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ARTIFICIAL TRANSMISSION LINE
DESIGN & CONSTRUCTION.

A

THESIS

PRESENTED TO THE FACULTY OF
THE COLLEGE OF ENGINEERING
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GENERAL . DESCRIPTION

An artificial transmission line is a combination of resistances, inductances and capacities, which will have approximately the same electrical properties as the actual line.

A long transmission line will have in addition to its resistance, the electrical properties of inductance and capacity in a considerable degree. In short lines the effect of these will be so small as to be negligible.

Inductance is a function of; 1st. the size of wire, 2nd. the arrangement of the circuit, and 3rd. the conditions and material surrounding or in the vicinity of the circuit.

If $D \equiv$ distance between conductors, expressed in inches.

$R =$ radius of conductor, expressed in inches.

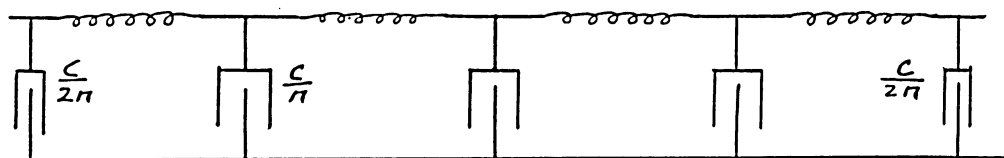
The inductance of a line per mile of conductor is:-

$$L = (0.74 \log_{10} \frac{D}{R} + 0.0805) 10^{-3} \text{ henrys for a copper line.}$$

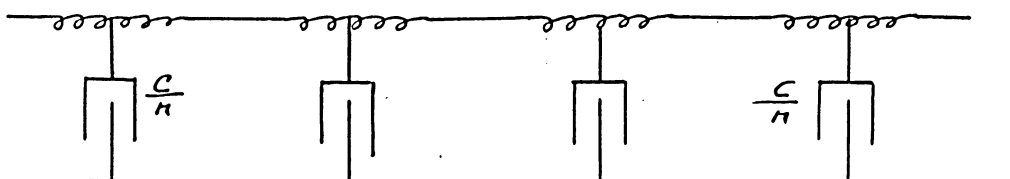
Capacity is a function of the same variables as inductance, but the value of capacity increases as the distance "D" decreases. Thus as two wires are brot closer together the capacity increases and the inductance decreases. The capacity of a line per mile of conductor between the conductor and neutral is:-

$$C = \frac{38.8}{\log_{10} \frac{D - R}{R}} 10^{-9} \text{ farads for a copper line.}$$

Among the effects which may occur in a long distance transmission line are; the increase or decrease of voltage along the line, the change of phase angle between the voltage and the current along the line, and the appearance of surges or impulses due to switching or other local changes. The resistance, capacity, and inductance of an actual transmission line are distributed uniformly thru-out its length. Artificial lines with variable linear constants are usually of the "lumpy" type, i.e., the inductance and capacity occur in alternately successive lumps. Lumpy lines may be divided into two classes, viz., "T" lines and "Π" (Pi) lines. A "T" line consists of sections each resembling a T , with a leak or condenser in the middle of each section of inductance. A "Π" line consists of sections each resembling a "Π" with a leak at each end of the inductance.



Π (Pi) Line
n sections.



T Line.

A lumpiness correction factor must be applied to lumpy lines depending upon the frequency of the circuit.

The line herein described is of the lumpy type consisting of 12 "T" sections, each representing 10 miles of actual line. The linear constants are variable and settings may be made for sizes of wire between #5 B. & S. to 300000 c.m. and for all spacings between 12 and 180 inches inclusive.

The artificial line distribution, after applying the proper lumpiness correction factor, will imitate correctly the distribution that would be found on the corresponding actual line in the steady state of operation. The artificial line may be connected as a single phase system and will represent accurately one phase of a three phase balanced system, as it may be assumed that whatever happens in one phase will also take place in the other two phases.

Also, if the impressed voltage on the model be $1/n$ of that on the actual line, the currents in the model system will be $1/n$ those corresponding in the actual system, and the powers in the model system will be $1/n^2$ those corresponding in the actual system, neglecting the corona effect. Thus, if 100 volts r.m.s. be impressed on the artificial line it may represent 100000 volts (star to neutral) on an actual line.

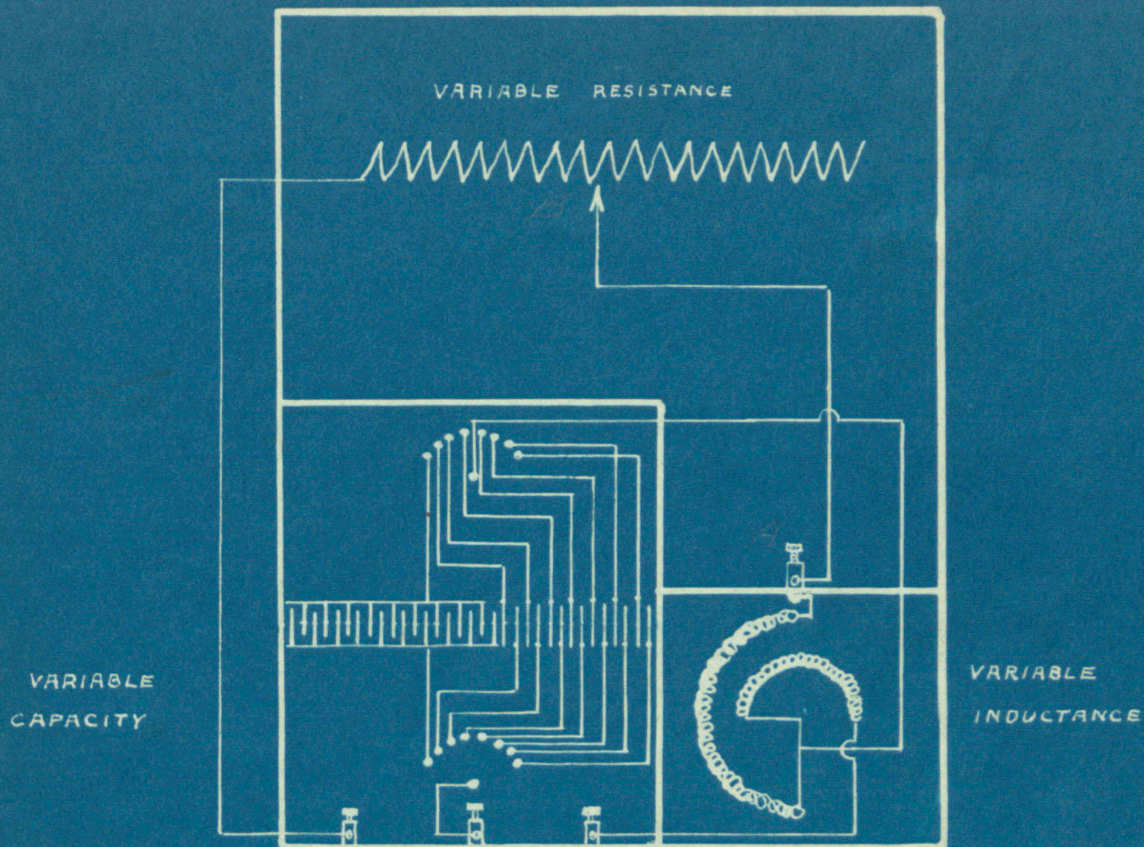
Then each milliampere observed would correspond to one ampere in the actual case, and each observed watt to one megawatt (1000 K.W.) in the actual case.

Page number 5 shows the connections of the artificial line variables in a 10 mile unit and the connections of several units as representing a "T" line.

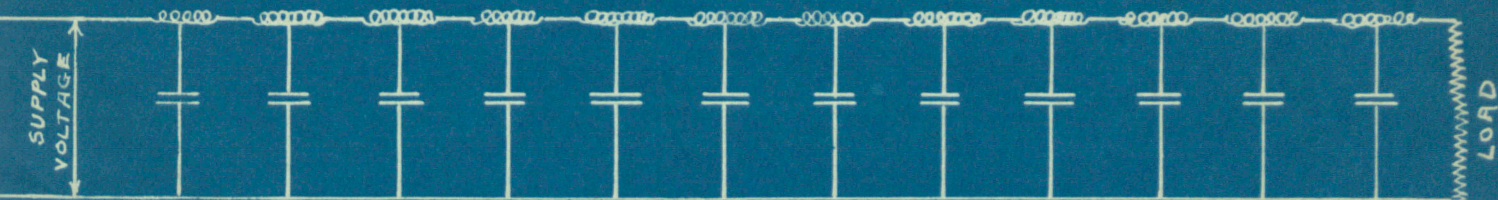
In order that the linear constants may vary for sizes of wire between #5 B&S to 300000 c.m. and for spacings between 12 and 180 inches ; a variable inductance of 24 millihenrys maximum, a condenser of 0.23 microfarads maximum, and a resistance of 17 ohms, approximately, are required.

Below is the detail design of the variable inductance (see page 6 and 7). It is wound of # 14 DCC wire, total resistance being 1.88 ohms @ 20 degrees centigrade. The outer coil has 220 turns, the inner coil has 460 turns, making a total of 680 turns. With the coils as designed it is possible to vary the inductance between 9 and 24 millihenrys, which more than covers the range required. The calibration curves of the 12 units are given on pages 12 to 23 inclusive.

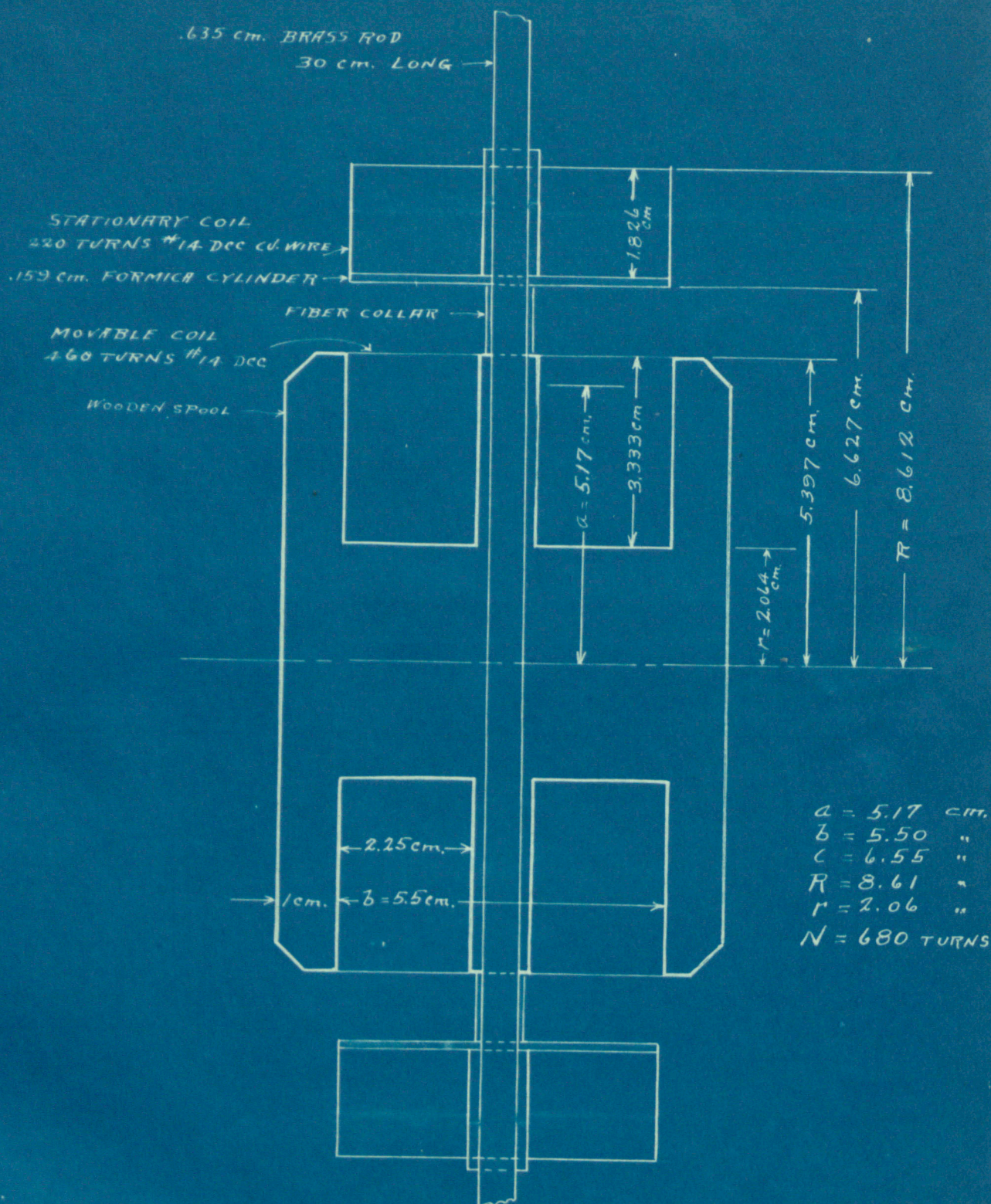
The line condenser is also variable, has a minimum capacity of approximately 0.0067m.f. and may be increased in steps of .0067m.f. up to 0.1 m.f. approx. Or the constant part (.14 m.f.) may be varied in steps of .0067 m.f. up to the maximum, 0.234m.f. (see page 26).



ARRANGEMENT OF RESISTANCE, INDUCTANCE AND CAPACITY IN A SINGLE UNIT.



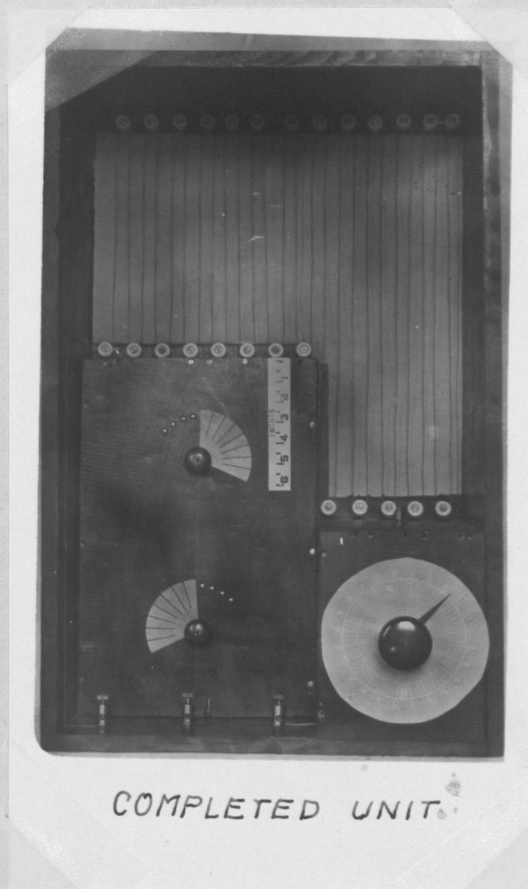
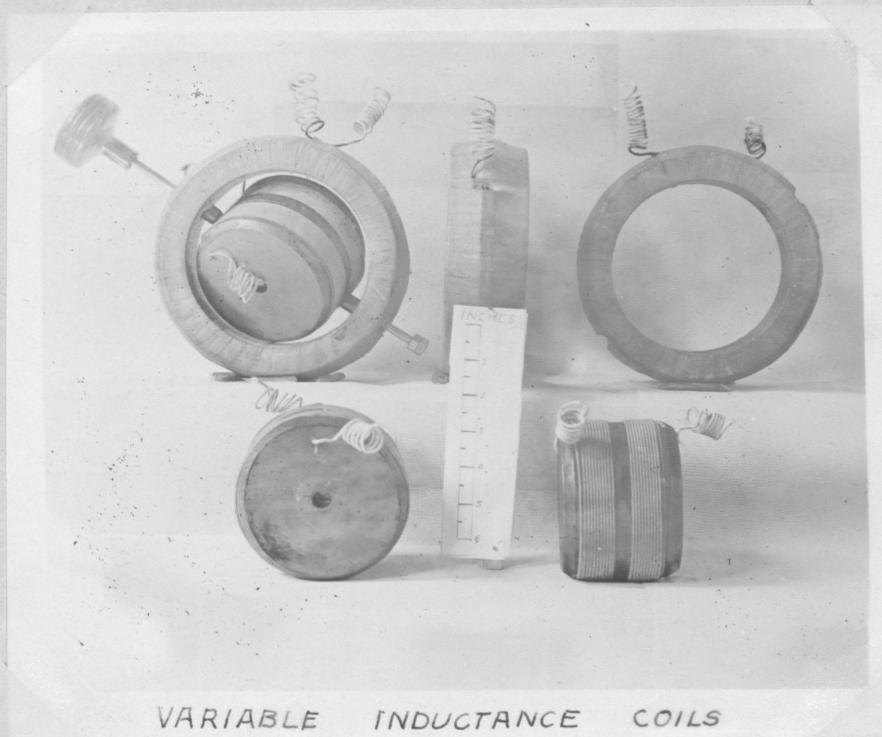
CONNECTIONS OF COMPLETE ARTIFICIAL LINE.



VARIABLE INDUCTANCE FOR ONE TEN MILE UNIT.

SCALE - FULL SIZE

C.B. JONES 6-4-17



Calculation of the mutual inductance of the two coils which are shown in detail on the following page. Brooks formula as applied to self inductance of coils is used and the mean radius (a) is estimated as follows :-

$$\text{Resultant} = 10.25 + 18.3 = 28.55 \text{ sq.cm.}$$

Summation of moments about "A" = 0

$$10.25 \times 7.7 + 18.3 \times 3.73 - 28.55a = 0$$

From which $a = 5.17 \text{ cm.}$

$$L = \frac{4\pi a^2 N^2 F' F''}{b+c+R} 10^{-6} \text{ mh.}$$

$$F' = \frac{10b + 12c + 2R}{10b + 10c + 1.4 R}$$

$$F'' = 0.5 \log_{10} \left(100 \frac{14 R}{2b + 3c} \right)$$

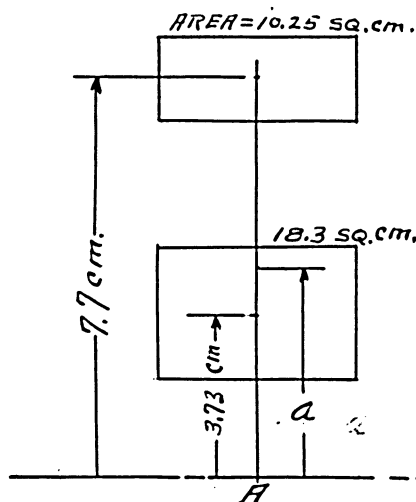
$$F' = \frac{10 \times 5.5 + 12 \times 6.55 + 2 \times 8.61}{10 \times 5.5 + 10 \times 6.55 + 1.4 \times 8.61} = 1.135$$

$$F'' = 0.5 \log_{10} \left(100 \frac{14 \times 8.61}{2 \times 5.5 + 3 \times 6.55} \right) = 1.008$$

$$L = \frac{4\pi \times 5.17^2 \times 680^2 \times 1.135 \times 1.008}{5.5 + 6.55 + 8.61} 10^{-6} = 27 \text{ millihenrys}$$

$$\begin{aligned} a &= 5.17 \text{ cm.} \\ b &= 5.50 \text{ cm} \\ c &= 6.55 \text{ cm} \\ R &= 8.61 \text{ cm} \\ N &= 680 \text{ turns} \end{aligned}$$

The actual value obtained from test on the coils as designed is approximately 24 mh. Therefore, there is an error of only eleven percent.



INDUCTANCE OF TRANSMISSION LINES

TEN-MILE SECTION TO NEUTRAL IN HENRIES

SPACING OF WIRE IN INS.	STRANDED 300,000 CM	STRANDED 250,000 CM	SOLID 0000	SOLID 000	SOLID 00	SOLID 0	SOLID 1	SOLID 2	SOLID 3	SOLID 4	SOLID 5	SOLID 6
12	.0128	.0131	.0135	.0139	.01428	.01462	.0150	.0154	.01575	.0161	.0165	.0169
18	.0141	.0144	.0148	.01516	.01555	.0159	.0163	.0167	.0170	.0174	.0178	.0181
24	.0150	.0153	.01575	.01613	.0165	.0169	.0172	.0176	.0180	.0184	.0187	.0191
36	.0163	.0166	.0170	.0174	.0178	.0181	.0185	.0189	.0193	.0196	.0200	.0204
48	.01725	.01754	.01795	.01835	.0187	.0191	.0195	.01985	.0202	.0206	.0210	.0213
60	.01795	.01825	.01867	.01905	.0194	.0198	.0202	.0206	.0209	.0213	.0216	.0220
72	.01855	.01885	.01925	.01962	.0200	.0204	.0208	.0211	.0215	.0218	.0222	.0226
84	.01905	.01935	.0198	.0202	.0205	.0209	.0212	.0216	.0220	.0224	.0228	.0231
96	.0195	.0198	.0202	.0206	.0210	.0213	.0217	.0220	.0224	.0228	.0232	.0236
108	.01985	.0202	.0206	.0210	.0213	.0217	.0220	.0224	.0228	.0232	.0236	.0239
120	.0202	.0205	.0209	.02126	.0216	.0220	.0224	.0228	.0232	.0235	.0239	.0243
132	.0205	.0208	.0212	.0216	.0220	.0223	.0227	.023	.0234	.0238	.0242	.0246
144	.0208	.0211	.0215	.02185	.0222	.0226	.0230	.0234	.0237	.0241	.0244	.0248
156	.0210	.0214	.02175	.0221	.0225	.0228	.0232	.0236	.0240	.0244	.0247	.0251
168	.0211	.0216	.0220	.02235	.0227	.0231	.0234	.0238	.0242	.0246	.0250	.0253
180	.0215	.0218	.02225	.0226	.0230	.0234	.0237	.0241	.0244	.0248	.0252	.0255

CALIBRATION OF INDUCTANCE COILS

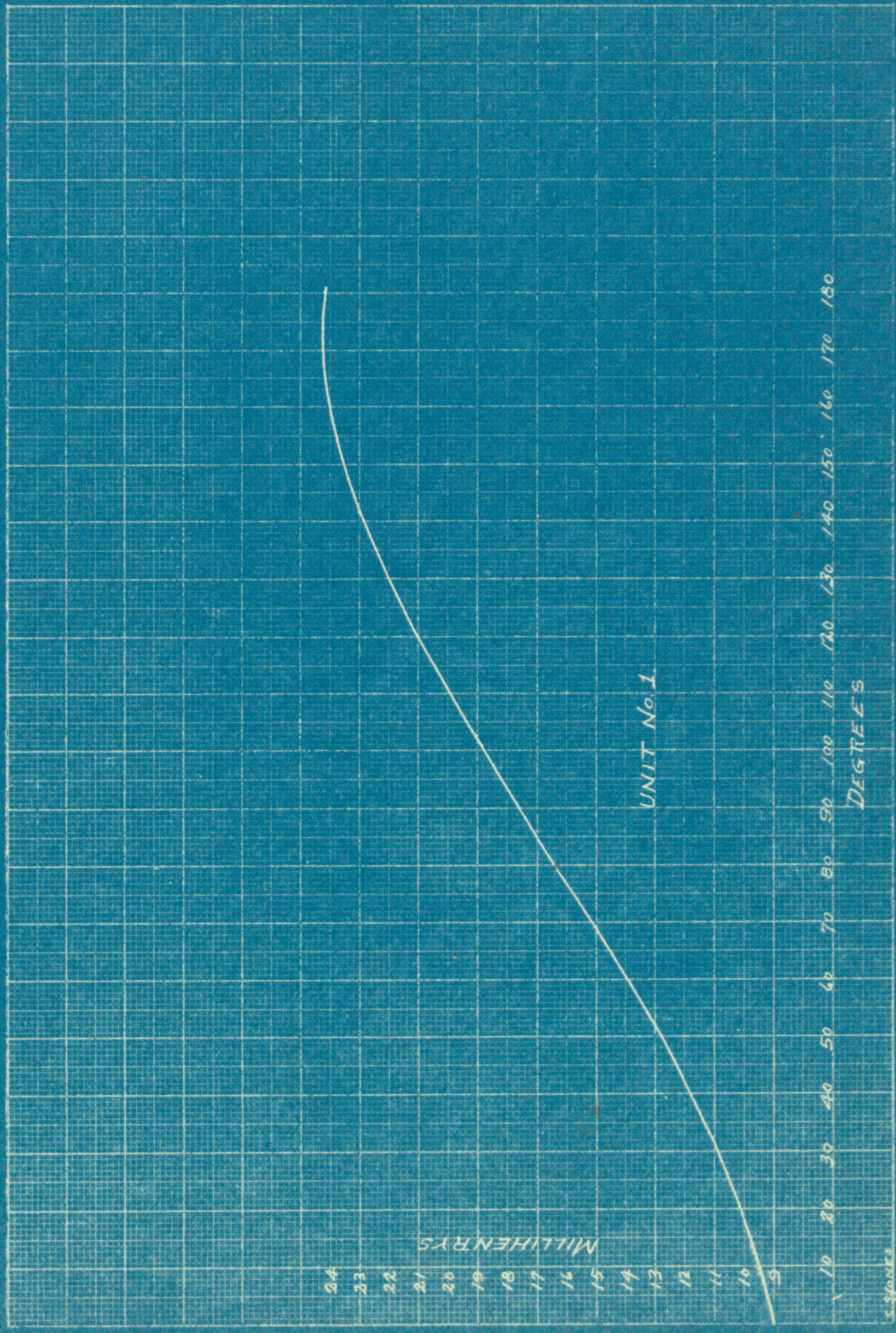
Unit #1		#2		#3	
Milli-henrys	Degrees	Milli-henrys	Degrees	Milli-henrys	Degrees
9	099.0	9.2	00.0	9.4	00.0
10	18.0	10	20.5	10	16.2
11	32.0	11	35	11	29.6
13	52.0	13	55	13	48.5
15	69.0	15	72	15	66
17	86.0	17	89	17	82.6
19	102.5	19	106.5	19	99.0
21	120.0	21	124	21	116
23	141.3	23	148	23	138
24	159.0	23.8	174	24	161
24.2	180.0			24.1	167
				24.8	180

