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ESTIMATION OF OD MATRIX FROM TRAFFIC COUNTS IN GREATER CINCINNATI AREA

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Abstract

The thesis is to estimate the OD matrix from two known data sets, an outdated OD matrix at traffic analysis zone (TAZ) level and the observed traffic counts on some links of the street network in the Greater Cincinnati Area, Ohio. The problem is treated as a bilevel programming problem. A Gauss-Seidel type method is applied to solve the problem, iteratively searching a descending direction and finding an appropriate step size.

The estimated OD matrix results in a traffic pattern closer to reality than the outdated OD matrix, which indicates that the estimation is robust. The estimated OD matrix is applied to estimate the racial composition of drivers on individual streets. The analysis of the number of drivers entering and leaving a TAZ helps examine the distribution of population and employment over time.

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Table of Contents

List of Tables

List of Figures

I. Introduction

The main purpose of this study is to estimate the origin-destination (OD) matrix from two known data sets, an outdated OD matrix at traffic analysis zone (TAZ) level and the observed traffic counts on some links of the street network in the Greater Cincinnati Area, Ohio. Combined with demographic data from the U.S. Census, the estimated OD matrix is then applied to estimate the racial composition of drivers on individual streets in the study area.

1.1 Background to the Problem

Why do we need to estimate an OD matrix? Let us first see what an OD matrix is. An origin-destination (OD) matrix, or an OD trip table, is a fundamental input in almost all transportation planning applications. The elements in the matrix represent the number of travel trips between different zones of a region during a specific period of time. The information contained in the table is often used to predict traffic flows on various routes and links of a road network.

However, the traffic demand data cannot be observed directly and thus the OD matrix cannot be obtained easily. Historically the OD matrix has been established through costly and time consuming interviews or surveys. Also, the trip tables soon become out-of-date if there is a rapid change in land-use development. Nevertheless, it is possible to obtain an OD matrix through cheaper and quicker means. One may obtain a rational estimate of trip table from traffic counts and other easily available information including an outdated OD matrix. That is why the estimation of an OD matrix is necessary in many cases.

Other problems come up. How to define the OD matrix estimation? And more specifically, how do we estimate?

The OD matrix estimation problem is defined by Cascetta and Nguyen (1988) as determining "an estimate of the O/D trip demand matrix T by efficiently combining traffic count based data and all other available information." The estimation is to find out an OD matrix which, when assigned to the transportation network, reproduces the observed traffic volumes on some links of the network (Abrahamson, 1998). Thus it can be treated as the "inverse" of the traffic assignment problem.

Traffic assignment may be defined as the process of allocating the demand for traffic between every pair of zones (OD zones) to a specific transportation network. It is one of the four steps in traditional transportation planning procedure, which include trip generation, trip distribution, modal split, and traffic assignment. From the assignment of the OD matrix the traffic flow on each network link may be obtained according to the selected assignment procedure.

But the OD matrix estimation problem is much more complicated to solve than assigning an OD matrix (known traffic demand) to the transportation network. Since the number of observed links is generally much less than the number of OD pairs in the trip matrix, there are multiple solutions to the problem. In order to determine a unique solution, one may ask for the most likely matrix that reproduces the observed flows. Besides link traffic counts, most matrix estimation techniques also rely on prior information in the form of an OD matrix – an externally generated trip table, or target trip table. The target OD matrix could be obtained by a sample survey or from an old (probably outdated) trip table. The added information contained in the target OD matrix is sufficient to select a single OD matrix that reproduces observed traffic counts (LeBlanc and Farhangian, 1982).

1.2 Scope and Contribution of the Thesis

This research is an extension of the Police Vehicle Stops Study which was carried out by the Department of Geography and the Division of Criminal Justice at the University of Cincinnati in the past two years. The Police Vehicle Stops Study is to determine in the City of Cincinnati if there is racial disparity in drivers the police stop, taking into account the racial characteristics of people driving vehicles. In order to obtain driving population, it is essential to estimate the traffic flows as well as their racial composition on individual streets by combining demographic data from the U.S. Census with traffic modeling methods used in transportation planning. Since the police vehicle stops data were collected in 2000, the population and traffic demand data should also be in the same period for time consistency. The most popular source for traffic demand data is the U.S. Department of Transportation which provides data at Traffic Analysis Zones (TAZs) level. However, the latest traffic demand data at TAZ level are for 1990. The 2000 data will not be available until the beginning of 2004. Since there have been a rapid change in land-use development and migration of population and employment, the traffic demand data for 1990 are obviously outdated and are not suitable for the situation in 2000.

Therefore, an estimation of the traffic demand (OD matrix) for 2000 is necessary to obtain more accurate results. The thesis is to estimate the OD matrix from two known data sets, an outdated OD matrix for 1990 at TAZ level and the observed traffic counts on some links of the street network in 2000 in the Greater Cincinnati Area, Ohio. To examine the robustness of the estimation, the estimated OD matrix will be assigned to the street network, and the result of assignment will be compared with that from the old OD matrix.

The literature shows that various approaches to estimating the OD matrix have been developed and tested. But few of the proposed methods are applied to large scale problems such as our case. This thesis applies an approach of mathematical programming and successfully solves the problem. Thus in terms of its practice the thesis has a contribution to the literature in the field of transportation planning.

In this thesis, the estimated OD matrix will be applied to estimate the racial composition of drivers on individual streets combined with demographic data for 2000 from the U.S. Census. The conventional method to obtain such information is through observation or video recording which is costly and time consuming. Furthermore, the accuracy is not high. The approach in this study is rather efficient and practical and thus is worth reference.

1.3 Organization of the Thesis

The thesis is arranged in the following way. This chapter introduces the motivation of the study and its importance. The next chapter describes the problem and provides an up-to-date review of the research advances in the study of the OD matrix estimation. Chapter three details the study area, data sources and data processing, followed by Chapter four that discusses methodology employed in the study. Chapter five presents and analyzes the results, and illustrates the applications of the estimated OD matrix. In Chapter six, we provide a summary of the thesis and propose some possible research directions for further study. The bibliography is list in the final chapter.

II. Problem Formulation and Literature Review

In this chapter, we first group and present the formulations, some properties and notations of the OD matrix estimation problem. We then provide an up-to-date review of the research advances in the study from various aspects.

2.1 Problem Specification

To formulate the OD matrix estimation problem more clearly, we use the following notation as does Chen (1994) and Florian and Chen (1992). Let *R(N,A)* represent a transportation network. Here, *N* is the set of nodes representing centroids of zones where trips are generated and/or where trips terminate. *A* denotes the set of corresponding directed links. Let *I* denote the index set of the origin-destination pairs of the network R. Denote $v = (v_a; a \in A)$ the link flow vectors, $\hat{v} = (\hat{v}_a; a \in \hat{A})$ the observed link flow vectors and $\hat{A}$ is a subset of *A*, $h = (h_k, k \in K_i, i \in I)$ the path flow vectors, where K_i representing the set of all paths between OD pair i, and $g = (g_i, i \in I)$ the demand vectors for all OD pairs. Note that a path is the route on which the flow between an OD pair is loaded. A path usually consists of many links.

As stated above, OD matrix estimation seeks to determine an estimate of the OD demand *g* using the observed link volumes and possibly prior information about the OD matrix. Additional data (prior information) and/or assumptions about the travel behavior are needed to determine a unique OD matrix. A crucial point in the estimation is the assignment technique used: which route(s) in the network *R* do trips between different zones take. There are two assumptions of assignment depending on the treatment of congestion: proportional assignment and nonproportional assignment.

In proportional assignment, it is assumed that the traffic volumes and the proportions of trips between each OD pair are independent. The link volumes (v_a) are proportional to the OD flow g_i. The proportion of trips between each OD pair that use a particular link for each link in the network depends on exogenously determined coefficients, i.e., route characteristics. The most popular method is "all-or-nothing" assignment in which all trips are loaded on the single least impedance path between each OD pair.

In contrast with proportional assignment assumption, a nonproportional assignment approach becomes more appropriate wherever network congestion effects are prominent. The cost for traveling on a link depends on the flow volume. This approach is based on the well-known Wardrop's first equilibrium principle (Wardrop, 1952): the traffic system is stable (in "equilibrium") when no traveler (user) can improve his travel time by switching to another route. The principle is referred to as user equilibrium. In this case, the proportions of trips between each OD pair will depend on the volume on all links and cannot be determined independently of the OD matrix estimation process.

A fundamental part of the prior information is the target OD matrix. The estimation is to minimize the distance between the estimated OD matrix and the target OD matrix subject to the flow constraints, the observed link volumes. However, the observed link flows might not be consistent, or might not satisfy the user equilibrium conditions because of the existence of measurement errors as well as temporal fluctuations. Thus, the estimation also seeks for small difference between the estimated link flows and the observed traffic volumes.

The mathematical formulation of the OD matrix estimation may be stated as follows:

$$\min F(g,v) = \gamma_1 F_1(g,\hat{g}) + \gamma_2 F_2(v,\hat{v}) \tag{2.1}$$

$$\text{s.t.} \begin{cases} v = assign(g) \\ v, g \geq 0 \end{cases}$$

where $F_1(g,\hat{g})$ and $F_2(v,\hat{v})$ are generalized distance measurement, or errors between the estimated trip demand g and the target OD matrix $\hat{g}$, and between the estimated flow pattern v and the observed traffic volumes $\hat{v}$, respectively. γ_1 and γ_2 are predetermined weights, which reflect the different confidence in the observed traffic counts $\hat{v}$ and the reference matrix $\hat{g}$. If the target OD matrix is very reliable γ_1 should be large compared to γ_2. In consequence, the estimate g would be close to $\hat{g}$, and then larger deviations between v and $\hat{v}$ would be accepted. In the above model, $v = assign(g)$ represents the results from proportional or nonproportional assignment. The constraints can be formulated in another form:

$$\sum_{i \in I} p_{ia} g_i = \hat{v}_a, \quad a \in \hat{A}$$

$$p_{ia} = P_{ia}(g), \quad a \in \hat{A}, \quad i \in I$$

where p_{ia}'s are the proportions which assign demand g_i to arc a, $P_{ia}(g)$ is a map from demand set G to $I \times A$, whose explicit functional form depends on the network assignments.

2.2 Statistical Inference Approaches

Various approaches to estimating the OD matrix have been developed and tested. Different assignment assumptions or different optimality criteria result in a large number of models for OD matrix estimation. There are three types of approaches, namely, statistical estimation methods, models based on maximum-entropy/minimum-information theory, and network equilibrium based techniques (Sherali, 1994).

The first category, statistical inference approaches, based on prior information, employ Maximum Likelihood (ML), General Least Squares (GLS) and Bayesian Inference techniques. Here the target OD matrix is assumed to come from a sample survey and the traffic volumes are also assumed to be generated by some probability distributions. The OD matrix is then estimated as parameters of the statistical distributions.

The Maximum Likelihood (ML) approach seeks the OD matrix g to maximize the likelihood of observing the target OD matrix $\hat{g}$ and the observed traffic counts $\hat{v}$ conditional on the true (estimated) OD matrix g, which is expressed as:

$$L(\hat{g},\hat{v} \mid g) = L(\hat{g} \mid g) \times L(\hat{v} \mid g)$$

The equation holds due to the independence assumption on the observed traffic counts and the target OD matrix. Spiess (1987) applies the ML and solve the problem using an algorithm of the cyclic coordinate ascent type.

The Generalized Least Squares approach assume that the target OD matrix $\hat{g}$ may be obtained from the estimated OD matrix g with a probabilistic error term and traffic counts may be obtained from a stochastic equation (Cascetta 1984, Bell 1991). This approach only requires the existence of dispersion matrices. The GLS estimator may be obtained by solving:

$$\begin{aligned} \min \quad & \tfrac{1}{2}(g-\hat{g})'Z^{-1}(g-\hat{g}) + \tfrac{1}{2}(v(g)-\hat{v})'W^{-1}(v(g)-\hat{v}) \\ \text{s.t.} \quad & g_i \geq 0 \end{aligned}$$

where Z and W represent the dispersion matrices of the error terms of the target OD matrix $\hat{g}$ and the observed traffic counts $\hat{v}$, respectively.

