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May 24, 1954

*I hereby recommend that the thesis prepared under my supervision by* Robert Lemlich  
*entitled* The Effect of Vibration on Heat Transfer

*be accepted as fulfilling this part of the requirements for the degree of* Doctor of Philosophy

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THE EFFECT OF VIBRATION

ON

HEAT TRANSFER

A dissertation submitted to the  
Graduate School of Arts and Sciences  
of the University of Cincinnati

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requirements for the degree of

DOCTOR OF PHILOSOPHY

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by

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„וַיְהִי כַשְׁמַע הָעָם אֶת קוֹל הַשּׁוֹפָר וַיִּרְעוּ הָעָם תְּרוּעָה גְדוֹלָה  
וַחֲפַל הַחֹמָה... וַיִּלְכְּדוּ אֶת הָעִיר׃“

יְהוֹשֻׁעַ  
כ:ו

"And it came to pass, when the people heard  
the sound of the trumpet, ..... the people shouted with  
a great shout .... the wall fell down .... and they  
took the city."

Joshua 6:20

(Old Testament,

King James Translation)

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ABSTRACT

In order to study the effects of vibration on heat transfer, an experimental investigation was carried out involving natural convection from electrically heated wires subjected to transverse vibration in air. Using this system, study was made of the individual vibrational variables, which included amplitude, frequency, and direction of vibration, together with other pertinent variables, namely temperature difference driving force, and wire diameter. Marked improvement in the coefficient of heat transfer was obtained by using vibration in the range of 39 to 122 cycles per second, even to the extent of tripling the film coefficient. Holding other variables constant, an increase in coefficient was observed for an increase in amplitude. A similar effect was obtained for an increase in frequency. No effect was observed for a change in the direction of vibration. In an effort to account for this latter observation the concept of a "stretched" film surrounding the entire path of vibration was proposed.

The effect of vibration on the coefficient was correlated in terms of a vibrational Reynolds number defined as twice the product of amplitude, frequency, diameter, and density, divided by viscosity. Using dimen-

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sional analysis, a pair of alternate correlations were developed for air or other diatomic gases with average deviations of 13% and 9% respectively in terms of improvement, and deviations of 8% and 6% respectively in terms of coefficient. By utilizing the limited data of an earlier investigator for a thicker cylinder vibrating in water, and combining it with the present data, a more extended correlation for fluids in general was also proposed.

### Note:

An article by the author based on this dissertation and certain further work appears in the journal *Industrial and Engineering Chemistry*, June 1955.

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NOMENCLATURE

|       |  |                                 |
|-------|--|---------------------------------|
| a     | = constant,  | dimensionless                   |
| A     | = area,  | ft. <sup>2</sup>                |
| c     | = heat capacity,                                       | BTU/lb. ° F                     |
| d     | = differential of,                                     |                                 |
| D     | = diameter,  | ft.                             |
| e     | = base of natural logarithms,                          | dimensionless                   |
| f     | = function   |                                 |
| F     | = frequency,   | sec <sup>-1</sup>               |
| g     | = gravitational acceleration,                          | ft./sec. <sup>2</sup>           |
| G     | = mass velocity,                                       | lb./hr. ft. <sup>2</sup>        |
| Gr    | = Grashof number,                                      | dimensionless                   |
| h     | = coefficient of heat transfer,                        | BTU/hr. ft. <sup>2</sup> ° F    |
| H     | = amplitude of vibration,<br>(See footnote on page 55) | ft.                             |
| k     | = thermal conductivity,                                | BTU/hr. ft. <sup>2</sup> °F/ft. |
| K, K' | = constants,   | dimensionless                   |
| L     | = length,  | ft.                             |
| M     | = mass,  | lb.                             |
| n     | = element number,                                      | dimensionless                   |
| Nu    | = Nusselt number                                       | dimensionless                   |
| Pr    | = Prandtl number,                                      | dimensionless                   |
| q     | = rate of heat transfer,                               | BTU/hr.                         |
| Q     | = electrical power,                                    | watts                           |
| R     | = residual heat transfer,                              | BTU/hr.                         |

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|            |  |                              |
|------------|--|------------------------------|
| Re         | = Reynolds number,                             | dimensionless                |
| t          | = temperature,                                 | ° F                          |
| T          | = absolute temperature,                        | ° R                          |
| U          | = overall coefficient of heat transfer         | BTU/hr. ft. <sup>2</sup> ° F |
| v          | = linear velocity,                             | ft./sec.                     |
| W          | = dimensionless quantity,                      | dimensionless                |
| x          | = thickness of laminar layer,                  | ft.                          |
| X          | = abscissa                                     |                              |
| Y          | = ordinate                                     |                              |
| α          | = constant,                                    | dimensionless                |
| β          | = volumetric coefficient of thermal expansion, | ° R <sup>-1</sup>            |
| γ, γ'      | = constants,                                   | dimensionless                |
| Δ          | = difference in,                               |                              |
| ε          | = emissivity,                                  | dimensionless                |
| θ          | = any function                                 |                              |
| κ          | = constant                                     | dimensionless                |
| μ          | = viscosity,                                   | lb./ft. sec.                 |
| ρ          | = density,                                     | lb./ft. <sup>3</sup>         |
| τ          | = time,  | hr.                          |
| φ, φ', φ'' | = function,                                    |                              |
| φ, φ'      | = function                                     |                              |
| ω          | = hypothetical angular velocity,               | radians/sec.                 |

Subscripts

|   |                                 |
|---|---------------------------------|
| b | in bulk fluid                   |
| f | average for film                |
| m | pertaining to element m         |
| n | pertaining to element n         |
| o | without vibration               |
| p | pertaining to element p         |
| r | due to radiation                |
| s | pertaining to wall surface      |
| w | for thermocouple wire           |
| x | at distance x from wall surface |

Superscripts

|   |  |
|---|--|
| ~ | root mean square   |
| - | arithmetic average   |
| ' | at same Pr x Gr as occurs with vibration<br>(when appended to a symbol representing other than<br>a function or a constant.) |

## INTRODUCTION

The unit operations controlled by films are among the most important in chemical engineering. These include heat transfer and the various types of mass transfer such as distillation, absorption, and extraction. The effects of many different variables on these operations have already been extensively investigated and reported in the literature. There is, however, an important set of variables about which little is known. These are the variables involving vibration. The scattered information that does exist in the literature is for the most part meager, qualitative, and uncorrelated.

Vibration can materially affect the rate of heat or mass transfer. Under certain circumstances substantial improvements have been reported as will be discussed more fully in a subsequent section. However, no satisfactory correlation has ever been offered by which the degree of improvement could be estimated, even for laboratory size equipment. In fact, no satisfactory analysis of the different variables involved has ever been presented. Accordingly, the purpose of the present investigation was to study the effect of vibration on a film type operation and analyze the effects of the different variables involved.

The particular operation chosen was that of heat transfer by natural convection, with the experimental work having been done with electrically heated wires in transverse vibration in air. The choice of this particular experimental system was made for the sake of simplicity and accuracy. Heat transfer studies from electrically heated wires involve simpler equipment than that required for mass transfer or other types of heat transfer study. Data can be obtained more readily and the use of electrical instruments for determining heat loads makes for greater precision. Furthermore, the study of natural convection offers another advantage in that no air rate determination is required, thus eliminating both the necessity for taking another measurement and the additional error associated with it. Finally, the type of vibration employed is not overly difficult to induce and measure. In short, the choice of this system allowed separate measurement and study of each of the different variables involved, which included amplitude, frequency, diameter, temperature difference driving force, and direction of vibration. This investigation also permitted some study of the effect of the nature of the fluid. Incorporation of this additional variable was accomplished by considering, in conjunction with the present experimental data for air, the limited data of an

earlier investigation dealing with vibration under water.

All film type operations are related since the resistances of the films, which control the rates of transfer, are determined to a large extent by similar variables. Thus, even though the investigation here represents only one operation, namely heat transfer by natural convection, the results should be applicable in a general way to other film type operations.

SUMMARY OF CONCLUSIONS

1. Vibration can materially increase the coefficient of heat transfer, a tripling of the coefficient having been obtained with the wires.
2. Holding other variables constant, an increase in the amplitude of vibration increases the heat transfer coefficient.
3. Holding other variables constant, an increase in the frequency of vibration also increases the heat transfer coefficient.
4. Holding other variables constant, an increase in wire diameter decreases the heat transfer coefficient.
5. Within the limits of experimental error, the direction of wire vibration has no effect on the heat transfer coefficient.
6. In general, the effects of vibration are in accord with the film concept.
7. The film about a vibrating wire is apparently

"stretched" to cover the entire path of vibration rather than carried back and forth by the wire.

8. The effect of the vibrational variables may be correlated in terms of a vibrational Reynolds number, defined as twice the product of diameter, frequency, amplitude and density, divided by viscosity.

9. The improvement in heat transfer coefficient for the central portions of the vibrating wires (which are essentially horizontal vibrating cylinders) in air may be expressed by either one of two somewhat different dimensionless correlations, depending on the precise definition of the term "improvement".

Correlation I: Defining "improvement" as the increase in heat transfer coefficient caused by vibration at the same product  $Gr \times Pr$  as prevailed without vibration, which for the present work in air is essentially equivalent to equality of film temperatures, then,

$$\text{Improvement} = \frac{h}{h_0} - 1 = \varphi \left[ \frac{(Re)(\beta \Delta t)^{0.16}}{(Gr)^{0.20}} \right]$$

where  $h$  and  $h_0$  are coefficients of heat transfer with and without vibration respectively,  $Re$  is Reynolds

number, Gr is Grashof number, Pr is Prandtl number,  $\beta$  is volumetric coefficient of thermal expansion,  $\Delta t$  is temperature difference driving force, and function  $\phi'$  is given in fig. 13. For improvements of over 10%, which covers most of the experimental range, this relationship may be expressed as

$$\frac{h}{h_0} = 0.75 + 0.00308 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Gr)^{0.410}}$$

Correlation II: Defining "improvement" as the increase in heat transfer coefficient caused by vibration at the same heat flux as prevailed without vibration, then,

$$\text{Improvement} = \frac{h}{h_0} - 1 = \phi \left[ \frac{(Re)(\beta \Delta t)^{0.16}}{(Gr)^{0.20}} \right]$$

where  $h_0$  is coefficient of heat transfer without vibration, and function  $\phi$  is given in fig. 14. A gain for improvements greater than 10%, which covers most of the experimental range, this relationship may be expressed as

$$\frac{h}{h_0} = 0.75 + 0.00432 \left[ \frac{(Re)^{1.86} (\beta \Delta t)^{0.297}}{(Gr)^{0.372}} \right]$$

10. The average deviations for correlations I and II

in terms of percentage improvement are 13% and 9% respectively. For a given  $h'_0$  or  $h_0$  the average deviations in  $h$  are 8% and 6% respectively. The results of control runs made with the wires stationary agree with those of previous investigators, the average deviation being about 5%. All of this indicates good experimental reproducibility.

11. For an approximate extension to other fluids (as well as for diatomic gases such as air) correlation I may be modified, thus,

$$\text{Improvement} = \frac{h}{h'_0} - 1 = \varphi' \left[ \frac{0.80(\text{Re})(\beta \Delta t)^{0.16}}{(\text{Pr})^{0.75}(\text{Gr})^{0.20}} \right]$$

As before, for improvements of over 10%,

$$\frac{h}{h'_0} = 0.75 + 0.0022 \left[ \frac{(\text{Re})^{2.05}(\beta \Delta t)^{0.328}}{(\text{Pr})^{1.54}(\text{Gr})^{0.410}} \right]$$

## PRELIMINARY HEAT TRANSFER CONSIDERATIONS

Before entering into the consideration of vibration and its effect on heat transfer, it is appropriate to review briefly some pertinent aspects of heat transfer itself, in particular, as applied to natural convection.

### Fundamentals:

For conduction, the rate of heat transfer  $q$  is expressed by Fourier's Law (62) as

$$q = -k A \frac{dt}{dL} \quad (1)$$

where  $t$  is temperature,  $k$  is thermal conductivity,  $A$  is area perpendicular to the direction of heat flow, and  $L$  is length parallel to heat flow. Convection, however, involves macroscopic motion, which precludes the direct application of Fourier's Law in the form of eq. 1. In its place a coefficient of heat transfer  $h$  is defined by

$$q = h A \Delta t \quad (2)$$

The resistance to convective heat transfer is measured by the reciprocal of this  $h$ , and is generally considered to be concentrated in a relatively stagnant surface film.

Numerous methods have been proposed for the estimation of  $h$  (49, 58, 108, 109, 110, 153). However, to date, no completely general theoretical method has proven satis-

factory, although significant strides have been taken in this direction. As an alternative, the technique of dimensional analysis has been employed to correlate the experimental data available. With this very useful tool supported by certain theoretical considerations, it has been shown that convective heat transfer can be expressed quantitatively in terms of certain dimensionless groups. These include the following:

Nusselt number - 
$$Nu = \frac{hD}{k} \quad (3)$$

where D is diameter or other characteristic linear dimension.

Prandtl number - 
$$Pr = \frac{c\mu}{k} \quad (4)$$

where c is heat capacity at constant pressure, and  $\mu$  is viscosity.

Reynolds number - 
$$Re = \frac{Dv\rho}{\mu} \quad (5)$$

where v is linear velocity and  $\rho$  is density.

Grashof number - 
$$Gr = \frac{D^3 \rho^2 g \beta \Delta t}{\mu^2} \quad (6)$$

where g is acceleration of gravity and  $\beta$  is volumetric coefficient of thermal expansion. It can be readily shown that for an ideal gas  $\beta = \frac{1}{T}$  where T is absolute temperature.

#### Natural Convective Crossflow:

Utilizing dimensional analysis, it can be shown that for a horizontal cylinder in natural convection,

$$\text{Nu} = f(\text{Gr}, \text{Pr}) \quad (7)$$

Based on the data of a number of different investigators (6, 44, 96, 130, 138, 148, 170), McAdams (108) presents the above relationship in the form

$$\text{Nu} = \phi(\text{Gr} \times \text{Pr}) \quad (8)$$

where fluid properties are evaluated at an average film temperature  $t_f$ , which is taken as the arithmetic mean of the surface temperature  $t_s$  and the bulk fluid temperature  $t_b$ . This function  $\phi$  is shown graphically in fig. 8, and is expressed empirically in a recent work of Bosworth (23) as

$$\text{Nu} = [0.63 + 0.35(\text{Gr} \times \text{Pr})^{1/6}]^2 \quad (8a)$$

Fig. 8 also shows a modification (108) of an earlier correlation of Rice (137, 138).

These relationships cover a variety of fluids and a wide range of diameters, including both wires and tubes. The success of correlations such as these are powerful testimony to the strength of dimensional analysis.

### SURVEY OF THE EFFECTS OF VIBRATION

Vibrations, both sonic and ultrasonic but particularly the latter, have found application in various phases of engineering and the physical sciences (42, 121). They have been utilized for such diverse purposes as the measurement of viscosity (144, 178), the determination of molecular weights of liquid polymers (174), the location of flaws in metals (32), the examination of filament structure (74), the setting of concrete (103), and the homogenization of milk (3). They have also been shown to increase the rate of chemical reactions, particularly oxidation (59, 60, 65, 140), to depolymerize high polymers (174), reduce liquid viscosities (1, 77, 87, 107, 155), emulsify liquids (21, 22, 104, 146, 177), break up flotation froth (161), and destroy microorganisms (81, 95, 139).

Applications of vibrations to the various unit operations are also to be found in the literature. However, little or no quantitative data or theory is available, particularly with regard to the film controlling operations of heat transfer and mass transfer. This is unfortunate since the limited data available indicates some very interesting possibilities along these lines.

The following discussion is divided and classified under the various unit operations with applications

to heat transfer discussed last and in greatest detail since they are the most pertinent to this dissertation.

Fluid Flow:

Kastner and Shih (84) studied the effect of vibration on fluid flow, in particular the manner in which the frequency of pulsating air flow affects the critical Reynolds number. These pulsations were obtained by mechanically interrupting the flow of air at frequencies ranging from about 3 to 33 cycles per second. It was found in every case that the pulsations lowered the critical Reynolds number, as might be expected from the additional turbulence produced by repeated interruptions in flow. Unfortunately, no measure of amplitude was taken so that no quantitative explanation or correlation could be offered.

An interesting and often disadvantageous effect of pulsation is the error it can induce in orifice-metering devices (9, 28). The pressure drop across an orifice is proportional to the square of the velocity through it, so that with pulsating flow the instrument would indicate the root-mean-square velocity rather than the mean velocity. Under certain circumstances the difference between the two is appreciable.

Distillation:

A patent by McKittrick and Cornish (112) claims an improvement in distillation by exposing the system to vibrations of 50 cycles per second to 50 megacycles per second. The apparatus described is of laboratory size and consists of a vertical glass tube fitted to a flask and topped with a condenser. The tube serves as the column and is packed with a single helix of wire. Vibrations are obtained from an automobile horn whose sound with a frequency of several hundred cycles per second is directed down the top of the column. For the system benzene-toluene at total reflux, a 55% reduction in the number of theoretical trays required for a given separation is reported. The authors suggest the improvement is due to an increased relative motion between phases induced by the vibration. This explanation seems quite reasonable in the light of the film concept according to which turbulence at a film tends to thin the film. Since a pair of such films exists at the liquid-gas interface any such thinning would increase the rate of mass transfer thus increasing the efficiency of the column.

Another laboratory scale investigation is described by Coffin and Funt (38). In their case, however, the frequency was 25,000 cycles per second, the sound intensity was much greater, and the column was not packed at all. For the system carbon tetrachloride-benzene at

total reflux, a 50% decrease in the number of required theoretical trays is reported. This is attributed in part to the visible turbulence in the liquid produced by the intense vibrations, although the experimental evidence indicates there is also another factor involved. This other factor may well be the additional turbulence in the gas phase produced by the vibration. Both these factors reduce film thickness and so improve column efficiency. A somewhat similar investigation by Thornton (165) using a rapidly oscillating pressure gave much the same results.

Along different lines, a patent by Decker and Hellmuth (45) suggests the use of ultrasound in the distillation of immiscible liquids. The vibration is said to keep the liquids so well mixed that the uniformity of vaporization is significantly improved.

#### Extraction:

A paper by Feick and Anderson (55) describes a pulsing packed column for the extraction of benzoic acid or acetic acid from toluene with water. Pulsations at frequencies ranging from about 3 to 17 cycles per second were introduced into the countercurrently flowing liquids through a reciprocating diaphragm on the column. The turbulence so induced broke up the toluene flow through

the packing into small droplets thus increasing the surface of contact, and also increased the relative motion between phases thus decreasing the film thickness. The net result was a reported improvement in extraction coefficient of 500% or more. Similar work has been performed by Von Berg and Weigandt (168).

A patent by Van Dyck (167) describes a liquid-liquid extraction column with perforated plates attached eccentrically to a rotating wheel. This gives the plates a reciprocating motion which in turn is said to move the liquid first in one direction and then in the other thus improving contact and efficiency.

Essentially the same end has been achieved by Cohen and Beyer (39) by using stationary sieve plates and pulsing the liquid through a tee in the stream entering the bottom of the column. Using pulses of 17 to 72 cycles per second on the system isoamyl alcohol-boric acid-water, the extraction efficiency was found to increase with both frequency and relative amplitude. As before, this increase in efficiency follows from the increased turbulence and droplet area caused by the pulses. No direct comparison to operation without pulsation is possible since the plate perforations were too small to allow counterflow of the phases by density difference alone.

Other work along the same general lines has been

carried out, largely for the United States Government, but much of the information has not been made public (3, 4, 17, 29, 35, 41, 52, 67, 69, 80, 98, 100, 132, 145, 159).

Crystallization:

Berlaga (18) showed that both the number of nuclei formed and the rate of crystal growth can be increased by subjecting the liquor to an ultrasonic field. Cassady (34) confirmed this observation with studies on the crystallization of alumina trihydrate from sodium aluminate solutions such as are used in the Bayer process for the production of alumina. Apparently the vibrations act as a disturbing element which not only increases the rate of nucleation, but also decreases the thickness of the liquid film on the growing crystals. Both these effects hinder the tendency to supersaturate which in turn increases the rate of crystallization. Similar work with Rochelle salt crystals (115), barium sulfate crystals (162), and thymol crystals (83) has been described. Several applications to the control of crystallization on an industrial scale have been patented (30, 131, 166).

Adsorption with Chemical Reaction:

Richardson (141) patented a proposal to increase the rate of diffusion at a mass transfer boundary with

vibration. The particular application involved is for the synthesis of ammonia from hydrogen and nitrogen. Finely divided catalyst suspended in a mixture of these two reaction gases is subjected to high intensity ultrasonic vibrations of about 50,000 cycles per second generated by a siren. The author claims the vibrations increase the rate of ammonia synthesis permitting the use of lower pressures and temperatures than would be possible otherwise.

The explanation offered is that both the catalyst particles and the molecules of the reaction gas oscillate at high frequency, but due to the greater inertia of the catalyst particles the amplitude of their oscillations is much less than that of the lighter gas molecules. This in turn induces relative movement between the catalyst and gas molecules which serves to separate the synthesized ammonia from the catalyst and bring fresh gas molecules in contact with the catalyst surface thus promoting the reaction. This explanation seems very reasonable and is in line with the concept of a film existing around each particle. Stated in another way the relative motion between catalyst particles and gas molecules serves to thin the film thus promoting mass transfer across it and increasing the rate of catalysis.

A patent by Stelling and Eklund (158), which

appears to be fundamentally similar to that just discussed, describes the application of 50 cycle per second vibrations at right angles to a reaction tube containing vanadium catalyst particles suspended in a gas. The vibrations are said to improve the conversion of sulfur dioxide to sulfur trioxide in the gas stream; the yield increasing with amplitude, as would be expected from the boundary layer considerations discussed above.

Dialysis:

The effects of ultrasonics on mass transfer by dialysis were studied by Rees (134). This work showed that the mass transfer coefficient increases with the frequency and the electrical power input to the ultrasonic generator. For a given frequency this electrical power input is a rough measure of the amplitude of vibration indicating that the rate of transfer increases with both frequency and amplitude. This observation is also in line with the film concept according to which the membrane is covered on each side by a film. Increased amplitude of vibration or increased frequency tend to increase the relative motion between liquid molecules and the membrane thus thinning the film and increasing the rate of transfer.

A serious deficiency in Rees' work is the lack

of any direct measure of amplitude. The use of electrical power input is at best a poor substitute for this very important variable and seriously limits further understanding of the phenomena.

Miscellany:

Partial separations between the constituents of mixtures have been described. While not film type phenomena, at least in the usual sense, these observations are of sufficient interest to be recounted here. Frei and Schiffer (64) report a qualitative separating action of ultrasonic standing waves in solutions of glycerol in water and solutions of hexane in heavier hydrocarbons. Unfortunately no quantitative data or explanation is offered and their conclusions appear to be specifically contradicted by Boyd and Zentner (25).

Passav (102) described the partial separation of a gas mixture consisting of hydrogen and carbon dioxide in a small glass tube by subjection to high intensity ultrasonics. Enrichments of several percent were reported. The extent of enrichment was further described as varying regularly with time over a cycle of about four hours. No satisfactory explanation was offered for this unusual observation.

Despite the lack of explanation for these pheno-

mena of separation one point stands out. The separations reported are between molecules that differ considerably in size. There are no reports of separation between molecules of similar size. This would seem to indicate that the vibration distinguishes between the molecules on the basis of their size and thus sorts them out in a manner that may well be similar to that in which standing sound waves cause dust particles in air to accumulate at the nodes and so effectively separate them from the very much smaller air molecules. These nodes are regions of minimum disturbance and the intense motion of the vibrating molecules at the antinodes tends to push the dust particles toward the quieter nodes. Several patents (26, 27, 75, 152), differing among themselves chiefly in design considerations rather than in principle, describe apparatus in which such separations of particles from gases can be carried out.

A patent by Schrey (150) suggests in very broad terms the application of vibrations to "increase the yield and rapidity of physical, chemical, and biological processes". The applications suggested range from heat transfer and diffusion to laundering and paper making. Unfortunately no explanations are offered nor is any quantitative data given. The patent is merely a list of suggestions with few details and as such contributes little to our knowledge.

Heat Transfer:

By and large, the work done with vibrations in heat transfer suffers from the same drawbacks as most of that done in the other unit operations. With one exception, to be discussed presently, measurements were not taken of all the pertinent variables, nor was a correlation of results offered. Nevertheless, a few interesting inventions and results have been reported.

Ormell (123) patented an oven involving mechanically vibrating shelves. The shelves are inclined and staggered one above the other so that granulated material entering the top of the oven flows from one shelf to the next, and so down through the oven which is heated by hot flue gases flowing countercurrently in direct contact with the material. However, no quantitative data were given. Among the applications suggested for this rather ingenious device are the roasting of ores and the destructive distillation of coal. The vibration serves a triple purpose. First, it facilitates the flow of particles through the oven. Secondly, the vibration increases the relative motion of the particles with respect to the hot gases, thus decreasing the film thickness and improving heat transfer and mass transfer. Finally, the constant jostling of the particles continually brings up fresh surface for exposure to the hot gases.

In a brief patent Andreas (2) proposed mechanical vibration of an entire heat exchanger. The method for producing the vibration is not described nor does it appear from the text that such a device was ever actually constructed. As before, no quantitative results are given although the claim is made that the improvement in heat transfer should be considerable. According to the film concept the suggestion has merit since it would result in greater relative motion between exchanger walls and fluids and hence result in thinner films. However, the lack of further information in the patent greatly limits its contribution to the field.

A more recent paper by West and Taylor (175) describes experimental results dealing with the effect of pulsations on heat transfer. Their apparatus consisted of a double-pipe steam-to-water heat exchanger and a double-pipe water-to-water heat exchanger. A reciprocating pump operating at 100 rpm forced water first through the inner tube of the steam exchanger where it was heated and then through the inner tube of the other exchanger where it was cooled. The normal pulsating effect of the reciprocating pump was deliberately increased by reducing the usual air damping action on the pump. Improvements in heat transfer coefficient of up to 70% were reported by virtue of the vibrations so induced in the equipment.