#4		#5		#6	
Milli-henrys	Degrees	Milli-henrys	Degrees	Milli-henrys	Degrees
9.1	00.0	9.2	00.0	9.2	00.0
10	25	10	36	10	19.5
11	38	11	50	11	33
13	57.7	13	69	13	53
15	74	15	86	15	69.5
17	92	17	103	17	86
19	109	19	120	19	103
21	127	21	138	21	120
23	150	23	161	23	141
24	175	24	180	24	157.5
24.9	180			24.2	180

CALIBRATION OF INDUCTANCE COILS

#7		#8		#9	
Milli-henrys	Degrees	Milli-henrys	Degrees	Milli-henrys	Degrees
9	00.0	9.1	00.0	9.1	00.0
8.9	7.5	10	32	9	9
10	38	11	45.5	10	33.5
11	50	13	65	11	51.5
13	69	15	83	13	71
15	85	17	99.5	15	88
17	102	19	117	17	105
19	118.5	21	135	19	121
21	136.5	23	153	21	139
23	159.5	23.8	180	23	162
23.8	180			23.9	180
23.9	189			24	189

#10		#11		#12	
Milli-henrys	Degrees	Milli-henrys	Degrees	Milli-henrys	Degrees
9.2	00.0	9	00.0	9.2	00.0
10	32	10	32	10	23
11	46	11	45.5	11	36
13	66	13	65	13	56
15	83	15	83	15	73
17	99	17	99	17	90
19	116	19	116	19	107
21	134	21	134.5	21	125
23	156	23	159	23	149
24	176	23.7	180	23.7	180
24.1	180				

