1^2 \left[J_1(\gamma_j r_1)\right]^2} \int_0^{r_1} r J_0(\gamma_j r) F(r)\, dr$$

The final solution is Eq. (4c) where A_j's are determined by (4d).

EXPRESSION FOR TORQUE ON SAMPLES

The shearing stress σ is given approximately by

$$(5)\quad \sigma = \mu \frac{\partial q_\theta}{\partial z}$$

$$(5a)\quad = \mu \left[\sum_{j=1}^{\infty} A_j \, J_0(\gamma_j r) \, \gamma_j \cos h(\gamma_j z) + \frac{\omega r_1}{h} \right]$$

The torque on the stationary sample is given by

$$(6)\quad T = \int_{r_o}^{r_1} 2\pi r \, dr \,.\, \sigma \,.\, r$$

$$= 2\pi\mu \sum_{j=1}^{\infty} A_j \gamma_j \int_{r_o}^{r_1} r^2 J_o(\gamma_j r) \, dr$$

$$+ \frac{2\pi\mu\omega r_1}{3h} \left[r_1^{\,3} - r_o^{\,3} \right]$$

which results from substituting (5a) into (6) and integrating.

Eq. (6) will give T in terms of μ, ω, h after A_j and $\int r^2 J_0(\gamma_j r)\, dr$ have been found. They may be found in a straight forward way by computing their values from the series which they involve. However this is extremely tedious.

h) EQUATIONS OF FLOW FOR VARIABLE VISCOSITY

The stress system produced by pressure and viscous forces may be represented by (52)

$$(\tau) = -p\, I + 2\mu\, \mathrm{Def}\, \bar{q}$$

Where

$$\mathrm{Def}\,\bar{q} = \begin{pmatrix} \frac{2}{3} u_x - \frac{1}{3} v_y - \frac{1}{3} w_z & \frac{1}{2}(u_y + v_x) & \frac{1}{2}(u_z + w_x) \\ \frac{1}{2}(v_x + u_y) & \frac{2}{3} v_y - \frac{1}{3} u_x - \frac{1}{3} w_z & \frac{1}{2}(v_z + w_y) \\ \frac{1}{2}(u_z + w_x) & \frac{1}{2}(v_z + w_y) & \frac{2}{3} w_z - \frac{1}{3} u_x - \frac{1}{3} v_y \end{pmatrix}$$

Computing the x-Component of Force for an element of volume and setting it equal to the inertia force

$$\rho \frac{Du}{Dt} = - p_x + 2 \frac{\partial}{\partial x} \left[\mu u_x - \frac{\mu}{3} (u_x + v_y + w_z)\right] + \frac{\partial}{\partial y} \left[\mu (v_x + u_y)\right] + \frac{\partial}{\partial z} \left[\mu(u_z + w_x)\right]$$

$$= -p_x + 2 \frac{\partial}{\partial x}(\mu u_x) + \frac{\partial}{\partial y}\left[\mu(v_x+u_y)\right] + \frac{\partial}{\partial z}\left[\mu(u_z+w_x)\right]$$

Since $\nabla \cdot \bar{q} = 0$.

This becomes

$$\rho \frac{Du}{Dt} = -p_x + 2 \frac{\partial \mu}{\partial x} u_x + 2 \frac{\partial \mu}{\partial y} u_y + 2 \frac{\partial \mu}{\partial z} u_z + 2 \mu \nabla^2 u + (\nabla \times (\mu \bar{\zeta}))_{x\text{-comp.}}$$

Adding the y and z components gives

$$\rho \frac{D\bar{q}}{Dt} = -\nabla p + 2 (\nabla\mu \cdot \nabla)\bar{q} + 2 \mu \nabla^2 \bar{q} + \nabla \times (\mu \nabla \times \bar{q})$$

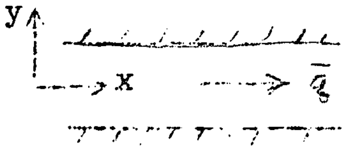
Neglecting Inertia Forces

$$\nabla p = 2\,(\nabla\mu\cdot\nabla)\bar{q} + 2\mu\nabla^2\,\bar{q} + \nabla\times(\mu\nabla\times\bar{q})$$

$$\nabla p = 2\,(\nabla\mu\cdot\nabla)\bar{q} + \mu\nabla^2\,\bar{q} - (\nabla\times\bar{q})\times\nabla\mu$$

See (53)

Consider the flow in one direction between two flat plates as shown



Where $\bar{q}$ is a function of y only and μ is a function of x and y

The above eq. becomes

1. $$\frac{\partial p}{\partial x} = 2\left(\frac{\partial\mu}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial\mu}{\partial y}\frac{\partial u}{\partial y}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial\mu}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

2. $$\frac{\partial p}{\partial y} = 2\left(\frac{\partial\mu}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial\mu}{\partial y}\frac{\partial v}{\partial y}\right) + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\partial\mu}{\partial x}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

Since $v = w = 0$, $\frac{\partial u}{\partial x} = 0$ and changing notation $u \rightarrow q$

1a $$\frac{\partial p}{\partial x} = \frac{\partial u}{\partial y}\frac{\partial q}{\partial y} + \mu\frac{\partial^2 q}{\partial y^2} = \frac{\partial}{\partial y}(\mu q')$$ where $q' = \frac{dq}{dy}$

2a $$\frac{\partial p}{\partial y} = \frac{\partial u}{\partial x}\frac{\partial q}{\partial y} = \frac{\partial}{\partial x}(\mu q')$$

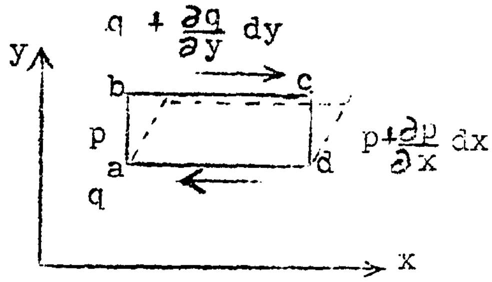
Thus the eq's for a one directional flow are

1a $$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}(\mu q')$$

2a $$\frac{\partial p}{\partial y} = \frac{\partial}{\partial x}(\mu q')$$

i) HEAT EQUATION

Consider an element of volume abcd moving with the fluid.



First computing the net work done on dV in time dt.

$\mu q'$ is the stress.

$$dW = -(\mu q')q\,dx\,dt + \left[\left(q + \frac{\partial q}{\partial y}dy\right)\left(\mu q' + \frac{\partial}{\partial y}(\mu q')\,dy\right)\right]dx\,dt$$

$$+\,pq\,dy\,dt - \left(p + \frac{\partial p}{\partial x}dx\right)q\,dy\,dt$$

$$= \left(\mu q'^2 dy + q\frac{\partial}{\partial y}(\mu q')\,dy + q'\frac{\partial}{\partial y}(\mu q')\,dy^2\right)dx\,dt$$

$$-\frac{\partial p}{\partial x}\,q\,dx\,dy\,dt$$

$$= \mu q'^2 dx\,dy\,dt \qquad \text{Since } \frac{\partial p}{\partial x} = \frac{\partial(\mu q')}{\partial y} \text{ by 1a}$$

Now

- Heat Out + Heat In + Heat Generated = Mass x Sp. Ht. x Temp. Rise

$$-k\frac{\partial \theta}{\partial y}dx\,dt + k\left[\frac{\partial \theta}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\right)dy\right]dx\,dt$$

$$+\frac{\mu q'^2}{J}dx\,dy\,dt = \rho\,c\,\frac{\partial \theta}{\partial x}\,q\,dx\,dy\,dt$$

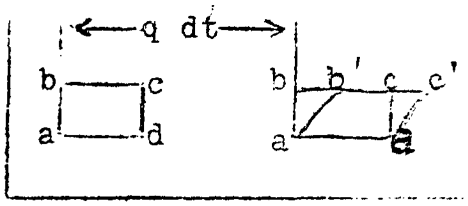
Neglecting Conduction in Direction of Flow

Hence

$$③\quad k\frac{\partial^2\theta}{\partial y^2} + \frac{\mu q'^2}{J} = \rho\, c\, q\frac{\partial\theta}{\partial x}$$

Note:

The heat carried into dV by virtue of the flow lead to terms of higher order thus



Heat carried into abcd by abb' $= \rho c\frac{1}{2}\, dy\left(\frac{\partial q}{\partial y}\, dy\right)\theta\, dt$

Heat carried out by dcc' $= \rho\, c\,\frac{1}{2}\, dy^2\left(\frac{\partial q}{\partial y} + \frac{\partial\left(\frac{\partial q}{\partial y}\right)}{\partial x}\, dx\right)\left(\theta + \frac{\partial\theta}{\partial x}dx\right)dt$

Net heat in $= -\rho c\,\frac{dy^2}{2}\left(\theta\frac{\partial q'}{\partial x}\, dx + q'\frac{\partial\theta}{\partial x}\, dx + \frac{\partial\theta}{\partial x}\frac{\partial q'}{\partial x}dx^2\right) dt$

$=$ Terms of Higher Order

Also if a coor. system is attached to abcd then the transformation eq's are

$$x = qt + x'$$

$$y = y'$$

and the $\nabla^2\theta$ has the same form in either set.

If we assume μ is a linear function of θ

$$\mu = b - a\theta$$

3 becomes

$$3a\qquad \frac{\partial^2\mu}{\partial y^2} - \frac{a}{kJ}\mu q'^2 - \frac{\rho c}{k}\, q\frac{\partial\mu}{\partial x} = 0$$

This is a special case of eq. on P 514 Ref. 53.

j) SOLUTION OF EQUATIONS OF FLOW AND HEAT EQUATION

Eliminating p from 1a and 2a gives

$$4. \qquad \frac{\partial^2(\mu q')}{\partial y^2} = \frac{\partial^2(\mu q')}{\partial x^2}$$

The two eq's. to be solved are

$$4. \qquad \frac{\partial^2(\mu q')}{\partial y^2} = \frac{\partial^2(\mu q')}{\partial x^2}$$

$$3a \quad \frac{\partial^2 \mu}{\partial y^2} - \frac{a}{kJ}\mu q'^2 - \frac{\rho c}{k} q \frac{\partial \mu}{\partial x} = 0$$

1st Method

The solution of 4 is $\mu q' = f_1(x + y) + f_2(x - y)$

Substituting this into 3a gives

$$\frac{1}{q'} f'' + \left[\mp\left(\frac{1}{q'}\right)' \oplus \frac{q''}{q'^2} - \lambda\frac{q}{q'}\right] f' - \left[\left(\frac{q''}{q'^2}\right)' + K q'\right] f = 0$$

Where $\lambda = \frac{\rho c}{k}$, $K = \frac{a}{kJ}$, $f' \equiv \frac{\partial f}{\partial(x \pm y)} \frac{\partial(x \pm y)}{\partial y}$

And where the L.H. side is the sum of the above coefficients acting on $f_1(x + y)$ and the coefficients with the encircled signs substituted, acting on $f_2(x - y)$.

Thus

$$Af_1'' + (B + C)f_1' + Ef_1 + Af_2'' + (-B+C)f_2' + Ef_2 = 0$$

Where

$A = \frac{1}{q'}$

$B = \left(\frac{1}{q'}\right)' - \frac{q''}{q'^2}$

$C = -\lambda\frac{q}{q'}$

$E = -\left(\frac{q''}{q'^2}\right)' - Kq'$

And

$q' \equiv \frac{dq}{dy}$ etc.

The foregoing equation is 2nd order with constant coefficients if y is held fixed.

$$\left\{AD^2 + (B + C)\ D + E\right\} f_1 = \left\{-AD^2 + (B-C)\ D - F\right\} f_2$$

Where D is the usual differential operator $\frac{d}{du}$

$$f_1 = \left\{-1 + \frac{2BD}{AD^2 + (B + C)\ D + E}\right\} f_2$$

$$\mu q' = f_1 + f_2 = \left\{\frac{2BD}{AD^2 + (B + C)\ D + E}\right\} f_2$$

Whose solution is

5 $\quad \mu q' = h_1 - h_2$

Where

$$h_1 = \frac{2Br_1}{A\ (r_1 - r_2)} \left[a_1(y)\ e^{r_1 x} + \int_0^x e^{r_1(x-t)} f_2(t-y)dt \right]$$

$$h_2 = \frac{2Br_2}{A\ (r_1 - r_2)} \left[a_2(y)\ e^{r_2 x} + \int_0^x e^{r_2(x-t)} f_2(t-y)dt \right]$$

And

$$r = \frac{-(B+C) \pm \sqrt{(B + C)^2 - 4\ AE}}{2A}$$

$a_1(y)$ and $a_2(y)$ are arbitrary functions of y

Now to get information on the undetermined functions in 5

Substituting 5 into 4 and setting x = 0, since this will be an identity gives

6 $$U'' + \frac{B + C}{A} V + \frac{E}{A} U = -\frac{2B(B + C)}{A^2} f_2\ (-y) + \frac{2B}{A} f_2'\ (-y)$$

Where

$$U = \frac{2B}{A\,(r_1 - r_2)} \left(r_1\, a_1(y) - r_2\, a_2(y) \right)$$

$$V = \frac{2B}{A\,(r_1 - r_2)} \left(r_1^2\, a_1(y) - r_2^2\, a_2(y) \right)$$

Similarly substituting 5 into 3a and setting $x = 0$ gives

$$7. \quad U' - V = \frac{2B}{A} f_2\,(-y) - 2\, f_2'\,(-y)$$

Now consider the "Bernouilli" condition at the intake

$$\frac{p}{\rho} + \frac{q^2}{2g} = \text{Constant}$$

Hence

$$\frac{\partial p}{\partial y} = -\frac{\rho}{g}\, qq'$$

But

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial x}(\mu q') \quad \text{By eq's. of flow}$$

$$= V + \frac{2B}{A}\, f_2\,(-y)$$

$$\therefore \quad V = -\frac{\rho}{g}\, qq' - \frac{2B}{A}\, f_2\,(-y)$$

Substituting this into 7 gives

$$U' + \frac{\rho}{g}\, qq' + \frac{2B}{A}\, f_2(-y) = \frac{2B}{A}\, f_2(-y) - 2f_2'\,(-y)$$

$$U = -\frac{\rho}{g}\,\frac{q^2}{2} - 2\, f_2\,(-y) + C_1$$

Putting these expressions for U and V into 6 gives an equation for $q(y)$ (The velocity distribution).

$$8 \quad -\frac{\rho}{g}(qq')' + \frac{\rho}{g}\, qq'(2\sigma + \lambda q) - (\sigma^2 - \sigma' - Kq'^2)\left[\frac{\rho}{g}\,\frac{q^2}{2} - C_1\right]$$

$$= 2\,(\sigma^2 - \sigma' - Kq'^2)\, f_2(-y) - 4\sigma f_2' - 2\, f_2''$$

Where $\sigma = \frac{q''}{q'}$

k) SOLUTION BY SEPARATION OF VARIABLES

$$4)\quad \frac{\partial^2(\mu q')}{\partial x^2} = \frac{\partial^2(\mu q')}{\partial y^2}$$

$$3a)\quad \frac{\partial^2 \mu}{\partial y^2} - K\mu q'^2 - \lambda q \frac{\partial \mu}{\partial x} = 0$$

Assume $\mu = X(x)\,Y(y)$

3a becomes

$$XY'' - Kq'^2 XY - \lambda q Y X' = 0$$

Which gives

$$9)\quad Y'' - (Kq'^2 - \gamma^2 q)\,Y = 0 \quad \text{and}$$

$$\lambda \frac{X'}{X} + \gamma^2 = 0$$

Where γ is the separation constant the X-Eq. can be integrated giving

$$X = Ce^{\frac{-\gamma^2}{\lambda}x}$$

Eq. 4 separates into

$$\frac{(Yq')''}{Yq'} = \frac{X''}{X} = \left(\frac{\gamma^2}{\lambda}\right)^2$$

Which gives

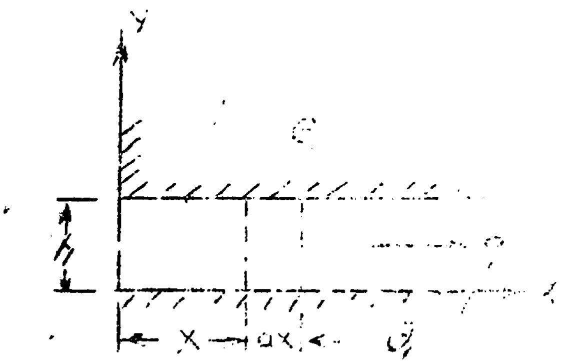
$$10)\quad Y = \frac{Ae^{\delta y} + Be^{-\delta y}}{q'} \quad \text{where } \delta = \frac{\gamma^2}{\lambda}$$

Substituting 10 into 9 gives

$$11)\quad \left\{\frac{\delta^2 - Kq'^2 + \lambda\delta q}{q'} - \left(\frac{q''}{q'^2}\right)' - 2\delta\frac{q''}{q'^2}\right\} Ae^{\delta y}$$

$$+ \left\{\frac{\delta^2 - Kq'^2 + \lambda\delta q}{q'} - \left(\frac{q''}{q'^2}\right)' + 2\delta\frac{q''}{q'^2}\right\} Be^{-\delta y} = 0$$

I.) SIMPLIFIED TREATMENT OF FILM TEMPERATURE USING "NEWTONIAN COOLING"



Consider two plates as shown, the upper plate moving with velocity V in z direction. Oil flows in x direction and the temperature θ varies with x only.

Heat Eq.

$$1)\quad \rho\, c\, Q\, \theta + \frac{1}{J}\, dF \cdot V = \rho\, c\, \left(\theta + \frac{d\theta}{dx}\, dx\right) Q + k(\theta - \theta_1)\, dx$$

Where Q = Rate of Flow
k = Film Coefficient

But

$$dF = \mu(x)\ \frac{V}{h}\, dx \quad \text{(Unit Width)}$$

1) Becomes

$$\frac{d\theta}{dx} + \left(\frac{k}{\rho c Q} + \frac{aV^2}{hJ\rho c Q} \right)\theta = \frac{k\theta_1}{\rho c Q} + \frac{bV^2}{\rho c Q h J}$$

Assuming $\mu = b - a\theta$

The Solution for which, with θ_o Inlet Temp. Is

$$\theta = \frac{B}{A}\left(1 - e^{-Ax}\right) + \theta_o e^{-Ax}$$

Where

$$B = \frac{k\theta hJ + bv^2}{\rho c Q h J}$$

$$A = \frac{-hJk + aV^2}{-\rho c Q h J}$$

Now computing the force per unit length

$$F = \frac{V}{h}\int_o^t \mu(x)\, dx$$ where t is the width of surface

$$\frac{V}{h}\int_o^t \left\{ b - \frac{aB}{A}\left(1 - e^{-Ax}\right) - a\theta_o e^{-Ax} \right\} dx$$

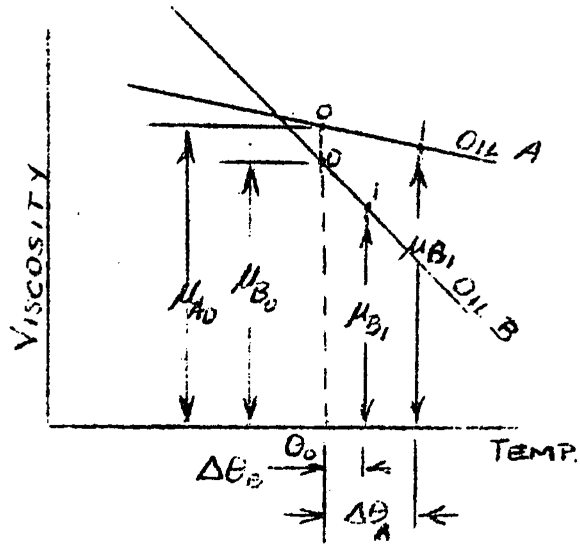
Finally

$$F = \frac{V}{hA}\left[\frac{k\mu_1}{\rho c Q}\ t + (a\theta_o - 1)\left(e^{-At} - 1\right) \right]$$

m) METHOD OF MEASURING FILM TEMPERATURE

Consider two tests run at the same speed and pressure but with two different oils whose viscosity indices differ i.e. oils which have viscosity curves as indicated below. Over a small temperature range the curves may be assumed linear without introducing appreciable error.

Since the torque T is proportional to the viscosity when the speed and film thickness are the same, an ordinate to the viscosity curve is proportional to the torque.



Consider the case where the inlet temperature θ_0 is the same in both tests. If the film temperature were θ_0 in both cases the measured torques would be in the ratio $\frac{\mu_{Ao}}{\mu_{Bo}}$. However, the oil temperature rises as it flows thru the film and the effective temperature increases to the point marked 1 above. Now the ratio of the torques (measured experimentally) will be $\frac{\mu_{A1}}{\mu_{B1}}$.

The temperature rise $\Delta\theta_A$ and $\Delta\theta_B$ are proportional to $\frac{T_A}{Q_A}$ and $\frac{T_B}{Q_B}$ respectively (from part d) where the Q's are the ra tes of flow and may be measured.

It follows that

$$\mu_{A1} = \mu_{Ao} - M_A \Delta\theta_B$$

$$\mu_{B1} = \mu_{Bo} - M_B \Delta\theta_B$$

where M_A and M_B are the slopes of the curves.

Dividing gives

$$1. \quad \frac{\mu_{A1}}{\mu_{B1}} = \frac{\mu_{Ao} - M_A \Delta\theta_A}{\mu_{Bo} - M_B \Delta\theta_B}$$

which is proportional to the torques, and can be measured.

Now

$$\Delta\theta_A \sim \frac{T_A}{Q_A}$$

$$\Delta\theta_B \sim \frac{T_B}{Q_B}$$

Hence $\frac{\Delta\theta_A}{\Delta\theta_B} = \frac{T_A\ Q_B}{T_B\ Q_A}$ which can be measured

Eliminating $\Delta\theta_A$ from 1 and solving for $\Delta\theta_B$ gives

$$\Delta\theta_B = \frac{\mu_{Bo} - \frac{\mu_{B1}}{\mu_{A1}}\mu_{Ao}}{M_B - M_A \frac{Q_B}{Q_A}}$$

In general torque readings in two tests are not made at exactly the same film thickness and inlet temperature. They can be converted to torques that correspond by multiplying by the ratio of film thicknesses or viscosities.

NOMENCLATURE

There has been some duplication in the use of symbols. Their meaning, however, is usually clear from the way they are used. In some cases the symbols have been defined at the place where they are used. In general, the following meanings have been attached to the symbols:

p	=	pressure
μ	=	coefficient of viscosity
$\bar{q}$	=	vector velocity
$q_r q_\theta q_z$	=	velocity components in the r, θ, and z directions
r, θ, z	=	standard cylindrical coordinates
r_o	=	inner radius of sample
r_1	=	outer radius of sample
p_o	=	pressure in inner cavity of samples
p_1	=	pressure outside samples
$\left\{\begin{matrix}Q\\R\end{matrix}\right\}$	=	rate of flow between samples
h	=	film thickness
ω	=	angular velocity of rotating sample
$\left\{\begin{matrix}\tau\\s\end{matrix}\right\}$	=	shearing stress
T	=	$\left\{\begin{matrix}\text{torque}\\\text{temperature}\end{matrix}\right\}$
V	=	linear velocity of sample
°u, v, w	=	x, y, z components of velocity
θ	=	temperature

° When u, v and w carry subscripts x, y or z, this means partial differentiation, e.g. $u_x = \frac{\partial u}{\partial x}$. In some cases primes (') have been used to designate differentiation.

NOMENCLATURE

(Continued)

$\bar{\xi}$ = vorticity vector

ρ = density

k = film coefficient of heat transfer; thermal conductivity

J = mechanical equivalent of heat

c = specific heat

Discussion

In many engineering discussions on Lubrication terms such as "Load Carrying Capacity", "Film Strength", and others are used. The writer feels that a few words about these may clarify them somewhat.

The Load Carrying Capacity of a journal bearing depends, as the load is increased at first, on the ability of the bearing to maintain a hydrodymamic film (where the laws of fluid mechanics hold) and then the ability to maintain thinner films where the laws of fluid machanics give way, and finally, with further increase of load, the Load Carrying Capacity depends on the physical and chemical nature of the molecular films which separate the two parts. Thus there are three regions of film thickness of interest. Let us designate them as follows:

a) Hydrodynamic Film
b) Semi-hydrodynamic Film
c) Molecular Films.

The Hydrodynamic Film essentially has no strength. Its ability to support a load is derived solely from the hydrodynamic pressure. The Load Carrying Capacity here is dependent on the ability of the equipment to maintain a continuous film. In journal bearings the clearance, width, speed, position of oil sources, and viscosity form the elements of a mechanism to produce and maintain a hydrodynamic film. Out of roundness of the journal or bearing, chatter, and other inaccuracies effect the ability of such a mechanism to maintain a film. Thus the machinery for maintaining a film is self-contained in a journal bearing. In contrast to this the ability to maintain a film is separated from other considerations in the Surface Quality Machine. Here the ability to maintain a hydrodynamic film rests in the ability of the super pressure pump to pump. Thus, this one variable, the ability to maintain a film, is divorced from the problem and does not interfere with other variables.

The ability to maintain a film has been studied very extensively. The plot of friction coefficient vs $\frac{\mu N}{P}$ (a dimensionless variable) which is so common, is in part a measure of the ability to maintain all three of the above types of films. The fundamental nature of each is disguised by the others.

The semi-hydrodynamic film may have some strength. For instance, Needs'(6) work indicates an apparent rigidity of a film of lubricant between two flat plates. There is much controversy, however, about the film thickness where hydrodynamic flow ceases.

The molecular films certainly have an inherent strength. These have been studied considerable by Langmuir, Blodgett (38) (39) and others.

The point to be brought out is that the ability to maintain a hydrodynamic film is divorced from the problem in the tests on the Surface Quality Machine, thus leaving the other variables unaffected by any change in ability to maintain a film.

The surface roughness of the mating parts of a bearing influence to some degree the ability to maintain a film. This was demonstrated in the tests on Sample No. 2a where a definite torque was produced by the oil flowing through the surface scratches. This indicates that the scratches have an influence on the direction of oil flow. Consequently, if very fine scratches (say 5 MI scratches) were oriented on the surface of a bearing in such a way as to drive the oil toward the center of the bearing on the loaded side, the ability to maintain a film would be increased. Such an orientation of scratches would resemble a herring-bone gear.

Dimensional inaccuracies are detrimental in all cases in a bearing. They tend to destroy the ability of the bearing to maintain a continuous film. It can be seen from Curve C-67 that the resistance to flow increases very rapidly at the thinner films. The performance of a bearing depends on its ability to keep the oil in the bearing. Consequently, a high resistance to flow is desirable. If the dimensional inaccuracies amount to anything at all continuous films of high resistance cannot be attained.

Surface Roughness has both advantages and disadvantages. One advantage of surface roughness is its aid in controlling the flow of oil in certain directions. That is, if the directional characteristics are such as to produce an apparent resistance to flow toward atmospheric pressure, the surface roughness will have a desirable effect. However, the surface roughness must be such as to allow very close clearances to be attained because of the high resistances associated with

Discussion (Continued)

thin films (see C-67). If the peaks of the surface roughness break through the film the purpose is defeated. Hence it appears that it is desirable to use surface scratches to/help prevent the oil from flowing out, and at the same time, to avoid sharp peaks which project through the film. A surface, super-finished partially, should exhibit good bearing qualities if properly ground beforehand. Another factor to be considered about surface finish is the heat transfer characteristics for such surfaces. The resistance to oil flow in a thin film varies directly with the viscosity and inversely with the cube of the thickness. Accordingly, at very thin films the rate of shear $\left(\frac{V}{h}\right)$ is large and so the rate of heat generation is also large. This tends to make the viscosity lower and hence lowers the resistance to flow and facilitates the "squeezing out" of the oil. A surface with good heat transfer properties is therefore desirable.

It has been shown that more friction is produced with the rough surfaces even though no metallic contact is made. This must be balanced with the above desirable effects. Since the resistance to flow varies inversely as the cube of the film thickness while the heat generated varies inversely as the first power of the film thickness it is advantageous to run at as thin a film as possible without having metallic contact, the gain in resistance more than off-setting the increased heat (especially with a lubricant with high viscosity index). However, there is no advantage in having a high resistance to flow outward if there is no way of continually

Discussion (Continued)

supplying oil in the high pressure region. This is where surface finish is important as an aid in supplying oil to the high pressure area.

In recent years there have been advances made in the design of machanisms to increase the ability of a bearing to maintain a continuous film. The Mitchell Thrust Bearing which uses pivoted pads is an illustration. The Filmatic Bearing of the Cincinnati Milling Machine Co. is also an illustration of the effort being made to augment a plain journal bearing's ability to maintain a continuous film. The test samples used in the Surface Quality Machine constitute a thrust bearing which is capable of carrying extremely heavy loads with the production of a very small amount of heat. The stability of this bearing increases very rapidly as the load is increased. This is shown by Curve C-8. The compensation pressure is proportional to the load and it is evident that at heavy loads the compensator pressure must be increased a large amount to change the film thickness appreciably. Under these loads the spindle is perfectly steady and has coefficients of friction of the order of .0002. Lower coefficients of friction can be easily obtained by making the diameter of the annular land larger, most of the load being carried by the pressure inside the samples and the friction arising in the thin film between the test surfaces. It should be emphasized that here, again, the ability to maintain a film is separated from the mating elements of the bearing.

Discussion (Continued)

From a study of the literature it is apparent that various investigators' results are in disagreement. For instance, Kingsbury's experiment with a tapered plug(1) indicates that Newton's Law for viscous shear holds down to .000025 in. while Hardy (55) puts the figure at .0004 in. It has been shown in this report that surface roughness will increase the stress while the temperature rise decreases it. With these two effects combined it is very easy to be misled by getting results which follow theory down to .000025in. This illustrates the advantage of definitely controlling the film thickness and the film temperature. The measurement of film temperature probably is one of the most elusive variables in the field. By allowing oil to continually flow through the surfaces the film temperature can be controlled, and, as mentioned earlier, can be measured. The method of maintaining a film works very satisfactorily. The film thickness can be set to any desired value within .0015 in. and will remain steady indefinitely. At films of the order of .0003in. a change in film thickness of several millionths of an inch would readily be detected if the spindle moved by that amount. Thus it seems quite feasible to apply this method for controlling films from, say, .000010in. to larger films. This degree of precision is far from impossible.

Summary

The tests on the Surface Quality Machine indicate that:

1. At films of about .00035 in the shearing stress is greater than would be predicted by Fluid Mechanics.

2. For rough finishes the shearing stress is greater than for smooth finishes when the film thickness becomes comparable with the surface roughness.

3. The shearing stress at about .00015 in is independent of the lubricating properties of the oil used, provided no contact occurs.

4. The directional properties of the surface scratches influence the flow of lubricant past them, the influence being greater at the thinner films.

5. The pressure affects the coefficient of friction only as it changes the viscosity of the oil in the range of film thicknesses studied.

6. The resistance to oil flow is less for the smooth surfaces than for the rough surfaces.

Summary (Continued)

7. The resistance to oil flow varies inversely as the cube of the film thickness as predicted by theory.

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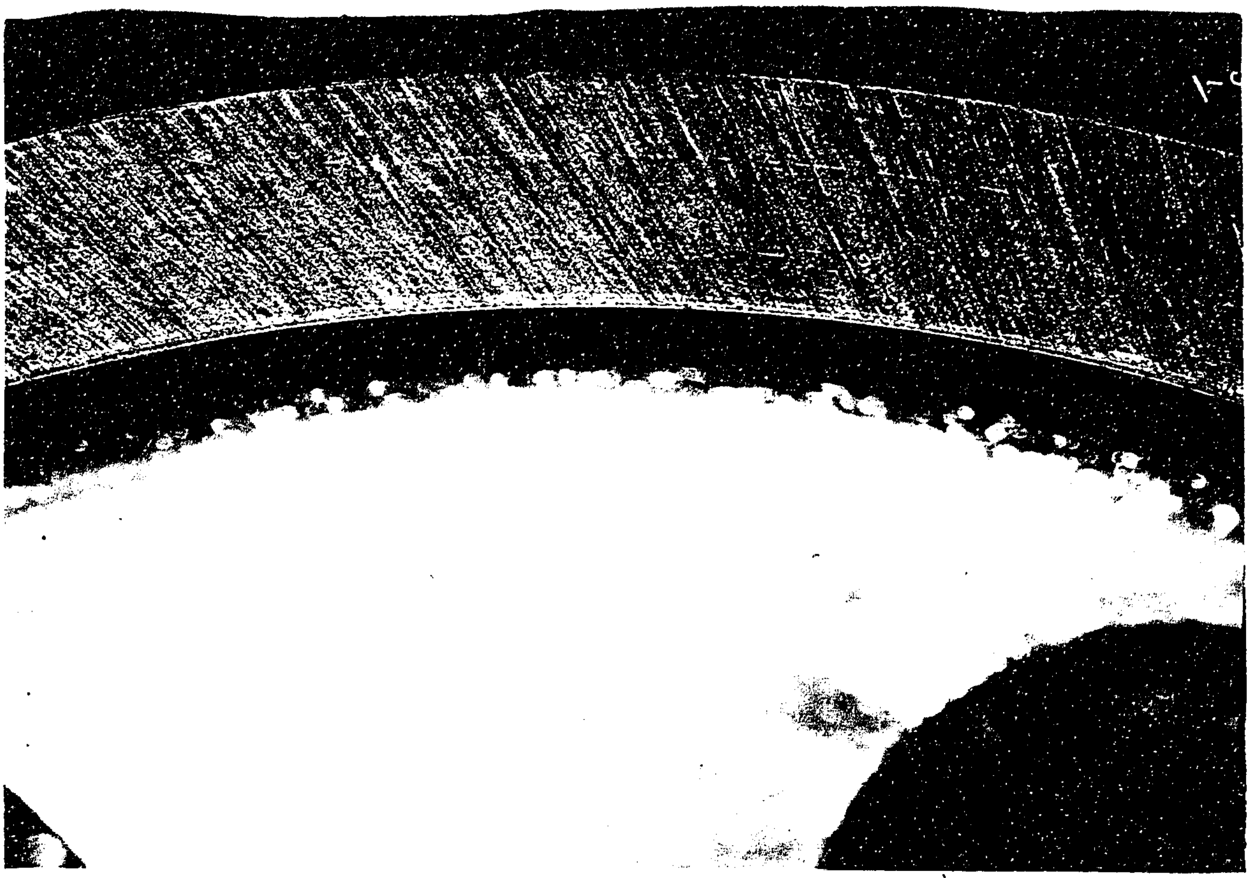
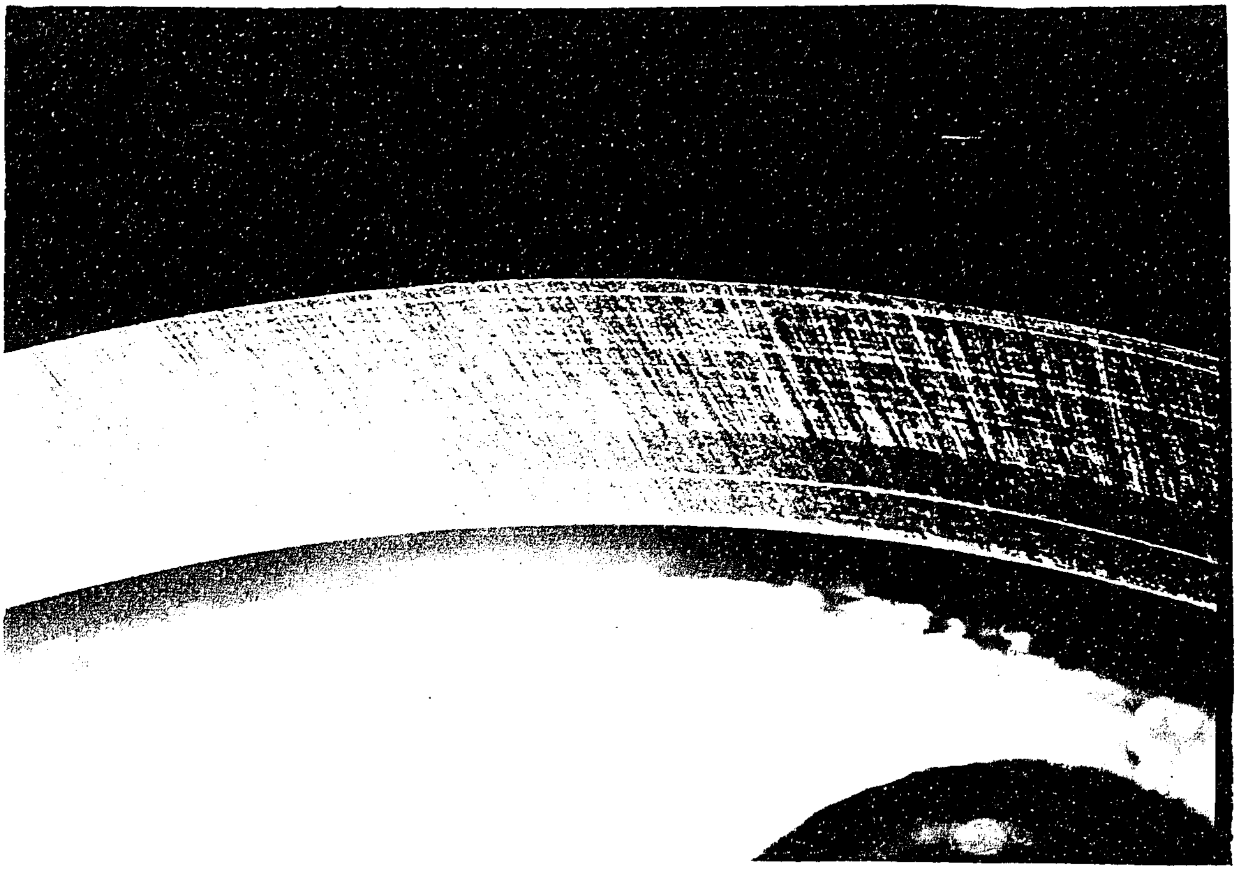