If the target OD matrix and traffic counts are assumed to follow multivariate normal (MVN) distribution, the GLS estimator can be shown to coincide with a ML estimator (Abrahamson, 1998).

The Bayesian inference approach considers the target OD matrix and the observed traffic counts as two sources of information about the estimated matrix g and combine them:

$$f(g \mid \hat{v}) \approx L(\hat{v} \mid g) \times \Pr(g)$$

where $f(g \mid \hat{v})$ is the posterior probability of observing g conditional on the observed traffic counts $\hat{v}$, $\Pr(g)$, a prior probability function of g, is a source of information about g from the target OD matrix, and $L(\hat{v} \mid g)$ represents the probability of the observed traffic counts as another source of information. The observed traffic counts are usually assumed to be a MVN distribution and the resulting estimated OD matrix also becomes MVN distributed (Maher, 1983).

The Bayesian Inference method is a statistical inference technique with properties in common with the ML and GLS approaches, which contain the sum of two parts relating to the target OD matrix $\hat{g}$ and the observed traffic counts $\hat{v}$. But in classical inference approaches (ML and GLS), g_i are assumed to be parameters of the likelihood function $L(\hat{g}, \hat{v} \mid g)$, while in Bayesian Inference approach they are random variables with given prior distributions (Abrahamson, 1998).

2.3 Models Based on Maximum-entropy/Minimum-information Theory

This category of approaches to estimate OD matrix attempt to determine the most likely OD matrix that is consistent with the information contained in observed traffic counts, by either maximizing the entropy or using a minimum-information based objective function with respect to a target OD matrix.

A "maximum entropy" OD matrix is an OD matrix with the largest number of associated microstates (most dispersed distribution of g_i's) which, when assigned to the network, reproduces observed traffic counts. Van Zuylen and Willumsen (1980) show that the matrix is obtained by solving the problem:

$$\max \; -\left(\sum_i g_i \ln g_i - g_i\right)$$

$$\text{s.t. } \sum_i g_i p_{ia} = \hat{v}_a \quad \text{all } a \in \hat{A} \subset A$$

A "minimum information" OD matrix is an OD matrix that adds as little information as possible to the information in the target OD matrix, while considering the equations relating the observed traffic counts with the estimated OD volumes. The estimated matrix is obtained from minimizing the following function:

$$I = \sum_i g_i \ln(g_i / \hat{g}_i)$$

When a target matrix is assumed equally probable, i.e. $\hat{g}_i = 1$, it becomes an entropy maximizing problem.

2.4 Network Equilibrium Based Techniques

In the case of congested networks, most general optimization problems of the previous two sections have a bilevel structure. The upper level is the OD matrix estimation problem to determine g and the lower level solves the equilibrium assignment problem.

Fisk (1988) extends the entropy model by Van Zuylen and Willumsen to the congested case by intruding the user-equilibrium conditions as constraints, and proposed a model with a bilevel structure that maximizes the entropy on the upper level and solves a user-equilibrium problem on the lower level.

Fisk (1989) examines several existing estimation approaches including Nguyen's equilibrium model, Van Zuylen and Willumsen's entropy maximizing model, and the combined distribution/assignment problem. It is shown that these formulations can be expected to produce the same results when observed traffic counts form an equilibrium flow pattern.

Yang et al. (1992) integrate equilibrium assignment of the OD matrix into the GLS technique using a Stachelberg leader-follower optimization model. The OD matrix estimation problem is formulated as a bilevel program with the GLS problem on the upper level and the equilibrium assignment problem on the lower level. The uncertainties in both the target OD matrix and the traffic counts are considered and a heuristic solution algorithm is provided.

Network equilibrium problems were also considered by Spiess (1990), Florian and Chen (1992) and Chen (1994). The OD matrix estimation techniques are based on mathematical programming concepts, mainly gradient based solutions. The related solution algorithms have been applied to large city networks.

III. Study Area and Data Processing

This chapter details the study area, data sources and data processing. The data for the study include target OD matrix, street network and traffic counts.

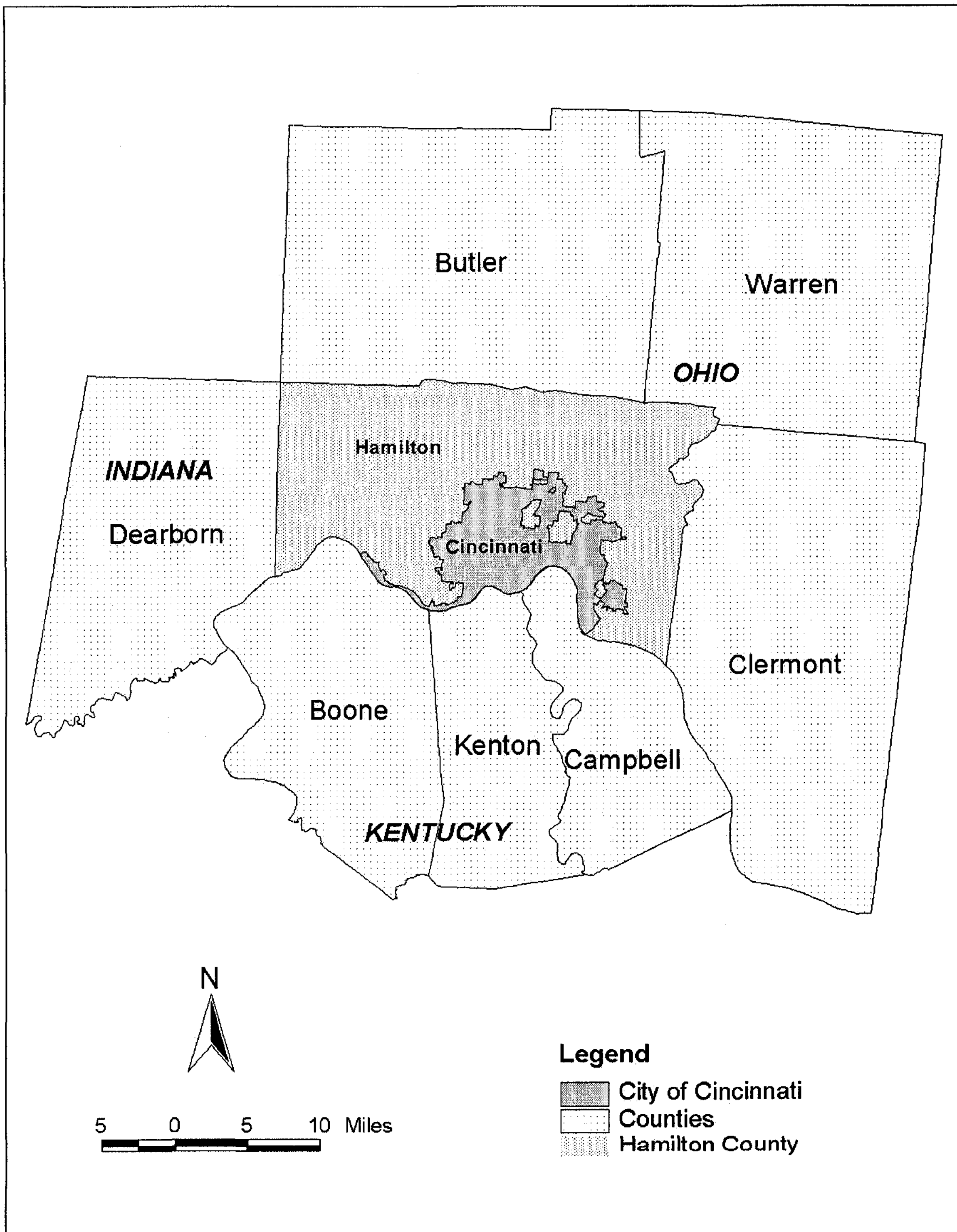
3.1 Study Area

This project updates outdated trip table and then applies it to estimate the number of commuters of different races in the streets in the City of Cincinnati. However, in order to minimize the boundary effect, the matrix estimation is performed for a much extensive territory – the Greater Cincinnati Area (Figure 3-1). The area includes four Ohio counties (Butler, Clermont, Hamilton and Warren), three counties in Kentucky (Boone, Campbell and Kenton) and Dearborn County in Indiana.

3.2 Target OD Matrix

The target OD matrix for this study is an outdated OD matrix that consists of information for 1990, which has been used in previous study of traffic assignment in Hamilton County by Vlassova (2002). According to Vlassova (2002), the source of the matrix was the Urban Element Component of the Census Transportation Planning Package (CTPP) (BTS, 1995), Part 3 of which contains journey-to-work information taken from sampled data (long census form) collected from about one in six households. These data were then grouped by residence of commuters and the values referring to the morning rush hour period (6:30am to 8:29am) were extracted for the processing. The number of commuters is the aggregate number of vehicles used to travel to work in the peak period by workers 16 years and over.

Figure 3-1: Study area



Transportation Analysis Zones (TAZs) are delineated by metropolitan planning organizations to organize the data in CTPP. TAZs polygons are the aggregation of Census blocks for the purposes of transportation modeling. The Greater Cincinnati Area has 997 TAZs, whose areas range from 0.01 to 40 square mile depending on the density of network and/or population. Figure 3-2 shows the TAZ boundaries in the study area. The figure also illustrates 11 exit points that represent the commuters whose trips begin in the study area but destinations are outside. They are located at the intersections of the border and the highways that commuters are most likely to take when traveling outside the area. Thus these points represent the destinations for outbound trips. The trips generated outside the area but ended in the area are not considered because they are widely dispersed and unlikely influence the traffic pattern within the City of Cincinnati.

The original trip table used by Vlassova (2002) contains about 36,000 records, each representing traffic flows between a pair of TAZs. Flow directions are not considered. Most traffic flows outside Highway 275 loop were removed from the table as they either bypass or pass the City through interstate highways and unlikely affect the traffic pattern of local streets within the City if excluded (Figure 3-3). That is to say, those flows would not change the flow pattern of on the streets with observed traffic counts (all within the City). The information from traffic counts has nothing to do with those records and would leave them untouched during traffic demand adjustment procedure. After elimination, the based OD matrix to be updated contains 25509 records, including flows that begin and/or end within the circle of 275 and flows passing streets in the city highway loop. The matrix reflects the movement of nearly 222,500 commuters, while the total number of flows in the old matrix is about 340,000.