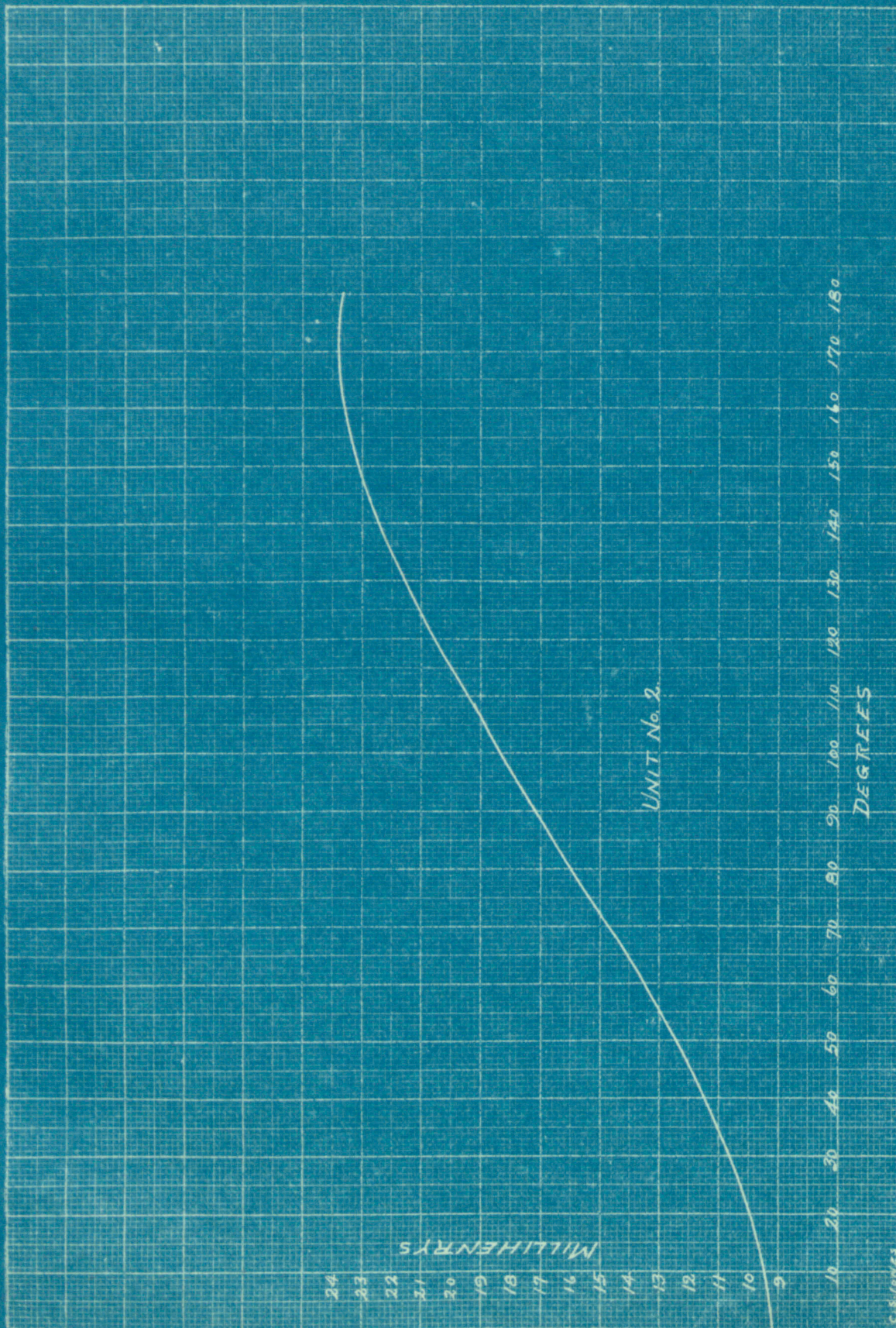


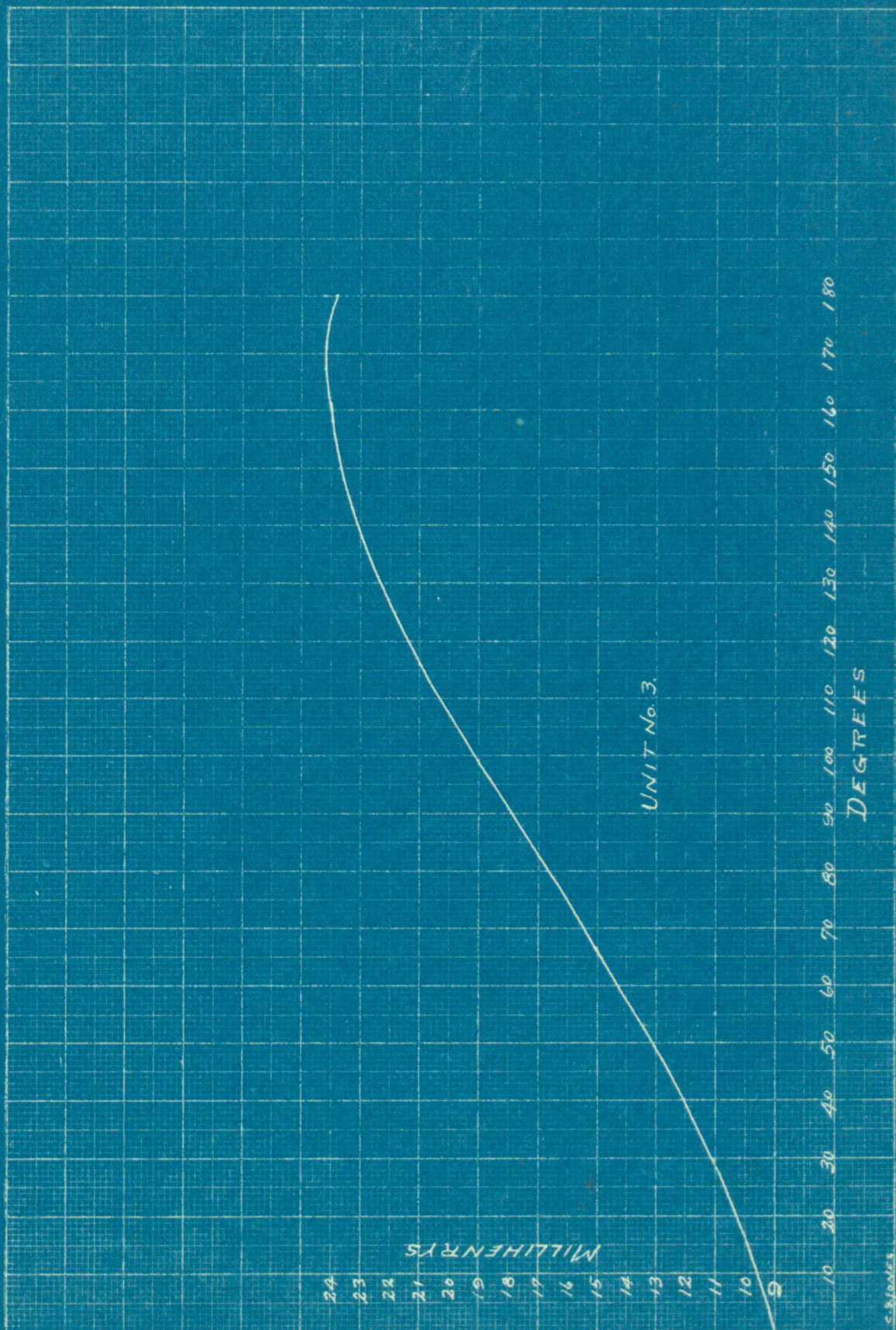
UNIT No. 1

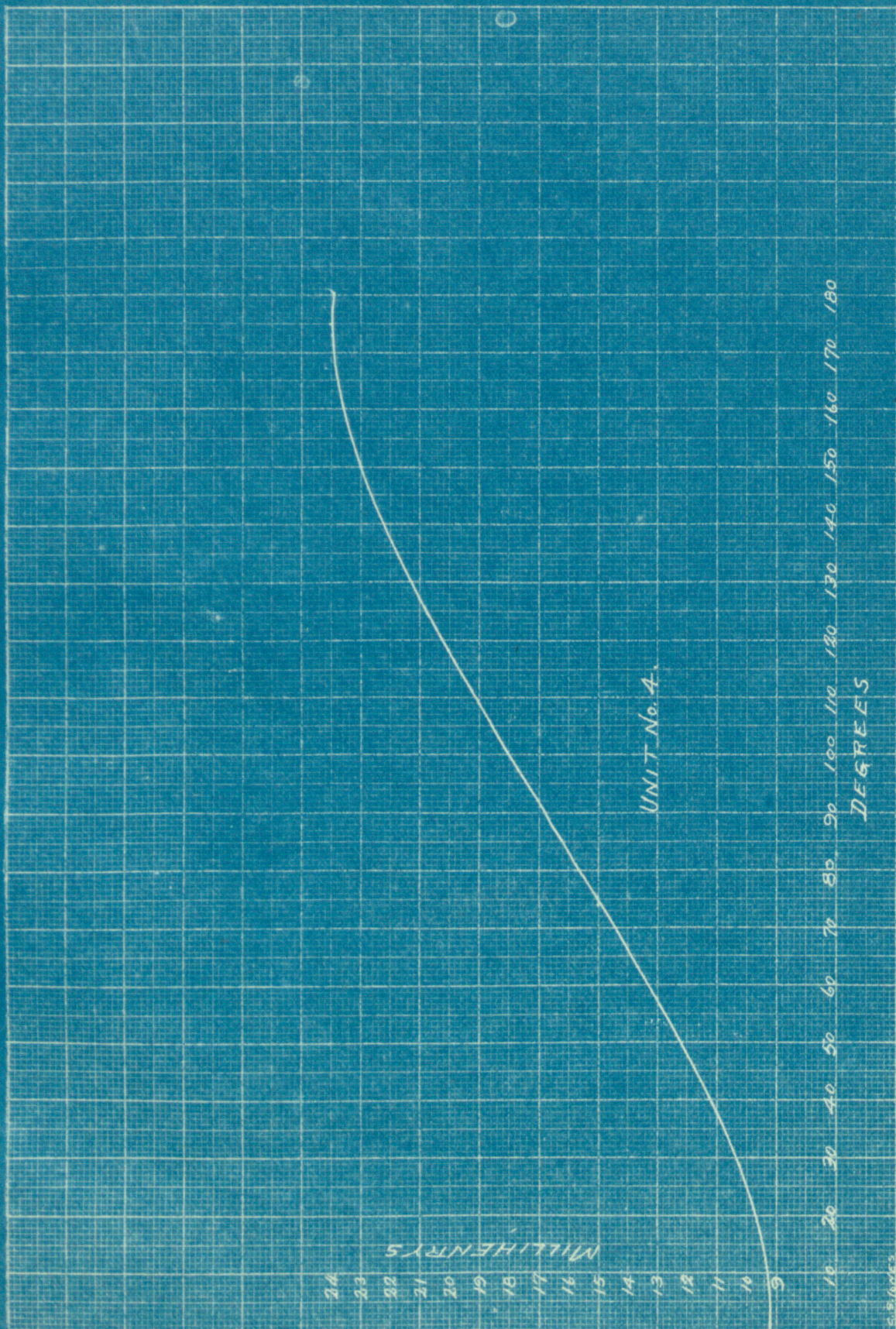
DEGREES

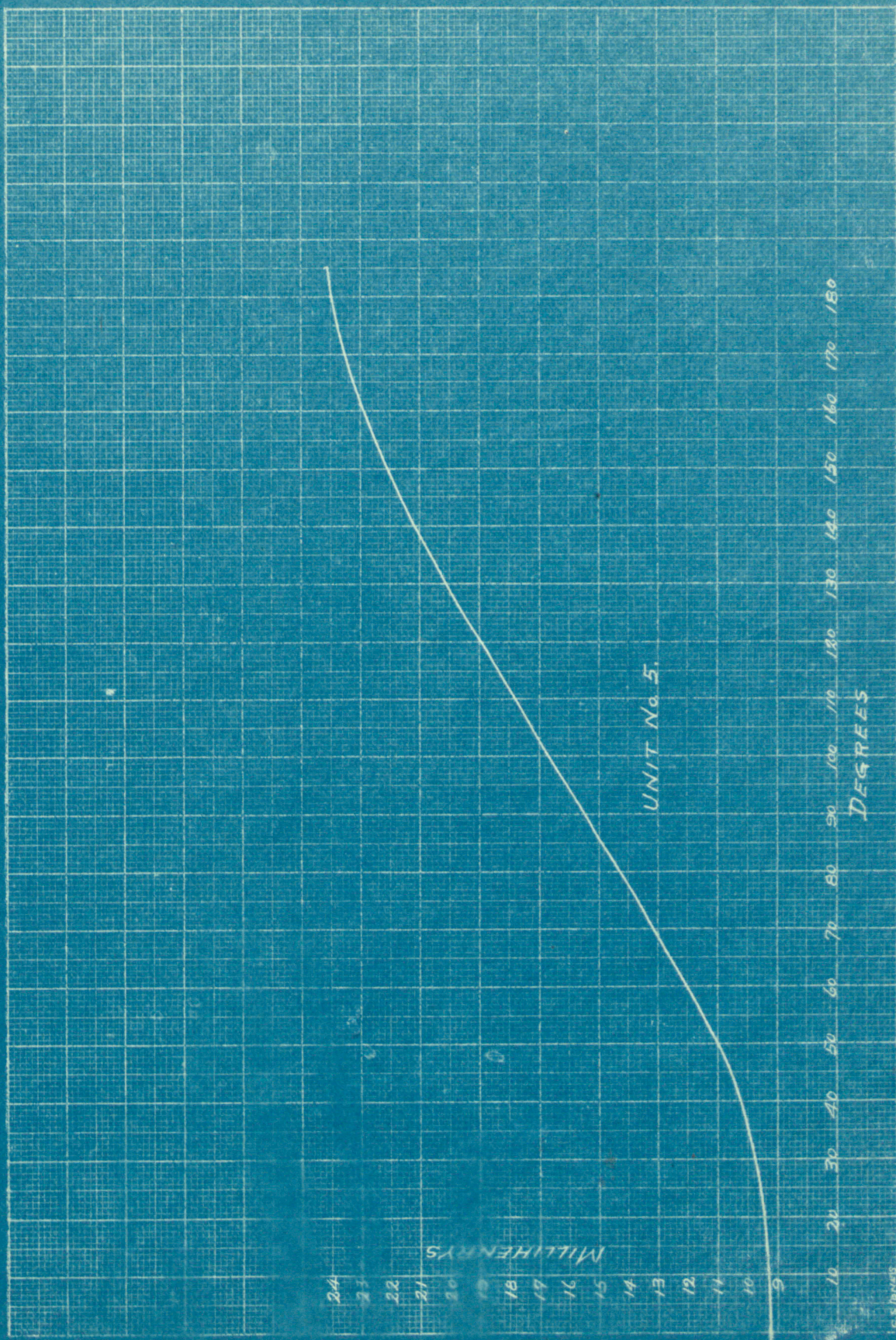
MILLIHENRYS

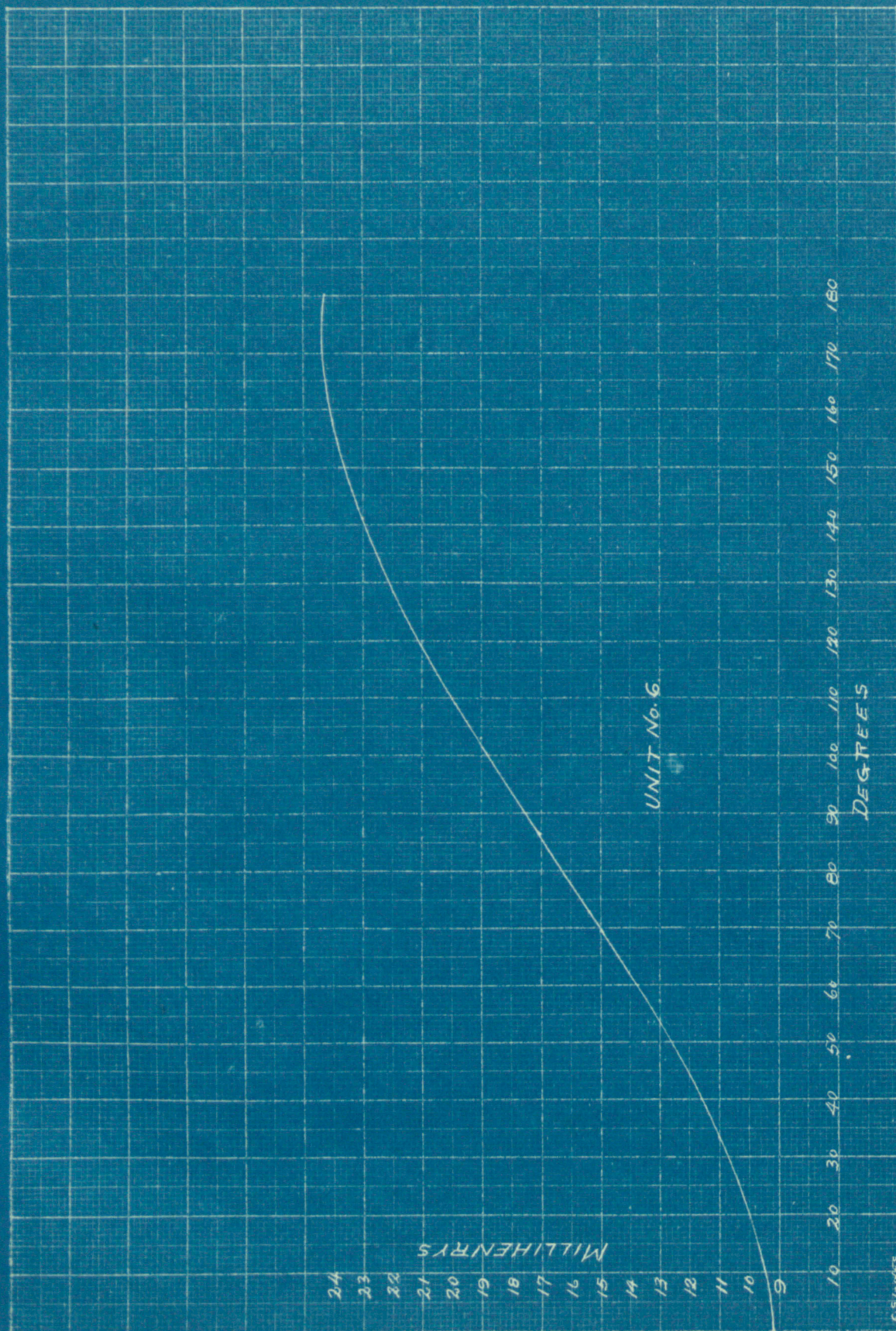
6-1-17

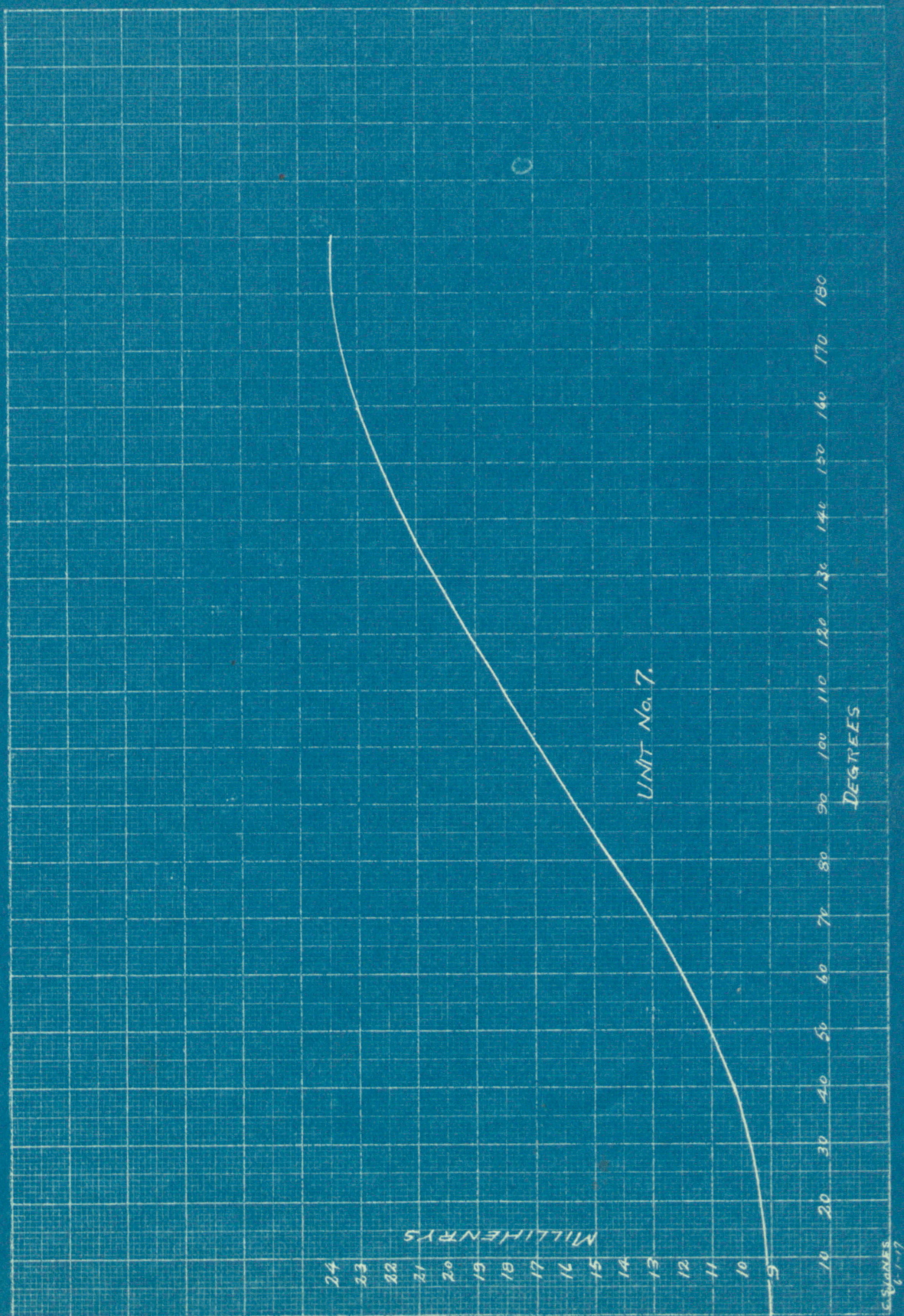


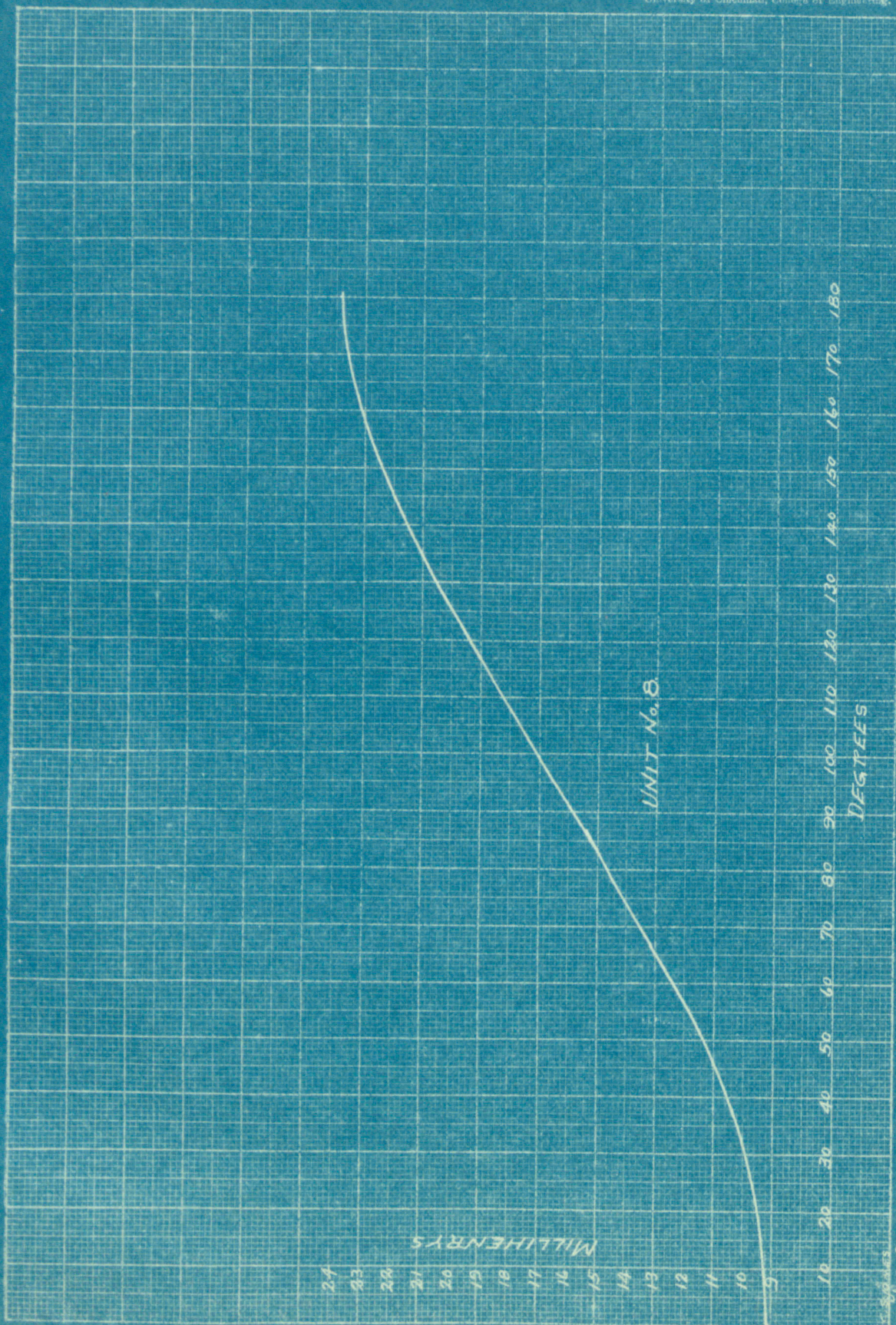


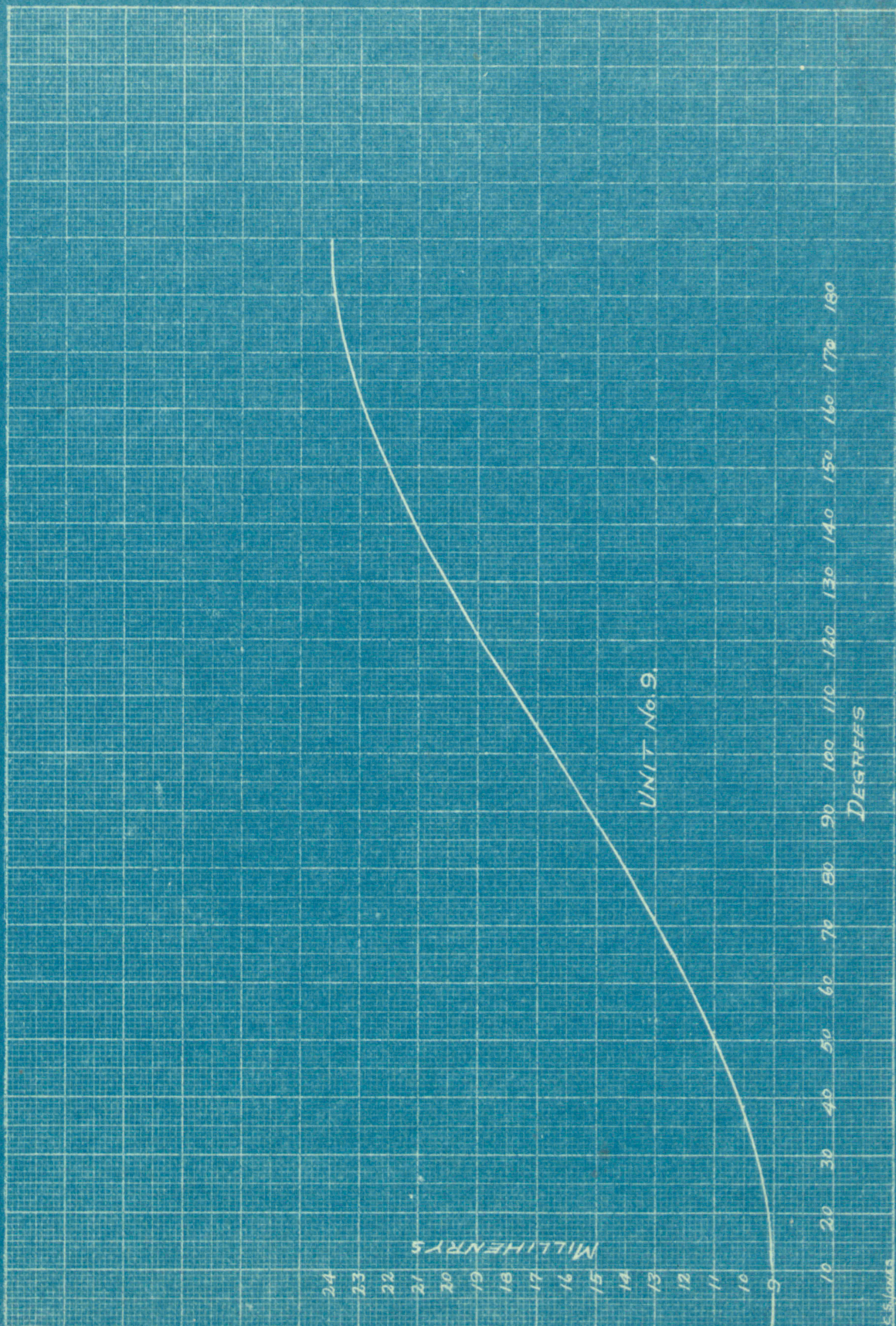


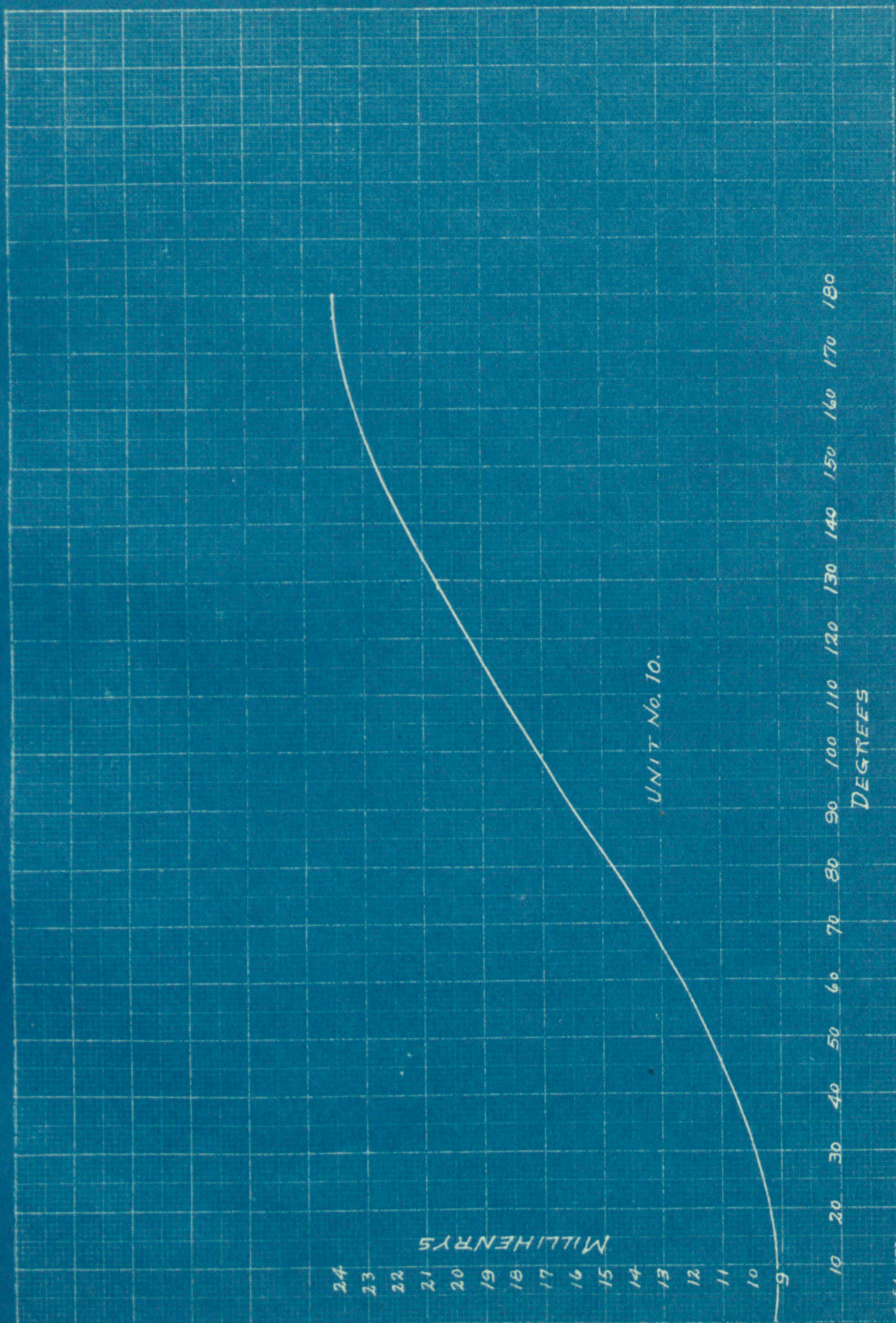


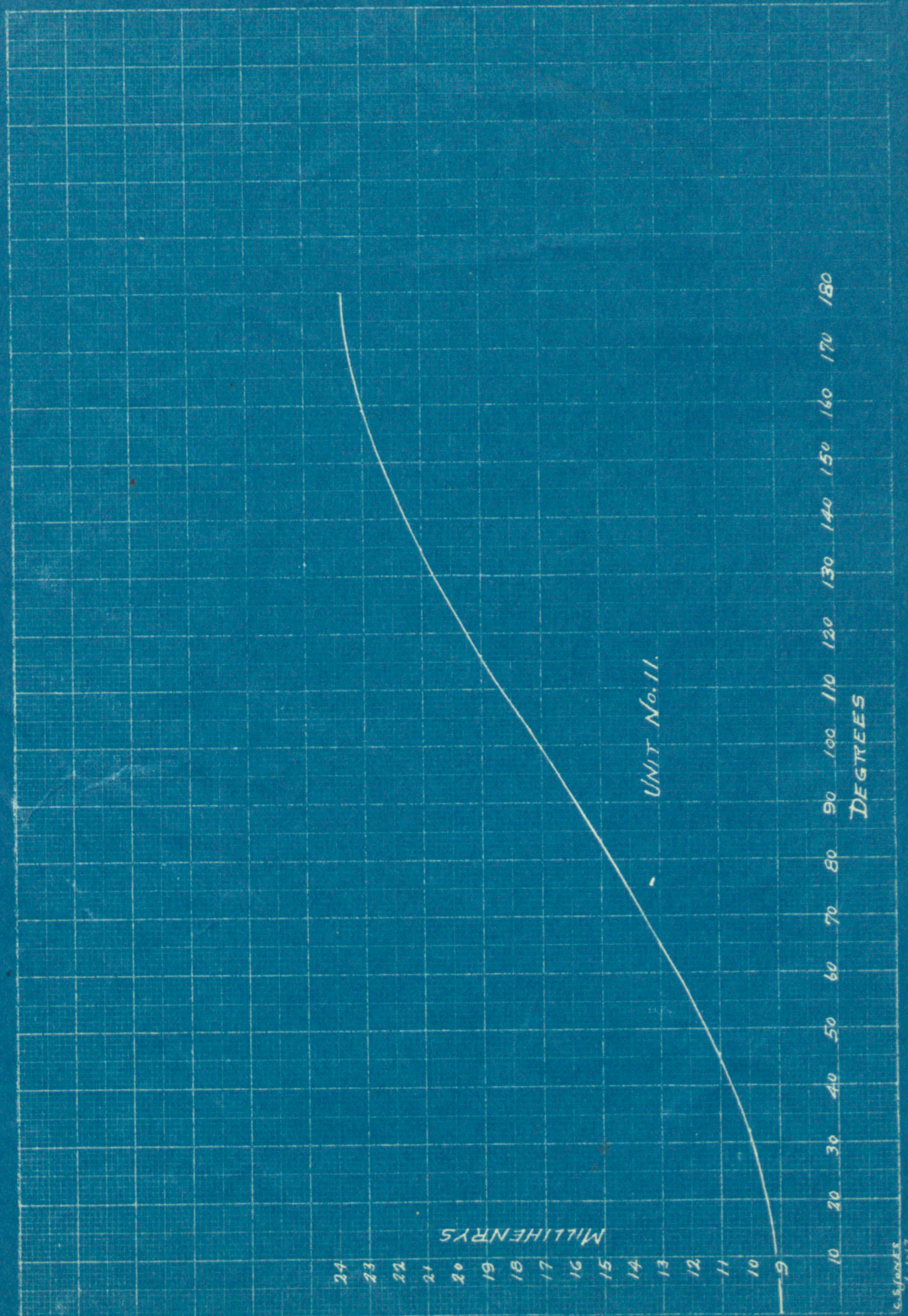


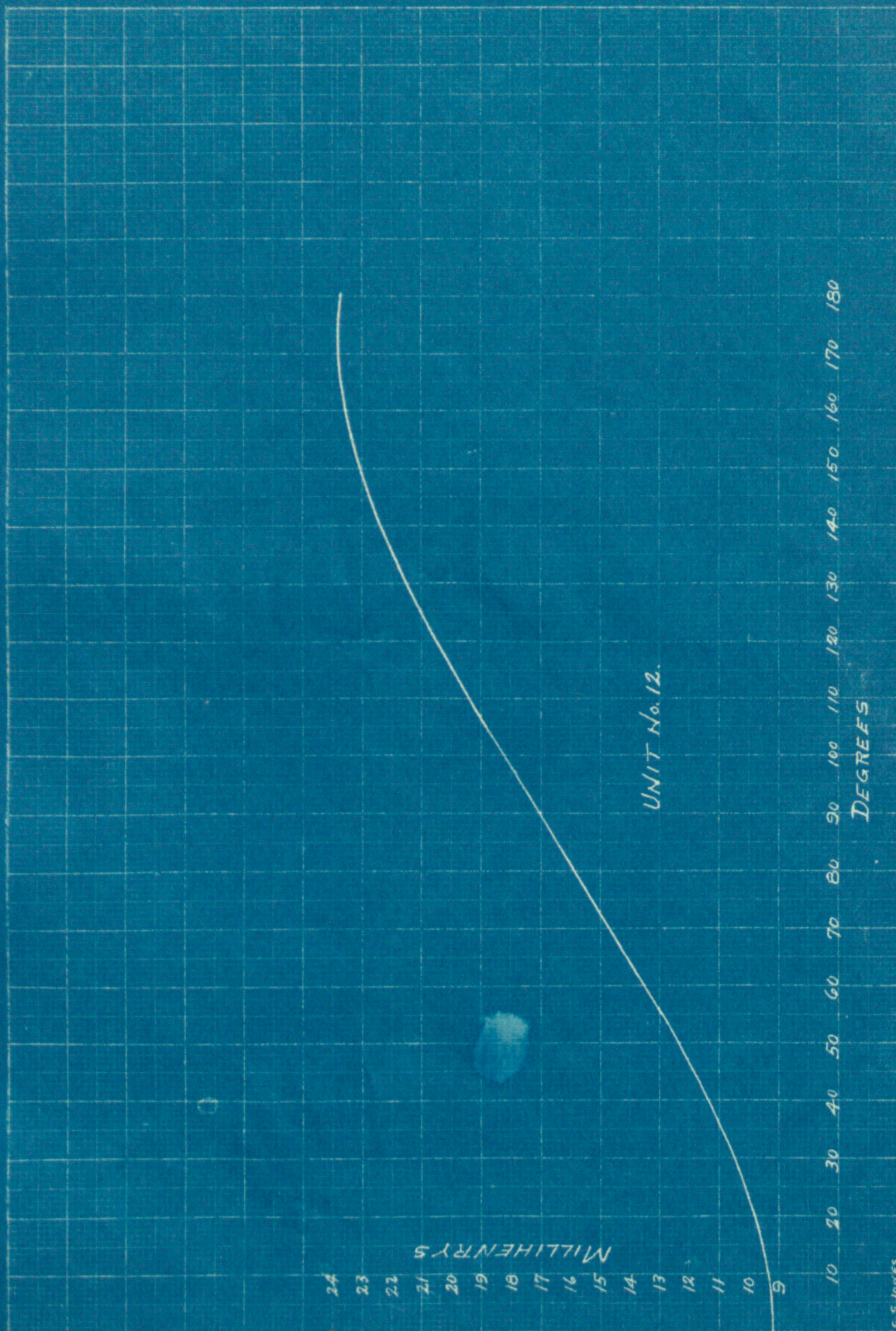












The condensers are clamped with bolts ^{between} half inch double boards. The boards are hard maple and oak veneered. Leads from the condenser plates are connected to brass terminals upon which a brass plate may be moved, by which the capacity may be varied. Refer to pictures shown on page number 26. Calibrations of condensers are given on pages 29 to 40 inclusive.

Connection diagrams for calibrating the condensers and inductance coils are shown on page 28. The Inductance is calibrated by balancing the resistance of the circuit with a D.C. galvanometer, then switching over to alternating current and balancing the reactance with a standard variable inductance, using a vibration galvanometer. The condensers are calibrated by the ballistic galvanometer method.

The line resistance is obtained by the use of Nicrome resistance wire, which has a resistance of 446 ohms per 1000 feet. Approximately ^{40 FT.} are used in each ten mile unit.

The condenser curves below are not correct, due to the fact that temperature and humidity were not taken into consideration. Since these curves were made the condensers have been impregnated.

Design of Line Condenser.

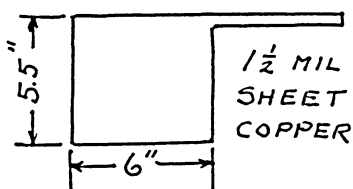
To determine the dielectric constant of the insulating material to be used in the condensers, a condenser of 16 sections, 9 x 10 copper plates, with 10 mil Formica insulation sheets was used. The capacity was measured with a ballistic galvanometer and found to be 0.292 microfarads.

$$K = \frac{C \times t}{0.000225 \times n \times A}$$

C = capacity in M. F.
t = distance between plates in mils.
n = number of sections.
A = area in square inches.
0.000225 = a constant.

$$K = \frac{0.292 \times 10}{0.000225 \times 16 \times 90} = 9.02$$

Condenser for artificial line.