This improvement was attributed to two factors. First, the peak forward motion of a pulsation represents a peak in longitudinal flow which in turn decreases the film thickness. Secondly, the subsequent minimum longitudinal flow, corresponding to the pause between pulsations, results in radial diversion of much of the excess longitudinal kinetic energy thus increasing turbulence and decreasing film thickness.

These investigators varied the flow Reynolds number from 30,000 to 85,000. They report that similar unpublished investigations (117, 171) at Reynolds numbers of 26 to 1375 showed no improvement. No explanation was offered but it appears to the present writer that this may be due to weaker pulses occasioned by the use of smaller piston strokes for circulating the water at these lower flow rates. Pulses that are too weak result in insufficient additional turbulence and therefore may not affect the film appreciably.

In the investigation no attempt was made to vary the speed of the pump so as to vary the frequency of pulsation. Furthermore, the only measure of amplitude offered was the ratio of maximum air volume to minimum air volume in the air chamber over a period of one pulsation. The relationship between this ratio and the magnitude of peak forward motion during a pulse (which is the amplitude) is

not known. All that can be said is that an increase in the ratio is likely to increase the amplitude, all other factors being held constant. While better than nothing, such a loose qualitative relationship is not suited to a quantitative understanding of what takes place. This, together with the failure to investigate the very important variable of frequency, makes the presentation of any sort of correlation impossible and seriously limits the applicability of the results to further understanding of the phenomena.

The only attempt to measure and correlate quantitatively the effects of vibration on heat transfer is that of Martinelli and Boelter (105, 106). They subjected a horizontal,  $\frac{3}{4}$  inch diameter, one foot long, electrically heated tube in natural convection under water to vertical sinusoidal vibration. The work was limited to this one diameter and one direction of vibration. Amplitudes up to 0.10 inch and frequencies up to 40 cycles per second were employed. Improvements in heat transfer coefficient of several-fold were reported.

An analytical derivation was presented involving a number of assumptions, the most important of which are as follows;

1. The vibrating body consists of a vertical plate, infinitely long and wide.

2. A viscous layer of the surrounding fluid exists next to the plate.
3. The fluid in this layer flows only in the vertical direction.
4. The fluid is incompressible and  $\frac{1}{1 + \beta t} \approx 1 - \beta t$ .
5. The variation of temperature gradient in a vertical direction is small.

These assumptions deserve some comment. Assumption number 1 is obviously highly idealized. The vibrating body is a cylinder, which is far from being an infinite plane. Similarly, assumption number 3 would scarcely hold for a cylinder, whose vertical motion would certainly force fluid above and below it to move in a direction having some horizontal components. Furthermore, since the temperature gradient occurs across the film which encircles the cylinder, assumption number 5 does not apply either. Finally, assumption number 4 restricts consideration to liquids since gases are compressible and have much larger values of  $\beta$  than liquids, so that  $\frac{1}{1 + \beta t}$  does not equal  $1 - \beta t$ . In short, these assumptions, particularly numbers 1, 3, 5, are highly restrictive and do not really apply to the case at hand.

Nevertheless the authors proceeded with an involved derivation based on hydrodynamic considerations to-

gether with these assumptions, and obtained the following relationship:

$$\frac{H\omega D\rho}{2\mu\sqrt{2}} \cdot \frac{\sqrt{\frac{\omega D^2\rho}{\mu\sqrt{2}}}}{\text{Nu}} = \text{Nu} \sqrt{\left(\frac{x\tilde{v}_x}{\mu}\right)^2 - \frac{1}{36}\left(\frac{\text{Gr}^2}{\text{Nu}^6}\right)} \quad (9)$$

where H is amplitude of vibration,  $\omega$  is a hypothetical angular velocity of vibration in radians per second, x is the thickness of the laminar layer, and  $\tilde{v}_x$  is the root mean square velocity at distance x from the surface.

Eq. 9 cannot give a solution for h since no method is presented for evaluating x and  $\tilde{v}_x$ . Accordingly, the authors assumed a constant value for the quantity

$\frac{x\tilde{v}_x}{\mu}$  and tried to compare eq. 9 with the trend of experimental data. This proved unsuccessful which is not surprising in view of the many assumptions involved. In an effort to remedy the situation, they made certain apparently arbitrary changes in eq. 9 and introduced certain empirical constants to fit the data giving another equation,

$$\frac{H\omega D\rho}{2\mu\sqrt{2}} = \text{Nu} \sqrt{12000 - 20 \frac{(\text{Gr} \times \text{Pr})^2}{(\text{Nu})^8}} \quad (9a)$$

A test of this equation with the data of the present experiments resulted in poor agreement. In view of the assumptions and empirical nature of the equation this

is not unexpected. In fact, the authors indicated that according to their correlation no appreciable improvement in  $h$  is possible by vibration in air unless very large frequencies (or amplitudes) are employed. This is not true at all, as proved by the experimental work of this dissertation which is discussed later. Furthermore, according to a recent communication from Boelter (20) "Professor W. E. Mason of our department reproduced some of the experiments and could not check R. C. M. [Martinelli's] results". Letters written to Mason brought little clarification.

All in all, while apparently a step in the right direction, this work of Martinelli and Boelter suffers from the double handicap of a weak correlation and too narrow a range of variables, only one diameter and one direction of vibration having been studied. In addition, there appears to be some doubt as to the validity of their experimental results.

## DESCRIPTION OF APPARATUS

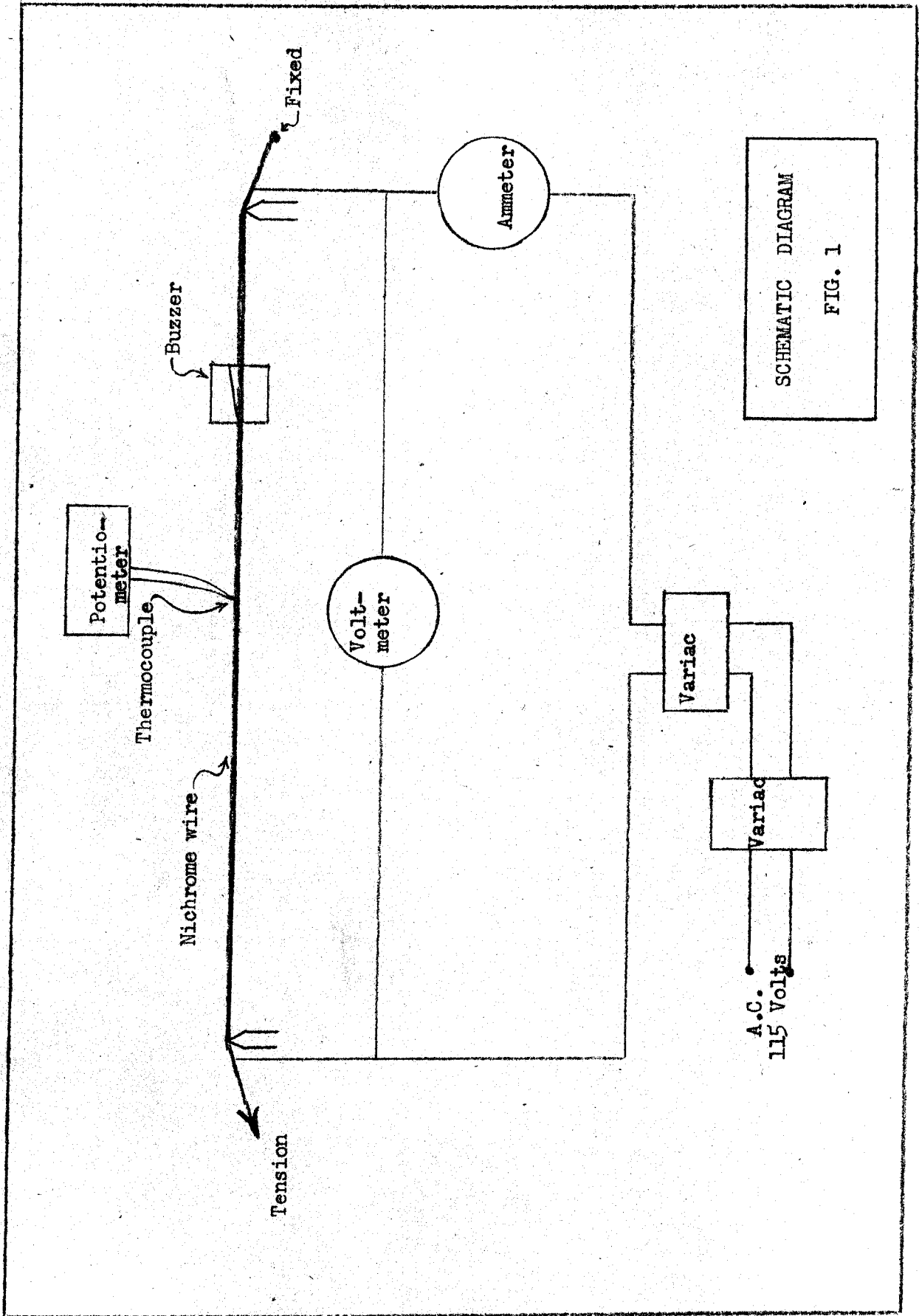
### General:

The apparatus consisted essentially of equipment and instruments to stretch electrically heated wires, vibrate them at various amplitudes and frequencies, and measure the different variables involved. A schematic diagram is shown in figure 1.

### Main Frame and Wires:

Three different nichrome wire sizes were employed as the heated element, namely 12, 18, and 22 gage. Measurement with a micrometer showed diameters of 0.0810, 0.0396, and 0.0253 inches respectively. The frame for supporting the particular wire in use consisted essentially of two wooden posts mounted vertically on a flat board as a base, and braced to insure rigidity. The base board was clamped firmly to the top of a desk and the entire setup located in a small, interior, windowless room with a well fitting door. When this door was closed and the ventilating blower shut off no stray air currents were perceptible.

Such elimination of stray air currents is of the utmost importance in natural convection studies. Had the apparatus been set up in a large laboratory where windows and doors are frequently open and people con-



stantly moving about, sizable errors would have been introduced. For example, from eq. 13\* a breeze of only 1 mile per hour would in itself result in a coefficient of over 12 BTU/hr.ft.<sup>2</sup>°F for the 0.0396 inch wire, which is of the same order of magnitude as the natural convection coefficients under investigation.

The tops of the posts were beveled and the wire was strung horizontally across the edges. These edges, 36.3 inches apart, then acted as two bridges between which the wire could vibrate. After passing over the right bridge the right end of the wire was fixed, while the left end of the wire, after passing over the left bridge, was wrapped around the movable jaw of a small vise. Tightening or loosening the vise varied the wire tension which in turn varied the frequency.

#### Vibrator:

The vibration inducing element consisted of an electric interrupter-type buzzer held in place with clamps. The armature of the buzzer was placed in contact with the wire at a point about nine inches to the left of the right-hand bridge so that vibrating the armature also vibrated the wire. The buzzer was actuated through a switch by either one 1½ volt dry cell, or by two such cells in series to give 3 volts. Insertion of a variable nichrome resistor in series with the cell

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\* Forced convective crossflow correlation (108) mentioned in a later section.

or cells permitted finer adjustment of buzzer voltage.

Electrical Equipment:

Electric current to the nichrome wires was drawn from 115 volt 60 cycle mains through two Variac variable autotransformers in series, which allowed a large step-down in voltage. A.C. was chosen because of its ready availability, ease of control, and lack of effect on the potentiometer. The current then passed through a switch and across the nichrome wire. Insulated copper wire was used for the leads and was soldered to the nichrome wire at points slightly outside the bridge gap so as not to interfere with vibration. The heated lengths for the 12, 18, and 22 gage wires were 39.2, 38.0, and 38.5 inches respectively.

Current was measured with either a 0-3-15 ampere Weston Electric ammeter or a 0-5 ampere Westinghouse ammeter, the choice depending on the magnitude of the current. Voltage was measured with a 0-15-150 volt Weston voltmeter. Voltage taps across the nichrome wire were soldered in together with the current taps at the same points, so that any voltage drops in the current leads would not affect voltmeter readings. The voltmeter was also used to check soldered connections for electrical soundness.

Vibration Measurement:

The frequency of vibration was determined by observing the wire under a variable, electronic, stroboscopic light with a scale readable to an accuracy of about one per cent. This stroboscopic light was calibrated from the patterns appearing on a marked, constant speed, rotating grindstone as the stroboscope frequency was varied. Details of this calibration procedure are given in the appendix and will be published shortly.

The amplitude of vibration was determined optically through a Leitz Wetzlar microscope fitted with a 32 mm. 0.10 objective and a 6x counting ocular eyepiece, giving a magnification of roughly twenty-fold. The eyepiece was calibrated by viewing the 0.0396 inch diameter wire and noting that it covered 21.3 scale divisions of the field. This corresponds to a count of 0.00186 inches per division. The scale in the eyepiece could be read with an accuracy of better than 0.2 divisions or about 0.0003 inch.

Temperature Determination:

The temperature of the nichrome wire was sensed by a thermocouple of 36 gage (0.005 inch diameter) wires, one of chromel and the other of alumel. These fine thermocouple wires were not wrapped around each other but were wrapped around the nichrome wire, twice each, and separated by a space along this wire of less than 0.3 inch.

This gave in effect a split thermocouple junction with the actual contact at the surface of the nichrome wire. The voltage generated by such a thermocouple is the same as would be generated by chromel and alumel alone in direct contact with each other at the same temperature (76). Initial attempts at welding the thermocouple wires to the nichrome wire failed. The thermocouple wires would break off adjacent to the weld after only a moment of vibration. As it was, the thermocouple wires would break off anyway after prolonged vibration, and necessitated renewal. The thermocouple was placed midway between the two bridges so as to be at the crest of the vibrating wire.

The thermocouple voltage was measured by a Wheelco model 310 potentiometer with the cold junction at room temperature. The galvanometer in this instrument cannot respond to 60 cycle A.C. and so is unaffected by any alternating voltage the thermocouple might pick up. The potentiometer scale could be read to about 0.02 millivolts, corresponding to about 1°F.

A survey of various methods for measuring surface temperature is given in the appendix. The thermocouple method of measuring surface temperatures was selected from the others discussed in this survey because it appeared to be the best and simplest. The technique

of using the wire itself as the resistance element (requiring a platinum or nickel wire) was not satisfactory. As described in the survey, repeated changes in wire tension associated with changes in frequency would have altered the electrical resistance of the wire and necessitated continual recalibrations. A plated junction, particularly on a wire of high electrical resistance such as nichrome (whose use is necessitated by the low current limit of the available Variacs), would have resulted in a local area of low electrical resistance right at the thermocouple junction. This in turn would have resulted in a local cold spot at the point where thermocouple voltage is generated. Methods that involve setting a thermocouple in a groove would have been difficult to apply to something as small as a wire. Furthermore, even if they could have been applied, the change in electrical resistance caused by removing some of the wire metal and substituting something else would have resulted in a local resistance irregularity and therefore a local temperature irregularity at the thermocouple junction. Other methods described such as interferometry require very special equipment and techniques.

As discussed in this survey on surface temperature measurement, the error in measuring surface temperature

directly with a thermocouple is small for natural convection to air. This was further checked in the following way:

A 12 inch length of 18 gage stainless steel hyperdermic tubing was suspended horizontally in ambient air in place of the nichrome wire. Threaded through the tube was a varnished chromel-alumel thermocouple made of the usual 0.005 inch wires. The junction of this electrically insulated thermocouple was centered longitudinally in the tube so that the temperature inside the tube could be measured. This was essentially the same as the temperature of the outer tube surface since calculation showed the thermal resistance of the tube wall to be negligible. Centered longitudinally on the outside of the tube were chromel and alumel wires wrapped around the tube in the usual way to give a split junction thermocouple measuring the outside surface temperature. The tube was then electrically heated to various temperatures. For these various levels of heat flux, the temperature indicated by the outer thermocouple was lower than that of the inner by only 3% of  $\Delta t$  (the difference between tube temperature and ambient temperature) corresponding to a difference of only 3% in h.

Attempts to properly vibrate this hyperdermic tubing failed. The vigorous motion caused the varnish

to short through electrically even when several coats were applied. Other surface coatings were tried but gave the same results. Finally after some difficulty a piece of thin plastic tubing was obtained through which the internal thermocouple was threaded. This in turn was threaded through an 11 inch length of 17 gage stainless steel hyperdermic tubing, which was the narrowest gage into which the plastic tubing would fit. An external thermocouple was attached as before and the whole assembly mounted and electrically heated to various temperatures as before. The assembly was first kept stationary and then vibrated at various amplitudes and frequencies. It was found that regardless of vibration or lack thereof the difference in temperature between internal and external thermocouples ranged from about 2% to 3% of  $\Delta t$  (and therefore about 2% to 3% of  $h$ ) with the external thermocouple always the lower. Thus the error is not only small but is roughly constant which tends to make it cancel out of the final correlations, as will be evident later.

The possibility of vibration itself generating any appreciable heat was also checked. It was found that vibrating the nichrome wires or the hyperdermic tubing without passing current through them resulted in no measurable wire or tubing temperature rise,

indicating no appreciable heat being generated by the vibration itself. Similarly, by disconnecting the linkage between buzzer armature and heated wire so that the wire remained stationary, it could be demonstrated that the sound of the buzzer had no effect on the wire temperature and therefore no effect on  $h$ .

It might also be pointed out that when taking temperature measurements there was a clear distance of about 6 inches from the central portion of the nichrome wire to the flat, horizontal, microscope supporting surface directly below it. This distance was more than enough to insure the absence of any measurable influence by this surface on the heat transfer coefficient. This was checked first by removing this surface so that there was a clear distance of about 2 feet to the desk below, and secondly by holding a flat, horizontal, surface directly below the wire, a distance of 1 inch away. In each case there was no measurable effect on the temperature of the heated wire.

Several thermocouples were checked at various temperatures against a calibrated mercury thermometer with a resulting agreement of about  $1^{\circ}$  F, which is about the precision of the potentiometer. Checking thermocouples directly against wet steam resulted in similar agreement.

### EXPERIMENTAL PROCEDURE

For each wire runs were carried out without vibration, with horizontal vibration, and with vertical vibration.

#### Stationary Wires:

The procedure for obtaining experimental data for a stationary wire began with certain preliminary adjustments. First the potentiometer was electrically zeroed against its standard cell and the cold junction compensating needle set for room temperature. Next, the ventilating blower for the room was shut off and the door closed to eliminate stray air currents. The wire was pulled taut by tightening the vise. Voltage was then applied across the wire and adjusted to the desired value with the Variacs. The time required to reach equilibrium varied from about three minutes for the 0.0253 inch wire to about fifteen minutes for the 0.0810 inch wire. In general, however, more time was allowed than this to be safe. The potentiometer was then balanced and its millivoltage reading converted to temperature. The ammeter and voltmeter readings were also taken. When reading the ammeter, the voltmeter was momentarily disconnected because it drew a small amount of current in parallel with the nichrome wire. The electric circuit was purely resistive so that the power factor equaled unity, and the product of amperes and volts gave the power dissipated in watts. As small changes in

room temperature took place the compensating needle was adjusted accordingly. The nichrome wire was occasionally gently polished with fine steel wool to keep its surface clean. No appreciable reduction in diameter resulted from this careful polishing.

Vibrating Wires:

The procedure when vibration was desired included the above details together with the necessary techniques for inducing and measuring the vibration. The position of the buzzer was adjusted to the wire to give good linkage between armature and wire. With the largest wire size the wire was sometimes tied to the armature with a piece of fine cord. Voltage from the dry cells was then applied to the buzzer but it was usually necessary to flick the wire near its center to make it break into vibration. The tension on the wire was adjusted with the vise to give the approximate desired frequency as determined by the stroboscope. Since heating the wire relaxed its tension and vibrating the wire cooled it with an opposite effect, readjustment of tension was necessary to obtain proper frequency and stable vibration.

Additional Adjustments for Vibration:

This matter of keeping stable vibration was sometimes troublesome. Theoretically a given stretched wire has

only one natural fundamental frequency. However, in practice this was not quite true for the system because the wire supports and buzzer supports were not absolutely rigid. As a result some energy from the moving armature found its way into these supports and was then fed back into the wire at a frequency different from the natural frequency of the wire. These frequencies beat against each other with the result that the amplitude of wire vibration sometimes tended to fluctuate appreciably. To eliminate this effect the wire tension and the linkage between armature and wire were both adjusted until the effect disappeared.

Amplitude of vibration was adjusted by varying linkage, tension, and buzzer voltage. For the case of horizontal vibration, the vibrating wire was sighted horizontally against a mark at the same level as the wire some distance away in order to be sure the wire was actually vibrating in a horizontal direction. Similarly, for vertical vibration the vibrating wire was sighted vertically against a mark directly below it.

#### Taking Data with Vibration:

After equilibrium had been reached the ammeter and voltmeter readings were taken and the wire temperature and room temperature recorded. The frequency was then determined by darkening the room and shining the stroboscopic

light on the vibrating wire. The stroboscope frequency was varied from its maximum on down until the wire appeared to be motionless or nearly so, at which point the scale reading was converted to true frequency from the calibration curve in the appendix. For wire frequencies below the scale range the stroboscope frequency was carefully reduced from its maximum until two nearly motionless wires seemed to appear and the true wire frequency obtained by dividing the frequency obtained from the calibration chart in half.

The amplitude for horizontal vibration was determined by slipping the microscope stage under the center of the wire and observing the motion by stroboscopic light through the calibrated grid in the eyepiece. The "apparent" amplitude, which was the clear distance in grid divisions between the inner edges of the vibrating wire or in other words one wire diameter less than the true amplitude, was noted and converted to true amplitude. Slight deviations of the microscope barrel from the vertical were of little consequence since they merely correspond to viewing the vibration at a slight angle from the normal to the plane of vibration, or in other words as a projection at a slight angle. The length of the image projection is proportional to the cosine of the angle of deviation, which for small angles is practically unity.

For example, for a barrel deviation of as much as  $5^\circ$  from the vertical the cosine would be 0.996; an error of only 0.4%.

The microscope was slipped in place only after the wire temperature reading was taken since its presence might induce some disturbance in the air which would affect the temperature at the thermocouple junction. However, the microscope was left in place until adjustments were complete for the next run and then removed. In this way two amplitude determinations could be taken for each run as a check.

For vertical vibration the procedure for amplitude determination was slightly different. For this case the barrel and stage were tilted backward until the barrel was horizontal. Slight deviations of the barrel from this horizontal position were of little importance for reasons similar to those pointed out above for the case with the barrel vertical. However, contrary to the case for the vertical barrel, it was not convenient to slip the microscope in and out because the stage interfered and could not be removed. Accordingly the microscope was left more or less permanently in position about one third of the way from the left end of the wire. With the stage vertical so as not to interfere with rising convection currents

this was sufficiently far removed from the thermocouple junction so as not to influence it. This fact was checked several times by removing the microscope and observing no change in potentiometer reading. The instantaneous arc assumed by a wire vibrating at its fundamental frequency is  $180^\circ$  of a sine wave. Thus by measuring the positions of the microscope and junction along the wire together with observing the amplitude through the microscope the true amplitude at the junction was readily calculated.

Barometric pressure was measured with a mercury manometer and air humidity determined with wet and dry bulb thermometers. These readings were taken only once each day since their range of fluctuation has little effect on the results, as will be evident later.

## VIBRATIONAL WAVE FORM

### Introduction:

In this work the wires are subjected to an external vibrating mechanism, namely the buzzer. Observation of the center of the wires through the microscope by stroboscopic light showed the vibration was essentially sinusoidal.

It was felt, however, that it would be of interest to get a direct visual picture of the wave form, and if possible record it photographically. This was done by allowing the wire to vibrate at right angles to a uniform magnetic field, picking up the induced e.m.f., amplifying it, and projecting it on an oscilloscope screen.

### Experimental Details:

The magnetic field was placed at the center of the wire to take advantage of maximum amplitude. It was obtained from two rectangular-faced alnico permanent magnets fitted together in the attracting position with opposite pole-pieces above and below the 0.0253 inch nichrome wire. The wire vibrated horizontally without approaching the edges of the pole-pieces even in its extreme positions, thus assuring vibration in an essentially uniform field of magnetic flux.

A short length of 0.003 inch varnished copper wire was carefully fixed against and parallel to the nichrome wire so that it executed every movement of the heavier wire without being in electrical contact with it. The emf induced in this copper wire was picked up by the amplifier. That generated in the nichrome wire itself was not used because it also vibrated near the main magnetic field of the buzzer which might have acted as a disturbing influence on the desired emf.

The amplifier output was fed to the vertical input of a DuMont oscillograph whose internal sweep supplied the horizontal coordinates for the patterns on the screen. Removing the alnico magnets or stopping the vibration removed the patterns from the screen, thus proving they were genuine and not the result of electrical cross-talk or some other spurious effect.

#### Results:

Oscillograms examined for various frequencies and amplitudes were all essentially sinusoidal.

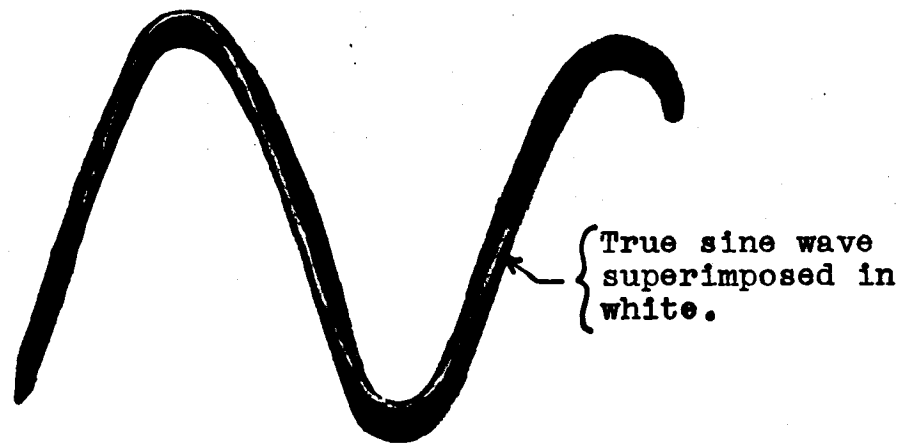
It should be pointed out that strictly speaking the oscillograms do not give the forms of the vibrations as such but rather the first derivatives. In other words, they do not represent a plot of distance against time, but rather a plot of velocity against time because the

motion or velocity of the wire generates the emf, not its position. However, since the derivative of a sine wave is another sine wave, the wave form comes through unchanged except for the usual magnification of imperfections that occurs when a derivative is taken.