Photomicrograph of Sample No. 2a

Upper Specimen

Profilometer Reading 50-55 MI

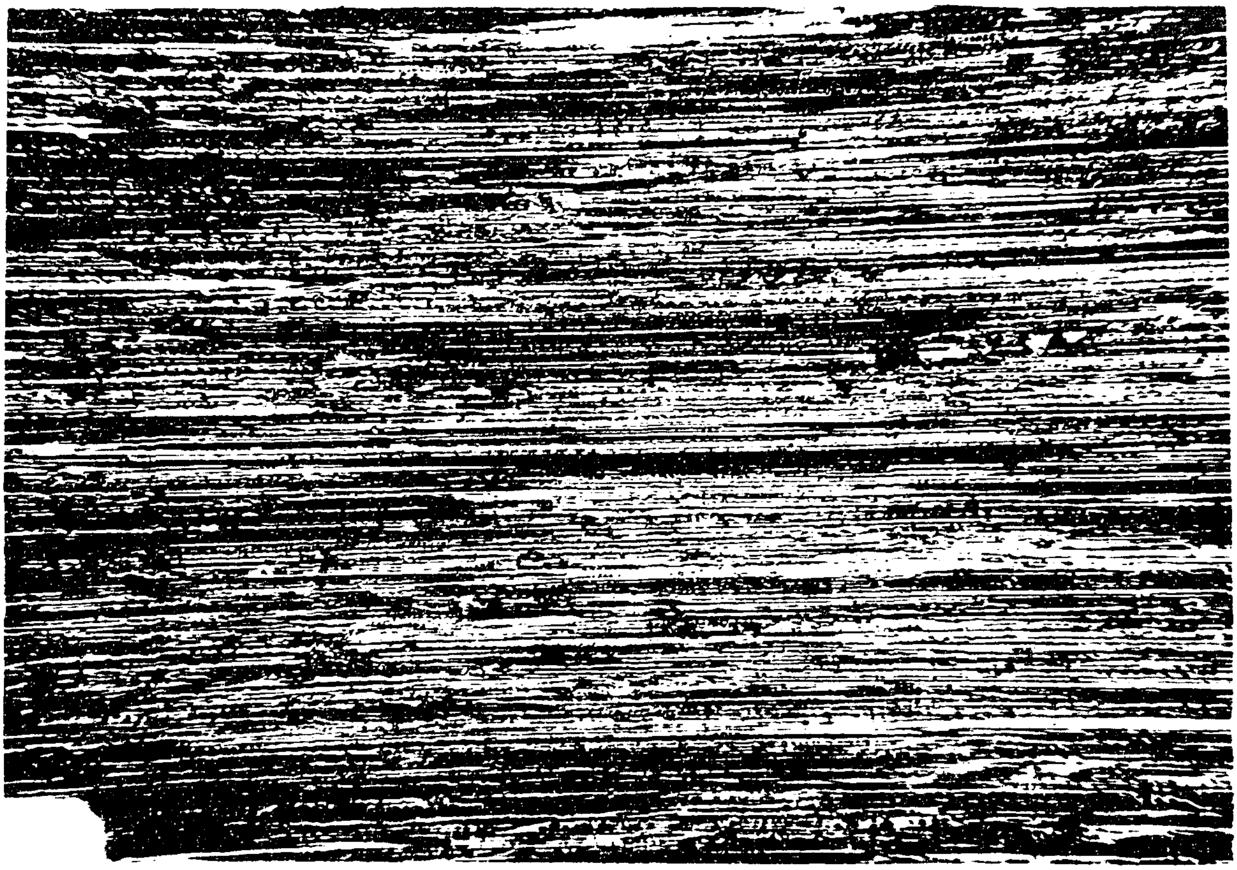
Magnification 10x



Photomicrograph of Sample No. 2a

Lower Specimen

Profilometer Reading	50-55 MI
Magnification	10x

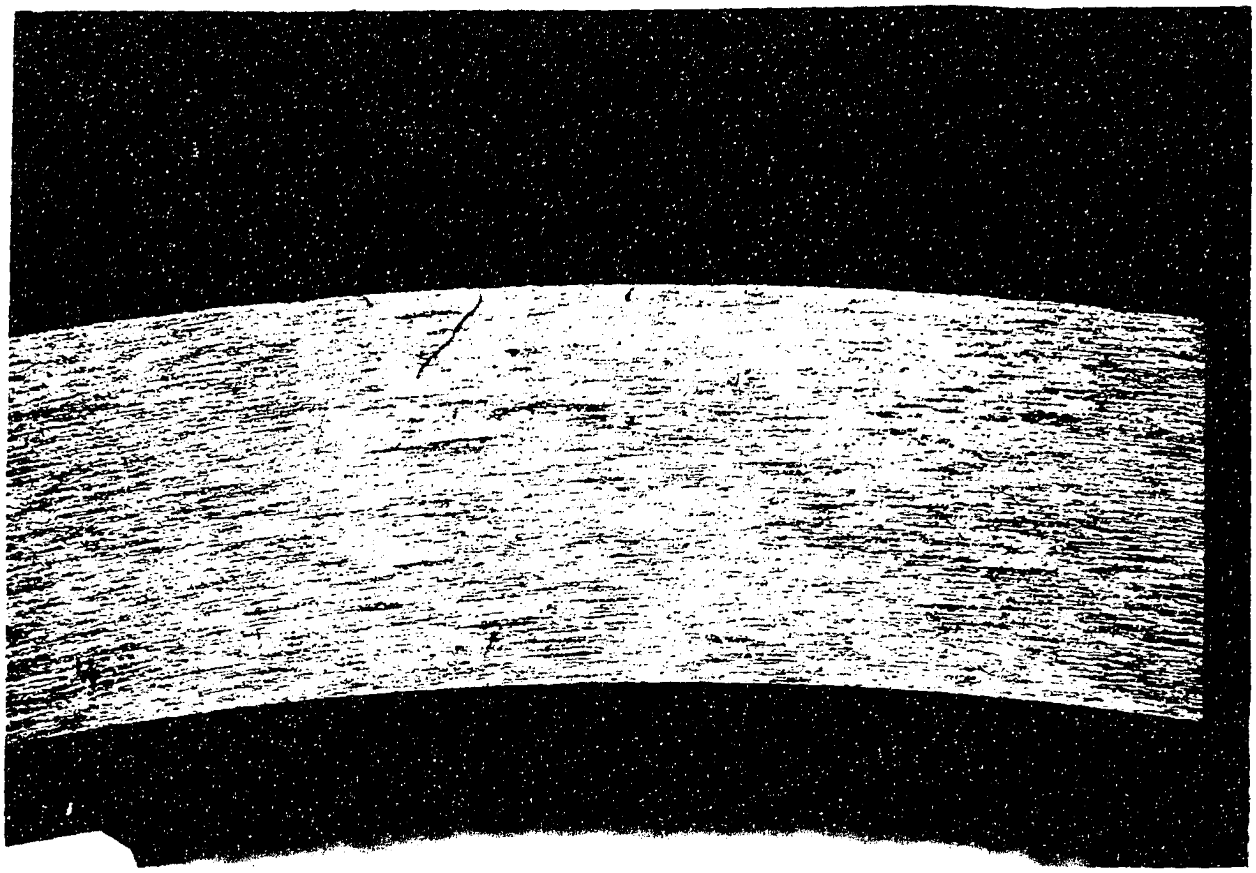


Photomicrograph of Sample No. 3a

Upper Cup

Profilometer Reading

Magnification 250x

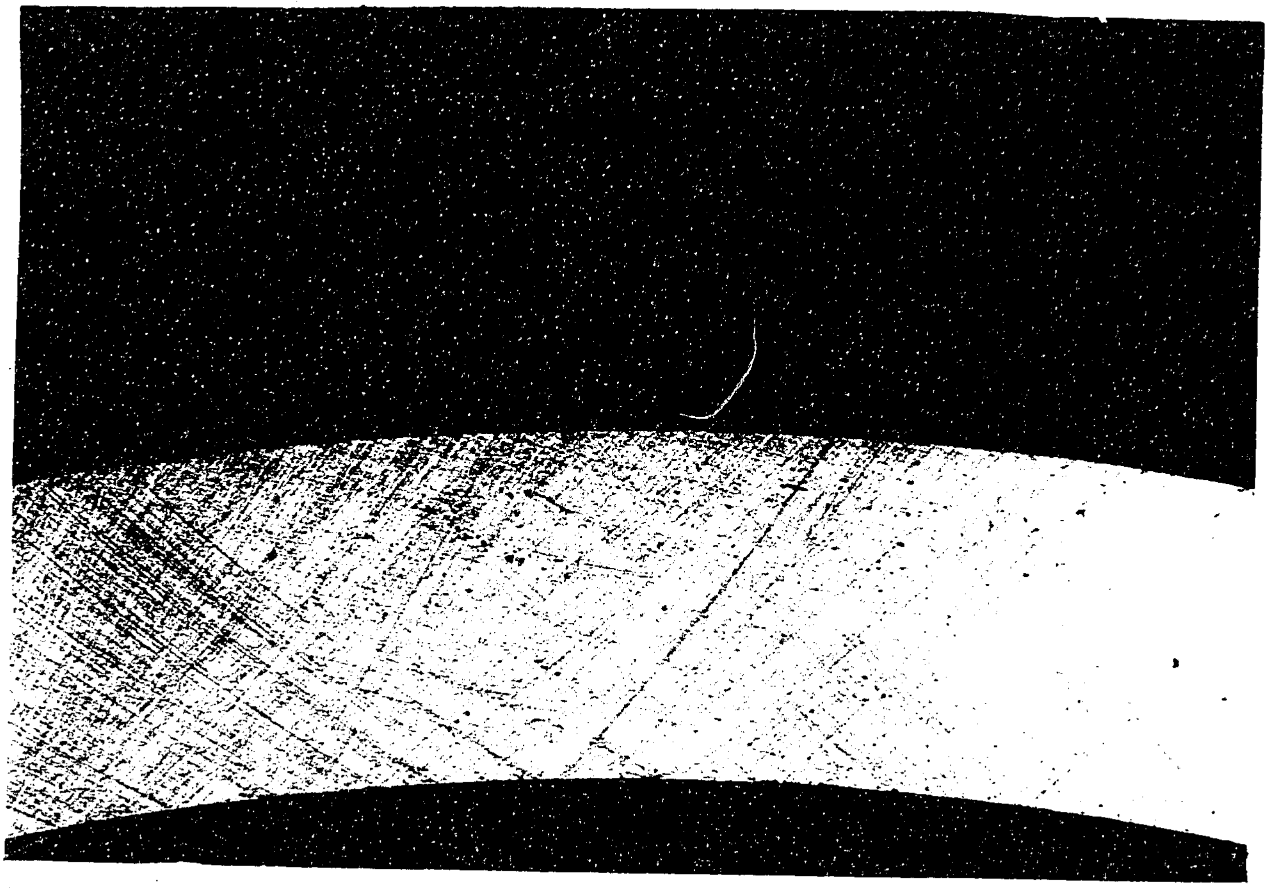


Photomicrograph of Sample No. 3a

Upper Cup

Profilometer Reading

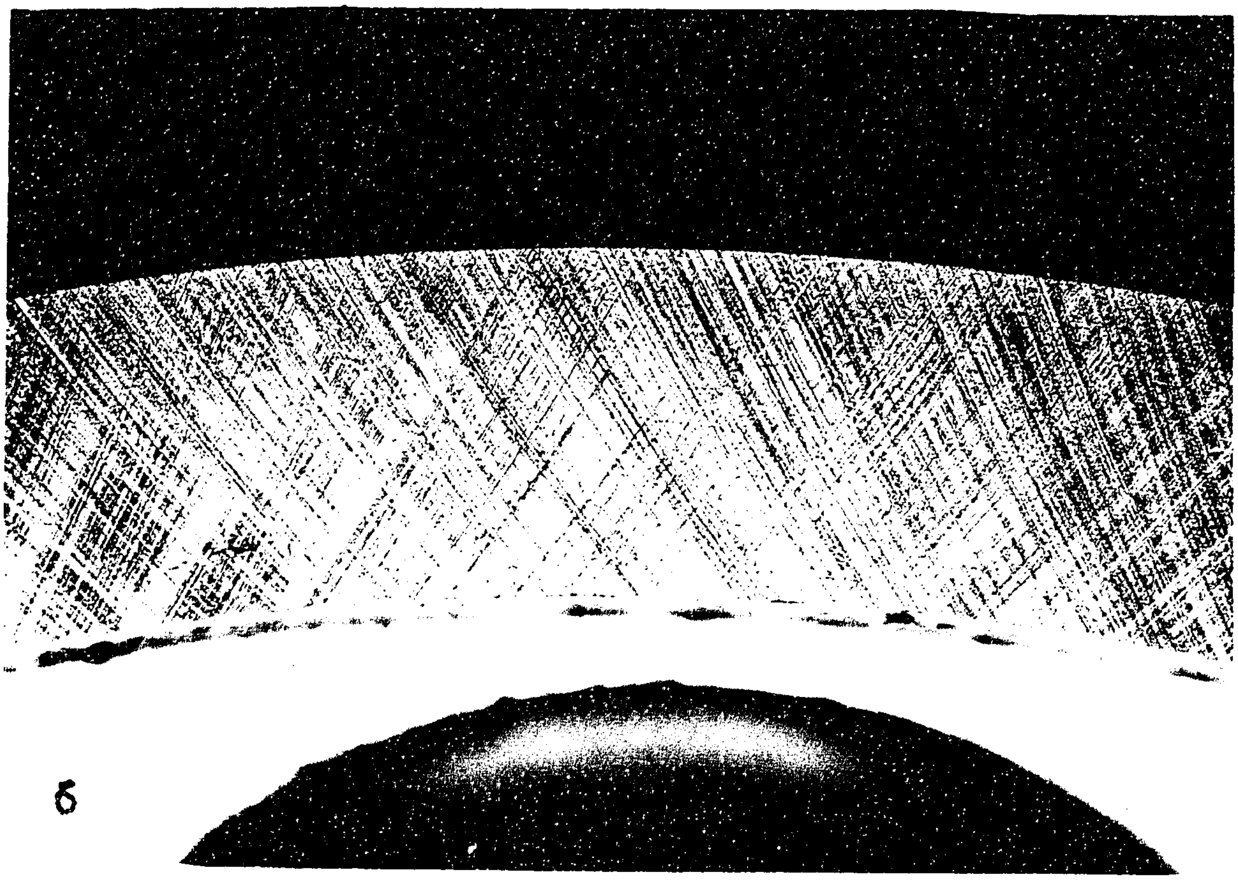
Magnification 17x



Photomicrograph of Sample No. 4b

Upper Specimen

Profilometer Reading	3-5 MI
Magnification	15x

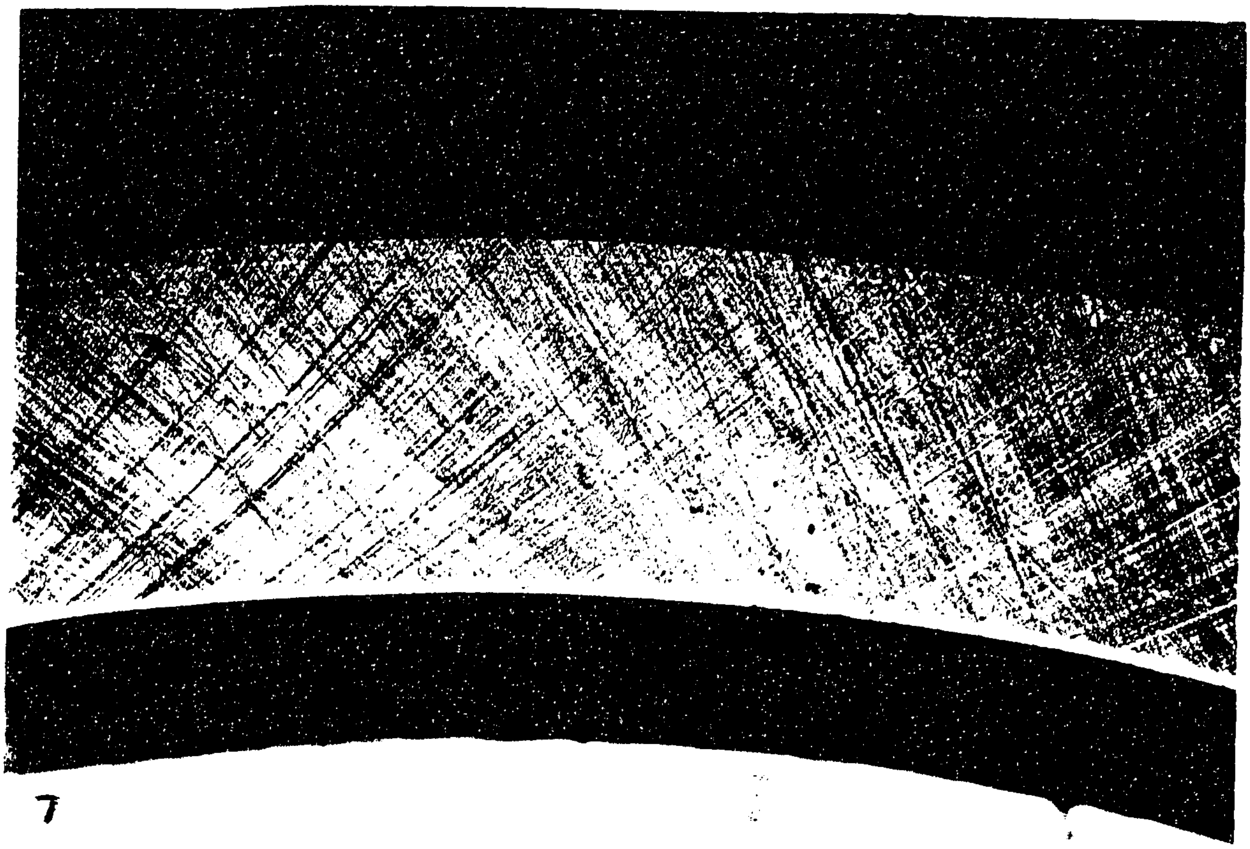


<u>Photomicrograph of Sample No. 5</u>

Upper Specimen

Profilometer Loading 30-60 MI

Magnification 15x



Photomicrograph of Sample No. 2b

Upper Specimen

Profilometer Reading 20-30 MI

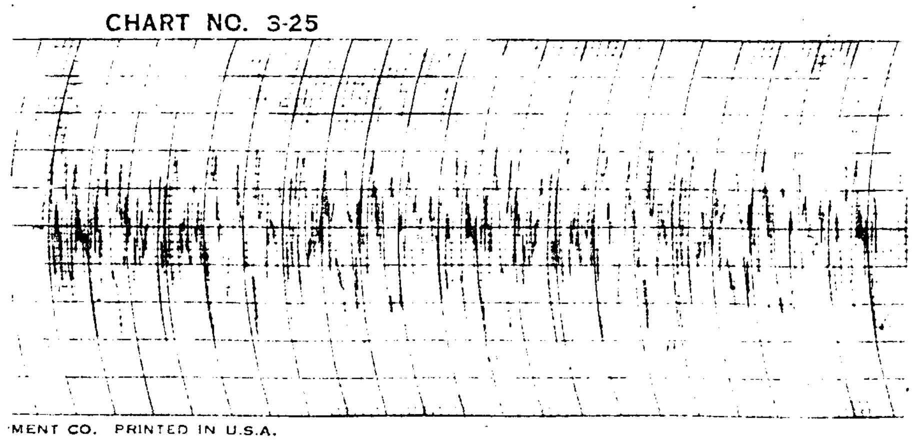
Magnification 15x

Photomicrograph of Sample No. 6

Upper Specimen

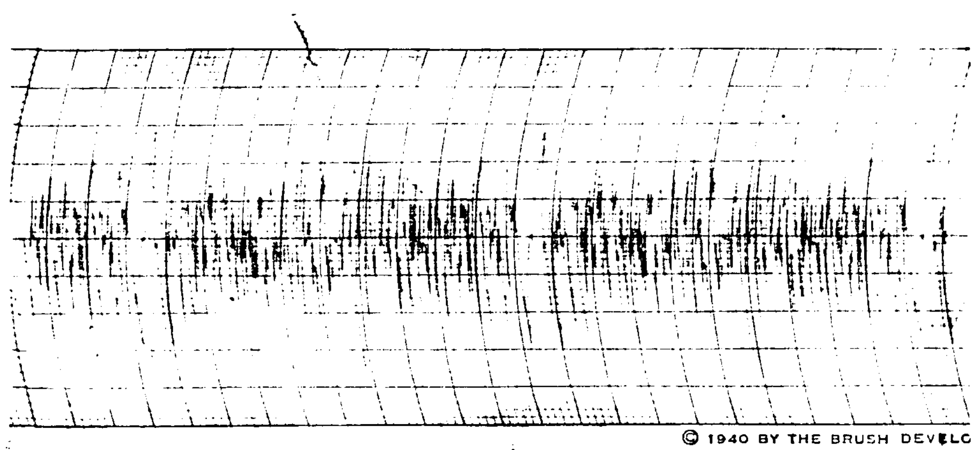
Profilometer Reading 20-30 MI

Magnification 15x



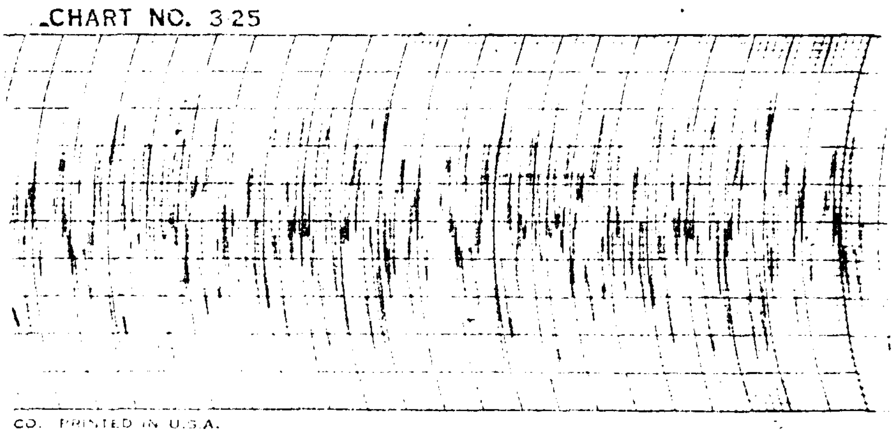
SAMPLE No. 2a (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - After sample was run on 75 - 80



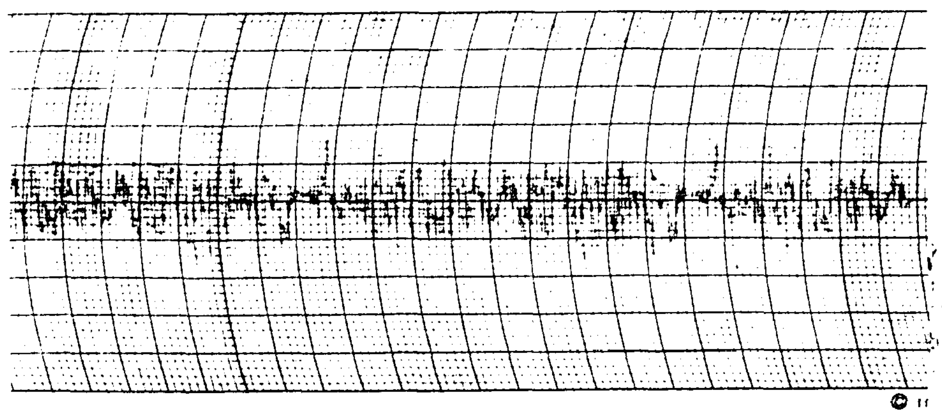
SAMPLE No. 2a (lower cup)

Path of stylus - $\perp$ to scratches
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - After sample was run on testa 75-80



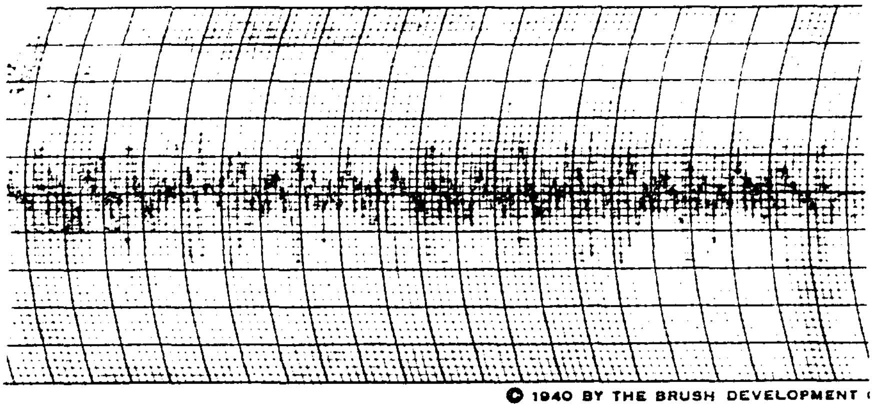
SAMPLE No. 2a (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - After 75-80



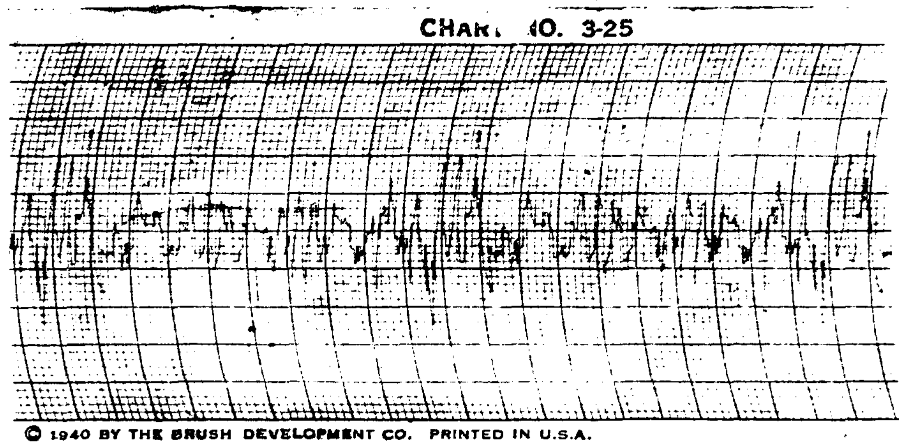
SAMPLE No. 2b (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



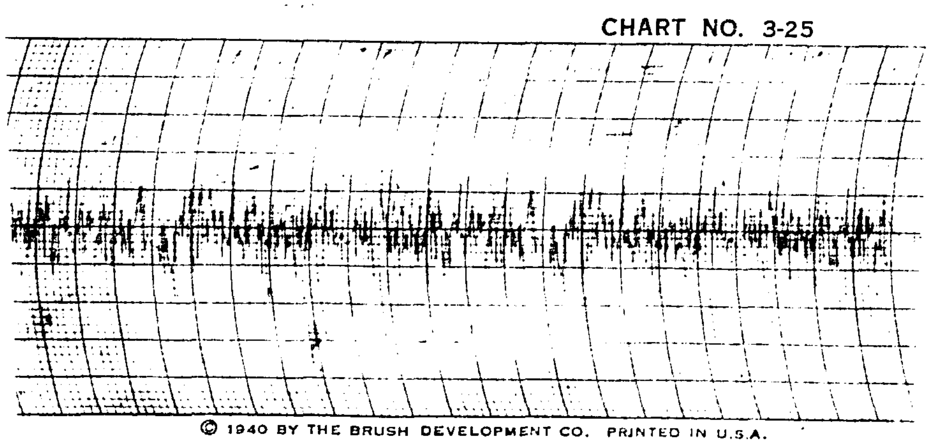
SAMPLE No. 2b (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



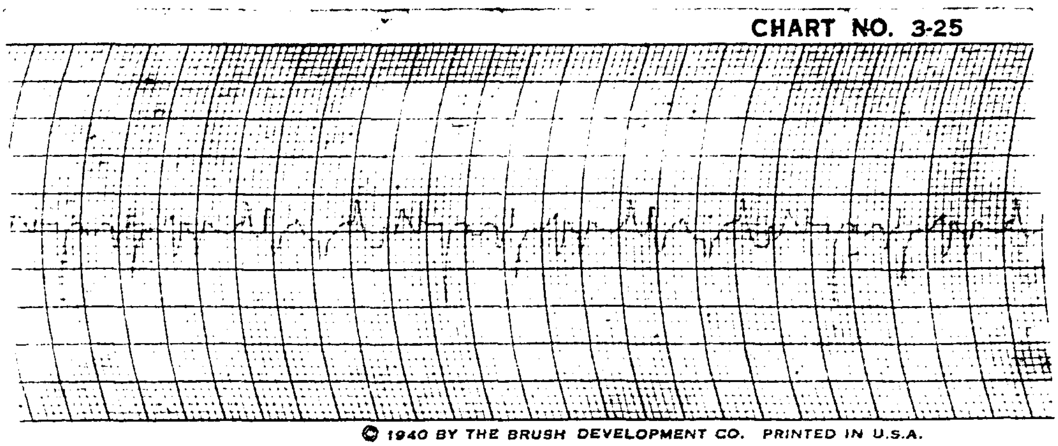
SAMPLE No. 3a (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - Before running



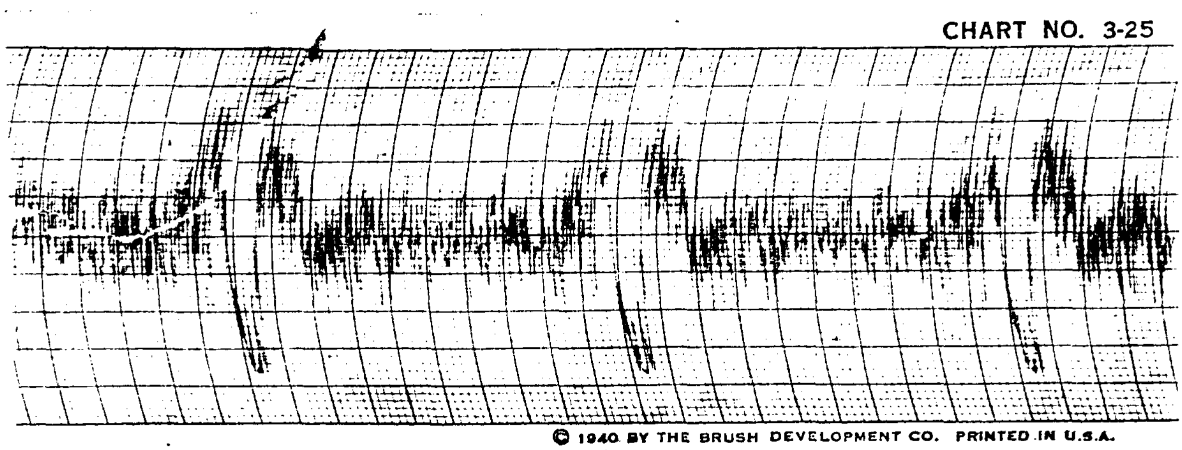
SAMPLE No. 3a (lower cup)

Path of stylus - radial
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - Before running



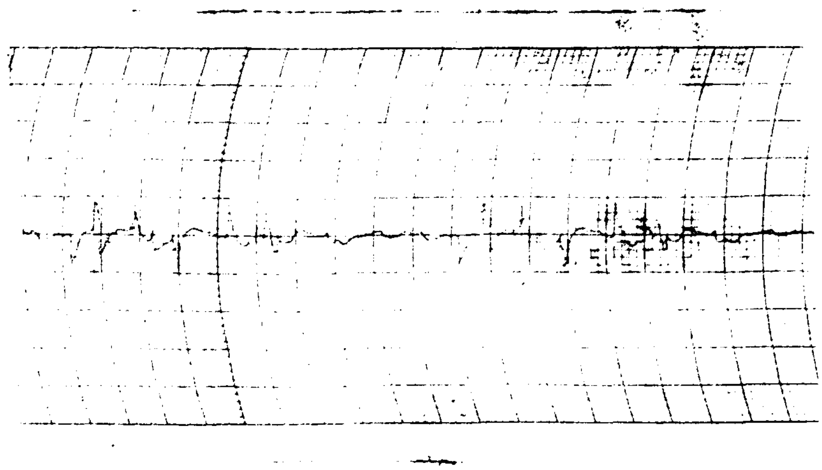
SAMPLE No. 3a (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - Before running



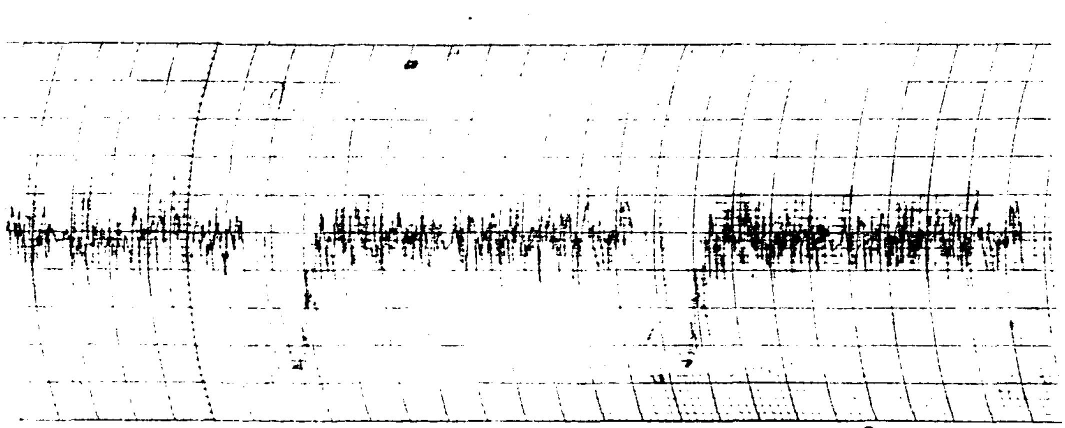
SAMPLE No. 3a (upper cup)

Path of stylus - radial
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - Before running



SAMPLE No. 3b (upper cup)

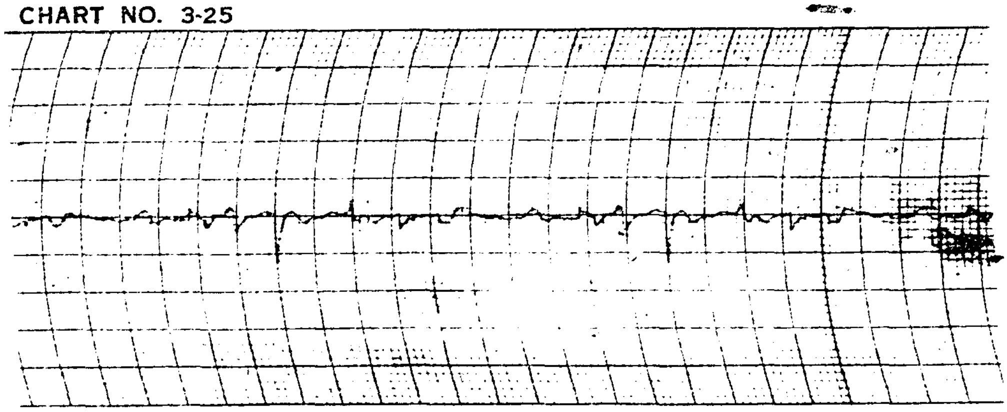
Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



SAMPLE No. 3b (upper cup)

Path of stylus - Radial
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks

CHART NO. 3-25



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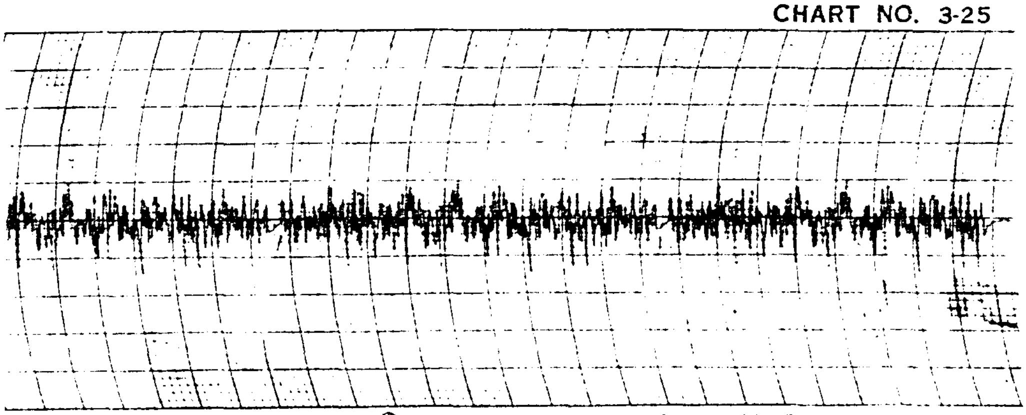
SAMPLE No. 3b (lower cup)

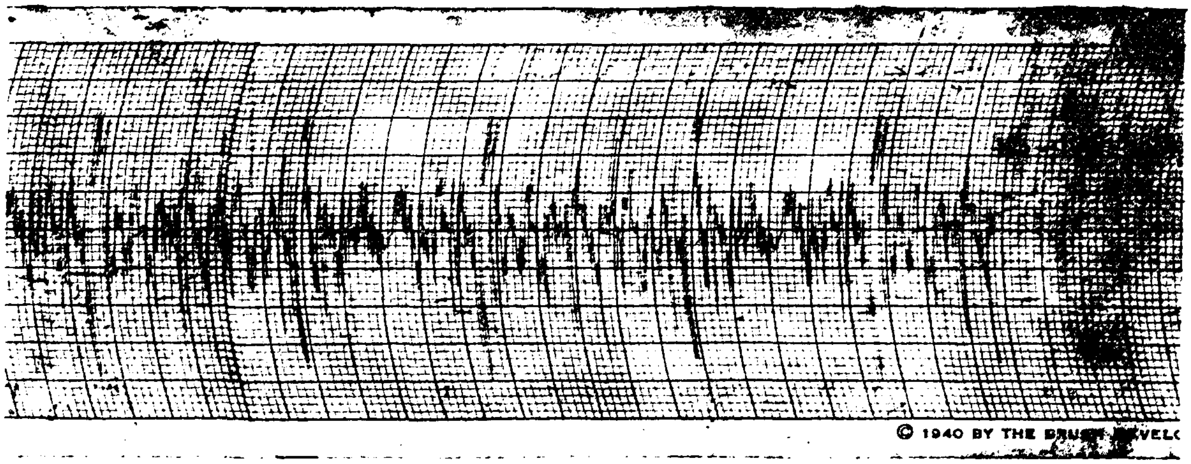
Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks

CHART NO. 3-25



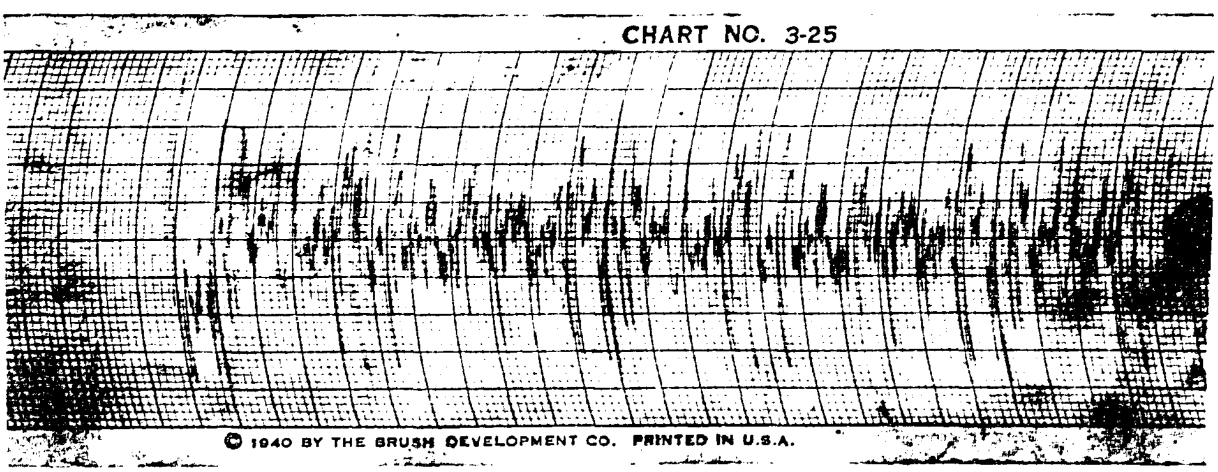
SAMPLE No. 3b (lower cup)

Path of stylus - radial
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



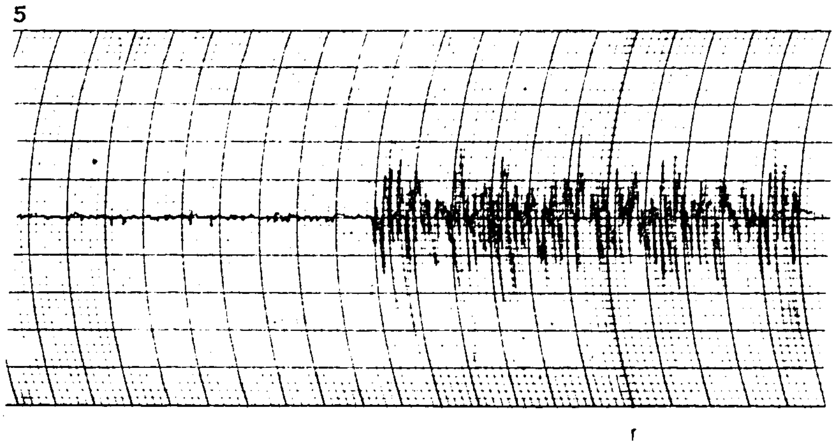
SAMPLE No. 4a (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 1 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



SAMPLE No. 4a (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 1 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



SAMPLE No. 4b (upper cup)

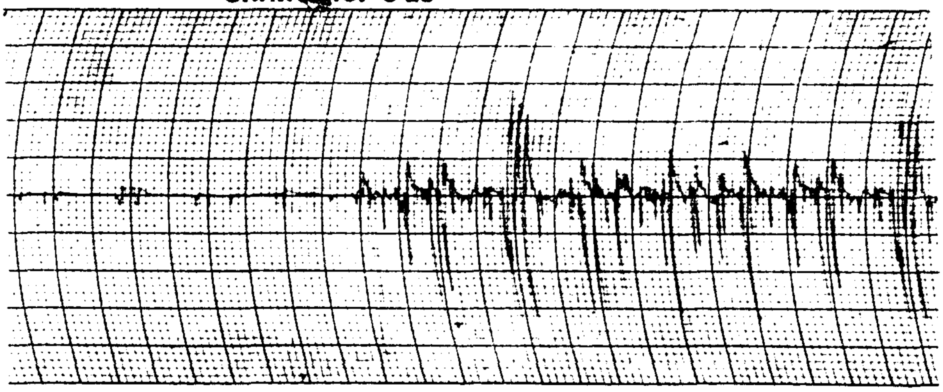
Path of stylus - tangential

Vertical Scale - 1 small division - $\frac{10 \text{ MI}}{1 \text{ MI}}$

Horizontal Scale - 1 small division - .0025 in.