Figure 3-2: Transportation Analysis Zones

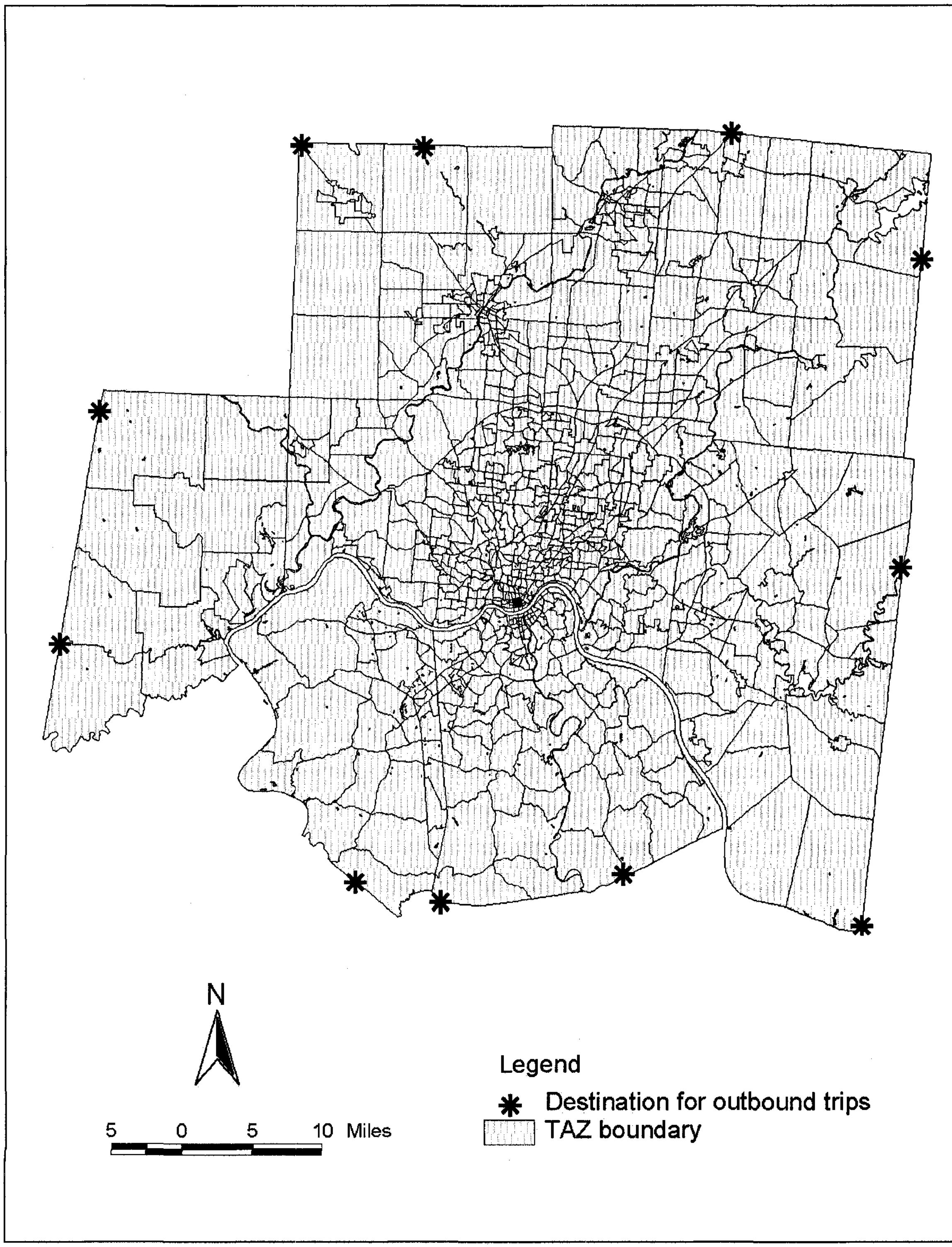
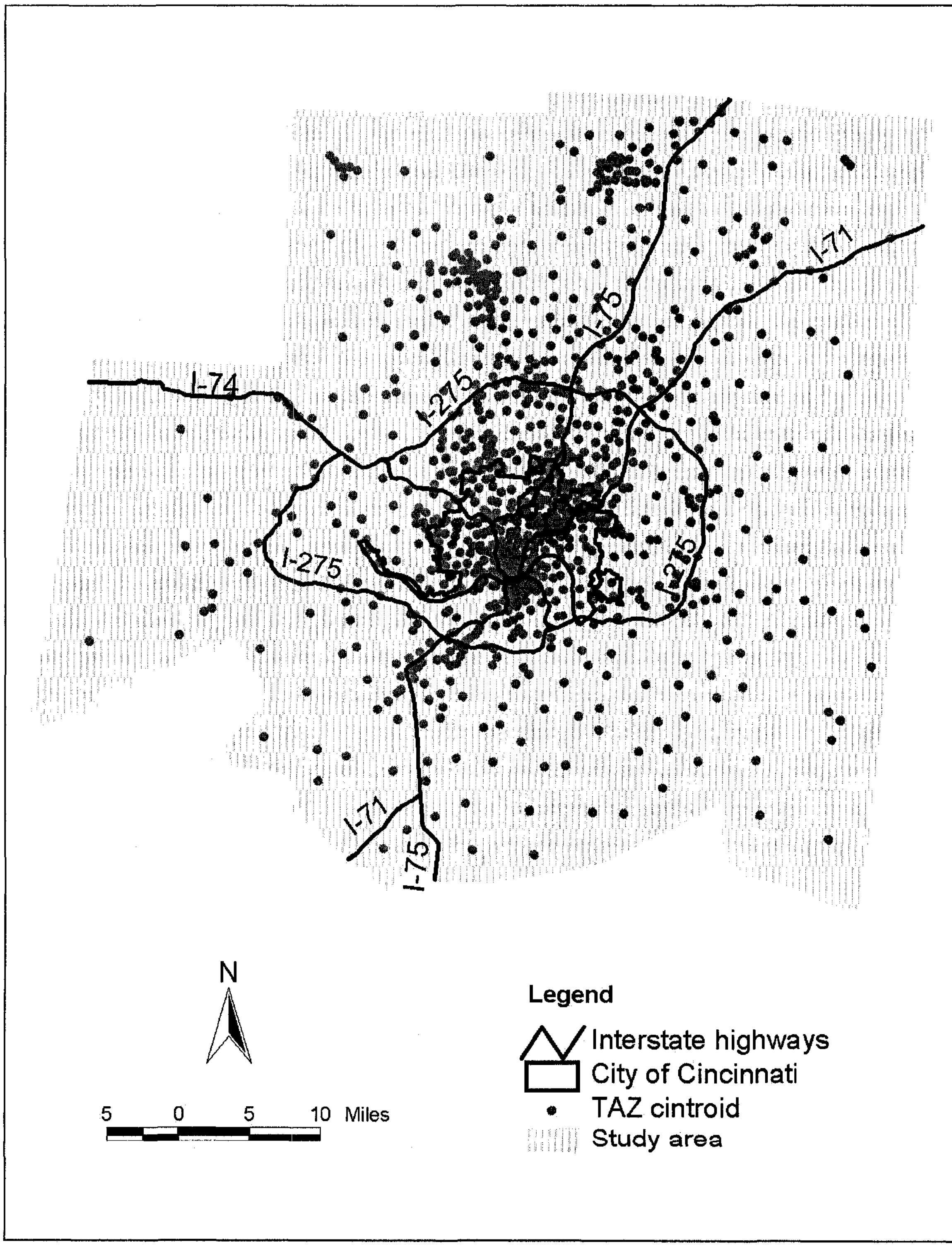


Figure 3-3: TAZ's and highways



3.3 Street Network

A complete street network by counties was downloaded from the official Bureau of Census Website. The downloaded TIGER/Liner files contained information on street category which is coded by a census feature class code (CFCC) to describe the road's characteristics. The CFCC, as used in the TIGER/Liner files, is a three-character code. The first character is a letter describing the feature class; the second character is a number describing the major category; and the third character is a number describing the minor category. Roads with code "A" (highways and local streets) were extracted (Table 3-1).

Table 3-1: Street categories

("*", the third character, can be any number among 0-9)

A1*	Primary Roads With Limited Access (interstate highway)
A2*	Primary Roads Without Limited Access (US highways)
A3*	Secondary and Connecting Roads (state highways)
A4*	Local, Neighborhood, and Rural Roads (local street)
A5*	Vehicular Trails
A6*	Road with Special Characteristics (ramp, etc)
A7*	Road as Other Thoroughfare (service road, etc)

In ArcView GIS, county street networks were merged into one file corresponding to the whole study area. The street category made it possible to classify free flow speed and maximum peak hour undirected capacity, which are needed to calculate link cost during traffic assignment procedure. This information was derived from Highway Capacity Manuals (NRC, 1998). Table 3-2 illustrates the simplified classifications of free flow speed and maximum peak hour undirected capacity (2-hour period). For instance, the capacity of freeway ranges from 750 to 1300 pcphpl (cars per hour per lane), depending on the level of service which measures maximum density of vehicles.

Table 3-2: Classifications of speed and capacity

Category	Free flow speed (ml/h)	2-hour capacity
A1	55	13600
A2	45	7200
A3	35	5400
A4	30	3200
A5	30	2000
A6	40	2400
A7	25	2000

The merged network consists of 98,232 links (street segments). It would be time-consuming to perform iterative traffic assignment on such a large-scale network. However, it's possible to reduce the network size by deleting some unnecessary links and merging links without intersecting streets.

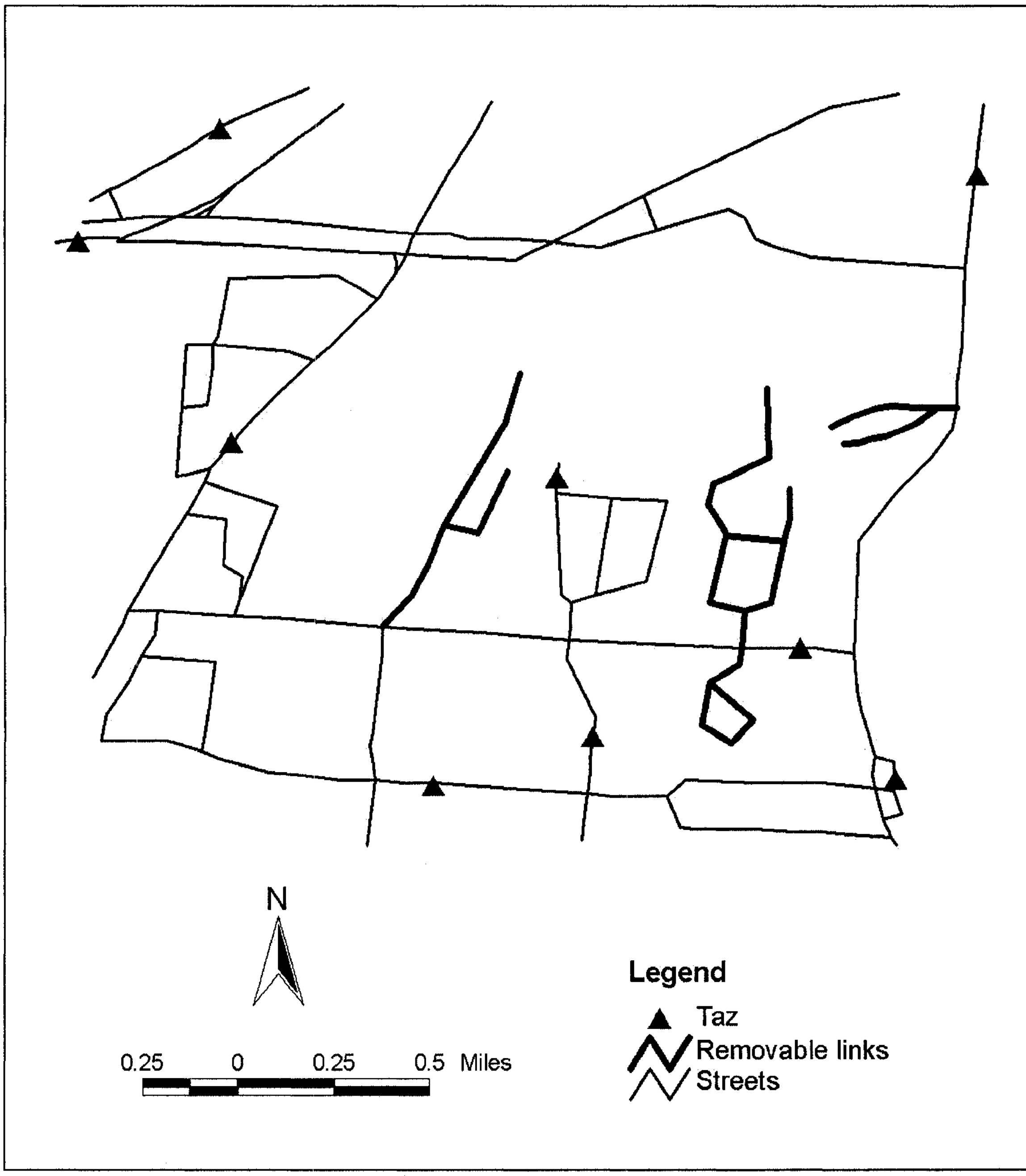
As each traffic analysis zone is represented by its centroid, the traffic flows may never be loaded on some street segments when assigned to the network. For example, after traffic assignment, no matter which assignment technique is used, the highlighted links in Figure 3-4 will unlikely load any traffic flow. Thus they can be eliminated from the network.