Photographing the oscillograms proved difficult. Fig. 2 shows the result of such an attempt for a vibration of 100 cycles per second frequency and 0.12 inch amplitude. In an effort to at least partially compensate for the photographic deficiencies involved and show the wave form more nearly as it appeared to the eye, a "retouched" tracing of fig. 2 is shown in fig. 3. The essentially sinusoidal shape of the wave form is evident in either figure.



WAVE FORM OSCILLOGRAM - FIG. 2



WAVE FORM OSCILLOGRAM TRACING - FIG. 3

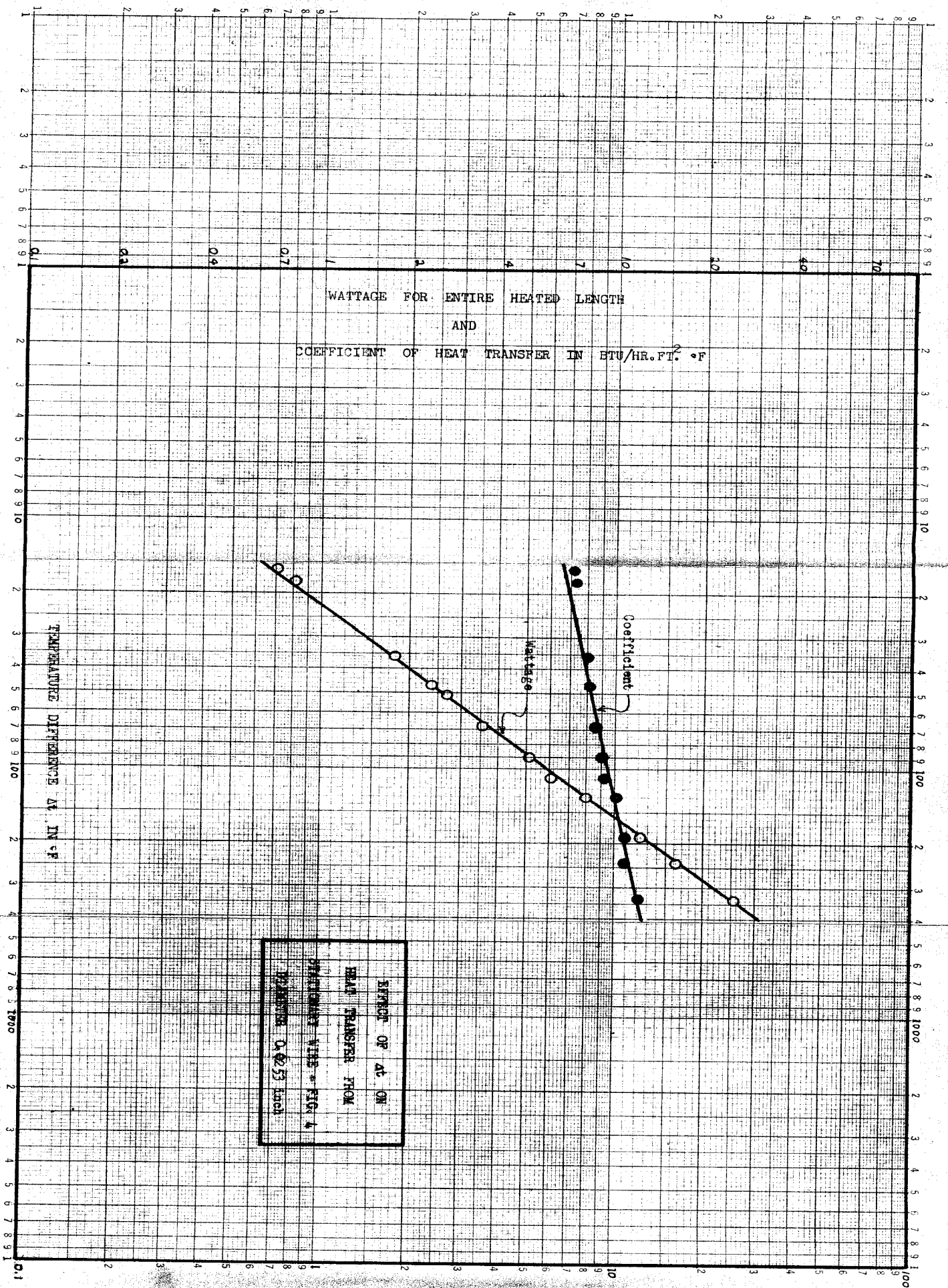
## DISCUSSION AND RESULTS

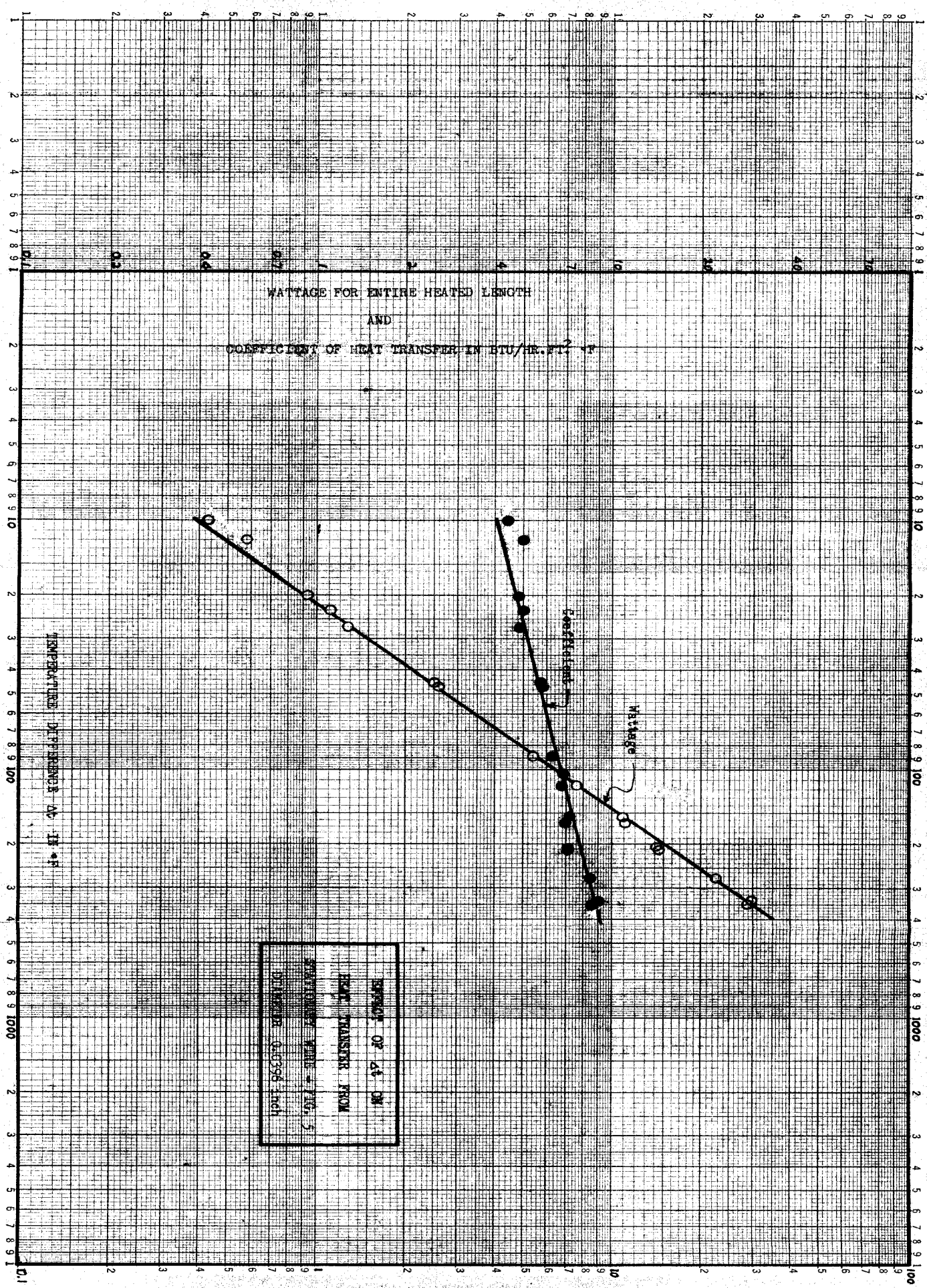
### Results with Stationary Wires:

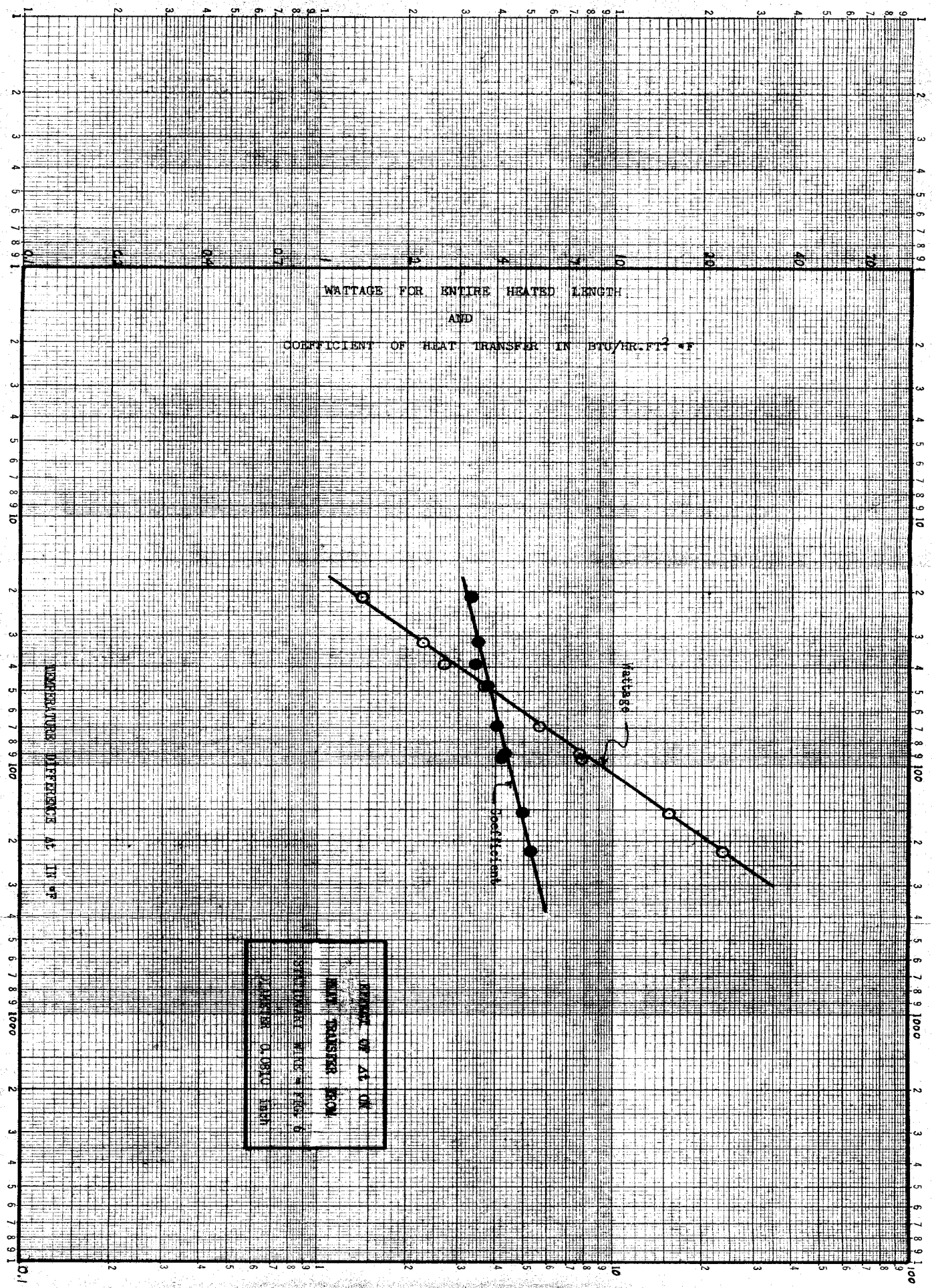
In order to study properly the effects of vibration on heat transfer, control runs had to be made first without vibration. These are shown in table 1. The wattage  $Q$  for the entire heated wire length is converted to  $q$  in BTU per hr. for a unit length of one foot, from which coefficient  $h$  is computed by eq. 2.

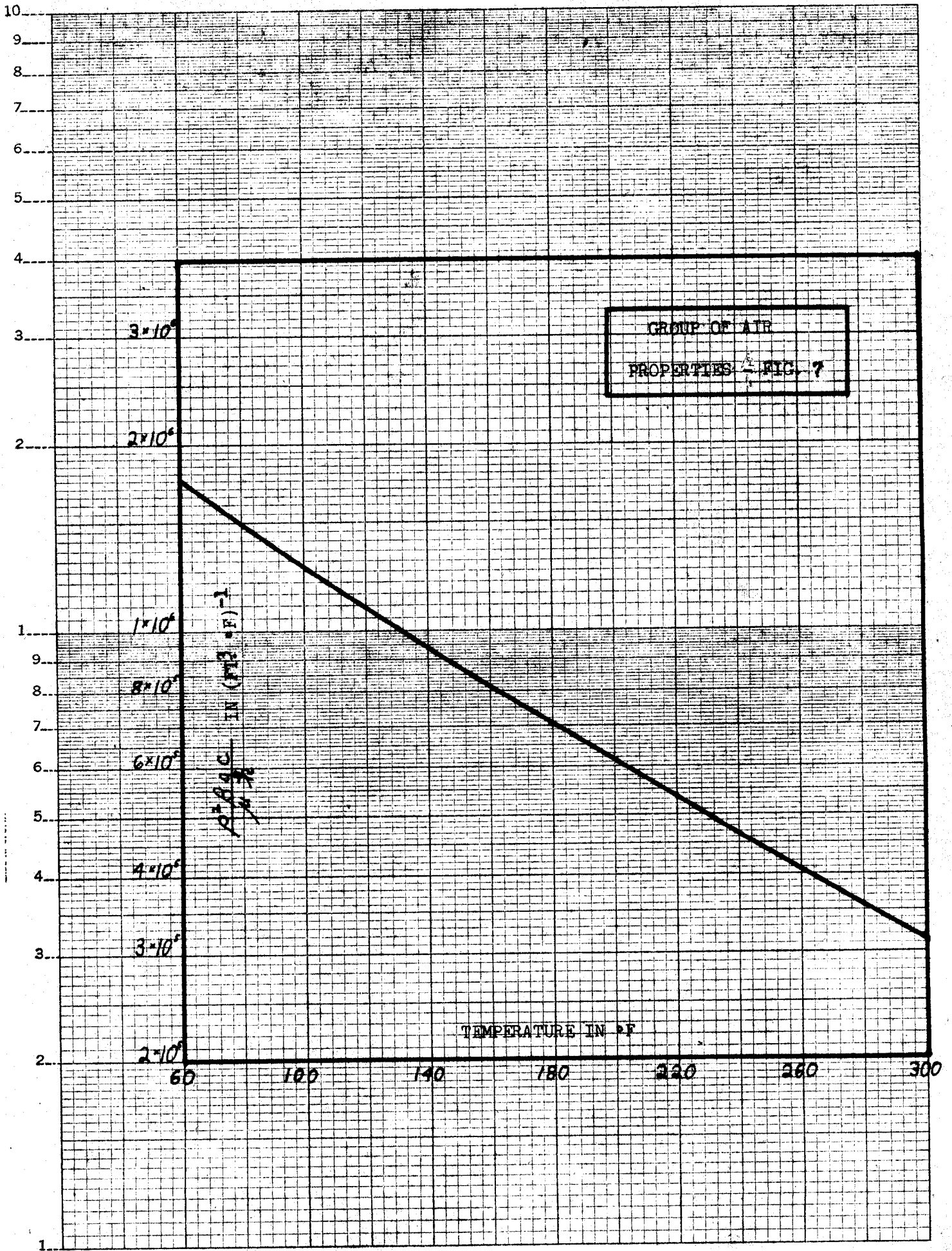
A plot of  $Q$  and  $h$  against  $\Delta t$  for each wire is given in figs. 4, 5, and 6. Inspection of these figures reveals a gradual increase in  $h$  with increasing  $\Delta t$ , which agrees with the results of previous investigators. This phenomena may be explained physically by the fact that an increase in  $\Delta t$  reduces local air density thus producing faster rising convection currents. This in turn makes for a greater local disturbance and a thinner film.

As discussed previously in the survey on heat transfer the usual way of presenting and comparing data for natural convection without vibration is by correlating  $Nu$  against  $(Gr \times Pr)$ , as indicated in eq. 8. To accomplish this here, first the quantity  $(\rho^2 \beta / \mu k)$  for air at one atmosphere pressure (108) was plotted against temperature in fig. 7. The humidity and the deviation of actual barometric pressure from standard have only a very small effect











and therefore were omitted from calculation at this point. With fig. 7 the product (Gr x Pr) was readily computed at  $t_f$  using eq. 10.

$$\text{Gr x Pr} = \frac{\rho^2 \beta g_c}{\mu k} (D^3 \Delta t) \quad (10)$$

The Nusselt number was then calculated with thermal conductivity data (118) at  $t_f$ , and  $\log_{10} \text{Nu}$  plotted against  $\log_{10}(\text{Gr x Pr})$  in fig. 8. Results of previously mentioned investigators (6, 44, 96, 130, 138, 148, 170) are also shown. Inspection of this figure shows the results of this investigation agree with the trend established by these earlier investigations and give about the same scatter. For an average curve (shown solid) the average deviation is about 5%. For further comparison the previously discussed curves of McAdams (108) and Rice (138) are also presented.

#### Effect of Alternation in Current on Temperature:

The wire temperature does not rise and fall appreciably with the rapid alternations in current. The mass of the wire and the thermal resistance of the film keep the wire temperature virtually constant with respect to time. This can be shown by considering the worst case (run 54) and making the unfavorable but simplifying assumption of a sudden interruption in current rather than

a more gradual sinusoidal change. Use of the well known unsteady state chart (108) for slender cylinders, together with this assumption, indicates a negligible change in temperature, too small to be read from the chart. By analytical computation the change in  $\Delta t$  amounts to less than 0.1%.

#### Vibrating Wires:

For vibrating wires, the effects on  $h$  of five major variables were investigated. These are  $\Delta t$ , amplitude<sup>\*</sup> of vibration, frequency, wire diameter, and direction of vibration; that is, whether vibration was horizontal or vertical. The overall range for  $\Delta t$  was from 7° to 365° F, for  $H$  from 0.055 to 0.231 inch, for  $F$  from 39 to 122 vibrations<sup>+</sup> per second, and for  $D$  from 0.0253 to 0.0810 inch. The runs are tabulated in table 2. Entries in column 10 of the table such as 6620/2 simply mean the stroboscopic light was set at twice the frequency of the vibrating wire to avoid the necessity for going below the instrument's scale. This point was discussed earlier. Amplitudes are tabulated in terms of microscope grid divisions directly.

Values of  $h$  are computed in the same manner as for the case without vibration. These values of  $h$  are for the center portion of the wire where the thermocouple is

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\*Defined as the distance between extreme positions of the wire centerline over the course of one complete cycle.

+cycles

located, and not for the wire as a whole since the amplitude of vibration varies along the length of the wire. However, because the center portion of the wire represents the top of a sine wave and as such is quite straight (flat) even at the crest of a vibration, these values of  $h$  which are calculated and later correlated are essentially for vibrating horizontal cylinders.

#### Electrical Resistance of Vibrating Wires:

The value of  $q$  used in calculation is essentially the average for the entire wire. No significant error is introduced in this way despite the fact that vibration increases  $h$  and therefore cools the center portion of the wire more than the ends. This is because the electrical resistance per unit length of wire varies only slightly from end to center due to the change of temperature with length. A change of as much as  $145^{\circ}$  F, which is the worst case (run 85) would involve a difference in electrical resistance (46) from end to center of about 2%. Since the entire wire is measured electrically, that is, both hot and cold portions are included, an average effective resistance is involved. This means the effective change in resistance is about half the 2%, or only about 1%. Since the heat generated anywhere along the wire equals the product of current squared times resistance, and since the

current must be uniform throughout the length of the wire, the maximum error in  $q$  is only 1%, which is negligible. The errors for the other runs are even smaller.

Conduction Along Vibrating Wires:

The heat dissipated anywhere along the wire equals that generated because no appreciable quantity of heat is conducted along the length of the wire, even for the wire with the largest diameter. This fact is proved for the worst case (run 137) by calculation using the method of relaxation (47). The method involves dividing the wire into 15 equal elements and writing a heat balance for each one. Details are given in the appendix but the results of the calculation may be summarized as follows:

$$\frac{\text{Net heat flow by conduction into central element}}{\text{Heat generated in central element}} = 0.3\%$$

$$\frac{\text{Greatest heat flow by conduction anywhere along wire}}{\text{Total heat dissipated by wire}} = 0.04\%$$

Additional confirmatory evidence of an experimental nature was also obtained. The electrical circuit was temporarily re-arranged so that current passed only through the left-hand third of the thickest wire. A similar wire was used as the electrical lead-in rather than a copper wire to prevent extraneous heat conduction along the lead wire. The thermocouple was located as usual

at the center of the main wire which was six inches from the heated section. The temperature of this heated section was then raised to 301° F. Nevertheless, the thermocouple junction only six inches away along the wire remained at room temperature as far as the potentiometer could detect, thus showing the heat conducted along the wire was negligible. By similar reasoning end effects are also without influence on the center.

Radiation:

The rate of heat transfer by radiation,  $q_r$ , is given by the well known Stefan-Boltzmann Law in the form

$$q_r = 0.174 \epsilon A \left[ \left( \frac{T_w}{100} \right)^4 - \left( \frac{T_b}{100} \right)^4 \right] \quad (11)$$

where  $A$  is wire surface in ft.<sup>2</sup>,  $\epsilon$  is emissivity,  $T_w$  is wire surface temperature in ° R, and  $T_b$  is environmental temperature in ° R. No value for  $\epsilon$  could be located for non-oxidized nichrome wire. However, using an  $\epsilon$  for nickel wire (129) of 0.1, computation by eq. 11 shows that for the case with the greatest radiative heat flux density (run 90)  $q_r/A$  is less than 3% of the total heat transferred per unit area. For other runs  $q_r/A$  is even smaller. Thus the contribution of radiation is trifling. Furthermore, as will be evident later, the final correlations deal with a ratio of coefficients rather than with coefficients them-

selves. Thus in most cases the effect of radiation virtually cancels out. For example, the effect on this ratio for the case mentioned above is about 0.1%, which, of course, is negligible. Accordingly, radiation may be safely neglected.

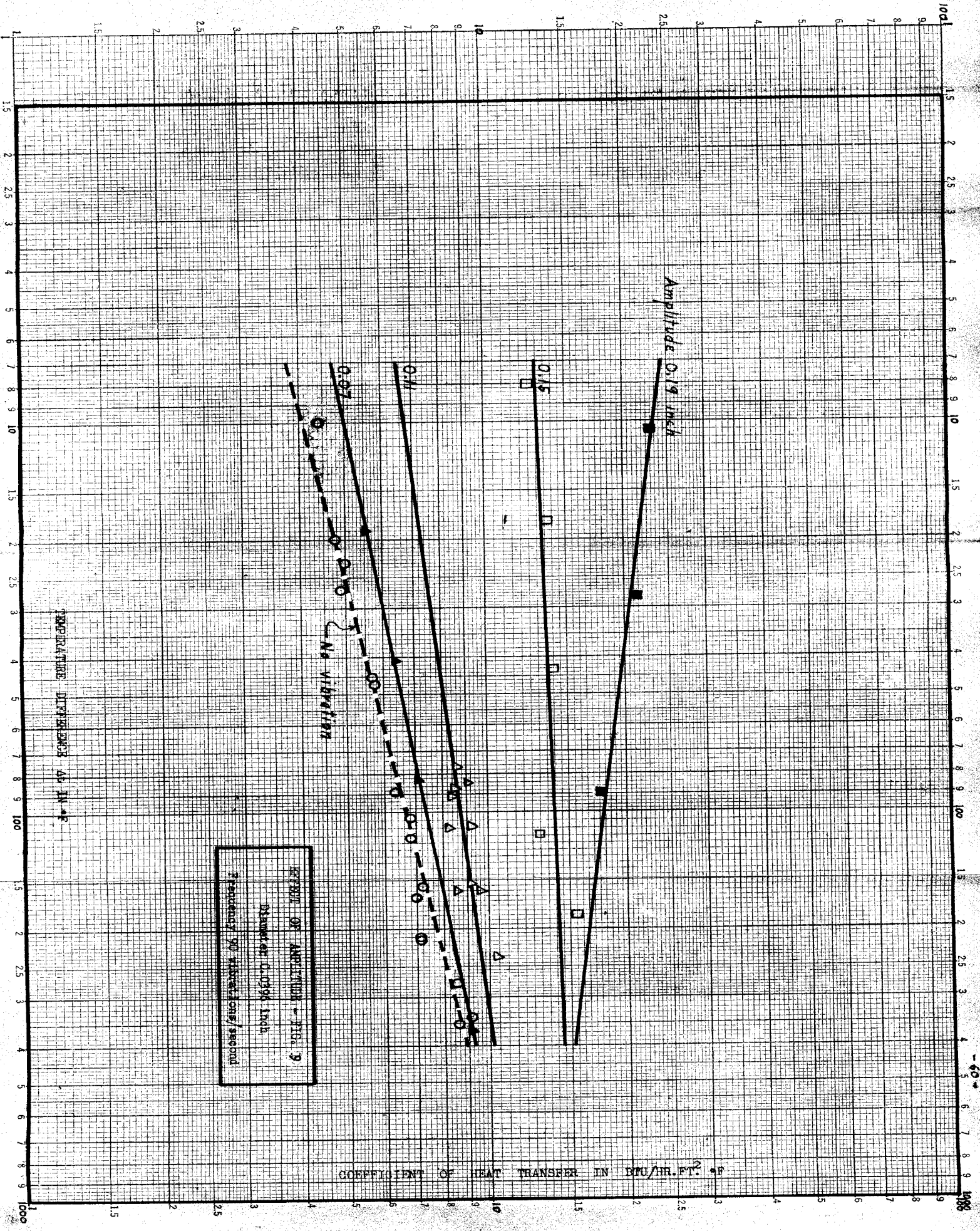
#### Effect of Vibration Variables:

Runs illustrating the effects on  $h$  of amplitude, frequency, and diameter, together with  $\Delta t$ , are listed in tables 3, 4, and 5 and plotted in figs. 9, 10, and 11 respectively. In general, these effects are in accord with the film concept, as will be shown subsequently in detail.