Remarks

CHART NO. 3-25



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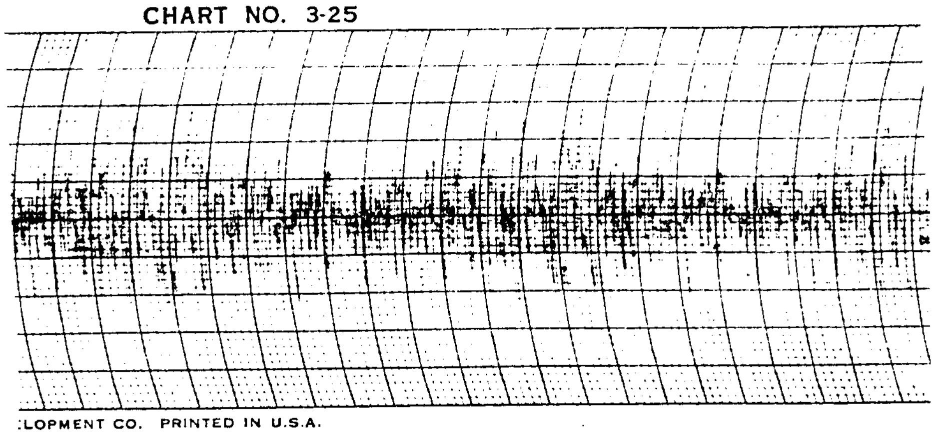
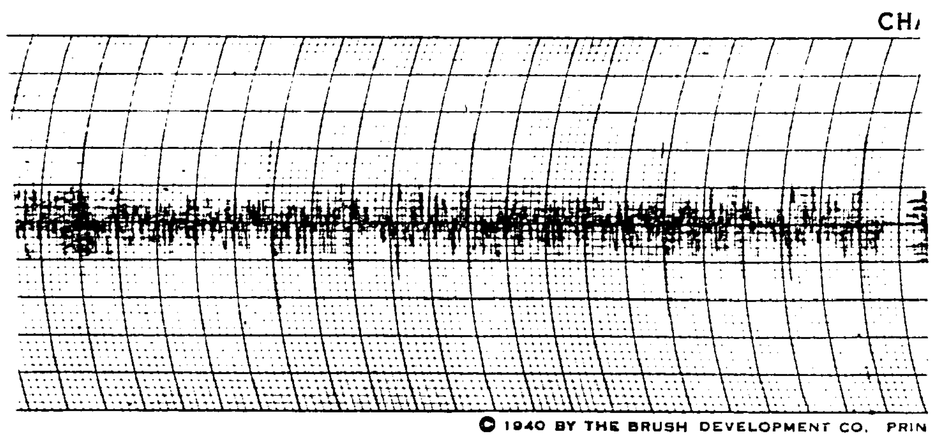
SAMPLE No. 4b (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
1 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



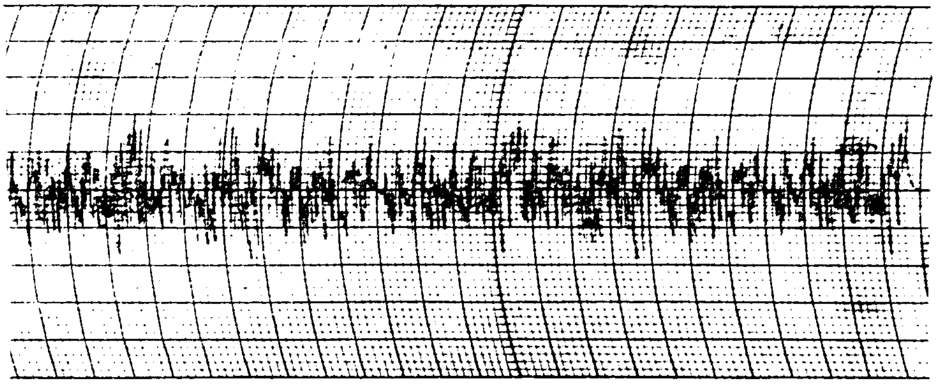
SAMPLE No. 5 (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



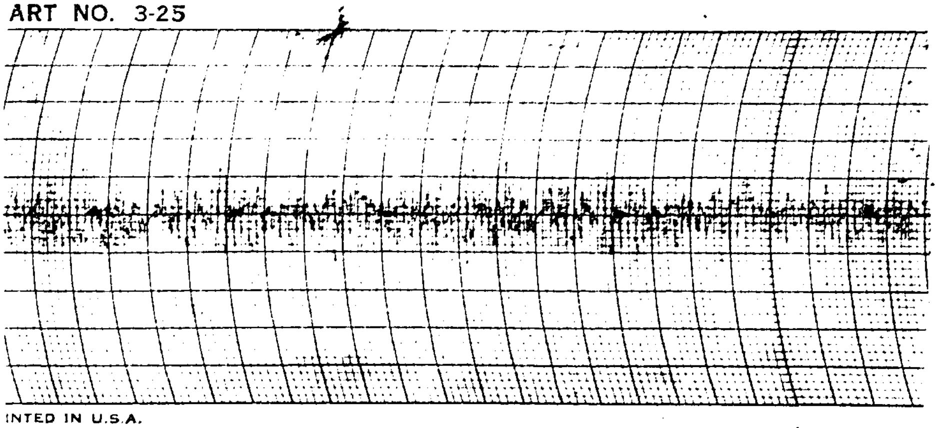
SAMPLE No. 5 (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



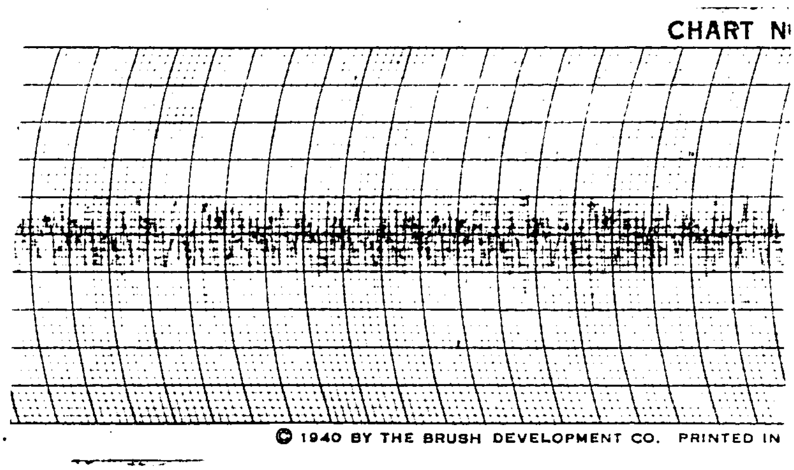
SAMPLE No. 5 (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks - rotated 90°



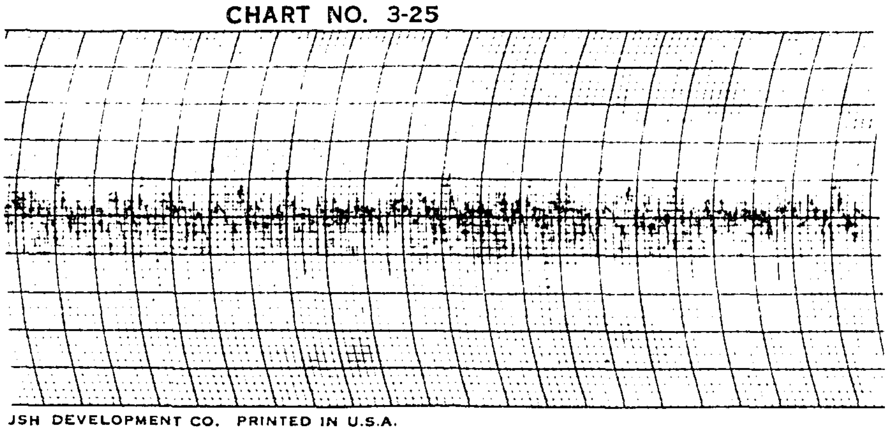
SAMPLE No. 6 (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



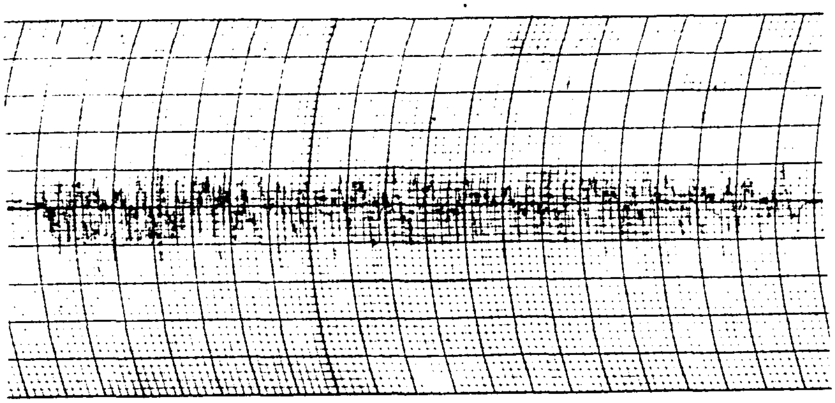
SAMPLE No. 6 (lower cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks



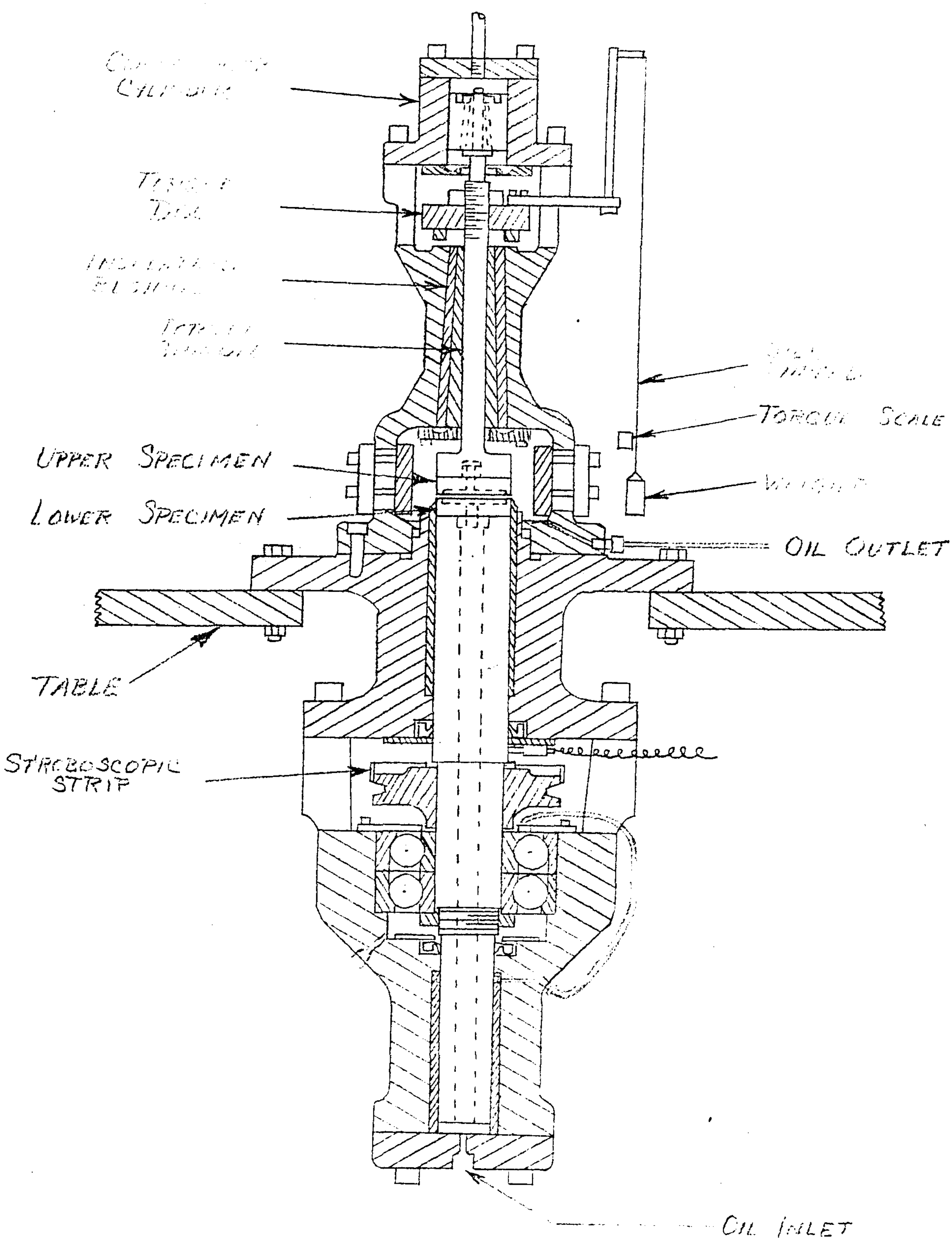
SAMPLE No. 7 (upper cup)

Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks

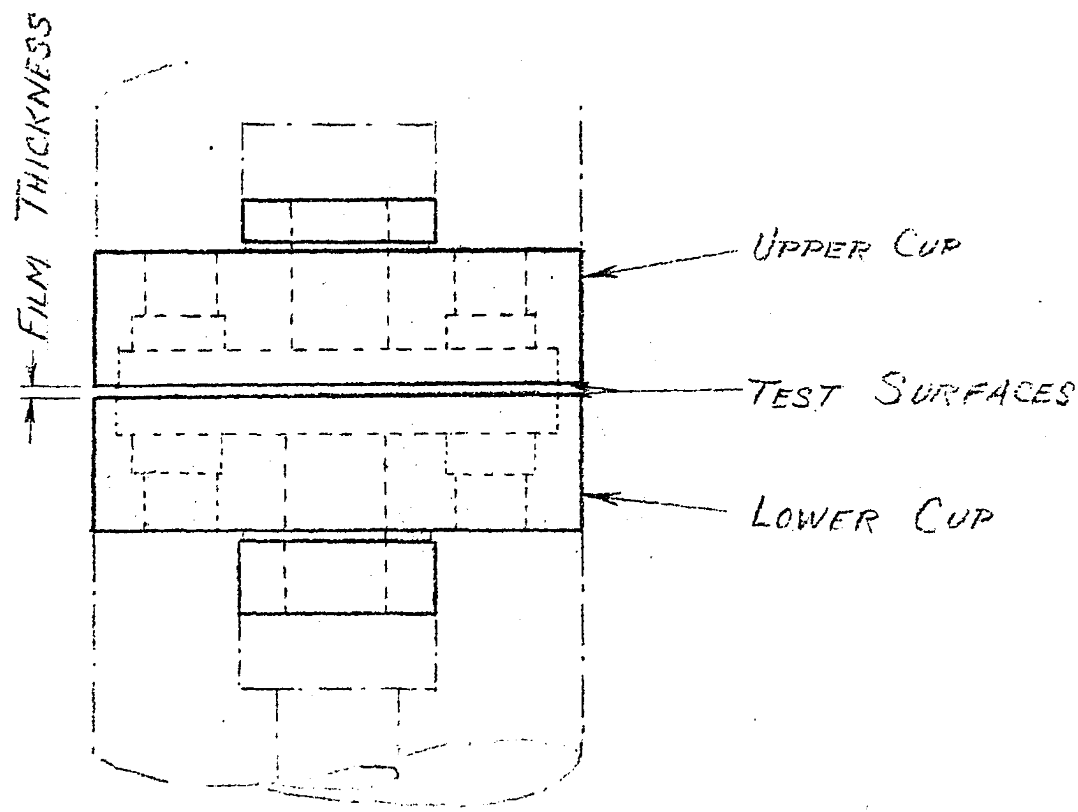


SAMPLE No. 7 (lower cup)

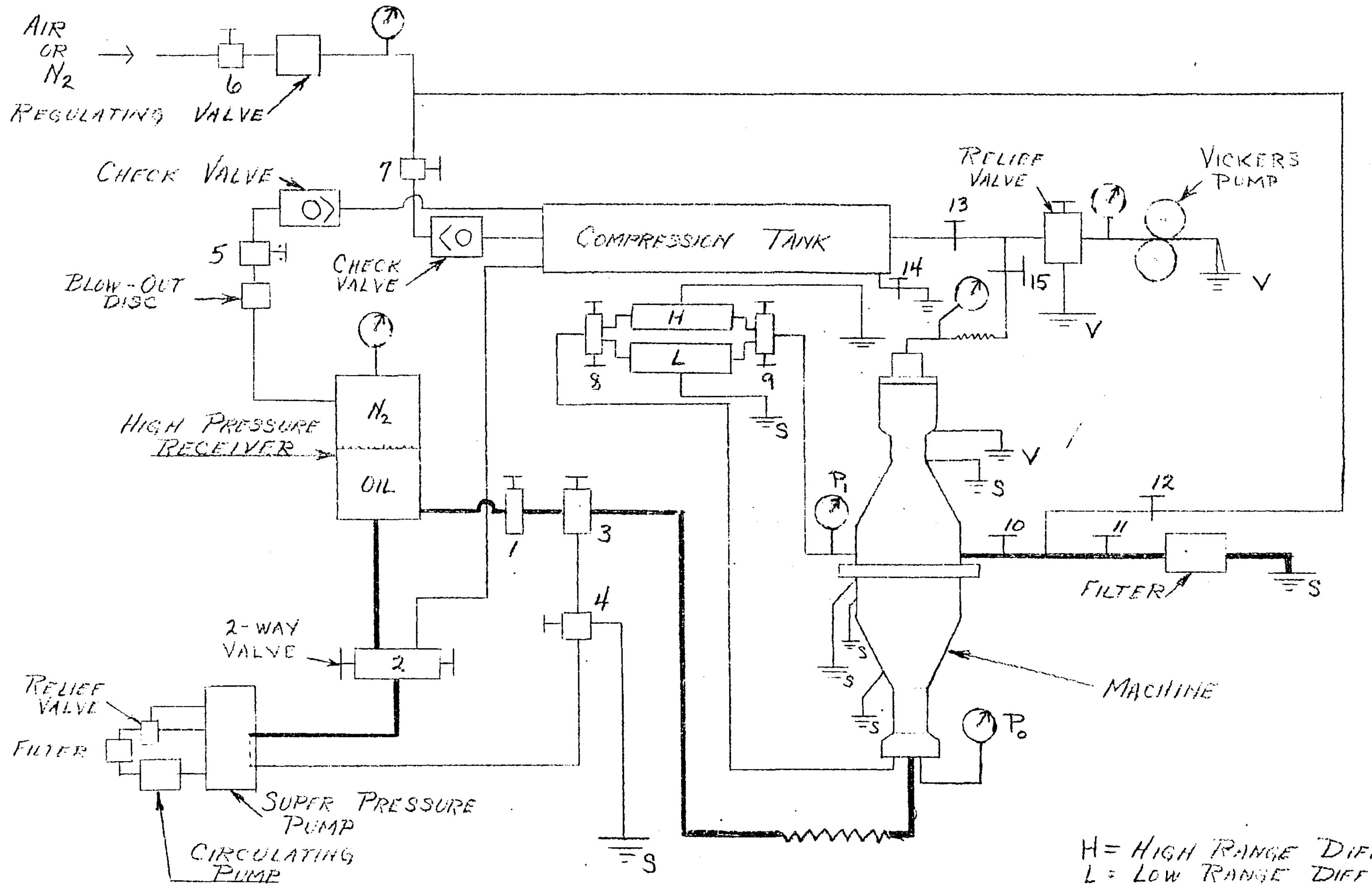
Path of stylus - tangential
Vertical Scale - 1 small division - 10 MI
Horizontal Scale - 1 small division - .0025 in.
Remarks.



SURFACE QUALITY TESTING MACHINE



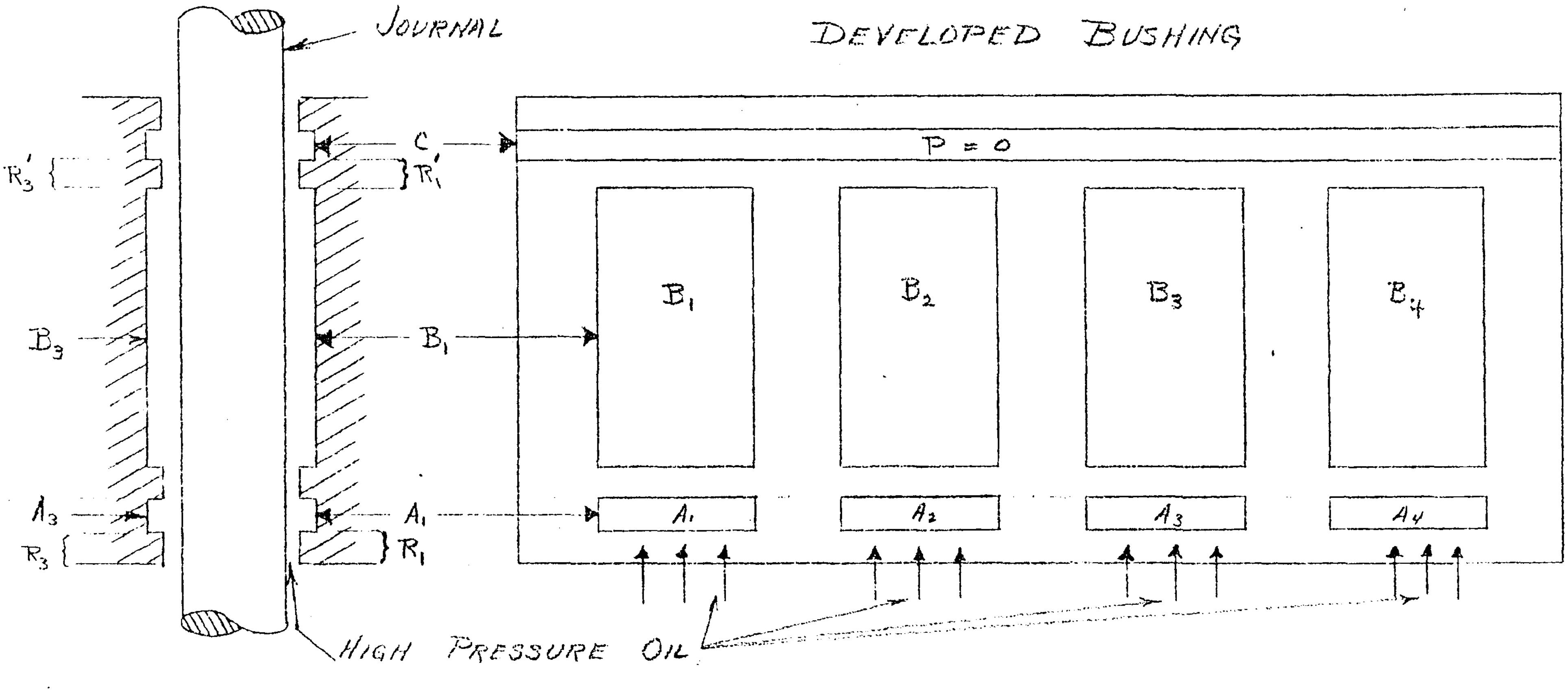
ARRANGEMENT OF TEST SAMPLES



H = HIGH RANGE DIFF'L GAGE
L = LOW RANGE DIFF'L GAGE

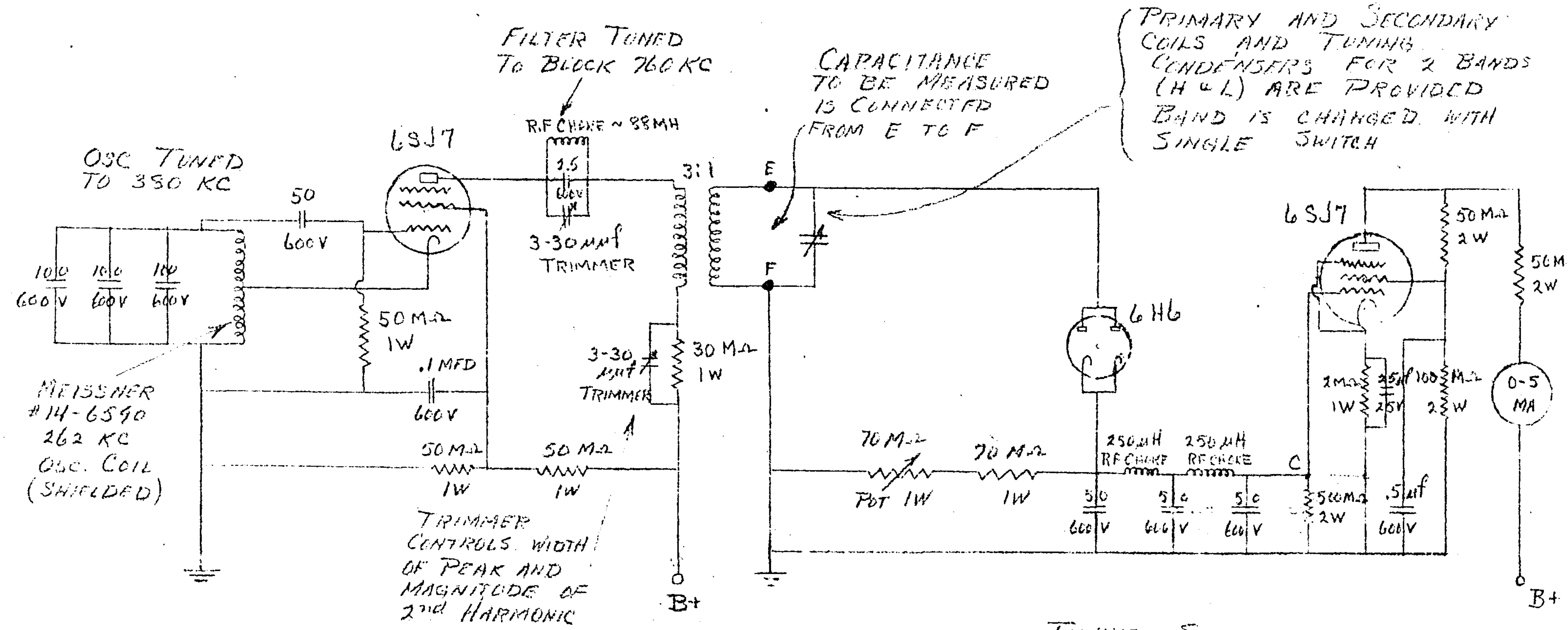
V = DRAINS TO VICKER'S RESERVOIR
S = " " SUP. PRES. "

HYDRAULIC CIRCUIT

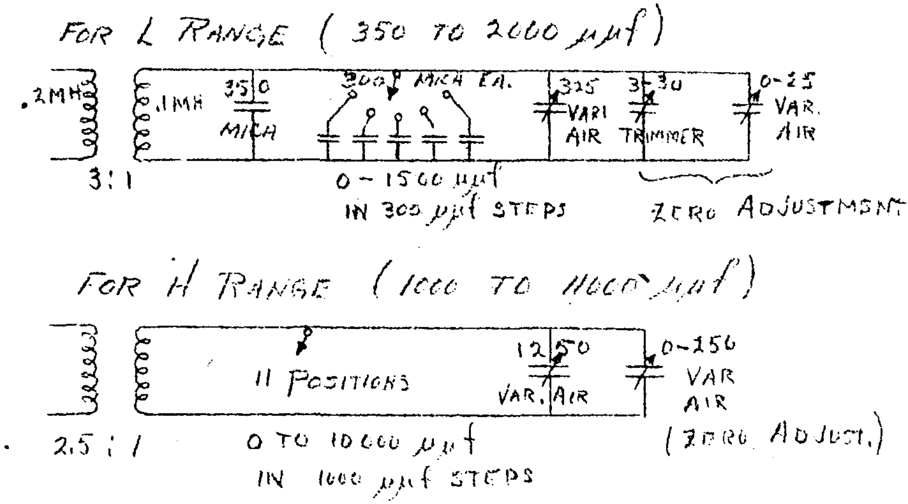


A_1 A_2 A_3 AND A_4 ARE CONNECTED TO B_3 B_4 B_1 AND B_2 RESPECTIVELY

PRINCIPLE OF "HYDROFILM" BEARING



TUNING SECTION



OSCILLATOR TERMINALS:

1. PLATE 6SJ7 OSC
2. " " AMP
3. 50 MΩ RES. TO (2)
4. B+
5. PLATE 6H6
6. PT. C OF FILTER
7. (−) END 6H6 LOAD POT.
8. GRD.

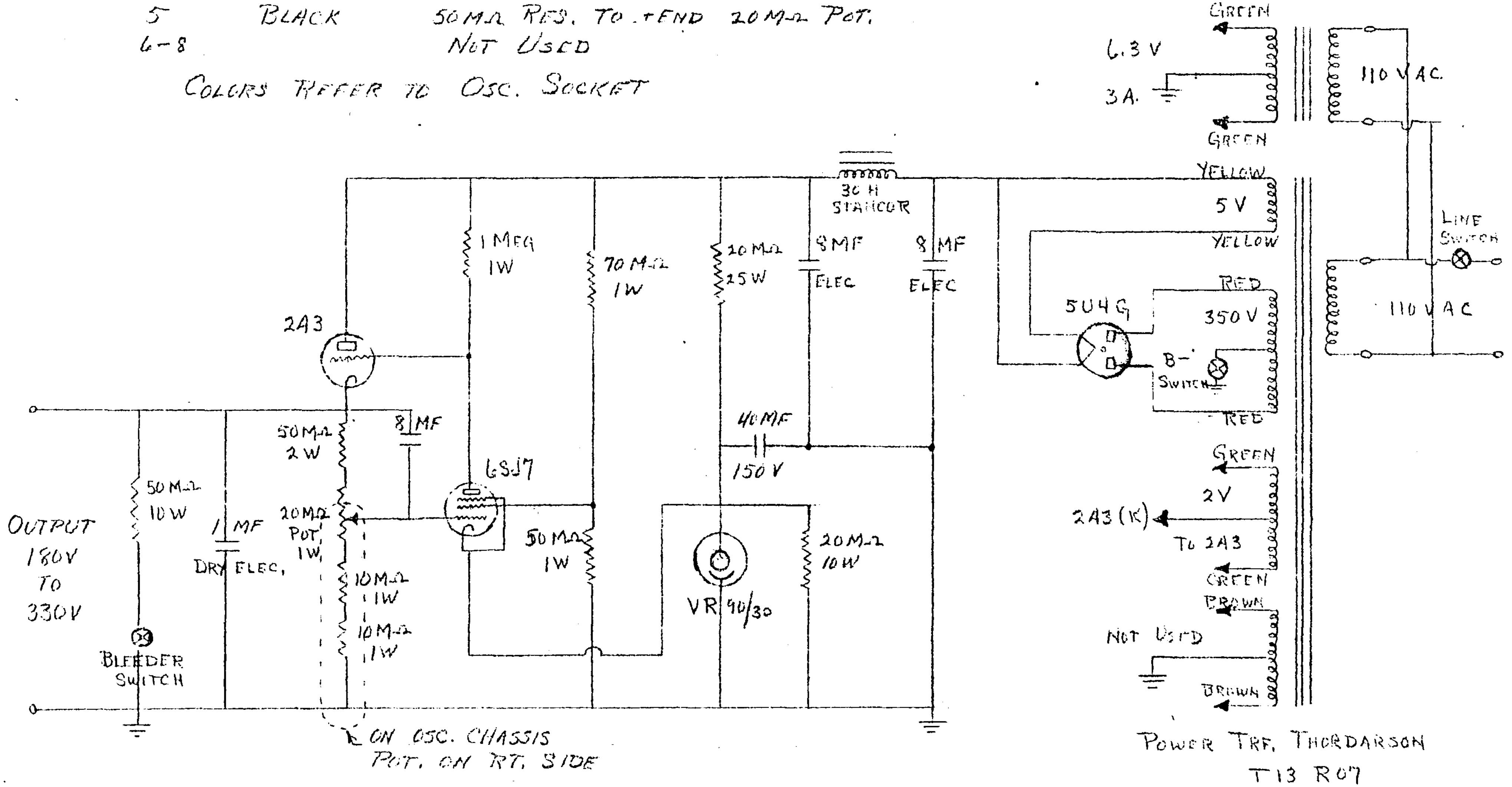
380 KC ELECTRON COUPLED
OSCILLATOR FOR MEASURING
CAPACITANCE

CONNECTION BET. POW. SUP. + OSCILLATOR

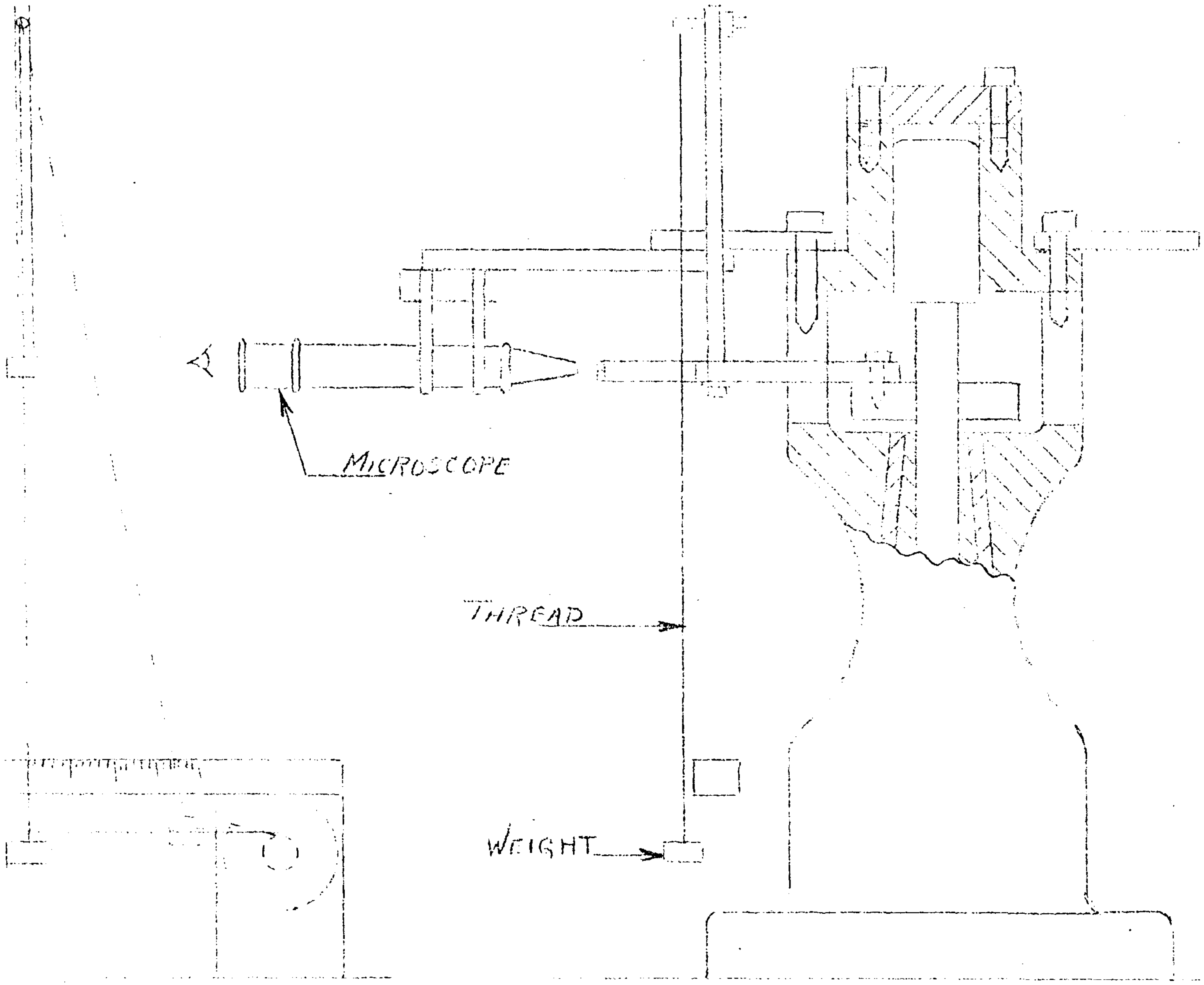
1	BLUE	GROUND
2-7	GREEN	6.3 V FILAMENT SUPPLY
3	BLACK	B+
4	RED-BLACK	GRID 6SJ7 TO CT 20M POT.
5	BLACK	50MΩ RES. TO +END 20MΩ POT.
6-8		NOT USED

COLORS REFER TO OSC. SOCKET

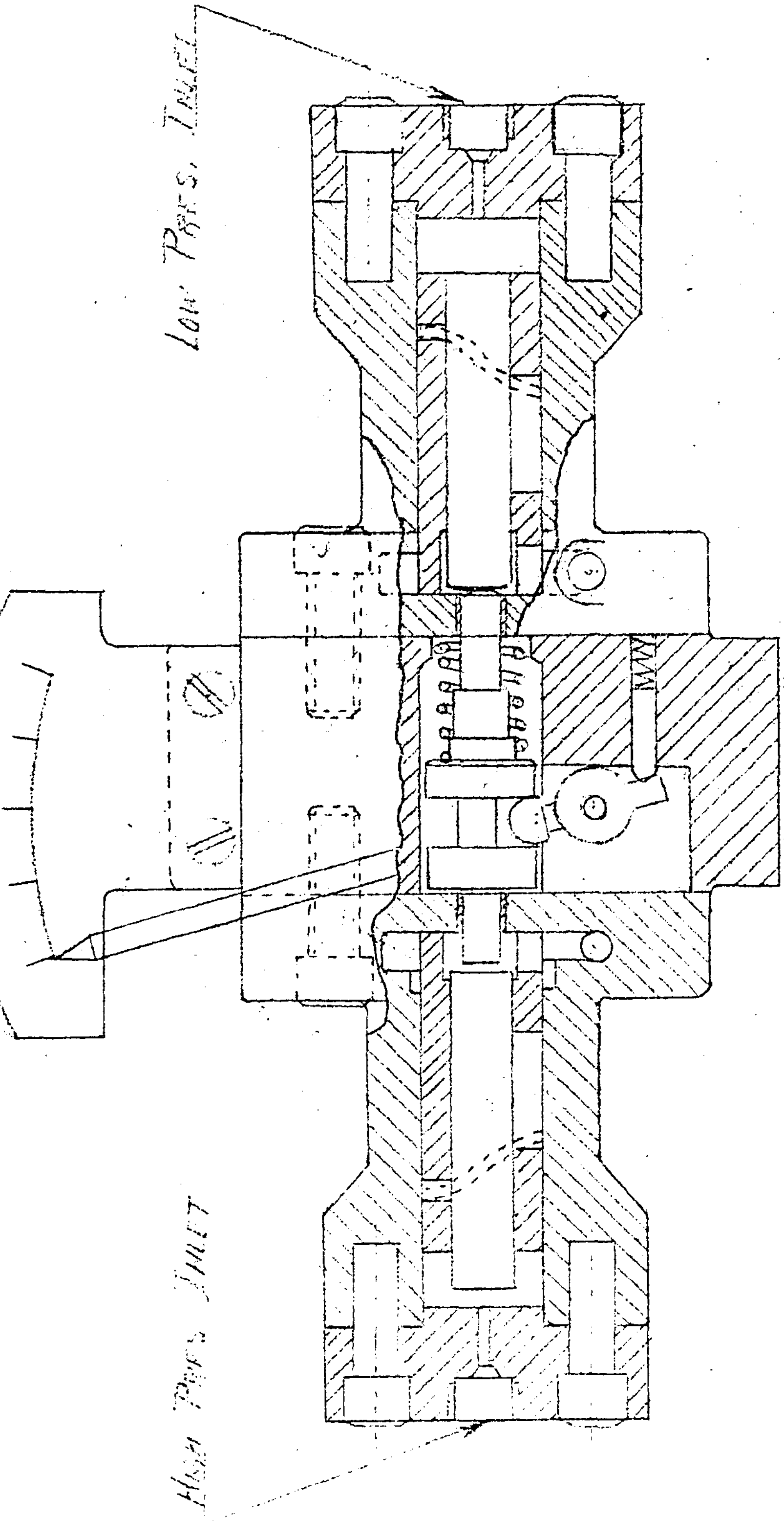
LOCATION OF POWER SUP. CONTROLS
LINE SWITCH - RIGHT
B(-) SWITCH - MIDDLE
BLEEDER SWITCH - LEFT



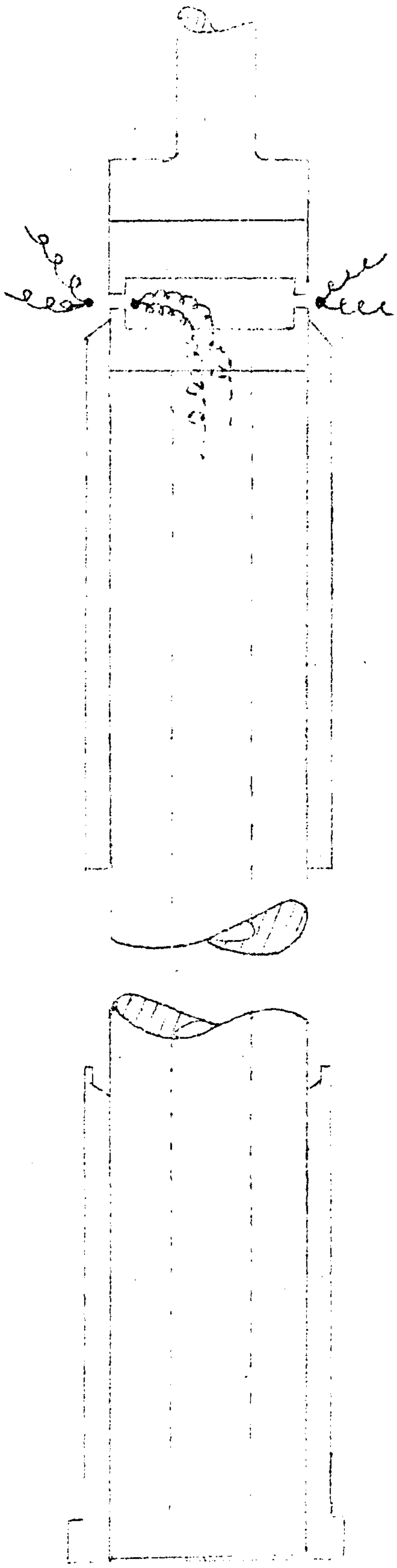
VOLTAGE REGULATED POWER SUPPLY (180V - 330V)