Most local streets and secondary roads outside Interstate-275 loop were also removed from the network with the assumption that the traffic flow pattern on observed counted streets would not change too much after the elimination.

The third approach to reduce the network dimension was to merge street segments between adjacent intersections if they are in the same street category.

After such 3-step process, the network size was decreased to totally 13,679 links. This significantly improved iterative traffic assignment algorithm.

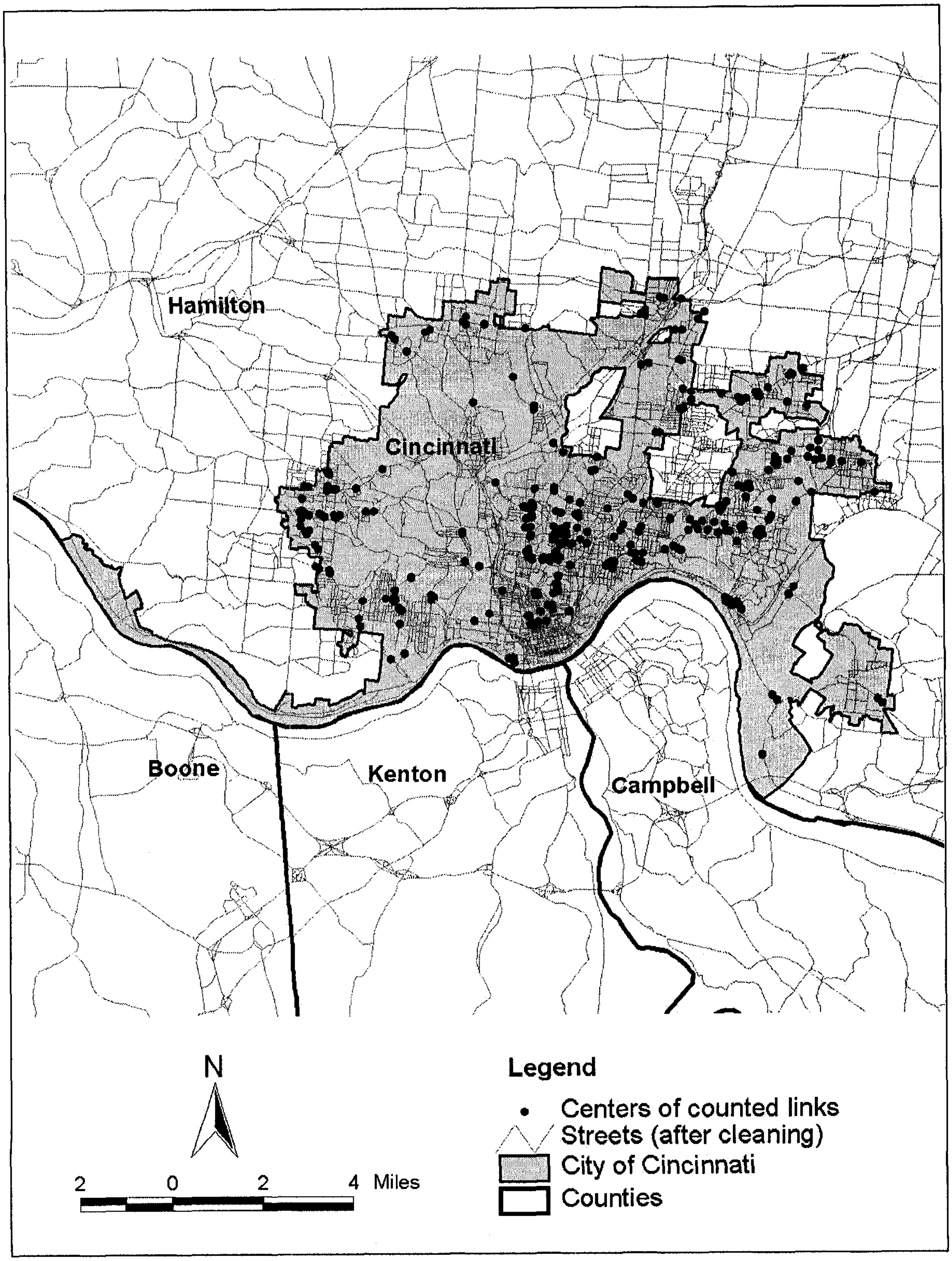
Figure 3-4: Removable links



3.4 Counted streets

The observed traffic counts were taken from 2000 traffic count record by Cincinnati Department of Public Services. It contains data on Average Daily Traffic (ADT) and its quarterly distribution. If ADT values in both directions on a street segment are known, they are combined as undirected flows. In the case of one direction, the undirected flow is obtained by multiplying ADT value by 2. To compute rush hour traffic using ADT, the proportion of rush hour flow for each counted street is assumed to be 0.115 of ADT which is the average proportion of 80 sampled counted streets. The number of counted links is 315, 2.3% of total number of links in the network (Figure 3-5).

Figure 3-5: Counted street locations



IV. Methodology

This chapter discusses methodology employed in the study, including mathematical formulation of the problem, the algorithm and traffic assignment technique.

4.1 Basic concepts in optimization problems

The model developed in this paper is stated as a constrained, nonlinear minimization problem. The problem is to choose the values of a set of I variables, $x = (x_1, x_2, \ldots, x_I)$, that minimize a nonlinear function (usually termed the *objective function*), yet complying with certain *constraints*. The set of all possible values of $x = (x_1, x_2, \ldots, x_I)$ that comply with all the constraints is called the *feasible region.* A general minimization program with J constraints can be written as

$$\min \ F(x_1, \ldots, x_I) \tag{4.1a}$$

$$\text{s.t. } C_j(x_1, \ldots, x_I) \geq b_j; \ \ j = 1, \ldots, J \tag{4.1b}$$

The special case of the mathematical programs is when they are unconstrained (without equation 4.1a). In this case, the first-order condition for a minimum at $x = x^* = (x_1^*, x_2^*, \ldots, x_I^*)$, the gradient of *F(x)* vanish at x^*. The gradient of F(x) with respect to x, $\nabla_x F(x)$, is the vector of partial derivative:

$$\nabla_x F(x) = \left(\frac{\partial F(x)}{\partial x_1}, \frac{\partial F(x)}{\partial x_2}, \ldots, \frac{\partial F(x)}{\partial x_I} \right)$$

At every point x, the gradient aims in the direction of the steepest (local) increase in F(x). Its first-order derivatives with respect to all of its variables must be equal to zero:

$$\nabla F(x^*) = 0, \text{ that is, } \frac{\partial F}{\partial x_i} = 0; \ \ \forall i \tag{4.2}$$

These are the necessary first-order conditions. Regarding the uniqueness of the minimum, if $F(x)$ is strictly convex everywhere, the minimum will be unique, global. A function is said to be everywhere *strictly convex* is all points on the straight line joining any two points on the graph of the function are above the graph.

In the case of constrained optimization problems, the necessary first-order optimality conditions are the Karush-Kuhn-Tucker (KKT) conditions. These conditions state that if a point x^* is an optimal solution to the problem, then there exist a set of quantities λ_j, called the *dual variables* as well as *Lagrange multipliers*, such that, at the solution

$$\frac{\partial F(x^*)}{\partial x_i} - \sum_j \lambda_j \frac{\partial C_j(x^*)}{\partial x_i} \geq 0; \quad \forall i \tag{4.3a}$$

$$x^* \left[\frac{\partial F(x^*)}{\partial x_i} - \sum_j \lambda_j \frac{\partial C_j(x^*)}{\partial x_i} \right] = 0; \quad \forall i \tag{4.3b}$$

$$\lambda_j \left[C_j(x^*) - b_j \right] = 0; \quad \forall j \tag{4.4a}$$

$$C_j(x^*) \geq b_j; \quad \forall j \tag{4.4b}$$

$$x_i \geq 0; \quad \forall i \tag{4.4c}$$

$$\lambda_i \geq 0; \quad \forall j \tag{4.4d}$$

For a maximization problem, the direction of Inequations (4.3a) and (4.4b) is reversed, to $\leq$.

The KKT conditions may be considered as the generalization of the conditions for optimality of unconstrained programs, Formula (4.2). When all constraints in the optimization problem are equalities, the KKT conditions may be expressed equivalently in a somewhat simpler form. In this case, the Lagrangian of the optimization problem may be formed:

$$G(x,\lambda) = F(x) - \sum_j \lambda_j \left[C_j(x^*) - b_j)\right] \quad (4.5)$$

Then with respect to the Lagrangian, the KKT may be stated as:

$$\frac{\partial G(x,\lambda)}{\partial \lambda_j} = 0; \quad \forall j \quad (4.6a)$$

$$x_i \frac{\partial G(x,\lambda)}{\partial x_j} = 0; \quad \forall i \quad (4.6b)$$

$$\frac{\partial G(x,\lambda)}{\partial x_j} \geq 0; \quad \forall i \quad (4.6c)$$

Regarding the existence of solutions to constrained optimization programs, if the feasible region is defined by linear (in)equalities, which will be our case, and is not empty, then the program will have at least one solution. If the objective function *F(x)* is strictly convex for all *x*, and if the feasible region is also convex, then the solution will be unique.

4.2 Optimization algorithms

The common algorithms for dealing with the minimization of a nonlinear multivariate convex function are descent methods. In each case, the algorithm generates a point $x^{l+1} = (x_1^{l+1},\ldots,x_I^{l+1})$ from $x^l = (x_1^l,\ldots,x_I^l)$, so that $F(x^{l+1}) < F(x^l)$. The core of any procedure is the calculation of x^{l+1} from x^l. This step can be written in standard form as:

$$x^{l+1} = x^l + \alpha_l d^l \quad (4.7)$$

where $d^l = (d_1^l,\ldots,d_I^l)$ is a descent direction vector and α_l is a nonnegative scalar known as the *step size.* d^l is a direction at point x^l in which the objective function F(x) is (locally) decreasing. The step size, α_l, determines how far the next point, x^{l+1}, will be along d^l. As the Equation (4.7) executed, each component of the variable vector is updated:

$$x_i^{l+1} = x_i^l + \alpha_l d_i^l \qquad \forall i$$

The convergence criteria used to terminate the algorithmic iterations can be based on the marginal contribution of successive iterations, that is, the closeness of $F(x^l)$ and $F(x^{l+1})$, or based on the change in the variables between successive iterations. These include, for example,

$$\left| F(x^l) - F(x^{l+1}) \right| \leq k \tag{4.8a}$$

$$\max_i \left\{ \left| \frac{\partial F(x^l)}{\partial x_i} \right| \right\} \leq k \tag{4.8b}$$

$$\max_i \left\{ \frac{x_i^l - x_i^{l-1}}{x_i^{l-1}} \right\} \leq k \tag{4.8c}$$

where k is a predetermined tolerance selected for each problem based on the desired degree of accuracy.

The method of steepest descent is perhaps the most intuitively logical minimization procedure. The direction of search is opposite to the gradient direction, while the move size is chosen so as to minimize *F(x)* in that direction. A important property of this method is that the successive search directions are perpendicular to each other, that is, $d^l \cdot d^{l+1} = 0$. The disadvantage of steepest descent method is that it requires a relatively large number of iterations.

Many other minimization approaches (i.e. Newton's method, quasi-Newton's methods) include second order methods in which the search direction is determined by using information regarding the second derivatives of the objective function. Most of these methods are not particularly applicable to the solution of the problems arising in urban networks studies. These problems are typically large (i.e., including many variables), and often preclude certain calculations such as the determination of the Hessian (the matrix of second derivatives).