#### Effect of Amplitude:

The effect of amplitude is illustrated in fig. 9. Holding other variables constant, this figure reveals an increase in  $h$  with an increase in amplitude. This is in accord with the film concept and follows from the increased disturbance and thinning of the film that results from an increase in amplitude. The term "disturbance" is used rather than "turbulence" because it is not really known whether the air motion is laminar, turbulent, or both.

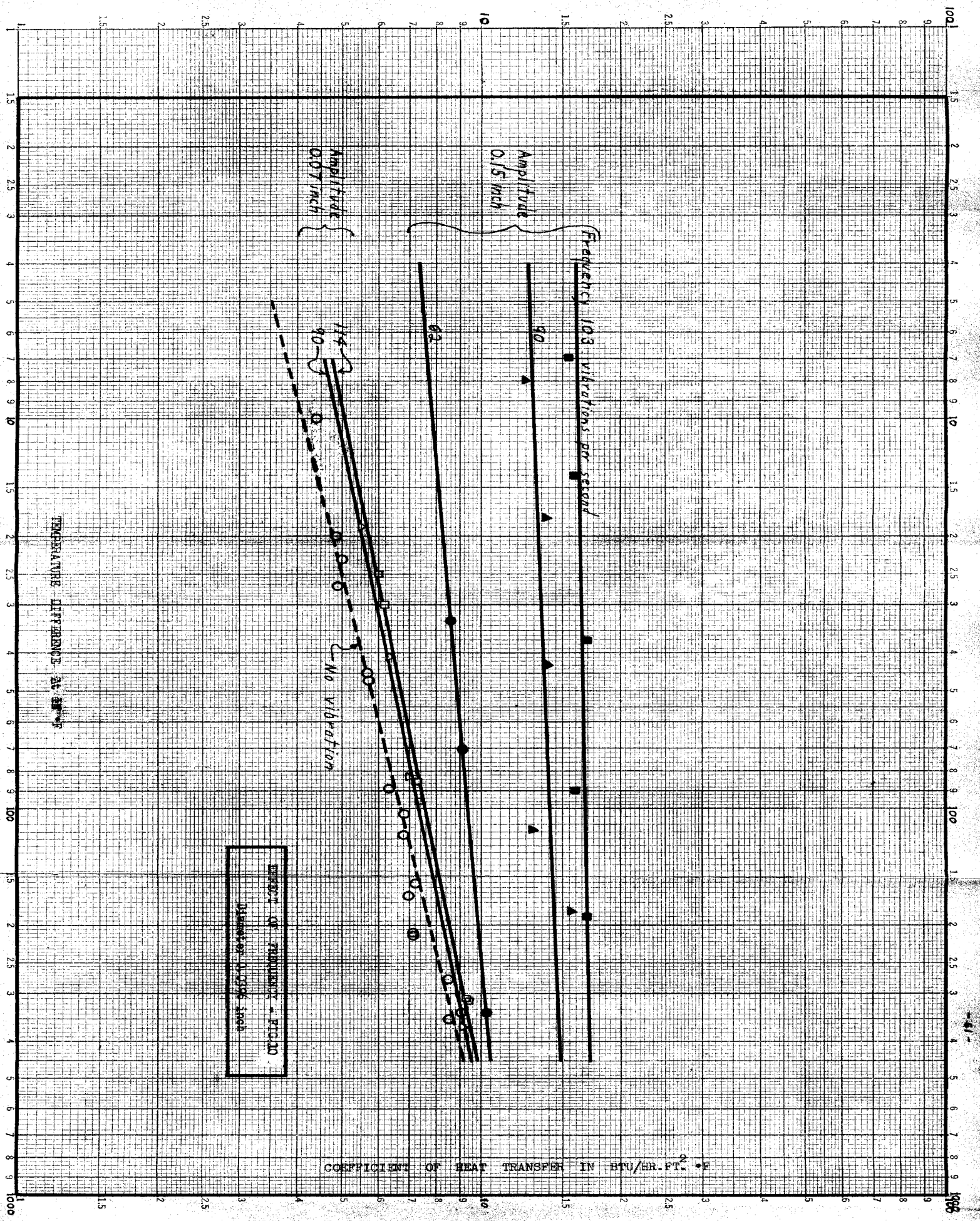
This figure also shows that a given increase in amplitude produces a smaller increase in  $h$  at higher values of  $\Delta t$  than at lower values of  $\Delta t$ . In other words, the



EXPERIMENT OF AMPLITUDE - FIG. 9  
 Diameter 0.076 inch  
 Frequency 90 vibrations/second

TEMPERATURE DIFFERENCE IN °F

COEFFICIENT OF HEAT TRANSFER IN BTU/HR.FT.<sup>2</sup>.°F





effect of amplitude diminishes as  $\Delta t$  increases. This can be explained in terms of the increased disturbance at the film due to natural convection currents produced by a high  $\Delta t$ . Under these conditions the additional disturbance caused by an increase in amplitude represents a smaller proportion of the total disturbance than it does for the situation with a low  $\Delta t$  and feebler natural convection currents. In the extreme this results in a negative slope at high amplitude in fig. 9. Further discussion of this point must be deferred until the Reynolds number for vibration is introduced.

#### Effect of Frequency:

The effect of frequency is illustrated in fig. 10. Holding the other variables constant, this figure shows that an increase in frequency increases  $h$ . This follows from the increased overall disturbance at the film produced by increasing the number of vibrations per second and hence the number of times per second a disturbance at the film is created. The figure also shows that frequency has a greater effect on  $h$  at a larger amplitude of vibration than at a smaller amplitude. This is to be expected since increasing the number of times per second that a large disturbance occurs results in a greater overall disturbance at the film than would result from correspondingly increasing the fre-

quency of a small disturbance.

Analogous with the case of amplitude discussed previously, fig. 10 shows that a given increase in frequency produces a smaller increase in  $h$  at higher values of  $\Delta t$  than at lower values. Again paralleling the case of amplitude, this may be explained by the decreased relative effect of the disturbance at the film due to vibration when compared to the increased natural convection disturbance resulting from the increased  $\Delta t$ .

Effect of Diameter:

The effect of diameter is illustrated in fig. 11. The frequency parameter could not be held constant over all three diameters because the 0.0810 inch diameter wire is comparatively much more massive than the other two thinner wires and therefore vibrated in a lower frequency range. Accordingly, the parameter chosen is the average velocity of vibration  $\bar{v}$  in feet per second. This  $\bar{v}$  is the distance covered by the wire in one vibration divided by the time elapsed. Thus,

$$\bar{v} = 2HF \tag{12}$$

Further mention will be made of  $\bar{v}$  later when the Reynolds number for vibration is introduced.

Dotted lines for the stationary wires ( $\bar{v} = 0$ ) are shown in fig. 11. The points for these lines are not

shown here for the sake of clarity but may be found in figs. 4, 5, and 6.

Inspection of fig. 11 shows that  $h$  increases as the diameter decreases, holding  $\bar{v}$  and  $\Delta t$  constant. This may be explained physically by the decreased hindrance to natural convection currents offered by a decrease in wire diameter. A thick wire tends to block rising natural convection currents thus diminishing their effectiveness in reducing film thickness. The figure, however, shows that the effect of diameter change decreases as the average velocity of vibration increases. In other words, the lines in the figure become closer together as  $\bar{v}$  increases. This follows from the decreased relative importance of diameter hindrance to natural convection currents (in affecting film thickness) when compared with the increased disturbance produced by increasing vibration.

Figure 11 also shows that an increase in average velocity causes a smaller increase in  $h$  at higher values of  $\Delta t$  than at lower values. This is due to the decreased relative effect on the film of the disturbance due to vibration when compared with the increased natural convection disturbance resulting from the increased  $\Delta t$ . This stems from the corresponding individual effects for amplitude and frequency discussed previously. As with

fig. 9 the slope is negative for high values of the parameter. Further discussion of this point must again be postponed to a later subsection.

#### Effect of Direction of Vibration:

The direction of vibration has no effect within the limits of experimental error. This can best be seen from figs. 13 and 14. The solid points represent vertical vibration and the hollow points represent horizontal vibration. No difference in trend between the two types of points can be discerned. Further discussion of these figures appears later.

#### Physical Picture:

With regard to a physical picture, all that is known with certainty is that the wire moves back and forth (or up and down), constantly accelerating or decelerating, in a non-homogenous field of rising convection currents. The actual film phenomena is undoubtedly quite complicated. Nevertheless, the experimental data do seem to indicate, at least as an approximation, that within the limits of the present range of variables the vibrating wire does not carry the film back and forth with it as represented in fig. 12-A, but rather that the film surrounds the entire vibrating path as represented in fig. 12-B. The



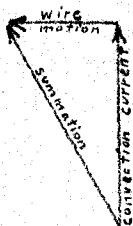
**Moving Film**

- A -



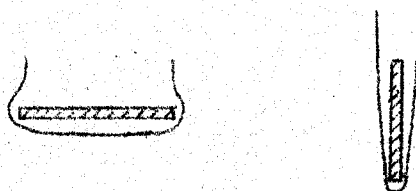
**Stretched Film**

- B -



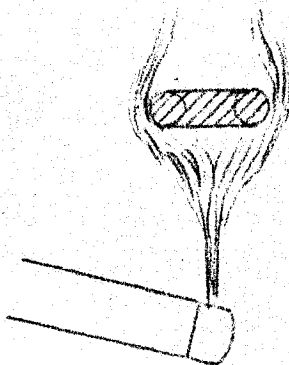
**Instantaneous Vector Summation**

- C -



**Flat Plate**

- D -



**Smoke Pattern**

- E -

**FILM  
CONSIDERATIONS**

**FIG. 12**

evidence to support this contention is threefold.

1. If the wire did carry its film back and forth there would be something of a vector summation of velocities at right angles between the horizontally moving wire and the rising convection current, as shown in fig. 12-C. This vector sum would never be less than the vertical current itself, and except for the two instants at either crest of the cycle, would always be greater. On the other hand, the vector summation for a vertically moving wire would result in a net addition half the time and in a cancelling subtraction during the other half. Thus the average effective velocity over a cycle for vertical vibration would be less than that for horizontal vibration. This would mean a thicker film and therefore a lower  $h$  for vertical vibration as compared with horizontal vibration. Now, as pointed out in the previous subsection, no difference could be discerned between the effects of horizontal and vertical vibration. Therefore it appears unlikely that the wire could have carried its film back and forth in the manner indicated in fig. 12-A. This means the wire must have "stretched" the film out, as indicated in fig. 12-B.

2. The shape within the stretched film (shown shaded) would probably have much the same rate of natural convective heat transfer in a horizontal position as in a vertical position. While the author knows of no data for

such a configuration, correlations (23) are available for plane surfaces (fig. 12-D) in air at moderate conditions of temperature and pressure which indicate that  $h$  for a vertical plate, as compared to that for a horizontal plate facing downward, as compared to that for one facing upward, is in the ratio 55:71:35 for laminar flow and in the ratio 13:17:8 for turbulent flow, all for fixed dimensions and  $\Delta t$ . Thus for laminar flow a horizontal plate loses heat from both surfaces at a rate proportional to  $71 + 35 = 106$ , while a vertical plate loses heat at a rate proportional to  $55 + 55 = 110$ . The relatively close agreement between the two sums is evident. The corresponding summations for turbulent flow are  $17 + 8 = 25$  and  $13 + 13 = 26$ . Here again the close agreement is evident. Thus, while this does not strictly prove that the shape shown in fig. 12-B has nearly the same coefficient in a vertical or horizontal position, it does represent strong evidence in that direction.

3. A simple experiment was conducted wherein a lit cigarette was held under one of the wires in horizontal vibration. The smoke appeared to curl up and around the vibrating path, as illustrated in fig. 12-E. No smoke was observed passing through the vibrating path. This would seem to indicate the existence of a stretched film.

It might also be pointed out that even on an intuitive basis it seems more reasonable for a rapidly

vibrating wire with moderate or small amplitude to stretch out its film rather than to carry it back and forth. The comparatively sudden and repeated changes in direction make it difficult to visualize a moving film that would keep up with the wire. In other words, while for a wire gently wafting over an extremely large horizontal amplitude of perhaps 10 feet this would probably be true, the concept of a stretched film appears more reasonable for the smaller amplitudes and higher frequencies employed in this investigation.

Admittedly, these arguments do not represent a rigorous proof of this stretched film concept. However, in the opinion of the author, they do constitute rather strong evidence in its favor.

#### Correlating the Data:

The data was taken with an eventual overall correlation as the goal in mind. A number of different attempts were made to arrive at such a correlation. The equation of Martinelli and Boelter discussed previously in the vibrations survey and modifications thereof were tried, but without success. Various other theoretical and empirical attempts were made but proved unsatisfactory.

Among these were attempts based on the stretched film concept. The internal dimensions (shaded portion) in

fig. 12-B are determined by the amplitude and wire diameter. In fact, the increase in  $h$  with increasing amplitude can even be explained qualitatively as being due in part to the resulting increased circumference of the stretched film. However, the thickness of the stretched film depends in part on the frequency, and no method for relating the two could be devised.

Attempts were also made to account for the resulting  $h$  in terms of a known natural convection effect added to a known forced convection effect. This was done by trying in some way to "add" an effect from eq. 8 for natural convection to an effect from the corresponding known (108) equation for forced convection which is of the form

$$Nu = (Pr)^{0.3} \phi'(Re) \quad (13)$$

This also failed.

Other unsuccessful attempts at correlation involve certain terms not yet defined, and are outlined in the following section. The only method of attack which did prove successful was that of dimensional analysis.

#### Dimensional Analysis and Form of General Correlation:

The coefficient  $h$  may be written in terms of variables as

$$h = \phi_1(c, \mu, k, D, \rho, \beta, \Delta t, g, H, F) \quad (14)$$

$$\text{or } \phi_2 (c, \mu, k, h, D, \rho, \beta, \Delta t, g, H, F) = 0 \quad (15)$$

where each subscripted  $\phi$  appearing both here and later represents some function. Now the various terms in eq. 14 can be arranged in dimensionless groups. The first step therefore is to find these groups.

The basic dimensions are heat  $q$ , time  $\tau$ , mass  $M$ , absolute temperature  $T$ , and length  $L$ . Then,

$$W = c^{a_1} \mu^{a_2} k^{a_3} h^{a_4} D^{a_5} \rho^{a_6} \beta^{a_7} (\Delta t)^{a_8} g^{a_9} H^{a_{10}} F^{a_{11}} \quad (16)$$

$$\text{or } W = \left(\frac{q}{MT}\right)^{a_1} \left(\frac{M}{L\lambda}\right)^{a_2} \left(\frac{q}{\lambda TL}\right)^{a_3} \left(\frac{q}{\lambda TL^2}\right)^{a_4} (L)^{a_5} \left(\frac{M}{L^3}\right)^{a_6} \left(\frac{1}{T}\right)^{a_7} \\ (T)^{a_8} \left(\frac{L}{\lambda^2}\right)^{a_9} (L)^{a_{10}} \left(\frac{1}{\lambda}\right)^{a_{11}} \quad (17)$$

where  $W$  represents some dimensionless quantity and each exponent  $a$  is a constant.

For each basic dimension,

$$q: \quad a_1 + a_3 + a_4 = 0 \quad (18)$$

$$\lambda: \quad -a_2 - a_3 - a_4 - 2a_9 - a_{11} = 0 \quad (19)$$

$$M: \quad -a_1 + a_2 + a_6 = 0 \quad (20)$$

$$T: \quad -a_1 - a_3 - a_4 - a_7 + a_8 = 0 \quad (21)$$

$$L: \quad -a_2 - a_3 - 2a_4 + a_5 - 3a_6 + a_9 + a_{10} = 0 \quad (22)$$

Solving for  $a_4, a_6, a_7, a_9,$  and  $a_{11}$  in terms of the others,

$$a_4 = -a_1 - a_3 \quad (23)$$

$$a_6 = a_1 - a_2 \quad (24)$$

$$a_7 = a_8 \quad (25)$$

$$a_9 = a_1 - 2a_2 - a_3 - a_5 - a_{10} \quad (26)$$

$$a_{11} = -a_1 + 3a_2 + 2a_3 + 2a_5 + 2a_{10} \quad (27)$$

Substituting in eq. 16,

$$W = c^{a_1} \mu^{a_2} k^{a_3} h^{-a_1-a_3} D^{a_5} \rho^{a_1-a_2} \beta^{a_8} (\Delta t)^{a_8} g^{a_1-2a_2-a_3-a_5-a_{10}} H^{a_{10}} F^{-a_1+3a_2+2a_3+2a_5+2a_{10}} \quad (28)$$

Grouping,

$$W = \left( \frac{c \rho g}{h F} \right)^{a_1} \left( \frac{\mu F^3}{\rho g^2} \right)^{a_2} \left( \frac{k F^2}{h g} \right)^{a_3} \left( \frac{D F^2}{g} \right)^{a_5} (\beta \Delta t)^{a_8} \left( \frac{H F^2}{g} \right)^{a_{10}} \quad (29)$$

Since  $DF^2$  has the same dimensions as  $g$  it can be substituted for  $g$  in any of the dimensionless groups of eq. 29. Similarly, substitution of  $Dk$  for  $h$ ,  $H$  for  $D$ ,  $\frac{\rho g}{DF}$  for  $\mu$ , and

$\frac{\rho^{2/3} g^{4/3}}{\mu^{2/3}}$  for  $F^2$ , leads to the groups

$$\left( \frac{c \mu}{k} \right), \left( \frac{D H F \rho}{\mu} \right), \left( \frac{h D}{k} \right), \left( \frac{D^3 \rho^2 g}{\mu^2} \right), (\beta \Delta t), \left( \frac{H}{D} \right)$$

so that

$$\frac{h D}{k} = \phi_5 \left[ \left( \frac{c \mu}{k} \right), \left( \frac{D H F \rho}{\mu} \right), \left( \frac{D^3 \rho^2 g}{\mu^2} \right), (\beta \Delta t), \left( \frac{H}{D} \right) \right] \quad (30)$$

From eq. 12 the quantity  $2HF$  is the average velocity  $\bar{v}$ . Accordingly, a factor of 2 may be included in the group  $\left(\frac{DHF}{\mu}\right)$  to give a type of Reynolds number.\*

$$Re = \frac{2DHF}{\mu} = \frac{D \bar{v}}{\mu} \quad (31)$$

Modifying eq. 30 slightly to include this factor of 2,

$$\frac{hD}{k} = \phi_4 \left[ \left(\frac{c\mu}{k}\right), \left(\frac{2DHF}{\mu}\right), \left(\frac{D^3 \rho^2 g}{\mu^2}\right), (\beta \Delta t), \left(\frac{H}{D}\right) \right] \quad (32)$$

The next major step is to determine the form of the function. Among the different forms investigated was that of writing  $\phi_4$  as a product of groups raised to powers, namely

$$\frac{hD}{k} = \chi \left(\frac{c\mu}{k}\right)^{a_1} \left(\frac{2DHF}{\mu}\right)^{a_2} \left(\frac{D^3 \rho^2 g}{\mu^2}\right)^{a_3} (\beta \Delta t)^{a_4} \left(\frac{H}{D}\right)^{a_5} \quad (33)$$

where  $\chi$  is some constant.

Now, when amplitude  $H$  is zero, that is, when the wire is stationary,  $\frac{hD}{k}$  must be some positive finite number since there will still be heat transferred by ordinary natural convection. This means that  $a_2 = -a_5$ . However, this in turn means that  $H$  is without effect on  $h$  since the net

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\*Note: It might be pointed out that this concept of a vibrational Reynolds number was devised independently by the present author before he managed to locate the obscure work of Martinelli and Boelter (105, 106) who used a somewhat similar idea, as indicated in eqs. 9 and 9a.

exponent on H is zero. This is contrary to experimental evidence presented earlier which shows that the amplitude has a marked effect on the coefficient. Therefore the form of the function given in eq. 33 is unsatisfactory and some other form must be devised.

Considering again eq. 32, when vibration is absent so that H equals zero this equation reduces to

$$\frac{hD}{k} = \phi_4 \left[ \left( \frac{c\mu}{k} \right), \left( \frac{D^3 \rho^2 g}{\mu^2} \right), (\beta \Delta t) \right] \quad (34)$$

However, as discussed in the heat transfer survey, previous investigators have shown that eq. 8 holds when vibration is absent, or

$$\frac{hD}{k} = \phi \left[ \left( \frac{c\mu}{k} \right) \times \left( \frac{D^3 \rho^2 g \beta \Delta t}{\mu^2} \right) \right] \quad (35)$$

This indicates that  $\left( \frac{D^3 \rho^2 g}{\mu^2} \right)$  and  $(\beta \Delta t)$  in eq. 34 can be combined to give the Grashof number. Substituting into eq. 32 and writing the effects of vibration as a "correction" to  $\phi(\text{Pr} \times \text{Gr})$ ,

$$\text{Nu} = \left[ \phi(\text{Pr} \times \text{Gr}) \right] \left[ 1 + \phi_5(\text{Re}, \text{Gr}, \text{Pr}, \beta \Delta t, \frac{H}{D}) \right] \quad (36)$$

Of the various forms of  $\phi_5$  examined with the experimental data, the form that proved best was

$$\phi_5 = \phi'_5 \left[ (\text{Re})^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(\text{Gr})^{\gamma_3}} \right] \quad (37)$$

where  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$  are constants. No effect for  $H/D$  could be found experimentally and  $Pr$  is virtually constant for air and other diatomic gases. Substituting eq. 37 into eq. 36,

$$Nu = \left\{ \phi(Pr \times Gr) \right\} \left\{ 1 + \phi'_5 \left[ (Re)^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(Gr)^{\gamma_3}} \right] \right\} \quad (38)$$

or

$$Nu = Nu'_0 \left\{ 1 + \phi'_5 \left[ (Re)^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(Gr)^{\gamma_3}} \right] \right\} \quad (39)$$

where  $Nu'_0$  is the Nusselt number for the same wire without vibration but at the same  $\Delta t$  as with vibration, that is, at the same value of  $Pr \times Gr$  since room temperature  $t_b$  varied only slightly and  $t_f = t_b + \frac{1}{2} \Delta t$ . Therefore, for corresponding values of thermal conductivity,  $k = k'_0$  where the subscript and prime have the same significance as before. Then, from the definition of Nusselt number,

$$\frac{Nu}{Nu'_0} - 1 = \frac{h}{h'_0} - 1 \quad (40)$$

where  $h'_0$  is correspondingly defined as the coefficient of heat transfer without vibration at the same  $\Delta t$ , which means the same  $Pr \times Gr$ , as with vibration, all for the same wire. Substituting eq. 40 into eq. 39,

$$\frac{h}{h'_0} - 1 = \phi'_5 \left[ (Re)^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(Gr)^{\gamma_3}} \right] \quad (41)$$

which is the form for correlation I.

A further step is possible, thus

$$q = hA \Delta t \quad (2)$$

Similarly  $q_0 = h_0 A \Delta t \quad (42)$

where  $q_0$  is the heat transferred without vibration at the same  $\Delta t$  (same  $Pr \times Gr$ ) as with vibration, all for the same wire.

Dividing eq. 2 by eq. 42,

$$\frac{q}{q_0} = \frac{h}{h_0} \quad (43)$$

or  $\frac{Q}{Q_0} = \frac{q}{q_0} = \frac{h}{h_0} \quad (44)$

where  $Q$  and  $Q_0$  are the wattages for the entire heated length of wire corresponding to  $q$  and  $q_0$  respectively.

Substituting in eq. 41,

$$\frac{Q}{Q_0} - 1 = \frac{q}{q_0} - 1 = \frac{h}{h_0} - 1 = \phi'_5 \left[ (Re)^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(Gr)^{\gamma_3}} \right] \quad (45)$$

Digressing for a moment, some of the more interesting of the "other unsuccessful attempts" at correlation mentioned at the conclusion of the previous subsection are enumerated here in the form of inequalities. Essentially, these involve attempts at relating the increase in coefficient or Nusselt number, or the decrease in resistance or reciprocal Nusselt number, directly to the Reynolds number. Certain quantities just defined or about to be defined are

involved, hence the appearance of these inequalities at this point in the discussion.

$$\text{Nu} \neq \text{Nu}_0' + \theta(\text{Re}) \quad (\text{A})$$

$$\text{Nu} \neq \text{Nu}_0 + \theta(\text{Re}) \quad (\text{B})$$

$$h \neq h_0' + \theta(\text{Re}) \quad (\text{C})$$

$$h \neq h_0 + \theta(\text{Re}) \quad (\text{D})$$

$$\frac{1}{h} \neq \frac{1}{h_0'} - \theta(\text{Re}) \quad (\text{E})$$

$$\frac{1}{h} \neq \frac{1}{h_0} - \theta(\text{Re}) \quad (\text{F})$$

$$\frac{1}{\text{Nu}} \neq \frac{1}{\text{Nu}_0'} - \theta(\text{Re}) \quad (\text{G})$$

$$\frac{1}{\text{Nu}} \neq \frac{1}{\text{Nu}_0} - \theta(\text{Re}) \quad (\text{H})$$

The symbol  $\theta$  represents any function. The significance of a  $_0$  subscript without a prime is explained immediately below. These inequalities are significant in that they point out apparently plausible hypotheses which actually are not valid.