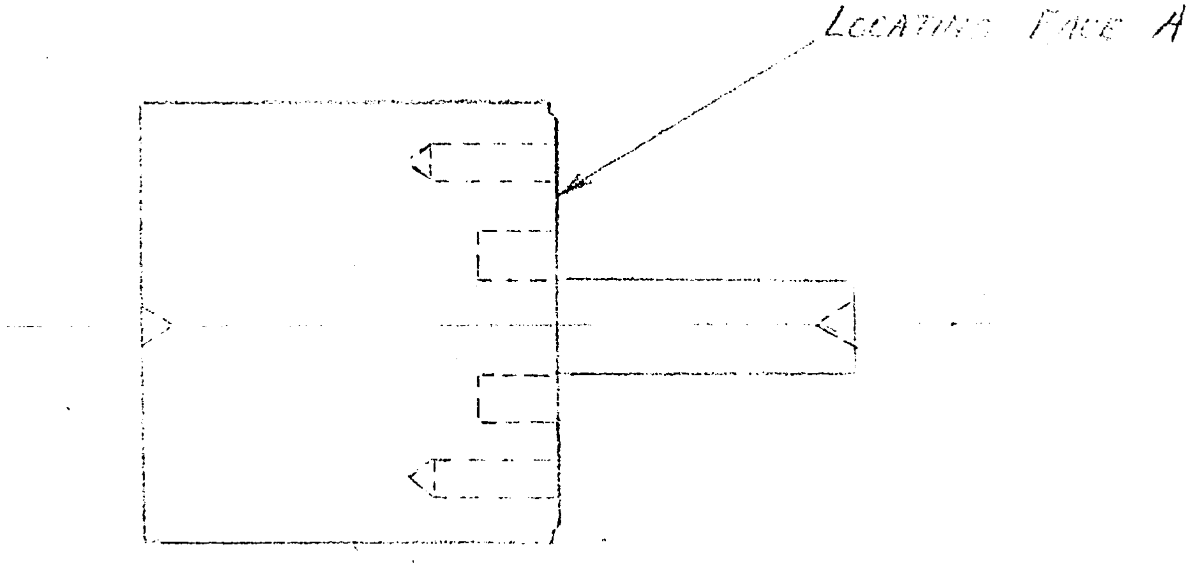
TORQUE MEASURING SET-UP



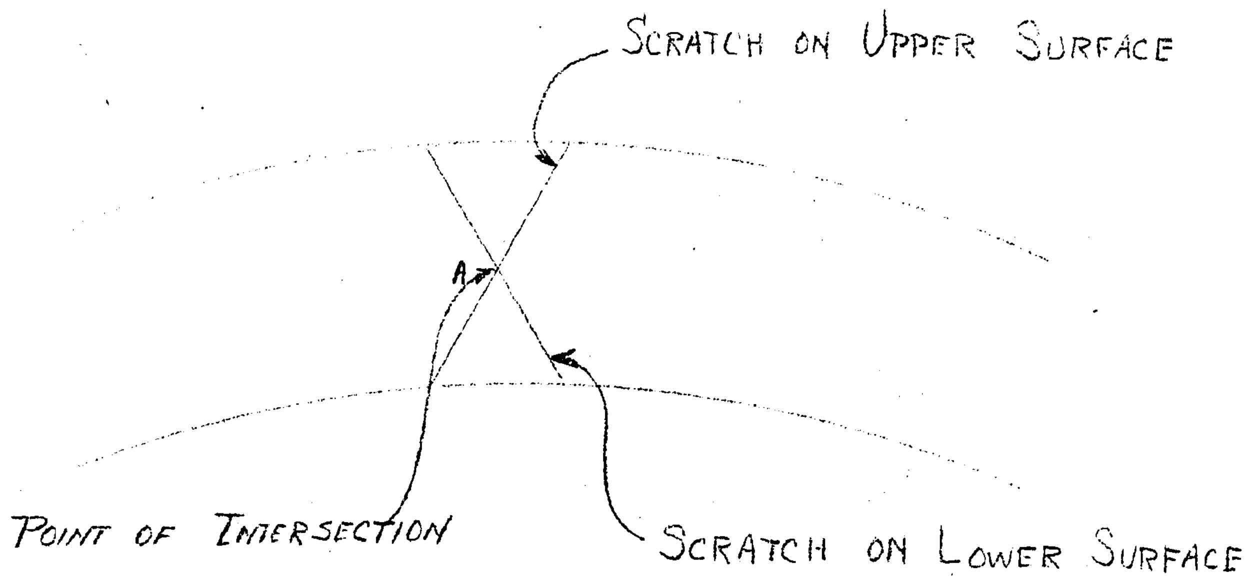
High Pressure Differential Gage



ARRANGEMENT, OF THERMOCOUPLES



MANDREL FOR GRINDING SAMPLES ON DRILL CENTERS



ARRANGEMENT OF SCRATCHES
ON SAMPLE Nº 2a

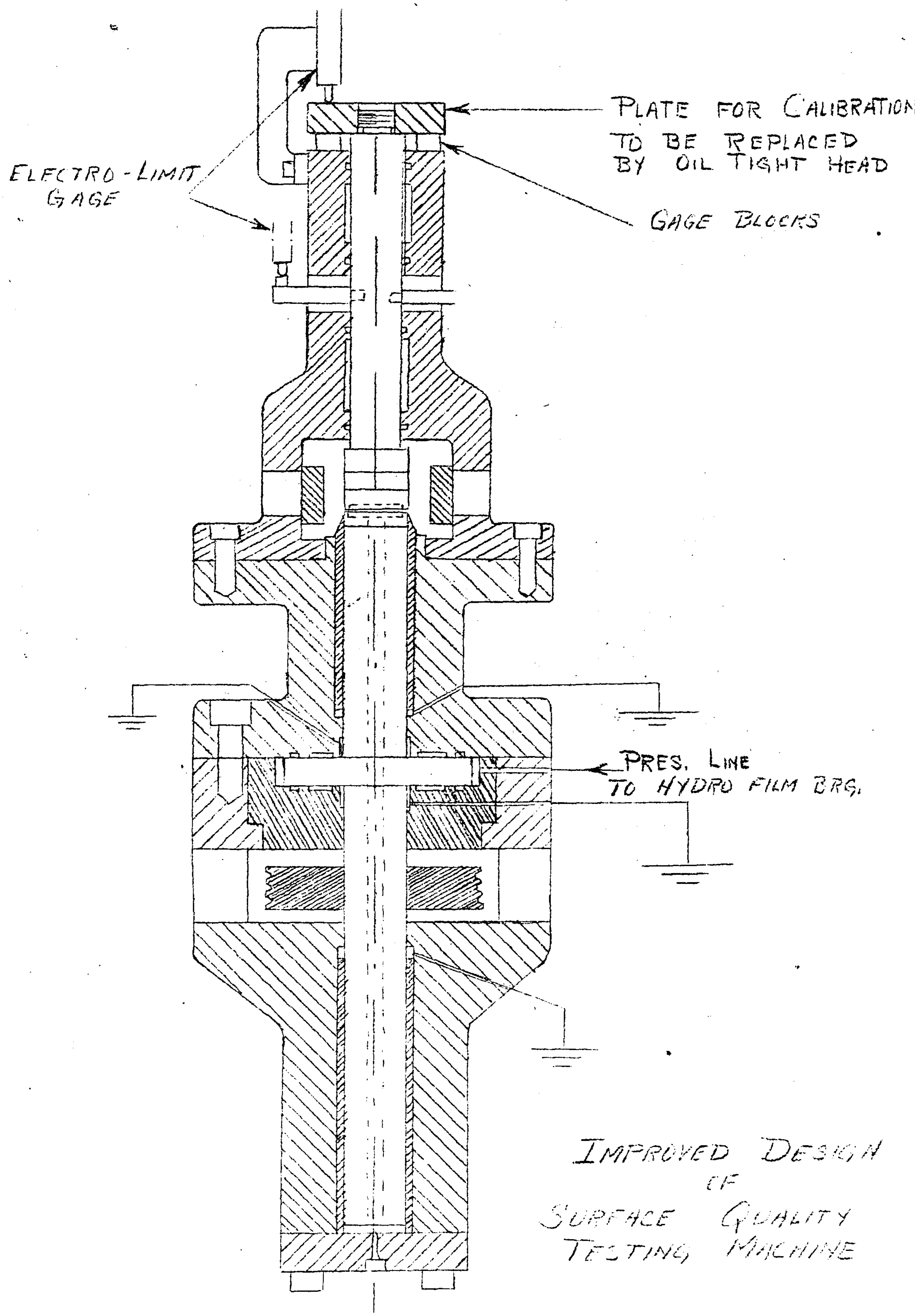
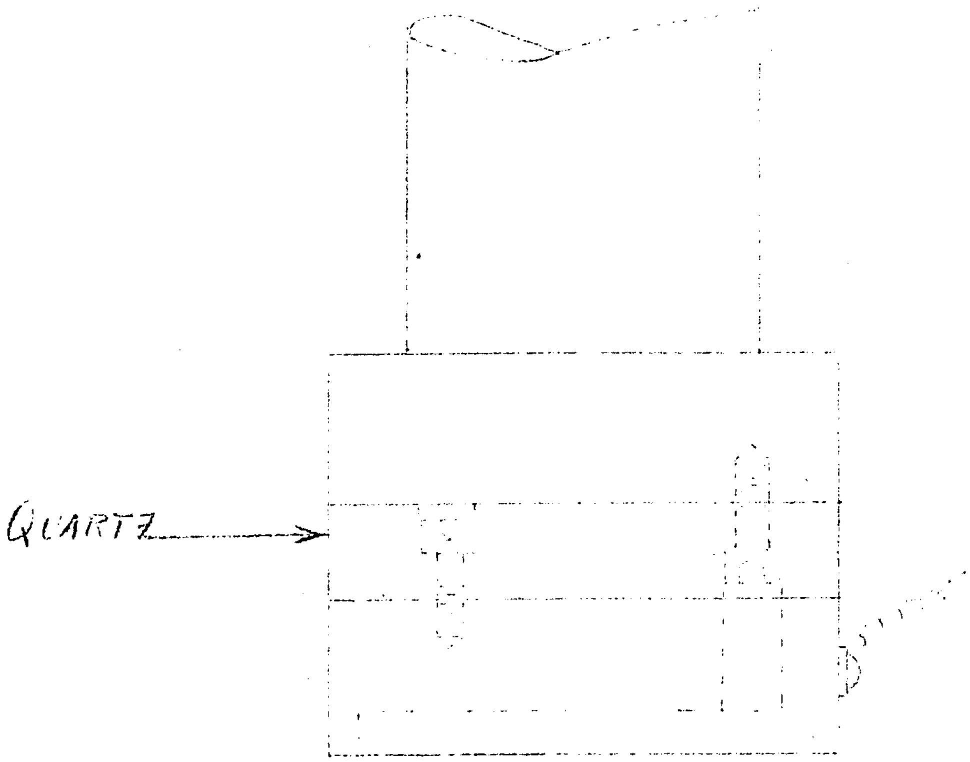
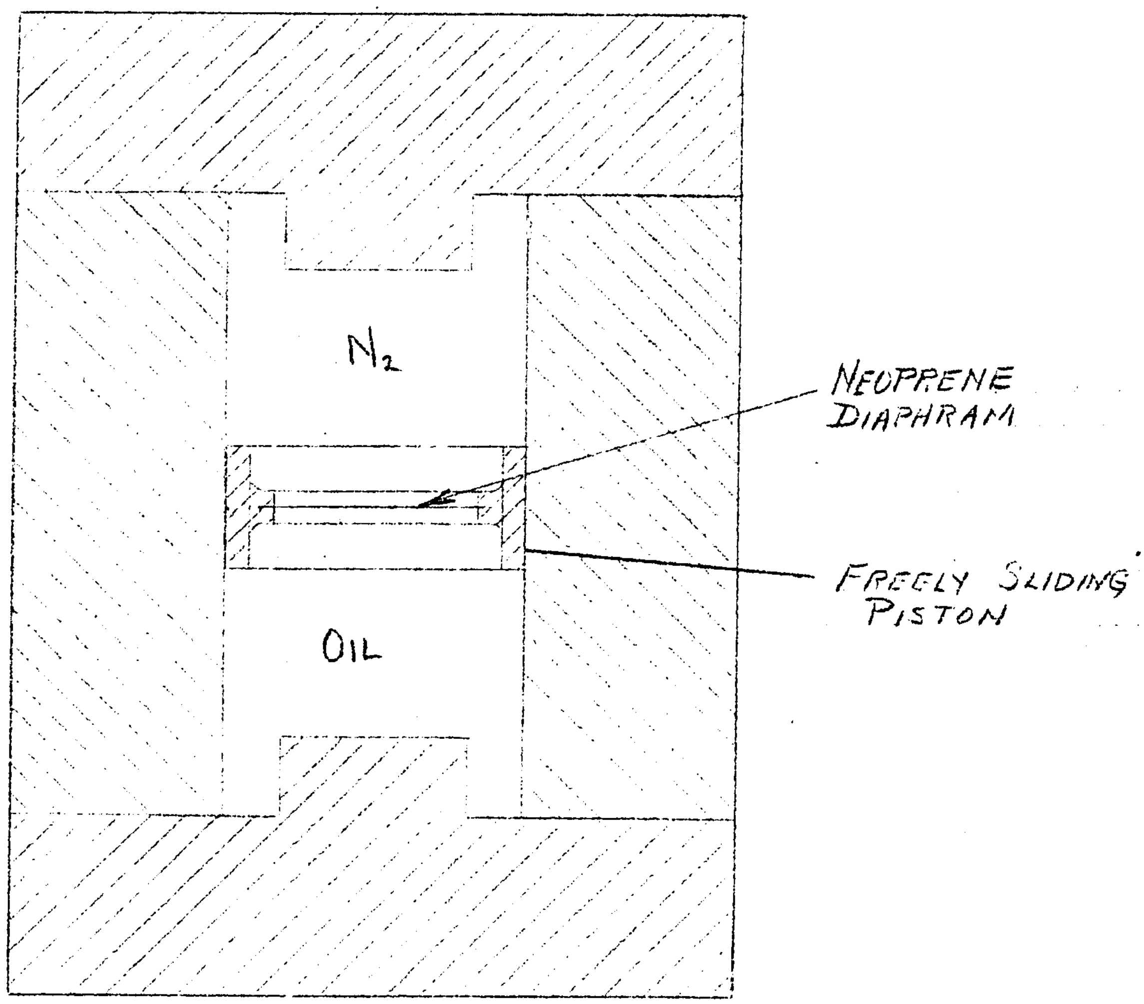


PLATE FOR CALIBRATION
TO BE REPLACED
BY OIL TIGHT HEAD
ELECTRO-LIMIT
GAGE
GAGE BLOCKS
PRES. LINE
TO HYDRO FILM BRG.
IMPROVED DESIGN
OF
SURFACE QUALITY
TESTING MACHINE

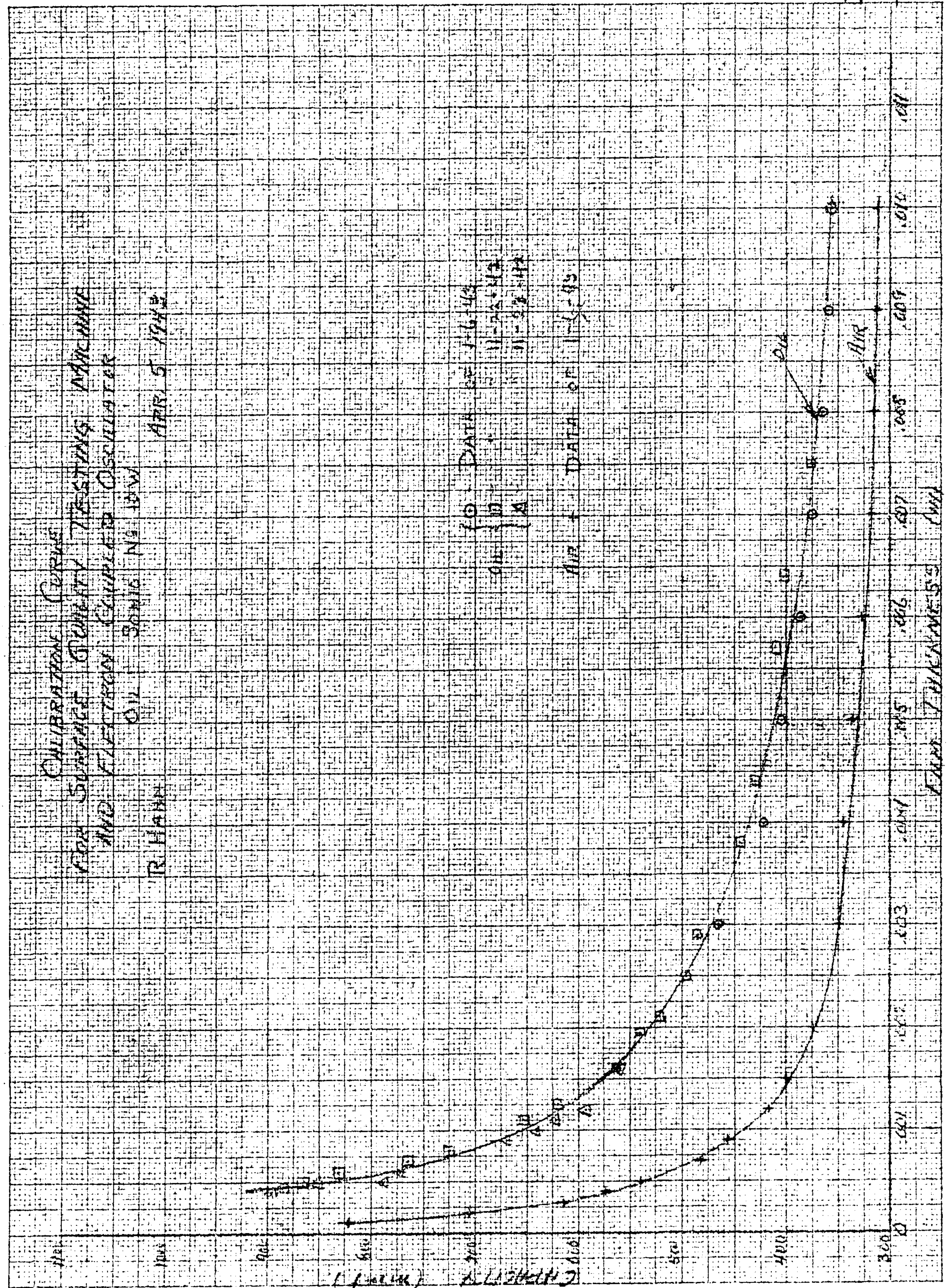


PROPOSED METHOD OF INSULATING SAMPLE



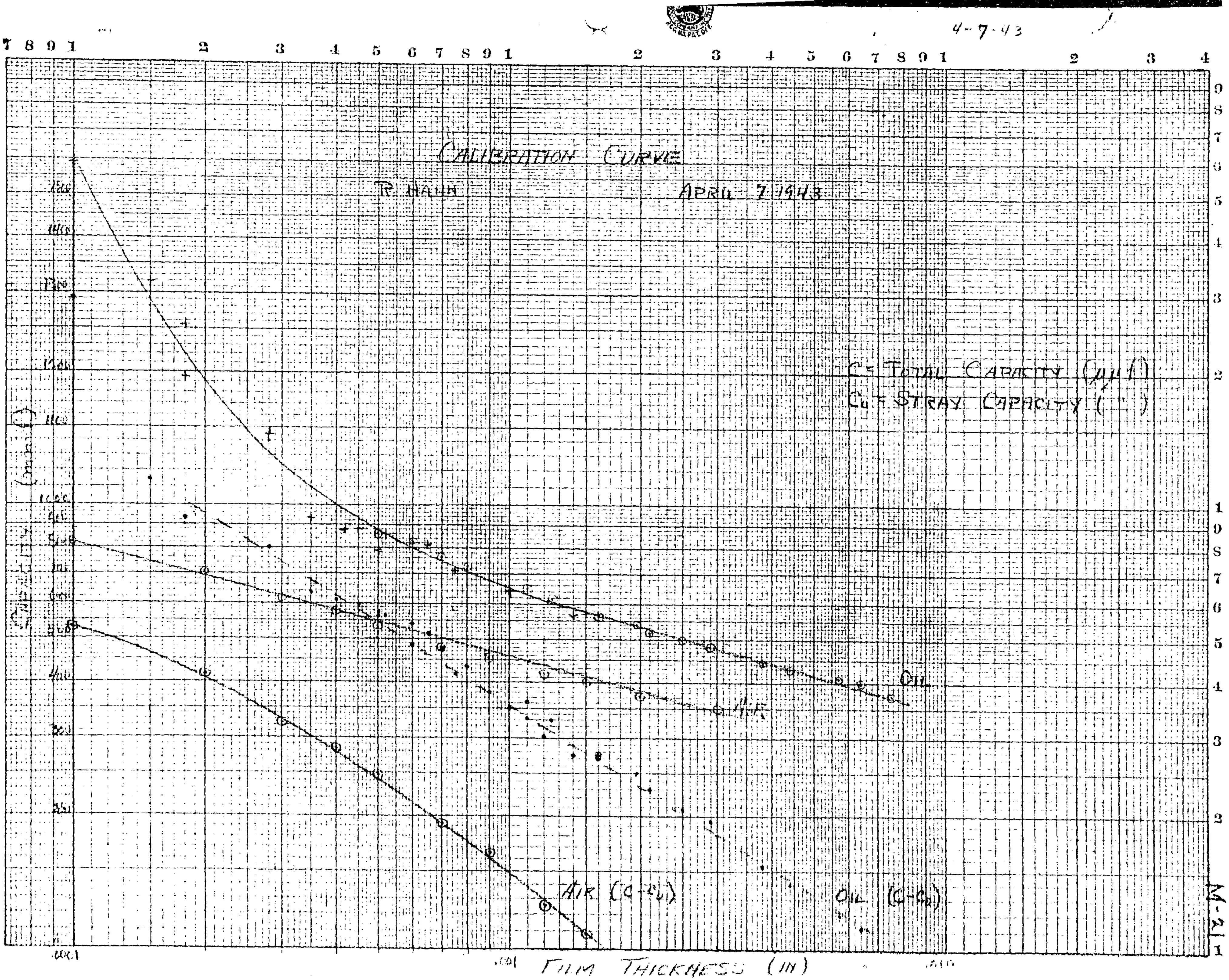
PROPOSED METHOD OF PREVENTING N_2 FROM DISSOLVING IN OIL

M-19

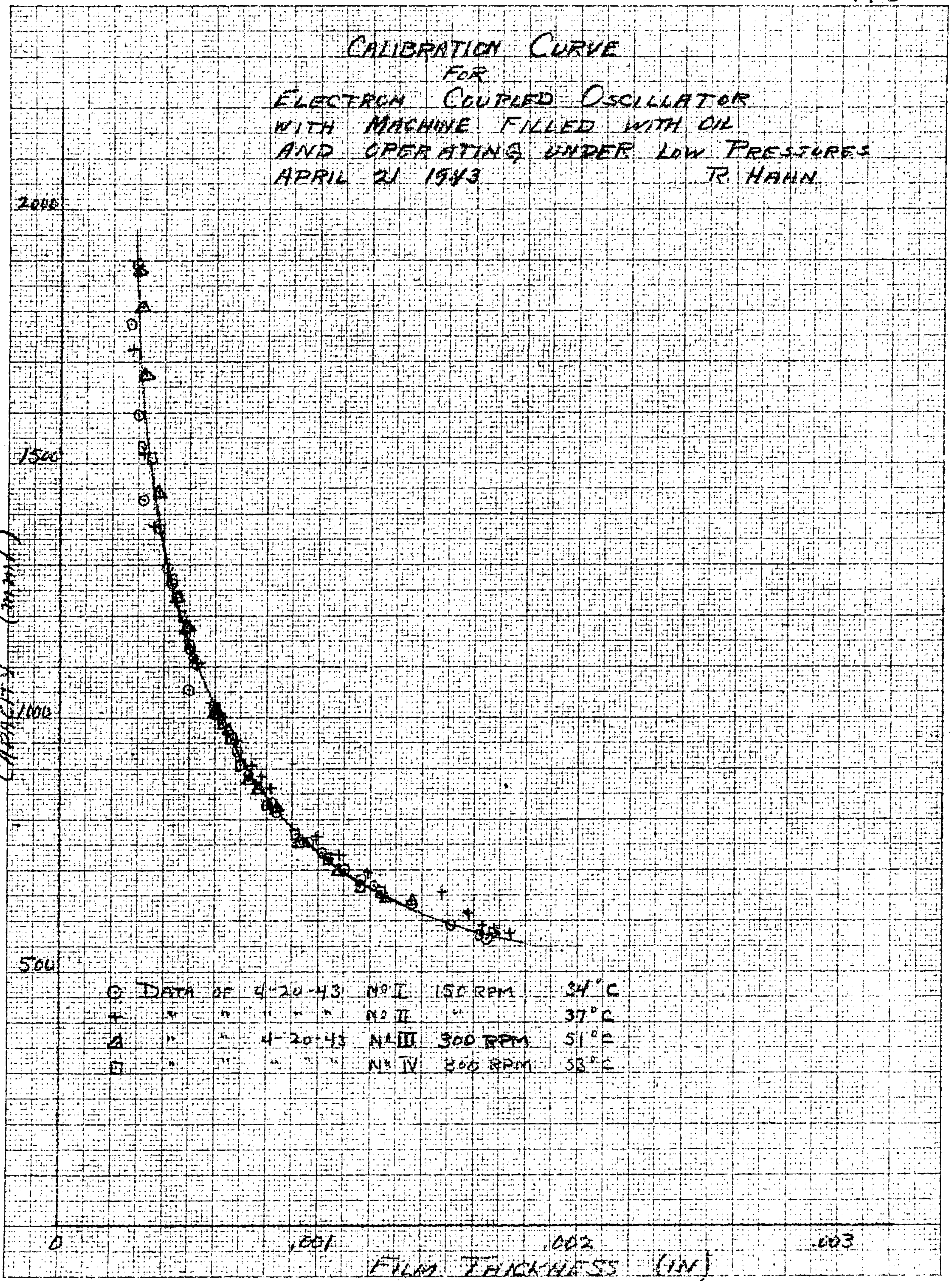


MADE IN U.S.A.

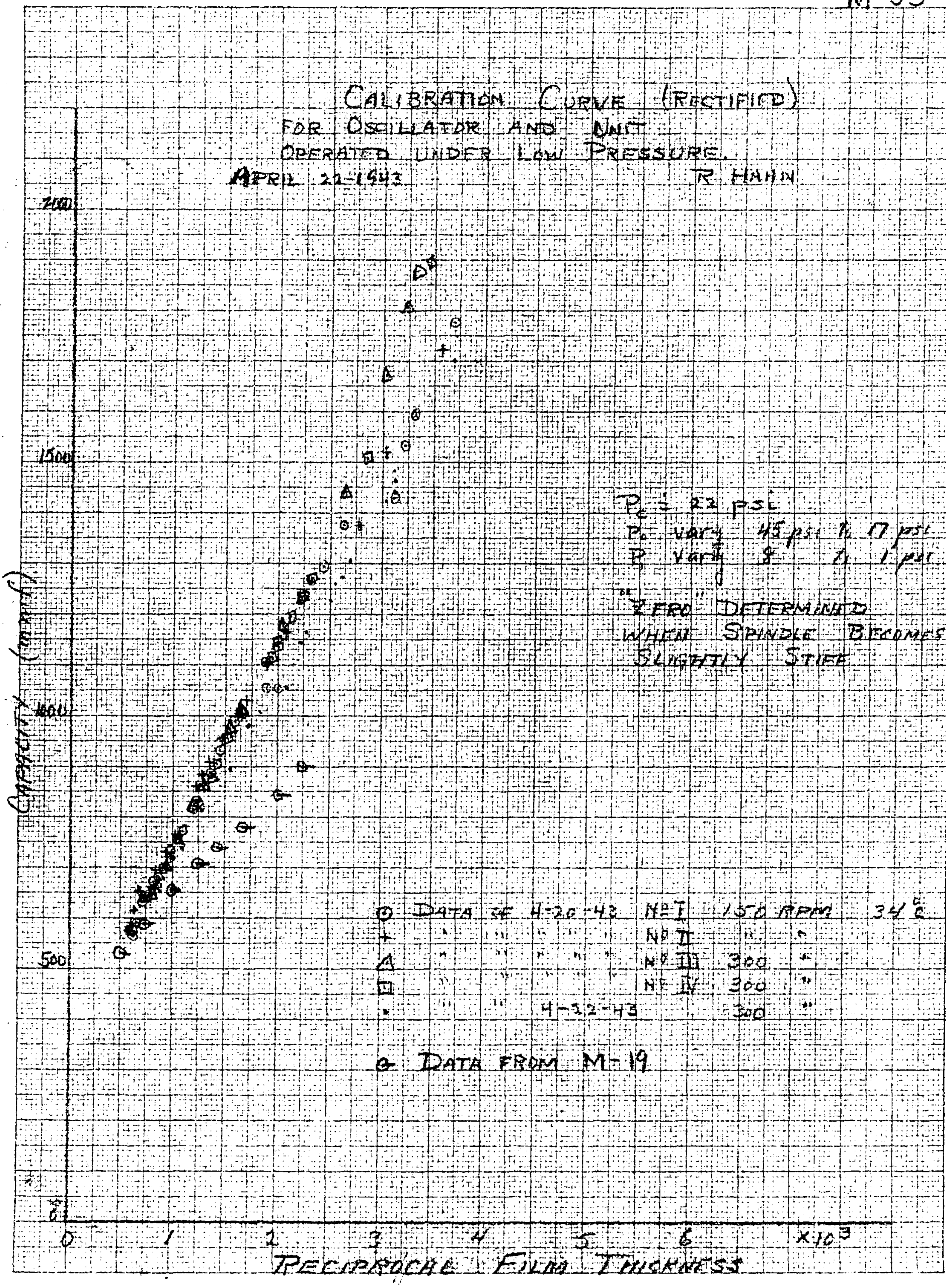
Form 63—Applied Sc.—500—5-41


M-21
4-7-43
CALIBRATION CURVE
R. HAHN
APRIL 7 1943
C = TOTAL CAPACITY (μμf)
C₀ = STRAY CAPACITY (")
OIL
AIR
AIR (C-C₀)
OIL (C-C₀)
FILM THICKNESS (IN)
.0001
.001
.010

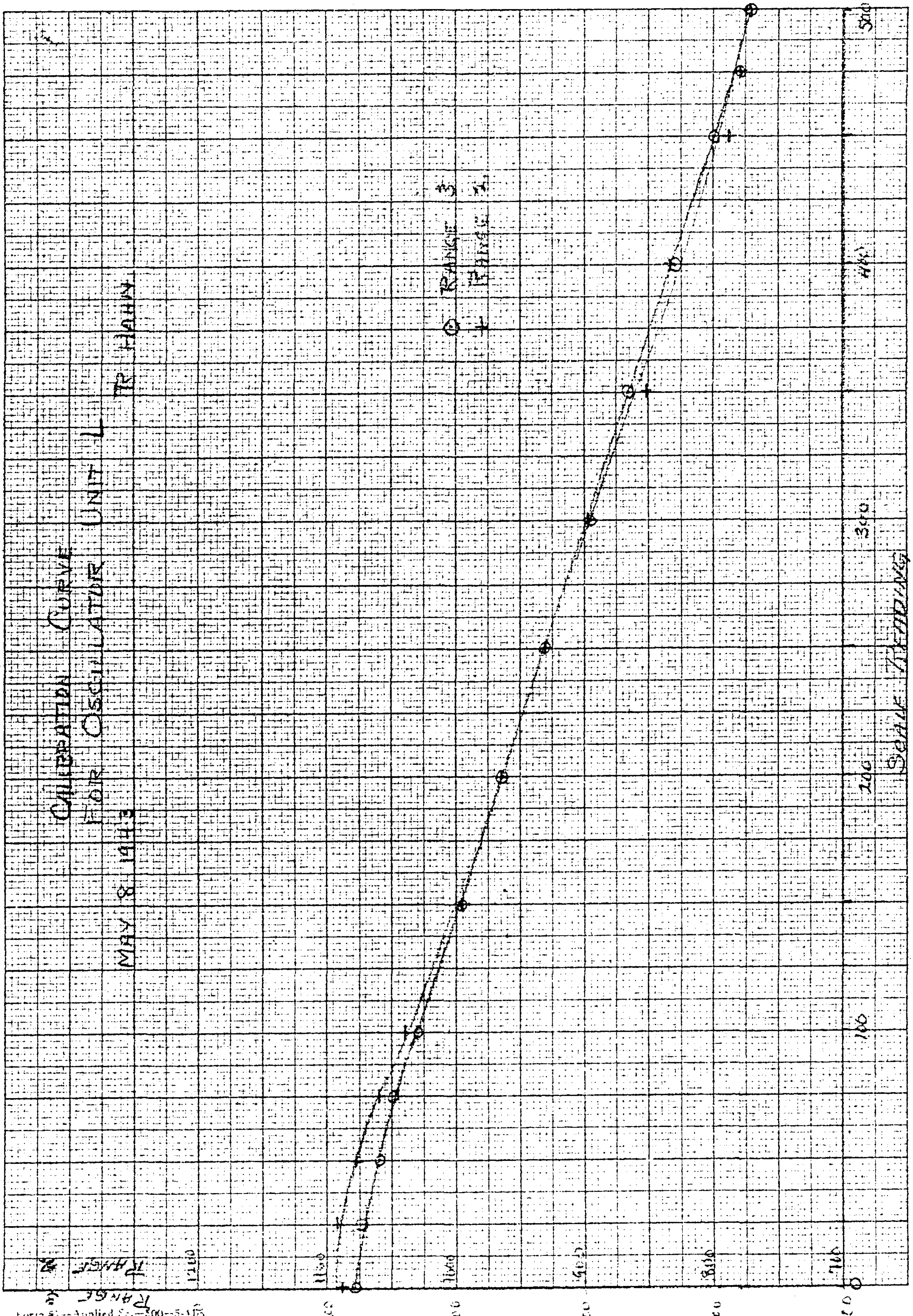
M-32


CALIBRATION CURVE
FOR
ELECTRON COUPLED OSCILLATOR
WITH MACHINE FILLED WITH OIL
AND OPERATING UNDER LOW PRESSURES
APRIL 21 1943
R. HAHN
2000
1500
1000
500
CAPACITY (mmmf)
0
.001
.002
.003
FILM THICKNESS (IN)
DATA OF 4-20-43 Nº I 150 RPM 34° C
" " " " " Nº II " 37° C
" " 4-20-43 Nº III 300 RPM 51° C
" " " " " Nº IV 800 RPM 53° C

M-33

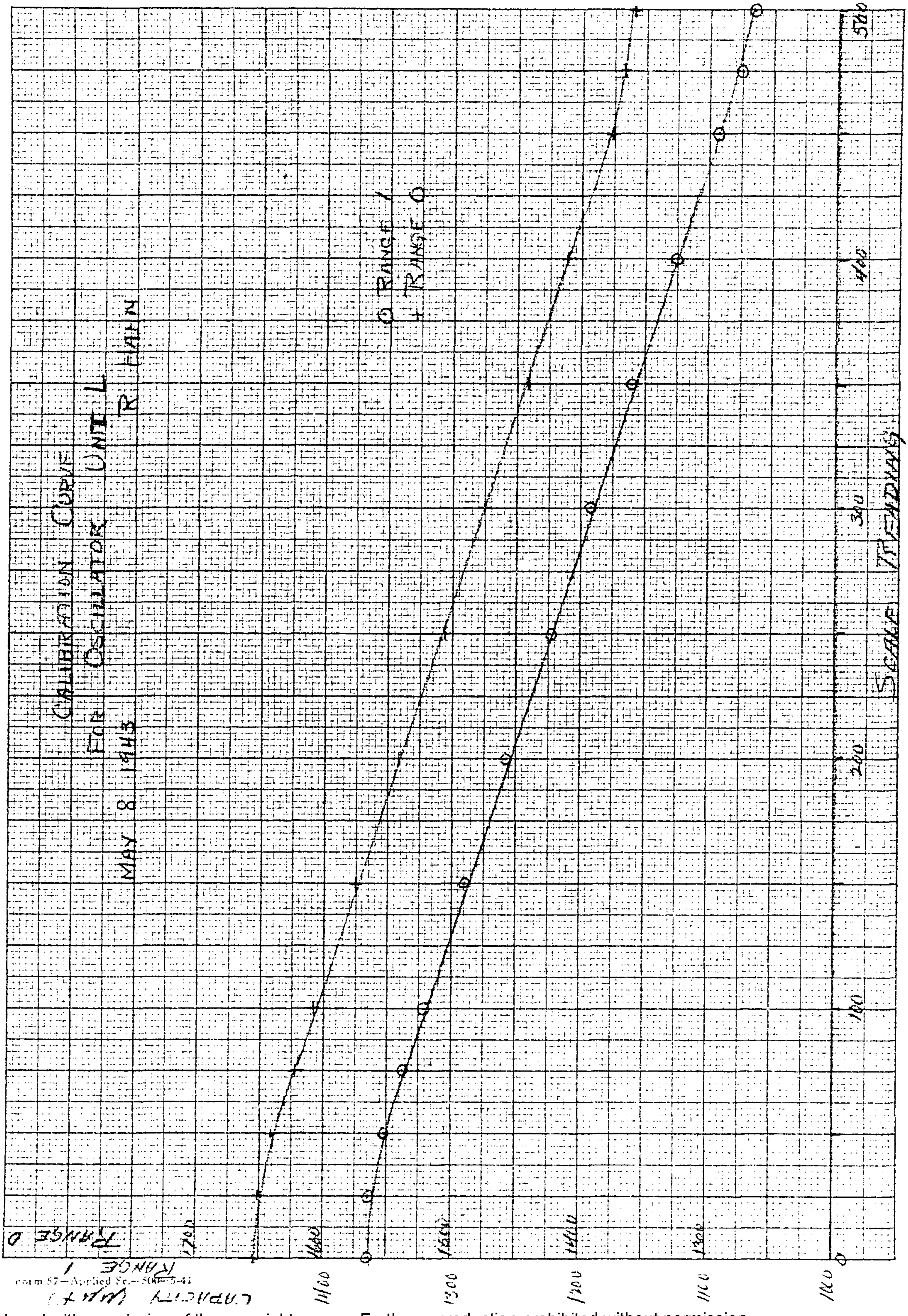

CALIBRATION CURVE (RECTIFIED)
FOR OSCILLATOR AND UNIT
OPERATED UNDER LOW PRESSURE.
APRIL 22-1943
R HAHN
2000
1500
1000
500
0
CAPACITY
0
1
2
3
4
5
6
x10^3
RECIPROCAL FILM THICKNESS
P_c = 22 psi
P_o vary 45 psi to 17 psi
P_i vary 8 to 1 psi
"ZERO" DETERMINED
WHEN SPINDLE BECOMES
SLIGHTLY STIFF
DATA OF 4-20-43 No I 150 RPM 34°C
No II
No III 300
No IV 300
4-22-43 300
DATA FROM M-19

M-36



Form 87--Applied Sc.--500--5-41

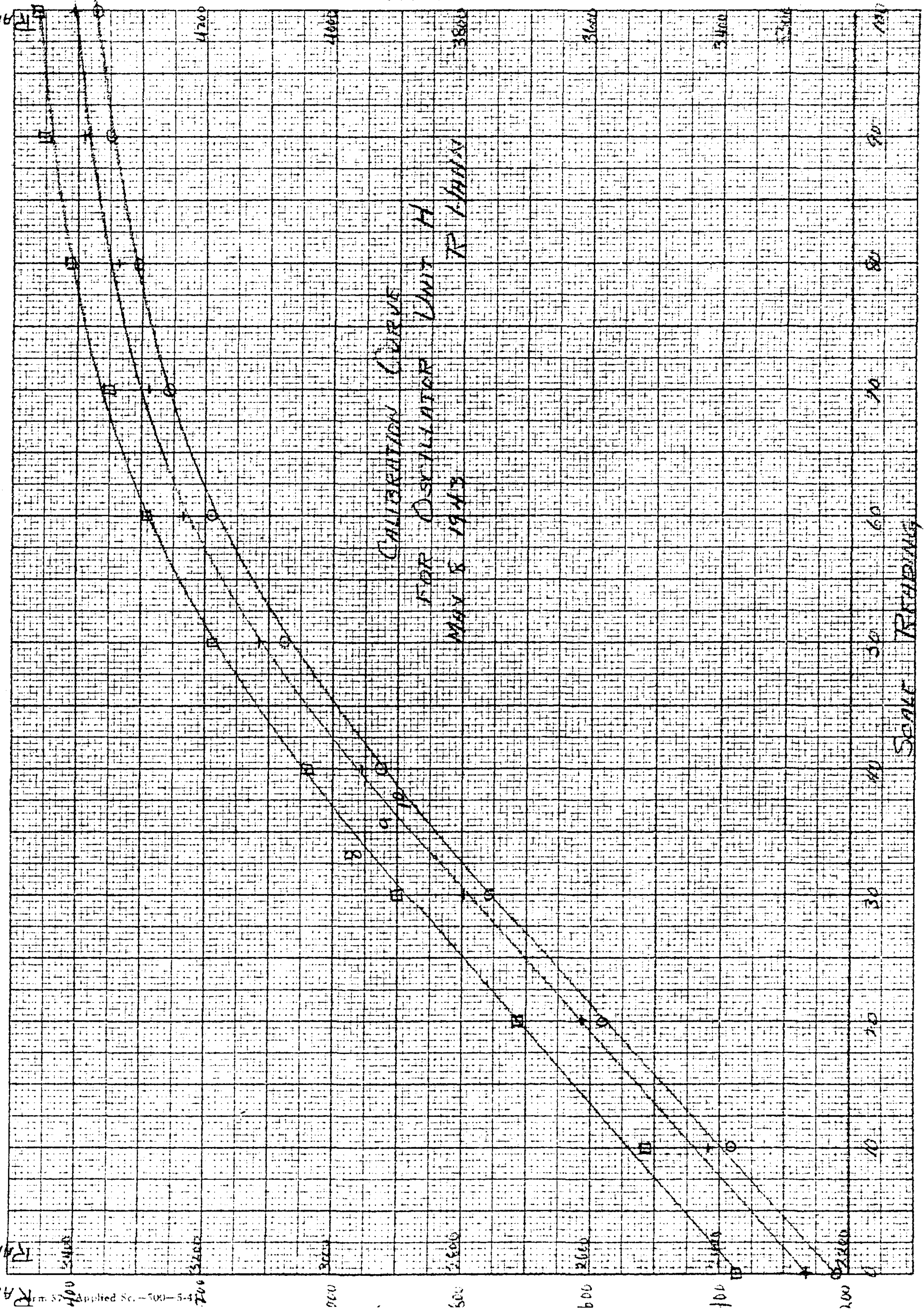
M-36

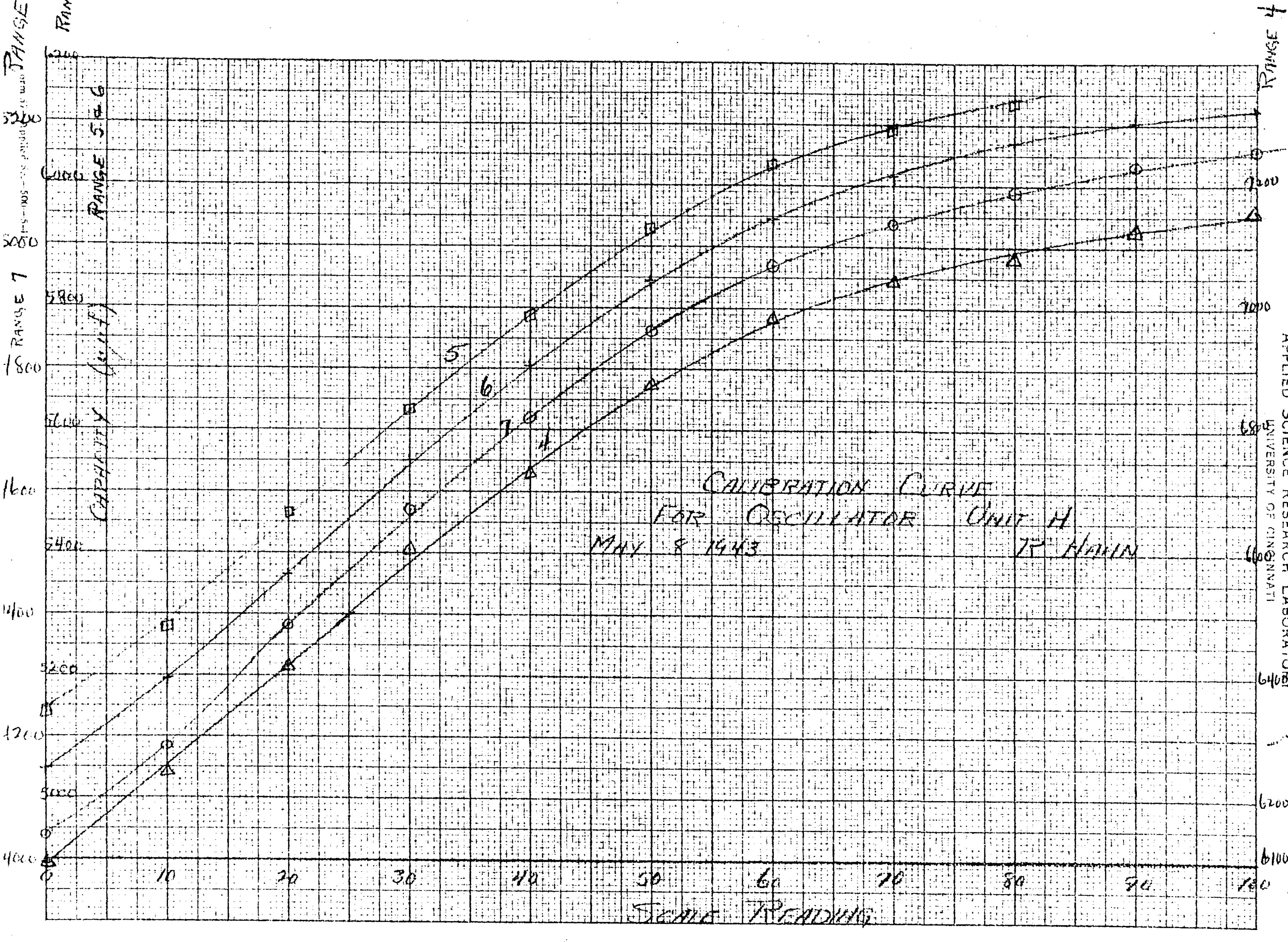


Form 57—Applied Sc.—500—3-41

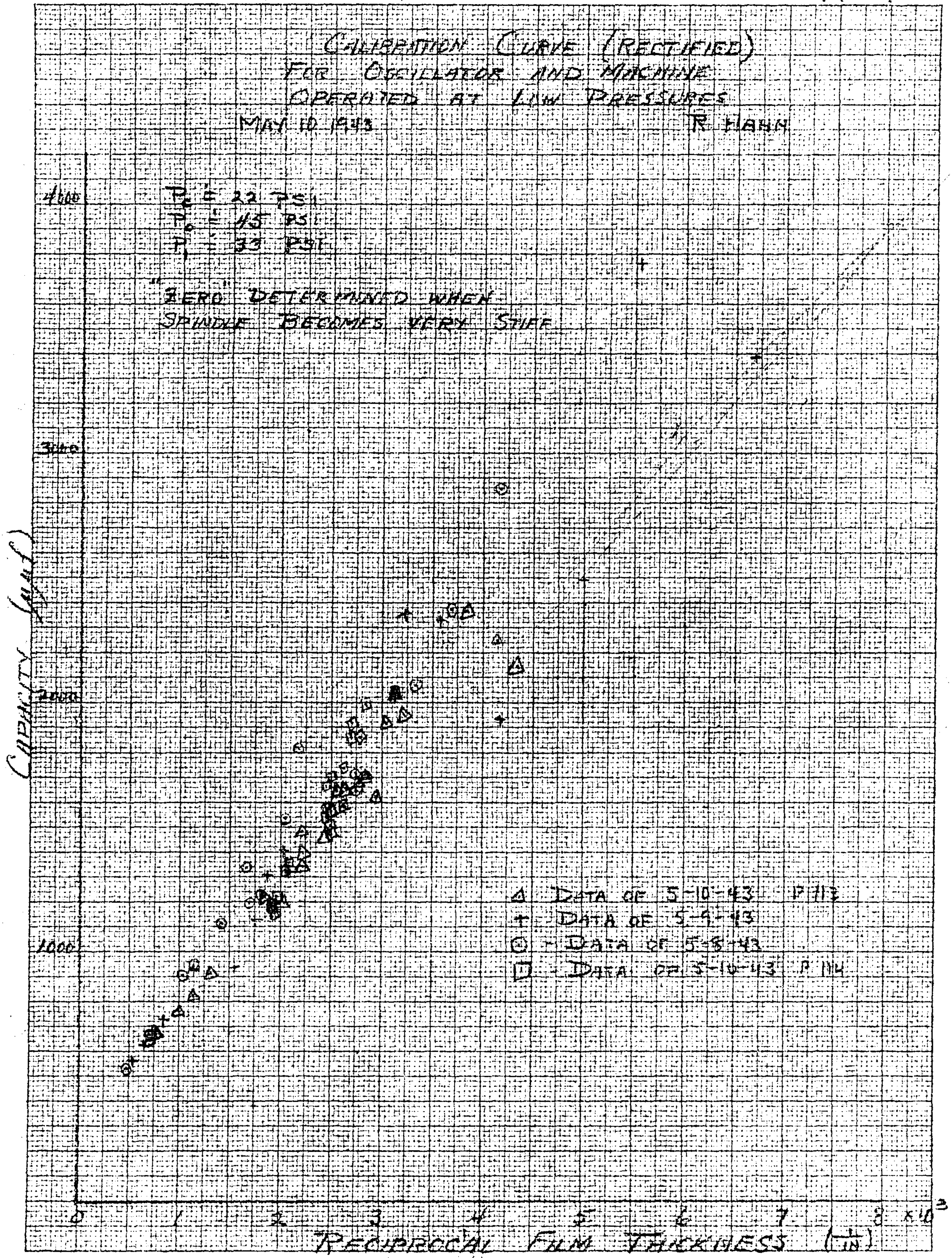
MADE IN U.S.A.