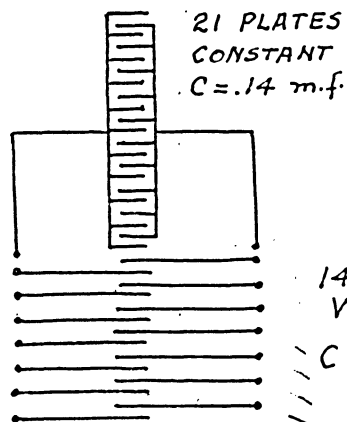


n = 35 copper plates.
t = 10 mil.
A = 6 x 5.5 = 33 sq. inches.
K = 9

$$C = \frac{0.000225 K n A}{t}$$

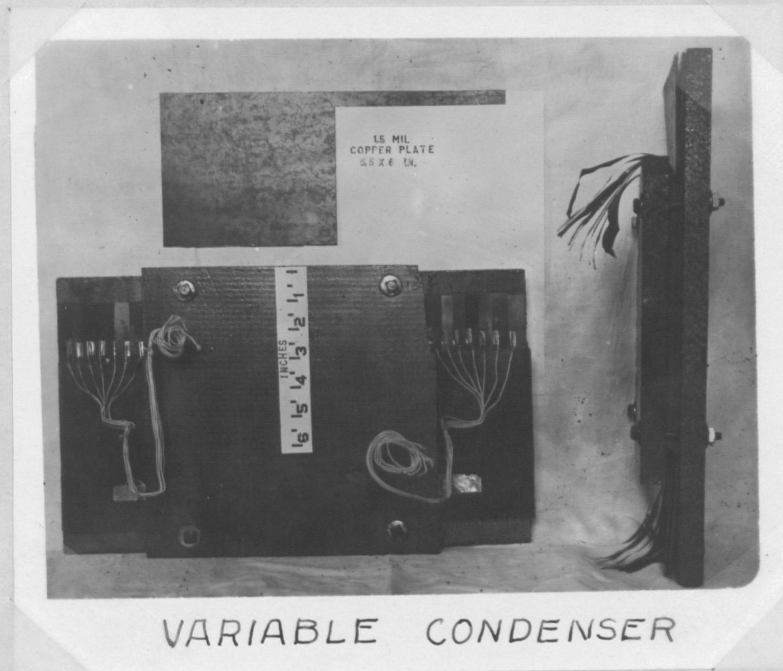
$$C = \frac{0.000225 \times 9 \times 35 \times 33}{10}$$

$$= 0.234 \text{ m.f.}$$

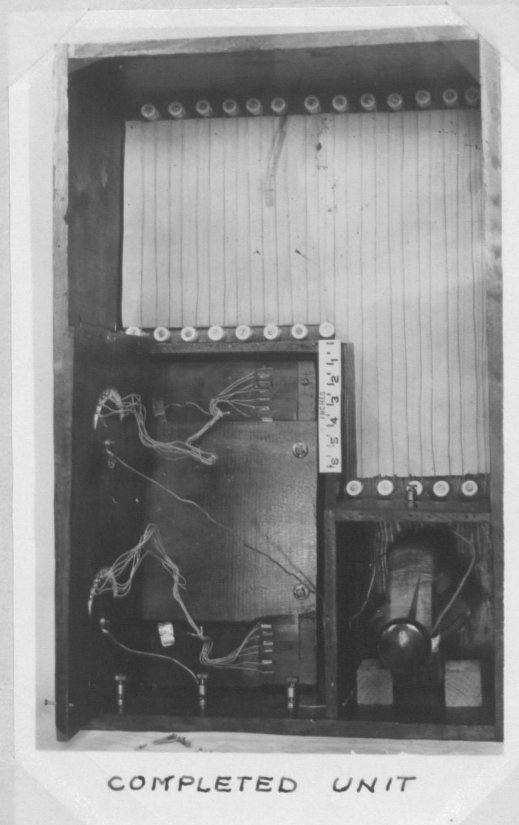


14 PLATES
VARIABLE

C = .0067 m.f. each.



VARIABLE CONDENSER
FOR ONE TEN-MILE UNIT.

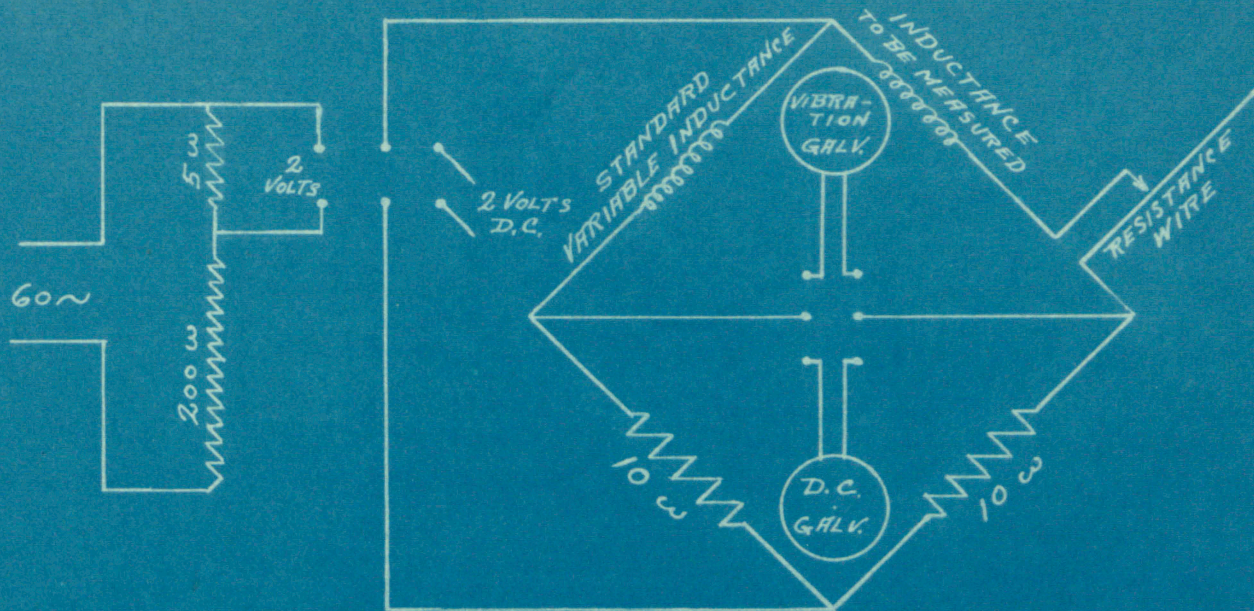


COMPLETED UNIT
REPRESENTING 10 MILES
OF ACTUAL LINE.

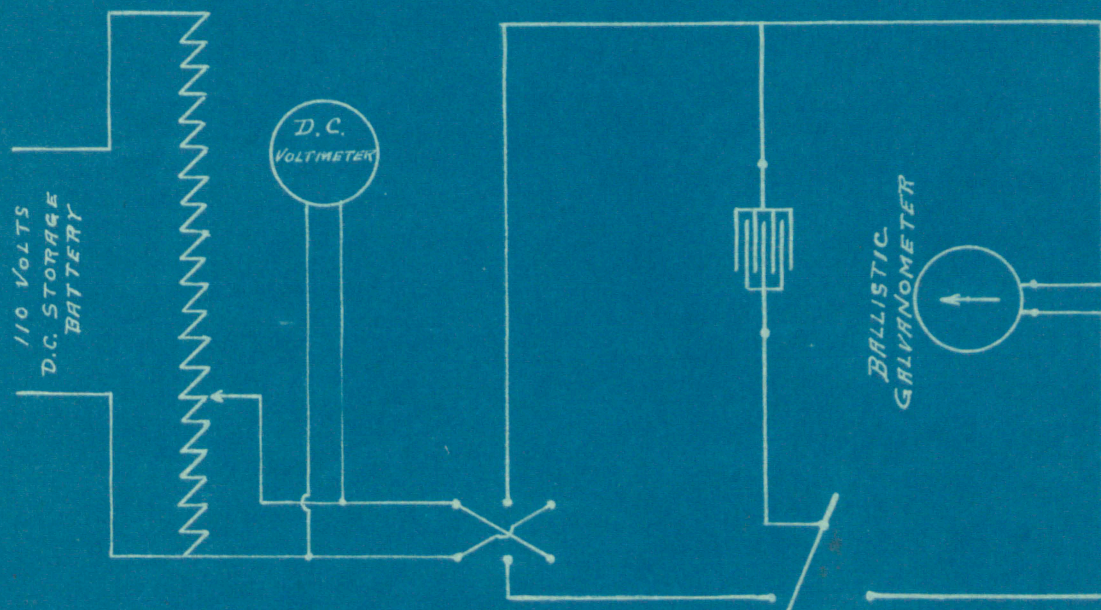
CAPACITY OF TRANSMISSION LINES

TEN-MILE SECTION TO NEUTRAL IN MICROFARADS

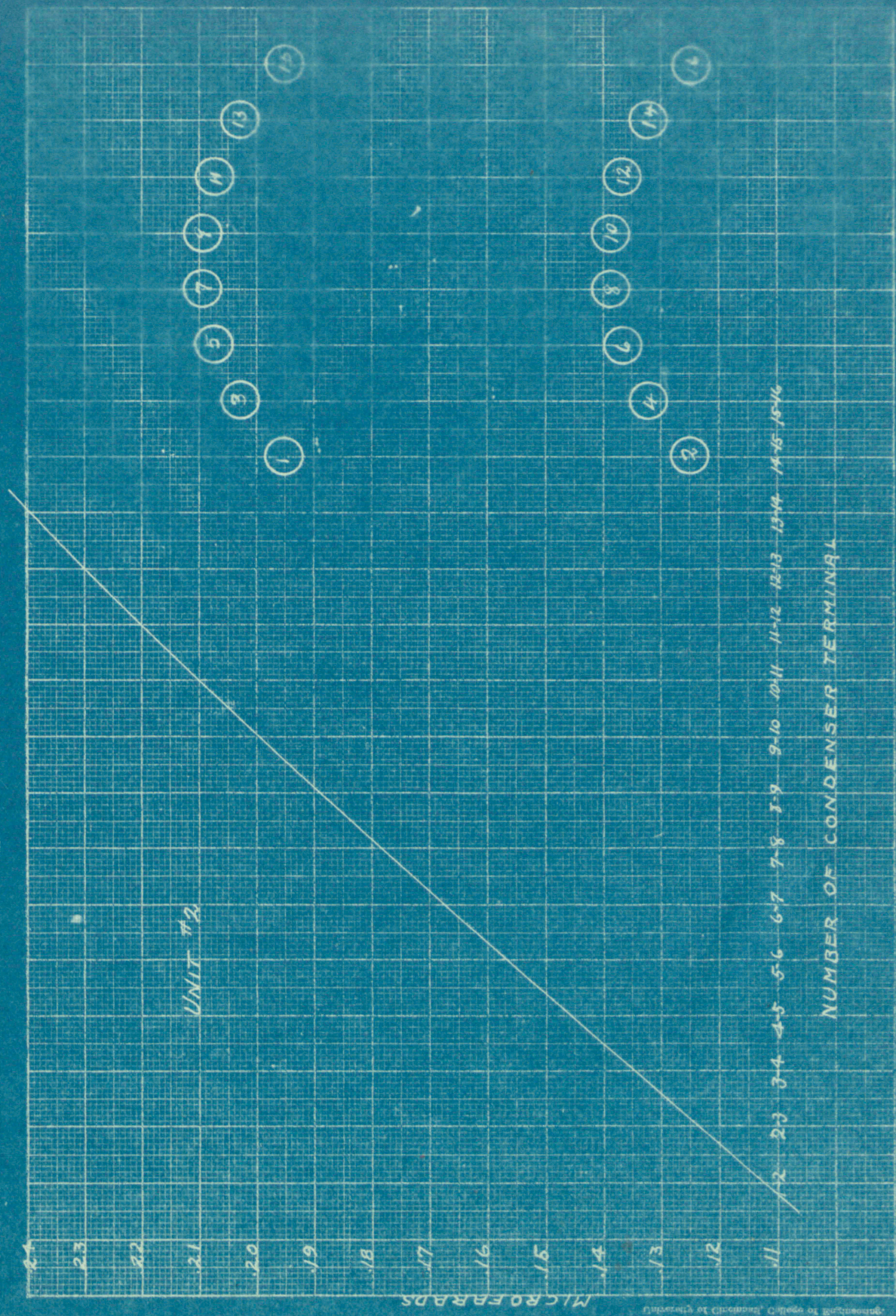
SPACING OF WIRES IN INS.	STRANDED 300,000 C.M.	STRANDED 250,000 C.M.	SOLID 0000	SOLID 000	SOLID 00	SOLID 0	SOLID 1	SOLID 2	SOLID 3	SOLID 4	SOLID 5	SOLID 6
12	.247	.27	.226	.220	.214	.208	.202	.	.192	.187	.183	.179
18	.222	.217	.205	.200	.195	.190	.183	.181	.177	.173	.169	.165
24	.207	.203	.192	.188	.183	.179	.175	.171	.167	.164	.160	.157
36	.1885	.185	.177	.173	.169	.165	.162	.159	.155	.152	.150	.147
48	.178	.175	.167	.164	.1605	.157	.154	.151	.148	.145	.143	.140
60	.170	.167	.161	.157	.154	.151	.148	.146	.143	.1405	.138	.135
72	.164	.162	.1555	.1525	.1495	.147	.144	.141	.139	.136	.134	.132
84	.159	.157	.151	.1485	.146	.143	.1405	.138	.135	.133	.131	.1285
96	.156	.154	.148	.145	.143	.140	.1378	.135	.1328	.1303	.1285	.1261
108	.153	.151	.1455	.1428	.140	.1375	.135	.1326	.1305	.1283	.1262	.124
120	.151	.1482	.143	.1405	.1375	.135	.133	.131	.1285	.1265	.1245	.1225
132	.148	.146	.141	.138	.1356	.1333	.1312	.129	.127	.1246	.123	.121
144	.146	.1438	.1386	.1362	.134	.1318	.1295	.1272	.125	.1233	.1215	.1193
156	.1435	.1417	.137	.135	.1325	.130	.128	.126	.124	.122	.120	.1182
168	.142	.1405	.1353	.133	.131	.129	.127	.1245	.1222	.1205	.119	.117
180	.141	.139	.1341	.1317	.1298	.1275	.1255	.1232	.1215	.1196	.1179	

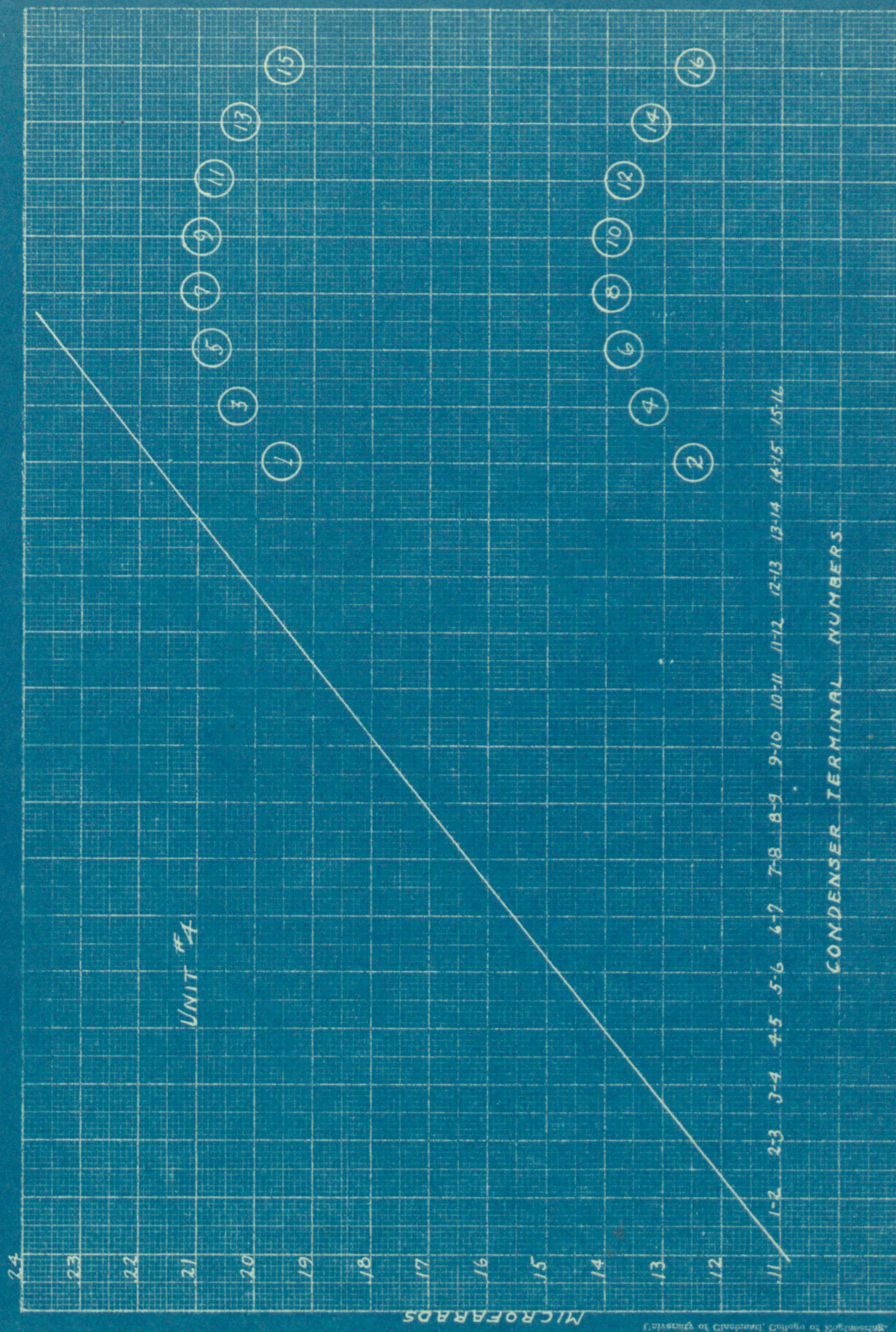


CONNECTION DIAGRAM FOR CALIBRATING INDUCTANCE COILS.

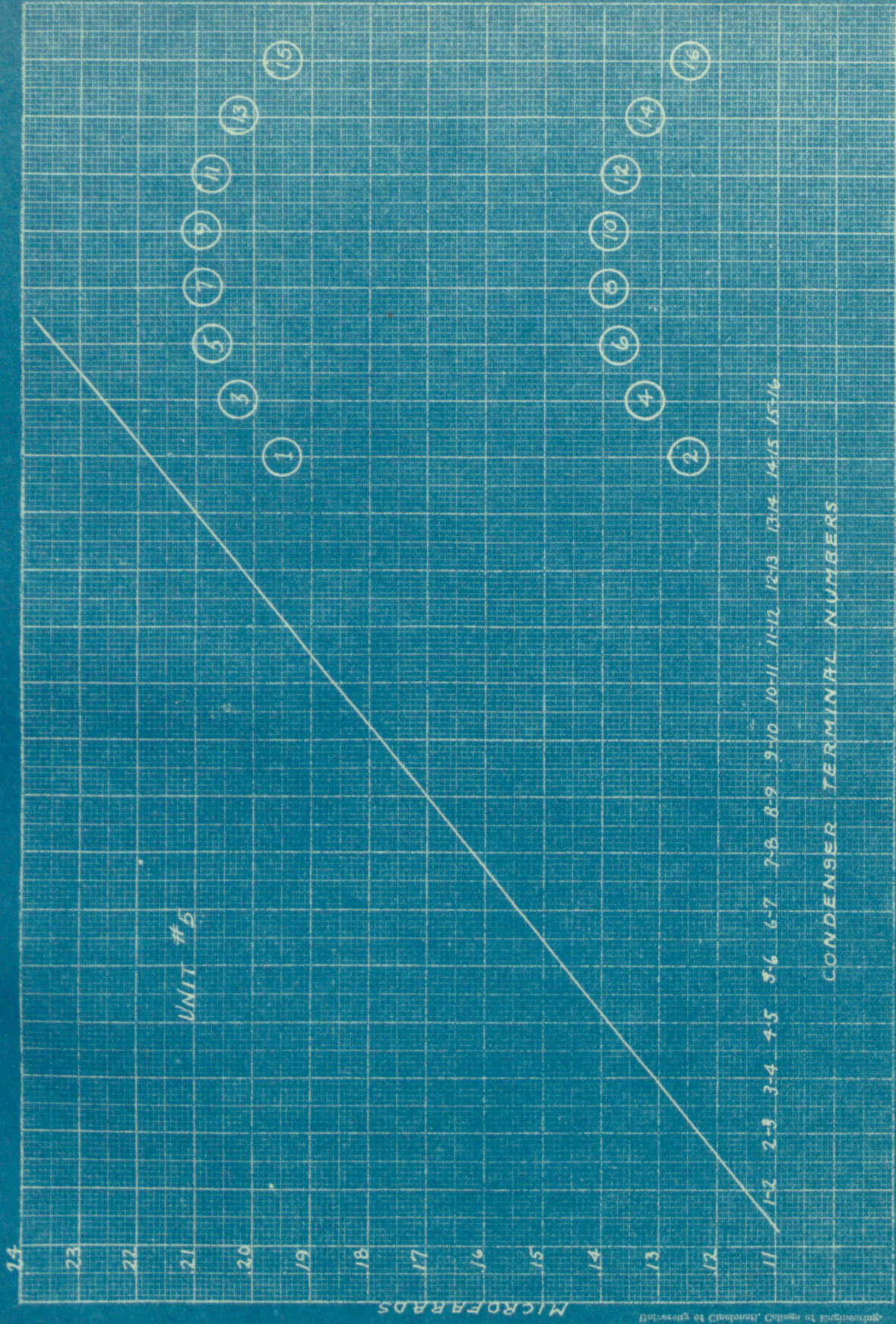


CONNECTION DIAGRAM FOR CALIBRATING CONDENSERS.