Returning now to the main trend of discussion, as an alternate to eqs. 41 and 45 the following may be written, which is the form for correlation II.

$$\frac{h}{h_0} - 1 = \phi_5^n \left[ (\text{Re})^{\gamma_1} \frac{(\beta \Delta t)^{\gamma_2}}{(\text{Gr})^{\gamma_3}} \right] \quad (46)$$

where  $h_0$  is the coefficient of heat transfer for the same wire without vibration but at the same heat flux (same  $q$ ) as with vibration, rather than at the same  $\Delta t$ . The quantities  $\gamma_1$ ,  $\gamma_2$ , and  $\gamma_3$  are constants. This eq. 46 could also be arrived at independently by dimensional analysis and subsequent reasoning similar to that for eq. 41 by first incorporating  $h_0$  in the right hand side of eq. 14 and utilizing the fact that  $Pr$  for diatomic gases is virtually constant so that  $c$  and  $k$  could be omitted from eq. 14.

No relationship could be found between  $h_1$  and  $h_0$  from which eqs. 41 and 46 could be further related.

As before, a further step is possible.

$$q = hA \Delta t \quad (2)$$

$$\text{Similarly, } q = h_0 A \Delta t_0 \quad (47)$$

where  $\Delta t_0$  is the temperature difference without vibration at the same heat flux as with vibration, all for the same wire diameter.

Combining eqs. 2 and 47,

$$\frac{h}{h_0} = \frac{\Delta t_0}{\Delta t} \quad (48)$$

Substituting in eq. 46,

$$\frac{\Delta t_0}{\Delta t} - 1 = \frac{h}{h_0} - 1 = \phi_5'' \left[ (Re) \gamma_1 \frac{(\beta \Delta t) \gamma_2}{(Gr) \gamma_3} \right] \quad (49)$$

Final Correlations:

All quantities are calculated at the average film temperature. Attempts at utilizing the bulk fluid temperature, that is, room temperature, for determining fluid properties correlated poorly.

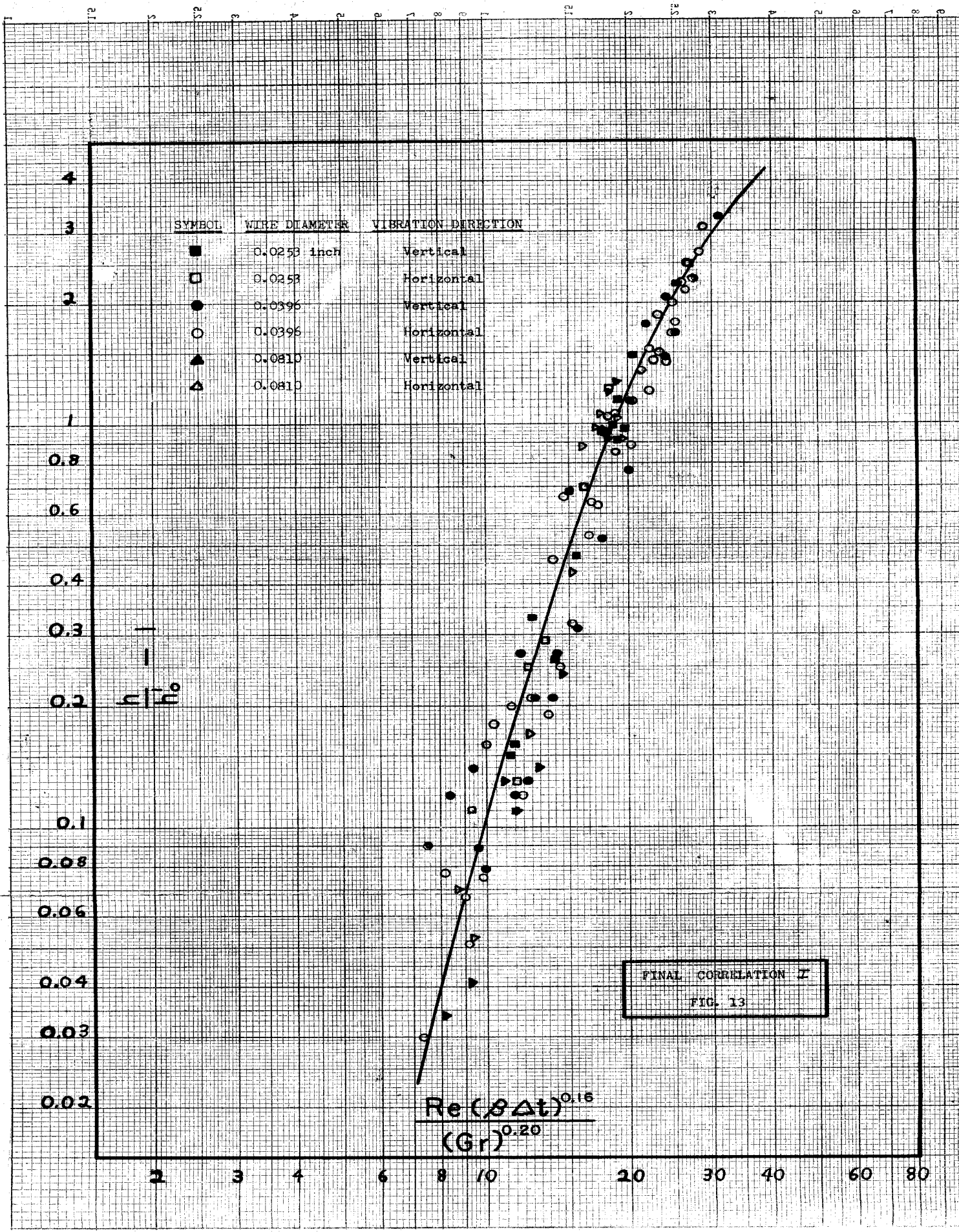
After many trial plots it was found the data correlated best when the ratios  $\gamma_1 : \gamma_2 : \gamma_3$  and  $\gamma_1 : \gamma_2 : \gamma_3$  were in the proportions 1.00 : 0.16 : 0.20.

Fig. 13 shows a plot of  $\left[ \frac{h}{h_0} - 1 \right]$  against  $\left[ \text{Re} \frac{(\beta \Delta t)^{0.16}}{(\text{Gr})^{0.20}} \right]$  for which the tabulated values are given in table 2.

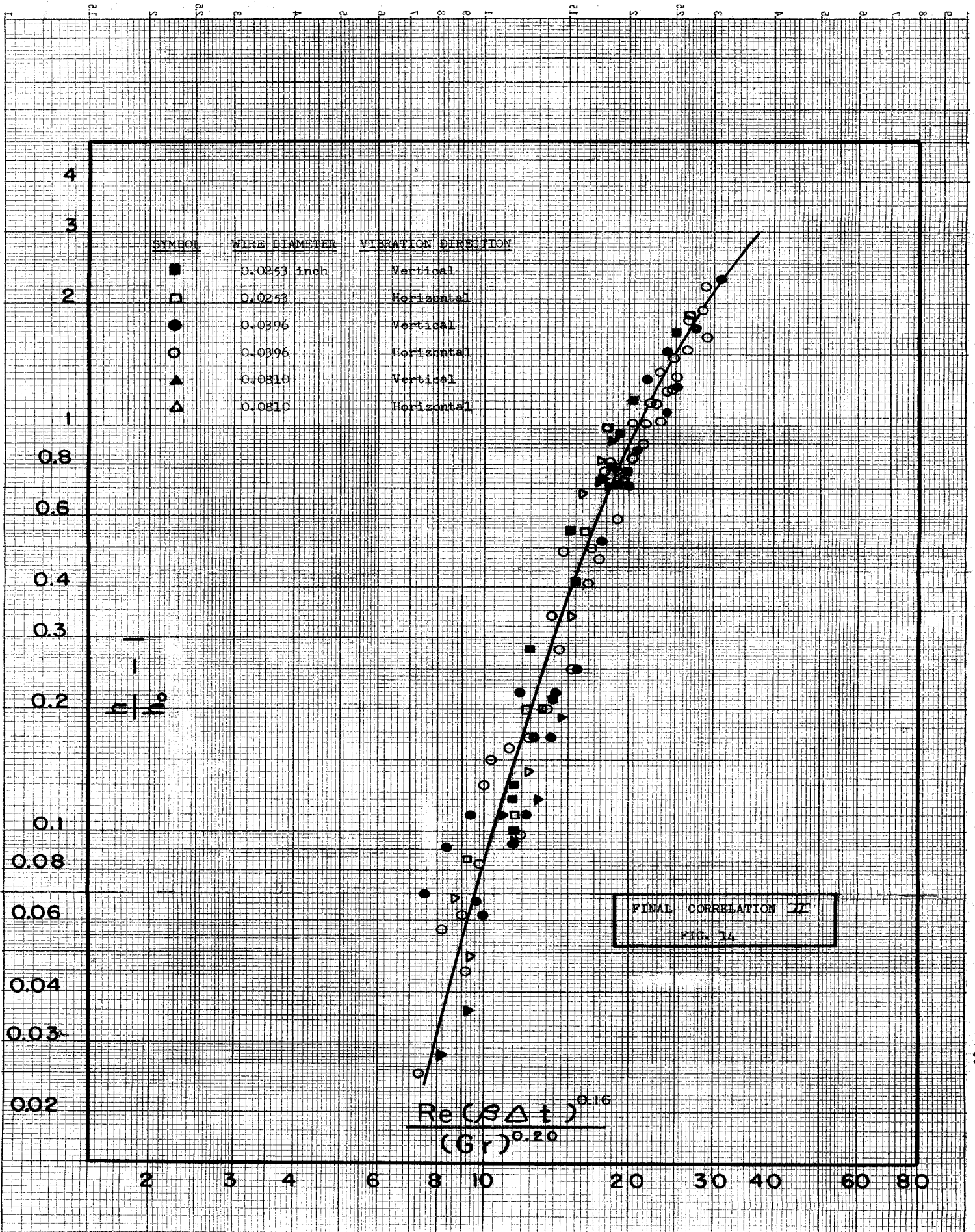
For convenience in calculating, the quantity  $h/h_0$  was computed directly from eq. 44.

Fig. 14 shows a plot of  $\left[ \frac{h}{h_0} - 1 \right]$  against  $\left[ \text{Re} \frac{(\beta \Delta t)^{0.16}}{(\text{Gr})^{0.20}} \right]$ . For convenience in calculation  $h/h_0$  was computed directly from eq. 48. In computing the abscissae for both figs. 13 and 14 the humidity and barometric pressure were included even though their effect in changing  $\rho$  is small. This was done because the overall effect of the abscissa is large; that is, the slope in these figures is steep.

The quantities  $\left[ \frac{h}{h_0} - 1 \right]$  and  $\left[ \frac{h}{h_0} - 1 \right]$  in figs. 13 and 14 are the improvements in coefficient resulting



FINAL CORRELATION Z  
FIG. 13



from vibration. For example, when  $\frac{h}{h_0} - 1$  equals 1.2 it represents a 120% increase in coefficient. Actually these figures show some increases of over 200% which corresponds to more than tripling the coefficient.

Having the final correlations in terms of the ratios of coefficients has other advantages besides that of convenience. Any errors in the determination of coefficient, such as errors in temperature measurement, tend to cancel each other in the ratio. This point was also mentioned earlier in connection with radiation. In addition, due to the generality inherent in the forms

$\frac{h}{h_0} - 1$  and  $\frac{h}{h_c} - 1$  which involve only the improvement caused by vibration, the correlations may also be applicable, at least in a general way, to bodies that are not cylindrical in shape.

Curves representing these final dimensionless correlations are drawn in figs. 13 and 14. These curves are not the same although they have the same trend. The average error, as measured by the average deviation of the experimental points from the curves, is 13% for correlation I and 9% for correlation II, where the % refers to percentage improvement in coefficient. For a given  $h_0$  or  $h_c$ , the average error in  $h$  itself would be even smaller; 8% for correlation I and 6% for correla-

tion II. In view of the different experimental variables and their ranges, this is considered quite satisfactory.

For improvements in coefficient of over 10%, which covers most of the experimental range, equations closely approximating these two final dimensionless correlations are as follows:

Correlation I:

$$\frac{h}{h_0} = 0.75 + 0.00308 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Gr)^{0.410}} \quad (50)$$

Correlation II:

$$\frac{h}{h_0} = 0.75 + 0.00432 \frac{(Re)^{1.86} (\beta \Delta t)^{0.297}}{(Gr)^{0.372}} \quad (51)$$

By using either of these correlations the effect of vibrations on heat transfer coefficient  $h$  can be estimated. If not already known the values of coefficient without vibration, namely  $h_0$  or  $h_c$ , may be estimated by any of the existing methods in the literature such as that of McAdams or Rice discussed earlier in the survey on heat transfer. Alternatively the solid curve of fig. 8 developed here for stationary wires may be used. Such a relationship, symbolized by eq. 8, may be combined with eqs. 40 and 50, thus

$$\frac{Nu}{\phi(Gr \times Pr)} = 0.75 + 0.00308 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Gr)^{0.410}} \quad (52)$$

Eq. 8 could be similarly substituted into eq. 51 but the result would involve dimensionless groups evaluated at two different temperatures.

Substituting eq. 8a (which represents McAdams' curve for  $\phi$ ) into eq. 52, and utilizing the fact (108) that as a close approximation for diatomic gases  $Pr = 0.74$ , results in the relation

$$Nu = \left[ 0.75 + 0.00308 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Gr)^{0.410}} \right] \left[ 0.63 + 0.333 (Gr)^{\frac{1}{6}} \right]^2 \quad (53)$$

Thus eq. 53 gives Nu for vibration in diatomic gases directly in terms of dimensionless groups for improvements greater than 10%.

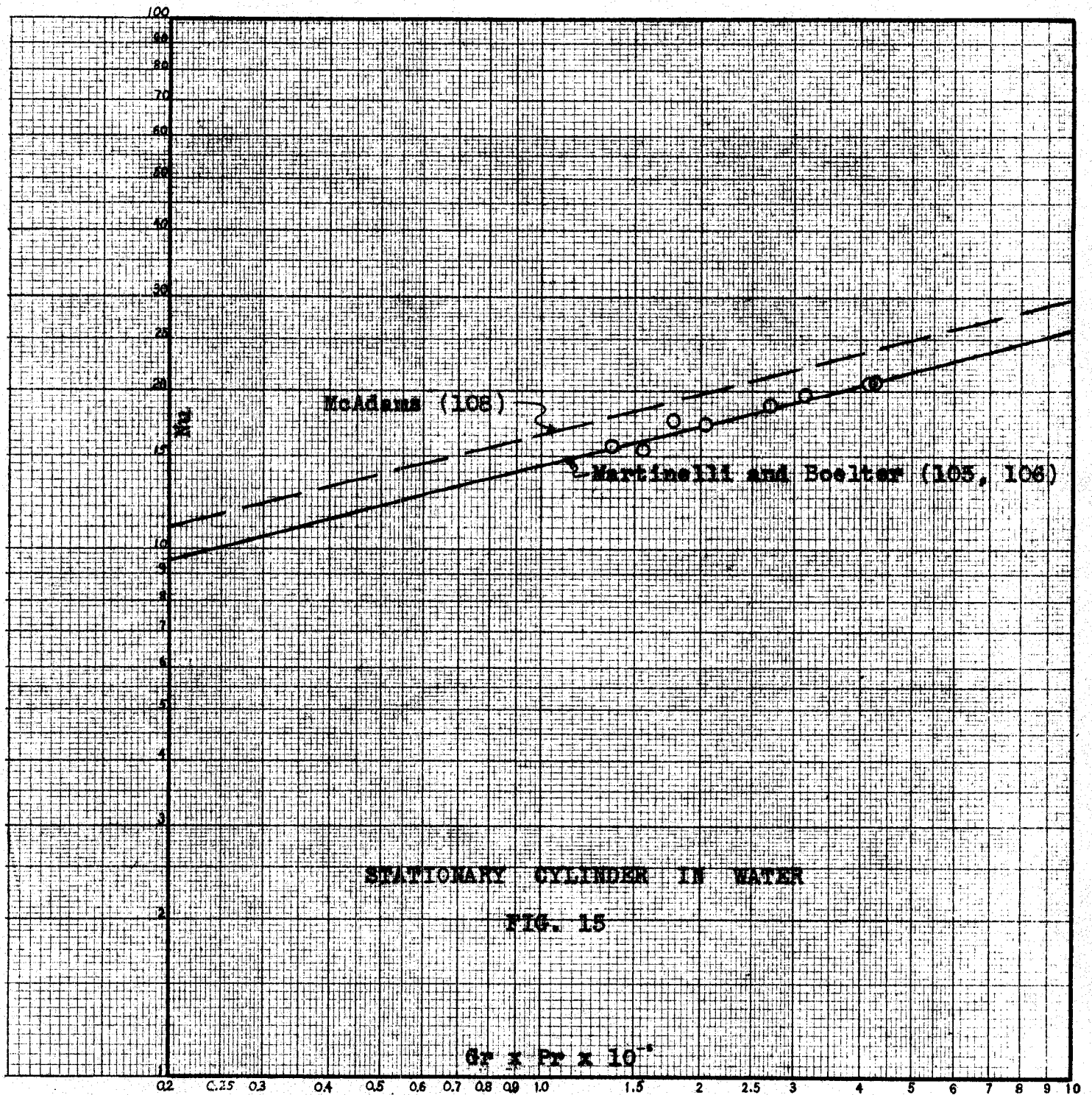
Referring back to the earlier discussion of figs. 9 and 11, the negative slopes in these plots now may be readily explained in terms of the Reynolds number and the final correlations. Since the room temperature varied only very moderately, an increase in  $\Delta t$  corresponded to an increase in average film temperature. This in turn decreased density and increased viscosity, so that for a fixed amplitude and frequency the Reynolds number decreased. Now, at high Re (corresponding to high parametric values in figs. 9 and 11) this small decrease in Re induces a considerable decrease in h, despite the increase in Gr occasioned by the increase in  $\Delta t$ . Stated in another way, at high levels of vibration the vibration

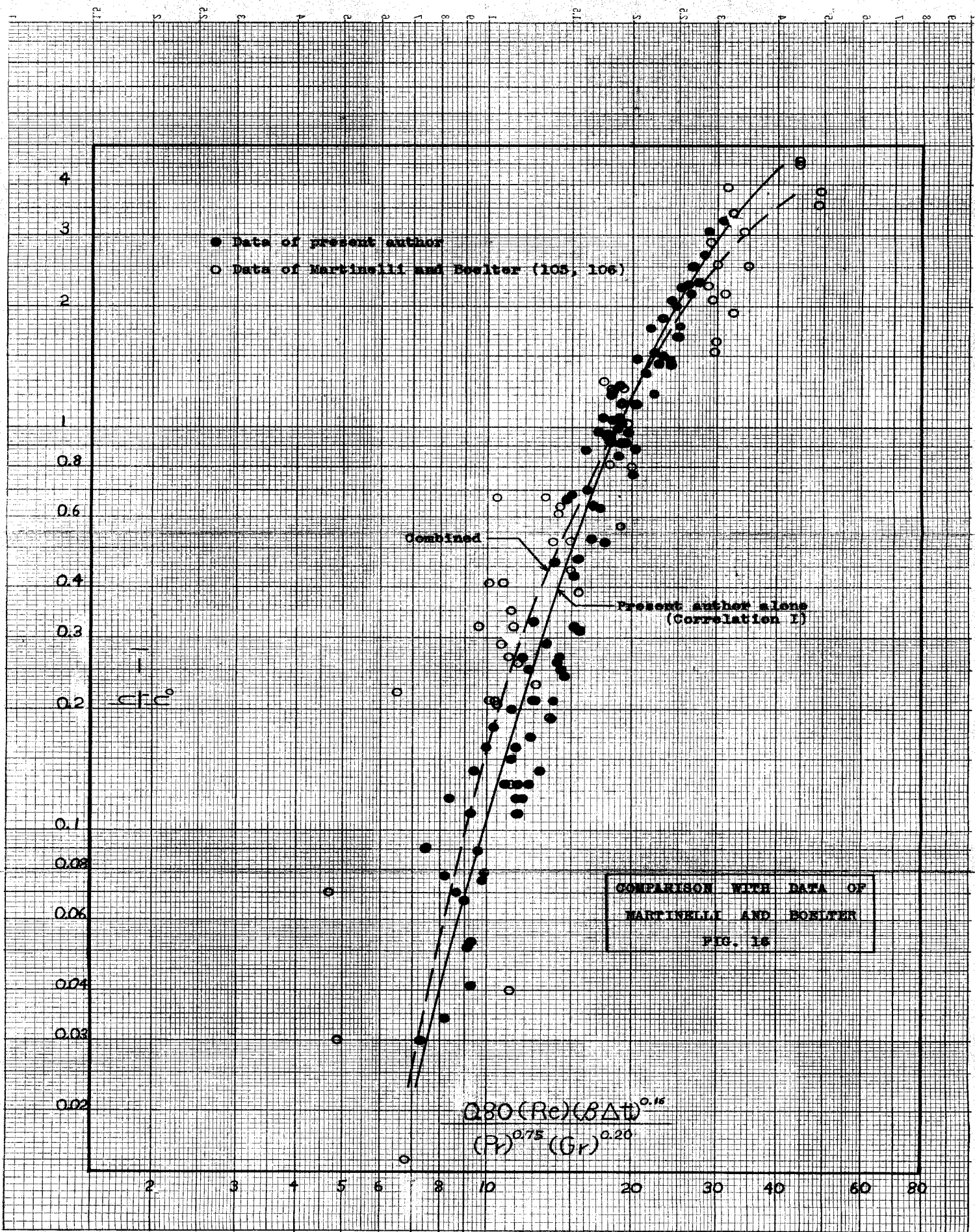
itself, which is characterized by the Reynolds number, has a greater effect on  $h$  than does the natural convection itself, which is characterized by the Grashof number. Thus the increase in  $\Delta t$  reduces the effect of the vibration and decreases  $h$ .

Extension to Other Fluids:

In an effort to extend the range to include other fluids the data of Martinelli and Boelter mentioned earlier was considered. A plot was prepared similar to that shown in fig. 13 where, as before, corresponding values of  $h$  and  $h_0$  were taken at the same  $Gr \times Pr$ . This was accomplished with the aid of the solid curve of fig. 15. The results so plotted did not correlate with the results for air. This, however, is not surprising. The correlation for air contains no allowance for a change in the nature of the fluid because  $Pr$  for air is virtually constant. Accordingly, after a number of trial plots, the Prandtl number was taken into account by multiplying the abscissa of fig. 13 by the quantity  $\left[0.80/(Pr)^{0.75}\right]$ . For air and other diatomic gases  $(Pr)^{0.75} = (0.74)^{0.75} = 0.80$ , so that the quantity reduces to unity leaving the correlation unchanged.

Fig. 16 shows this final plot for both the water data of Martinelli and Boelter and the air data of the





present investigator. The general trend of the results of Martinelli and Boelter are in agreement with the results of the present author although the former have the greater scatter. A curve based on the combined data and a curve based on the author's data alone are shown for comparison.

It is interesting to note that the trend of the water data appears to bear out the slight curvature at the upper end of the plot. A possible explanation for this curvature may be arrived at by considering the situation where  $Re$  increases to a very large value. Under these circumstances  $\frac{h}{h_0} - 1$  becomes very large so that  $\frac{h}{h_0} \gg 1$ . The effect of forced motion is then much greater than that of natural convection, and in the limit will predominate. Now, for simple forced convection at right angles to a cylinder (108), and for given physical properties, a plot of  $\log h$  against  $\log Re$  yields a slope dependent on  $Re$  but considerably less than the slope indicated at the upper end of either curve in fig. 16. Therefore it may well be that as the abscissa increases the slope in fig. 16 (or fig. 13) will decrease to a limit that equals the slope of the simple forced convection curve. Similar reasoning may also be applied to the slope in fig. 14.

Attempts at modifying correlation II, that is, the abscissa of fig. 14, to include the water data did not prove as successful. This fact, together with the rather non-fundamental nature of the definition for  $h_0$  [involving equality of  $q$  rather than equality of  $Gr \times Pr$ ] would seem to indicate that correlation I, despite its somewhat greater average error when based on air alone, is the stronger of the two correlations.

With regard to the two curves shown in fig. 16, the author believes that if a single correlation is to be selected for all fluids, liquid and gaseous, the curve based on his data alone should be chosen. This contention is supported by the greater scatter among the plotted results based on the work of Martinelli and Boelter, and the doubt cast on the validity of their data by Boelter and Mason. This latter point was discussed earlier in the survey. Therefore, modifying eq. 50 to include the effect of a change in the nature of the fluid, that is, the effect of  $Pr$ , yields

$$\frac{h}{h_0} = 0.75 + 0.0022 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Pr)^{1.54} (Gr)^{0.410}} \quad (54)$$

for improvements in coefficient of over 10%. Similarly modifying eq. 53 for improvements greater than 10%, yields,

$$Nu = \left[ 0.75 + 0.0022 \frac{(Re)^{2.05} (\beta \Delta t)^{0.328}}{(Pr)^{1.54} (Gr)^{0.410}} \right] \left[ 0.63 + 0.35(Gr \times Pr)^{\frac{1}{6}} \right]^2 \quad (55)$$

Thus the solid curve of fig. 16, or alternatively eqs. 54 or 55 make possible an estimation of the effect of vibration on heat transfer in other fluids as well as in air.

**APPENDIX**

## SURVEY OF SURFACE TEMPERATURE MEASUREMENT

### General:

One of the important experimental steps in this investigation is the determination of surface temperature. Accordingly, a survey of the various methods proposed in the literature for accomplishing such determinations is presented. Earlier reviews on the subject include those of Otmer and Coats (126) in 1928 and Colburn and Hougen (40) in 1930. The method finally selected together with reasons for the choice is discussed in detail in a previous section describing the experimental apparatus.

### Thermometer Method:

One of the first methods proposed is that of Callendar and Nicolson (31) who placed thermometers in holes drilled axially in the two inch thick wall of a pipe at varying distances from the center. The pipe carried steam inside and was cooled by water outside. The temperature of the inside wall surface was estimated by extrapolating the thermometer readings.

### Thermocouple Methods:

The commonest methods in use today depend on a thermocouple as the sensing element. Bowden (24) deter-

mined the temperature at the surface between sliding dissimilar metals by using the metals themselves as a thermocouple. This, however, is not a usual application. In general pressing or securing a thermocouple junction to the surface in question, or else in some way embedding the junction below the surface is usually involved. Unfortunately, simply securing (by welding, brazing, soldering, etc.) the junction directly to the surface can sometimes introduce error, depending on the situation.