M-37


CALIBRATION CURVE
FOR OSCILLATOR UNIT H
MAY 5 1943
SCALE READING


APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI
CALIBRATION CURVE
FOR OSCILLATOR UNIT H
MAY 8 1943
R HAHN
SCALE READING
CAPACITY (μμf)
RANGE 7
RANGE 5 & 6
RANGE 4
5
6
7
4
0
10
20
30
40
50
60
70
80
90
100

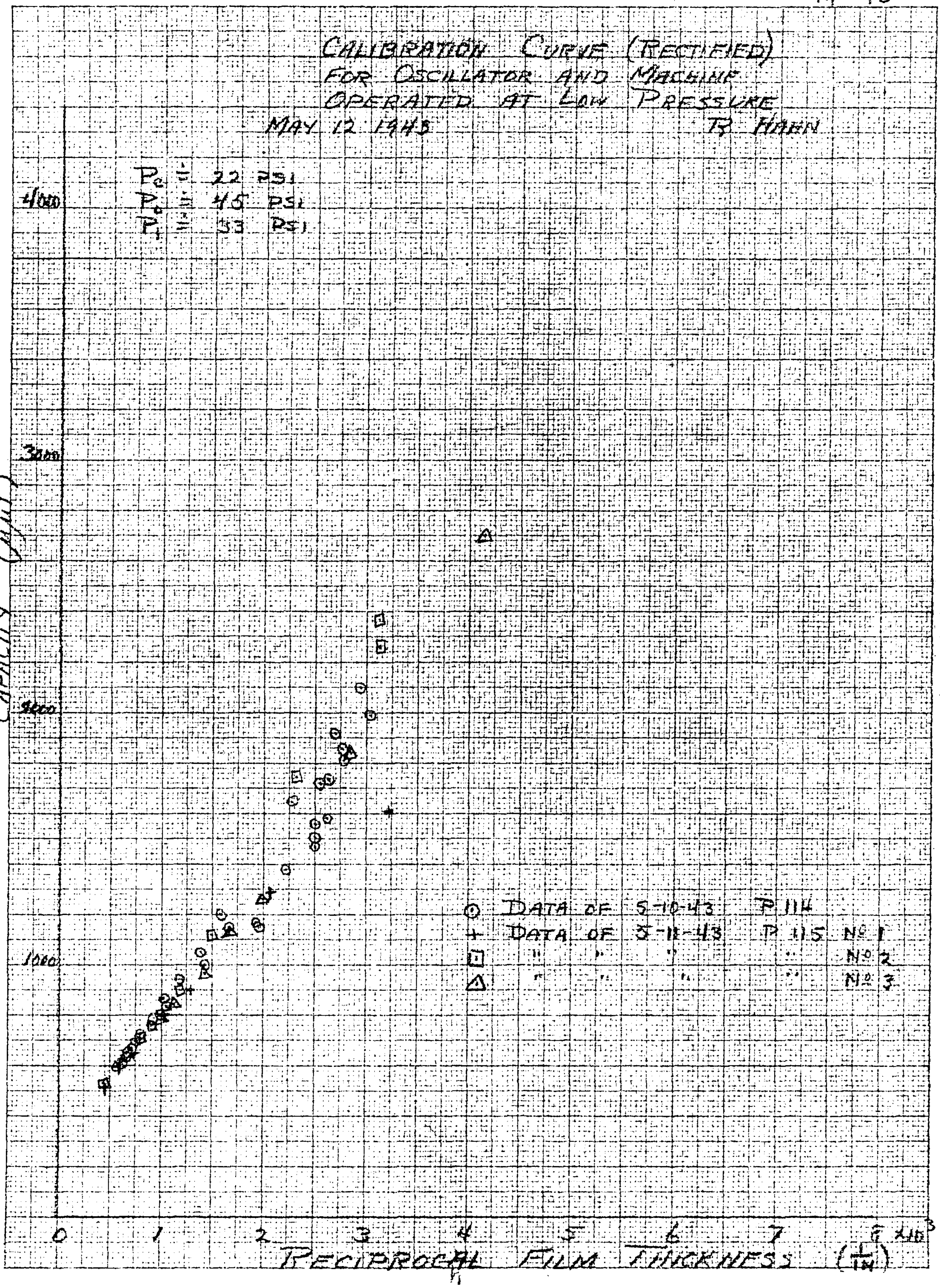
M-39



Form 63—Applied Sc.—500—5-41

APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

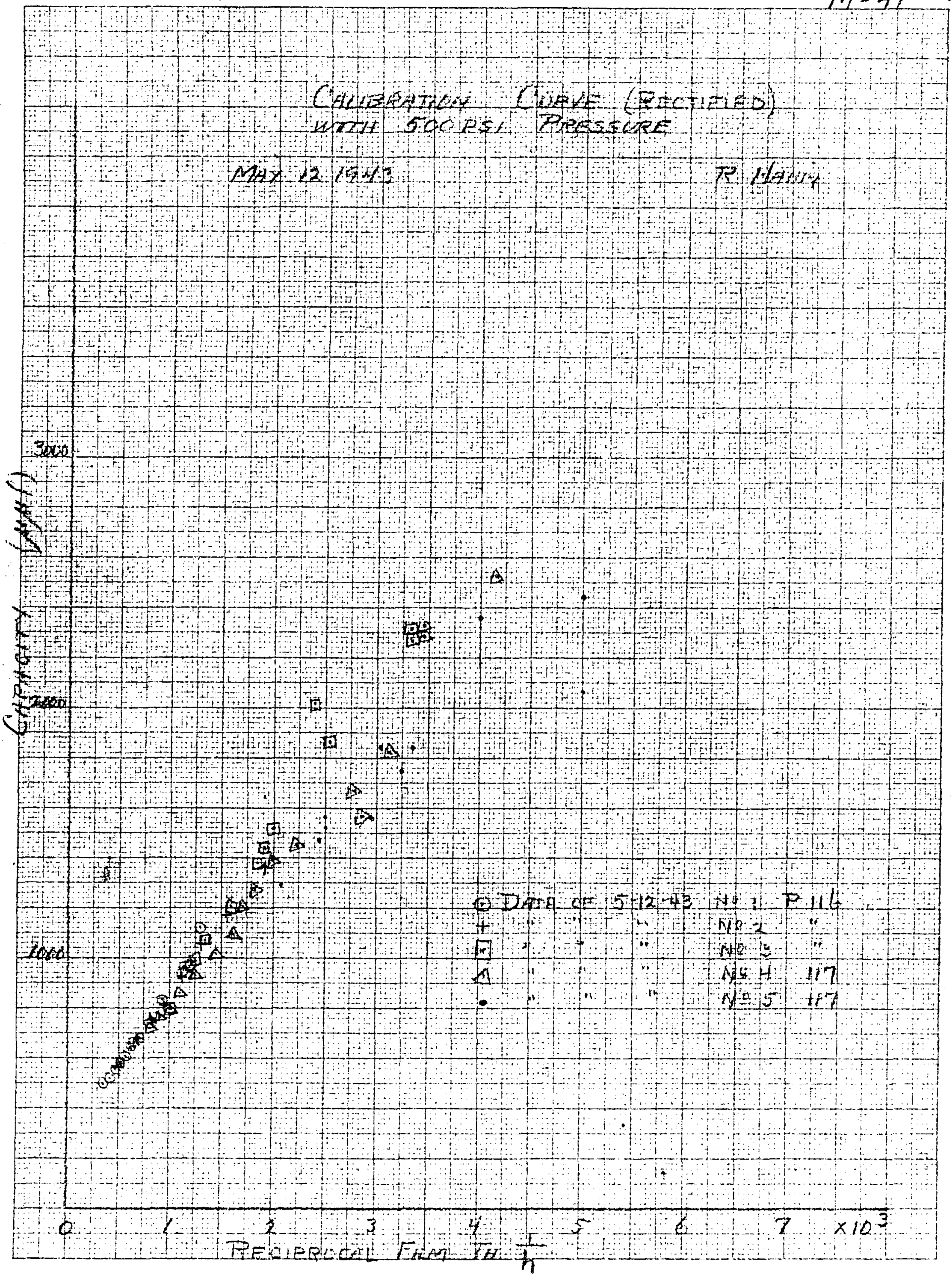
M-40



MADE IN U.S.A.

Form 63—Applied Sc.—500—5-41

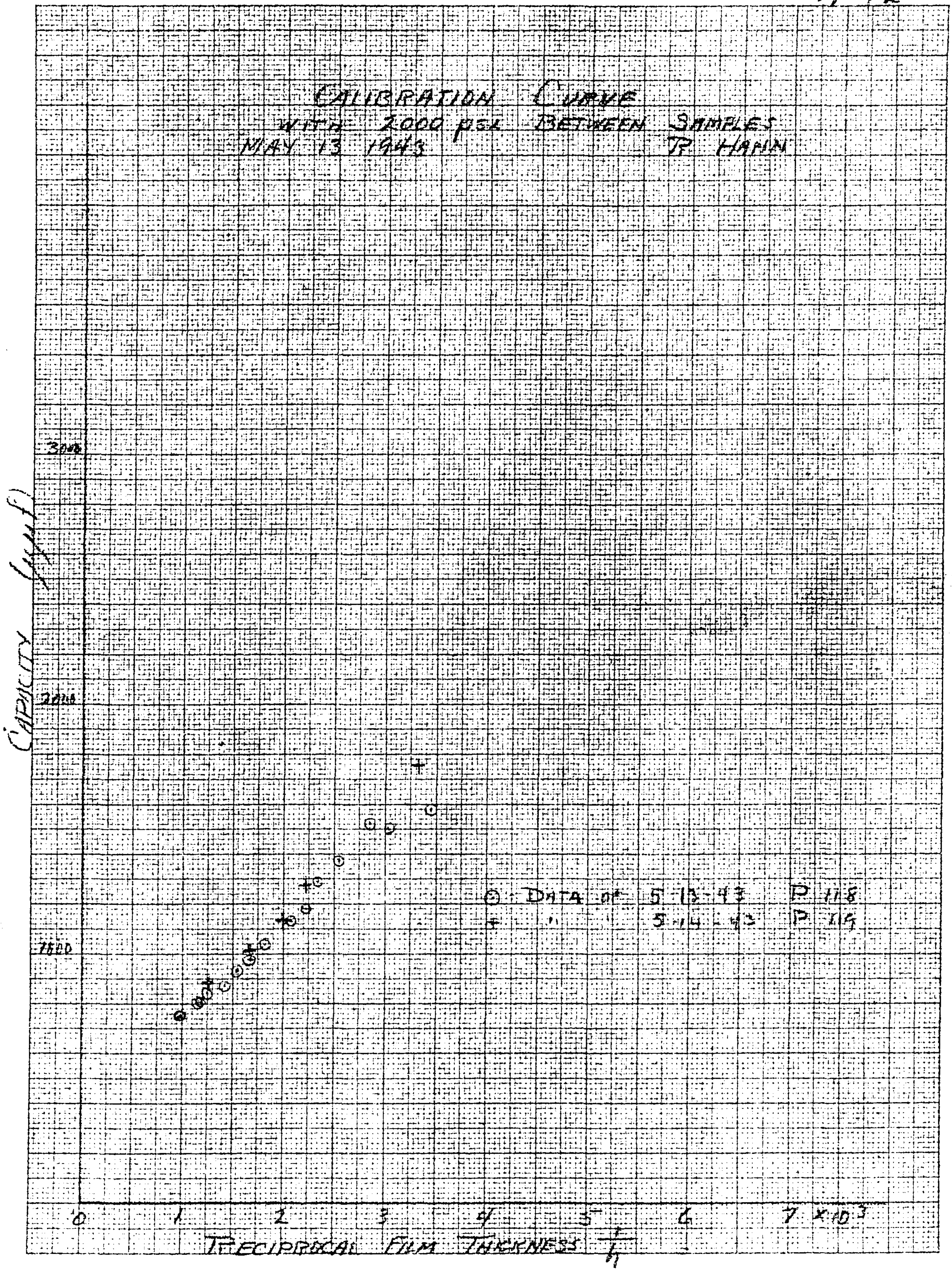
M-41



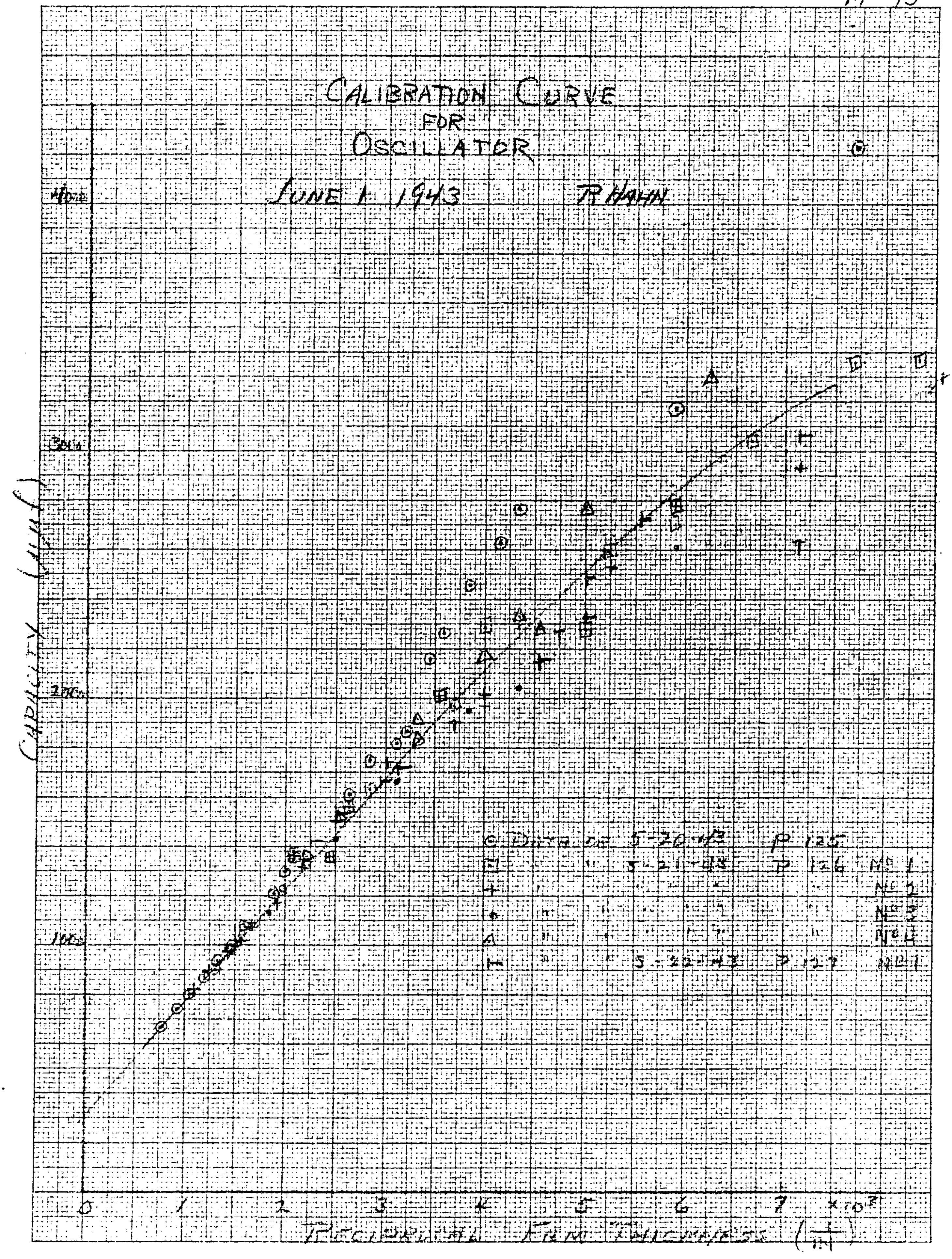
Form 65 - Applied Sc.—500—5-41

APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

M-42

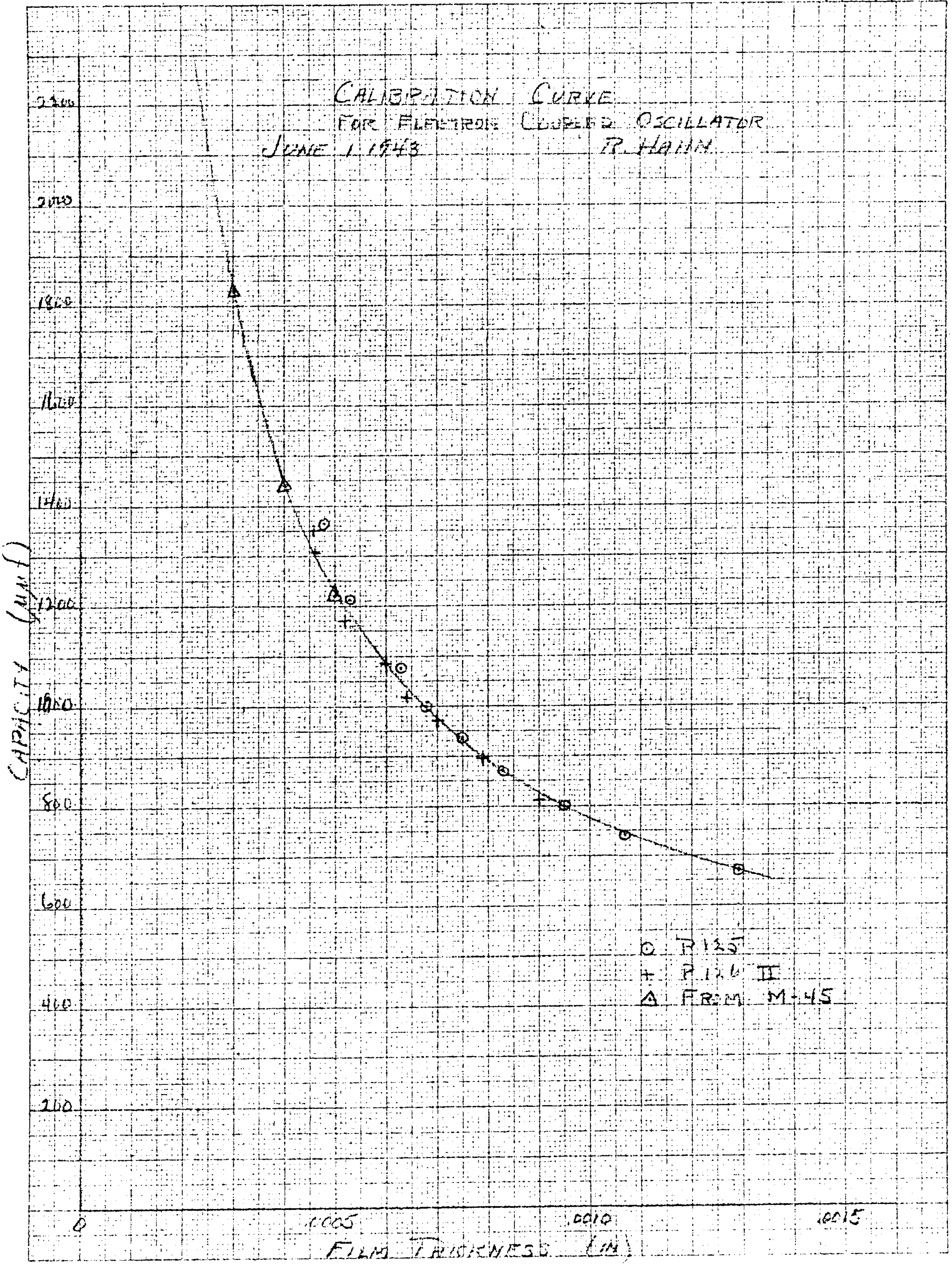


M-45



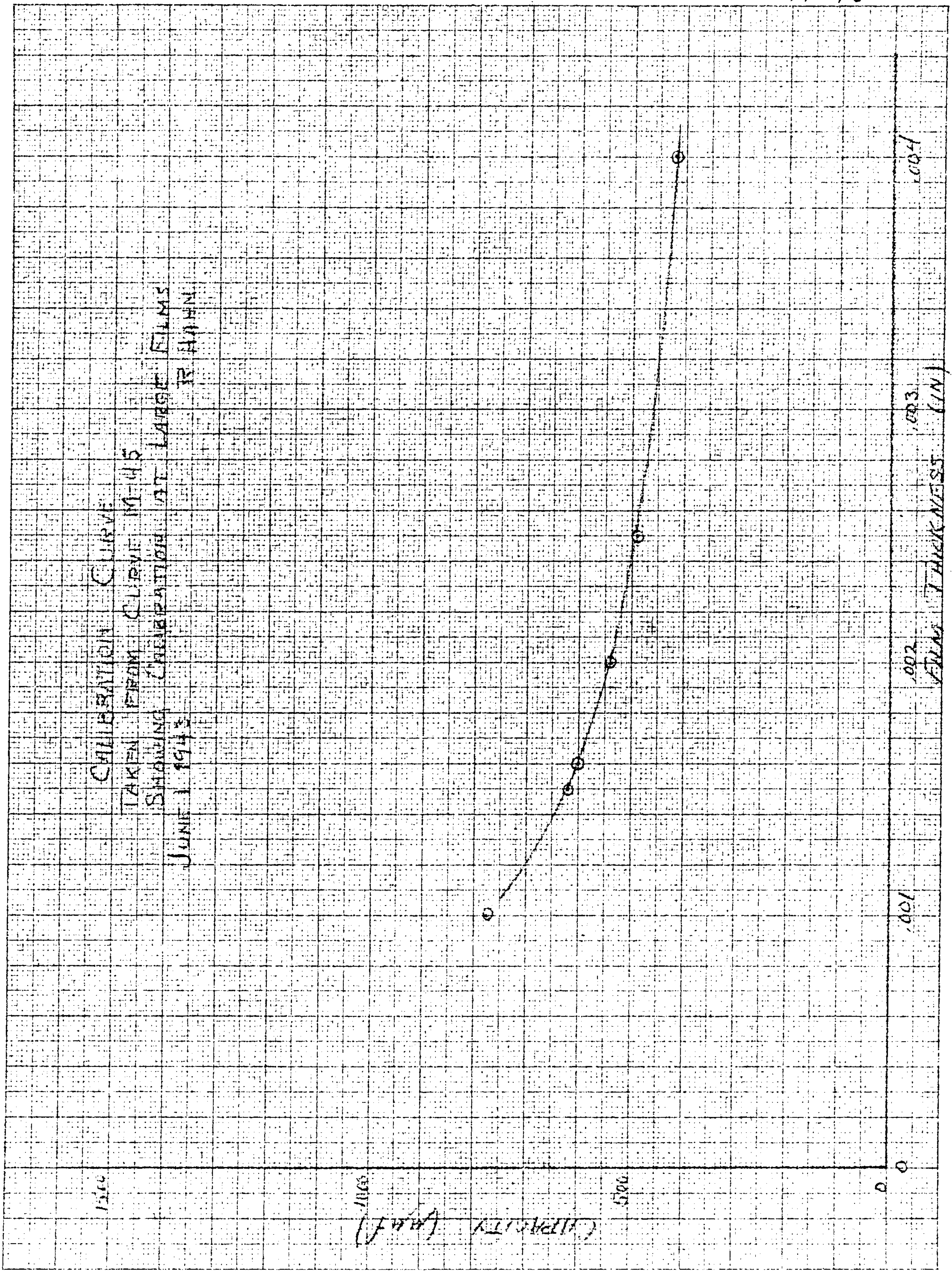
Form 65 – Applied Sc.—500—5-41

M-47



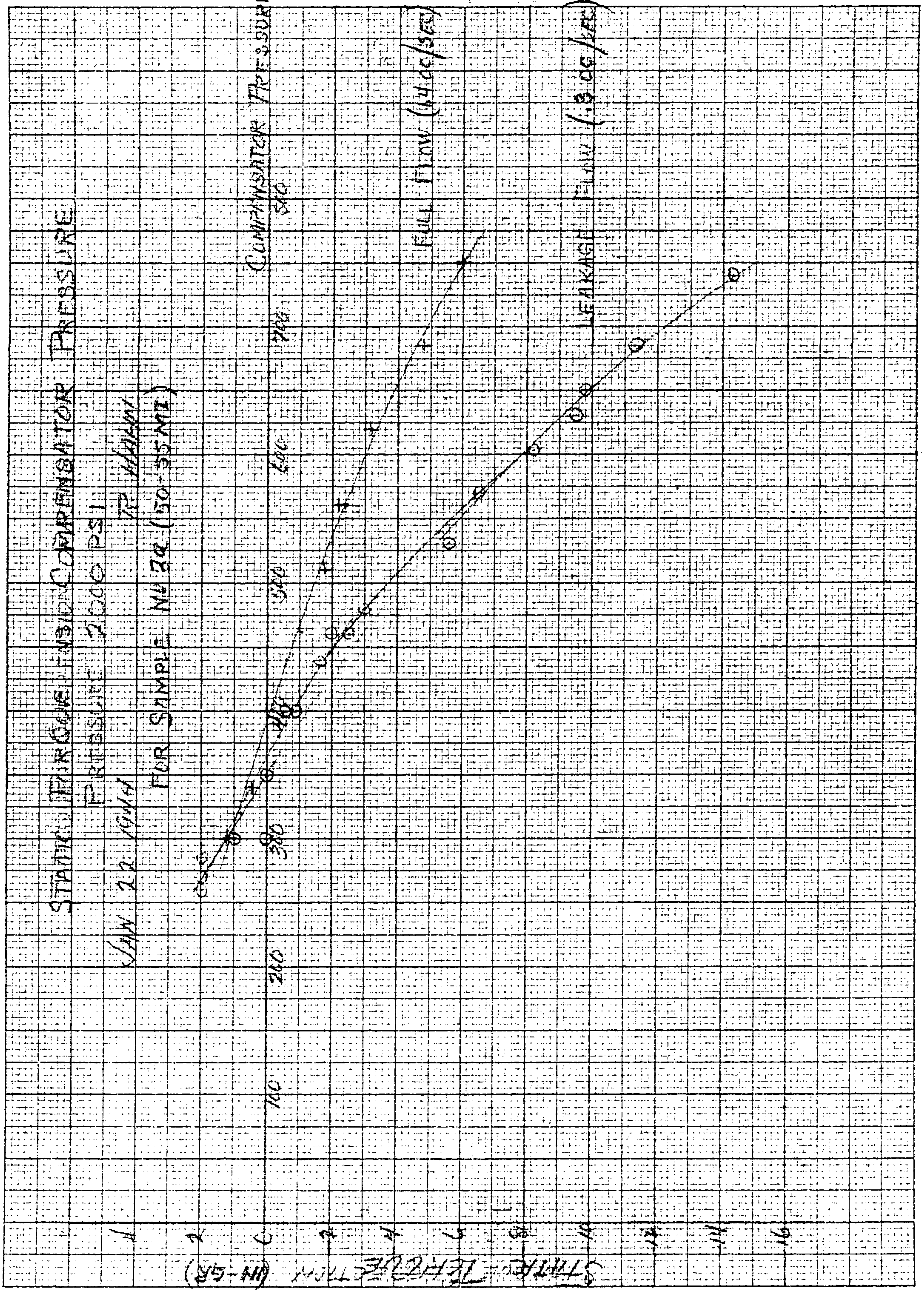
Form 63—Applied Sc.—500—5-41

M-48

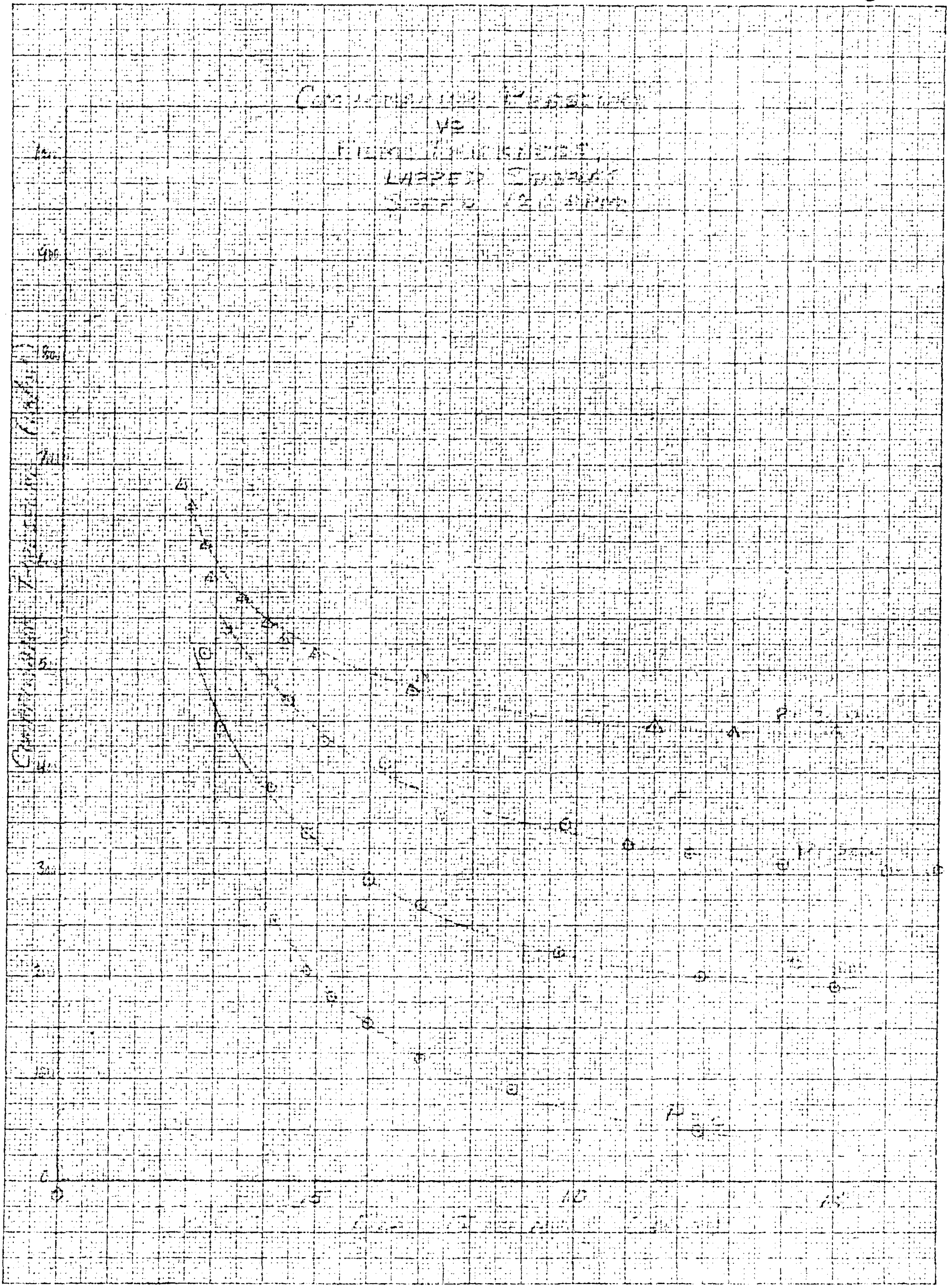


Form 63 – Applied Sc.—500—5-41

M-58

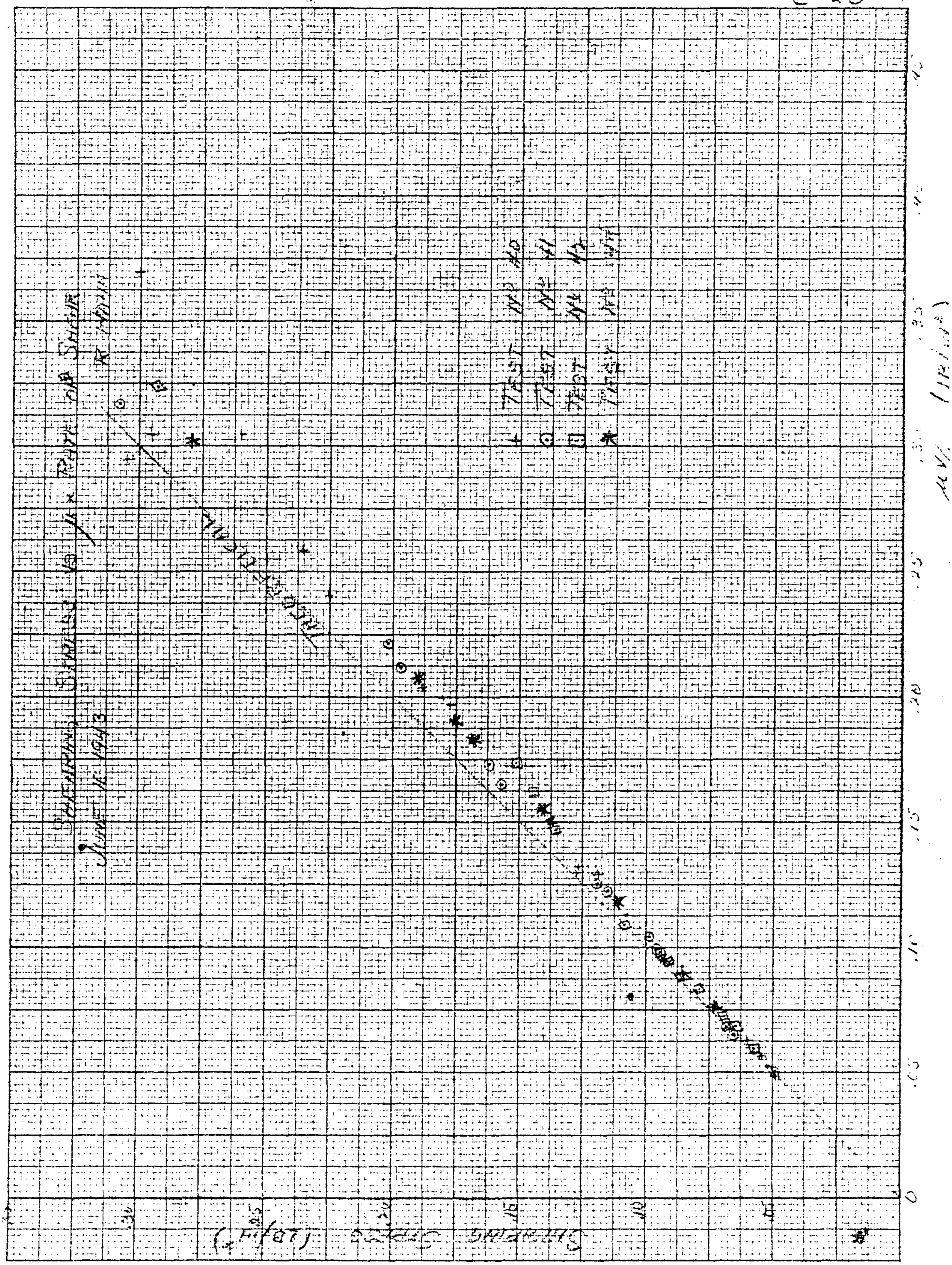

STATIC TORQUE VS COMPENSATOR PRESSURE
PRESSURE 2000 PSI
FOR SAMPLE No 2a (50-55 MI)
COMPENSATOR PRESSURE
100
200
300
400
500
600
700
800
FULL FLOW (.14 cc/SEC)
LEAKAGE FLOW (.3 cc/SEC)
2
0
2
4
6
8
10
12
14
16