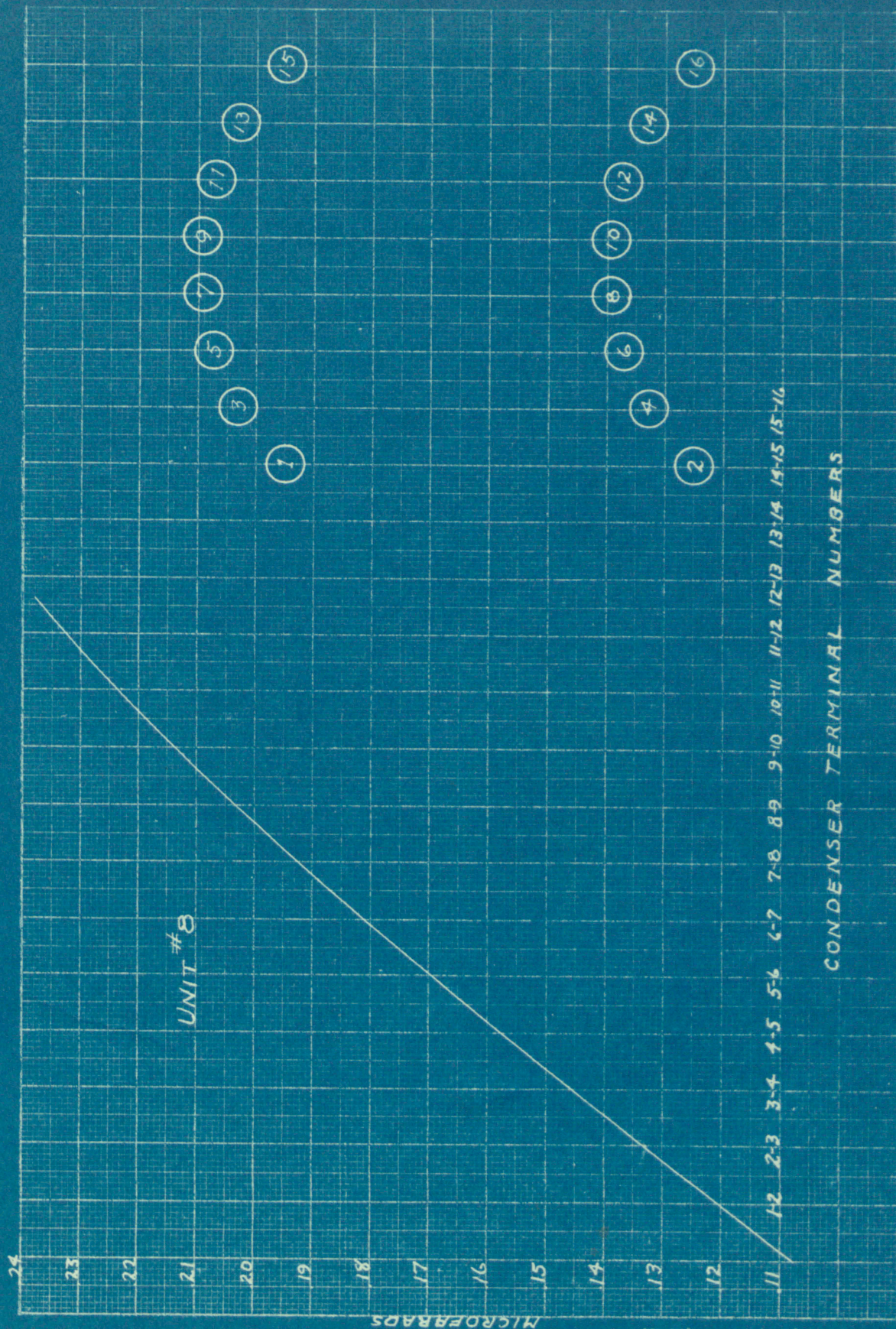


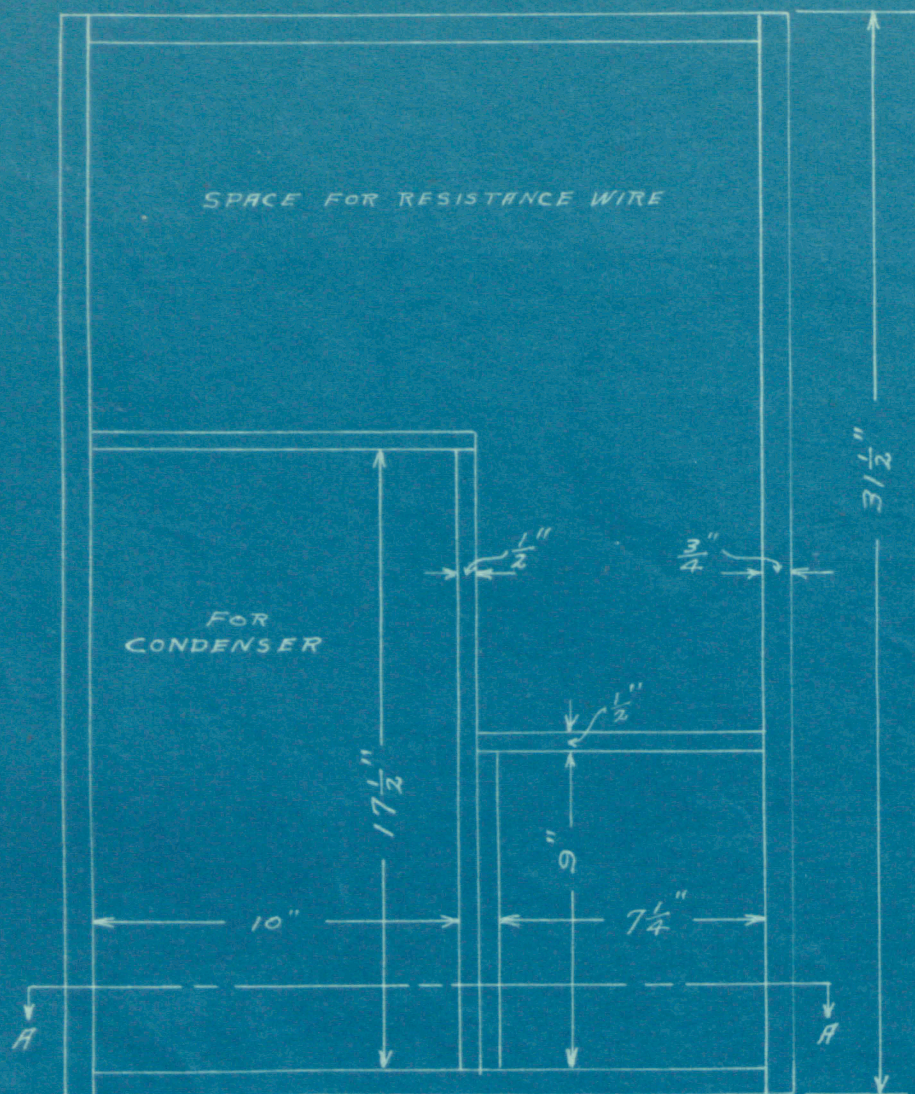


Approved for Release by NSA on 05-08-2014 pursuant to E.O. 13526

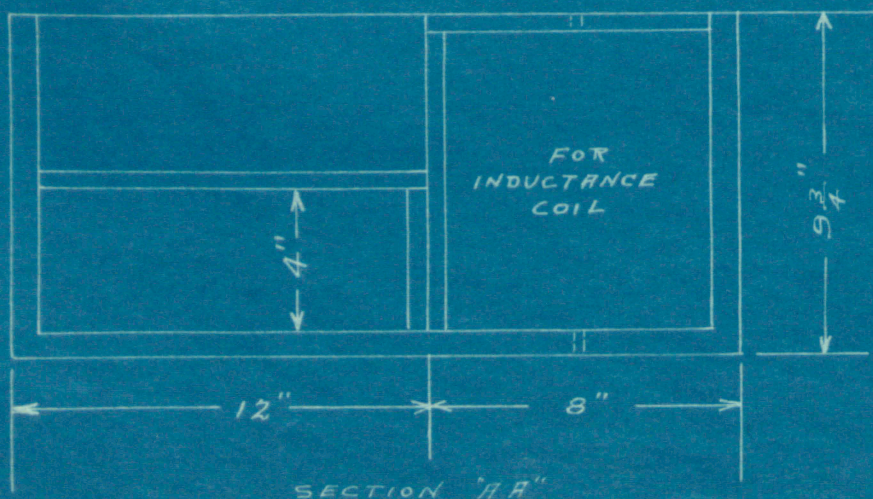


Manufactured by the General Electric Company, Schenectady, N.Y.





WOODEN BOX FOR ONE TEN MILE UNIT



SCALE $\frac{3}{16}$ " = 1"
C. C. JONES 6-2-17



SHOWING COMPLETE ARTIFICIAL
TRANSMISSION LINE

ARTIFICIAL TRANSMISSION LINE
 TWELVE UNITS * TEN MILES PER UNIT. MATERIAL
 AND ACTUAL COST OF SAME.

Sheet Copper, 10 mil 3-1/4 lbs at 60¢	\$ 1.95
Sheet Copper, 1-1/2 mil x 6" wide 7 lbs @ \$1	7.00
Copper Wire #14 DCC 115.75 lbs @ 52¢	60.47
Hard Maple for wooden spools 6"x 6" x 6'	6.50
Formica 10 mil, 190 sheets @ 9.5¢ per sheet	18.05
Formica 10 mil, 260 sheets @ 7.25¢ per sheet	18.85
Formica Cylinder 5-7/32" diam. 1/16" thick x 3 ft	gratis
Fiber Tubing 1/4" x 2-1/2'	.52
Rolled Brass 5/8" x 4' 4.5 lbs @ 47¢ for terminals	2.12
Brass Rod 1/4" x 11'	.95
Brass Nuts 24	.46
Spring Brass for condensers 12" x 1.5'	.76
Wooden Knobs for condensers 24	.65
Brass headed tacks for condensers 250	.57
Knobs, handles for variable inductance 24	1.55
Wood Screws 2 doz #2 3/8"	.05
Wood Screws 23 doz #6 1-3/4"	1.00
Bolts 1/4" x 2-1/2" & nuts 4 doz	.50
Insulators, porcelain 1/2" 300	3.00
Nichrome Resistance Wire 40 mil 446 ohms/1000', 2#	6.42
Wooden Boxes - 12 Units @ \$2.00	24.00
Total .	\$155.37

Cost Per Unit \$ 12.95

HYPERBOLIC SINES

	0.1		0.2		0.3		0.4		0.5	
60°	0.099917	60.082	0.19934	60.331	0.29777	60.746	0.39474	61.330	0.48978	62.085
61°	0.099912	61.081	0.19929	61.324	0.29764	61.731	0.39441	62.302	0.48916	63.042
62°	0.099907	62.079	0.19925	62.317	0.29751	62.715	0.39409	63.273	0.48855	63.998
63°	0.099902	63.077	0.19921	63.309	0.29738	63.698	0.39379	64.243	0.48795	64.950
64°	0.099897	64.075	0.19918	64.301	0.29725	64.680	0.39349	65.212	0.48738	65.900
65°	0.099893	65.073	0.19914	65.293	0.29712	65.661	0.39320	66.179	0.48681	66.848
66°	0.099889	66.071	0.19911	66.284	0.29700	66.641	0.39293	67.144	0.48627	67.794
67°	0.099885	67.069	0.19908	67.275	0.29689	67.621	0.39266	68.108	0.48575	68.738
68°	0.099881	68.066	0.19904	68.265	0.29678	68.599	0.39240	69.070	0.48523	69.679
69°	0.099877	69.064	0.19901	69.255	0.29667	69.577	0.39214	70.030	0.48473	70.617
70°	0.099873	70.062	0.19898	70.245	0.29657	70.555	0.39190	70.990	0.48426	71.554
71°	0.099869	71.059	0.19895	71.235	0.29647	71.532	0.39167	71.948	0.48380	72.489
72°	0.099865	72.057	0.19892	72.225	0.29637	72.508	0.39145	72.905	0.48338	73.422
73°	0.099861	73.054	0.19889	73.214	0.29628	73.483	0.39123	73.861	0.48296	74.354
74°	0.099858	74.051	0.19887	74.203	0.29620	74.458	0.39104	74.817	0.48257	75.284
75°	0.099855	75.048	0.19885	75.192	0.29612	75.432	0.39084	75.771	0.48220	76.212
76°	0.099852	76.045	0.19883	76.180	0.29604	76.406	0.39066	76.724	0.48184	77.138
77°	0.099850	77.042	0.19881	77.168	0.29596	77.379	0.39050	77.676	0.48152	78.062
78°	0.099847	78.039	0.19878	78.156	0.29590	78.351	0.39034	78.628	0.48123	78.986
79°	0.099845	79.036	0.19876	79.144	0.29584	79.322	0.39018	79.578	0.48095	79.909
80°	0.099843	80.033	0.19875	80.131	0.29578	80.294	0.39004	80.528	0.48067	80.830
81°	0.099841	81.030	0.19873	81.118	0.29573	81.266	0.38993	81.477	0.48044	81.750
82°	0.099839	82.026	0.19872	82.106	0.29569	82.238	0.38983	82.425	0.48023	82.669
83°	0.099838	83.023	0.19871	83.093	0.29566	83.209	0.38973	83.373	0.48004	83.587
84°	0.099837	84.020	0.19870	84.080	0.29563	84.180	0.38965	84.321	0.47987	84.505
85°	0.099836	85.017	0.19869	85.067	0.29561	85.150	0.38958	85.268	0.47972	85.422
86°	0.099835	86.014	0.19868	86.054	0.29559	86.120	0.38952	86.215	0.47960	86.338
87°	0.099834	87.011	0.19868	87.041	0.29557	87.090	0.38948	87.162	0.47952	87.254
88°	0.099833	88.008	0.19867	88.028	0.29555	88.060	0.38946	88.108	0.47947	88.170
89°	0.099832	89.004	0.19867	89.014	0.29553	89.030	0.38944	89.054	0.47945	89.085
90°	0.099831	90.000	0.19867	90.000	0.29552	90.000	0.38942	90.000	0.47943	90.000

Example of the use of Table: $\sinh(0.3/75^\circ) = 0.29612/75^\circ 432 = 0.29612/75^\circ 25'55''$

Note. — $\sinh p/\delta = 0/\delta$ when $p = 0$

HYPERBOLIC COSINES

	.0.1		0.2		0.3		0.4		0.5	
60°	0.99751	0.249	0.99012	0.999	0.97810	2.267	0.96191	4.075	0.94219	6.460
61°	0.99736	0.244	0.98952	0.979	0.97674	2.221	0.95948	3.998	0.93838	6.343
62°	0.99721	0.238	0.98893	0.957	0.97541	2.173	0.95711	3.914	0.93465	6.217
63°	0.99707	0.232	0.98835	0.934	0.97411	2.123	0.95478	3.826	0.93099	6.083
64°	0.99693	0.226	0.98780	0.910	0.97285	2.070	0.95251	3.733	0.92741	5.941
65°	0.99679	0.220	0.98725	0.885	0.97163	2.014	0.95031	3.635	0.92393	5.789
66°	0.99666	0.214	0.98672	0.859	0.97044	1.955	0.94816	3.532	0.92054	5.631
67°	0.99653	0.207	0.98621	0.832	0.96927	1.894	0.94608	3.423	0.91725	5.463
68°	0.99641	0.200	0.98571	0.804	0.96814	1.830	0.94407	3.310	0.91407	5.288
69°	0.99629	0.193	0.98523	0.775	0.96706	1.764	0.94213	3.193	0.91100	5.106
70°	0.99617	0.185	0.98477	0.744	0.96601	1.696	0.94026	3.072	0.90805	4.916
71°	0.99606	0.177	0.98433	0.713	0.96500	1.626	0.93846	2.946	0.90521	4.718
72°	0.99596	0.169	0.98391	0.681	0.96404	1.553	0.93674	2.816	0.90248	4.513
73°	0.99586	0.161	0.98350	0.648	0.96313	1.479	0.93510	2.682	0.89989	4.302
74°	0.99576	0.153	0.98312	0.614	0.96227	1.402	0.93354	2.544	0.89742	4.085
75°	0.99567	0.144	0.98276	0.580	0.96145	1.324	0.93207	2.403	0.89508	3.861
76°	0.99559	0.135	0.98242	0.545	0.96068	1.244	0.93069	2.258	0.89288	3.631
77°	0.99551	0.126	0.98210	0.509	0.95996	1.162	0.92938	2.111	0.89081	3.396
78°	0.99544	0.117	0.98181	0.472	0.95929	1.078	0.92816	1.960	0.88889	3.155
79°	0.99537	0.108	0.98154	0.435	0.95866	0.993	0.92705	1.807	0.88711	2.910
80°	0.99531	0.099	0.98128	0.397	0.95808	0.908	0.92603	1.652	0.88548	2.660
81°	0.99525	0.090	0.98105	0.359	0.95756	0.821	0.92509	1.494	0.88399	2.406
82°	0.99520	0.080	0.98085	0.320	0.95710	0.732	0.92425	1.333	0.88266	2.149
83°	0.99515	0.070	0.98067	0.281	0.95670	0.643	0.92351	1.170	0.88147	1.888
84°	0.99511	0.060	0.98051	0.242	0.95635	0.552	0.92287	1.006	0.88044	1.624
85°	0.99508	0.050	0.98037	0.203	0.95605	0.461	0.92233	0.841	0.87957	1.357
86°	0.99506	0.040	0.98026	0.163	0.95580	0.369	0.92189	0.675	0.87886	1.087
87°	0.99504	0.030	0.98018	0.123	0.95560	0.277	0.92154	0.508	0.87830	0.816
88°	0.99502	0.020	0.98012	0.082	0.95545	0.185	0.92128	0.340	0.87790	0.544
89°	0.99501	0.010	0.98009	0.041	0.95537	0.093	0.92112	0.171	0.87766	0.272
90°	0.99500	0.000	0.98007	0.000	0.95534	0.000	0.92106	0.000	0.87758	0.000

Example of the use of Table: $\cosh 0.5/81^\circ = 0.88399/2.24' 22''$

Note.— $\cosh p/s = 1.0/0^\circ$ when $p = 0$ whatever the value of s

HYPERBOLIC TANGENTS

	0.1		0.2		0.3		0.4		0.5	
60°	0.10017	59.833	0.20133	59.332	0.30444	58.479	0.41037	57.255	0.51983	55.625
61°	0.10018	60.837	0.20140	60.345	0.30473	59.510	0.41107	58.305	0.52128	56.700
62°	0.10019	61.841	0.20148	61.360	0.30501	60.542	0.41176	59.359	0.52271	57.781
63°	0.10020	62.845	0.20156	62.375	0.30528	61.575	0.41244	60.417	0.52412	58.867
64°	0.10020	63.849	0.20164	63.391	0.30555	62.610	0.41311	61.479	0.52552	59.959
65°	0.10021	64.853	0.20171	64.408	0.30580	63.647	0.41376	62.549	0.52689	61.058
66°	0.10022	65.857	0.20179	65.425	0.30605	64.686	0.41441	63.612	0.52824	62.164
67°	0.10023	66.862	0.20186	66.443	0.30630	65.727	0.41504	64.685	0.52957	63.276
68°	0.10024	67.866	0.20193	67.461	0.30655	66.769	0.41565	65.760	0.53085	64.391
69°	0.10025	68.871	0.20199	68.480	0.30678	67.813	0.41623	66.837	0.53209	65.511
70°	0.10026	69.877	0.20206	69.501	0.30701	68.859	0.41680	67.918	0.53330	66.638
71°	0.10026	70.882	0.20212	70.522	0.30722	69.906	0.41735	69.002	0.53446	67.771
72°	0.10027	71.888	0.20217	71.544	0.30742	70.955	0.41788	70.089	0.53560	68.909
73°	0.10028	72.893	0.20223	72.566	0.30762	72.005	0.41838	71.179	0.53669	70.052
74°	0.10028	73.898	0.20228	73.589	0.30781	73.056	0.41886	72.272	0.53773	71.200
75°	0.10029	74.904	0.20234	74.612	0.30799	74.108	0.41932	73.368	0.53872	72.351
76°	0.10029	75.910	0.20239	75.635	0.30816	75.162	0.41975	74.466	0.53965	73.507
77°	0.10030	76.916	0.20243	76.659	0.30831	76.217	0.42016	75.566	0.54054	74.667
78°	0.10030	77.922	0.20246	77.684	0.30846	77.273	0.42054	76.668	0.54138	75.831
79°	0.10031	78.928	0.20250	78.709	0.30860	78.329	0.42088	77.771	0.54215	76.999
80°	0.10031	79.934	0.20254	79.734	0.30872	79.386	0.42120	78.876	0.54285	78.170
81°	0.10032	80.940	0.20257	80.759	0.30884	80.445	0.42150	79.983	0.54349	79.344
82°	0.10032	81.946	0.20260	81.785	0.30894	81.506	0.42177	81.092	0.54407	80.520
83°	0.10032	82.953	0.20263	82.812	0.30904	82.567	0.42201	82.203	0.54459	81.699
84°	0.10033	83.960	0.20265	83.838	0.30912	83.628	0.42222	83.315	0.54503	82.881
85°	0.10033	84.967	0.20267	84.864	0.30920	84.689	0.42239	84.427	0.54540	84.065
86°	0.10033	85.974	0.20268	85.891	0.30926	85.751	0.42252	85.540	0.54571	85.251
87°	0.10033	86.981	0.20270	86.918	0.30930	86.813	0.42263	86.654	0.54596	86.438
88°	0.10033	87.988	0.20270	87.946	0.30933	87.875	0.42274	87.768	0.54616	87.626
89°	0.10033	88.994	0.20271	88.973	0.30934	88.937	0.42279	88.883	0.54628	88.813
90°	0.10033	90.000	0.20271	90.000	0.30934	90.000	0.42280	90.000	0.54631	90.000

Example of the use of Table: $\tanh 0.2/70^\circ = 0.20206/69.501 = 0.20206/69.30.04''$

Note. — $\tanh p/8 = 0/8$ when $p = 0$

CORRECTING FACTOR $\frac{\sinh \theta}{\theta}$

47

	0.1		0.2		0.3		0.4		0.5	
60°	0.99917	0.082	0.99670	0.331	0.99257	0.746	0.98685	1.330	0.97956	2.085
61°	0.99912	0.081	0.99645	0.324	0.99213	0.731	0.98603	1.302	0.97832	2.042
62°	0.99907	0.079	0.99625	0.317	0.99170	0.715	0.98523	1.273	0.97710	1.998
63°	0.99902	0.077	0.99605	0.309	0.99127	0.698	0.98448	1.243	0.97590	1.950
64°	0.99897	0.075	0.99585	0.301	0.99083	0.680	0.98373	1.212	0.97476	1.900
65°	0.99893	0.073	0.99570	0.293	0.99040	0.661	0.98300	1.179	0.97364	1.848
66°	0.99889	0.071	0.99555	0.284	0.99000	0.641	0.98235	1.144	0.97254	1.794
67°	0.99885	0.069	0.99540	0.275	0.98963	0.621	0.98165	1.108	0.97150	1.738
68°	0.99881	0.066	0.99520	0.265	0.98927	0.599	0.98100	1.070	0.97046	1.679
69°	0.99877	0.064	0.99505	0.255	0.98891	0.577	0.98035	1.030	0.96946	1.617
70°	0.99873	0.062	0.99490	0.245	0.98857	0.555	0.97975	0.990	0.96852	1.554
71°	0.99869	0.059	0.99475	0.235	0.98823	0.532	0.97918	0.948	0.96760	1.489
72°	0.99865	0.057	0.99460	0.225	0.98790	0.508	0.97863	0.905	0.96676	1.422
73°	0.99861	0.054	0.99445	0.214	0.98760	0.483	0.97808	0.861	0.96592	1.354
74°	0.99858	0.051	0.99435	0.203	0.98733	0.458	0.97758	0.817	0.96514	1.284
75°	0.99855	0.048	0.99425	0.192	0.98707	0.432	0.97710	0.771	0.96440	1.212
76°	0.99852	0.045	0.99415	0.180	0.98680	0.406	0.97665	0.724	0.96368	1.138
77°	0.99850	0.042	0.99405	0.168	0.98653	0.379	0.97625	0.676	0.96304	1.062
78°	0.99847	0.039	0.99395	0.156	0.98633	0.351	0.97585	0.628	0.96246	0.986
79°	0.99845	0.036	0.99385	0.144	0.98613	0.322	0.97545	0.578	0.96190	0.909
80°	0.99843	0.033	0.99375	0.131	0.98593	0.294	0.97512	0.528	0.96134	0.836
81°	0.99841	0.030	0.99365	0.118	0.98577	0.266	0.97483	0.477	0.96088	0.750
82°	0.99839	0.026	0.99360	0.106	0.98563	0.238	0.97458	0.425	0.96046	0.669
83°	0.99838	0.023	0.99355	0.093	0.98553	0.209	0.97433	0.373	0.96008	0.587
84°	0.99837	0.020	0.99350	0.080	0.98543	0.180	0.97413	0.321	0.95974	0.505
85°	0.99836	0.017	0.99345	0.067	0.98537	0.150	0.97395	0.268	0.95944	0.422
86°	0.99835	0.014	0.99340	0.054	0.98530	0.120	0.97380	0.215	0.95920	0.338
87°	0.99834	0.011	0.99335	0.041	0.98523	0.090	0.97370	0.162	0.95904	0.254
88°	0.99833	0.008	0.99335	0.028	0.98517	0.060	0.97365	0.108	0.95894	0.170
89°	0.99832	0.004	0.99335	0.014	0.98510	0.030	0.97360	0.054	0.95890	0.085
90°	0.99831	0.000	0.99335	0.000	0.98507	0.000	0.97355	0.000	0.95886	0.000

Example of the use of Table: $\frac{\sinh 0.3/80^\circ}{0.3/80^\circ} = 0.98593/0.294 = 0.98593/0.017.38''$

Note. $\frac{\sinh \theta}{\theta} = 1.0/0^\circ$ when $\theta = 0/8$

CORRECTIVE FACTOR $\frac{\tanh \theta}{\theta}$

NOTE.—ALL THE ANGLES FOUND IN THIS TABLE ARE NEGATIVE

48

	0.1		0.2		0.3		0.4		0.5	
		θ^-		θ^-		θ^-		θ^-		θ^-
60°	1.0017	0.167	1.0067	0.668	1.0148	1.521	1.0259	2.745	1.0397	4.375
61°	1.0018	0.163	1.0070	0.655	1.0158	1.490	1.0278	2.695	1.0425	4.300
62°	1.0019	0.159	1.0074	0.640	1.0167	1.458	1.0294	2.641	1.0454	4.220
63°	1.0020	0.155	1.0078	0.625	1.0176	1.425	1.0311	2.583	1.0482	4.133
64°	1.0020	0.151	1.0082	0.609	1.0185	1.390	1.0328	2.521	1.0510	4.041
65°	1.0021	0.147	1.0086	0.592	1.0193	1.353	1.0344	2.456	1.0538	3.942
66°	1.0022	0.143	1.0090	0.575	1.0202	1.314	1.0360	2.388	1.0564	3.836
67°	1.0023	0.138	1.0093	0.557	1.0210	1.273	1.0376	2.315	1.0591	3.724
68°	1.0024	0.134	1.0097	0.539	1.0218	1.231	1.0391	2.240	1.0617	3.609
69°	1.0025	0.129	1.0100	0.520	1.0226	1.187	1.0406	2.163	1.0642	3.489
70°	1.0026	0.123	1.0103	0.499	1.0234	1.141	1.0420	2.082	1.0666	3.362
71°	1.0026	0.118	1.0106	0.478	1.0241	1.094	1.0434	1.998	1.0689	3.229
72°	1.0027	0.112	1.0109	0.456	1.0247	1.045	1.0447	1.911	1.0712	3.091
73°	1.0028	0.107	1.0112	0.434	1.0254	0.995	1.0460	1.821	1.0734	2.948
74°	1.0028	0.102	1.0114	0.411	1.0260	0.944	1.0472	1.728	1.0755	2.800
75°	1.0029	0.096	1.0117	0.388	1.0266	0.892	1.0483	1.632	1.0774	2.649
76°	1.0029	0.090	1.0120	0.365	1.0272	0.838	1.0494	1.534	1.0793	2.493
77°	1.0030	0.084	1.0122	0.341	1.0277	0.783	1.0504	1.434	1.0811	2.333
78°	1.0030	0.078	1.0123	0.316	1.0282	0.727	1.0513	1.332	1.0828	2.169
79°	1.0031	0.072	1.0125	0.291	1.0287	0.671	1.0522	1.229	1.0843	2.001
80°	1.0031	0.066	1.0127	0.266	1.0291	0.614	1.0530	1.124	1.0857	1.830
81°	1.0032	0.060	1.0129	0.241	1.0295	0.555	1.0538	1.017	1.0870	1.656
82°	1.0032	0.054	1.0130	0.215	1.0298	0.494	1.0544	0.908	1.0881	1.480
83°	1.0032	0.047	1.0132	0.188	1.0301	0.433	1.0550	0.797	1.0892	1.301
84°	1.0033	0.040	1.0133	0.162	1.0304	0.372	1.0555	0.685	1.0901	1.119
85°	1.0033	0.033	1.0134	0.136	1.0307	0.311	1.0560	0.573	1.0908	0.935
86°	1.0033	0.026	1.0134	0.109	1.0309	0.249	1.0563	0.460	1.0914	0.749
87°	1.0033	0.019	1.0135	0.082	1.0310	0.187	1.0566	0.346	1.0919	0.562
88°	1.0033	0.012	1.0135	0.054	1.0311	0.125	1.0568	0.232	1.0923	0.374
89°	1.0033	0.006	1.0135	0.027	1.0311	0.063	1.0570	0.117	1.0926	0.187
90°	1.0033	0.000	1.0136	0.000	1.0311	0.000	1.0570	0.000	1.0926	0.000

EXAMPLE OF THE USE OF TABLE:— $\frac{\tanh 0.4/71^\circ}{0.4/71^\circ} = 1.0434 \sqrt{1.998} = 1.0434 \sqrt{1.59'53''}$

NOTE. $\frac{\tanh \theta}{\theta} = 1.0/0^\circ$ WHEN $\theta = 0^\circ$

DISCUSSION OF TRANSIENT CONDITIONS.