This error can stem from two sources. First, the thermocouple leads pass through the surrounding fluid. Therefore heat can be conducted along these leads causing the junction temperature to differ from the true surface temperature. The magnitude of this error has been shown by Nusselt (122A) to be a function of  $(D h k_w / k_s)$  where  $k_w$  and  $k_s$  are the thermal conductivities of the thermocouple wire and surface materials respectively,  $D$  is the thermocouple wire diameter, and  $h$  is the coefficient of heat transfer to the surrounding fluid. This problem is discussed by Bailey (8) and a somewhat similar problem is analyzed by Rizika and Rohsenow (142).

The second source of error involves the local disturbing effect of the leads on the fluid at the junction. This disturbance can alter the fluid flow pattern at the point where the temperature measurement is being made.

Large diameter thermocouple wires tend to increase this source of error since they increase this disturbance. The effect is also likely to be greater in forced convection than in natural convection due to the increased coefficients.

In short the error may be large or small depending on circumstances. For the case of a small diameter thermocouple attached to a hot surface in natural convection with air, for which situation  $h$  is small, the error would not be great. Colburn and Hougen (40) soldered a copper-constantan thermocouple directly to the surface of a steam drum with lead wires extending into the surrounding air and reported an error of only 2% in temperature. Kratz and Broderick (92) fastened a thermocouple with adhesive vellum to a surface  $40^{\circ}\text{C}$  hotter than the surrounding air. An error of only  $0.5^{\circ}\text{C}$  was reported.

Other investigators who employed thermocouples on cylindrical surfaces include Comings (40A) whose surface was a  $3/8$  inch cylinder in air and Farber and Scorah (54) who used a thermocouple of 0.013 inch wires welded to a 0.040 inch wire which was heated electrically under water.

Rice (137) soldered junctions into holes in copper tubes of 4.3 to 11.4 cm. outside diameter in air. Kreisinger and Barkley (93) and Smith (154) describe a

method for peening thermocouple wires into surfaces.

For pressing against boiler walls, King and Blackie (88) employed a thermocouple backed by a copper disk acting as a radiation screen. Other arrangements for pressing the junction to a heated wall are described by Dutton and Marley (48) and Fairweather (53). Means for applying external heat to the thermocouple to balance heat losses from the junction are given by Bayer and Buss (13) and Sasaki (147). Thornton (164) discusses industrial methods of using thermocouples to measure surface temperatures in a boiler. Temperature measurements of the far surface of a wall (separated from the surface in question by the wall thickness) can sometimes be made and the results adjusted where necessary by calculation involving the wall's thermal resistance. Langmuir (97) did this for plane surfaces, while Kirst (89) and Bromley (31) used this method for hollow cylinders.

Although as shown above the error involved with air, where the coefficient of heat transfer is low, is small, the error involved in cases such as condensing vapors, where the coefficient is very large, may be considerable. For such cases and others, various methods have been suggested toward eliminating the effect of conduction along the leads and the local disturbing effect of the junction and leads. The conduction effect

can be practically eliminated by submerging the leads from the junction in an isothermal zone (108) for one or more inches of their length. This zone is often available on or preferably under the surface. Jordan (82) and Webster (173) threaded a thermocouple through a thin tube which was then attached to the surface. Unfortunately, the auxiliary tube presents a disturbing effect on the main surface. Othmer and Coates (126) present a method used later in investigating condensation coefficients (124) where one thermocouple lead is embedded in similar metal plated on the surface in question. The junction is thus the entire interface between the main surface and the plate rather than just a small area subject to the disturbing influence of the leads. Limitations to this method have been pointed out by Colburn and Hougen (40). Reiher (135) and others (51, 68, 76, 86, 143, 169) embed the wires in grooves cut in the surface or otherwise submerge the leads below the surface. These grooves are cemented over and smoothed down. Thus advantage can be taken of an isothermal zone without greatly disturbing the surface. Hebbard and Badger present a method (71, 73) that has been used (72, 116, 128), sometimes with modification (11). In principle it involves inserting a thermocouple junction in a hole drilled through the chord of a tube wall, so that the junction is between the inner and

outer surfaces of the tube. The leads are carried around to the other side of the tube in grooves. Thus the surface is substantially free of disturbing influences.

Knoblauch and Hencky (90) suggest bringing the leads out axially through the wall itself. This was done for hollow cylinders (78, 79) with holes drilled axially through the metal, and for a plate (120) with a hole drilled in the edge. Othmer and White (127) propose a method used in vapor condensation studies (125) where the leads are brought out through a solid metal rod.

#### Resistance Methods:

The second set of methods commonly employed depends on change of electrical resistance with change in temperature. Among the metals, platinum and nickel are most often used. The temperature obtained by this method is an average for the entire resistance element, be it tube, wire, or some other shape. However, this average is frequently sufficiently close to the surface temperature that little error is introduced. Furthermore, when local surface temperature irregularities occur, as in dropwise condensation, this average can be advantageous.

Ayrton and Kilgour (6) studied heat transfer from electrically heated platinum wires in air, using the wires themselves as resistance thermometers. Similar

studies were undertaken by others (44, 96, 111, 119, 130, 133, 163). Bedi (16) and others (5, 43) studied the change of wire resistance with traction. If the tension on a wire used as a resistance thermometer is altered, recalibration may be necessary. Even bending is not advisable (76). Tube wall surface temperatures have been determined by using the tube as a resistance thermometer (10, 12, 19, 31, 70). This is generally more difficult to accomplish than with wires since accurate measurement of a much lower resistance is usually required. Giesecke (66) and others (114) suggest attaching a resistance wire to the surface in question. This technique is likely to disturb the surface in much the same manner as direct attachment of a thermocouple might. Becker, et.al. (15) discuss thermistors which have a large change in resistance for a given change in temperature. Fischer and Norris (57) used thermistor beads to determine surface temperatures on the nose of a V-2 rocket in flight.

#### Radiation Methods:

The third set of methods in common use involves radiation, either in the optical region or the infra-red. An advantage here is that no physical contact with the surface is necessary. In the optical pyrometer, the visibly glowing surface is compared in intensity with a

glowing standard. This instrument is widely used in metallurgy.

Wire temperatures have also been measured by optical pyrometers (46, 50, 54). In a radiation pyrometer (101) radiation usually impinges on a thermocouple or thermopile. Neubert (122) determined temperatures between 300° and 500° C with a phosphorescent screen excited to glowing by blue or ultraviolet light. The infra-red radiation of the hot surface was then directed on to the screen where a dark image was formed. The extinction of the phosphorescence was proportional to the temperature distribution of the surface. Fischer (56) and Schwabe (151) determined surface temperatures above 250° C by infra-red photography. The changes in refractive index of the surrounding fluid with changes in temperature have been used (149) to estimate temperature distributions around heated bodies by means of so called Schlieren photographs. Kennard (85) presented a method using an interferometer and monochromatic light whereby interference fringes were determined and temperatures calculated.

#### Extensometric Methods:

The temperature change of a body can be determined extensometrically by measuring its change in length.

This has been done by Stanton (157) and others (156, 160, 179). This method usually requires frequent recalibration and is not widely used.

Melting Point Methods:

Surface temperatures also can be approximately measured by coating a portion of the surface with a thin layer of a material with a known melting or softening point. This may be done with Tempilstiks (172).

Indirect Methods:

Certain methods exist whose purposes are to avoid the necessity of making the surface temperature measurement directly. In heat transfer studies the method of Wilson (176) has been widely applied (14, 33, 61, 63, 91, 113, 136) in both condensation and convection studies. In essence it involves plotting  $\frac{1}{U}$  against  $\frac{1}{G^{0.8}}$  where  $U$  is the overall heat transfer coefficient and  $G$  is the mass rate of flow, and then extrapolating to where  $\frac{1}{G^{0.8}}$  equals zero while holding constant the thermal resistances that do not involve  $G$ . Chu, et. al. (37) presents a modification of Wilson's method. For the case where the thermal resistance of one fluid and the separating wall is negligible compared to the thermal resistance of the other fluid, the wall surface temperature can be taken as essen-

tially equal to the temperature of the first fluid. An example of this is steam transferring heat to air through a tube wall (36).

CALIBRATION OF THE STROBOSCOPIC LIGHT

The stroboscopic light was calibrated over its frequency range against an A.C. driven (constant speed) grinding wheel by a technique that is simpler and more accurate than the usual cumbersome one requiring a machine with a different speed for each frequency.

The method is based on the various spoke-like patterns which appear when the stroboscopic light shines on the side of a rotating wheel that has been previously marked with chalk along one radius. For example, when the FPM (flashes per minute) of the light equal the RPM of the wheel, one radial line or spoke will appear. A single line will also appear when the ratio of FPM to RPM is any one of the values  $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/5$ , etc. Similarly, two opposite radial lines will appear when this ratio is  $2/1$ ,  $2/3$ ,  $2/5$ ,  $2/7$ , etc. The general rule applicable to any pattern is as follows: The number of radial lines appearing (always equally spaced) will equal the numerator of the fraction representing the ratio, provided that fraction is reduced to its lowest terms. For example, the ratio  $6/10$  can be reduced to  $3/5$ . The numerator is three; therefore three radial lines will appear.

The procedure, therefore, consisted of first marking the side of the grinding wheel with chalk, starting it

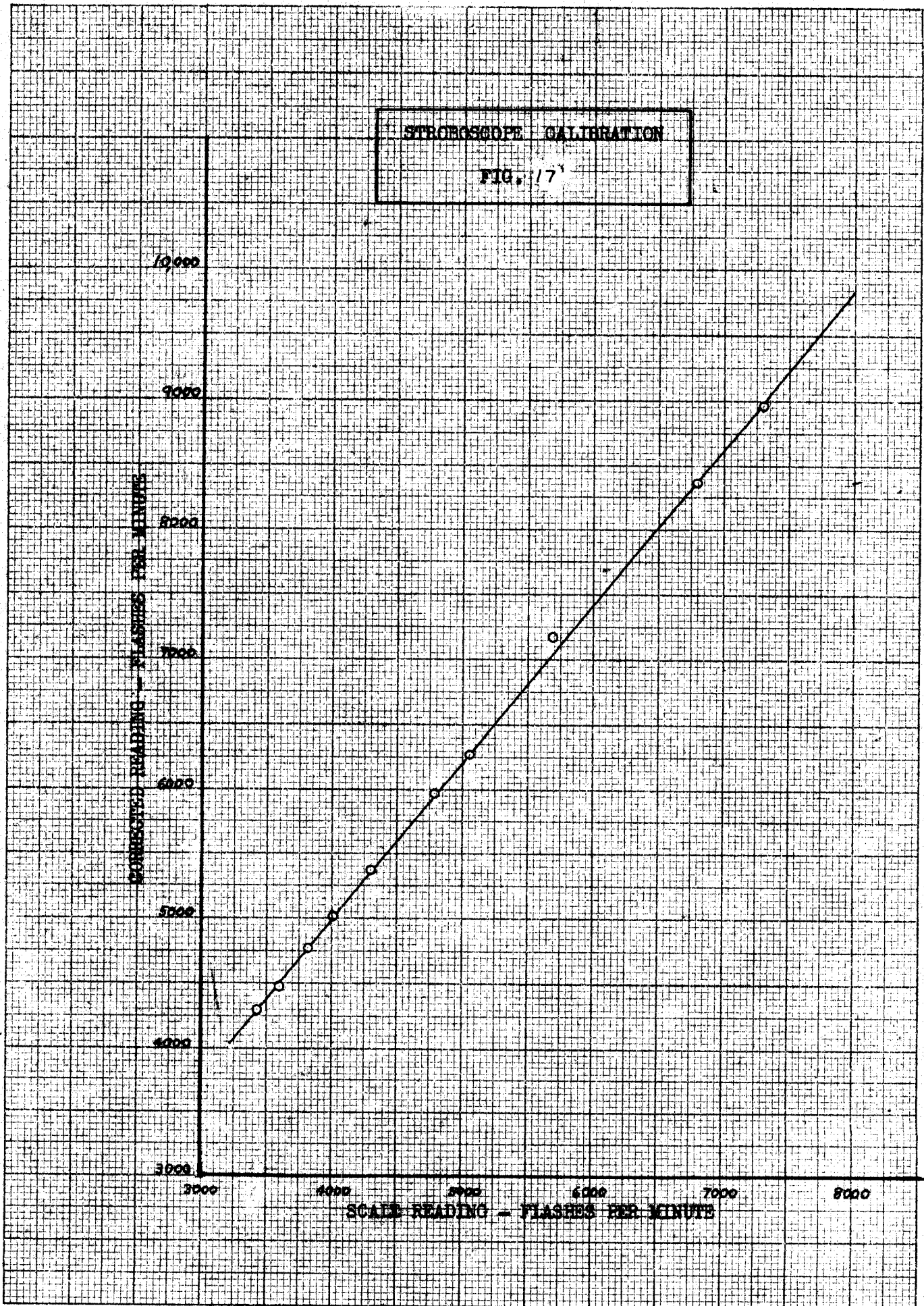
up, setting the stroboscope for maximum frequency, and shining it on the rotating chalk mark. The stroboscope frequency was then gradually decreased and every pattern of seven or fewer lines recorded together with the stroboscope scale reading. Limiting the recorded patterns to those of seven or fewer spokes provided a sufficient number of points for calibration without necessitating consideration of the more complex patterns. The rotating wheel was carefully clocked at 3583 RPM using a revolution-counter and stopwatch. This RPM was then multiplied by the appropriate ratio of FPM to RPM to give the true FPM for the corresponding scale reading. The results of the calibration are given in the table below and shown in the figure following.

STROBOSCOPE CALIBRATION - TABLE A

| <u>Number of Radial Lines per Pattern</u> | <u>Ratio of FPM to RPM</u> | <u>Stroboscope Scale Reading in FPM</u> | <u>Ratio x 3583 = True FPM</u> |
|---|----------------------------|---|--------------------------------|
| 6   | 6/5                        | 3430                                    | 4300                           |
| 5   | 5/4                        | 3600                                    | 4479                           |
| 4   | 4/3                        | 3820                                    | 4777                           |
| 7   | 7/5                        | 4010                                    | 5016                           |
| 3   | 3/2                        | 4300                                    | 5375                           |
| 5   | 5/3                        | 4790                                    | 5972                           |
| 7   | 7/4                        | 5060                                    | 6270                           |
| 2   | 2/1                        | 5690                                    | 7166                           |
| 7   | 7/3                        | 6800                                    | 8360                           |
| 5   | 5/2                        | 7300                                    | 8958                           |

A brief paper by the author describing this technique of calibration has been accepted for publication (99) and will appear shortly.

STROBOSCOPE CALIBRATION  
FIG. 17



RELAXATION CALCULATION

The worst case for conduction along the wire is for the run with the largest product of wire cross-section times temperature driving force along the wire. This is run 137 for the thickest wire of 0.0810 inch diameter.

The information required from table 2 is 21.5 watts for a heated length of 39.2 inches, room temperature of 76°F,  $\Delta t$  at center of 115°F, frequency of 53.5 vibrations per second, amplitude at center of 96.1 scale divisions,  $h$  at center of 9.21 BTU/hr.ft.<sup>2</sup> °F, and  $h_0$  of 5.20 BTU/hr.ft.<sup>2</sup> °F which equals  $h$  at the ends where there is no vibration. Thermal conductivity  $k$  for nichrome (45) is 7.4 BTU/hr.ft.<sup>2</sup> °F/ft.

The method is as follows:

The 36.3 inches of vibrating wire is divided into 15 equal elements, each 2.42 inches long. Then a heat balance is written around each element. For element  $n$  bordered by elements  $m$  and  $p$  the balance is

$$q_{mn} + Q_n = q_{np} + q_n$$

where  $q_{mn}$  is the rate of heat transfer by conduction from element  $m$  to element  $n$ ,  $q_{np}$  is the rate of heat transfer by conduction from element  $n$  to element  $p$ ,  $Q_n$  is the rate of electrical heat generation in element  $n$ , and  $q_n$  is the rate of heat dissipation by convection

for element  $n$ . A residual  $R_n$  may be defined as

$$R_n = q_{mn} + Q_n - q_{np} - q_n$$

This residual should be zero if the other terms in the above equation are known exactly. In practice a close approach of  $R_n$  to zero is satisfactory in so far as the solution is concerned.

Now, by Fourier's Law, with due regard for sign,

$$q_{mn} = -\left(\frac{kA}{L}\Delta t\right)_{mn}$$

Substituting,

$$q_{mn} = -\frac{7.4\pi \times (0.081)^2 \times 12}{4 \times 114 \times 2.42} (\Delta t)_{mn}$$

or

$$q_{mn} = -0.001313 (\Delta t)_{mn}$$

Also,

$$q_n = -(hA\Delta t)_n$$

Substituting,

$$q_n = -\frac{\pi \times 0.081 \times 2.42}{12 \times 12} (h\Delta t)_n$$

or

$$q_n = -0.00428 (h\Delta t)_n$$

The heat generated electrically is

$$Q_n = \frac{21.5 \times 2.42 \times 56.92 \times 60}{39.2 \times 1000} = 4.53 \text{ BTU/hr.}$$

The arc of the vibrating wire is a sine wave at any instant. Accordingly, the position of the center

of each element corresponds to a certain number of angular degrees as shown in column 2 of table B. From this the average amplitude of each element can be calculated knowing the amplitude at the center, as in column 3. Trial and error calculations using figure 14\* are outlined in the remainder of the table. The elements are numbered consecutively starting with the one located at the left end. Thus element number 8 is the central element. Since the wire is symmetrical about its center only computations for the first eight elements are necessary; the remaining seven follow by symmetry. The values of  $t_f$  in column 17 are sufficiently close to those of column 11, and the residuals  $R_n$  in column 21 are sufficiently close to zero that further trials are unnecessary.

As shown in column 22,

$$\frac{\text{Net heat flow by conduction into central element}}{\text{Heat generated in central element}} = 0.3\%$$

Also,

$$\frac{\text{Greatest heat flow by conduction anywhere along wire}}{\text{Total heat dissipated by wire}} = \frac{0.0262}{15 \times 4.53} = 0.04\%$$

In conclusion, this demonstrates that the heat conducted along the wire is negligible.

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\* For convenience in calculation, a straight line approximation was actually used for the correlation shown in fig. 14, relating the ordinate Y to the abscissa X by  $Y = 0.000071 X^{3.71}$ .

RELAXATION - TABLE B

| 1         | 2               | 3                      | 4                       | 5                             | 6                     | 7  | 8   | 9                   | 10              |
|-----------|-----------------|------------------------|-------------------------|-------------------------------|-----------------------|--|---|---------------------|-----------------|
| Element n | Angular degrees | Amplitude in divisions | Assumed $\Delta t$ , °F | Average film temp. $t_f$ , °F | Viscosity, centipoise | Specific volume $\frac{1}{\rho}$ , ft. <sup>3</sup> /lb. | $\frac{Re(\Delta t)^{0.16}}{(Gr)^{0.20}}$ | $\frac{h}{h_0} - 1$ | $\Delta t$ , °F |
| 1         | 6               | 10.0                   | 204                     | 178                           | 0.0209                | 16.3   | 1.80                                      | <0.01               | 204             |
| 2         | 18              | 29.7                   | 176                     | 164                           | 0.0206                | 15.9   | 5.47                                      | 0.0137              | 201             |
| 3         | 30              | 48.0                   | 160                     | 156                           | 0.0204                | 15.7   | 9.00                                      | 0.0655              | 192             |
| 4         | 42              | 64.3                   | 144                     | 148                           | 0.0202                | 15.5   | 12.3                                      | 0.170               | 174             |
| 5         | 54              | 77.7                   | 132                     | 142                           | 0.0200                | 15.4   | 15.0                                      | 0.327               | 154             |
| 6         | 66              | 87.8                   | 123                     | 138                           | 0.0199                | 15.3   | 17.2                                      | 0.500               | 136             |
| 7         | 78              | 94.0                   | 117                     | 135                           | 0.0199                | 15.2   | 18.5                                      | 0.630               | 125             |
| 8         | 87              | 96.0                   | 115                     | 134                           | 0.0198                | 15.1   | 19.0                                      | 0.685               | 121             |

| 11                            | 12                    | 13   | 14  | 15                  | 16              | 17                            | 18                                      | 19                 | 20              | 21              | 22                |
|-------------------------------|-----------------------|--|---|---------------------|-----------------|-------------------------------|---|--------------------|-----------------|-----------------|-------------------|
| Average film temp. $t_f$ , °F | Viscosity, centipoise | Specific volume $\frac{1}{\rho}$ , ft. <sup>3</sup> /lb. | $\frac{Re(\Delta t)^{0.16}}{(Gr)^{0.20}}$ | $\frac{h}{h_0} - 1$ | $\Delta t$ , °F | Average film temp. $t_f$ , °F | Coef. $h$ , BTU/hr. ft. <sup>2</sup> °F | $q_{in}$ , BTU/hr. | $q_n$ , BTU/hr. | $R_n$ , BTU/hr. | net $q_{net}$ , % |
| 178                           | 0.0209                | 16.3   | 1.80                                      | <0.01               | 204             | 178                           | 5.20                                    | 4.54               | 4.54            | -0.008          | -0.2 %            |
| 177                           | 0.0209                | 16.3   | 5.31                                      | 0.0127              | 201             | 177                           | 5.26                                    | 0.0034             | 4.53            | -0.010          | -0.2              |
| 172                           | 0.0209                | 16.1   | 8.70                                      | 0.0585              | 193             | 172                           | 5.50                                    | 0.0113             | 4.53            | -0.015          | -0.1              |
| 163                           | 0.0206                | 15.9   | 11.9                                      | 0.157               | 176             | 164                           | 6.01                                    | 0.0216             | 4.54            | -0.007          | 0.1               |
| 153                           | 0.0203                | 15.6   | 14.7                                      | 0.305               | 156             | 154                           | 6.79                                    | 0.0262             | 4.54            | 0.012           | 0.3               |
| 144                           | 0.0201                | 15.4   | 16.9                                      | 0.472               | 139             | 145                           | 7.65                                    | 0.0232             | 4.53            | 0.005           | 0.1               |
| 139                           | 0.0200                | 15.3   | 18.3                                      | 0.600               | 128             | 140                           | 8.31                                    | 0.0111             | 4.53            | 0.002           | 0.3               |
| 137                           | 0.0199                | 15.2   | 18.8                                      | 0.660               | 123             | 138                           | 8.63                                    | 0.0059             | 4.54            | 0.002           | 0.3               |

SAMPLE CALCULATIONS

Stationary Wires:

The following illustrates the calculations involved in preparing table 1 and figs. 4, 5, 6, and 8:

Run number 10: Wire diameter 0.0253 inch.

Temp. diff.  $\Delta t = t_s - t_b = 139 - 71 = 68^\circ\text{F}$

Wattage Q for the entire heated length of 38.5 inches is 3.54 watts.

Substituting in eq. 2 and making the proper unit conversions,

$$h = \frac{3.54 \times 56.92 \times 60 \times 12 \times 12}{1000 \times 38.5 \times \pi \times 0.0253 \times 68} = 8.35 \text{ BTU/hr. ft.}^2 \text{ }^\circ\text{F}$$

Q and h are then plotted against  $\Delta t$  in fig. 4.

$$t_f = \frac{t_s + t_b}{2} = \frac{139 + 71}{2} = 105^\circ\text{F}$$

Reading at  $105^\circ\text{F}$  from thermal conductivity data (118) and fig. 7,

$$k = 0.0157 \text{ BTU/hr. ft.}^2 \text{ }^\circ\text{F/ft.}$$

$$\frac{\rho^2 \beta g c}{\mu k} = 1.22 \times 10^6 (\text{ft.}^3 \text{ }^\circ\text{F})^{-1}$$

By the definition of Nusselt number using the diameter of  $0.0253/12 = 0.00211 \text{ ft.}$ ,

$$\text{Nu} = \frac{8.35 \times 0.00211}{0.0157} = 1.12$$

$$\log_{10} \text{Nu} = 0.050$$

Substituting in eq. 10,

$$\text{Gr} \times \text{Pr} = 1.22 \times 10^6 \times (0.00211)^3 \times 68 = 0.778$$

$$\log_{10} (\text{Gr} \times \text{Pr}) = - 0.109$$

The quantity 0.050 is then plotted against - 0.109 in fig. 8.

Vibrating Wires:

The following will illustrate the calculations involved in preparing table 2 and figs. 9, 10, 11, 13, and 14.

Run number 98: Wire diameter 0.0396 inch. Q for the entire heated length of 38.0 inches is 5.94 watts. The calculations for  $\Delta t$  and h are similar to those for the stationary wire.

$$\text{Temp. diff. } \Delta t = t_s - t_b = 141 - 75 = 66^\circ\text{F}$$

From eq. 2,

$$h = \frac{5.94 \times 56.92 \times 60 \times 12 \times 12}{1000 \times 38.0 \times \pi \times 0.0396 \times 66} = 9.33 \text{ BTU/hr. ft.}^2 \text{ }^\circ\text{F}$$

$$t_f = \frac{1}{2} (t_s + t_b) = \frac{1}{2} (141 + 75) = 108^\circ\text{F}$$

The barometric pressure is 30.2 inches of mercury. The humidity as determined from a psychrometric chart (7) is 0.005 lb. moisture per lb. dry air. Reading the specific volume at  $t_f$  from this chart and multiplying by the small pressure and humidity correction of

$$\frac{29.9}{30.2 \times 1.005} \text{ gives } 1/\rho = 14.3 \text{ ft.}^3 \text{ per lb. of moist air.}$$

The stroboscope scale reading is 4380. Reading

from the stroboscope calibration chart in the appendix and dividing by 60 to convert to seconds gives a frequency  $F = 90.9$  vibrations per second. The amplitude is 60.8 microscope scale divisions. Using the scale calibration of 0.00186 inches per division, the average velocity  $\bar{v}$  is calculated from eq. 12.

$$\bar{v} = 2HF = 2 \times \frac{60.8 \times 0.00186}{12} \times 90.9$$

$$\bar{v} = 1.71 \text{ feet per second}$$

The viscosity (94) at a temperature of 103°F is 0.0192 centipoise. Substituting in eq. 31 and converting units to make the result dimensionless,

$$Re = \frac{D \bar{v} \rho}{\mu} = \frac{0.0396 \times 1.71 \times 1488}{12 \times 14.3 \times 0.0192}$$

$$Re = 30.6$$

As discussed earlier in the survey on heat transfer  $\beta$  equals the reciprocal absolute temperature for gases which approach ideality. This is the case for air under the conditions of this experiment, so that

$$\beta = \frac{1}{T_f} = \frac{1}{460 + 103} = 0.00176/^\circ R$$

By the definition of Grashof number,

$$Gr = \frac{D^3 \rho^2 g \beta \Delta t}{\mu^2}$$

Substituting and converting units to make the result dimensionless,

$$Gr = \frac{(0.0396)^3 \times 32.2 \times 0.00176 \times 66 \times (1488)^2}{1728 \times (14.3)^2 \times (0.0192)^2}$$

$$Gr = 3.95$$

$$\beta \Delta t = 0.00176 \times 66 = 0.116$$

$$Re \frac{(\beta \Delta t)^{0.16}}{(Gr)^{0.20}} = 30.6 \times \frac{(0.116)^{0.16}}{(3.95)^{0.20}} = 16.5$$

From eq. 44,

$$\frac{h}{h_j} = \frac{Q}{Q_j} = \frac{5.94}{3.87} = 1.53$$

where  $Q_j$  is read from fig. 5 at  $\Delta t$  of  $66^\circ$  F.

$$\text{Then, } \frac{h}{h_j} - 1 = 0.53$$

which is then plotted against 16.5 in fig. 13.