C-8



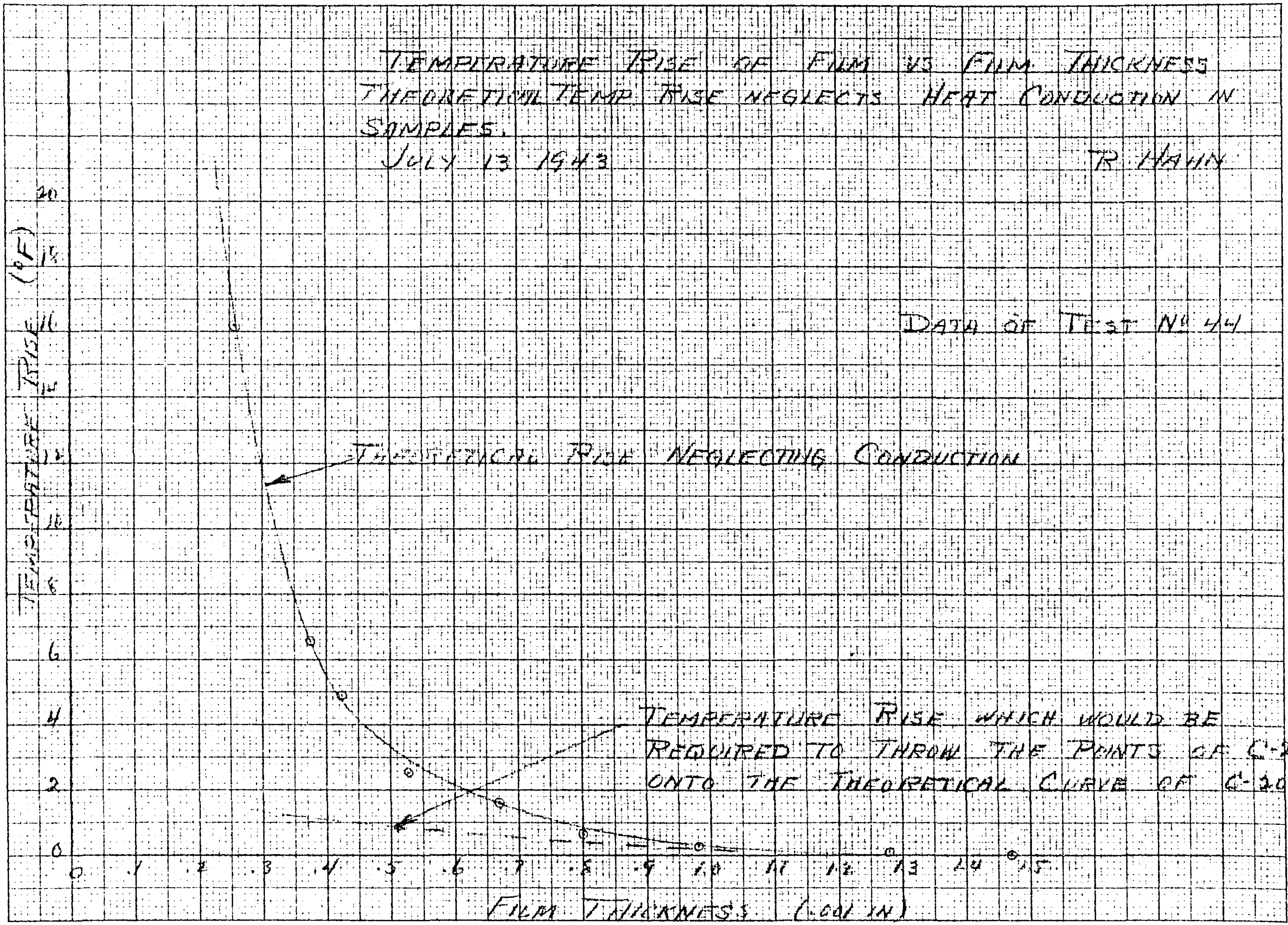
Form 63—Applied Sc.—500—5-41

C-20



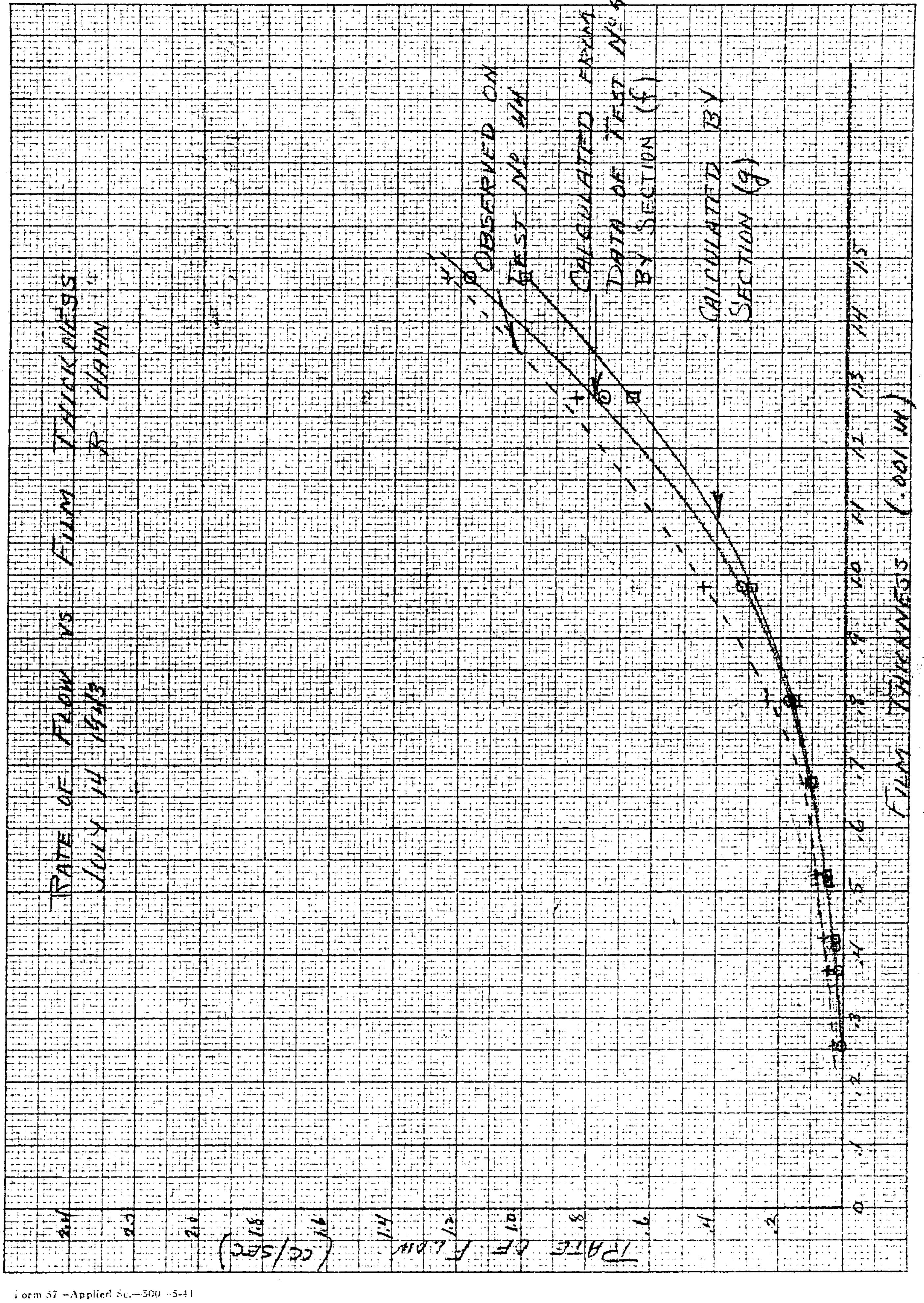
Form 57—Applied Sc.—500—5-41

MADE IN U.S.A.

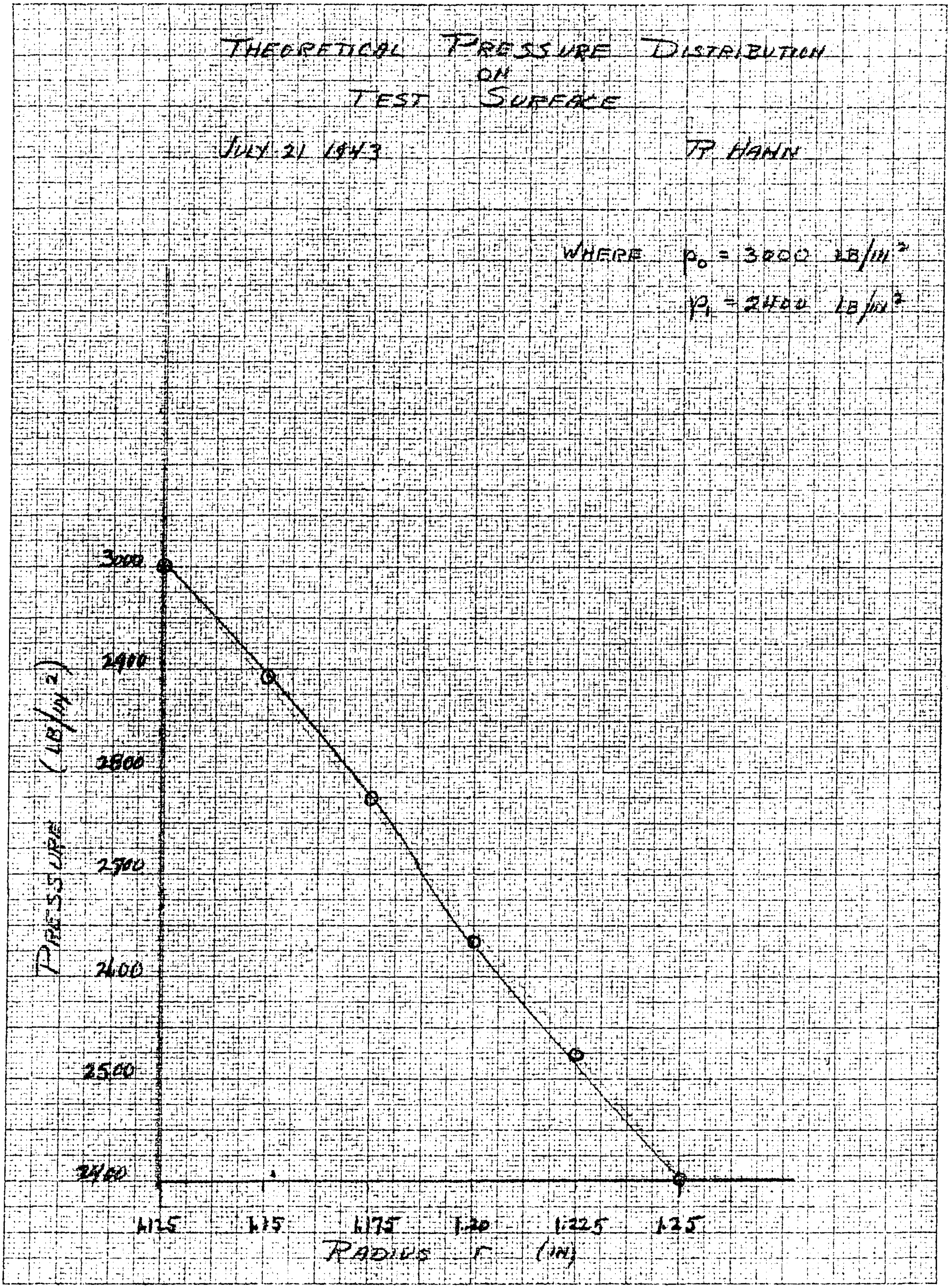


TEMPERATURE RISE OF FILM VS FILM THICKNESS
THEORETICAL TEMP RISE NEGLECTS HEAT CONDUCTION IN SAMPLES.
JULY 13 1943
R HAHN
DATA OF TEST No 44
THEORETICAL RISE NEGLECTING CONDUCTION
TEMPERATURE RISE WHICH WOULD BE REQUIRED TO THROW THE POINTS OF C-20 ONTO THE THEORETICAL CURVE OF C-20
TEMPERATURE RISE (°F)
FILM THICKNESS (.001 IN)
0
2
4
6
20
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
1.5

C-24

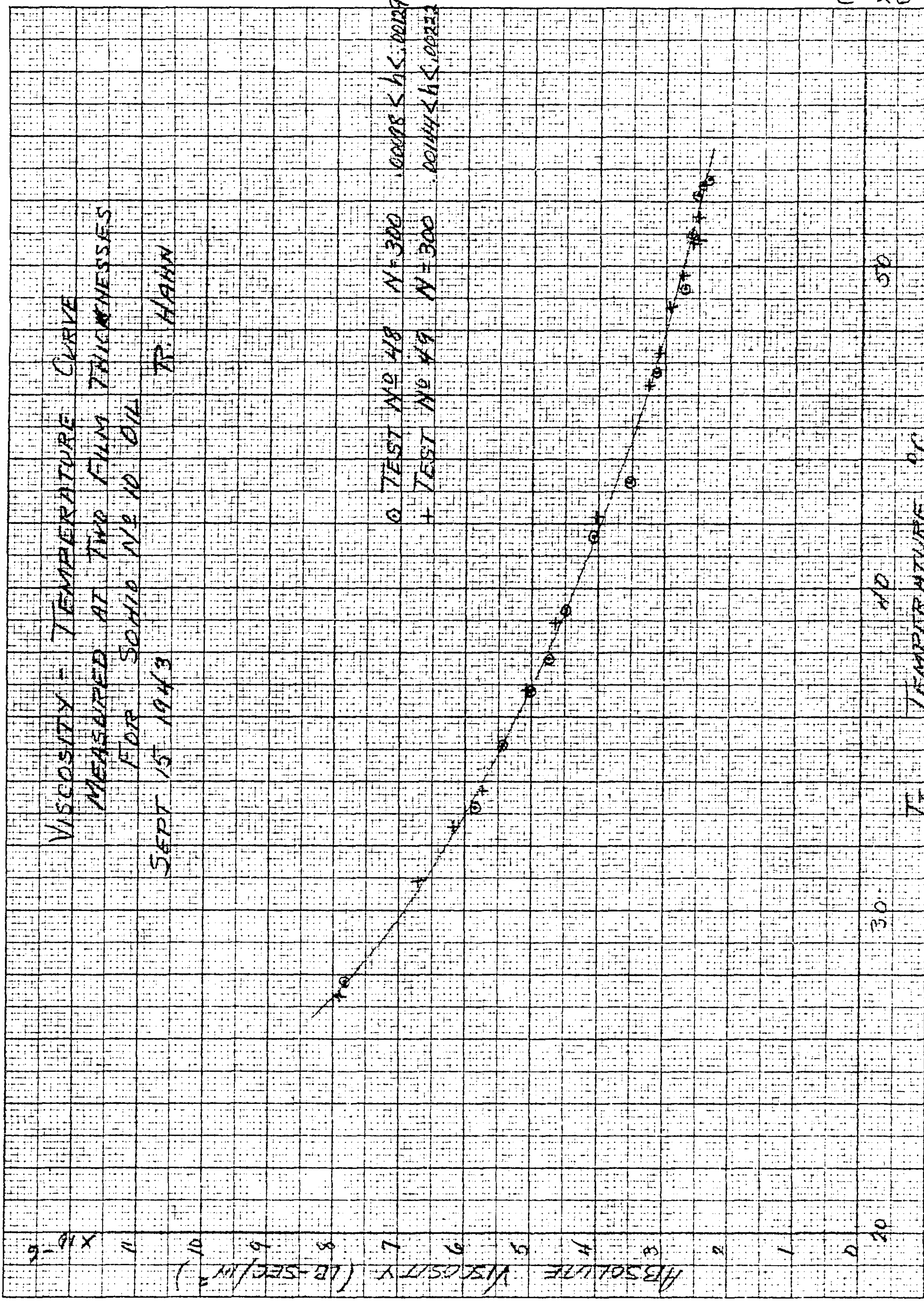


Form 57 —Applied Sc.—500 —5-41


THEORETICAL PRESSURE DISTRIBUTION
ON
TEST SURFACE
JULY 21 1943
T. P. HAHN
WHERE p_o = 3000 LB/IN^2
p_1 = 2400 LB/IN^2
PRESSURE (LB/IN^2)
3000
2900
2800
2700
2600
2500
2400
1.125
1.15
1.175
1.20
1.225
1.25
RADIUS r (IN)

APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

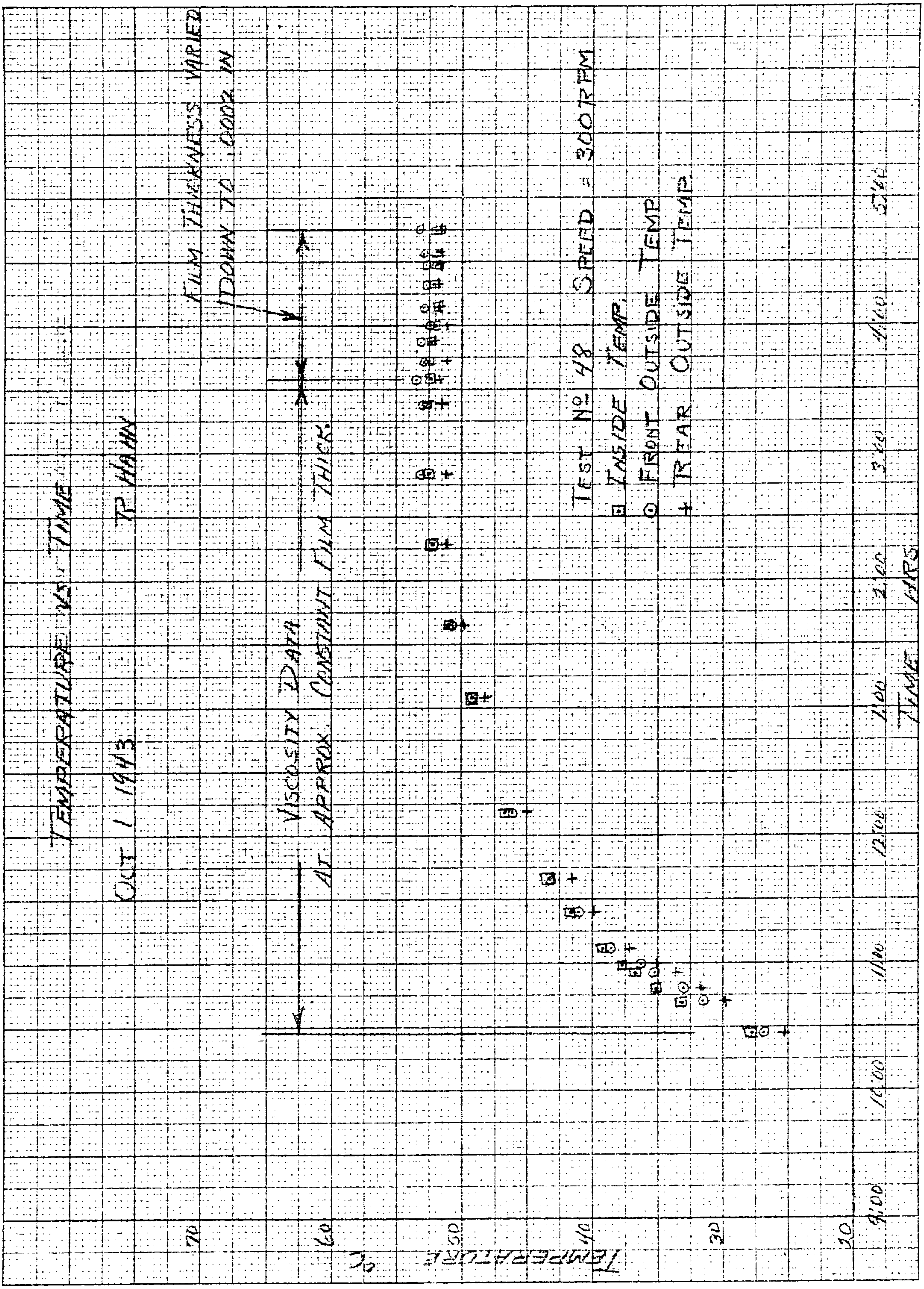
C-26



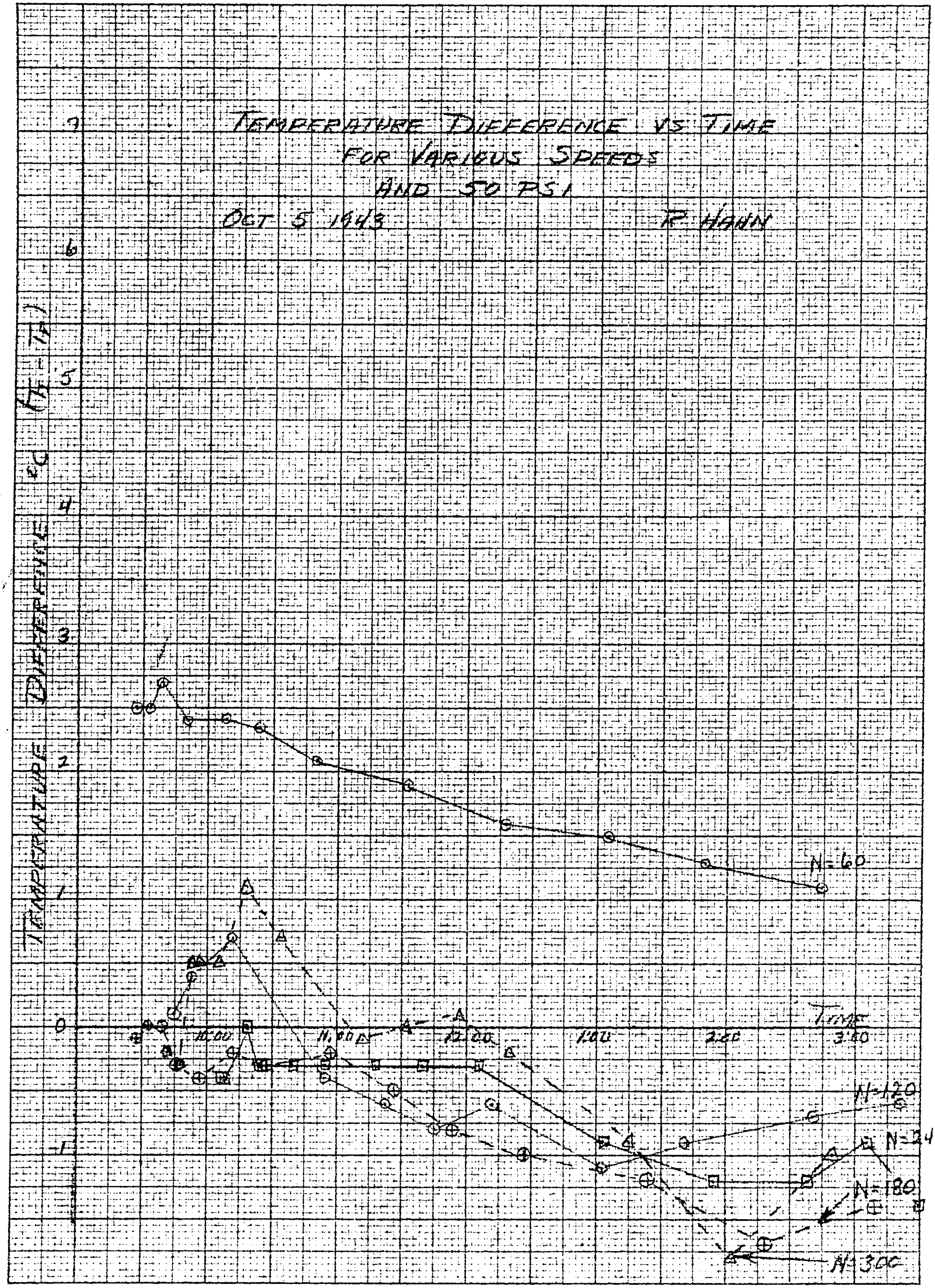
MADE IN U.S.A.

Form 57— Applied Sc —300 5-41

C-28


TEMPERATURE VS. TIME
OCT 1 1943
P HAHN
VISCOSITY DATA
AT APPROX. CONSTANT FILM THICK.
FILM THICKNESS VARIED
DOWN TO .0002 IN
TEST No 48 SPEED = 300 RPM
□ INSIDE TEMP.
⊙ FRONT OUTSIDE TEMP
+ REAR OUTSIDE TEMP
TIME HRS
TEMPERATURE °C
70
60
50
40
30
20
9:00
10:00
11:00
12:00
1:00
2:00
3:00
4:00
5:00

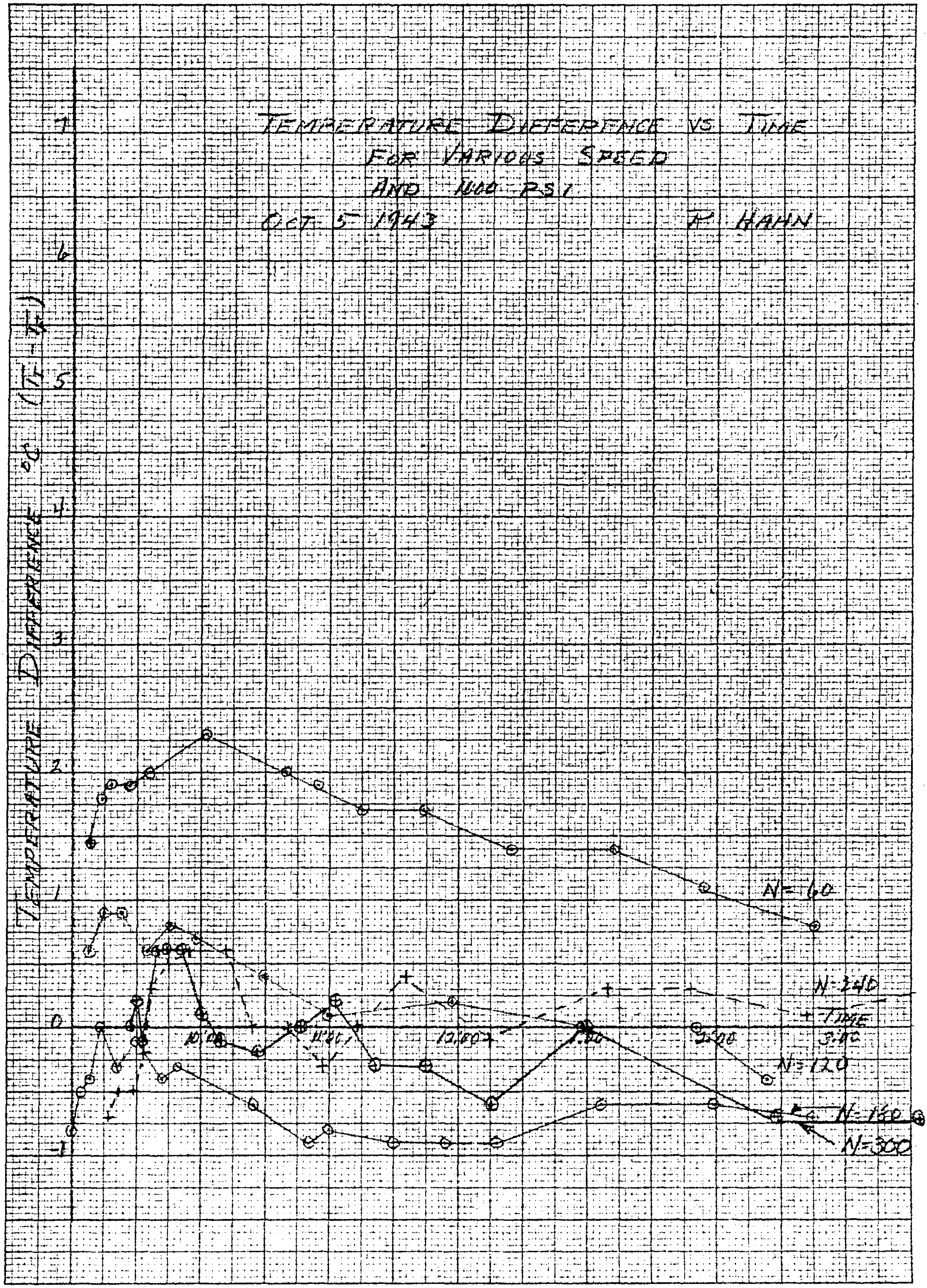
C-31



Form 57—Applied Sc.—500—5-41

MADE IN U.S.A.

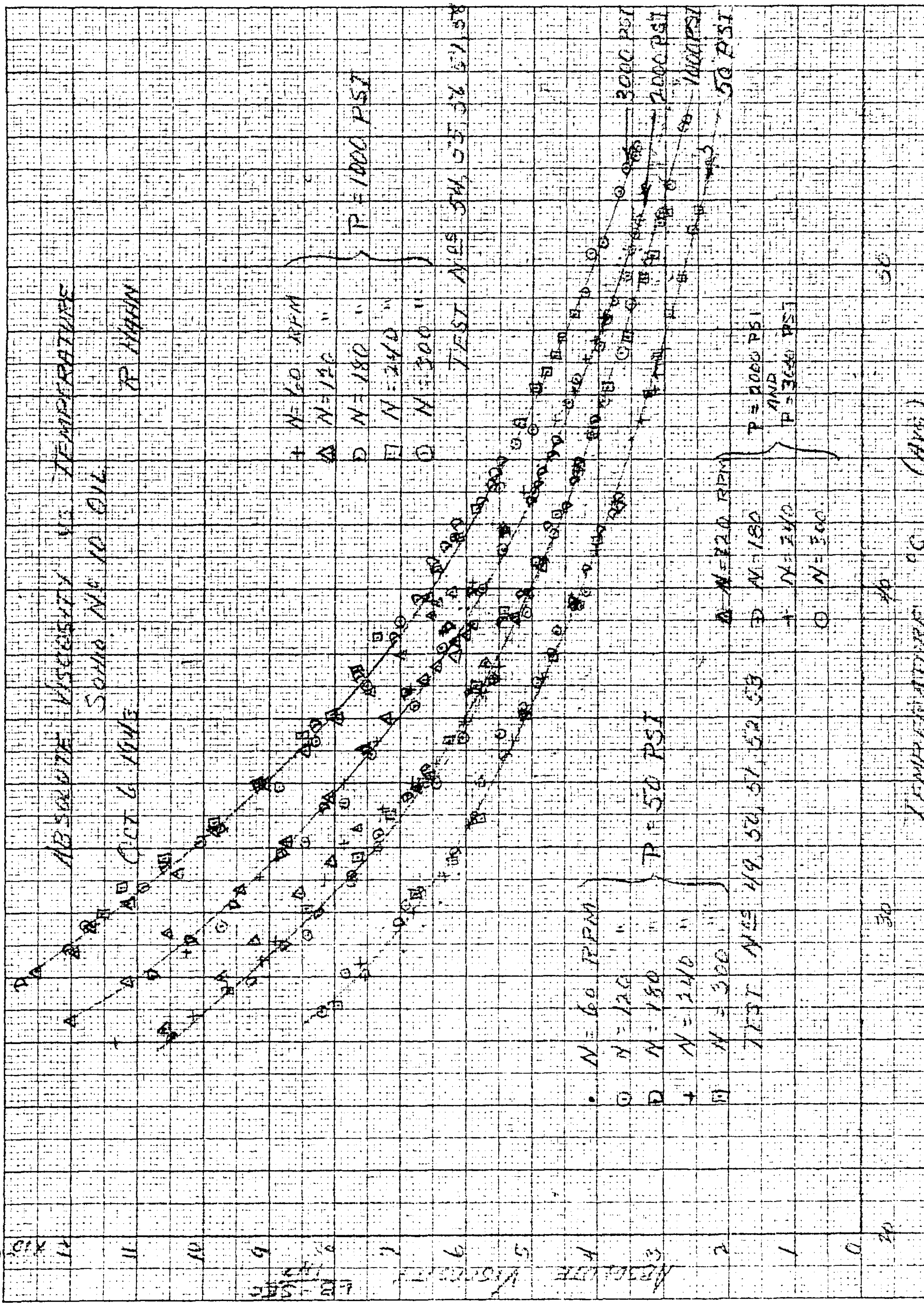
C-32



Form 57—Applied Sc.—500—5-41

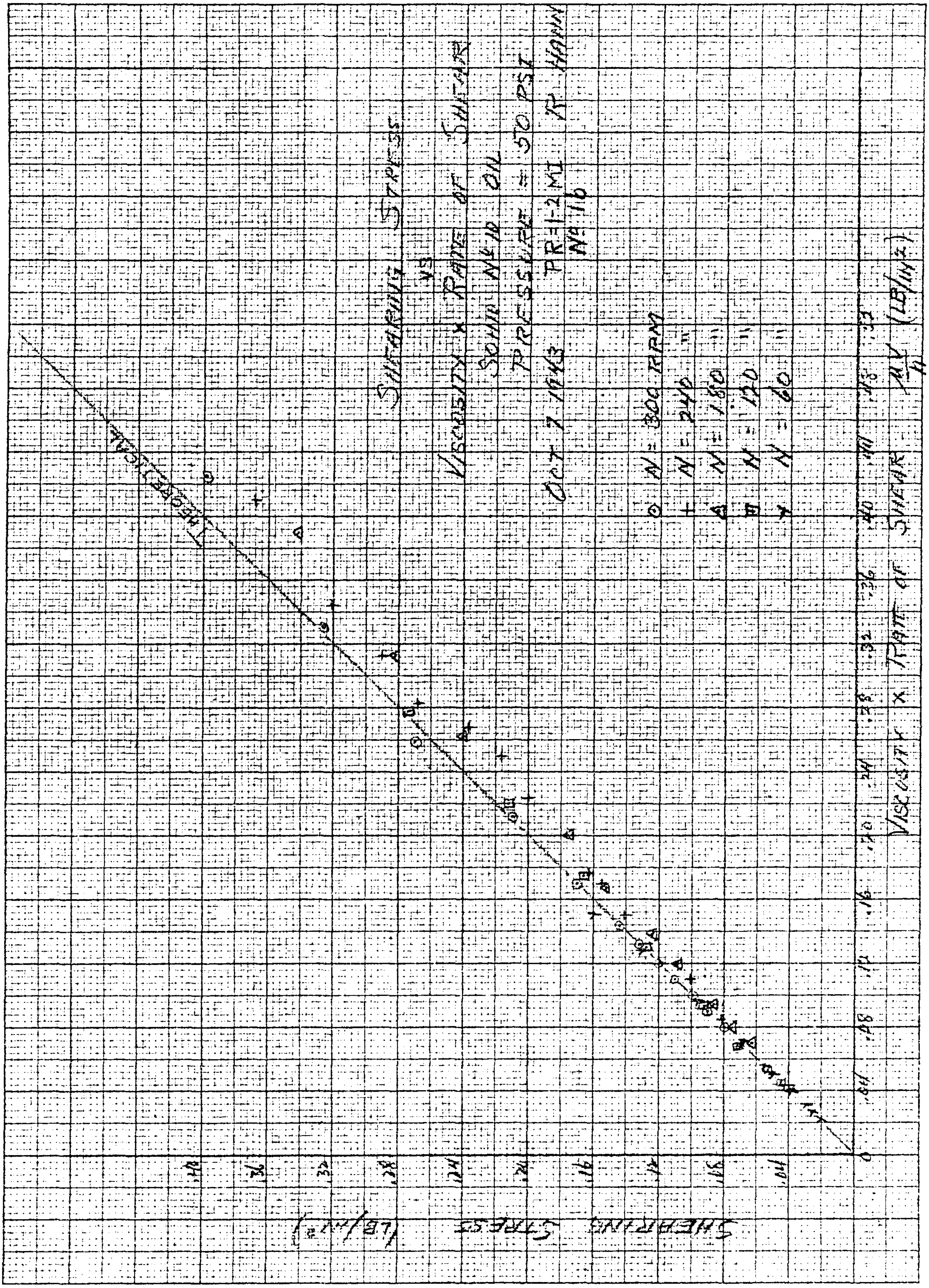
APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

C-34



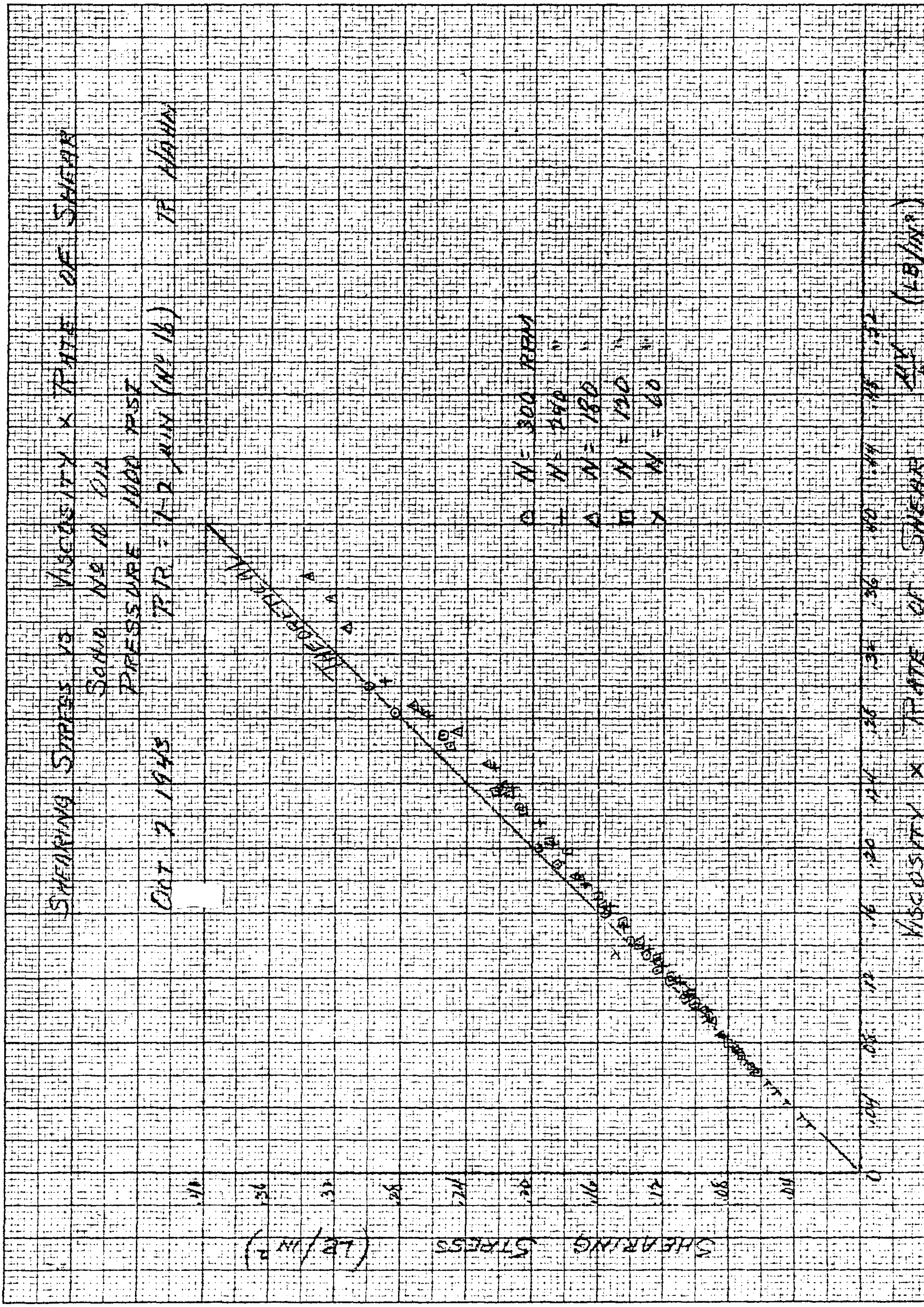
Form 57 - Applied Co. - 5M - 5-41

C-35



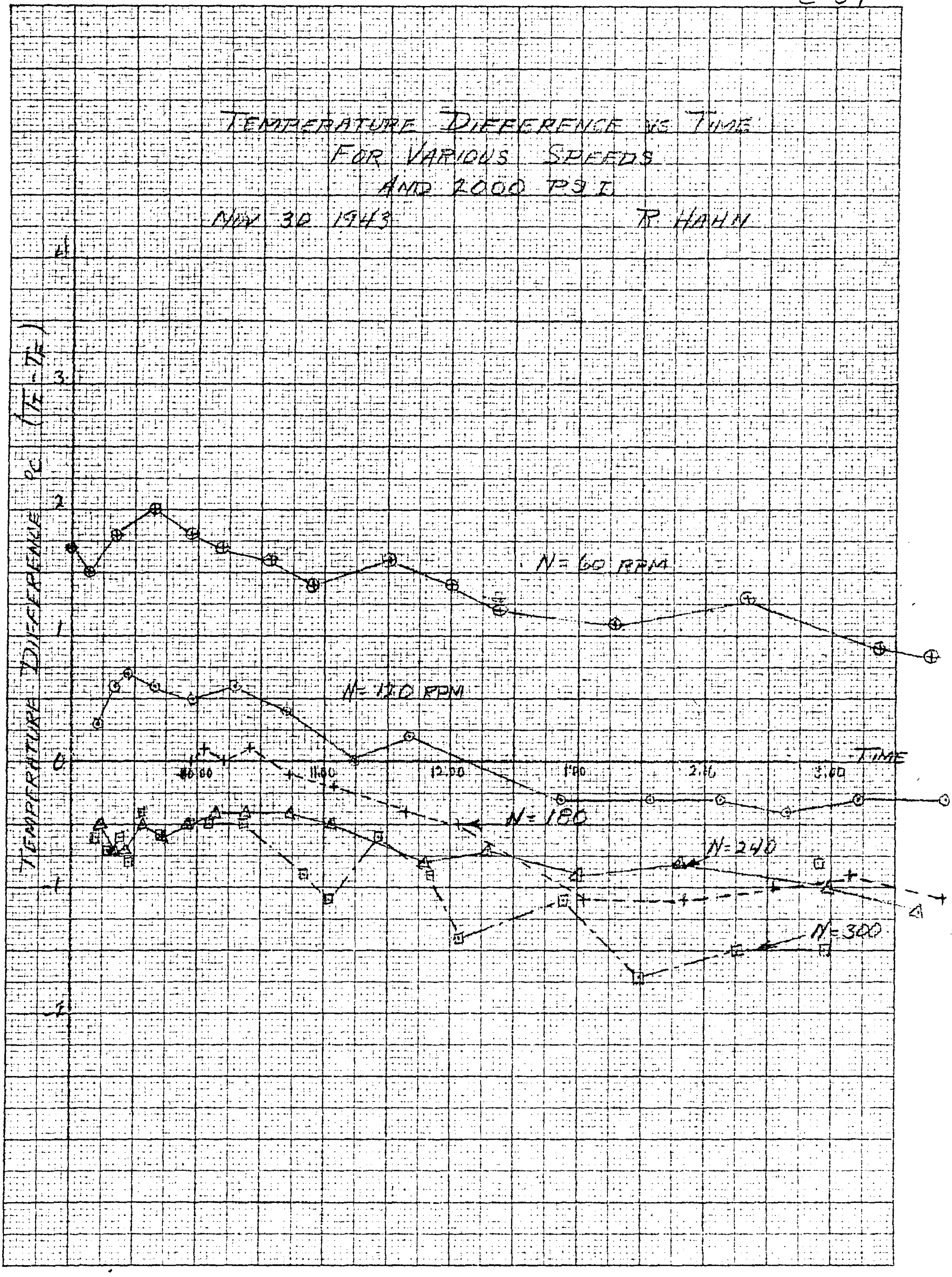
Form 57—Applied Sc.—500—5-41

C-36


SHEARING STRESS vs VISCOSITY x RATE OF SHEAR
SOHIO No 10 OIL
PRESSURE 1000 PSI
RR = 1-2, RUN (No 16)
OCT 7 1943
F HAHN
○ N = 300 RPM
+ N = 240 "
△ N = 180 "
□ N = 120 "
Y N = 60 "
VISCOSITY x RATE OF SHEAR ZN/h (LB/IN²)
SHEARING STRESS (LB/IN²)