The general solution of an electric current problem includes besides the permanent term (constant or periodic) a transient term, which disappears after a time depending upon the circuit conditions.

In a circuit containing resistance and inductance or capacity only the transient terms of current and voltage are not sufficiently large and of long duration to cause harmful nor even appreciable effects, and it is only in circuits containing inductance and capacity that excessive values of current and potential differences may be reached by the transient term, sometimes with serious results. No transient term results from the resistance, but only those circuit constants which represent storage of energy, electrostatically by capacity, magnetically by the inductance, give rise to transient phenomena, and the more the resistance predominates the less the severity and duration of the transient term. (Hence the charging resistance on lightning arresters).

If a circuit contains only one energy-storing constant, inductance or capacity, the transient term, which connects the initial with the stationary condition of the circuit, necessarily can be a steady logarithmic term only, or a gradual approach. An oscillation can occur only with the existence of two energy-storing constants (capacity and inductance) which permits a surge of energy from one to the other.

When voltage is applied to a transmission line or when there is a line disturbance, such as a surge or a short circuit, the line conditions do not adjust themselves instantaneously. Therefore, from one condition to the other there is a transient state in which energy surges back and forth and ultimately settles into a steady state.

This surging during transit is due to an overcharge of the line condenser at time of transit, i.e., the unstable condition when voltage is applied or the circuit condition in any way changed, allows the line condenser to be overcharged, and after which it must necessarily discharge. As an explanation of this overcharge, imagine the line condenser being charged, now at the instant the condenser e.m.f. is equal to the impressed e.m.f. some value of current still exists and this current continues to decrease but cannot cease flowing instantaneously due to the inductance of the line. This decreasing current produces a decreasing magnetic flux (iL) which generates an e.m.f., thus the line condenser is overcharged. The condenser discharges below an e.m.f. equal to that of the impressed e.m.f.. This alternate overcharge and discharge continues until a state of equilibrium is reached. These surges oscillate back and forth, and due to the energy dissipated gradually decreases to zero, the line conditions then being in the steady state.

GENERAL EQUATIONS
OF
TRANSMISSION LINE
IN
SPACE ONLY.

Consider an alternating current power transmission line of the three phase type, with the wires symmetrically placed at the vertices of an equilateral triangle. The system may be regarded as being divisible into three independent single phase lines, each operated at star voltage to the neutral. It is then only necessary in deriving the formulae, to consider a single phase line, since the three individual single phase lines will have equal wire inductances, and also equal wire capacities.

The capacity in a transmission line is distributed, and if the line is open at the receiver end, there will be a charging current flowing.

Let r_c = insulation resistance of line condenser.

x_c = condensive reactance of line condenser.

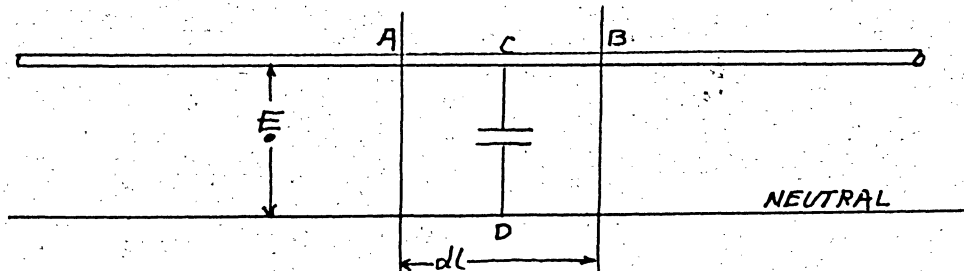
$$\text{then } 1 = \frac{E}{Z} = \frac{E}{r_c - jx_c} = \frac{E(r_c + jx_c)}{r_c^2 + x_c^2}$$

$$1 = E \frac{r_c}{r_c^2 + x_c^2} + jE \frac{x_c}{r_c^2 + x_c^2} = E g_c + jE b_c$$

the conductance (g) of the line condenser is infinitely small and may be neglected, therefore,

$$1 = jE b_c = j \frac{E}{x_c} = jE 2\pi f c.$$

The figure shows an elemental section of a transmission line, single phase to neutral:-



The capacity, CD , is lumped in the center of an elemental length, dl , which corresponds to a distributed capacity when dl approaches zero.

Complex quantities will be represented by a period placed above or below the letter: as, $\dot{E} = e_1 + je_2$

Let $\dot{I} = i_1 - ji_2$ current flowing in line.

$\dot{E} = e_1 - je_2$ voltage between line and neutral.

$Z = r + jx$ impedance per unit length of line.

$Y = g + jb$ Capacity admittance per unit length of line.

Then the impedance of the distance dl will be Zdl and the capacity admittance will be Ydl . Therefore, the voltage drop, dE , in the distance dl is

$$(1) \quad d\dot{E} = \dot{I}Zdl$$

and the current dI flowing thru the capacity in distance dl is

$$(2) \quad d\dot{I} = \dot{E}Ydl$$

Equations (1) and (2) may be written

$$(3) \quad \frac{d\dot{E}}{dl} = \dot{I}Z$$

(4) $\frac{d\dot{I}}{dl} = \dot{E}Y$ and are known as the fundamental differential equations of the transmission line.

Differentiating (3) and (4) with regard to l we obtain

$$(5) \quad \frac{d^2\dot{E}}{dl^2} = Z \frac{d\dot{I}}{dl}$$

$$(6) \quad \frac{d^2\dot{I}}{dl^2} = Y \frac{d\dot{E}}{dl}$$

Substituting the values of $\frac{d\dot{I}}{dl}$ and $\frac{d\dot{E}}{dl}$ from (3) and (4) into (5) and (6) we obtain

$$(7) \quad \frac{d^2\dot{E}}{dl^2} = ZY\dot{E}$$

$$(8) \quad \frac{d^2 \dot{I}}{dl^2} = ZY \dot{I}$$

From equations (1) and (2) if dl increases, $d\dot{E}$ and $d\dot{I}$ increase, or l is measured in the direction of rising power. Therefore, the receiving end of the line is called $l = 0$ and l will be measured toward the generator.

Equations (7) and (8) when integrated, give the values of E and I in terms of the length of the line, and since they do not contain t as a variable, are only applicable after the conditions have reached a steady state.

These equations are of the general form

(9) $\frac{d^2 z}{dx^2} - kz = 0$ and it can be shown (see "Differential Equations" by Forsyth) that the general equation can be integrated by the substitution

$$(10) \quad z = e^{\pm ax}$$

If this is a solution it should satisfy equation (9). Differentiating (10) with regard to x we have

$$\frac{dz}{dx} = \pm a e^{\pm ax}$$

and

$$(11) \quad \frac{d^2 z}{dx^2} = a^2 e^{\pm ax}$$

Substituting (11) in (9) we have $e^{\pm ax} (a^2 - k) = 0$ or

$$(12) \quad a^2 - k = 0$$

$$(13) \quad a = \pm \sqrt{k}$$

This shows that $z = e^{\pm ax}$ is a solution provided $a = \pm \sqrt{k}$, and since the original differential equation is of the second order the complete solution must contain two arbitrary constants, and is given by,

$$(14) z = \dot{A}_1 \epsilon^{+\sqrt{K}x} + \dot{A}_2 \epsilon^{-\sqrt{K}x}$$

The constants $\dot{A}_1$ and $\dot{A}_2$ are determined by the end conditions or initial conditions

Since $K = ZY$ we have for the solution of (8)

$$(15) \dot{I} = \dot{A}_1 \epsilon^{\sqrt{ZY} \ell} - \dot{A}_2 \epsilon^{-\sqrt{ZY} \ell}$$

and differentiating

$$(16) \frac{d\dot{I}}{d\ell} = \dot{A}_1 \sqrt{ZY} \epsilon^{\sqrt{ZY} \ell} + \dot{A}_2 \sqrt{ZY} \epsilon^{-\sqrt{ZY} \ell}$$

From (4) $\frac{d\dot{I}}{d\ell} = \dot{E} Y$ Substituting this value in (16) we have

$$(17) \dot{E} = \sqrt{\frac{Z}{Y}} [\dot{A}_1 \epsilon^{\sqrt{ZY} \ell} + \dot{A}_2 \epsilon^{-\sqrt{ZY} \ell}] \text{ which is the solution of (7)}$$

But (18) $Z = r + jx$

(19) $Y = g + jb$

Equations (15) and (17) may be thrown into another form by substituting

$$(20) V = \sqrt{ZY} = \alpha + j\beta = \sqrt{(r+jx)(g+jb)} \text{ Squaring (20)}$$

(21) $(\alpha^2 - \beta^2) + 2j\alpha\beta = (rg - xb) + j(xg + rb)$ The real terms may be equated, as also the imaginaries.

$$(22) \begin{cases} \alpha^2 - \beta^2 = rg - xb \\ 2\alpha\beta = xg + rb \end{cases}$$

Squaring and adding

$$(\alpha^2 + \beta^2)^2 = (rg - xb)^2 + (xg + rb)^2$$

$$(23) \begin{aligned} &= r^2g^2 + x^2g^2 + b^2x^2 + b^2r^2 + 2rgbx - 2rgbx \\ &= (r^2 + x^2)g^2 + (r^2 + x^2)b^2 \\ &= (r^2 + x^2)(g^2 + b^2) \end{aligned}$$

But the absolute value of Z , call it z is equal to $\sqrt{r^2 + x^2}$, and the absolute value of Y , call it y is equal to $\sqrt{g^2 + b^2}$

$$(24) \begin{aligned} (\alpha^2 + \beta^2)^2 &= z^2 y^2 \\ \alpha^2 + \beta^2 &= zy \end{aligned}$$

$$\beta^2 = zy - \alpha^2 \text{ Substitute this value in (22)}$$

$$\alpha^2 + \alpha^2 - zy = rg - xb$$

$$\alpha^2 = \frac{1}{2}(rg - xb + zy)$$

$$(25) \begin{cases} \alpha = \sqrt{1/2 (ZY + Rg - Xb)} \\ \beta = \sqrt{1/2 (ZY - Rg + Xb)} \end{cases}$$

Equations (15) and (17) now become.

$$(26) \quad I = \dot{A}_1 e^{(\alpha + j\beta)l} - \dot{A}_2 e^{-(\alpha + j\beta)l}$$

$$(27) \quad \dot{E} = \sqrt{\frac{Z}{Y}} [\dot{A}_1 e^{(\alpha + j\beta)l} + \dot{A}_2 e^{-(\alpha + j\beta)l}]$$

But $e^{\pm j\beta l} = \cos \beta l \pm j \sin \beta l$ (See "Engineering Mathematics - Steinmetz") (26) and (27) now become

$$(28) \quad I = \dot{A}_1 e^{\alpha l} (\cos \beta l + j \sin \beta l) - \dot{A}_2 e^{\alpha l} (\cos \beta l - j \sin \beta l)$$

$$(29) \quad \dot{E} = \sqrt{\frac{Z}{Y}} [\dot{A}_1 e^{\alpha l} (\cos \beta l + j \sin \beta l) + \dot{A}_2 e^{\alpha l} (\cos \beta l - j \sin \beta l)]$$

We are now ready to determine the integration constants $\dot{A}_1$ and $\dot{A}_2$

Let $\dot{I}_0 = i_0 - j i_0 =$ terminal current of line
 $\dot{E}_0 = e_0 - j e_0 =$ " " voltage " "

Then when $l = 0$ (at terminal)

$$I = \dot{I}_0 \quad \text{and} \quad \dot{E} = \dot{E}_0$$

Substituting in (28) and (29)

$$(30) \quad \begin{cases} \dot{I}_0 = \dot{A}_1 - \dot{A}_2 \\ \dot{E}_0 = \sqrt{\frac{Z}{Y}} (\dot{A}_1 + \dot{A}_2) \end{cases} \quad \text{from which}$$

$$\begin{aligned} \dot{A}_1 - \dot{A}_2 &= \dot{I}_0 \\ \dot{A}_1 + \dot{A}_2 &= \dot{E}_0 / \sqrt{\frac{Z}{Y}} \end{aligned}$$

$$\text{adding } 2\dot{A}_1 = \dot{I}_0 + \dot{E}_0 \sqrt{\frac{Y}{Z}}$$

$$\text{subtracting } 2\dot{A}_2 = -\dot{I}_0 + \dot{E}_0 \sqrt{\frac{Y}{Z}}$$

$$(31) \quad \dot{A}_1 = \frac{1}{2} \{ \dot{I}_0 + \dot{E}_0 \sqrt{\frac{Y}{Z}} \}$$

$$(32) \quad \dot{A}_2 = \frac{1}{2} \{ -\dot{I}_0 + \dot{E}_0 \sqrt{\frac{Y}{Z}} \}$$

Substituting these values in (28) and (29) we get the complete equations for current and voltage at any point of the line.

$$(33) \quad I = \frac{1}{2} \{ (\dot{I}_0 + \dot{E}_0 \sqrt{\frac{Y}{Z}}) e^{+\alpha l} (\cos \beta l + j \sin \beta l) + (\dot{I}_0 - \dot{E}_0 \sqrt{\frac{Y}{Z}}) e^{-\alpha l} (\cos \beta l - j \sin \beta l) \}$$

$$(34) \quad \dot{E} = \frac{1}{2} \{ (\dot{E}_0 + \dot{I}_0 \sqrt{\frac{Z}{Y}}) e^{+\alpha l} (\cos \beta l + j \sin \beta l) + (\dot{E}_0 - \dot{I}_0 \sqrt{\frac{Z}{Y}}) e^{-\alpha l} (\cos \beta l - j \sin \beta l) \}$$

SAMPLE PROBLEMS
STEADY STATE
OF
TRANSMISSION LINE.

NOTE.— These equations are only applicable at a time at which the conditions in the transmission line have reached a steady state.

Example No. 1.

Assume a line 120 miles long delivering 200 Amperes at 60000 Volts to neutral at receiver end of line, 3 Phase, 60~, Power-factor .90, Wire 250000 cm., 6' spacing. Copper Wire.

$$r = 0.224 \text{ ohms per mile @ } 25^\circ \text{C.}$$

$$L = 1.88 \times 10^{-3} \text{ henrys per mile}$$

$$C = 0.016 \cdot 10^{-6} \text{ farads per mile}$$

$$X = 2\pi fL = 2\pi 60 \times 1.88 \times 10^{-3} = 0.71$$

$$Z = r + jX = 0.224 + j0.71$$

$$Z = 0.75$$

And neglecting conductance ($g = 0$)

$$b = 2\pi fC = 2\pi 60 \times 0.016 \times 10^{-6} = 6 \times 10^{-6}$$

$$Y = jb = j 6 \times 10^{-6}$$

$$Y = 6 \times 10^{-6}$$

$$a = \left[\frac{1}{2} (ZV + rI - bX) \right]^{1/2}$$

$$\beta = \left[\frac{1}{2} (ZV - rI + bX) \right]^{1/2}$$

$$V = a + j\beta = \sqrt{ZY}$$

$$\frac{V}{Z} = \sqrt{\frac{Y}{Z}}$$

$$\frac{V}{Y} = \sqrt{\frac{Z}{Y}} = Z_0 = \text{Surge impedance.}$$

$$a^2 = \frac{1}{2} (0.75 \times 6 - 0.71 \times 6) 10^{-6} = 0.125 \times 10^{-6}$$

$$a = 0.35 \times 10^{-3}$$

$$\beta^2 = \frac{1}{2} (0.75 \times 6 + 0.71 \times 6) 10^{-6} = 4.37 \times 10^{-6}$$

$$\beta = 2.1 \times 10^{-3}$$

$$V = a + j\beta = (0.35 + j2.1) 10^{-3}$$

$$\frac{\sqrt{Y}}{Z} = \frac{V}{Z} = \frac{(0.35 + j2.1) 10^{-3}}{0.224 + j0.71} = (2.9 + j0.4) 10^{-3}$$

$$\frac{\sqrt{Z}}{Y} = \frac{V}{Y} = \frac{(0.35 + j2.1) 10^{-3}}{0 + j6 \times 10^{-6}} = (0.35 - j0.058) 10^{+3}$$

$$I = \frac{1}{2} \left[(I_0 + E_0 \sqrt{\frac{Y}{Z}}) E^{+a1} (\cos \beta l + j \sin \beta l) + (I_0 - E_0 \sqrt{\frac{Y}{Z}}) E^{-a1} (\cos \beta l - j \sin \beta l) \right]$$

$$E = \frac{1}{2} \left[(E_0 + I_0 \sqrt{\frac{Z}{Y}}) E^{+a1} (\cos \beta l + j \sin \beta l) + (E_0 - I_0 \sqrt{\frac{Z}{Y}}) E^{-a1} (\cos \beta l - j \sin \beta l) \right] 10^3$$

Receiver end of line

$$I = 200 \text{ amps } I_0 = 180 - 87j$$

$$E = 60 \times 10^3 \text{ Volts } E_0 = 60 \times 10^3 + j0$$

$$\frac{1}{2} [I_0 + E_0 \sqrt{\frac{Y}{Z}}] = \frac{1}{2} [(180 - j87) + 60 \cdot 10^3 (2.9 + j0.4) 10^{-3}] = 177 - j31.5$$

$$\frac{1}{2} [I_0 - E_0 \sqrt{\frac{Y}{Z}}] = \frac{1}{2} [(180 - j87) - 60 \cdot 10^3 (2.9 + j0.4) 10^{-3}] = 3 - j55.5$$

$$\frac{1}{2} [\bar{E}_0 + \bar{I}_0 \sqrt{Z}] = \frac{1}{2} [60 \cdot 10^3 + (180 - j87)(0.35 - j0.058)10^3] = (59 - j20.5)10^3$$

$$\frac{1}{2} [\bar{E}_0 - \bar{I}_0 \sqrt{Z}] = \frac{1}{2} [60 \cdot 10^3 - (180 - j87)(0.35 - j0.058)10^3] = (1 + j20.5)10^3$$

$$\text{For } l = 60 \text{ miles } E^{+a_1} = E^{-0.21} = 1.021 \quad E^{-a_1} = E^{-0.21} = 0.98$$

$$\beta l = 2.1 \times 10^{-3} \times 60 = 0.126 \quad \cos \beta l = \cos 0.126 = 0.99 \quad \sin \beta l = \sin 0.126 = 0.126$$

$$\bar{I} = (177 - j31.5)E^{+a_1}(\cos \beta l + j \sin \beta l) + (3 - j55.5)E^{-a_1}(\cos \beta l - j \sin \beta l)$$

$$\bar{E} = (59 - j20.5)E^{+a_1}(\cos \beta l + j \sin \beta l) + (1 + j20.5)E^{-a_1}(\cos \beta l - j \sin \beta l) \cdot 10^3$$

By substituting a_1 & βl for various lengths " l " in these equations values for $\bar{I}$ & $\bar{E}$ are obtained.

$$\bar{I} = (177 - j31.5)1.021(0.99 + j0.126) + (3 - j55.5)0.98(0.99 - j0.126)$$

$$\bar{I} = 179 - j63.4$$

$$\bar{E} = [(59 - j20.5)1.021(0.99 + j0.126) + (1 + j20.5)0.98(0.99 - j0.126)] \cdot 10^3$$

$$\bar{E} = (65.62 + j7.23)10^3$$

$$\text{For } l = 120 \text{ miles } E^{+a_2} = E^{+0.42} = 1.042 \quad E^{-a_2} = E^{-0.42} = 0.959$$

$$\beta l = 2.1 \times 10^{-3} \times 120 = 0.252 \quad \cos \beta l = \cos 0.252 = 0.968 \quad \sin \beta l = \sin 0.252 = 0.249$$

$$\bar{I} = (177 - j31.5)1.042(0.968 + j0.249) + (3 - j55.5)0.959(0.968 - j0.249)$$

$$\bar{I} = 177 - j38$$

$$\bar{E} = [(59 - j20.5)1.042(0.968 + j0.249) + (1 + j20.5)0.959(0.968 - j0.249)] \cdot 10^3$$

$$\bar{E} = (70.74 + j13.41)10^3$$

Receiver End $l = 0$

$$\bar{I} = 180 - j87$$

$$i = 200 \text{ amps}$$

$$\tan \theta = 0.483$$

$$\theta_1 = 26^\circ$$

$$\bar{E} = 60 \cdot 10^3$$

$$e = 60 \cdot 10^3 \text{ Volts}$$

$$\theta_2 = 0$$

$$\theta_1 - \theta_2 = \theta = 26^\circ$$

$$\cos \theta = .90 \text{ power factor lag}$$

Middle of line $l = 60 \text{ miles}$

$$\bar{I} = 179 - j63.4$$

$$i = 190 \text{ amps}$$

$$\tan \theta_1 = 0.352$$

$$\theta_1 = 19.4^\circ$$

$$\bar{E} = (65.62 + j7.23)10^3$$

$$e = 66 \cdot 10^3 \text{ Volts}$$

$$\tan \theta_2 = 0.111$$

$$\theta_2 = -6.33^\circ$$

$$\theta_1 - \theta_2 = \theta = 25.73^\circ$$

$$\cos \theta = .901 \text{ p.f. lag.}$$

Generator End of line $l = 120 \text{ miles}$

$$\bar{I} = 177 - j38$$

$$i = 181 \text{ amps}$$

$$\tan \theta_1 = 0.214$$

$$\theta_1 = 12.08^\circ$$

$$\bar{E} = (70.74 + j13.41)10^3$$

$$e = 72.7 \cdot 10^3 \text{ Volts}$$

$$\tan \theta_2 = 0.190$$

$$\theta_2 = -10.76^\circ$$

$$\theta_1 - \theta_2 = \theta = 23.84^\circ$$

$$\cos \theta = .915 \text{ P.F. Lag.}$$

Solutions of equations (7) and (8) can be obtained in a different form.

$$(7) \frac{d^2 \bar{E}}{dl^2} = ZY\bar{E}$$

$$(8) \frac{d^2 \bar{I}}{dl^2} = ZY\bar{I}$$

These equations are of the general form

$$(35) \frac{d^2 z}{dx^2} - Kz = 0$$

$$(36) \text{ let } z = \cosh ax + \sinh ax$$

Differentiating gives $\frac{dz}{dx} = a \sinh ax + a \cosh ax$

$$\frac{d^2 z}{dx^2} = a^2 [\cosh ax + \sinh ax] = a^2 z \quad (37)$$

Substituting the above values in the general formula we obtain

$$\begin{aligned} \bar{I}_{60} &= (180 - j87)(.994 + j.0026) \\ &\quad + (2.86 + j.405)10^3 \times 6 \times 10^4 (.0208 + j.1257) \\ &= 179.8 - j64.0 \end{aligned}$$

$$\begin{aligned} \bar{E}_{60} &= (350 - j58.3)(180 - j87)(.0208 + j.1257) \\ &\quad + 6 \times 10^4 (.994 + j.0026) \\ &= 65955 + j6605 \end{aligned}$$

For generator end of line $l = 120$ and $\sqrt{ZY}l = (.042 + j.252)$

$$\begin{aligned} \sinh(.042 + j.252) &= \sinh.042 \cos.252 + j \cosh.042 \sin.252 \\ &= .042 \times .9684 + j 1.00088 \times .249 \\ &= .0407 + j.249 \end{aligned}$$

$$\begin{aligned} \cosh(.042 + j.252) &= \cosh.042 \cos.252 + j \sinh.042 \sin.252 \\ &= 1.00088 \times .9684 + j .042 \times .249 \\ &= .9692 + j.01046 \end{aligned}$$

Substituting these values in the general solution we obtain

$$\begin{aligned} \bar{I}_{120} &= (180 - j87)(.9692 + j.01046) \\ &\quad + (2.86 + j.405)10^3 \times 6 \times 10^4 (.0407 + j.249) \\ &= 176.3 - j38.9 \end{aligned}$$

$$\begin{aligned} \bar{E}_{120} &= (350 - j58.3)(180 - j87)(.0407 + j.249) \\ &\quad + 6 \times 10^4 (.9692 + j.01046) \\ &= 70760 + j13380 \end{aligned}$$

Summarizing we have.