The study of transportation network equilibrium problems involves equivalent mathematical programs which include exclusive linear constraints. Thus, the discussion of constrained minimization is limited to these types of constraints.

The basic framework outlined above for unconstrained minimization can be used to describe algorithms for constrained optimization as well. The added difficulty in constrained optimization is to maintain feasibility. In other words, the search direction has to point in a direction in which at least some feasible points lie, and given such a search direction, the step size would not cause infeasibility.

4.3 Mathematical formulation

Wardrop's first principle, referred to as user equilibrium, indicates that, if all the users perceive costs in the same way, "under equilibrium conditions traffic arranges itself in congested networks such that all routes between any origin-destination pair have equal and minimum costs while all unused routs have greater or equal costs." This principle has been characterized by Sheffi (1985) in equivalent formulations. Chen (1994) gives similar variational inequality formulation: An arc (or path) flow pattern v is a user equilibrium flow if v satisfied the following variational inequality:

$$c(v,\psi)^T(v - v') \leq 0 \quad \text{for all } v' \in \Omega \tag{4.9}$$

for given parameter ψ, where Ω is the set of all feasible arc flows.

Generally, some transportation planning problems may be formulated as

$$\min_{v,\psi} F(v,\psi) \tag{4.10a}$$

$$\text{s.t.}\quad c(v,\psi)^T(v-v')\le 0,\quad \forall v'\in\Omega \tag{4.10b}$$

Note that such problem is a type of bilevel programming problem. The variational inequality constraint (4.10b) is equivalent to an optimization problem if the cost function $c(v,\psi)$ is monotone (Chen, 1994). In a similar way, we formulate the OD matrix estimation problem in the case of congested network:

Find an OD matrix g such that g solves

$$\min F(g,v)=\gamma_1 F_1(g,\hat{g})+\gamma_2 F_2(v,\hat{v}) \tag{4.11a}$$

$$\text{s.t.}\quad \sum_{k\in K_i} h_k - g_i = 0, \tag{4.11b}$$

$$h_k \ge 0, \tag{4.11c}$$

$$v_a = \sum_{i\in I}\sum_{k\in K_i}\delta_{ak}h_k \tag{4.11d}$$

$$c(v)(v-v')\le 0, \tag{4.11e}$$

for all $k\in K_i, i\in I$ and for $\forall v'$ satisfies (4.11b)-(4.11d).

Chen (1994) has proved that the problem (4.11a)-(4.11e) has at least one solution under the assumption that "the cost function $c(v)$ is continuously differentiable and strictly monotone." Similar to Chen (1994) and Florian and Chen (1992), we specify $F_1(g,\hat{g})$ and $F_2(v,\hat{v})$ as quadratic functions and let $\gamma_1=\gamma_2=\frac{1}{2}$. Then the variational inequality (4.11e) is equivalent to an optimization problem and the OD matrix estimation problem may be stated as the following bilevel programming problem:

$$\min F(g,v)=\frac{1}{2}\left[\ \sum_{i\in I}(g_i-\hat{g}_i)^2+\sum_{a\in\hat{A}}(v_a-\hat{v}_a)^2\ \right] \tag{4.12a}$$

$$\text{s.t.}\qquad g_i\ge 0,\quad i\in I, \tag{4.12b}$$

where v solves the following problem for fixed g

$$\min f(v,g) = \sum_{a\in A} \int_0^{v_a} c_a(t)dt \tag{4.13a}$$

$$\text{s.t.} \quad \sum_{k\in K_i} h_k - g_i = 0, \quad i \in I \tag{4.13b}$$

$$h_k \geq 0, \quad k \in K_i, \quad i \in I \tag{4.13c}$$

$$v_a = \sum_{i\in I}\sum_{k\in K_i} \delta_{ak} h_k, \quad a \in A \tag{4.13d}$$

When substituting h_k by $p_k g_i$, using p and g as essential variables, and assuming the demand to be positive, we can simplify the problem alternatively in its marginal function form:

$$\min F(g,v) = \frac{1}{2}[\ \sum_{i\in I}(g_i - \hat{g}_i)^2 + \sum_{a\in \hat{A}}(v_a - \hat{v}_a)^2\ ] \tag{4.14a}$$

$$\text{s.t.} \quad \sum_{a\in A} \int_0^{v_a} c_a(t)dt - V(g) = 0, \tag{4.14b}$$

$$\sum_{k\in K_i} p_k - 1 = 0 \qquad i \in I \tag{4.14c}$$

$$g > 0, p \geq 0, \tag{4.14d}$$

where the constraints

$$v_a = \sum_{i\in I}\sum_{k\in K_i} \delta_{ak} h_k, \quad a \in A \tag{4.15}$$

are considered to be exogenous and the marginal function *V(g)* is defined by

$$V(g) = \inf\{\ \sum_{a\in A} \int_0^{v_a} c_a(t)dt \mid v \textit{ satisfies } (4.14c)\ (4.14d) \textit{ and } (4.15)\ \} \tag{4.16}$$

The marginal function V(g) is differentiable under assumption that the cost function c(v) is continuously differentiable and strictly monotone (Chen, 1994). The gradient can be computed by

$$\nabla V(g)_i = \sum_{a\in A} p_{ia} c_a(v_a), \quad i \in I \tag{4.17}$$

where p_{ia} are the arc proportions of the equilibrium assignment for given g defined by $\sum_{k\in K_i} \delta_{ak} p_k$.

4.4 Gauss-Seidel Type Algorithm

Chen (1994) proposed an Augmented Lagrangean Method to solve the OD matrix estimation problem (4.14a)-(4.14d) and proved the convergence properties of the algorithm. Chen (1994) also pointed out the algorithm is impractical for large network since it needs to enumerate explicitly the paths or path proportions of the network. A Gauss-Seidel type method was then proposed to solve the problem. The bilevel structure of the problem is decomposed of two one-level optimization problems by fixing the upper and lower variables in turn at each iteration. The path information may be obtained implicitly from equilibrium link flows.

Let (g^l, p^l) be a feasible solution of (4.14a)-(4.14d). Consider two problems related to (g^l, p^l) defined as follows:

$$(P_{p^l}) \quad \min F(g, p^l) = \tfrac{1}{2}[\sum_{i\in I}(g_i - \hat{g}_i)^2 + \sum_{a\in A}(v_a(p) - \hat{v}_a)^2] \tag{4.18a}$$

$$s.t. \qquad g \geq 0, \tag{4.18b}$$

and

$$(P_{g^l}) \quad \min F(g^l, p) = \tfrac{1}{2}[\sum_{i\in I}(g_i^l - \hat{g}_i)^2 + \sum_{a\in A}(v_a(p) - \hat{v}_a)^2] \tag{4.19a}$$

$$s.t. \qquad \sum_{a\in A}\int_0^{v_a} c_a(t)dt - V(g^{l+1}) = 0, \tag{4.19b}$$

$$\sum_{k\in K_i} p_k - 1, \qquad i \in I \tag{4.19c}$$

$$p \geq 0. \tag{4.19d}$$

(P_{p^l}) is obtained from (4.14a)-(4.14d) by fixing the variable $p = p^l$ and relaxing the gap constraints while (P_{g^l}) is obtained by fixing the variable $g = g^l$. The Gauss-Seidel type algorithm is designed to work alternatively on these two subproblems and is described below (Chen 1994 and Florian and Chen 1992).

- Initialization

Choose an initial OD matrix g^1; solve the lower level equilibrium problem with $g = g^1$. p^1 is obtained implicitly by the solution $v(g^1, p^1)$. Set l:=1.

- l th iteration.

1. g^l is given and p^l is implicitly known from v^l. Calculate

$$b_i^l = \frac{\partial F(g, p^l)}{\partial g_i}\bigg|_{g=g^l}$$

$$= g_i^l - \hat{g}_i + \sum_{a \in A} p_{ia}^l (v_a^l - \hat{v}_a) \quad (4.20)$$

2. Solve the direction finding subproblem of (P_{p^l}) at g^l:

$$\min \sum_{i \in I} b_i^l x_i \quad (4.21a)$$

$$s.t. \quad x_i \geq 0, i \in I \quad (4.21b)$$

A solution x^l can be computed by

$$x_i^l = \begin{cases} 2g_i^l & \text{if } b_i^l < 0 \\ g_i^l & \text{if } b_i^l = 0 \\ 0 & \text{if } b_i^l > 0 \end{cases} \quad (4.22)$$

3. Find the descent direction of (P_{p^l}) at g^l by $d^l = x^l - g^l$. If

$$\nabla_g F(g^l, p^l) d^l \geq 0, \quad (4.23)$$

then terminate the algorithm. Otherwise, let

$$g^{l+1} = g^l + \lambda^l d^l, \tag{4.24}$$

where the step size λ^l is determined by

$$F(g^l + \lambda d^l, p^l) - F(g^l, p^l) \leq \frac{\lambda}{2} \nabla F(g^l, p^l) d^l, \tag{4.25a}$$

$$0 \leq \lambda \leq 1. \tag{4.25b}$$

Chen (1994) shows that λ^l at each iteration can be determined by

$$0 \leq \lambda^l \leq \min\left(1, \frac{\sum_{i \in I} (g_i^l - \hat{g}_i) + \sum_{a \in \hat{A}} (v_a^l - \hat{v}_a)(v_a^l - v_a(x^l))}{\sum_{i \in I} (g_i^l - x_i^l)^2 + \sum_{a \in \hat{A}} (v_a^l - v_a(x^l))^2}\right) \tag{4.26a}$$

where

$$v_a(x^l) = \sum_{i \in I} \sum_{k \in K_i} \delta_{ak} p_k^k x_i^l, \tag{4.26b}$$

for $a \in \hat{A}$, which needs an additional assignment.

4. Solve the problem $(P_{g^{l+1}})$ and denote the link flow solution by v^{l+1}. p^{l+1} is determined implicitly from v^{l+1}. Go to Step 1.

Chen (1994) proved that all (g^l, p^l)'s generated by the algorithm are feasible solutions of the problem (4.14a)-(4.14d) and that if the algorithm terminates at (g^l, p^l), then (g^l, p^l) satisfies the necessary optimality conditions of the problem.

4.5 Traffic Assignment Technique

The problem of finding the user equilibrium over a transportation network involves the assignment of OD flows to the network links so that travel time on all used paths for any OD pair equals the minimum travel time between the origin and destination. Some heuristic methods have been developed for finding the user-equilibrium flow pattern. Two most common methods are capacity restraint methods and incremental assignment techniques, which are still widely used today (Sheffi, 1985 and Oppenheim, 1995).

At the core of the heuristic approaches lies the network loading mechanism. Network loading is the process of assigning the OD entries to the network for specific (constant) link costs. The process follows the route-choice criterion, which underlies any traffic assignment model. The UE flow pattern is the result of each motorist using the minimum cost path between his origin and his destination. Accordingly, the network loading mechanism used in all the algorithms designed to solve the UE problem assigns each OD flow to the shortest cost path connecting this OD pair (Sheffi 1985). This procedure is known as "all-or-nothing" (AON) assignment.

In the AON procedure the trips from any origin zone to any destination zone are loaded onto a single, minimum cost, path between them. Any other path connecting this OD pair is not assigned any flow. During this process, the link costs are assumed to be fixed (i.e., not flow dependent) at some value. The only computational difficulty with the procedure involves finding the minimum cost path connecting each OD pair.

Incremental assignment assigns a portion of the OD matrix at each iteration. The travel costs are then updated and an additional portion of the OD matrix is loaded onto the network. The procedure is outlined below:

Step 1: *Initialization.* Divide each OD entry into several portions (i.e. set $g_i^n = g_i * p^n$ where $\sum_i p^i = 1$). Set n: =1. Set $\{t_a^n\}$=predetermined link costs, free flow costs (travel times). On the basis of link costs $\{t_a^n\}$ carry out an AON assignment of the matrix g^n. Let v_a^n be the flow on link *a*.