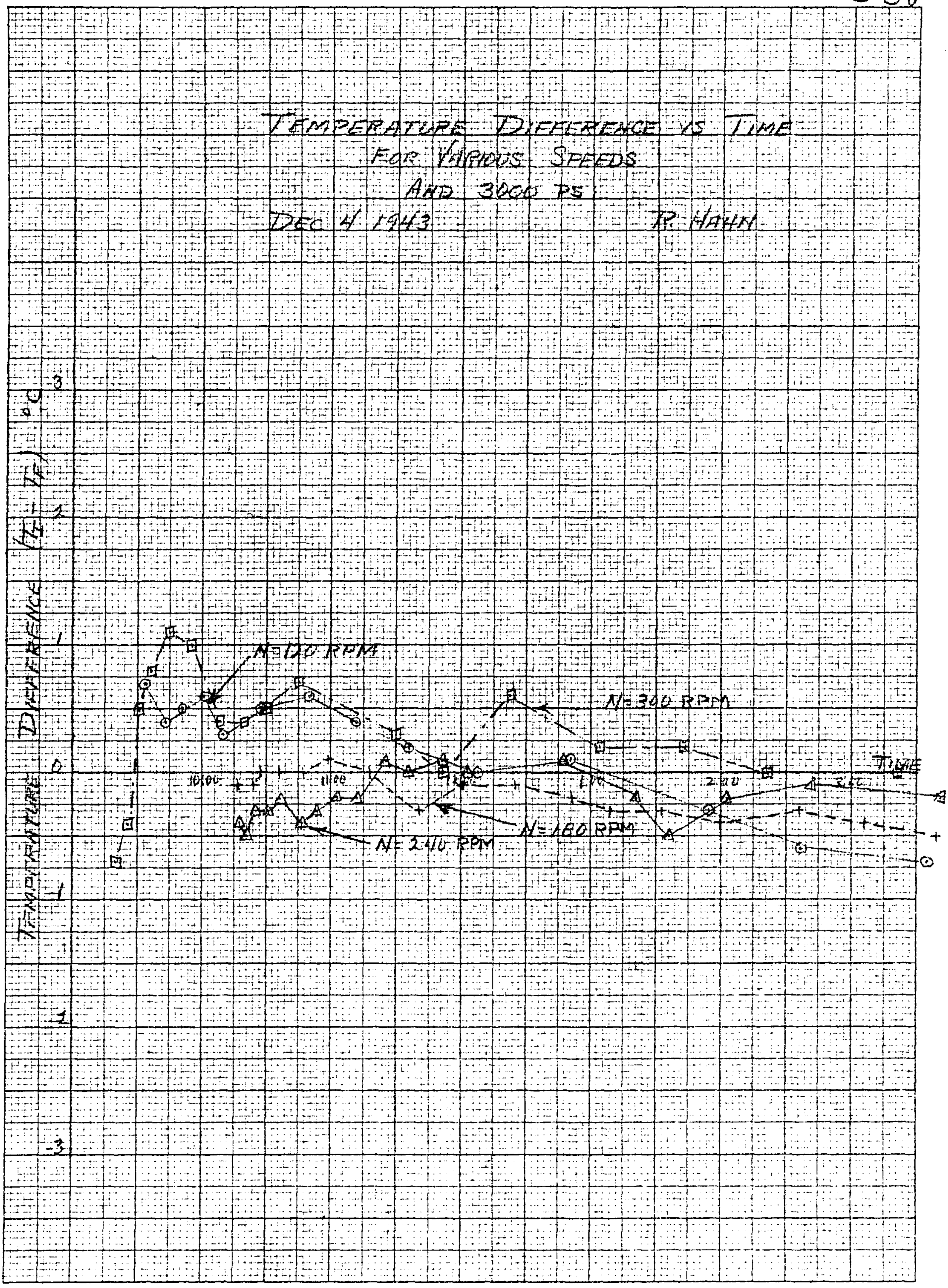
APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

C-37



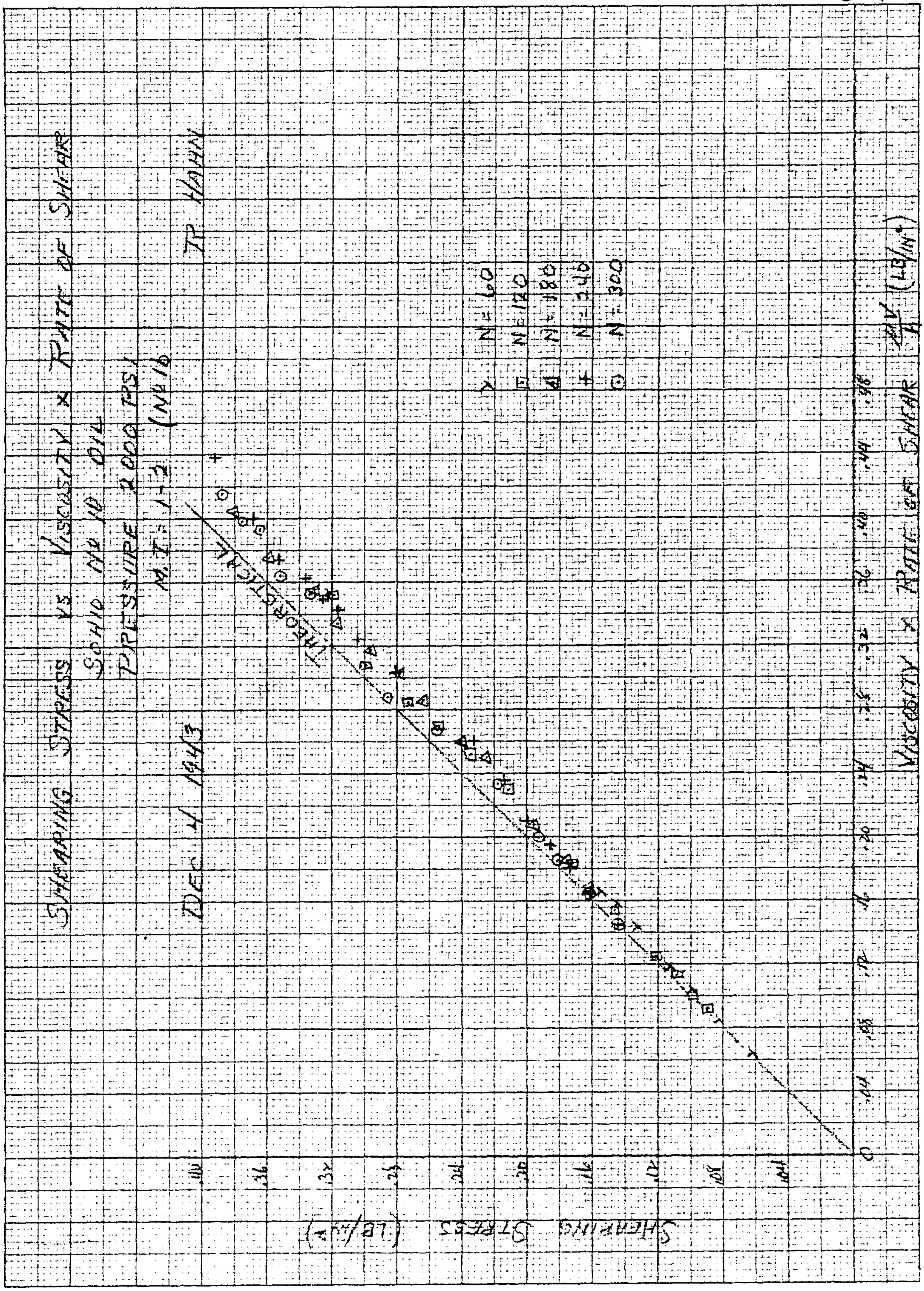
Form 57 — Applied Sc.—500—5-41

C-38



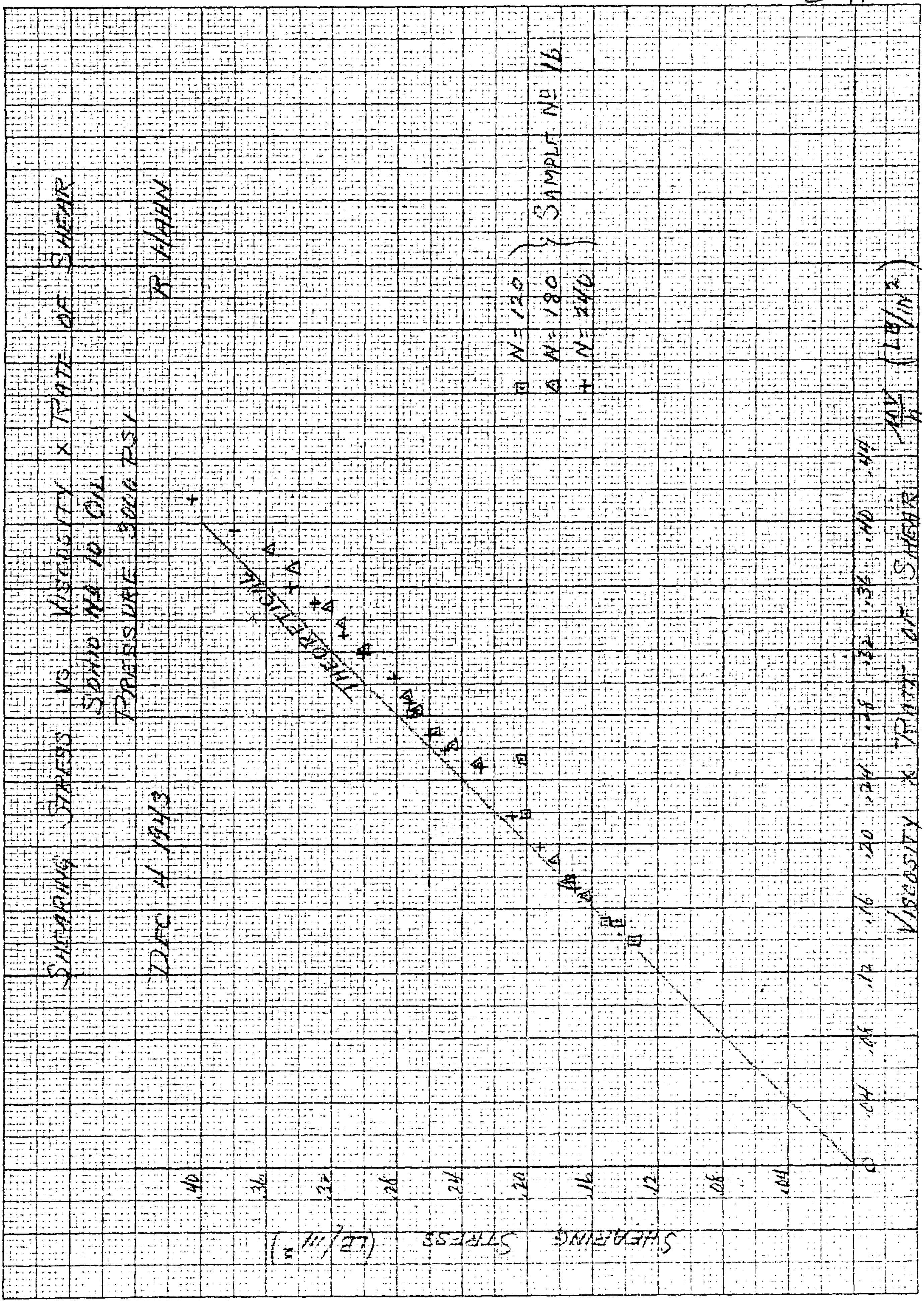
Form 57—Applied Sc.--500--5-41

C-40



Form 57 - Applied Sc. 500-5-41

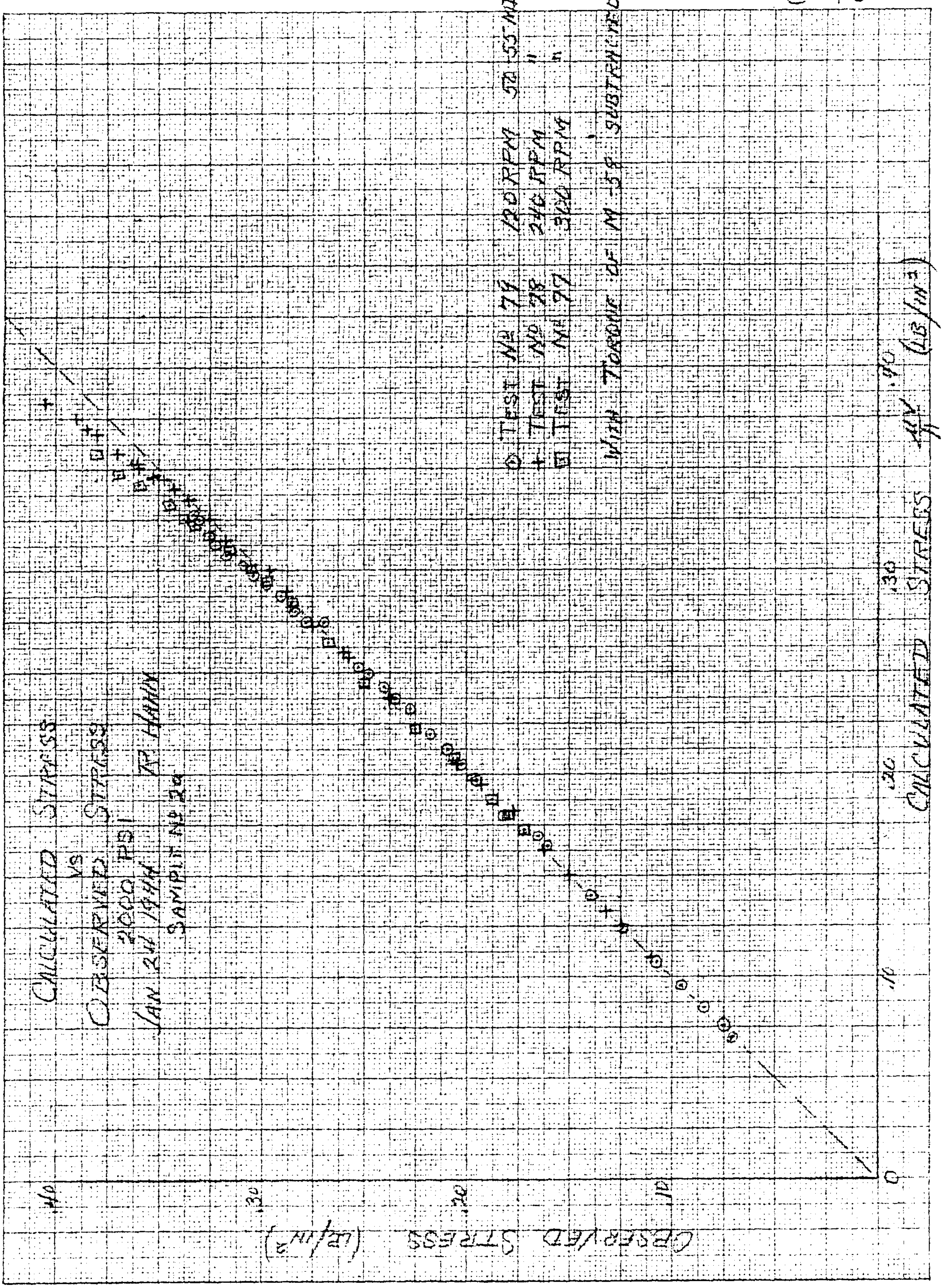
C-41



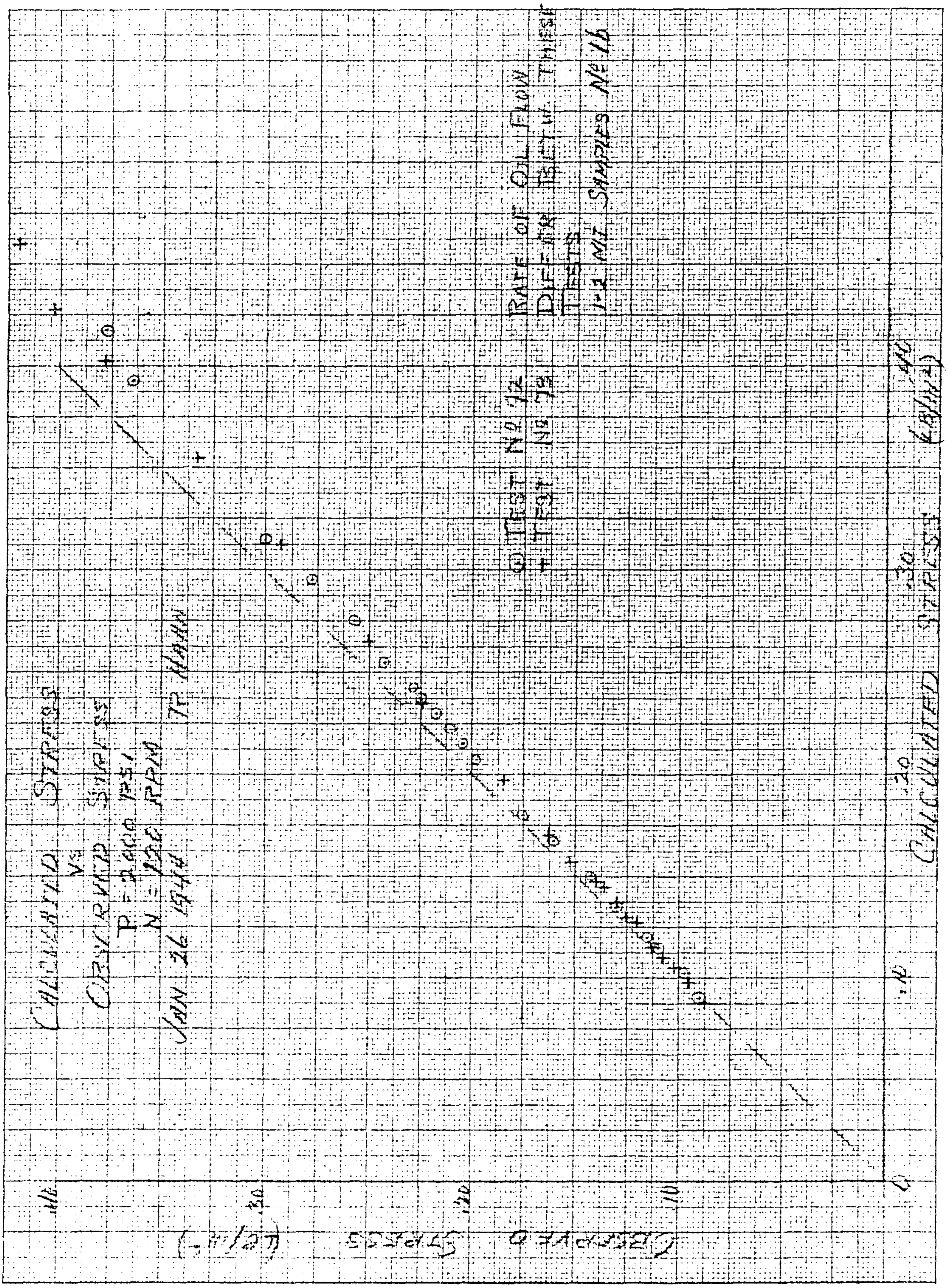
Form 57—Applied Sc.—500—5-41

APPLIED SCIENCE RESEARCH LABORATORY
UNIVERSITY OF CINCINNATI

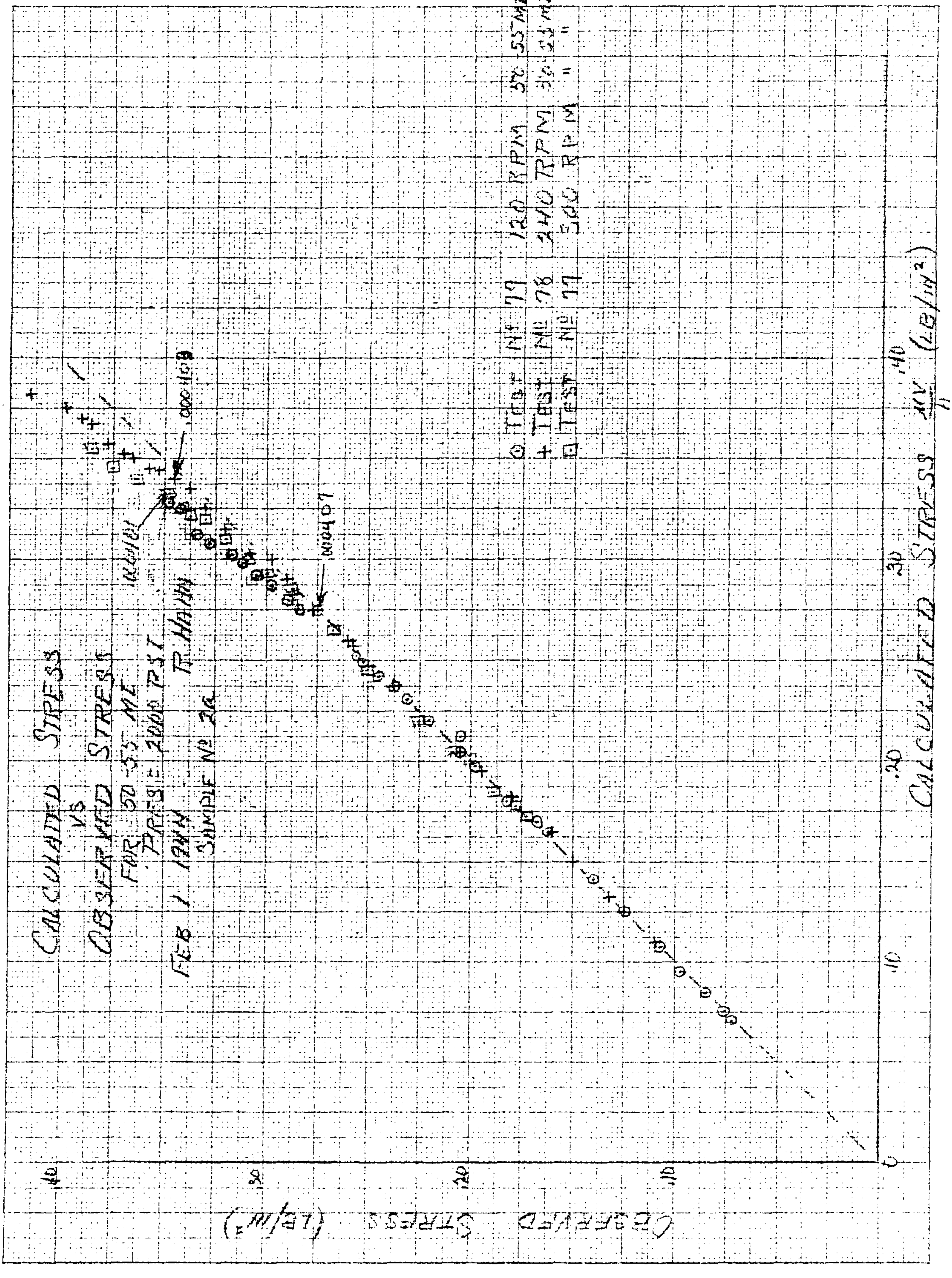
C-46



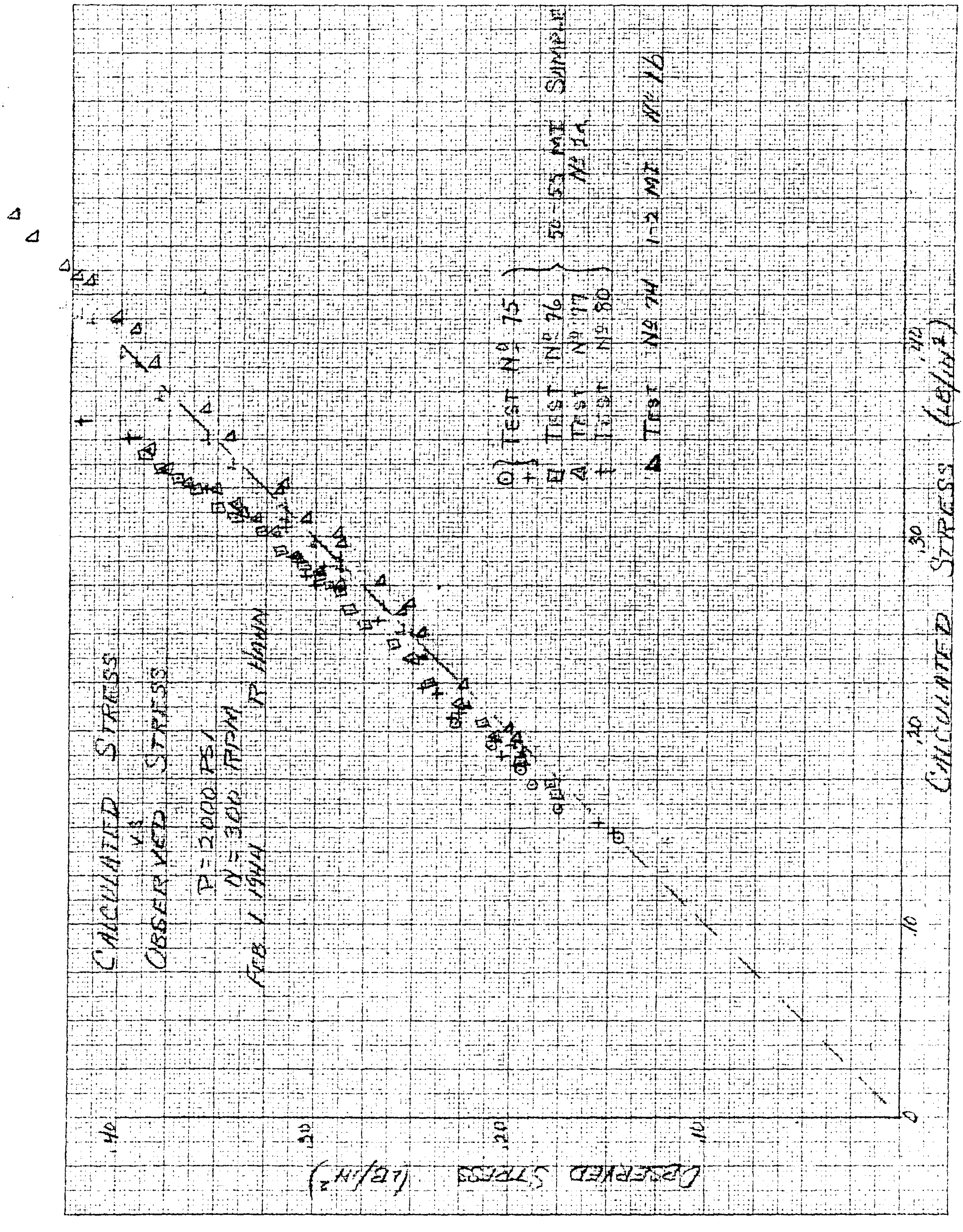
C-47


CALCULATED STRESS
VS
OBSERVED STRESS
P = 2400 PSI
N = 120 RPM
JAN 26 1944
TP HAHN
⊙ TEST No 12
+ TEST No 72
RATE OF OIL FLOW
DIFF'N BET'N THESE
TESTS
1-2 MI SAMPLES No 16
.10
.20
.30
.40
CALCULATED STRESS (LB/IN²)
.10
.20
.30
OBSERVED STRESS (LB/IN²)

C-48

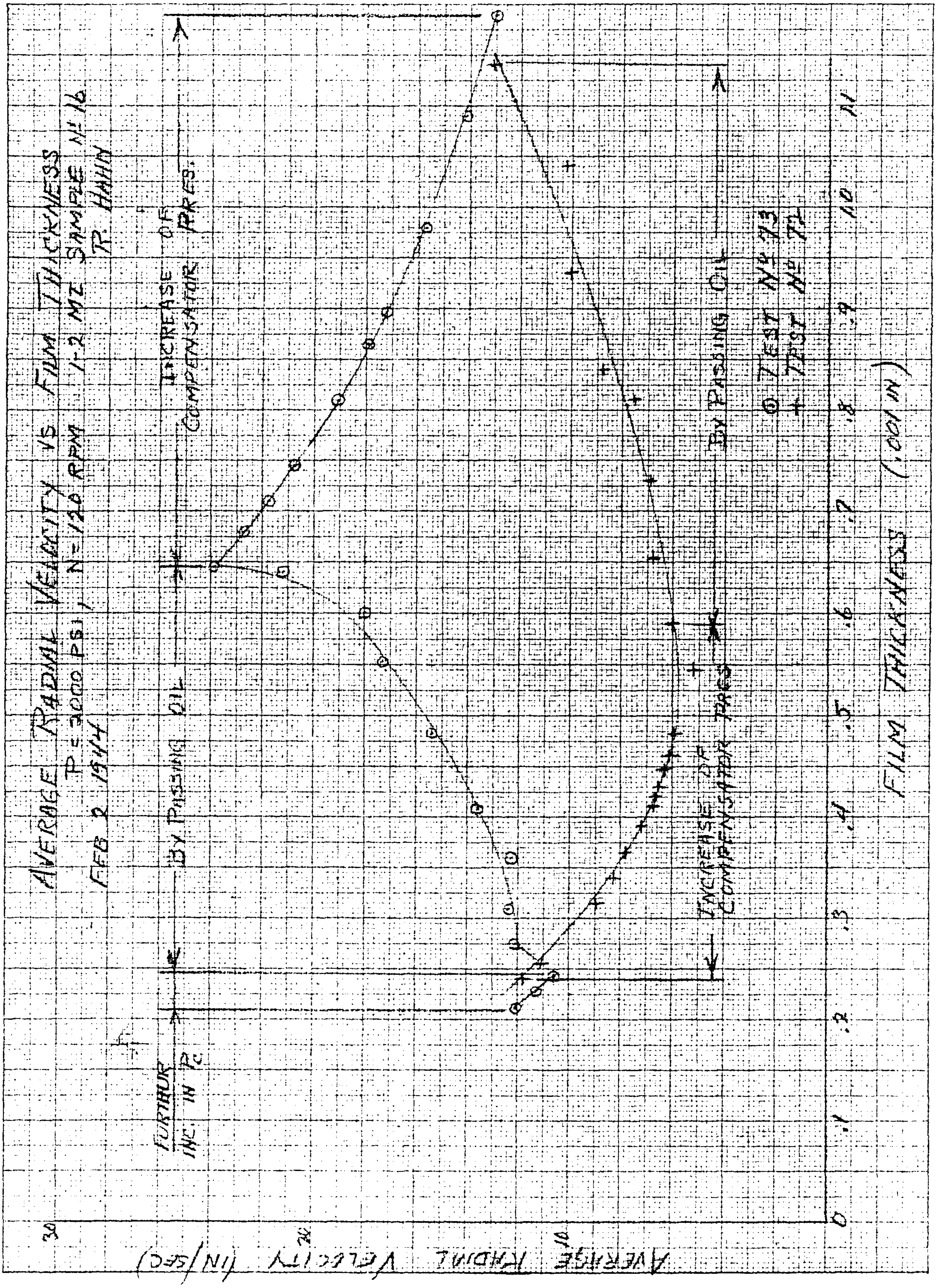

CALCULATED STRESS
VS
OBSERVED STRESS
FOR 50-55 ME
PRES = 2000 PSI
FEB 1 1944 R. HAHN
SAMPLE № 26
⊙ TEST № 77 120 RPM 50-55 ME
+ TEST № 78 240 RPM 50-55 ME
⊡ TEST № 77 500 RPM " "
CALCULATED STRESS AV (LB/IN²)
OBSERVED STRESS (LB/IN²)
0
10
20
30
40

C-47



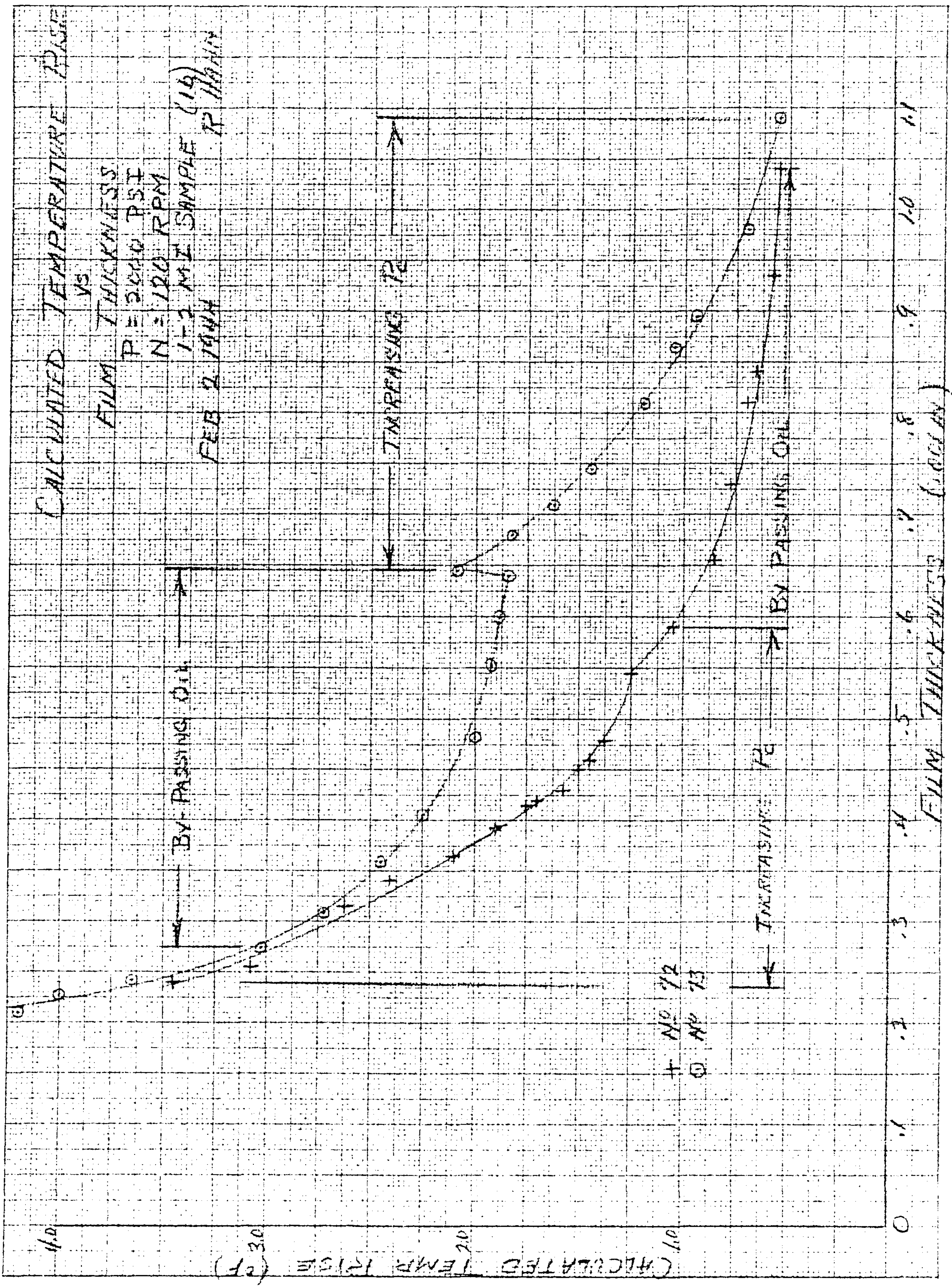
Applied Sciences—1000—6-43

C-50

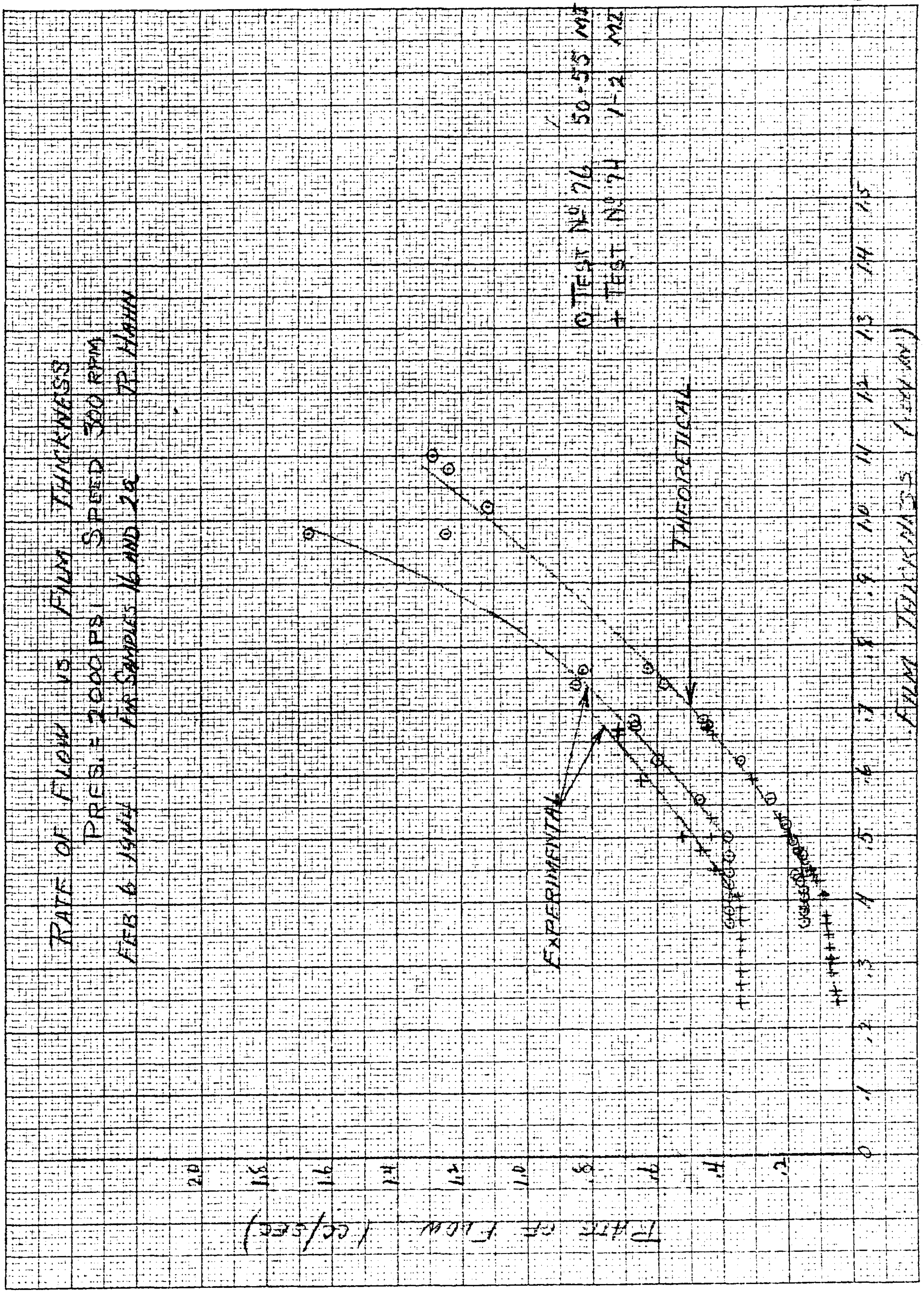


Applied Science 1000-6-43

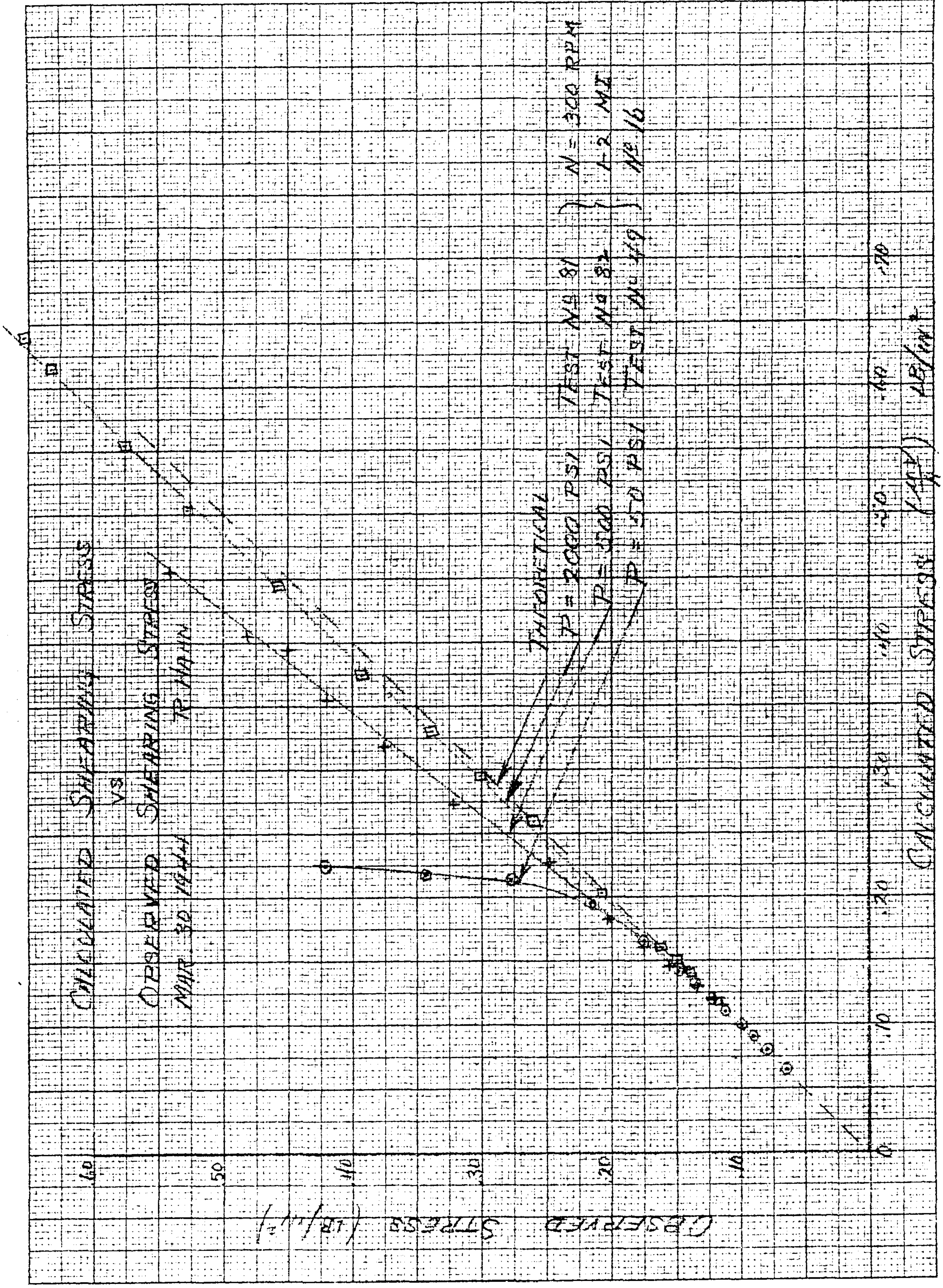
C - 51


CALCULATED TEMPERATURE RISE
VS
FILM THICKNESS
P = 2000 PSI
N = 120 RPM
1-2 MIL SAMPLE (1B)
BY PASSING OIL
INCREASING P2
BY PASSING OIL
INCREASING P
+ No 72
⊙ No 73
FILM THICKNESS (.001 IN)
CALCULATED TEMP RISE (°F)
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.0
2.0
3.0
4.0