Receiving end of line $l = 0$

$\bar{I} = 180 - j87$	$i = 200 \text{ amps}$	$\tan \theta_1 = -.483$	$\theta_1 = -26^\circ$
$\bar{E} = 6 \times 10^4$	$e = 60000 \text{ volt}$		$\theta_2 = 0$
			$\theta_1 - \theta_2 = \theta = -26^\circ$

Power factor .9 lag.

Middle of line $l = 60$

$\bar{I} = 180 - j64$	$i = 191 \text{ amps}$	$\tan \theta_1 = -.356$	$\theta_1 = -19^\circ 36'$
$\bar{E} = 65950 + j6605$	$e = 66300 \text{ volts}$	$\tan \theta_2 = +.1$	$\theta_2 = 5^\circ 42'$
			$\theta_1 - \theta_2 = \theta = -25^\circ 18'$

Power factor 0.904 lag.

Generator end of line

$l = 120$

$\bar{I} = 176.3 - j38.9$	$i = 180$	$\tan \theta_1 = -.22$	$\theta_1 = -12^\circ 25'$
$\bar{E} = 70760 + j13383$	$e = 72200$	$\tan \theta_2 = +.189$	$\theta_2 = 10^\circ 42'$
			$\theta_1 - \theta_2 = \theta = -23^\circ 7'$

Power factor 0.92 lag.

GENERAL EQUATIONS
OF
TRANSMISSION LINE IN
SPACE AND TIME.

The preceding equations were developed under the assumption that the voltage and current has reached a steady state, that is, that the voltage, current and frequency were independent of the time t .

However, when any of the conditions are changed, such as switching on or off the current in a transmission line, or in changing the value of current, transient conditions occur which depend on the time t . These transients occur and die away in a very short time, but under certain conditions the amplitudes may reach destructive proportions, and thus it is important that they should be discussed.

We shall now develop the most general equations of the transmission line, solving the equations for both space and time.

As before, considering an elemental section of the line it is clear that $de = rdl \cdot i + Ldl \cdot \frac{di}{dt}$ from which

$$(1) \frac{de}{dl} = ri + L \frac{di}{dt} \quad di = gdl \cdot e + Cdl \frac{de}{dt} \text{ from which}$$

$$(2) \frac{di}{dt} = ge + C \frac{de}{dt} \text{ Differentiating (1) with respect to } t, \text{ and (2) with respect to } l \text{ we have.}$$

$$\frac{d}{dt} \left(\frac{de}{dl} \right) = r \frac{di}{dt} + L \frac{d^2 i}{dt^2} \quad (1A)$$

$$\frac{d^2 i}{dl^2} = g \frac{de}{dl} + C \frac{d}{dl} \left(\frac{de}{dt} \right) \quad (2A)$$

Substituting (1A) and (1) in (2A) we have.

$$(3) \frac{d^2 i}{dl^2} = gr i + (rC + gL) \frac{di}{dt} + LC \frac{d^2 i}{dt^2}$$

and in the same manner.

$$(4) \frac{d^2 e}{dl^2} = rge + (rC + gL) \frac{de}{dt} + LC \frac{d^2 e}{dt^2}$$

(5) These differential equations may be solved by terms of the form $i = \sum A_N \cdot E^{-al - bt}$

Where E is the base of Napierian logarithms.

$$\frac{di}{dt} = -b A E^{-al - bt}$$

$$\frac{d^2 i}{dt^2} = b^2 A E^{-al - bt}$$

$$\frac{de}{dl} = -a A E^{-al - bt}$$

$$\frac{d^2 e}{dl^2} = a^2 A E^{-al - bt}$$

Substituting these values in (3).

$$a^2 A E^{-al - bt} = gr A E^{-al - bt} - (rC + gL) b A E^{-al - bt} + LC b^2 A E^{-al - bt}$$

which gives the condition that (5) shall hold.

$a^2 = gr - b(rC + gL) + LC b^2$ which may be written

$$(6) a^2 = (bL - r)(bC - g)$$

The quantities i and e must always be real; but since a and b appear as exponents they may be complex.

in which case the integration constants A must be such complex quantities that the imaginaries in A cancel.

To get the form in which the solution for e appears substitute (5) in (1)

$$\frac{de}{dt} = r A E^{-al-bt} - b L A E^{-al-bt}$$

rewriting and integrating

$$\frac{de}{dt} = (r - Lb) A E^{-al-bt}$$

$$\int de = (r - Lb) \int A E^{-al-bt} dt \text{ gives}$$

$$(8) e = \frac{(r - Lb)}{a} A E^{-al-bt}$$

In the exponential function e^{-al-bt} , let $a = k + jk$

$b = p + jq$ then

$$(12) E^{-al-bt} = E^{-(kL + jkL - pt - jqL)} = E^{-(kL + pt) - j(kL + qt)}$$

the latter of which can be expressed as a trigonometric function of the angle $(kL + qt)$

(13) Taking $kL + qt = \text{constant}$ gives the relation between L and t for constant phase of the oscillation. With change of time t the phase thus changes in position L in the circuit, e.g. moves along the circuit. Differentiating (13) with respect to t

$$K \frac{dL}{dt} + q = 0 \quad \frac{dL}{dt} = -\frac{q}{K} \text{ which is the speed of propagation of an electric phenomenon along the circuit.}$$

Since (6) is a quadratic several pairs of combinations of a and b exist which in general are complex imaginary numbers. rewriting (11)

$$a^2 = (bL - r)(bc - g) \text{ substituting the following}$$

$$(17) a = a_1 \sqrt{Lc}$$

$$(18) a = k + jK$$

$$a_1 = k_1 + jK_1$$

$$b = p + jq$$

$$(k_1 + jK_1)^2 = \frac{[(p + jq)L - r]}{L} \frac{[(p + jq)c - g]}{c}$$

$$(20) (k_1 + jK_1)^2 = \left[(p + jq) - \frac{r}{L} \right] \left[(p + jq) - \frac{g}{c} \right] \text{ equating reals and imaginaries we have}$$

$$(21) k_1^2 - K_1^2 = (p - \frac{r}{L})(p - \frac{g}{c}) - q^2$$

$$k_1 K_1 = \frac{1}{2} (2p - \frac{r}{L} - \frac{g}{c})$$

$$(22) u = \frac{1}{2} (\frac{r}{L} + \frac{g}{c})$$

$$(23) m = \frac{1}{2} (\frac{r}{L} - \frac{g}{c}) \quad (24) s = u - p \text{ into}$$

(21) gives

$$(25) k_1^2 - K_1^2 = s^2 - q^2 - m^2$$

$$k_1 K_1 = s q$$

$$(26) p^2 - q^2 = k_1^2 - K_1^2 + m^2$$

$$s q = k_1 K_1$$

so that

In (25) by squaring 1st equation and adding four times the second, can be written

$$h_1^4 - 2h_1^2 K_1^2 + 4h_1^2 K_1^2 + K_1^4 = (s^2 - g^2 - m^2)^2 + 4s^2 g^2$$

$$\left. \begin{aligned} h_1^2 + K_1^2 &= \sqrt{(s^2 - g^2 - m^2)^2 + 4s^2 g^2} \\ &= \sqrt{(s^2 + g^2 - m^2)^2 + 4g^2 m^2} = R_1^2 \end{aligned} \right\} \quad (27)$$

Likewise (26) may be written

$$\left. \begin{aligned} s^2 + g^2 &= \sqrt{(h_1^2 - K_1^2 + m^2)^2 + 4h_1^2 K_1^2} \\ &= \sqrt{(h_1^2 + K_1^2 + m^2)^2 - 4h_1^2 m^2} = R_0^2 \end{aligned} \right\} \quad (28)$$

Substituting 17 & 18 gives by 25, 26, 27, & 28 from 25 & 17 & 18.

$$h_1^2 = LC(K_1^2 + s^2 - g^2 - m^2) \text{ \& from 27}$$

$$h_1^2 = LC(R_1^2 - K_1^2)$$

$$\text{add } 2h_1^2 = LC(R_1^2 + s^2 - g^2 - m^2)$$

$$\left. \begin{aligned} h_1 &= \sqrt{LC} \sqrt{\frac{1}{2}(R_1^2 + s^2 - g^2 - m^2)} \\ K_1 &= \sqrt{LC} \sqrt{\frac{1}{2}(R_1^2 - s^2 + g^2 + m^2)} \\ R_1^2 &= \sqrt{(s^2 + g^2 - m^2)^2 + 4g^2 m^2} \end{aligned} \right\} \quad (29)$$

In similar way

$$\left. \begin{aligned} s &= \frac{1}{\sqrt{LC}} \sqrt{\frac{1}{2}(R_2^2 + h^2 - k^2 + LCm^2)} \\ g &= \frac{1}{\sqrt{LC}} \sqrt{\frac{1}{2}(R_2^2 - h^2 + k^2 - LCm^2)} \\ R_2^2 &= \sqrt{(h^2 + k^2 + LCm^2)^2 - 4LCk^2 m^2} \end{aligned} \right\} \quad (30)$$

$$\text{where } R_0^2 = \frac{R_2^2}{LC}$$

We have assumed in the preceding that, $a = (+h + jk)$ and $b = (+u + s + jg)$ and we have made the condition that these shall satisfy (1) and therefore the original differential equation. But any one of the expressions $(\pm h \pm jk)$ & $(u \pm s \pm jg)$ hold providing also that the

second equation of (24) or (25) is satisfied.

$$hK = \sqrt{LC} \cdot sq \quad (31)$$

that is if h & K have the same sign, s & q must have the same sign, and inversely if s & q have opposite signs, h & K must have opposite signs. The corresponding values of a & b then are:-

$$\left. \begin{array}{ll} (1) \ a = +h + jk & b = a - s + jq \\ \quad \quad = +h - jk & \quad = a - s + jq \\ (2) \ a = -h - jk & b = a - s - jq \\ \quad \quad = -h + jk & \quad = a - s + jq \\ (3) \ a = -h + jk & b = a + s - jq \\ \quad \quad = -h - jk & \quad = a + s + jq \\ (4) \ a = +h - jk & b = a + s - jq \\ \quad \quad = +h + jk & \quad = a + s + jq \end{array} \right\} \quad (32)$$

Substituting (1) of (32) in (5) & (8) i.e., $i = A e^{-al-bt}$, $e = \frac{bL-r}{a} A e^{-al-bt}$

We have:-

$$i = A_1 e^{-(h+jk)(-(a-s-jq)t} + A_1' e^{-(h-jk)(-(a-s+jq)t}$$

$$= e^{-hl-(a-s)t} (A_1 e^{-jk t + jq t} + A_1' e^{jk t - jq t})$$

$$= e^{-hl-(a-s)t} \left\{ A_1 [\cos(qt-kl) + j \sin(qt-kl)] \right.$$

$$\left. + A_1' [\cos(qt-kl) - j \sin(qt-kl)] \right\}$$

$$= e^{-hl-(a-s)t} \left\{ (A_1 + A_1') \cos(qt-kl) + j(A_1 - A_1') \sin(qt-kl) \right\}$$

hence A_1 and A_1' must be conjugate

Complex imaginary quantities and

Writing:-

$$\left. \begin{aligned} C_1 &= A_1 + A_1' \\ C_1' &= j(A_1 - A_1') \end{aligned} \right\} \quad (34)$$

$$\begin{aligned} e_1 &= \frac{(u-s-jg)L-r}{h+jk} A_1 e^{-(h+jk)L-(u-s-jg)t} \\ &\quad + \frac{(u-s+jg)L-r}{h-jk} A_1' e^{-(h-jk)L-(u-s+jg)t} \\ &= e^{-hl-(u-s)t} \left\{ \frac{[(u-s-jg)L-r](h-jk)}{h^2+k^2} A_1 e^{j(gt-kl)} \right. \\ &\quad \left. + \frac{[(u-s+jg)L-r](h+jk)}{h^2+k^2} A_1' e^{-j(gt-kl)} \right\} \end{aligned}$$

$$i_1 = e^{-hl-(u-s)t} \{ C_1 \cos(gt-kl) + C_1' \sin(gt-kl) \} \quad (35)$$

$$\begin{aligned} e_1 &= e^{-hl-(u-s)t} \left\{ \left(\frac{[(u-s)L-r]h-gkL}{h^2+k^2} - j \frac{[(u-s)L-r]k+ghL}{h^2+k^2} \right) \right. \\ &\quad \left. A_1 [\cos(gt-kl) + j \sin(gt-kl)] \right. \\ &\quad \left. + \left(\frac{[(u-s)L-r]h-gkL}{h^2+k^2} + j \frac{[(u-s)L-r]k+ghL}{h^2+k^2} \right) \right. \\ &\quad \left. A_1' [\cos(gt-kl) - j \sin(gt-kl)] \right\} \quad (36) \end{aligned}$$

Introducing the following notations in (36)

$$\left. \begin{aligned} c_1 &= \frac{gkL + h[r-(u-s)L]}{h^2+k^2} = \frac{gk + h(m+s)L}{h^2+k^2} \\ c_1' &= \frac{k[r-(u-s)L] - ghL}{h^2+k^2} = \frac{k(m+s) - gh}{h^2+k^2} L \end{aligned} \right\} \quad (37)$$

$$\begin{aligned} e_1 &= e^{-hl-(u-s)t} \left\{ (-c_1 + jc_1') A_1 [\cos(gt-kl) + j \sin(gt-kl)] \right. \\ &\quad \left. + (-c_1 - jc_1') A_1' [\cos(gt-kl) - j \sin(gt-kl)] \right\} \\ &= e^{-hl-(u-s)t} \left\{ [-c_1(A_1 + A_1') + jc_1'(A_1 - A_1')] \cos(gt-kl) \right. \\ &\quad \left. + [-jc_1(A_1 - A_1') - c_1'(A_1 + A_1')] \sin(gt-kl) \right\} \quad (38) \end{aligned}$$

$$e_1 = E^{-hl-(u-s)t} \left\{ (c_1' C_1' - c_1 C_1) \cos(qt - kl) - (c_1 C_1' + c_1' C_1) \sin(qt - kl) \right\} \quad (39)$$

The second group of values of a and b in (32) differ from the first only in the reversal of the signs of h and k and thus i_2 and e_2 are derived from i_1 and e_1 by reversing the signs of h and k . Using the same notation for c_1 and c_1' makes it necessary to reverse the sign of the integration constants C so that

$$\left. \begin{aligned} C_2 &= -(A_2 + A_2') \\ C_2' &= -j(A_2 - A_2') \end{aligned} \right\} \quad (40)$$

$$i_2 = -E^{hl-(u-s)t} \left\{ C_2 \cos(qt + kl) + C_2' \sin(qt + kl) \right\} \quad (41)$$

$$e_2 = E^{hl-(u-s)t} \left\{ (c_1' C_2' - c_1 C_2) \cos(qt + kl) - (c_1 C_2' + c_1' C_2) \sin(qt + kl) \right\} \quad (42)$$

The third group of (32) differs from the first by the reversal of the signs of h and s .

$$\left. \begin{aligned} C_3 &= \frac{gk - h(m-s)}{h^2 + k^2} L \\ C_3' &= \frac{k(m-s) + gh}{h^2 + k^2} L \end{aligned} \right\} \quad (43)$$

$$\left. \begin{aligned} C_3 &= A_3 + A_3' \\ C_3' &= j(A_3 - A_3') \end{aligned} \right\} \quad (44)$$

$$i_3 = E^{hl-(u+s)t} \left\{ C_3 \cos(qt - kl) + C_3' \sin(qt - kl) \right\} \quad (45)$$

$$e_3 = E^{hl-(u+s)t} \left\{ (c_2' C_3 - c_2 C_3') \cos(qt - kl) - (c_2 C_3' + c_2' C_3) \sin(qt - kl) \right\} \quad (46)$$

The fourth group of (32) differs from the third group by reversal of sign of h and k . Retaining notations C_2 and C'_2 , and introducing

$$(47) \left. \begin{aligned} C_4 &= -(A_4 + A'_4) \\ C'_4 &= j(A_4 - A'_4) \end{aligned} \right\}$$

$$(48) \quad i_4 = -E^{-kl-(u+0)t} \{ C_4 \cos(qt+kl) + C'_4 \sin(qt+kl) \}$$

$$(49) \quad e_4 = E^{-kl-(u+0)t} \{ (C'_2 C'_4 - C_2 C_4) \cos(qt+kl) - (C_2 C'_4 + C'_2 C_4) \sin(qt+kl) \}$$

This gives for the general expression of the equations of the transmission line, in both space and time, the sum of the above four components:

$$(50) \quad \begin{aligned} i &= E^{-kl-(u-0)t} \{ C_1 \cos(qt-kl) + C'_1 \sin(qt-kl) \} \\ &- E^{-kl-(u-0)t} \{ C_2 \cos(qt+kl) + C'_2 \sin(qt+kl) \} \\ &+ E^{-kl-(u+0)t} \{ C_3 \cos(qt-kl) + C'_3 \sin(qt-kl) \} \\ &- E^{-kl-(u+0)t} \{ C_4 \cos(qt+kl) + C'_4 \sin(qt+kl) \} \end{aligned}$$

$$(51) \quad \begin{aligned} e &= E^{-kl-(u-0)t} \{ (C'_1 C'_3 - C_1 C_3) \cos(qt-kl) - (C'_1 C_3 + C_1 C'_3) \sin(qt-kl) \} \\ &+ E^{-kl-(u-0)t} \{ (C'_1 C'_2 - C_1 C_2) \cos(qt+kl) - (C'_1 C_2 + C_1 C'_2) \sin(qt+kl) \} \\ &+ E^{-kl-(u+0)t} \{ (C'_2 C'_3 - C_2 C_3) \cos(qt-kl) - (C'_2 C_3 + C_2 C'_3) \sin(qt-kl) \} \\ &+ E^{-kl-(u+0)t} \{ (C'_2 C'_4 - C_2 C_4) \cos(qt+kl) - (C'_2 C_4 + C_2 C'_4) \sin(qt+kl) \} \end{aligned}$$

$$\text{where } C_1 = \frac{gh + h(m+0)}{R^2 + K^2} L$$

$$C'_1 = \frac{K(m+0) - gh}{R^2 + K^2} L$$

$$u = \frac{1}{2} \left(\frac{z}{L} + \frac{z}{L} \right)$$

$$h = \sqrt{2} c \sqrt{\frac{1}{2} (R_1^2 + \rho^2 - g^2 - m^2)}$$

$$K = \sqrt{2} c \sqrt{\frac{1}{2} (R_1^2 - \rho^2 + g^2 + m^2)}$$

$$R_1^2 = \sqrt{\rho^2 + g^2 - m^2}^2 + 4g^2 m^2$$

$$h^2 + K^2 = L^2 C R_1^2$$

$$C_1 = A_1 + A'_1$$

$$C_2 = -(A_2 + A'_2)$$

$$C_3 = A_3 + A'_3$$

$$C_4 = -(A_4 + A'_4)$$

$$C_2 = \frac{gh - h(m-0)}{R^2 + K^2} L$$

$$C'_2 = \frac{K(m-0) + gh}{R^2 + K^2} L$$

$$m = \frac{1}{2} \left(\frac{z}{L} - \frac{z}{L} \right)$$

$$A = \sqrt{2} c \sqrt{\frac{1}{2} (R_2^2 + h^2 - K^2 - L^2 C m^2)}$$

$$g = \sqrt{2} c \sqrt{\frac{1}{2} (R_2^2 - h^2 + K^2 - L^2 C m^2)}$$

$$R_2^2 = \sqrt{(h^2 + K^2 + L^2 C m^2)^2 - 4L^2 C K^2 m^2}$$

$$\rho^2 + g^2 = \frac{R_2^2}{L^2 C}$$

$$C'_1 = j(A_1 - A'_1)$$

$$C'_2 = -j(A_2 - A'_2)$$

$$C'_3 = j(A_3 - A'_3)$$

$$C'_4 = -j(A_4 - A'_4)$$

SAMPLE PROBLEMS
TRANSIENT CONDITIONS
OF
TRANSMISSION LINE.

Line Constants assumed for Problem.

Assume 100 000 volts at generator sine wave star connected

$E = \text{max voltage} = 100\,000\sqrt{2} = 141\,400$ volts spacing of wire = 8 ft = 96 in. frequency 60 cycles size wire 000 copper $X_c =$ Condensare reactance $X =$ inductive reactance $L =$ inductance $C =$ capacity $\theta = 2 \pi \text{ ft.}$ $\theta_0 =$ phase at moment of starting. Charging current amps/mile/100 000 volts phase wire to neutral = 0.548 amps (Standard Handbook)

Charging current for 120 miles = 66 amperes.
Inductive reactance ohms/mile = .776 (Stand. Handbook)
Inductive reactance 120 miles $X = 93.2$ ohms #000 wire ohms per mile @ 25°C = .333 $\therefore$ for 120 miles $x = 40$ ohms
 $X_c = \frac{100\,000}{66} = 1520$ ohms.

Transient conditions of start of transmission line open at receiving end.