Step 2: *Updating*. Set $t_a^n = t_a(v_a^{n-1})$ using the function recommended by Bureau of Public Roads (FHWA, 1973):

$$t_a^n = t_a^0(1 + 0.15(v_a^{n-1} / C_a)^4)$$

where t_a^{n+1} is link travel time, t_a^0 free flow link travel time, and C_a link capacity.

Step 3: *Incremental loading*. Perform AON assignment based on t_a^n, but using only a portion of trip demand for each OD pair. This yields a flow pattern $\{ w_a^n \}$.

Step 4: *Flow summation*. Set $v_a^n = v_a^{n-1} + w_a^n$, $\forall a$.

Step 5: *Stopping rule*. If all increments have been loaded then stop; else go to step 2.

In general, as the number of increments grows, the incremental assignment algorithm may generate a flow pattern closer to the UE condition. However, even a large number of increments will not always produce the user-equilibrium flow pattern. Some solution algorithms have been developed for the UE program, the most important of which is the convex combinations method (Sheffi, 1985).

V. Implementation and Applications

This chapter presents and analyzes the results from the estimation of the OD matrix, and illustrates the applications of the estimated OD matrix.

5.1 Implementation and Computational Results

The Gauss-Seidel type method (GST) described in the previous section was implemented by macro commands of ArcView GIS and statistical software SAS. Incremental traffic assignment as a substitute of equilibrium assignment was implemented in ArcView GIS. At the same time the proportion of each OD flow that passes the observed links was recorded. The computation of the gradient of the objective function with respect to the demand at current equilibrium solution was executed in SAS.

The results are illustrated in the following tables and graphs. The computational results in Table 5-1 indicate that the objective function value decreases at later iteration. Predicted traffic flows on observed links are the results of assigning estimated OD matrix to the network. Figure 5-1 to Figure 5-6 illustrate the line fit of predicted flows and observed flows. At later iteration, the correlation between these two types of flows becomes stronger. R^2 is an indicator of how much of the variation in the data is explained by the model. The values of R^2 in Table 5-1 imply that the observed flows alone can explain more variation of predicted flows as the algorithm converges.

Table 5-1: Computational results

	$F_1(g)$	$F_2(v)$	$F(g,v)$	R^2	Correlation
Initial	0	447,167,832	447,167,832	0.148	0.384
Iter. 1	332,063	200,499,450	200,831,513	0.332	0.577
Iter. 2	834,552	139,924,579	140,759,131	0.440	0.663
Iter. 3	1,190,024	114,453,508	115,643,532	0.495	0.703
Iter. 4	1,893,272	96,700,767	98,594,039	0.546	0.739
Iter. 5	1,957,714	85,907,283	87,864,997	0.598	0.773