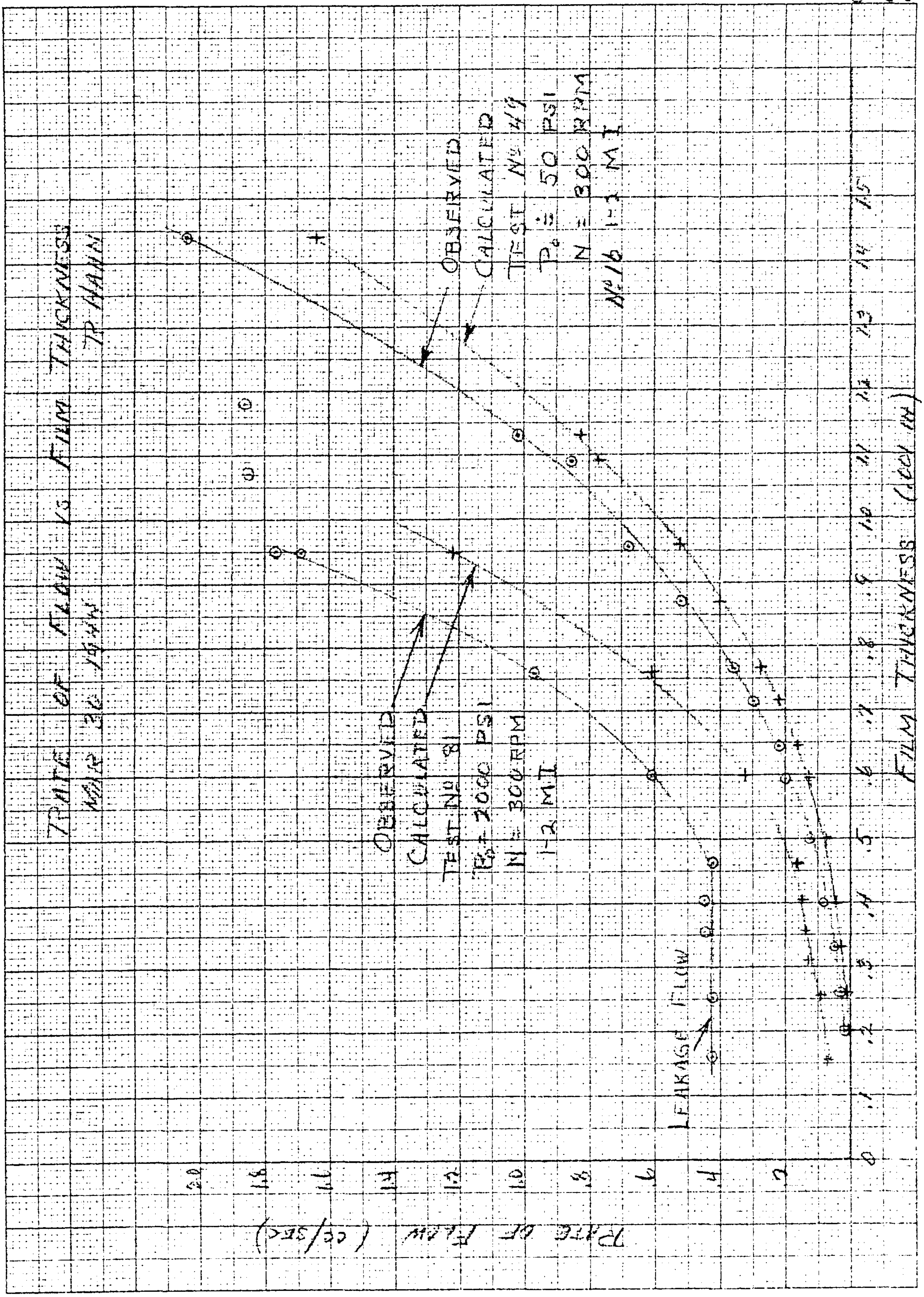
C-53


RATE OF FLOW VS FILM THICKNESS
PRES. = 2000 PSI SPEED 300 RPM
FEB 6 1944 FOR SAMPLES 16 AND 26 R. HAHN
⊙ TEST No. 76 50-55 MI
+ TEST No. 74 1-2 MI
THEORETICAL
EXPERIMENTAL
FILM THICKNESS (1/1000 IN.)
RATE OF FLOW (CC/SEC)
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4 1.5
.2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.0

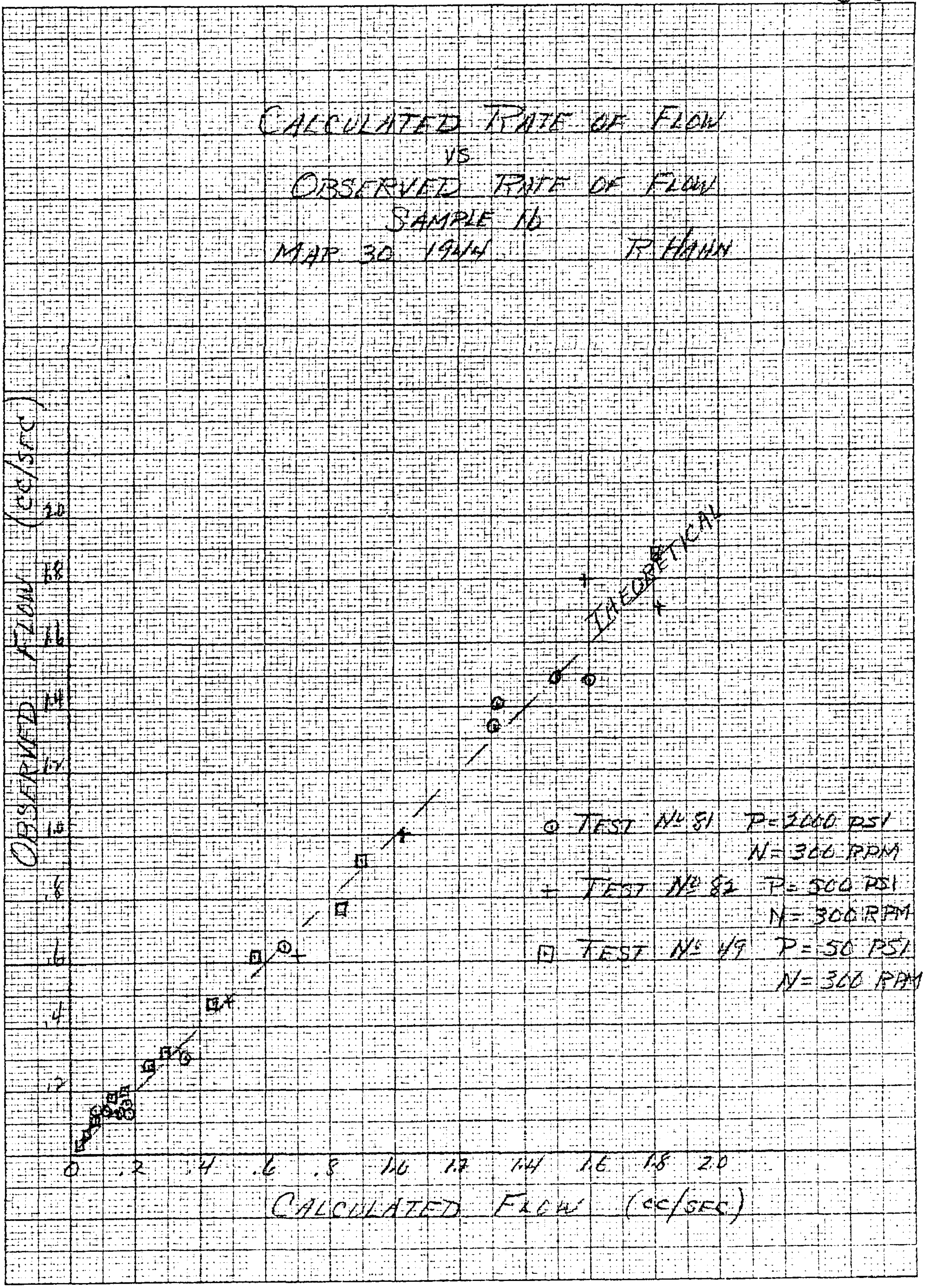
C-54



Form 57—Applied Sc.—500—5-41


RATE OF FLOW VS FILM THICKNESS
P HAHN
OBSERVED
CALCULATED
TEST No 49
P0 = 50 PSI
N = 300 RPM
No 16 1-3 MI
OBSERVED
CALCULATED
TEST No 81
P0 = 2000 PSI
N = 300 RPM
1-2 MI
LEAKAGE FLOW
FILM THICKNESS (.001 in)
RATE OF FLOW (cc/sec)

C-56



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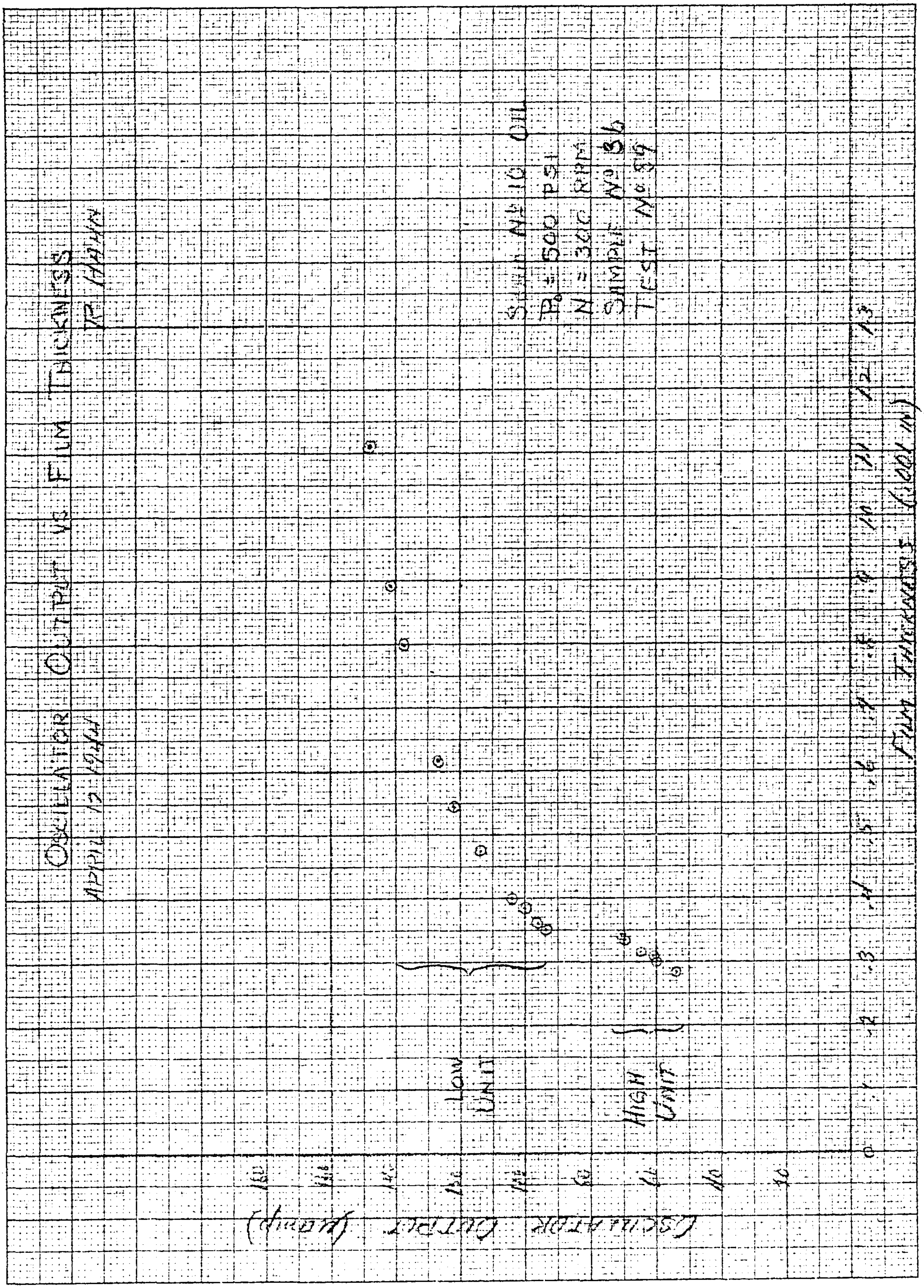
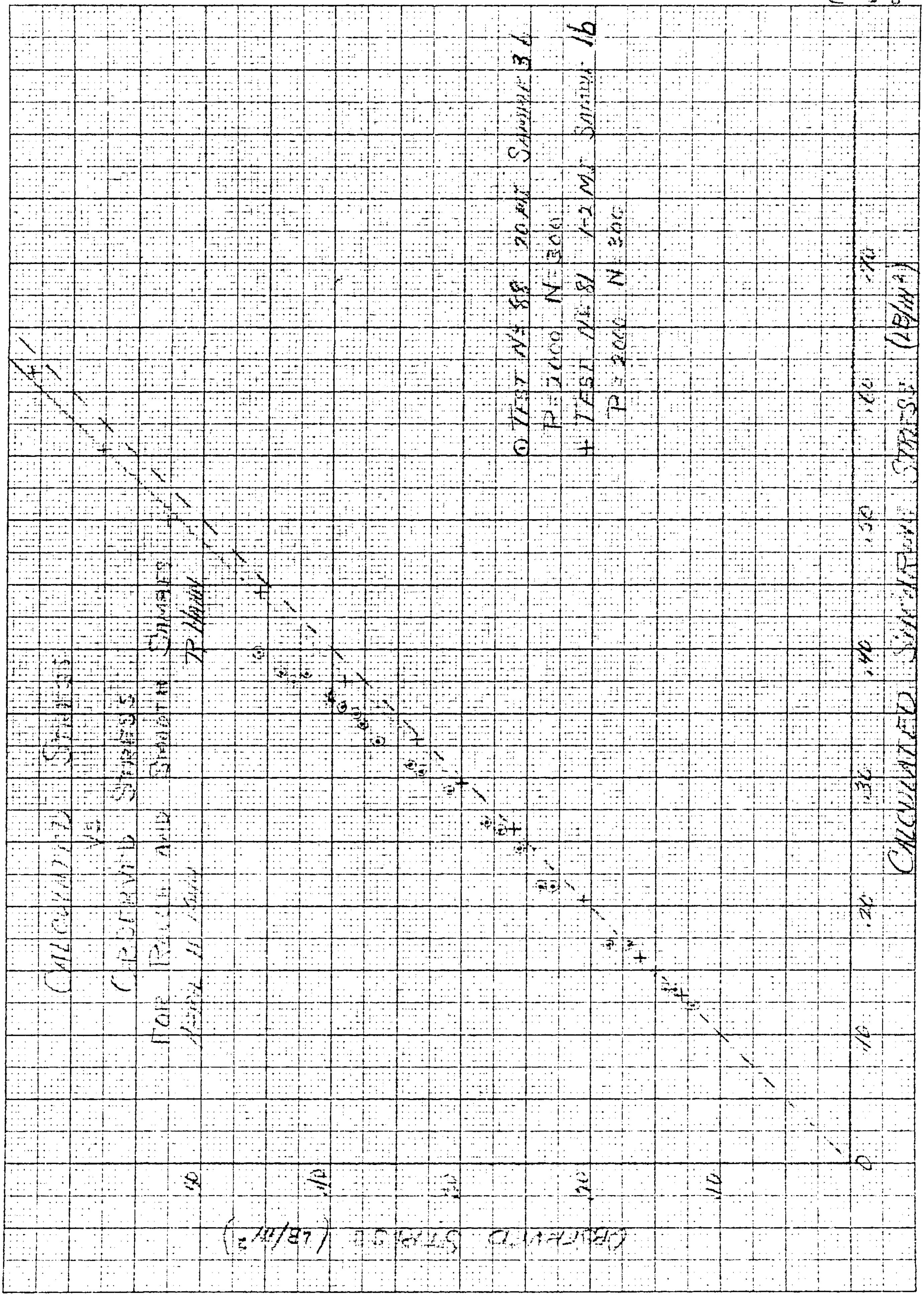
C-57



MADE IN U.S.A.

Form 57--Applied Sc.- 500--5-41

C-58



Form 87 - Applied Sci - 200—8-41

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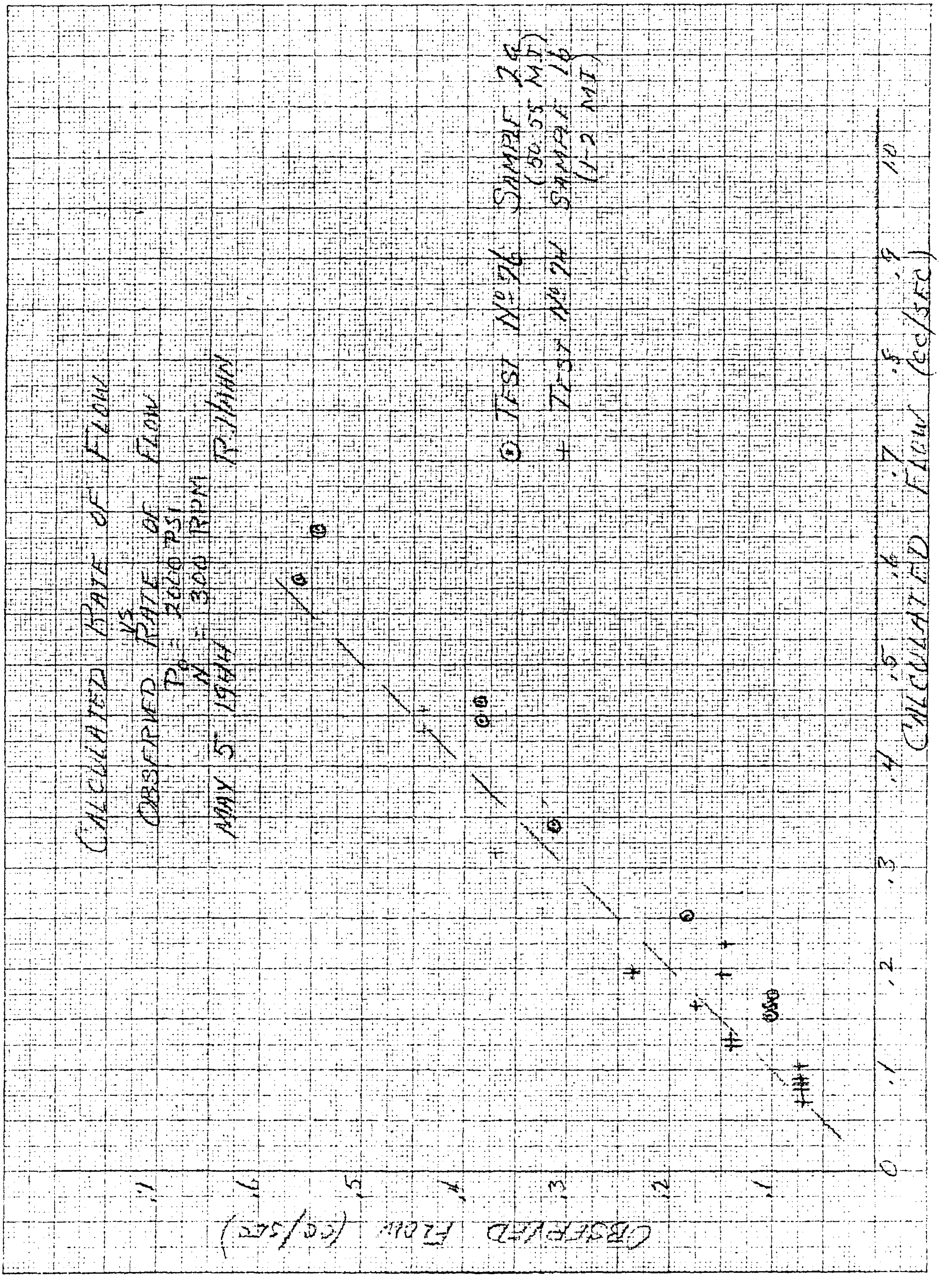
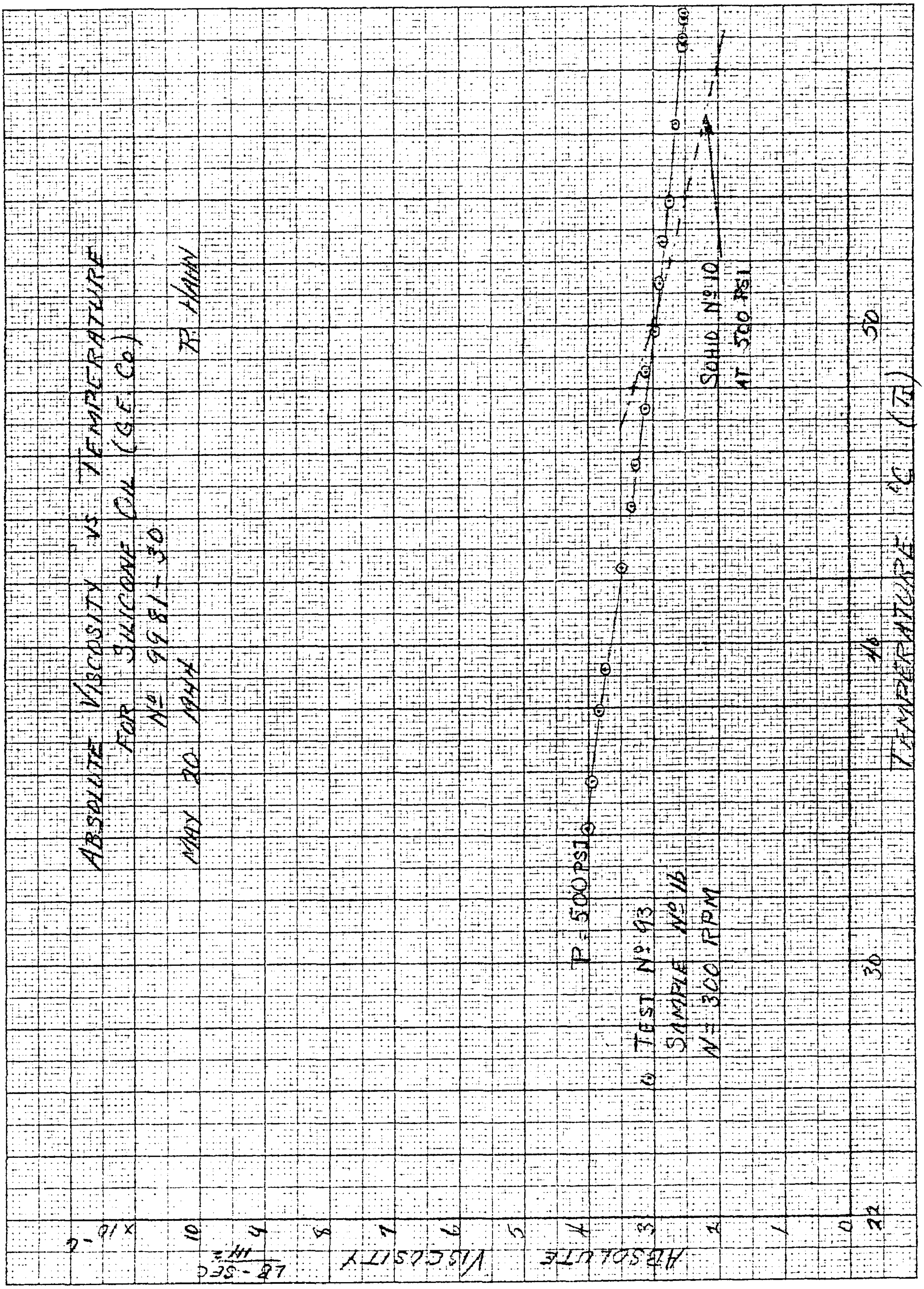
C-61



C-62



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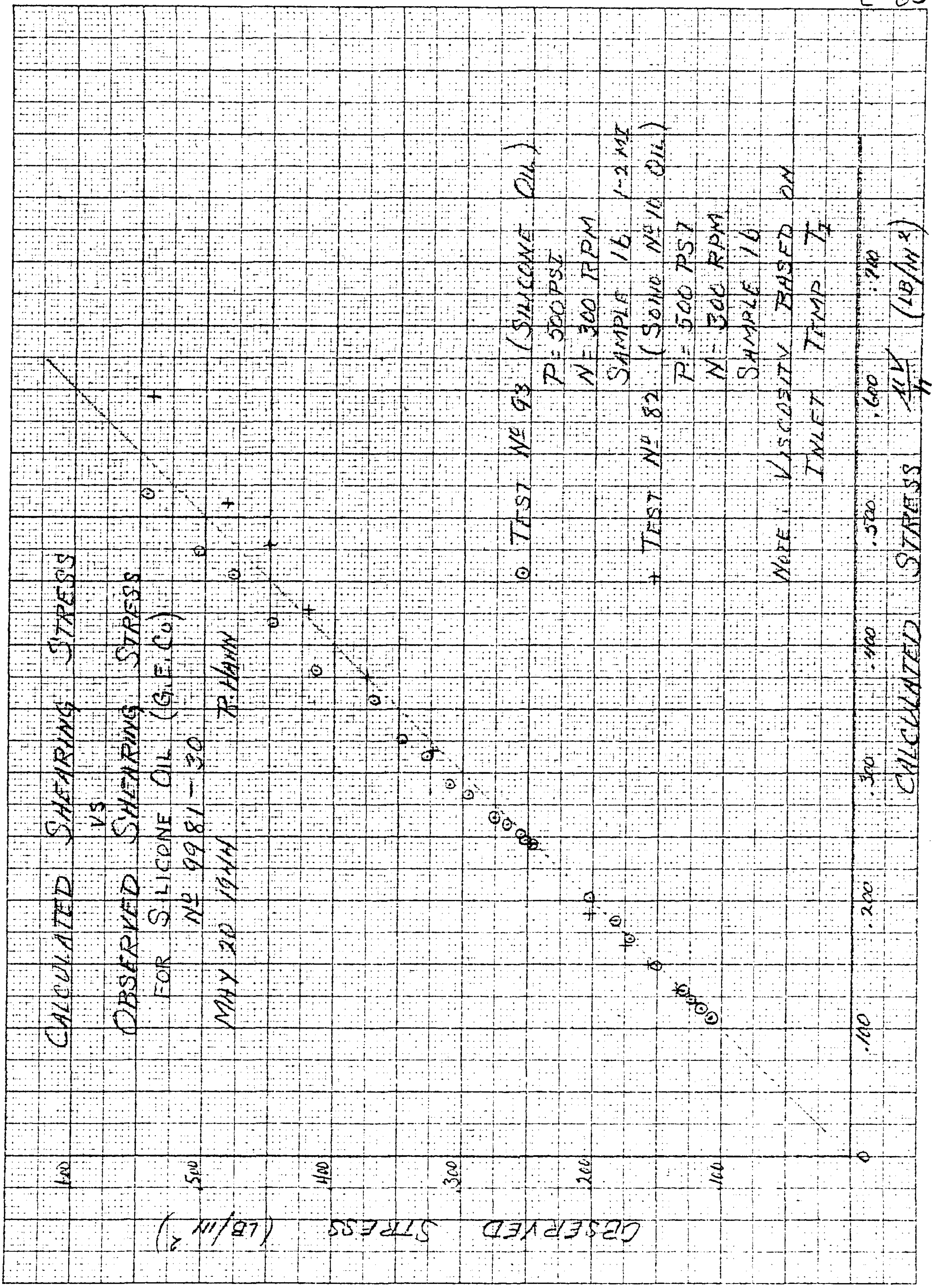
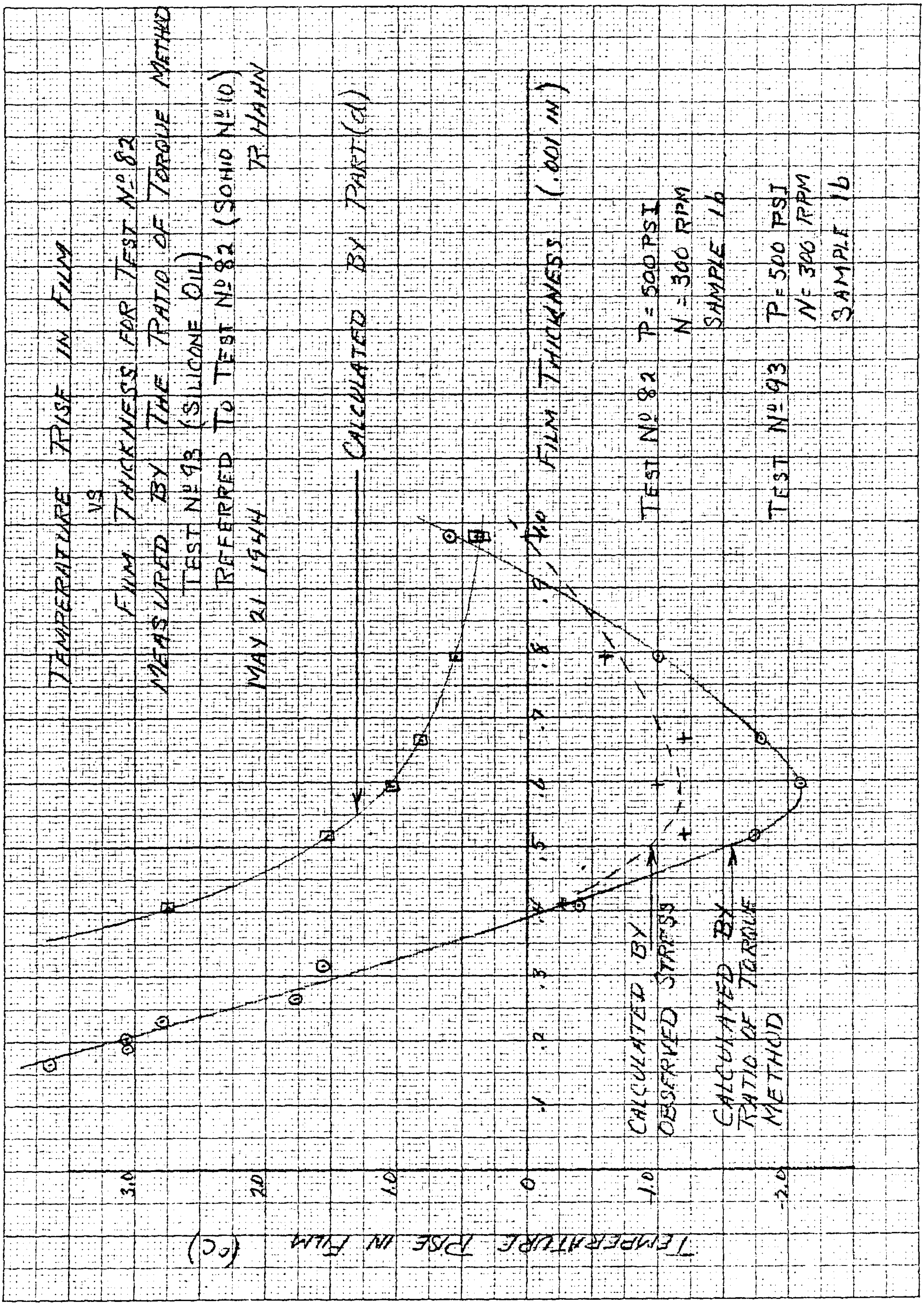
C-63



Form 57—Applied Sci.—500—5-41


TEMPERATURE RISE IN FILM
VS
FILM THICKNESS FOR TEST No 82
MEASURED BY THE RATIO OF TORQUE METHOD
TEST No 93 (SILICONE OIL)
REFERRED TO TEST No 82 (SOHIO No 10)
MAY 21 1944
B HAHN
CALCULATED BY PART (d)
FILM THICKNESS (.001 IN)
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0
TEST No 82 P = 500 PSI
N = 300 RPM
SAMPLE 1b
TEST No 93 P = 500 PSI
N = 300 RPM
SAMPLE 1b
CALCULATED BY OBSERVED STRESS
CALCULATED BY RATIO OF TORQUE METHOD
3.0
2.0
1.0
0
1.0
-2.0
TEMPERATURE RISE IN FILM (°C)

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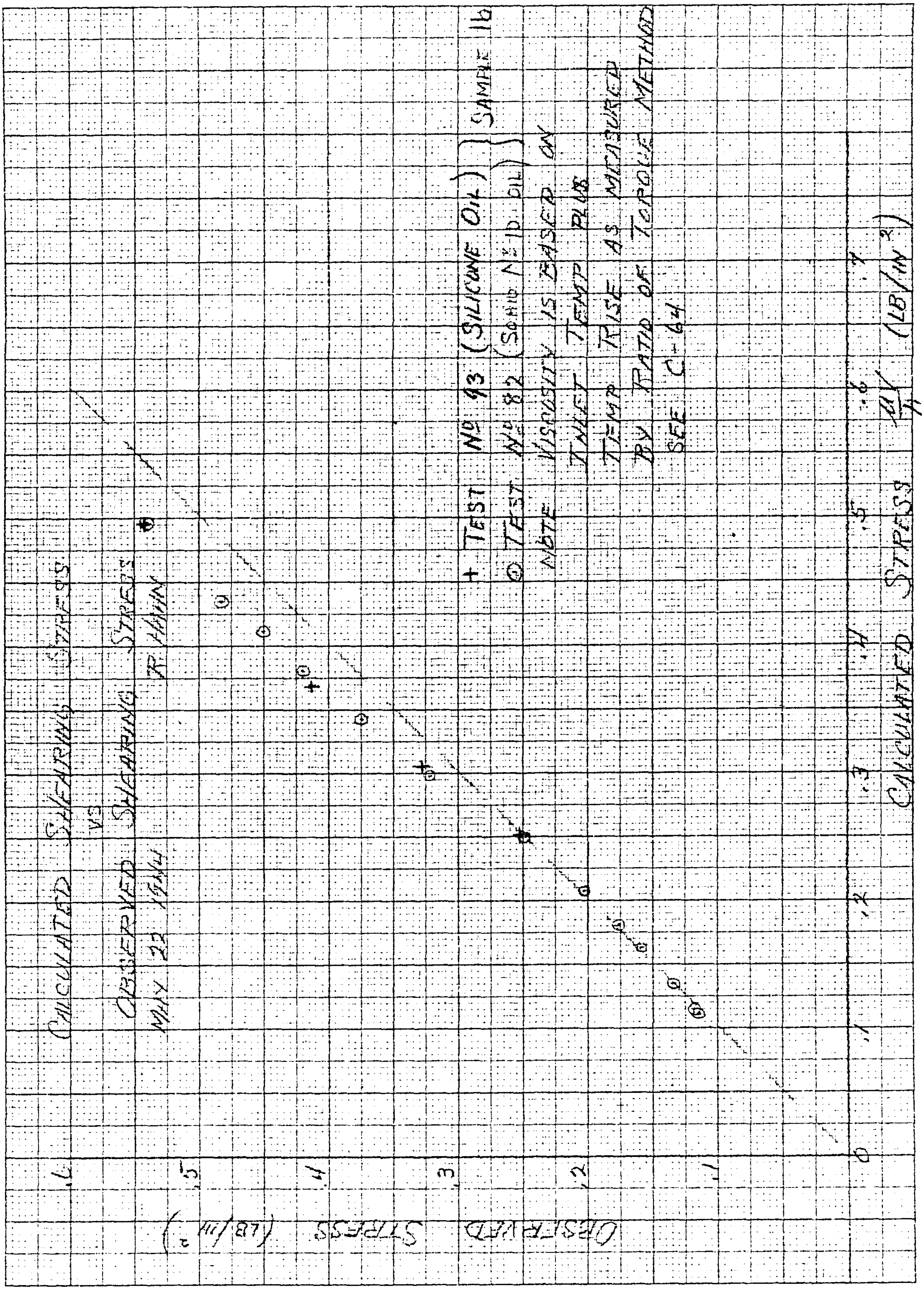

20 × 20 to the inch, 10th lines heavy.
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Form 51 – Applied Sci.–500 – 5-41

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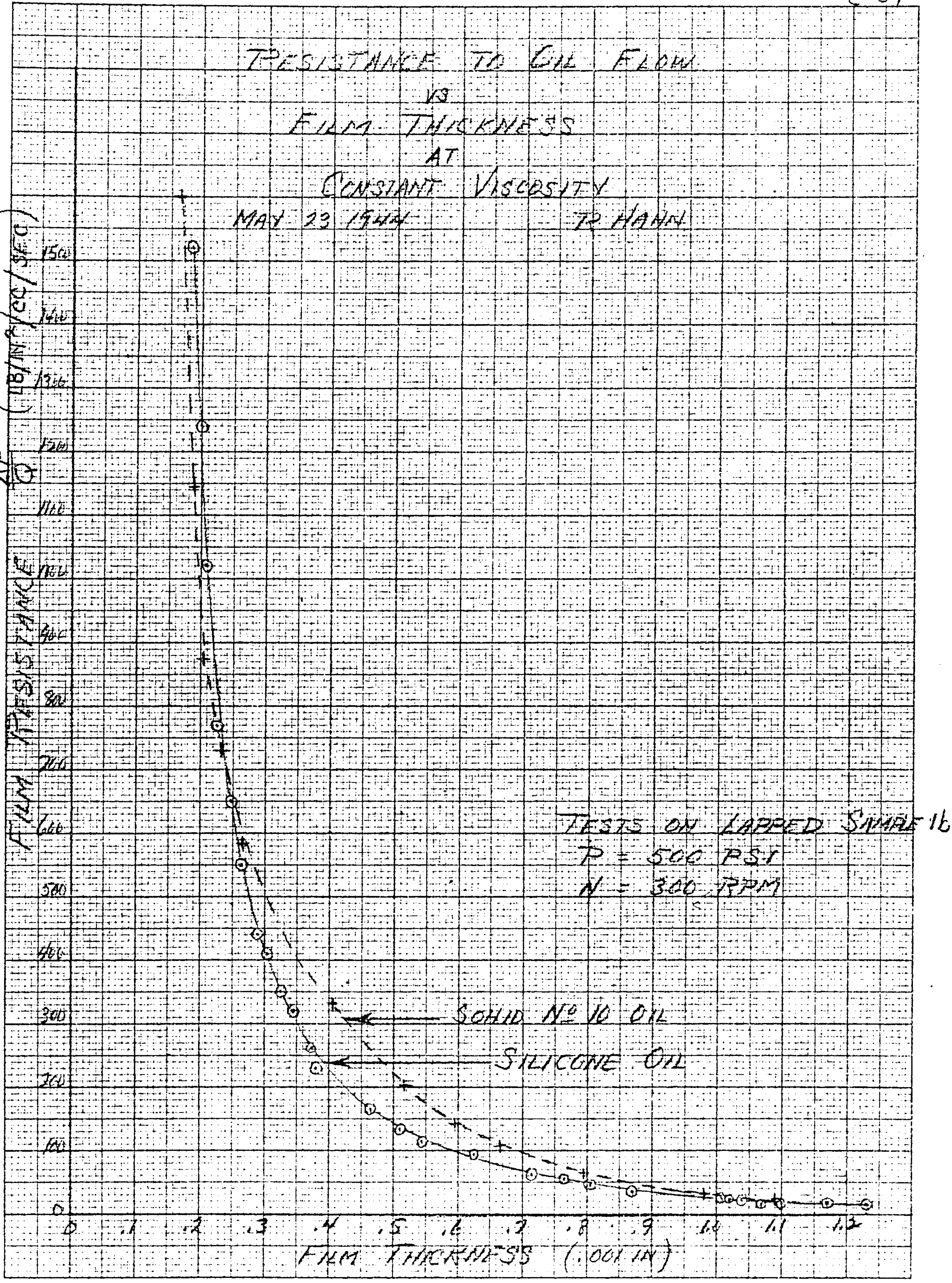
C-67



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