The stationary condition of charging current is from eq (6) p 107 Phenomena Transient $i = \frac{E}{X_c} \sin(\theta - \theta_0)$ and stationary condition of potential is from Eq. (6) p 107 $E_i = E \cos(\theta - \theta_0)$. The terms representing that component of oscillation which depends upon the phase of impressed e.m.f. or the point of the impressed sinif. wave at which the oscillation begins is given by Eq. (7) p 108

$$i'' = -\frac{E}{X_c} e^{-\frac{2\theta}{\sqrt{X_c X_c}}} \left\{ \cos \sqrt{\frac{X_c}{X}} \theta \cdot \sin \theta_0 - \left[\sqrt{\frac{X_c}{X}} \cos \theta_0 + \frac{1}{2\sqrt{X_c X_c}} \sin \theta_0 \right] \sin \sqrt{\frac{X_c}{X}} \theta \right\}$$

$$e_i'' = -E e^{-\frac{2\theta}{\sqrt{X_c X_c}}} \left\{ \cos \theta_0 \cos \sqrt{\frac{X_c}{X}} \theta + \left[\frac{1}{2\sqrt{X_c X_c}} \cos \theta_0 + \sqrt{\frac{X_c}{X}} \sin \theta_0 \right] \sin \sqrt{\frac{X_c}{X}} \theta \right\}$$

$$i = i' + i''$$

$$e = e' + e''$$

$$i' = -93 \sin(\theta - \theta_0)$$

$$e' = 141\,400 \cos(\theta - \theta_0)$$

$$i'' = -93 e^{-\frac{2\theta}{\sqrt{X_c X_c}}} \left\{ \cos \sqrt{\frac{X_c}{X}} \theta \sin \theta_0 - 4.03 \cos \theta_0 \sin 4.03 \theta \right\}$$

$$= -93 e^{-.215\theta} \left\{ \cos 4.03 \theta \sin \theta_0 - 4.03 \cos \theta_0 \sin 4.03 \theta \right\}$$

since $\frac{1}{2\sqrt{X_c X_c}} = .053$ and can be neglected

$$e'' = -141\,400 e^{-.215\theta} \left\{ \cos 4.03 \theta \cos \theta_0 + (.05 \cos \theta_0 + .25 \sin \theta_0) \sin 4.03 \theta \right\}$$

$\therefore$ oscillations produced in starting the transmission line are

$$i = i' + i'' = -93 \left\{ \sin(\theta - \theta_0) + e^{-.215\theta} [\sin \theta_0 \cos 4.03 \theta - 4.03 \cos \theta_0 \sin 4.03 \theta] \right\}$$

$$e = e' + e'' = 141\,400 \left\{ \cos(\theta - \theta_0) - e^{-.215\theta} [\cos 4.03 \theta \cos \theta_0 + (.05 \cos \theta_0 + .25 \sin \theta_0) \sin 4.03 \theta] \right\}$$

The maximum values occur when $\theta_0 = 0$ or

$$i = -93(\sin \theta - 4.03 E^{-.215\theta} \sin 4.03 \theta$$

$$e = 141400 \{ \cos \theta - E^{-.215\theta} [\cos 4.03 \theta + .05 \sin 4.03 \theta] \}$$

The ~~max~~ ^{min} values occur when $\theta_0 = 90^\circ$ and are

$$i = 93 \{ \cos \theta - E^{-.215\theta} \cos 4.03 \theta \}$$

$$e = 141400 \{ \sin \theta - E^{-.215\theta} .25 \sin 4.03 \theta \}$$

Transient conditions when opening Circuit under full load

The general equations are (Eq [18] p 113)

$$i = \frac{E}{X_c} \sin(\theta - \theta_0) + i_0 E^{-\frac{A}{X_c} \theta} \cos \sqrt{\frac{X_c}{X}} \theta$$

$$e = E \cos(\theta - \theta_0) + i_0 \sqrt{X X_c} E^{-\frac{A}{X_c} \theta} \sin \sqrt{\frac{X_c}{X}} \theta \text{ substituting values we have}$$

$$i = -93 \sin(\theta - \theta_0) + i_0 E^{-.215\theta} \cos 4.03 \theta$$

$$e = 141400 \cos(\theta - \theta_0) + 374 i_0 E^{-.215\theta} \sin 4.03 \theta$$

Assume full load is 200 amperes

$$i_0 = 200 \sqrt{2} = 283 \text{ amperes, substituting this value}$$

$$i = -93 \sin(\theta - \theta_0) + 283 E^{-.215\theta} \cos 4.03 \theta$$

$$e = 141400 \cos(\theta - \theta_0) + 105840 E^{-.215\theta} \sin 4.03 \theta$$

The values are a maximum for $\theta_0 = 0$ and non inductive circuit $i_0 = 283$

$$i = -93 \sin \theta + 283 E^{-.215\theta} \cos 4.03 \theta$$

$$e = 141400 \cos \theta + 105840 E^{-.215\theta} \sin 4.03 \theta$$

Transient conditions when rupturing a line under short circuit.

$$Z = 40 + j 93.2$$

$$Z = 101 \text{ ohms}$$

$$i_0 = \frac{141400}{101} \cos(\theta_0 + \gamma) = 1395 \cos(\theta_0 + \gamma) \text{ where } \tan \gamma = \frac{x}{r}$$

$$i = -\frac{E}{X_c} \sin(\theta - \theta_0) + E E^{-\frac{A}{X_c} \theta} \left\{ \left[\frac{\cos(\theta_0 + \gamma)}{Z} - \frac{\sin \theta_0}{X_c} \right] \cos \sqrt{\frac{X_c}{X}} \theta + \frac{\cos \theta_0}{\sqrt{X X_c}} \sin \sqrt{\frac{X_c}{X}} \theta \right\}$$

$$e = E \cos(\theta - \theta_0) - E E^{-\frac{A}{X_c} \theta} \left\{ \cos \theta_0 \cos \sqrt{\frac{X_c}{X}} \theta - \frac{\cos(\theta_0 + \gamma)}{\sqrt{X X_c}} \sin \sqrt{\frac{X_c}{X}} \theta \right\}$$

from eqs (22) p 114. Substituting the values we have

$$i = -93 \sin(\theta - \theta_0) + 1395 E^{-.215\theta} \{ [\cos(\theta_0 + \gamma) - .067 \sin \theta_0] \cos 4.03 \theta + .27 \cos \theta_0 \sin 4.03 \theta \}$$

$$e = 141400 \{ \cos(\theta - \theta_0) - E^{-.215\theta} [\cos \theta_0 \cos 4.03 \theta - 3.72 \cos(\theta_0 + \gamma) \sin 4.03 \theta] \}$$

These values are a maximum when $\frac{d}{d\gamma} = 0$

$$\frac{d}{d\gamma} = K \sin(\theta + \gamma) = 0 \text{ so that values are a maximum for } \theta_0 = -\gamma$$

$$\text{But } \tan \gamma = \frac{x}{r} = \frac{93.2}{40} = 2.33$$

$$\gamma = 67^\circ$$

$$\therefore \theta_0 = -\gamma = 67^\circ$$

Substituting these values

$$i = -93 \sin(\theta + 67) + 1395 E^{-.215\theta} \{ \cos 4.03 \theta + .105 \sin 4.03 \theta \} \text{ neglecting } .067 \cdot 93 \cos 4.03 \theta$$

$$e = 141400 \{ \cos(\theta + 67) - E^{-.215\theta} [3.9 \cos 4.03 \theta - 3.72 \sin 4.03 \theta] \}$$

START. OPEN AT Receiver									Opening Receiver Under Full-Load		
i	$e \cdot 10^3$	θ	Radians	$\sin \theta$	$E^{-215\theta}$	$\sin 4.03\theta$	$\cos \theta$	$\cos 4.03\theta$	i	$e \cdot 10^3$	
217	35.1	10	.174	.174	.963	.643	.985	.766	193	204.5	
310	110	20	.349	.342	.928	.985	.940	.174	13.9	229.6	
244	186	30	.523	.500	.894	.866	.866	-.500	-172.0	199.2	
50	223	40	.698	.643	.860	.342	.766	-.940	-289.0	139.0	
-177	201	50	.872	.766	.829	-.342	.643	-.940	-291.2	60.8	
-340	127	60	1.047	.866	.798	-.866	.500	-.500	-193.5	-2.0	
-372	294	70	1.220	.940	.769	-.985	.342	+174	-49.0	-32.0	
-271	-55.5	80	1.396	.985	.740	-.643	.174	.766	+69.5	-25.0	
-93	-101.5	90	1.570	1.000	.713	0	0	1.000	109.0	0	
+74	-98.7	100	1.745	.985	.686	+643	-.174	.766	57.5	+22.1	
145	-64.6	110	1.920	.940	.661	.985	-.342	.174	-54.8	20.5	
128	-25.6	120	2.094	.866	.638	.866	-.500	-.500	-170.5	-12.1	
7.5	-9.3	130	2.269	.766	.614	.342	-.643	-.940	-234.0	-58.6	
-136	-29.8	140	2.443	.643	.591	-.342	-.766	-.940	-217	-129.0	
-232	-82.2	150	2.618	.500	.570	-.866	-.866	-.500	-127	-174.0	
-235	-146	160	2.792	.342	.549	-.985	-.940	+174	-4.8	-190.0	
-142	-196	170	2.967	.174	.525	-.643	-.985	.766	+97.9	-175.0	
0	-283	180	3.141	0	.509	0	-1.00	1.000	144.0	-144.4	
+134	-192	190	3.316	-.174	.490	+643	-.985	.766	122.0	-106.0	
205	-145	200	3.490	-.342	.472	.985	-.940	.174	55.0	-84.0	
194	-90.2	210	3.665	-.500	.455	.866	-.866	-.500	-17.7	-80.0	
116	-50	220	3.840	-.643	.438	.342	-.766	-.940	-57.2	-92.0	
-17.2	-35	230	4.014	-.766	.422	-.342	-.643	-.940	-48.0	-106.0	
-61	-42	240	4.189	-.866	.406	-.866	-.500	-.500	+23.0	-108.0	
-57	-58	250	4.360	-.940	.392	-.985	-.342	+174	107.0	-89.1	
+1	-65.5	260	4.538	-.985	.377	-.643	-.174	.766	173.0	-50.0	
93	-51.4	270	4.712	-1.000	.364	0	0	1.000	196.0	0	
176	-13.3	280	4.887	-.985	.350	+643	+174	.766	167.5	+48.0	
212	+40	290	5.061	-.940	.337	.985	.342	.174	104.0	83.0	
186	93.7	300	5.236	-.866	.325	.866	.500	-.500	34.7	101.0	
111	132.5	310	5.409	-.766	.313	.342	.643	-.94	-12.0	102.0	
21	148	320	5.585	-.643	.301	-.342	.766	-.94	-20.2	97.0	
-47	143	330	5.760	-.500	.290	-.866	.866	-.50	+53.5	96.0	
-71	126	340	5.934	-.342	.279	-.985	.940	+174	45.5	104.0	
-48	110	350	6.108	-.174	.267	-.643	.985	.766	74.0	121.0	
0	105	360	6.283	0	.258	0	1.00	1.000	75.0	141.4	

Note:—Table Continued on page following.

$141.400 \cos \theta \times 10^3$	$105.840 \cos 215^\circ \sin \theta \times 10^3$	$105.840 \sin 4.03^\circ \times 10^3$	$283 e^{-215^\circ} \cos 4.03^\circ$	$283 \cos 4.03^\circ$	$-935 \sin \theta$	4.03°	$.215^\circ \theta$	$\theta + 67^\circ$	$\sin(\theta + 67^\circ)$	$\cos(\theta + 67^\circ)$
139	65.5	68	209	217	-16.1	40		77	.974	.225
133	96.6	104	45.7	49.3	-31.8	80		87	.998	.052
122	77.2	91.5	-126	-141	-46.5	120		97	.992	-.122
108	31.1	36.2	-229	-266	-59.8	160		107	.956	-.292
90.8	-30	-36.2	-220	-266	-71.2	200	.187	117	.891	-.454
70.6	-73	-91.5	-113	-141	-80.5	240		127	.798	-.602
48.3	-80	-104	+38.4	+49.3	-87.4	280		137	.682	-.731
24.6	-50.3	-68	161	217	-91.5	320		147	.545	-.839
0	+0	+0	202	283	-93	360		157	.391	-.92
-24.6	46.7	68	149	217	-91.5	40		167	.225	-.974
-48.3	68.8	104	32.6	49.3	-87.4	80	.413	177	.052	-.998
-70.6	58.5	91.5	-90	-141	-80.5	120		187	-.122	-.992
-90.8	22.2	36.2	-163	-266	-71.2	160		197	-.292	-.956
-108	-21.4	-36.2	-157	-266	-59.8	200		207	-.454	-.891
-122	-52	-91.5	-80.3	-141	-46.5	240		217	-.602	-.798
-133	-57.2	-104	27	+49.3	-31.8	280		227	-.731	-.682
-139	-35.8	-68	114	217	-16.1	320		237	-.834	-.545
-141.4	+0	+0	144	283	+0	360		247	-.92	-.391
-139	33.4	68	106	217	16.1	40	.713	257	-.974	-.225
-133	49.2	104	+23.2	49.3	31.8	80		267	-.998	-.052
-122	41.6	91.5	-64.2	-141	46.5	120		277	-.992	+.122
-108	15.9	36.2	-117	-266	59.8	160	.826	287	-.956	.292
-90.8	-15.3	-36.2	-112	-266	71.2	200	.862	297	-.891	.454
-70.6	-37.1	-91.5	-57.2	-141	80.5	240	.90	307	-.798	.602
-48.3	-40.8	-104	19.3	49.3	87.4	280	.937	317	-.682	.731
-24.6	-25.6	-68	81.6	217	91.5	320	.976	327	.545	-.839
-0	+0	+0	103	283	93	360	1.012	337	-.391	.92
+24.6	23.8	68	76	217	91.5	40	1.05	347	.225	.974
48.3	35.1	104	16.6	49.3	87.4	80	1.088	357	-.052	.998
70.6	29.7	91.5	-45.8	-141	80.5	120	1.125	367	.122	.992
90.8	11.3	36.2	-83.2	-266	71.2	160	1.161	377	.292	.956
108	-10.9	-36.2	-80	-266	59.8	200	1.20	387	.454	.891
122	-26.5	-91.5	-41	-141	46.5	240	1.237	397	.602	.798
133	-29	-104	13.7	+49.3	31.8	280	1.275	407	.731	.682
139	-18.1	-68	58	217	16.1	320	1.32	417	.839	.545
141.4	+0	0	73	283	0	360	1.356	427	.92	.391

RUPTURING SHORT CIRCUIT

θ	$1395^{\circ} \times \cos 4.03^{\circ}$	$105^{\circ} \times 1395^{\circ} \times \sin 4.03^{\circ}$	$1395^{\circ} \times \cos 4.03^{\circ}$	$146^{\circ} \times \sin 4.03^{\circ}$	$-93 \sin(\theta + 69^{\circ})$	i
10	1070	94	1030	90	-90	1030
20	243	145	225	135	-93	267
30	-697	127	-623	113	-92	-602
40	-1310	50	-1130	43	-89	-1176
50	-1310	-50	-1085	-41	-83	-1209
60	-697	-127	-562	-101	-74	-731
70	+243	-145	+187	-111	-63	+13
80	1070	-94	792	-70	-51	670
90	1395	0	993	0	-36	957
100	1070	+94	734	+64	-21	777
110	243	145	160	96	-5	251
120	-697	127	-444	81	+11	-352
130	-1310	50	-805	31	27	-747
140	-1310	-50	-773	-29	42	-760
150	-697	-127	-397	-72	56	-413
160	+243	-145	+133	-80	68	+121
170	1070	-94	562	-49	78	591
180	1395	0	710	0	86	796
190	1070	+94	525	+46	91	662
200	243	145	115	68	93	276
210	-697	127	-317	58	92	-167
220	-1310	50	-574	22	89	-463
230	-1310	-50	-552	-21	83	-490
240	-697	-127	-283	-51	74	-260
250	+243	-145	+952	-57	63	+101
260	1070	-94	403	-35	51	419
270	1395	0	507	0	36	543
280	1070	+94	374	+33	-21	386
290	243	145	82	49	-5	126
300	-697	127	-227	41	-11	-197
310	-1310	50	-410	16	-27	-421
320	-1310	-50	-394	-15	-42	-451
330	-697	-127	-222	-37	-56	-295
340	+243	-145	+68	-40	-68	-40
350	1070	-94	286	-25	-78	173
360	1395	0	360	0	-86	274

TRANSIENT CONDITIONS OF TRANSMISSION LINE

STARTING CONDITIONS WITH LINE OPEN AT RECEIVING END.

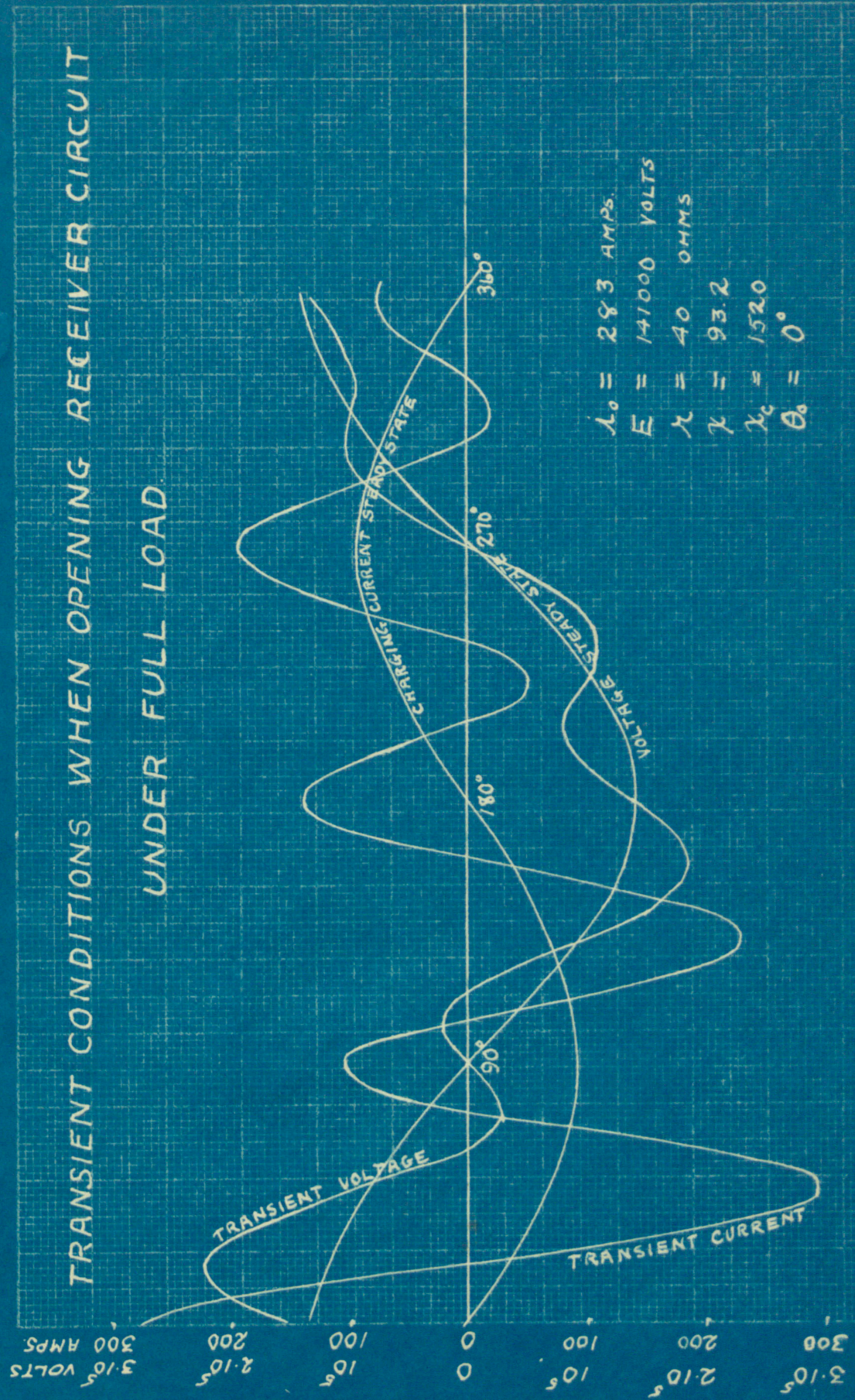
$\theta_0 = 0$ — MAXIMUM CASE

2X10⁵ VOLTS
400 AMPS
2X10⁵ VOLTS
400 AMPS

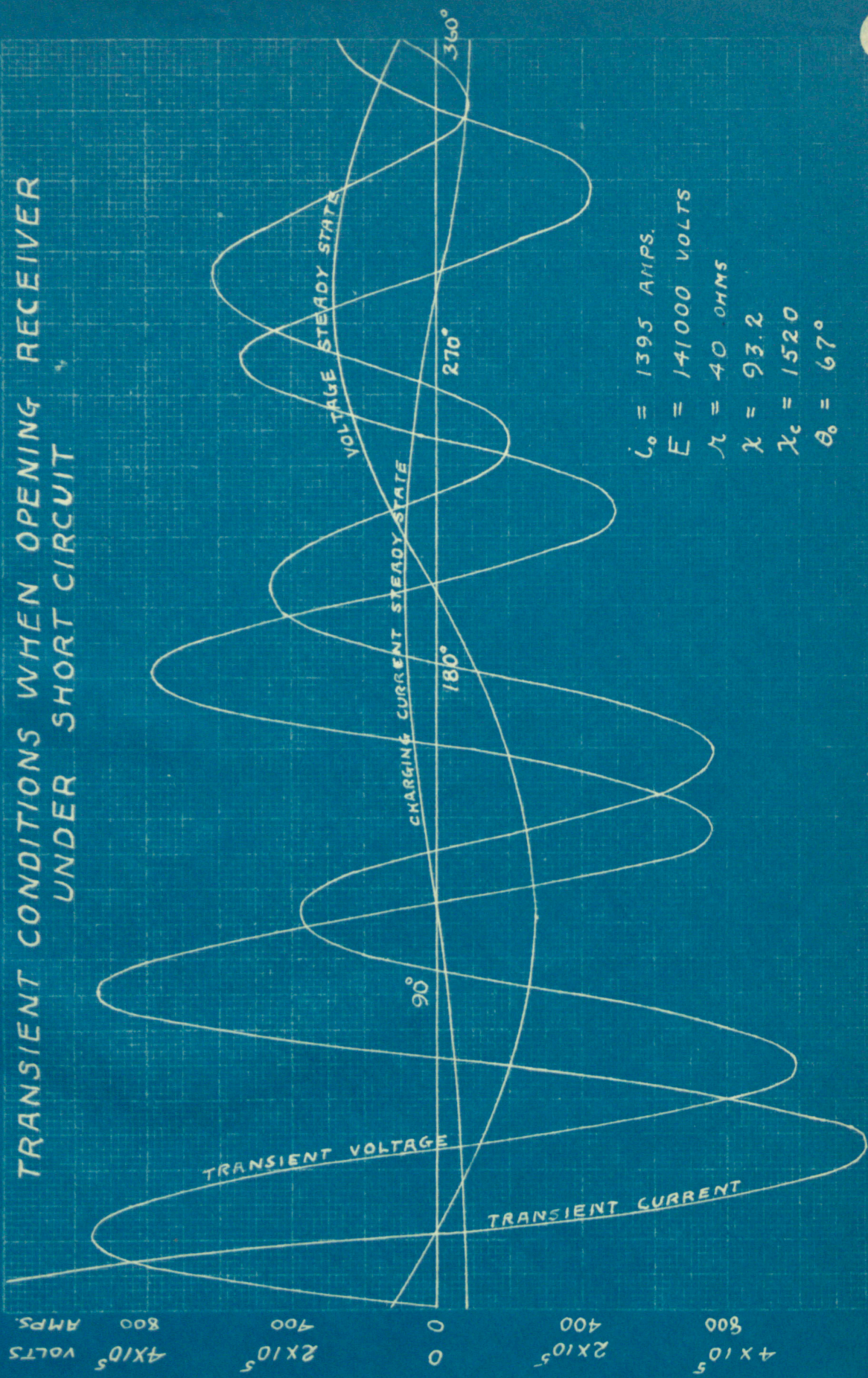


$E = 141000$ VOLTS
 $R = 40$
 $X = 93.2$
 $X_c = 1520$
 $\theta_0 = 0$

TRANSIENT CONDITIONS WHEN OPENING RECEIVER CIRCUIT
UNDER FULL LOAD



TRANSIENT CONDITIONS WHEN OPENING RECEIVER
UNDER SHORT CIRCUIT



$I_0 = 1395$ AMPS.
 $E = 141000$ VOLTS
 $r = 40$ OHMS
 $\chi = 93.2$
 $\chi_c = 1520$
 $\theta_0 = 67^\circ$