Alternatively,  $h_j$  could be read directly from fig. 5 at  $\Delta t$  of  $66^\circ$  F and then used to compute the ratio  $h/h_j$ .

From eq. 48,

$$\frac{h}{h_o} = \frac{\Delta t_o}{\Delta t} = \frac{93}{66} = 1.41$$

where  $\Delta t_o$  is read from fig. 5 at  $Q$  of 5.94 watts.

Then,

$$\frac{h}{h_o} - 1 = 0.41$$

which is plotted against 16.5 in fig. 14.

Alternatively,  $h_o$  could be read directly from fig. 5 at  $Q$  of 5.94 watts and then used to compute the ratio  $h/h_o$ .

However, because of the comparatively flat slope of the coefficient line in fig. 5, for the sake of accuracy this alternate method was not used.

**TABULATED RESULTS**

STATIONARY WIRES

TABLE 1

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| Run  | Wire Diam., Inches | Q, Watts | Wire surface temp. t <sub>s</sub> , °F | Room temp. t <sub>r</sub> , °F | Temp. diff. Δt, °F | Coef. h <sub>2</sub> , BTU/hr.ft <sup>2</sup> .°F | Average film temp. t <sub>f</sub> , °F | Thermal Cond. k, BTU/hr.ft.°F | $\frac{P_{Bc}}{(t_s - t_r)^{-1}}$ | Gr x Pr | log(Gr x Pr) | Nu    | log <sub>10</sub> Nu |
|------|--------------------|----------|--|--------------------------------|--------------------|---|--|-------------------------------|-----------------------------------|---------|--------------|-------|----------------------|
| 1    | 0.0253             | 0.71     | 92                                     | 76                             | 16                 | 7.15  | 84                                     | 0.0153                        | 1.45x10 <sup>6</sup>              | 0.218   | -0.662       | 0.985 | -0.007               |
| 2    | "                  | 1.79     | 112                                    | 76                             | 36                 | 7.99  | 94                                     | 0.0155                        | 1.33x10 <sup>6</sup>              | 0.449   | -0.348       | 1.09  | 0.038                |
| 3    | "                  | 2.68     | 128                                    | 77                             | 51                 | 8.45  | 103                                    | 0.0157                        | 1.24x10 <sup>6</sup>              | 0.593   | -0.227       | 1.14  | 0.056                |
| 4    | "                  | 5.10     | 167                                    | 77                             | 90                 | 9.10  | 122                                    | 0.0161                        | 1.07x10 <sup>6</sup>              | 0.903   | -0.044       | 1.19  | 0.076                |
| 5    | "                  | 8.00     | 208                                    | 78                             | 130                | 9.87  | 143                                    | 0.0167                        | 9.13x10 <sup>5</sup>              | 1.11    | 0.045        | 1.25  | 0.097                |
| 6    | "                  | 12.3     | 267                                    | 78                             | 189                | 10.4  | 173                                    | 0.0174                        | 7.46x10 <sup>5</sup>              | 1.32    | 0.121        | 1.27  | 0.104                |
| 7    | "                  | 25.4     | 416                                    | 78                             | 338                | 12.1  | 247                                    | 0.0191                        | 4.46x10 <sup>5</sup>              | 1.42    | 0.152        | 1.33  | 0.124                |
| 8    | "                  | 0.82     | 88                                     | 70                             | 18                 | 7.35  | 79                                     | 0.0151                        | 1.50x10 <sup>6</sup>              | 0.253   | -0.597       | 1.03  | 0.116                |
| 9    | "                  | 6.07     | 180                                    | 71                             | 109                | 8.94  | 126                                    | 0.0162                        | 1.03x10 <sup>6</sup>              | 1.05    | 0.021        | 1.16  | 0.065                |
| 10   | "                  | 3.54     | 139                                    | 71                             | 68                 | 8.35  | 105                                    | 0.0157                        | 1.22x10 <sup>6</sup>              | 0.778   | -0.109       | 1.12  | 0.049                |
| 11   | "                  | 16.3     | 312                                    | 72                             | 240                | 10.9  | 192                                    | 0.0178                        | 6.54x10 <sup>5</sup>              | 1.47    | 0.167        | 1.29  | 0.111                |
| 12   | "                  | 2.38     | 122                                    | 75                             | 47                 | 8.14  | 99                                     | 0.0153                        | 1.27x10 <sup>6</sup>              | 0.285   | -0.545       | 1.12  | 0.050                |
| 13   | 0.0396             | 0.43     | 88                                     | 78                             | 10                 | 4.42  | 83                                     | 0.0152                        | 1.46x10 <sup>6</sup>              | 0.524   | -0.281       | 1.05  | -0.018               |
| 14   | "                  | 1.27     | 105                                    | 78                             | 27                 | 4.89  | 92                                     | 0.0154                        | 1.36x10 <sup>6</sup>              | 1.32    | 0.121        | 1.05  | 0.020                |
| 15   | "                  | 5.39     | 167                                    | 78                             | 89                 | 6.28  | 123                                    | 0.0162                        | 1.06x10 <sup>6</sup>              | 3.38    | 0.529        | 1.28  | 0.108                |
| 16   | "                  | 7.38     | 194                                    | 78                             | 116                | 6.79  | 136                                    | 0.0165                        | 9.62x10 <sup>5</sup>              | 4.01    | 0.603        | 1.36  | 0.134                |
| 17   | "                  | 11.1     | 244                                    | 78                             | 166                | 6.94  | 161                                    | 0.0171                        | 8.11x10 <sup>5</sup>              | 4.83    | 0.684        | 1.34  | 0.128                |
| 18   | "                  | 10.8     | 230                                    | 74                             | 156                | 7.19  | 152                                    | 0.0169                        | 8.60x10 <sup>5</sup>              | 4.81    | 0.682        | 1.40  | 0.174                |
| 19   | "                  | 29.6     | 412                                    | 74                             | 338                | 9.09  | 213                                    | 0.0190                        | 4.61x10 <sup>5</sup>              | 5.59    | 0.747        | 1.58  | 0.197                |
| 20   | "                  | 22.5     | 350                                    | 75                             | 275                | 8.50  | 213                                    | 0.0183                        | 5.65x10 <sup>5</sup>              | 5.60    | 0.748        | 1.53  | 0.185                |
| 21   | "                  | 2.60     | 123                                    | 76                             | 47                 | 5.74  | 100                                    | 0.0156                        | 1.27x10 <sup>6</sup>              | 2.14    | 0.330        | 1.21  | 0.083                |
| 22   | "                  | 0.93     | 96                                     | 76                             | 20                 | 4.84  | 86                                     | 0.0153                        | 1.43x10 <sup>6</sup>              | 1.03    | 0.013        | 1.04  | 0.019                |
| * 23 | "                  | 0.58     | 88                                     | 76                             | 12                 | 5.01  | 82                                     | 0.0152                        | 1.47x10 <sup>6</sup>              | 0.633   | -0.199       | 1.09  | 0.037                |
| 24   | "                  | 14.2     | 279                                    | 71                             | 208                | 7.10  | 175                                    | 0.0174                        | 7.32x10 <sup>5</sup>              | 5.46    | 0.737        | 1.35  | 0.129                |
| 25   | "                  | 14.5     | 284                                    | 72                             | 212                | 7.10  | 178                                    | 0.0175                        | 7.20x10 <sup>5</sup>              | 5.47    | 0.738        | 1.34  | 0.127                |
| 26   | "                  | 28.8     | 426                                    | 76                             | 350                | 8.55  | 251                                    | 0.0192                        | 4.33x10 <sup>5</sup>              | 5.44    | 0.736        | 1.47  | 0.167                |
| 27   | "                  | 2.50     | 115                                    | 70                             | 45                 | 5.68  | 93                                     | 0.0154                        | 1.35x10 <sup>6</sup>              | 2.18    | 0.339        | 1.22  | 0.085                |
| 28   | "                  | 6.85     | 174                                    | 70                             | 104                | 6.84  | 122                                    | 0.0161                        | 1.07x10 <sup>6</sup>              | 4.00    | 0.602        | 1.40  | 0.146                |
| 29   | "                  | 1.11     | 93                                     | 70                             | 23                 | 5.01  | 82                                     | 0.0152                        | 1.47x10 <sup>6</sup>              | 1.21    | 0.083        | 1.09  | 0.037                |
| 30   | 0.0810             | 2.70     | 123                                    | 84                             | 39                 | 3.41  | 104                                    | 0.0157                        | 1.23x10 <sup>6</sup>              | 14.7    | 1.167        | 1.47  | 0.166                |
| 31   | "                  | 1.41     | 105                                    | 84                             | 21                 | 3.31  | 95                                     | 0.0155                        | 1.32x10 <sup>6</sup>              | 8.51    | 0.930        | 1.40  | 0.146                |
| 32   | "                  | 5.64     | 153                                    | 84                             | 69                 | 4.02  | 119                                    | 0.0161                        | 1.09x10 <sup>6</sup>              | 23.1    | 1.363        | 1.69  | 0.228                |
| 33   | "                  | 7.92     | 178                                    | 85                             | 93                 | 4.20  | 132                                    | 0.0164                        | 9.91x10 <sup>5</sup>              | 28.3    | 1.452        | 1.73  | 0.238                |
| 34   | "                  | 15.5     | 239                                    | 85                             | 154                | 4.96  | 162                                    | 0.0171                        | 8.01x10 <sup>5</sup>              | 37.8    | 1.578        | 1.96  | 0.292                |
| 35   | "                  | 23.6     | 306                                    | 85                             | 221                | 5.26  | 196                                    | 0.0179                        | 6.36x10 <sup>5</sup>              | 43.1    | 1.635        | 1.98  | 0.297                |
| 36   | "                  | 3.68     | 121                                    | 76                             | 48                 | 3.78  | 100                                    | 0.0156                        | 1.27x10 <sup>6</sup>              | 18.7    | 1.272        | 1.63  | 0.213                |
| 37   | "                  | 7.73     | 165                                    | 76                             | 89                 | 4.28  | 121                                    | 0.0161                        | 1.08x10 <sup>6</sup>              | 29.5    | 1.470        | 1.79  | 0.253                |
| 38   | "                  | 2.28     | 108                                    | 76                             | 32                 | 3.51  | 92                                     | 0.0154                        | 1.35x10 <sup>6</sup>              | 13.3    | 1.224        | 1.54  | 0.187                |

\* Wild run.

VIBRATING WIRES

TABLE 2

| 1   | 2                     | 3                            | 4           | 5                                    | 6                            | 7                       | 8  | 9                                 | 10                | 11                    | 12                                 | 13                              | 14   |
|-----|-----------------------|------------------------------|-------------|--------------------------------------|------------------------------|-------------------------|--|-----------------------------------|-------------------|-----------------------|------------------------------------|---------------------------------|--|
| Run | Wire diam.,<br>Inches | Direction<br>of<br>Vibration | Q,<br>Watts | Wire surface<br>temp. t <sub>s</sub> | Room Temp.<br>t <sub>p</sub> | Temp<br>diff.<br>Δt, °F | Humidity,<br>lbH <sub>2</sub> O/lb dry air | Barometric<br>Pressure,<br>In. Hg | Strob.<br>reading | Freq. f,<br>Vib./sec. | Amplitude<br>in scale<br>divisions | Average<br>Velocity<br>ft./sec. | Average<br>film temp.<br>t <sub>f</sub> , °F |
| 39  | 0.0253                | vertical                     | 2.45        | 99                                   | 75                           | 24                      | 0.005                                      | 30.1                              | 5050              | 104.3                 | 66.8                               | 2.16                            | 87   |
| 40  | "                     | "                            | 7.70        | 145                                  | 75                           | 70                      | 0.005                                      | 30.1                              | 4930              | 102.1                 | 70.6                               | 2.24                            | 110  |
| 41  | "                     | "                            | 19.4        | 228                                  | 75                           | 153                     | 0.005                                      | 30.1                              | 4920              | 101.9                 | 85.2                               | 2.69                            | 152  |
| 42  | "                     | "                            | 2.58        | 94.5                                 | 76                           | 18.5                    | 0.005                                      | 30.1                              | 4440              | 92.0                  | 97.5                               | 2.78                            | 85   |
| 43  | "                     | "                            | 8.64        | 201                                  | 76                           | 125                     | 0.005                                      | 30.1                              | 4520              | 93.9                  | 52.6                               | 1.53                            | 139  |
| 44  | "                     | "                            | 2.66        | 116                                  | 76                           | 40                      | 0.005                                      | 30.1                              | 4530              | 94.0                  | 50.1                               | 1.46                            | 96   |
| 45  | "                     | "                            | 2.66        | 122                                  | 77                           | 45                      | 0.005                                      | 30.0                              | 4410              | 91.3                  | 48.9                               | 1.38                            | 100  |
| 46  | "                     | "                            | 2.65        | 119                                  | 77                           | 42                      | 0.005                                      | 30.0                              | 4410              | 91.3                  | 58.6                               | 1.66                            | 98   |
| 47  | "                     | "                            | 9.70        | 184                                  | 77                           | 107                     | 0.005                                      | 30.0                              | 4440              | 91.9                  | 71.1                               | 2.02                            | 131  |
| 48  | "                     | "                            | 9.63        | 211                                  | 77                           | 134                     | 0.005                                      | 30.0                              | 4470              | 92.7                  | 53.6                               | 1.54                            | 88   |
| 49  | "                     | "                            | 2.18        | 98                                   | 78                           | 20                      | 0.005                                      | 30.1                              | 4750              | 98.2                  | 75.5                               | 2.30                            | 88   |
| 50  | "                     | "                            | 2.19        | 103                                  | 78                           | 25                      | 0.005                                      | 30.1                              | 5000              | 103.3                 | 67.2                               | 2.15                            | 91   |
| 51  | "                     | "                            | 2.48        | 105                                  | 74                           | 31                      | 0.009                                      | 29.5                              | 6620/2            | 68.0                  | 83.4                               | 1.76                            | 90   |
| 52  | "                     | horizontal                   | 1.48        | 101                                  | 76                           | 25                      | 0.010                                      | 30.3                              | 4800              | 99.4                  | 48.8                               | 1.50                            | 89   |
| 53  | "                     | "                            | 2.67        | 123                                  | 76                           | 47                      | 0.010                                      | 30.3                              | 4800              | 99.4                  | 35.6                               | 1.10                            | 100  |
| 54  | "                     | "                            | 2.73        | 96                                   | 78                           | 18                      | 0.010                                      | 30.3                              | 4310              | 89.3                  | 108.0                              | 2.99                            | 87   |
| 55  | "                     | "                            | 2.68        | 103                                  | 77                           | 26                      | 0.007                                      | 30.1                              | 4040              | 84.0                  | 80.0                               | 2.08                            | 90   |
| 56  | "                     | "                            | 6.86        | 151                                  | 78                           | 73                      | 0.007                                      | 30.1                              | 3980              | 82.9                  | 78.6                               | 2.02                            | 115  |
| 57  | "                     | "                            | 2.41        | 117                                  | 78                           | 39                      | 0.007                                      | 30.1                              | 5300              | 109.4                 | 44.2                               | 1.50                            | 118  |
| 58  | "                     | "                            | 6.79        | 180                                  | 79                           | 101                     | 0.007                                      | 30.1                              | 5050              | 104.4                 | 46.9                               | 1.52                            | 130  |
| *59 | 0.0396                | vertical                     | 2.34        | 86.5                                 | 76                           | 10.5                    | 0.007                                      | 30.3                              | 4330              | 89.8                  | 99.6                               | 2.77                            | 81   |
| 60  | "                     | "                            | 5.79        | 104                                  | 76                           | 28                      | 0.007                                      | 30.3                              | 4460              | 92.1                  | 104.4                              | 2.98                            | 90   |
| 61  | "                     | "                            | 15.1        | 165                                  | 76                           | 89                      | 0.007                                      | 30.3                              | 4250              | 88.1                  | 98.5                               | 2.68                            | 121  |
| 62  | "                     | "                            | 2.52        | 94                                   | 74                           | 20                      | 0.007                                      | 30.2                              | 4220              | 87.6                  | 75.5                               | 2.05                            | 84   |
| 63  | "                     | "                            | 6.20        | 127                                  | 75                           | 52                      | 0.007                                      | 30.2                              | 4200              | 87.2                  | 74.6                               | 2.02                            | 101  |
| 64  | "                     | "                            | 14.5        | 213                                  | 75                           | 138                     | 0.007                                      | 30.2                              | 4200              | 87.2                  | 73.4                               | 1.98                            | 144  |
| 65  | "                     | "                            | 32.2        | 383                                  | 76                           | 307                     | 0.007                                      | 30.2                              | 4350              | 90.1                  | 67.0                               | 1.87                            | 230  |
| 66  | "                     | "                            | 1.44        | 97                                   | 72                           | 25                      | 0.006                                      | 29.4                              | 5160              | 106.7                 | 37.3                               | 1.23                            | 85   |
| 67  | "                     | "                            | 6.87        | 168                                  | 72                           | 96                      | 0.006                                      | 29.4                              | 5300              | 109.4                 | 36.8                               | 1.25                            | 120  |
| 68  | "                     | "                            | 28.5        | 387                                  | 72                           | 315                     | 0.006                                      | 29.4                              | 5350              | 110.3                 | 38.3                               | 1.31                            | 230  |
| 69  | "                     | "                            | 2.65        | 116                                  | 72                           | 44                      | 0.006                                      | 29.4                              | 4300              | 89.3                  | 30.4                               | 0.84                            | 94   |
| 70  | "                     | "                            | 7.15        | 173                                  | 72                           | 101                     | 0.006                                      | 29.4                              | 4310              | 89.5                  | 29.5                               | 0.82                            | 123  |

\*Wild run - omitted  
from figs. 13, 14, 16.

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TABLE 2  
(continued)

| 1   | 15                       | 16                                     | 17   | 18  | 19                                   | 20                                    | 21                                   | 22      | 23       |
|-----|--------------------------|--|------|---|--------------------------------------|---------------------------------------|--------------------------------------|---------|----------|
| Run | Viscosity,<br>centipoise | Specific<br>volume<br>$1/p, ft.^3/lb.$ | Re   | Re $\frac{(\Delta t)}{(Gr)^{0.20}}$<br>0.16 | $h_p$<br>BTU/hr. ft. <sup>2</sup> °F | $h_o'$<br>BTU/hr. ft. <sup>2</sup> °F | $h_p$<br>BTU/hr. ft. <sup>2</sup> °F | $h/h_o$ | $h/h_o'$ |
| 39  | 0.0186                   | 13.8                                   | 26.5 | 19.2  | 16.4                                 | 8.34                                  | 7.61                                 | 1.96    | 2.15     |
| 40  | 0.0192                   | 14.3                                   | 25.5 | 18.6  | 17.7                                 | 9.89                                  | 8.87                                 | 1.79    | 1.99     |
| 41  | 0.0203                   | 15.4                                   | 27.0 | 19.7  | 20.3                                 | 11.5                                  | 10.3                                 | 1.77    | 1.97     |
| 42  | 0.0185                   | 13.7                                   | 34.4 | 25.1  | 22.4                                 | 8.30                                  | 6.95                                 | 2.70    | 3.22     |
| 43  | 0.0200                   | 15.1                                   | 16.0 | 11.6  | 11.1                                 | 10.1                                  | 9.95                                 | 1.10    | 1.11     |
| 44  | 0.0188                   | 14.0                                   | 17.4 | 12.5  | 10.7                                 | 8.36                                  | 8.01                                 | 1.28    | 1.33     |
| 45  | 0.0189                   | 14.1                                   | 16.2 | 11.6  | 9.50                                 | 8.39                                  | 8.19                                 | 1.13    | 1.16     |
| 46  | 0.0189                   | 14.2                                   | 19.5 | 14.0  | 10.1                                 | 8.35                                  | 8.03                                 | 1.21    | 1.26     |
| 47  | 0.0197                   | 14.9                                   | 21.7 | 15.6  | 14.6                                 | 10.3                                  | 9.61                                 | 1.41    | 1.52     |
| 48  | 0.0201                   | 15.2                                   | 15.8 | 11.5  | 11.5                                 | 10.3                                  | 10.1                                 | 1.12    | 1.15     |
| 49  | 0.0188                   | 13.8                                   | 24.1 | 20.5  | 17.5                                 | 8.14                                  | 7.05                                 | 2.15    | 2.48     |
| 50  | 0.0187                   | 13.9                                   | 26.1 | 19.0  | 14.2                                 | 8.24                                  | 7.44                                 | 1.72    | 1.91     |
| 51  | 0.0187                   | 14.2                                   | 20.8 | 15.0  | 12.8                                 | 8.29                                  | 7.66                                 | 1.55    | 1.68     |
| 52  | 0.0186                   | 13.8                                   | 18.4 | 13.3  | 9.51                                 | 7.93                                  | 7.40                                 | 1.20    | 1.29     |
| 53  | 0.0189                   | 14.1                                   | 12.9 | 9.26  | 9.13                                 | 8.41                                  | 8.20                                 | 1.085   | 1.11     |
| 54  | 0.0186                   | 13.7                                   | 36.8 | 27.0  | 24.4                                 | 8.43                                  | 6.96                                 | 2.89    | 3.50     |
| 55  | 0.0187                   | 13.9                                   | 25.2 | 18.3  | 16.5                                 | 8.26                                  | 7.41                                 | 2.00    | 2.23     |
| 56  | 0.0193                   | 14.5                                   | 22.6 | 16.2  | 15.1                                 | 9.75                                  | 8.90                                 | 1.55    | 1.70     |
| 57  | 0.0194                   | 14.6                                   | 16.6 | 12.3  | 9.94                                 | 8.25                                  | 7.96                                 | 1.20    | 1.25     |
| 58  | 0.0197                   | 14.9                                   | 16.2 | 11.7  | 10.8                                 | 9.74                                  | 9.55                                 | 1.11    | 1.13     |
| 59  | 0.0184                   | 13.6                                   | 54.4 | 30.6  | 23.1                                 | 5.56                                  | 4.45                                 | 4.16    | 5.58     |
| 60  | 0.0187                   | 13.9                                   | 56.6 | 31.0  | 21.4                                 | 6.52                                  | 5.02                                 | 3.29    | 4.26     |
| 61  | 0.0195                   | 14.6                                   | 46.2 | 25.1  | 17.6                                 | 7.81                                  | 6.57                                 | 2.25    | 2.68     |
| 62  | 0.0185                   | 13.7                                   | 39.7 | 21.8  | 13.1                                 | 5.66                                  | 4.72                                 | 2.31    | 2.77     |
| 63  | 0.0190                   | 14.1                                   | 37.1 | 20.1  | 12.4                                 | 6.62                                  | 5.79                                 | 1.87    | 2.14     |
| 64  | 0.0201                   | 15.2                                   | 31.9 | 17.6  | 10.9                                 | 7.17                                  | 7.17                                 | 1.52    | 1.52     |
| 65  | 0.0222                   | 17.4                                   | 23.8 | 14.1  | 10.9                                 | 8.91                                  | 8.58                                 | 1.22    | 1.27     |
| 66  | 0.0185                   | 14.0                                   | 23.2 | 12.7  | 5.99                                 | 5.11                                  | 4.95                                 | 1.17    | 1.21     |
| 67  | 0.0195                   | 14.9                                   | 21.2 | 11.5  | 7.43                                 | 6.80                                  | 6.65                                 | 1.093   | 1.12     |
| 68  | 0.0222                   | 17.7                                   | 16.3 | 9.60  | 9.40                                 | 8.80                                  | 8.63                                 | 1.067   | 1.089    |
| 69  | 0.0188                   | 14.2                                   | 15.4 | 8.35  | 6.26                                 | 5.74                                  | 5.60                                 | 1.091   | 1.12     |
| 70  | 0.0195                   | 15.0                                   | 13.7 | 7.47  | 7.35                                 | 6.87                                  | 6.75                                 | 1.07    | 1.09     |