Figure 5-1: The fit of initial flows vs. observed flows

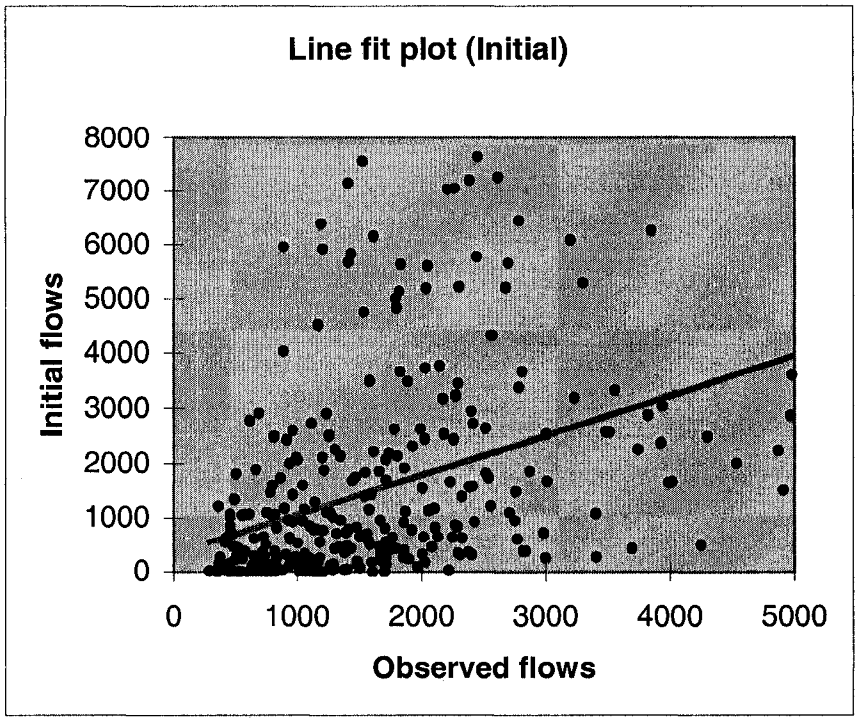


Figure 5-2: The fit of predicted flows vs. observed flows at 1st iteration

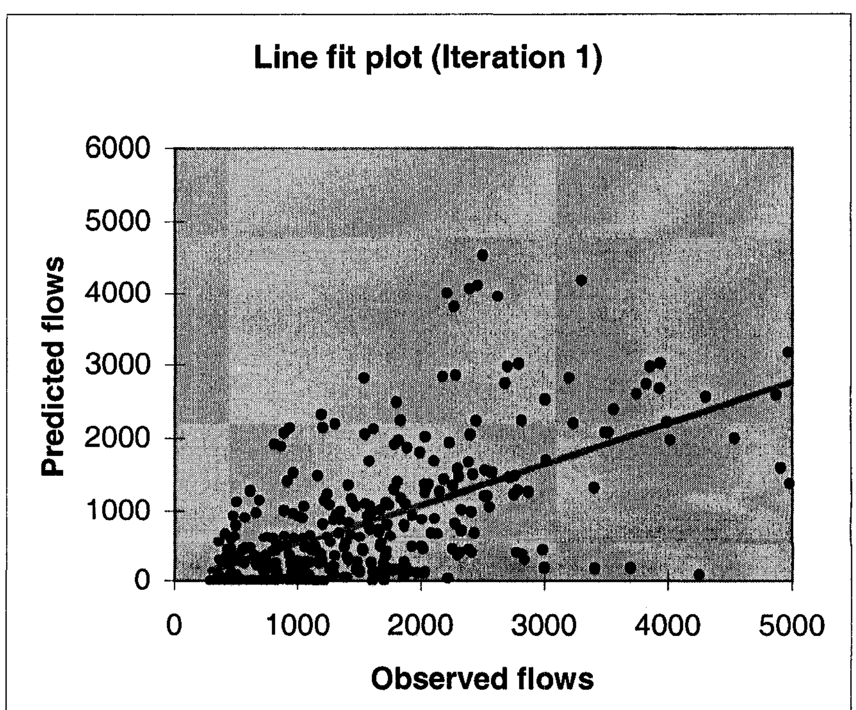


Figure 5-3: The fit of predicted flows vs. observed flows at 2nd iteration

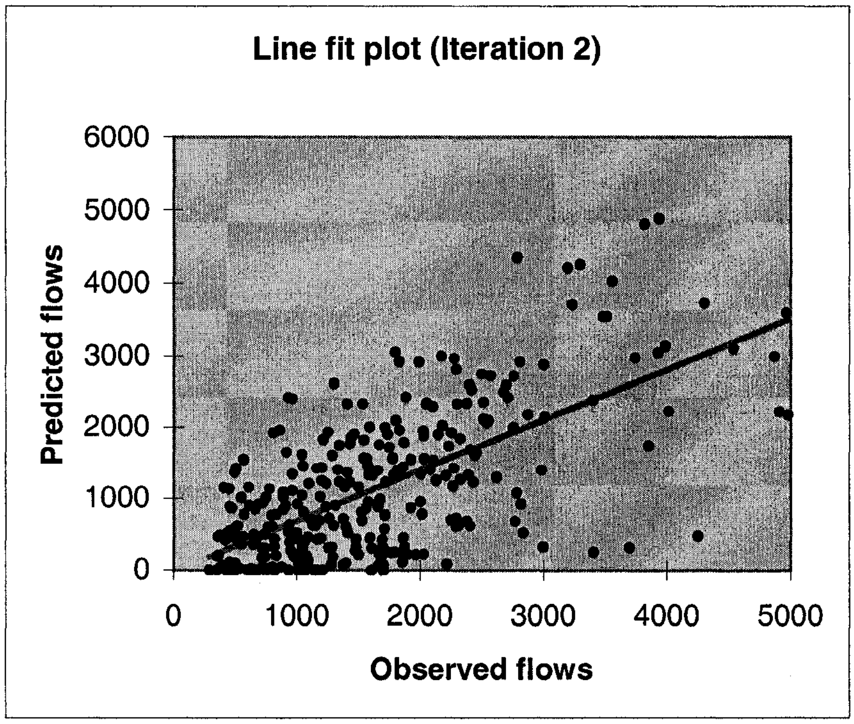


Figure 5-4: The fit of predicted flows vs. observed flows at 3rd iteration

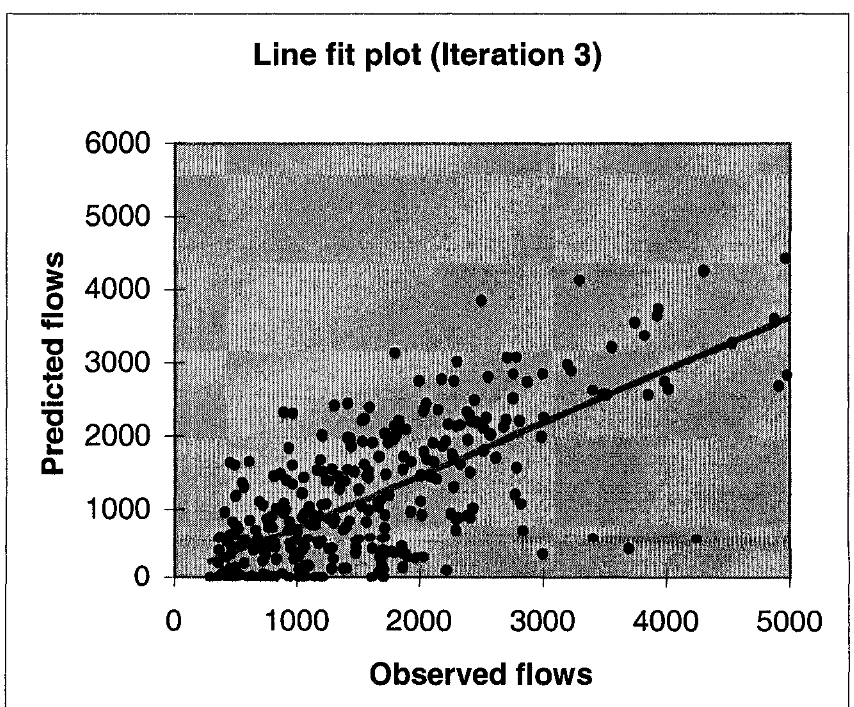


Figure 5-5: The fit of predicted flows vs. observed flows at 4th iteration

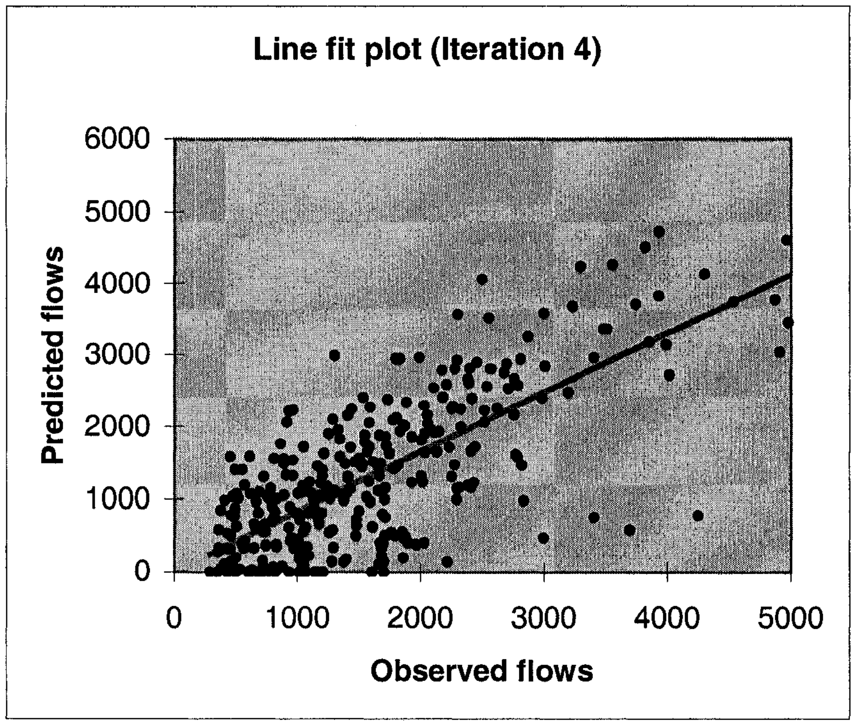
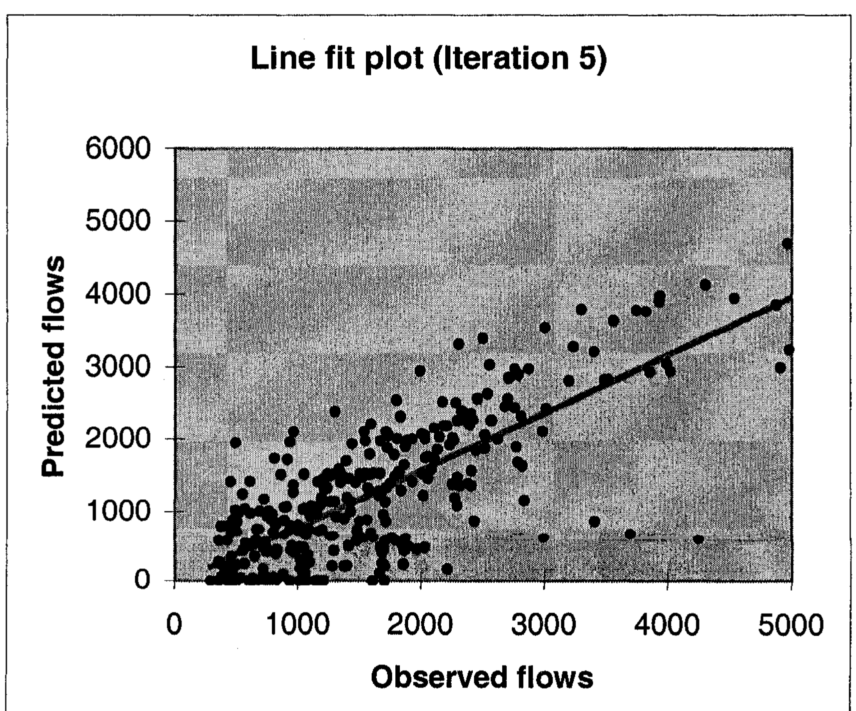


Figure 5-6: The fit of predicted flows vs. observed flows at 5th iteration

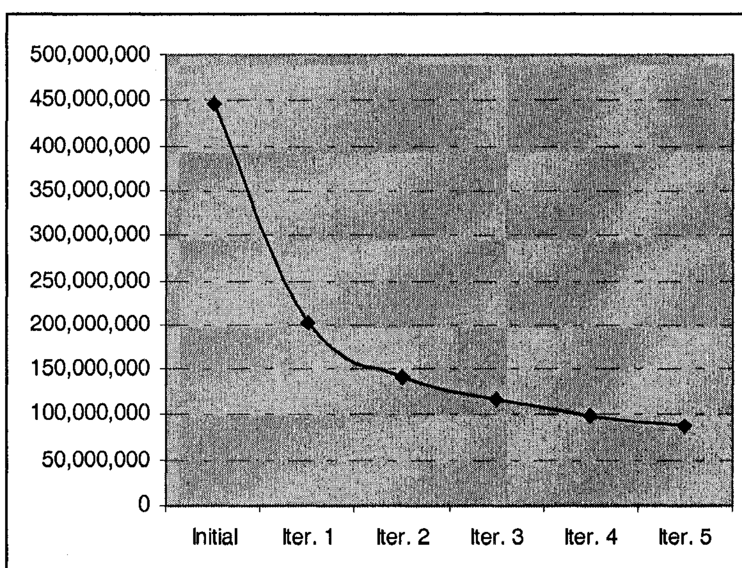


The stopping criterion for solving the problem could be based on the values of the objective function, as mentioned in Formula (4.8a) and (4.8b). However, this function is merely a mathematical construct that lacks behavioral or economic meaning. Consequently, the convergence criterion should be based on the relevant figures of merit, which in this case consist of the flows. A possible measure of the closeness of a particular solution to the problem is the similarity of successive flows which is based on the change in flows. The algorithm can terminate if

$$\frac{\sqrt{\sum_a (g_a^{n+1} - g_a^n)^2}}{\sum_a g_a^n} \leq k \tag{5.1}$$

The tolerance k in this project was predetermined as 0.001.

Figure 5-7: The reduction in the value of F(g, v)



The convergence criterion was not met and the algorithm continued. At the fifth iteration, the left side of (5.1) is 0.0014, which is very close to the convergence criterion. The algorithm stopped at this point because of the significant computational cost. The convergence pattern is demonstrated in Figure 5-7 in terms of the reduction in the value of the objective function from iteration to iteration. The asymptotic pattern shown in this figure is typical of most optimization algorithms. The marginal contribution of each successive iteration becomes smaller and smaller as the algorithm proceeds (Table 5-1, Table 5-2). According to the formulation of the objective function, the estimated OD matrix produces a traffic pattern closer to the observed traffic counts, thus closer to the reality. Therefore, the estimation is effective. This confirms that the estimated OD matrix is more appropriate for traffic study for 2000 than the OD matrix for 1990.

Table 5-2: Marginal contribution of each successive iteration

Iter. 1	Iter. 2	Iter. 3	Iter. 4	Iter. 5
0.551	0.299	0.178	0.147	0.109

5.2 Estimate Street-level Drivers

The estimated OD matrix and initial OD matrix were applied to determine street-level commuter flows and their racial composition in the study area, respectively. The results from these two approaches were then compared with sampled streets with observed racial counts.

The racial composition was estimated in traffic assignment algorithms. Before assignment, proportion of drivers of different races for each origin TAZ was added to the trip table. There are several steps to obtain the information about racial proportion for each TAZ. First, the demographic profiles at block group level including age and race were extracted from the U.S. Census 2000. The driver population is comprised of all people between 15 and 74. The information was then added to the attribute table of the block group shapefile. The Union process in ArcView GIS was used to produce a new theme containing the features and attributes of black group layer and TAZ layer. The boundaries of block groups may not correspond to the TAZ boundaries. With assumption that the demographic features are homogeneous in each block group, it is possible to aggregate block group's demographic data to TAZ boundaries. The proportions of white and black drivers in the TAZ are then obtained.

In incremental assignment algorithm, the proportions of black and white drivers in origin TAZ are retrieved and the number of black and white drivers in the flow of each OD pair is calculated by multiplying the flow value by the proportion of each of the two races in the TAZ.

When all flows in the trip table are assigned to the network, the proportions of white and black drivers in each street segment are calculated by dividing the drivers in each race by the total number of drivers. Both results from initial OD matrix and estimated OD matrix are compared with observed race counts on 111 locations shown in Figure 5-8. With contrast to the initial assignment using a trip table for 1990, the final assignment uses the updated trip table. It would be expected to have a closer correspondence with the observed racial proportion.

The proportions of white drivers resulting from assigning the estimated OD matrix to the street network have a stronger correlation with the observed proportions of white drivers than those from initial OD matrix (Table 5-3). The same pattern appears in the predicted and observed proportions of black drivers. These findings support the guess about the correspondence of predicted racial proportions with the observed racial proportions.

Figure 5-8: Race counts locations

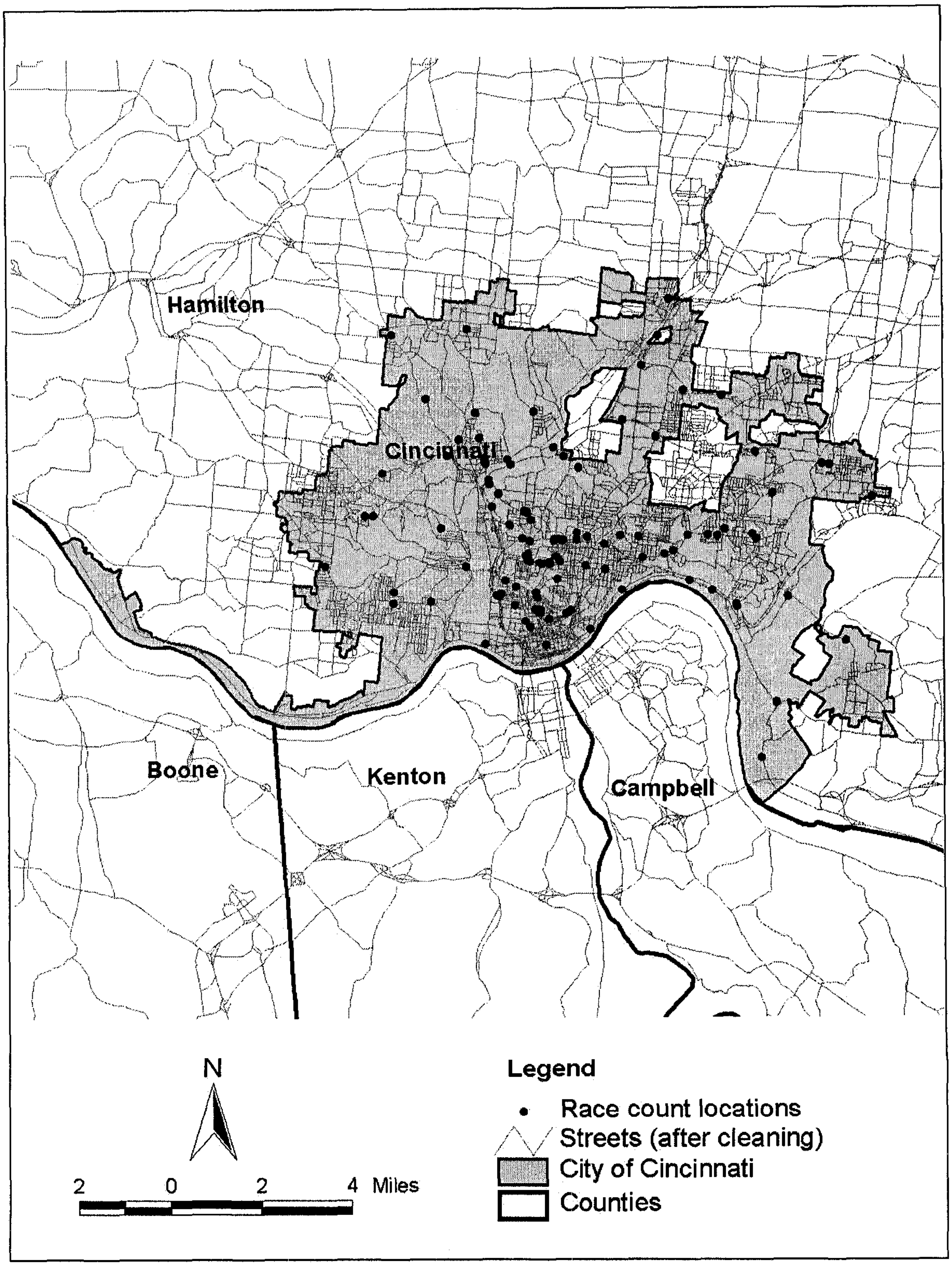


Table 5-3: Correlation between predicted proportions and observed proportions

	Counted white prop.		Counted black prop.	
Estimated OD matrix	White prop.	0.642	Black prop.	0.630
Initial OD matrix	White prop.	0.605	Black prop.	0.587

Paired two samples for means t-test is also performed to examine the closeness of predicted proportions and observed proportions. Table 5-4 and Table 5-5 demonstrate the results of t-test for the proportions of white drivers obtained from the initial OD matrix vs. counted proportions of white drivers, and t-test for the proportions of white drivers acquired from estimated OD matrix vs. counted proportions of white drivers. Both tests show that the predicted proportions are significant different from those counts. However, with a smaller absolute t-statistic and a much lager p-value, the proportions from estimated OD matrix have a closer correspondence with the observed proportions.

Table 5-4: Difference between predicted proportions from initial OD matrix and observed proportions

t-Test: Paired Two Sample for Means

	0.686	*0.8569*
Mean	0.6659	0.7662
Variance	0.0415	0.0160
Observations	110	110
Pearson Correlation	0.6060	
Hypothesized Mean Difference	0	
df	109	
t Stat	-6.4966	
P(T<=t) one-tail	1.26E-09	
t Critical one-tail	1.6590	
P(T<=t) two-tail	2.53E-09	
t Critical two-tail	1.9820	

Table 5-5: Difference between predicted proportions from estimated OD matrix and observed proportions

t-Test: Paired Two Sample for Means

	0.686	*0.873*
Mean	0.6659	0.7304
Variance	0.0415	0.0280
Observations	110	110
Pearson Correlation	0.6432	
Hypothesized Mean Difference	0	
df	109	
t Stat	-4.2281	
P(T<=t) one-tail	2.46E-05	
t Critical one-tail	1.6590	
P(T<=t) two-tail	4.92E-05	
t Critical two-tail	1.9820	

5.3 Work hour Drivers

As the original OD matrix for 1990, the estimated OD matrix has 3 attributes including origin and destination TAZ ID, and flow for each OD pair. This enables us to calculate the number of drivers entering and leaving each TAZ during rush hour. The difference between these two numbers implies a pattern of land use, employment and population distribution. By comparing such a pattern in 2000 with that in 1990, we can find out the overall change of population and employment during the ten-year's period.

The number of drivers entering each TAZ and the number of drivers leaving each TAZ are obtained by summarizing the OD matrix by destination TAZ ID and origin TAZ ID, respectively. The difference between these two numbers in each TAZ is calculated for both 2000 and 1990 from the following equations.

$$\text{DrvDiff1990} = \text{inDrivers1990} - \text{outDrivers1990}$$

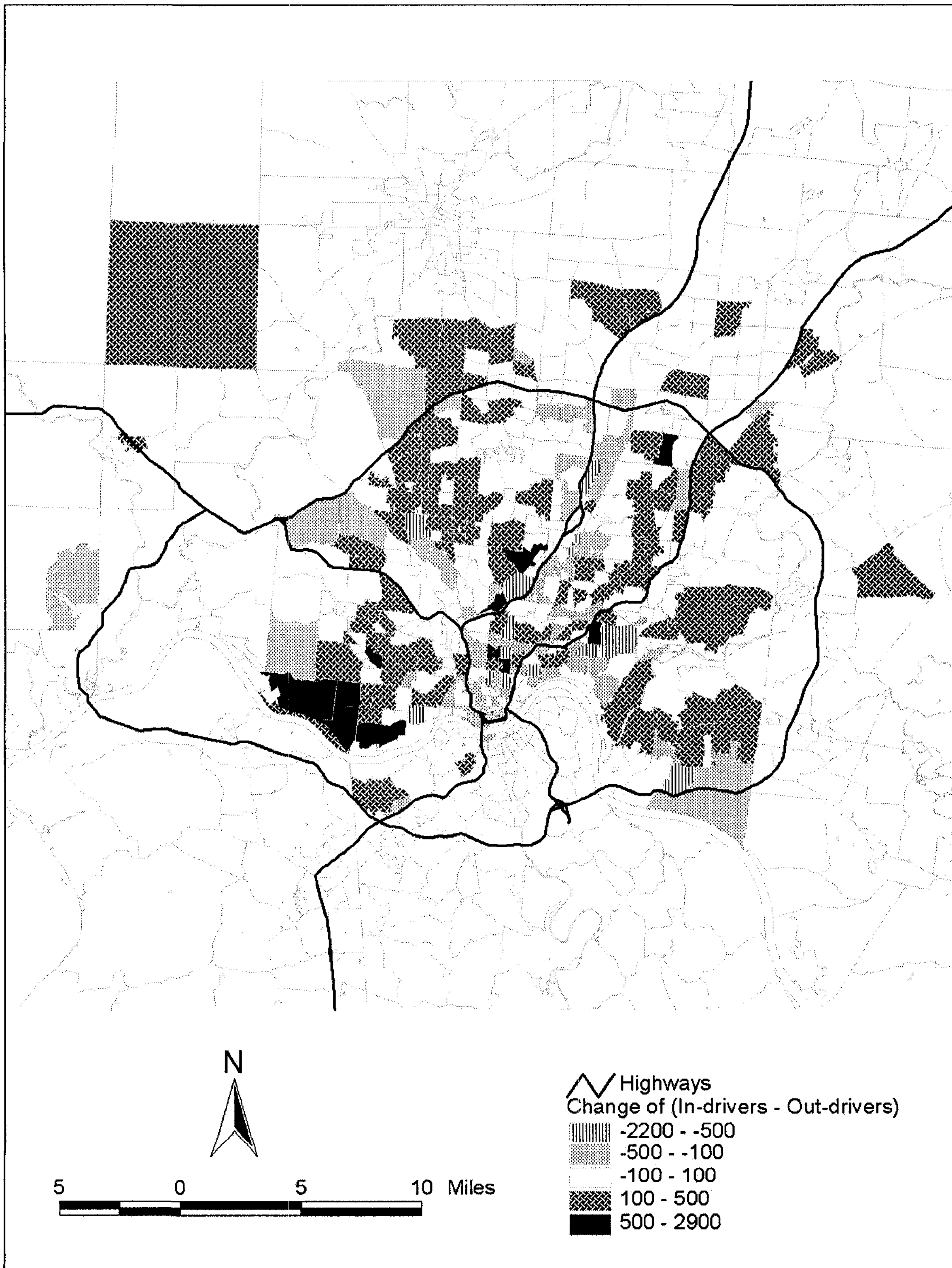
$$\text{DrvDiff2000} = \text{inDrivers2000} - \text{outDrivers2000}$$

The disparity of DrvDiff2000 and DrvDiff1990 tells us the change of the number of "surplus" flow moving toward each TAZ. The comparison of the patterns in 1990 and 2000 is demonstrated in Figure 5-9. Most TAZs have a positive change. Moreover, these TAZs spread out around the City of Cincinnati and Hamilton County. Such a distribution pattern reflects the out-migration of population and the out-spreading of work place from the City and the core county, Hamilton County in the MSA, Greater Cincinnati Area, to surrounding areas.

During 1990-2000, the population of the City decreased from 364,040 to 331,285, while the population of the County decreased from 866,228 to 845,303. In the same period, most surrounding counties had increases in their populations. As the City and County have the majority of employment in the Greater Cincinnati Area, more people travel from outside the City and County to work place inside the City and County during rush hour. Thus many TAZs have positive gains in the differentiation of the number of drivers. At more aggregated level, the gains the City and the County are about 17,000 and 7,000, respectively. This indicates that the City of Cincinnati is still the center of employment although it saw the loss of population.

The pattern of change in the counties surrounding Hamilton County is not evident because the observed traffic counts, the flow constraints for the optimization problem, are all located in the City.

Figure 5-9: Out-migration from Hamilton to the neighboring counties



VI. Conclusions and Recommendations

This research studied the bilevel programming problem occurring in the transportation planning context and applied it to estimate the origin-destination (OD) matrix on traffic analysis zone (TAZ) level in Greater Cincinnati Area, Ohio from two known data sets, an outdated OD matrix for 1990 and observed traffic counts in 2000 on some links of the street network. The objective of the study was to obtain a more accurate OD matrix that is closer to the situation in 2000 where the available OD matrix for 1990 is out-of-date.

The problem was set up as a mathematical programming problem that minimizes the distance between the estimated OD matrix and outdated target OD matrix, and the difference between the estimated link flows and the observed flows, subject to the flow constraints. A Gauss-Seidel type method was applied to solve the problem, iteratively searching a descending direction and finding an appropriate step size. The bilevel structure of the problem was decomposed of two one-level optimization problems by fixing the upper and lower variables in turn at each iteration. The upper level problem estimates the OD matrix with the assumption that link flow volumes were given. The lower level problem is the equilibrium assignment problem that determines a link flow pattern of the equilibrium type. In our case, the equilibrium assignment was approximated by incremental assignment.

The solution to the problem was obtained after five iterations of the algorithm when the convergence criterion was approached. At later iteration, the correlation between predicted flows and the counted flows on the observed links become stronger. The marginal contribution to the objective function of each successive iteration becomes smaller and smaller as the algorithm proceeds. The computational results indicate that the estimation of the OD matrix is robust and the estimated OD matrix is closer to the reality in 2000 than the OD matrix for 1990.

When the estimated OD matrix was applied to determine street-level commuter flows and their racial composition in the study area, the racial proportions have a closer correspondence with the observed proportions than those from initial OD matrix. This confirms that the estimated OD matrix from outdated OD matrix is accurate when information about traffic counts on network links is incorporated.

The estimated OD matrix was employed to calculate the number of drivers entering and the number of drivers leaving each TAZ during rush hour. The difference between these two numbers implies a pattern of land use, employment and population distribution. The comparison of the patterns in 2000 and 1990 reflects the out-migration of population and the out-spreading of work place from the City and the core County in the MSA, Greater Cincinnati Area, to surrounding areas, although the City and County faced the loss of population.

This thesis applied an approach of mathematical programming and successfully solved a large-scale problem of OD matrix estimation in transportation planning. The estimated OD matrix was applied to estimate the racial composition of drivers on individual streets through transportation modeling procedure. The conventional method to obtain such information is through observation or video recording which is costly and time consuming. Furthermore, the accuracy is not high. The approach in this study is rather efficient and practical and is worth reference. Therefore, the thesis has its practical contribution to the literature.

Several limitations of the study exist.

First, to achieve a user-equilibrium flow pattern, an incremental traffic assignment technique was applied. However, as with other heuristic methods, such an approach may not converge, or may produce a set of flows that is not in agreement with the user-equilibrium criterion. In general, as the number of increments grows, the incremental assignment algorithm may generate a flow pattern closer to the user-equilibrium condition. But a large number of increments would be associated with a considerable computational effort.

Some transportation planning software, including Quick Response System (QRS), EMME/2, and TransCAD, has built-in function to perform user-equilibrium traffic assignment. However, there exist some difficulties in the estimation of the OD matrix, i.e., recording the proportion of a flow passing the each observed link, file conversion between GIS software and transportation planning software, and network reconstruction.

The traffic counts during rush hour in this study, which provided essential information to update the OD matrix, were estimated from Average Daily Traffic (ADT) by multiplying a single percentage. However, each link has very different traffic distribution related to its location, street category, etc. Thus, the rough counts data used in this project may result in invalid estimation.

Finally, the street network had been rather simplified in the study because turn and intersection delay, one-way direction, separated or limited access, were not considered in traffic assignment.

Therefore, it is necessary in further study not only to derive a user-equilibrium flow pattern, but to improve data accuracy and data processing techniques.

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