TABLE 2  
(continued)

| 1   | 2      | 3          | 4    | 5   | 6  | 7   |
|-----|--------|------------|------|-----|----|-----|
| 71  | 0.0396 | vertical   | 3.36 | 95  | 73 | 22  |
| 72  | "      | "          | 6.66 | 123 | 74 | 49  |
| 73  | "      | "          | 13.8 | 192 | 75 | 117 |
| 74  | "      | "          | 2.48 | 94  | 76 | 18  |
| 75  | "      | "          | 1.80 | 106 | 76 | 30  |
| 76  | "      | "          | 5.98 | 162 | 77 | 85  |
| 77  | "      | "          | 27.7 | 387 | 77 | 310 |
| 78  | "      | "          | 6.99 | 165 | 78 | 87  |
| 79  | "      | "          | 6.91 | 173 | 78 | 95  |
| 80  | "      | "          | 6.94 | 163 | 78 | 85  |
| 81  | "      | horizontal | 0.97 | 78  | 70 | 8   |
| 82  | "      | "          | 2.40 | 90  | 72 | 18  |
| 83  | "      | "          | 14.3 | 187 | 73 | 114 |
| 84  | "      | "          | 5.81 | 117 | 74 | 43  |
| 85  | "      | "          | 27.8 | 258 | 74 | 184 |
| 86  | "      | "          | 1.02 | 92  | 73 | 19  |
| 87  | "      | "          | 2.52 | 114 | 73 | 41  |
| 88  | "      | "          | 5.70 | 156 | 73 | 83  |
| 89  | "      | "          | 14.2 | 268 | 69 | 199 |
| 90  | "      | "          | 32.2 | 434 | 69 | 365 |
| 91  | "      | "          | 14.2 | 265 | 69 | 196 |
| 92  | "      | "          | 1.00 | 83  | 71 | 12  |
| 93  | "      | "          | 2.62 | 103 | 71 | 32  |
| 94  | "      | "          | 6.06 | 148 | 72 | 76  |
| 95  | "      | "          | 14.5 | 248 | 72 | 176 |
| 96  | "      | "          | 29.8 | 391 | 72 | 319 |
| 97  | "      | "          | 1.49 | 92  | 75 | 17  |
| 98  | "      | "          | 5.94 | 141 | 75 | 66  |
| 99  | "      | "          | 13.8 | 230 | 75 | 155 |
| 100 | "      | "          | 32.0 | 393 | 75 | 318 |
| 101 | "      | "          | 2.60 | 107 | 75 | 32  |
| 102 | "      | "          | 6.18 | 118 | 75 | 43  |
| 103 | "      | "          | 1.04 | 77  | 70 | 7   |
| 104 | "      | "          | 6.01 | 107 | 70 | 37  |
| 105 | "      | "          | 2.16 | 84  | 70 | 14  |

TABLE 2  
(continued)

| 1   | 2      | 3          | 4    | 5   | 6  | 7   | 8     | 9    | 10     | 11    | 12    | 13   | 14  |
|-----|--------|------------|------|-----|----|-----|-------|------|--------|-------|-------|------|-----|
| 71  | 0.0396 | vertical   | 3.36 | 95  | 73 | 22  | 0.009 | 30.2 | 5770   | 119.0 | 70.4  | 2.59 | 84  |
| 72  | "      | "          | 6.66 | 123 | 74 | 49  | 0.009 | 30.2 | 5770   | 119.0 | 65.1  | 1.20 | 99  |
| 73  | "      | "          | 13.8 | 192 | 75 | 117 | 0.009 | 30.2 | 5950   | 122.5 | 58.1  | 2.21 | 134 |
| 74  | "      | "          | 2.48 | 94  | 76 | 18  | 0.009 | 30.0 | 5690/2 | 58.7  | 124.3 | 2.26 | 85  |
| 75  | "      | "          | 1.80 | 106 | 76 | 30  | 0.006 | 30.0 | 5730   | 117.9 | 36.7  | 1.34 | 91  |
| 76  | "      | "          | 5.98 | 162 | 77 | 85  | 0.006 | 30.0 | 5750   | 118.6 | 35.4  | 1.30 | 120 |
| 77  | "      | "          | 27.7 | 387 | 77 | 310 | 0.006 | 30.0 | 5830   | 120.0 | 36.0  | 1.34 | 232 |
| 78  | "      | "          | 6.99 | 165 | 78 | 87  | 0.006 | 30.0 | 5750   | 118.6 | 34.4  | 1.26 | 122 |
| 79  | "      | "          | 6.91 | 173 | 78 | 95  | 0.006 | 30.0 | 5550   | 114.4 | 28.8  | 1.02 | 126 |
| 80  | "      | "          | 6.94 | 163 | 78 | 85  | 0.006 | 30.0 | 5570   | 114.8 | 46.8  | 1.56 | 121 |
| 81  | "      | horizontal | 0.97 | 78  | 70 | 8   | 0.009 | 30.1 | 4310   | 89.3  | 83.6  | 2.31 | 74  |
| 82  | "      | "          | 2.40 | 90  | 72 | 18  | 0.009 | 30.1 | 4310   | 89.3  | 84.0  | 2.32 | 81  |
| 83  | "      | "          | 14.3 | 187 | 73 | 114 | 0.009 | 30.1 | 4290   | 89.1  | 80.6  | 2.23 | 130 |
| 84  | "      | "          | 5.81 | 117 | 74 | 43  | 0.009 | 30.1 | 4310   | 89.3  | 82.5  | 2.28 | 96  |
| 85  | "      | "          | 27.8 | 258 | 74 | 184 | 0.009 | 30.1 | 4400   | 91.2  | 79.2  | 2.24 | 166 |
| 86  | "      | "          | 1.02 | 92  | 73 | 19  | 0.004 | 30.0 | 4350   | 90.2  | 37.8  | 1.06 | 83  |
| 87  | "      | "          | 2.52 | 114 | 73 | 41  | 0.004 | 30.0 | 4310   | 89.3  | 35.7  | 0.99 | 94  |
| 88  | "      | "          | 5.70 | 156 | 73 | 83  | 0.004 | 30.0 | 4330   | 89.9  | 37.1  | 1.03 | 115 |
| 89  | "      | "          | 14.2 | 268 | 69 | 199 | 0.002 | 30.2 | 4370   | 90.7  | 38.7  | 1.09 | 169 |
| 90  | "      | "          | 32.2 | 434 | 69 | 365 | 0.002 | 30.2 | 4400   | 91.2  | 35.8  | 1.01 | 252 |
| 91  | "      | "          | 14.2 | 265 | 69 | 196 | 0.002 | 30.2 | 4400   | 91.2  | 37.8  | 1.07 | 167 |
| 92  | "      | "          | 1.00 | 83  | 71 | 12  | 0.005 | 30.2 | 4330   | 89.9  | 59.0  | 1.64 | 77  |
| 93  | "      | "          | 2.62 | 103 | 71 | 32  | 0.005 | 30.2 | 4320   | 89.8  | 57.6  | 1.60 | 87  |
| 94  | "      | "          | 6.06 | 148 | 72 | 76  | 0.005 | 30.2 | 4300   | 89.3  | 57.0  | 1.58 | 110 |
| 95  | "      | "          | 14.5 | 248 | 72 | 176 | 0.005 | 30.2 | 4300   | 89.3  | 57.1  | 1.58 | 160 |
| 96  | "      | "          | 29.8 | 391 | 72 | 319 | 0.005 | 30.2 | 4310   | 89.3  | 58.9  | 1.63 | 232 |
| 97  | "      | "          | 1.49 | 92  | 75 | 17  | 0.005 | 30.2 | 4380   | 90.9  | 58.8  | 1.55 | 84  |
| 98  | "      | "          | 5.94 | 141 | 75 | 66  | 0.005 | 30.2 | 4380   | 90.9  | 60.8  | 1.71 | 106 |
| 99  | "      | "          | 13.8 | 230 | 75 | 155 | 0.005 | 30.2 | 4390   | 90.9  | 59.0  | 1.66 | 153 |
| 100 | "      | "          | 32.0 | 393 | 75 | 318 | 0.005 | 30.2 | 4400   | 91.0  | 60.5  | 1.70 | 234 |
| 101 | "      | "          | 2.60 | 107 | 75 | 32  | 0.005 | 30.2 | 4400   | 91.2  | 59.1  | 1.67 | 91  |
| 102 | "      | "          | 6.18 | 118 | 75 | 43  | 0.005 | 30.2 | 4970   | 102.9 | 76.1  | 2.43 | 97  |
| 103 | "      | "          | 1.04 | 77  | 70 | 7   | 0.003 | 30.2 | 4970   | 102.9 | 79.6  | 2.54 | 74  |
| 104 | "      | "          | 6.01 | 107 | 70 | 37  | 0.003 | 30.2 | 4980   | 103.0 | 80.1  | 2.56 | 89  |
| 105 | "      | "          | 2.16 | 84  | 70 | 14  | 0.003 | 30.2 | 5000   | 103.3 | 80.6  | 2.58 | 77  |

|     | 1      | 15   | 16   | 17   | 18   | 19   | 20   | 21    | 22    | 23 |
|-----|--------|------|------|------|------|------|------|-------|-------|----|
| 71  | 0.0185 | 13.7 | 50.2 | 27.5 | 15.8 | 5.79 | 4.82 | 2.74  | 3.29  |    |
| 72  | 0.0189 | 14.0 | 44.4 | 24.0 | 14.1 | 6.79 | 5.74 | 2.08  | 2.46  |    |
| 73  | 0.0198 | 14.9 | 36.6 | 20.0 | 12.2 | 7.16 | 6.93 | 1.71  | 1.77  |    |
| 74  | 0.0185 | 13.8 | 43.4 | 24.0 | 14.3 | 5.65 | 4.78 | 2.53  | 3.06  |    |
| 75  | 0.0187 | 14.0 | 25.2 | 13.8 | 6.23 | 5.33 | 5.15 | 1.17  | 1.21  |    |
| 76  | 0.0195 | 14.7 | 22.4 | 12.2 | 7.30 | 6.60 | 6.49 | 1.11  | 1.13  |    |
| 77  | 0.0223 | 17.5 | 16.9 | 9.96 | 9.29 | 8.75 | 8.60 | 1.062 | 1.079 |    |
| 78  | 0.0195 | 14.7 | 21.6 | 11.8 | 8.33 | 6.83 | 6.55 | 1.22  | 1.27  |    |
| 79  | 0.0196 | 14.9 | 17.2 | 9.36 | 7.55 | 6.83 | 6.64 | 1.11  | 1.14  |    |
| 80  | 0.0195 | 14.7 | 28.6 | 15.6 | 8.46 | 6.79 | 6.48 | 1.25  | 1.31  |    |
| 81  | 0.0182 | 13.5 | 46.0 | 25.9 | 12.6 | 4.74 | 3.90 | 2.65  | 3.23  |    |
| 82  | 0.0184 | 13.7 | 45.2 | 24.8 | 13.8 | 5.60 | 4.61 | 2.47  | 3.00  |    |
| 83  | 0.0197 | 15.0 | 37.0 | 20.2 | 13.0 | 7.42 | 6.88 | 1.83  | 1.89  |    |
| 84  | 0.0188 | 14.0 | 40.8 | 22.1 | 14.0 | 6.55 | 5.50 | 2.14  | 2.53  |    |
| 85  | 0.0206 | 15.8 | 33.7 | 18.8 | 15.7 | 8.75 | 7.61 | 1.79  | 2.06  |    |
| 86  | 0.0185 | 13.7 | 20.5 | 11.3 | 5.58 | 4.81 | 4.65 | 1.16  | 1.20  |    |
| 87  | 0.0188 | 14.0 | 18.4 | 10.0 | 6.39 | 5.65 | 5.50 | 1.13  | 1.16  |    |
| 88  | 0.0193 | 14.5 | 18.1 | 9.79 | 7.43 | 6.58 | 6.63 | 1.083 | 1.075 |    |
| 89  | 0.0207 | 15.8 | 16.3 | 9.45 | 7.43 | 7.10 | 7.06 | 1.045 | 1.051 |    |
| 90  | 0.0227 | 17.9 | 12.2 | 7.32 | 9.15 | 8.92 | 8.89 | 1.025 | 1.03  |    |
| 91  | 0.0206 | 15.7 | 16.2 | 9.00 | 7.52 | 7.09 | 7.05 | 1.062 | 1.067 |    |
| 92  | 0.0183 | 13.5 | 32.7 | 18.2 | 8.60 | 4.75 | 4.21 | 1.81  | 2.04  |    |
| 93  | 0.0186 | 13.8 | 30.8 | 16.7 | 8.50 | 5.66 | 5.19 | 1.50  | 1.64  |    |
| 94  | 0.0192 | 14.3 | 28.1 | 15.2 | 8.29 | 6.63 | 6.30 | 1.25  | 1.32  |    |
| 95  | 0.0205 | 15.7 | 24.1 | 13.5 | 8.56 | 7.10 | 7.20 | 1.20  | 1.19  |    |
| 96  | 0.0223 | 17.8 | 20.2 | 11.9 | 9.71 | 8.84 | 8.61 | 1.098 | 1.12  |    |
| 97  | 0.0185 | 13.6 | 32.2 | 17.8 | 9.10 | 5.15 | 4.67 | 1.77  | 1.95  |    |
| 98  | 0.0192 | 14.3 | 30.6 | 16.5 | 9.33 | 6.61 | 6.11 | 1.41  | 1.53  |    |
| 99  | 0.0203 | 15.5 | 25.9 | 14.3 | 9.24 | 7.21 | 7.37 | 1.28  | 1.25  |    |
| 100 | 0.0223 | 17.9 | 21.0 | 12.4 | 10.4 | 8.93 | 8.65 | 1.17  | 1.21  |    |
| 101 | 0.0187 | 13.8 | 31.7 | 17.3 | 8.45 | 5.75 | 5.20 | 1.47  | 1.63  |    |
| 102 | 0.0189 | 14.0 | 45.2 | 24.6 | 14.9 | 6.70 | 5.57 | 2.23  | 2.68  |    |
| 103 | 0.0182 | 13.4 | 51.0 | 28.9 | 15.4 | 4.81 | 3.78 | 3.20  | 4.08  |    |
| 104 | 0.0186 | 13.8 | 48.8 | 26.4 | 16.9 | 6.65 | 5.36 | 2.54  | 3.15  |    |
| 105 | 0.0183 | 13.5 | 51.4 | 28.4 | 16.0 | 5.47 | 4.37 | 2.93  | 3.66  |    |

TABLE  
(contin)

| 1   | 2      | 3          | 4    | 5   | 6  | 7   | 8     | 9    | 10     | 11    | 12    | 13   | 14  |
|-----|--------|------------|------|-----|----|-----|-------|------|--------|-------|-------|------|-----|
| 106 | 0.0396 | horizontal | 30.9 | 264 | 70 | 190 | 0.003 | 30.2 | 4950   | 102.3 | 80.2  | 2.54 | 165 |
| 107 | "      | "          | 13.7 | 160 | 71 | 89  | 0.003 | 30.2 | 5020   | 103.8 | 78.9  | 2.54 | 116 |
| 108 | "      | "          | 6.21 | 116 | 75 | 41  | 0.005 | 29.8 | 3630   | 75.4  | 99.5  | 2.32 | 96  |
| 109 | "      | "          | 15.2 | 183 | 75 | 108 | 0.005 | 29.8 | 3650   | 76.0  | 96.5  | 2.27 | 129 |
| 110 | "      | "          | 32.5 | 261 | 75 | 186 | 0.005 | 29.8 | 3650   | 76.0  | 120.4 | 2.84 | 168 |
| 111 | "      | "          | 2.75 | 106 | 73 | 33  | 0.007 | 29.8 | 5940/2 | 61.2  | 75.6  | 1.44 | 90  |
| 112 | "      | "          | 6.14 | 144 | 74 | 70  | 0.007 | 29.8 | 6000/2 | 61.8  | 75.5  | 1.44 | 109 |
| 113 | "      | "          | 32.2 | 401 | 75 | 326 | 0.007 | 29.8 | 6020/2 | 61.9  | 72.3  | 1.39 | 238 |
| 114 | "      | "          | 2.55 | 119 | 75 | 44  | 0.007 | 29.8 | 5700/2 | 58.7  | 45.0  | 0.82 | 97  |
| 115 | "      | "          | 2.54 | 90  | 70 | 20  | 0.007 | 30.2 | 5740   | 118.5 | 64.2  | 2.36 | 80  |
| 116 | "      | "          | 13.6 | 170 | 74 | 96  | 0.007 | 30.2 | 5780   | 119.0 | 64.9  | 2.39 | 122 |
| 117 | "      | "          | 32.3 | 310 | 74 | 236 | 0.007 | 30.2 | 5710   | 117.5 | 65.0  | 2.36 | 192 |
| 118 | "      | "          | 1.05 | 83  | 75 | 8   | 0.007 | 30.2 | 5550   | 114.3 | 67.6  | 2.40 | 79  |
| 119 | "      | "          | 6.15 | 120 | 75 | 45  | 0.007 | 30.2 | 5410   | 111.5 | 65.5  | 2.26 | 98  |
| 120 | 0.0810 | vertical   | 2.73 | 89  | 80 | 19  | 0.010 | 29.7 | 4490/2 | 46.5  | 96.4  | 1.39 | 90  |
| 121 | "      | "          | 5.72 | 118 | 82 | 36  | 0.010 | 29.7 | 4150/2 | 43.0  | 104.7 | 1.40 | 100 |
| 122 | "      | "          | 4.77 | 116 | 82 | 34  | 0.010 | 29.7 | 4150/2 | 43.0  | 100.4 | 1.34 | 99  |
| 123 | "      | "          | 7.80 | 124 | 82 | 52  | 0.010 | 29.7 | 4130/2 | 42.9  | 107.3 | 1.43 | 108 |
| 124 | "      | "          | 2.82 | 92  | 80 | 22  | 0.008 | 29.7 | 3790/2 | 39.5  | 113.1 | 1.39 | 91  |
| 125 | "      | "          | 2.70 | 116 | 80 | 36  | 0.008 | 29.7 | 3740/2 | 38.9  | 51.4  | 0.62 | 98  |
| 126 | "      | "          | 7.50 | 163 | 80 | 83  | 0.008 | 29.7 | 4200/2 | 43.5  | 56.7  | 0.76 | 122 |
| 127 | "      | "          | 5.64 | 137 | 76 | 61  | 0.009 | 29.7 | 4200/2 | 43.5  | 76.1  | 1.03 | 107 |
| 128 | "      | "          | 3.35 | 114 | 76 | 37  | 0.009 | 29.7 | 4150/2 | 43.0  | 82.6  | 1.10 | 95  |
| 129 | "      | "          | 24.1 | 280 | 78 | 202 | 0.009 | 29.7 | 4100/2 | 42.5  | 77.0  | 1.01 | 179 |
| 130 | "      | horizontal | 2.86 | 102 | 80 | 22  | 0.007 | 30.3 | 4100/2 | 42.6  | 92.0  | 1.22 | 91  |
| 131 | "      | "          | 5.14 | 127 | 80 | 47  | 0.007 | 30.3 | 4060/2 | 42.1  | 88.3  | 1.15 | 104 |
| 132 | "      | "          | 5.74 | 114 | 73 | 41  | 0.009 | 29.8 | 4040/2 | 42.0  | 92.6  | 1.20 | 94  |
| 133 | "      | "          | 7.99 | 124 | 74 | 50  | 0.009 | 29.8 | 4360/2 | 45.1  | 95.6  | 1.34 | 99  |
| 134 | "      | "          | 7.68 | 159 | 75 | 84  | 0.009 | 29.8 | 4850/2 | 50.1  | 49.0  | 0.76 | 117 |
| 135 | "      | "          | 3.68 | 121 | 76 | 45  | 0.009 | 29.8 | 4790/2 | 49.6  | 43.5  | 0.67 | 99  |
| 136 | "      | "          | 2.77 | 98  | 76 | 22  | 0.009 | 29.8 | 5230/2 | 54.0  | 84.2  | 1.41 | 87  |
| 137 | "      | "          | 21.5 | 191 | 76 | 115 | 0.009 | 29.8 | 5190/2 | 53.5  | 86.1  | 1.59 | 134 |
| 138 | "      | "          | 2.52 | 108 | 76 | 32  | 0.010 | 29.8 | 4040/2 | 42.0  | 65.7  | 0.86 | 92  |
| 139 | "      | "          | 5.30 | 135 | 78 | 57  | 0.010 | 29.8 | 4100/2 | 42.5  | 73.8  | 0.97 | 107 |

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TABLE 2  
(continued)

|     | 15     | 16   | 17   | 18   | 19   | 20   | 21   | 22    | 23    |
|-----|--------|------|------|------|------|------|------|-------|-------|
| 106 | 0.0206 | 15.8 | 38.4 | 21.4 | 16.9 | 8.90 | 7.15 | 1.90  | 2.36  |
| 107 | 0.0194 | 14.5 | 44.4 | 24.0 | 16.0 | 7.20 | 6.58 | 2.22  | 2.43  |
| 108 | 0.0188 | 14.2 | 42.6 | 23.1 | 15.7 | 6.66 | 5.50 | 2.36  | 2.86  |
| 109 | 0.0197 | 15.2 | 37.4 | 20.4 | 14.6 | 7.23 | 6.82 | 2.02  | 2.14  |
| 110 | 0.0207 | 16.2 | 41.6 | 23.3 | 18.1 | 8.99 | 7.25 | 2.03  | 2.50  |
| 111 | 0.0187 | 14.0 | 26.9 | 14.6 | 8.66 | 5.84 | 5.23 | 1.49  | 1.66  |
| 112 | 0.0192 | 14.5 | 25.6 | 13.8 | 9.10 | 6.77 | 6.21 | 1.34  | 1.46  |
| 113 | 0.0224 | 17.6 | 17.3 | 10.3 | 10.3 | 8.92 | 8.72 | 1.15  | 1.18  |
| 114 | 0.0189 | 14.2 | 15.1 | 8.16 | 6.01 | 5.69 | 5.58 | 1.057 | 1.077 |
| 115 | 0.0184 | 13.7 | 16.0 | 25.1 | 13.2 | 5.68 | 4.72 | 2.32  | 2.79  |
| 116 | 0.0195 | 14.8 | 40.8 | 22.1 | 14.7 | 7.31 | 6.65 | 2.01  | 2.21  |
| 117 | 0.0213 | 16.5 | 33.1 | 18.9 | 14.2 | 8.97 | 7.70 | 1.59  | 1.85  |
| 118 | 0.0184 | 13.7 | 46.8 | 26.5 | 13.6 | 4.83 | 5.88 | 2.82  | 3.50  |
| 119 | 0.0189 | 14.1 | 41.8 | 22.6 | 14.2 | 6.64 | 5.81 | 2.14  | 2.44  |
| 120 | 0.0187 | 14.1 | 52.8 | 19.0 | 7.09 | 3.63 | 3.12 | 1.95  | 2.27  |
| 121 | 0.0189 | 14.4 | 51.6 | 18.4 | 7.84 | 4.08 | 3.56 | 1.92  | 2.20  |
| 122 | 0.0189 | 14.3 | 49.6 | 17.7 | 6.91 | 3.98 | 3.54 | 1.74  | 1.96  |
| 123 | 0.0191 | 14.6 | 51.4 | 18.2 | 7.40 | 4.32 | 3.86 | 1.71  | 1.92  |
| 124 | 0.0187 | 14.1 | 52.8 | 17.4 | 6.32 | 3.66 | 3.23 | 1.73  | 1.96  |
| 125 | 0.0189 | 14.3 | 23.2 | 8.25 | 3.70 | 3.60 | 3.58 | 1.028 | 1.034 |
| 126 | 0.0195 | 14.9 | 26.4 | 9.35 | 4.45 | 4.30 | 4.27 | 1.036 | 1.041 |
| 127 | 0.0191 | 14.5 | 37.2 | 13.0 | 4.56 | 4.09 | 4.00 | 1.12  | 1.14  |
| 128 | 0.0188 | 14.2 | 41.4 | 14.6 | 4.46 | 3.75 | 3.59 | 1.19  | 1.24  |
| 129 | 0.0209 | 16.4 | 29.8 | 10.9 | 5.89 | 5.30 | 5.20 | 1.11  | 1.13  |
| 130 | 0.0187 | 13.9 | 47.2 | 16.9 | 6.41 | 3.62 | 3.24 | 1.77  | 1.98  |
| 131 | 0.0190 | 14.2 | 42.8 | 15.2 | 5.39 | 4.02 | 3.78 | 1.34  | 1.43  |
| 132 | 0.0188 | 14.1 | 45.6 | 16.0 | 6.90 | 4.10 | 3.68 | 1.68  | 1.88  |
| 133 | 0.0189 | 14.3 | 49.6 | 17.4 | 7.87 | 4.33 | 3.82 | 1.82  | 2.06  |
| 134 | 0.0194 | 14.7 | 26.8 | 9.39 | 4.50 | 4.29 | 4.27 | 1.049 | 1.053 |
| 135 | 0.0189 | 14.3 | 24.8 | 8.73 | 4.03 | 3.78 | 3.77 | 1.068 | 1.07  |
| 136 | 0.0186 | 14.0 | 54.6 | 19.5 | 6.21 | 3.60 | 3.24 | 1.73  | 1.92  |
| 137 | 0.0188 | 15.1 | 53.4 | 19.0 | 9.21 | 5.20 | 4.53 | 1.77  | 2.03  |
| 138 | 0.0187 | 14.1 | 32.5 | 11.5 | 3.88 | 3.55 | 3.50 | 1.094 | 1.11  |
| 139 | 0.0191 | 14.5 | 35.1 | 12.4 | 4.59 | 4.02 | 3.94 | 1.14  | 1.17  |

RUNS FOR FIG. 9 - TABLE 3

Frequency = 90 vibrations/sec.

| Amplitude-inches | Run numbers |
|------------------|-------------|
| 0.19             | 59-61       |
| 0.15             | 81-85       |
| 0.11             | 92-101      |
| 0.07             | 86-88, 90   |
| 0                | 13-29       |

RUNS FOR FIG. 10 - TABLE 4

| Amplitude - inches | Frequency- vib./sec. | Run numbers  |
|--------------------|----------------------|--------------|
| 0.15               | 103                  | 103 - 107    |
| 0.15               | 90                   | 81 - 85      |
| 0.15               | 62                   | 111 - 113    |
| 0.07               | 114                  | 66-68, 75-77 |
| 0.07               | 90                   | 86-88, 90    |
| 0                  | 0                    | 13-29        |

RUNS FOR FIG. 11 - TABLE 5

| Average velocity - ft/sec. | Diameter-inches | Run numbers             |
|----------------------------|-----------------|-------------------------|
| 2.8                        | 0.0253          | 41, 42, 54              |
| 2.8                        | 0.0396          | 59, 60, 110             |
| 1.5                        | 0.0253          | 43-45, 48, 52, 57, 58   |
| 1.5                        | 0.0396          | 94, 95, 111, 112        |
| 1.5                        | 0.0810          | 120, 121, 123, 124, 126 |
| 0                          | 0.0253          | 1 - 12                  |
| 0                          | 0.0396          | 13 - 29                 |
| 0                          | 0.0810          | 30 - 38                 |

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